Entanglement and renormalization-group flow

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Work with K. Boutivas, D. Katsinis, G. Pastras

Introduction

- Quantum entanglement denotes a phenomenon in which particles (like photons or electrons) become interconnected, sharing a single quantum state regardless of the distance separating them.
- Measuring a property of one particle instantly determines the state of the other.
- The 2022 Nobel Prize in Physics recognized experiments that confirmed this phenomenon, rejecting the idea of "hidden variables". (Alain Aspect, John F. Clauser, Anton Zeilinger)
- Entanglement is essential for building multi-qubit quantum gates and quantum computers. Important applications in cryptography and quantum sensing.
- Entanglement entropy is a quantitative measure of the quantum entanglement between two parts of a composite system.It tracks the information lost or the "uncertainty" that arises when you can only observe one part of an entangled pair.

- Consider a quantum mechanical system with many degrees of freedom, such as a spin chain or a quantum field.
- Assume it is in the ground state $|\Psi\rangle$, which is a pure state.
- The density matrix of the total system is $\rho_{\rm tot} = |\Psi\rangle\langle\Psi|$.
- The von Neumann entropy $S_{\rm tot} = -{\rm tr}\rho_{\rm tot} \log \rho_{\rm tot}$ vanishes.
- Now divide the total system into subsystems A and B and assume that B is inaccessible to A.
- Trace out the part B of the Hilbert space in order to obtain the reduced density matrix of A: $\rho_{\rm A} = {\rm tr}_{\rm B}\rho_{\rm tot}$.
- The entropy $S_{\rm A} = -{\rm tr}_{\rm A}\rho_{\rm A} \log \rho_{\rm A}$ is a measure of the entanglement between A and B.
- It is nonvanishing and $S_{\rm A} = S_{\rm B}$.

- For spatially separated systems in the ground state in a static background, the leading contribution is proportional to the area of the entangling surface between A and B:

$$S_A \sim \frac{\partial A}{\epsilon^{d-1}} + \text{subleading terms.}$$

- Massless scalar field in 3+1 dimensions (spherical entangling surface):

$$S_A = s\,(R/\epsilon)^2 + a\,\log(R/\epsilon) + d$$

$s \simeq 0.3$ (scheme-dependent) (Srednicki 1993)
$a = -\frac{1}{90}$ (universal)
(Solodukhin 2008, Lohmayer et al. 2009)

- Conformal field theory in 1+1 dimensions, with central charge c. System of length L, divided into pieces of lengths ℓ and $L-\ell$:

$$S_A = \frac{c}{6}\log\left(\frac{2L}{\pi\epsilon}\sin\frac{\pi\ell}{L}\right) + \bar{c}_1'$$

$\bar{c}_1'$ scheme-dependent. (Korepin 2004, Calabrese, Cardy 2004)

- Entanglement entropy in curved backgrounds
  de Sitter space (Maldacena, Pimentel 2013)
- Relation between entanglement entropy and c-, F- and a-functions
  $1+1$ d: $\mathrm{R}\,\Delta\mathrm{S}'(\mathrm{R}) \equiv$ c-function. (Casini, Huerta 2012)
  $3+1$ d: $\mathrm{R}^2\,\Delta\mathrm{S}'' - \mathrm{R}\,\Delta\mathrm{S}' \equiv$ a-function. (Casini, Teste, Torroba 2017)

References

Entanglement and expansion

- K. Boutivas, G. Pastras and N. Tetradis
  "Entanglement and expansion"
  arXiv:2302.14666 [hep-th], JHEP 05 (2023) 199
- K. Boutivas, D. Katsinis, G. Pastras and N. Tetradis
  "Entanglement in Cosmology"
  arXiv:2310.17208 [gr-qc], JCAP 04 (2024) 017

Entanglement of squeezed states

- D. Katsinis, G. Pastras and N. Tetradis
  "Entanglement of Harmonic Systems in Squeezed States"
  arXiv:2304.04241 [hep-th], JHEP 10 (2023) 039
- D. Katsinis, G. Pastras and N. Tetradis
  "Entanglement Entropy of a Scalar Field in a Squeezed State"
  arXiv:2403.03136 [hep-th], JHEP 10 (2024) 173

Entanglement in de Sitter space

- K. Boutivas, D. Katsinis, G. Pastras and N. Tetradis
  "A Numerical Calculation of Entanglement Entropy in dS Space"
  arXiv:2407.07811 [hep-th], Phys. Rev. D 111 (2025) 6, 065010
- K. Boutivas, D. Katsinis, G. Pastras and N. Tetradis
  "Entanglement Entropy as a Probe Beyond the Horizon"
  arXiv:2407.07824 [hep-th], Phys. Rev. Lett. 135 (2025) 6, 061602

Entanglement in the Einstein universe

- K. Boutivas, D. Katsinis, G. Pastras, N. Prountzos and N. Tetradis
  "Entanglement on a Sphere"
  arXiv:2507.01174 [hep-th], Phys. Rev. D 112 (2025) 6, 065020

Entanglement in Anti-de Sitter space

- K. Boutivas, D. Katsinis, I. Papadimitriou, G. Pastras and N. Tetradis
  "Entanglement Entropy of a Scalar Field in Anti-de Sitter Space"
  arXiv:2505.15980 [hep-th], JHEP 11 (2025) 064
- K. Boutivas, D. Katsinis, G. Pastras, N. Smeyers and N. Tetradis
  "Boundary Conditions and Entanglement in Anti-de Sitter Space"
  arXiv:2608.04229 [hep-th]

Computing c- and a-functions from entanglement

- K. Boutivas, D. Katsinis, G. Pastras and N. Tetradis
  "Computing c- and a-functions from Entanglement"
  arXiv:2509.03259 [hep-th], Phys. Rev. D 113 (2026) 10, 105026

Plan

- Entanglement entropy in an expanding background
- Entanglement entropy in various curved backgrounds
- c- and a-functions through entanglement
- Prospects

Entanglement and expansion

- Consider a free scalar field $\phi(\tau, \mathrm{x})$ in a FRW background

$$\mathrm{ds}^2 = \mathrm{a}^2(\tau)\left(\mathrm{d}\tau^2 - \mathrm{dr}^2 - \mathrm{r}^2\mathrm{d}\Omega^2\right).$$

- With the definition $\phi(\tau, \mathrm{x}) = \mathrm{f}(\tau, \mathrm{x})/\mathrm{a}(\tau)$, the action becomes

$$\mathrm{S} = \frac{1}{2}\int \mathrm{d}\tau\, \mathrm{d}^3\mathrm{x}\left(\mathrm{f}'^2 - (\nabla \mathrm{f})^2 + \left(\frac{\mathrm{a}''}{\mathrm{a}} - \mathrm{a}^2\mathrm{m}^2\right)\mathrm{f}^2\right).$$

  The field $\mathrm{f}(\tau, \mathrm{x})$ has a canonically normalized kinetic term.

- For de Sitter: $\mathrm{a}(\tau) = -1/(\mathrm{H}\tau)$ with $-\infty < \tau < 0$, and

$$\mathrm{S} = \frac{1}{2}\int \mathrm{d}\tau\, \mathrm{d}^3\mathrm{x}\left(\mathrm{f}'^2 - (\nabla \mathrm{f})^2 + \frac{2\kappa}{\tau^2}\mathrm{f}^2\right),$$

  where $\kappa = 1 - \mathrm{m}^2/2\mathrm{H}^2$.

de Sitter era

- Oscillator with time-dependent frequency

$$\omega^2(\tau) = \omega_0^2 - \frac{2\kappa}{\tau^2}.$$

- The solution of the Schrödinger equation can be expressed as

$$\mathrm{F}(\tau, \mathrm{f}) = \frac{1}{\sqrt{\mathrm{b}(\tau)}} \exp\left(\frac{\mathrm{i}}{2}\frac{\mathrm{b}'(\tau)}{\mathrm{b}(\tau)}\mathrm{f}^2\right) \mathrm{F}^0\left(\int \frac{d\tau}{\mathrm{b}^2(\tau)}, \frac{\mathrm{f}}{\mathrm{b}(\tau)}\right),$$

where $\mathrm{F}^0(\tau, \mathrm{f})$ is a solution with constant frequency ω_0.

- For the Bunch-Davies vacuum, $\mathrm{b}(\tau)$ is

$$\mathrm{b}^2(\tau) = -\frac{\pi}{2}\omega_0\tau\left(\mathrm{J}_\nu^2\left(-\omega_0\tau\right) + \mathrm{Y}_\nu^2\left(-\omega_0\tau\right)\right).$$

and tends to 1 as $\tau \to -\infty$.

- For $\kappa > 0$ and $\tau \to 0^-$, we have $\Delta \mathrm{f}/\Delta\pi \to 0$: Squeezed state.

Transition from dS with $a(\tau) \sim -1/\tau$ to radiation domination (RD) with $a(\tau) \sim \tau$

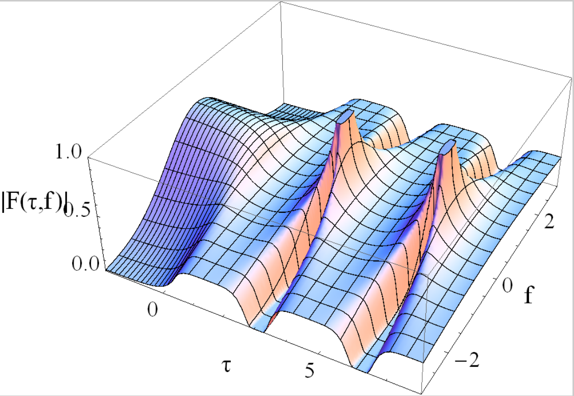

Figure: Left plot: The amplitude of the 'ground-state' wave function for the transition from a dS to a RD background at $\tau = 0.5$, for $\omega_0 = 1$, H $= 2$.

Entanglement entropy of two quantum oscillators

- Hamiltonian (we switched from f to x)

$$H = \frac{1}{2}\left[p_1^2 + p_2^2 + k_0(x_1^2 + x_2^2) + k_1(x_1 - x_2)^2 - \lambda(\tau)(x_1^2 + x_2^2)\right].$$

- The Hamiltonian can be rewritten as

$$H = \frac{1}{2}\left[p_+^2 + p_-^2 + w_+^2(\tau)x_+^2 + w_-^2(\tau)x_-^2\right],$$

$x_\pm = \frac{x_1 \pm x_2}{\sqrt{2}}$, $\omega_{0+}^2 = k_0, \omega_{0-}^2 = k_0 + 2k_1$, $w_\pm^2(\tau) = \omega_{0\pm}^2 - \lambda(\tau)$.

- The ‘ground state’ is the tensor product of the ‘ground states’ of the two decoupled canonical modes:

$$\psi_0(x_+, x_-) = \left(\frac{\Omega_+\Omega_-}{\pi^2}\right)^{\frac{1}{4}} \exp\left[-\frac{1}{2}\left(\Omega_+ x_+^2 + \Omega_- x_-^2\right) + \frac{i}{2}\left(G_+ x_+^2 + G_- x_-^2\right.\right.$$

$$\Omega_\pm(\tau) \equiv \frac{\omega_{0\pm}}{b^2(\tau;\omega_{0\pm})}, \quad G_\pm(\tau) \equiv \frac{b'(\tau;\omega_{0\pm})}{b(\tau;\omega_{0\pm})}.$$

- Express the wave function in terms of x_1, x_2.
- The reduced density matrix is given by

$$\rho(x_2, x_2') = \int_{-\infty}^{+\infty} dx_1 \psi_0(x_1, x_2)\psi_0^*(x_1, x_2').$$

- The Gaussian integration gives

$$\rho(x_2, x_2') = \sqrt{\frac{\gamma - \beta}{\pi}} \exp\left(-\frac{\gamma}{2}(x_2^2 + x_2'^2) + \beta x_2 x_2'\right) \exp\left(i\frac{\delta}{2}(x_2^2 - x_2'^2)\right),$$

where γ, β, δ are functions of $\Omega_\pm$, $G_\pm$.
- The eigenfunctions of the reduced density matrix satisfy

$$\int_{-\infty}^{+\infty} dx_2' \rho(x_2, x_2') f_n(x_2') = p_n f_n(x_2).$$

- One finds

$$f_n(x) = H_n(\sqrt{\alpha}x) \exp\left(-\frac{\alpha}{2}x^2\right) \exp\left(i\frac{\delta}{2}x^2\right),$$

where $\alpha = \sqrt{\gamma^2 - \beta^2}$ and H_n is a Hermite polynomial.

- The eigenvalues p_n are

$$p_n = \sqrt{\frac{2(\gamma-\beta)}{\gamma+\alpha}}\left(\frac{\beta}{\gamma+\alpha}\right)^n = (1-\xi)\xi^n,$$

where

$$\xi = \frac{\beta}{\gamma+\alpha}.$$

- The entanglement entropy can be calculated as

$$S = -\sum_{n=0}^{\infty}(1-\xi)\xi^n \log\left[(1-\xi)\xi^n\right] = -\log(1-\xi) - \frac{\xi}{1-\xi}\log\xi.$$

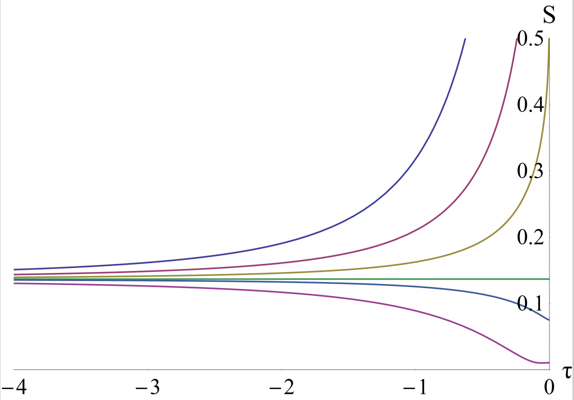

Figure: Left plot: The entanglement entropy in a dS background as a function of conformal time τ for $\omega_+ = 1$, $\omega_- = 2$ and κ =1, 0.5, 0.2, 0, −0.1, −0.5 (from top to bottom), where $\kappa = 1 - \mathrm{m}^2/2\mathrm{H}^2$.

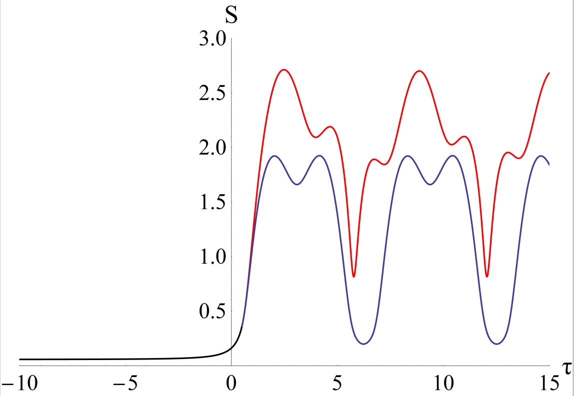

Figure: Left plot: The entanglement entropy as a function of conformal time τ for $\omega_+ = 1$, $\omega_- = 1.5$, H = 2 and $\tau_0 = 0.5$. The black line corresponds to a dS background, with a transition at τ_0 to either a RD era (blue line) or to a MD era (red line).

A volume effect

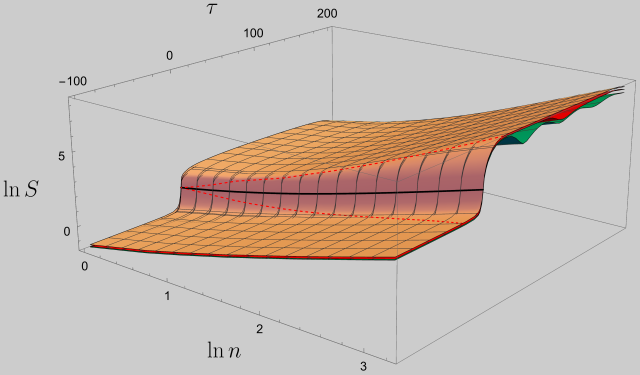

Figure: The entanglement entropy for a spherical region as a function of the entangling radius at various times for $H\epsilon = 1$. The radius of the spherical lattice is $L = N\epsilon$. Results for $N = 200$ (brown), $N = 100$ (red), $N = 50$ (green). We indicate the entropy at the dS to RD transition (black curve) and the location of the comoving horizon (dashed, red curve).

Summary

- Modes that start as quantum fluctuations in the Bunch-Davies vacuum are expected to freeze upon horizon exit and transmute into classical stochastic fluctuations.
- This is only part of the picture. Even though its classical features are dominant, the field never loses its quantum nature.
- The various modes evolve into squeezed states.
- The squeezing triggers an enhancement of quantum entanglement. The effect is visible in the entanglement entropy.
- The entanglement entropy survives during the eras of radiation or matter domination. A volume effect appears during these eras.
- Interpretation of the entropy as thermodynamic? A quantum-mechanical realization of reheating after inflation?
- Can the entropy of the Universe be attributed to the presence of the cosmological horizon?
- Observable consequences? A look behind the horizon?
- Weakly interacting, very light fields that stay coherent during the cosmological evolution (gravitational waves).

Entanglement entropy of massive fields in curved backgrounds

- Define a canonically normalized field by absorbing the curvature-dependent coefficient of the kinetic term.
- Expand in real spherical harmonics $\mathrm{Y}_{\ell \mathrm{m}}(\theta, \phi)$ in order to obtain a system of $(1+1)$-dimensional, decoupled fields $\phi_{\ell \mathrm{m}}(\mathrm{t}, \mathrm{r})$.
- Discretize the radial direction in order to obtain a system of coupled harmonic oscillators for each sector.
- Derive the Hamiltonian and deduce the coupling matrix $\mathrm{K}_{\mathrm{i},\mathrm{j}}$.
- Define the quantum state by selecting the appropriate vacuum.
- Compute the reduced density matrix and its eigenvalues for a subsystem extending from the origin to a radius R.
- Repeat for every (ℓ, m)-sector.
- Compute the entanglement entropy by adding all the sectors.
- The $\ell = 0$ sector corresponds to the discretized Hamiltonian of the $(1+1)$-dimensional field theory.

Summary

- In $3+1$ dimensions, the divergent terms in the entanglement entropy within a spherical entangling surface of proper area A are

$$S_{EE} = \frac{c_1}{4\pi}\frac{A}{\epsilon^2} + \left(c_2 + c_3 \frac{A}{4\pi}\mu^2 + c_4 \frac{A}{4\pi a^2}\right)\log\frac{a}{\epsilon} + c_{IR}\frac{A}{4\pi a^2}\log\frac{R^{tot}}{a} + \text{finite},$$

  with a the curvature scale of the background.
- For dS space $a = 1/H$, for AdS space a is the AdS length a_{AdS}, for the Einstein universe it is the radius of the spatial sphere.
- A dependence on the size of the entire system R^{tot} appears in dS space and in the Einstein universe.
- The universal coefficients are

$$\begin{array}{rllll} R\times S^3: & c_2 = -\frac{1}{90}, & c_3 = -\frac{1}{6}, & c_4 = \frac{1}{6}, & c_{IR} = \frac{1}{6} \\ dS: & c_2 = -\frac{1}{90}, & c_3 = -\frac{1}{6}, & c_4 = \frac{1}{3}, & c_{IR} = \frac{1}{3} \\ AdS: & c_2 = -\frac{1}{90}, & c_3 = -\frac{1}{6}, & c_4 = -\frac{1}{3}, & c_{IR} = 0. \end{array}$$

Comments

- The values of c_3 and c_4 are such that the corresponding contributions cancel for a field whose mass arises from a non-minimal coupling to gravity $\frac{1}{6}\mathcal{R}\phi^2$. The coefficient of the logarithmic term for the resulting Weyl-invariant theory is given by c_2 and is related to the A-type conformal anomaly. (Solodukhin 2008)
- The values of c_4 for dS and AdS are opposite, as they are related through the analytic continuation $H^2 \to -1/a^2_{AdS}$.
- In all spaces, $c_3 = -\frac{1}{6}$. (Hertzberg, Wilczek 2010)
- A dependence on the size of the entire system R^{tot} appears in dS space and in the Einstein universe. The entanglement entropy acts as a probe beyond the horizon. No infrared effect in AdS.

c-, F- and a- functions

- Consider the entanglement entropy for a spherical entangling surface.
- The evolution from the UV to the IR corresponds to R changing from 0 to ∞.
- $1+1$ d: $\mathrm{R}\,\Delta \mathrm{S}'(\mathrm{R}) \to$ c-function. (Casini, Huerta 2012)
  $3+1$ d: $\mathrm{R}^2\,\Delta \mathrm{S}''(\mathrm{R}) - \mathrm{R}\,\Delta \mathrm{S}'(\mathrm{R}) \equiv$ a-function. (Casini, Teste, Torroba 2017)
- The contribution to the entropy from the UV fixed point, as well as UV divergent terms related to the deformations of the conformal theory must be subtracted.
- The relation is also valid in the static patch of dS space (Abate, Torroba 2024) and in AdS space (Abate, Salazar, Torroba 2026).
- The relation makes use of the finite part of the entanglement entropy.

Methodology

- We consider a free, massive, scalar field in (3+1)-dimensional flat space.
- We expand the field in real spherical harmonic moments
- The radial coordinate is discretized on a lattice of concentric spherical shells with radii $r_j = j\epsilon$, where $j = 1, 2, \ldots, N$.
- The Hamiltonian reads

$$H = \frac{1}{2\epsilon} \sum_{\ell,m} \sum_{j=1}^{N} \left[\pi_{\ell m,j}^2 + \left(\phi_{\ell m,j+1} - \phi_{\ell m,j}\right)^2 + \frac{\ell\left(\ell+1\right)}{j^2} \phi_{\ell m,j}^2 + \bar{\mu}^2\, \phi_{\ell m,j}^2 \right].$$

  with $\bar{\mu} = \mu\epsilon$. We set $\epsilon = 1$.
- This form implements Dirichlet boundary conditions, with vanishing field at the origin and at a radius $r = L$. Possible zero modes are thus eliminated.
- The $\ell = 0$ sector corresponds to the discretized Hamiltonian of the $(1+1)$-dimensional field theory.

A c-function in $1+1$ dimensions

- Consider a free, massive scalar field in $1+1$ dimensions.
- In the deep UV the mass becomes irrelevant and we have one massless degree of freedom.
- In the deep IR the massive field decouples and there are no degrees of freedom.
- The c-function must interpolate between the two limits in a monotonic fashion.
- Consider a system of length L, divided into subsystems of lengths R and L − R.
- Impose Dirichlet boundary conditions, with the field vanishing at the ends. No zero modes in the spectrum.
- For $L \to \infty$, the entanglement entropy in the massless theory is

$$S = \frac{1}{6}\log\left(\frac{2R}{\epsilon}\right) + \bar{c}_1'$$

(Holzhey, Larsen, Wilczek 1994, Korepin 2004, Calabrese, Cardy 2004)

- If the contribution from the UV fixed point is subtracted, the finite part of the entropy must depend only on μR for $\mu\epsilon \to 0$. The constant part does not contribute to the entropy.
- The form of the entropy is

$$S(R,\mu) = \frac{1}{6}\ln\frac{R}{\epsilon} + S_{fin}(\mu R)$$

- For $x = \mu R \to 0$

$$S_{fin} = \frac{1}{6}x^2\log(x) + \left(\frac{\gamma_E}{6} - \frac{5}{36}\right)x^2.$$

- For $x = \mu R \to \infty$

$$S_{fin} = -\frac{1}{6}\log(x).$$

- An effective c-function can be defined as

$$c(x) = 6x\, S'(x,\mu\epsilon) \to 1 + 6x\, S'_{fin}(x).$$

- It interpolates between 1 and 0.

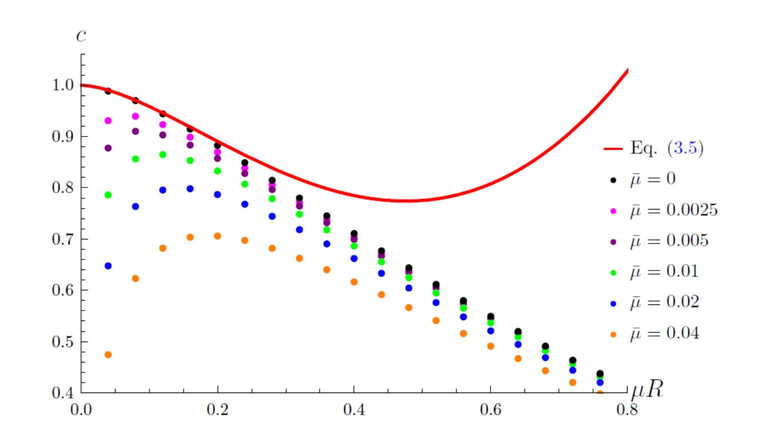

Figure: The numerical results for the c-function for various values of $\bar{\mu} = \mu\epsilon$. The black points correspond to the extrapolation to the continuum limit $\epsilon \to 0$. The solid red line corresponds to the analytical expression valid for small μR.

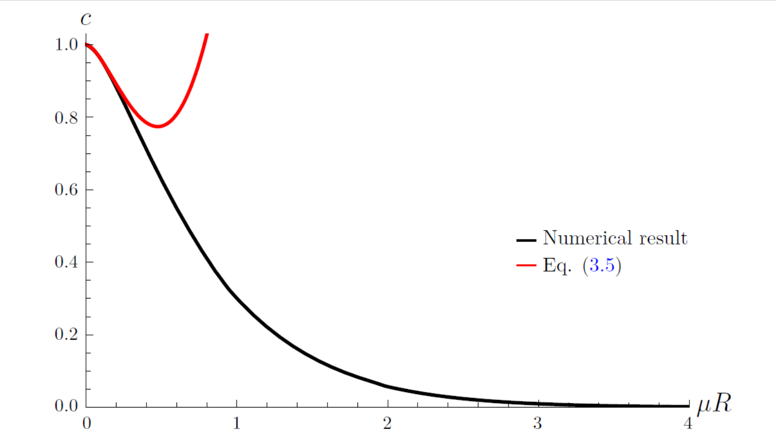

Figure: The black line depicts the numerical result for the c-function in the continuum. The red line corresponds to the analytical expression valid for small μR.

An a-function in $3+1$ dimensions

- The entropy has the form

$$S(R,\mu) = c\frac{R^2}{\epsilon^2} - \frac{1}{90}\ln\frac{R}{\epsilon} + \frac{1}{6}\mu^2 R^2 \ln \mu\epsilon + S_{\rm fin}(\mu R).$$

- An effective a-function can be defined as

$$a(x) = -45\left(x^2\, S''(x,\mu\epsilon) - x\, S'(x,\mu\epsilon)\right) \to 1 - 45\left(x^2\, S''_{\rm fin}(x) - x\, S'_{\rm fin}(x)\right).$$

- It interpolates between 1 and 0.

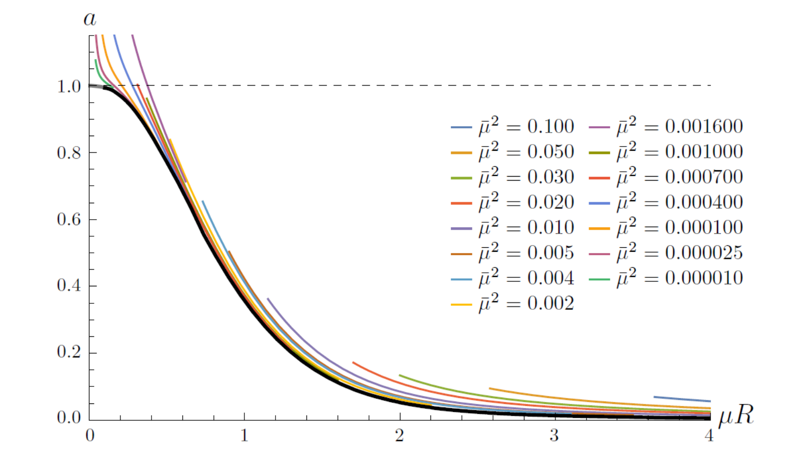

Figure: The colored lines depict the numerical results for the a-function for various values of $\bar{\mu} = \mu\epsilon$. The black line corresponds to the extrapolation to the continuum limit $\epsilon \to 0$.

Prospects

- Open questions
  - Calculation of the F-function in (2+1)-dimensional flat space.
  - Calculation of c-, F- and a-functions in dS and AdS space.
  - Entanglement and holography.
- Real challenges
  - Entanglement entropy of interacting theories.
  - Calculation of a-, F- and a-functions interpolating between nontrivial fixed points.
  - Entanglement during collapse (e.g. Vaidya background).
  - Entanglement entropy in geometries with event horizons (e.g. Schwarzschild background).
- How to measure entanglement entropy?
  - Cosmology
  - Analogue gravity

How to measure entanglement entropy

Verification of the area law of mutual information in a quantum field simulator #3

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Experimental verification of the area law of mutual information in a quantum field simulator

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Theoretical understanding of the scaling of entropies and the mutual information has led to significant advances in the research of correlated states of matter, quantum field theory, and gravity. Measuring von Neumann entropy in quantum many-body systems is challenging as it requires complete knowledge of the density matrix. In this work, we measure the von Neumann entropy of spatially extended subsystems in an ultra-cold atom simulator of one-dimensional quantum field theories. We experimentally verify one of the fundamental properties of equilibrium states of gapped quantum many-body systems, the area law of quantum mutual information. We also study the dependence of mutual information on temperature and the separation between the subsystems. Our work is a crucial step toward employing ultra-cold atom simulators to probe entanglement in quantum field theories.

Quantum field simulator for dynamics in curved spacetime

#13

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Quantum field simulator for dynamics in curved spacetime

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The observed large-scale structure in our Universe is seen as a result of quantum fluctuations amplified by spacetime evolution [1]. This, and related problems in cosmology, asks for an understanding of the quantum fields of the standard model and dark matter in curved spacetime. Even the reduced problem of a scalar quantum field in an explicitly time-dependent spacetime metric is a theoretical challenge [2–4] and thus a quantum field simulator can lead to new insights. Here, we demonstrate such a quantum field simulator in a two-dimensional Bose-Einstein condensate with a configurable trap [5, 6] and adjustable interaction strength to implement this model system. We explicitly show the realisation of spacetimes with positive and negative spatial curvature by wave packet propagation and confirm particle pair production in controlled power-law expansion of space. We find quantitative agreement with new analytical predictions for different curvatures in time and space. This benchmarks and thereby establishes a quantum field simulator of a new class. In the future, straightforward upgrades offer the possibility to enter new, so far unexplored, regimes that give further insight into relativistic quantum field dynamics.

densate of potassium-39 with configurable de
bution and additional dynamic control of a
actions. With that we implement curved me
phononic field of the form (see methods: 'Cu
metric')

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\left(\frac{\mathrm{d}u^2}{1-\kappa u^2} + u^2\mathrm{d}\varphi$$

This corresponds to the standard cosmolo
of a $2+1$ dimensional homogeneous and is
verse, the Friedmann-Lemaître-Robertson-W
ric (FLRW) in reduced circumference coordi
This metric is parametrised by intrinsic and e
vature: the intrinsic curvature, κ, is the curv
spatial part of the metric, while the extrins
arises from the time dependence of the scale
In our atomic implementation both parame
ture and scale factor, can be adjusted indep

Phonons in the central region of a h
trapped Bose-Einstein condensate experien
with $\kappa < 0$. In cosmological settings, this
bolic two-dimensional spatial geometry with
ordinate of infinite range, as depicted in
Through the Poincaré transformation, the in
bolic space is mapped to a finite disc, perf
to be implemented in finite-size ultracold