Image of Jena: Entdecke-Deutschland.de, 2024

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Functional Renormalization of QCD in 1+1 dimensions

Eric OEVERMANN
Friedrich Schiller University Jena

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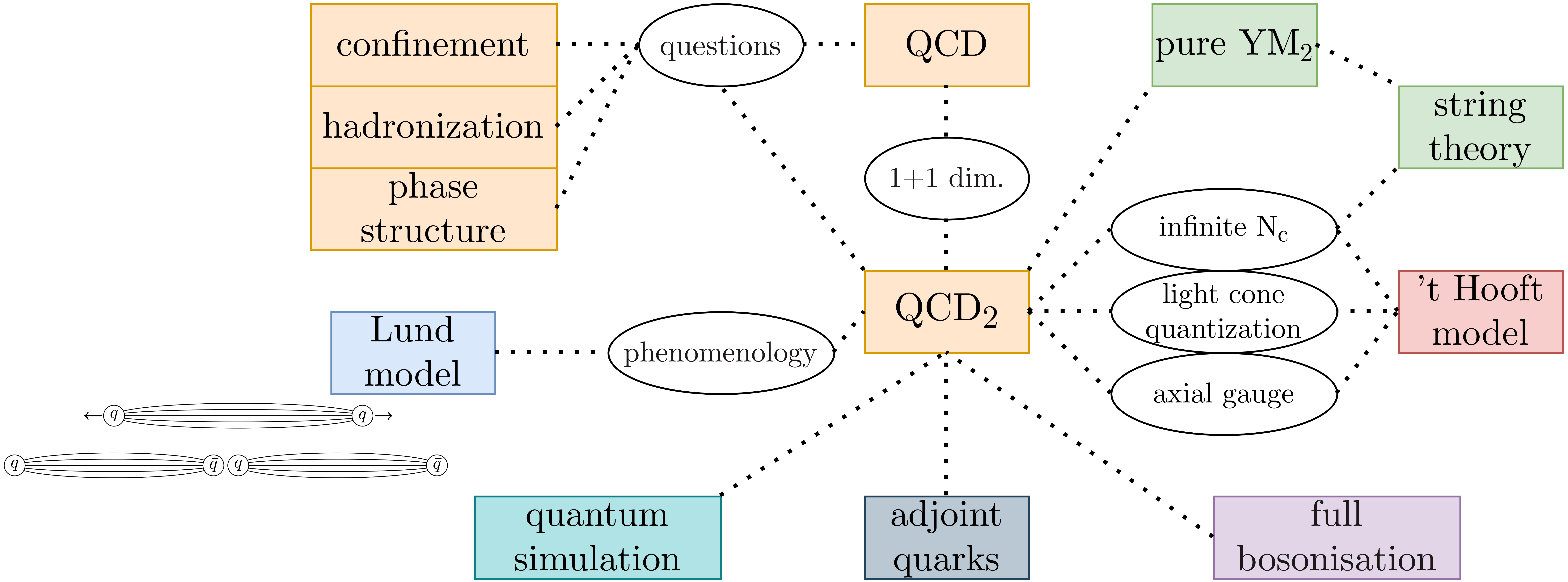
Contextualization



For image and rererences see: EO, Adrian Koenigstein, Stefan Floerchinger, Phys.Rev.D 111 (2025)

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   - QCD in 1+1
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   - Fierz completeness
2. **Non-relativistic theory**
   - Access full two-body quark-antiquark bound state and scattering spectrum

Functional renormalization of QCD in 1+1 dimensions: four-fermion interactions from quark-gluon dynamics

EO, Koenigstein, Floerchinger, Phys.Rev.D 111 (2025) : first FRG paper on QCD in 1+1

QCD in 1+1 dimensions

Microscopic action

$$S = \int \mathrm{d}^2x \left[\bar{\psi} \left[\gamma^{\mu} \left(\partial_{\mu} - \mathrm{i}\, A_{\mu} \right) + m \right] \psi + \frac{1}{2g^2} \mathrm{tr}\left(F_{\mu\nu} F^{\mu\nu} \right) \right]$$

A. Gauge coupling g of positive energy dimension: **super-renormalizable theory**

B. **No propagating gauge DoF**

C. Quarks: 2 components, **no spin/angular DoF**

Need **gauge fixing** and **IR regularization**

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Technical steps on gauge fixing:

- Background & fluctuation field splitting
  $A = \bar{A} + a$
- Gauge fixing term & ghosts

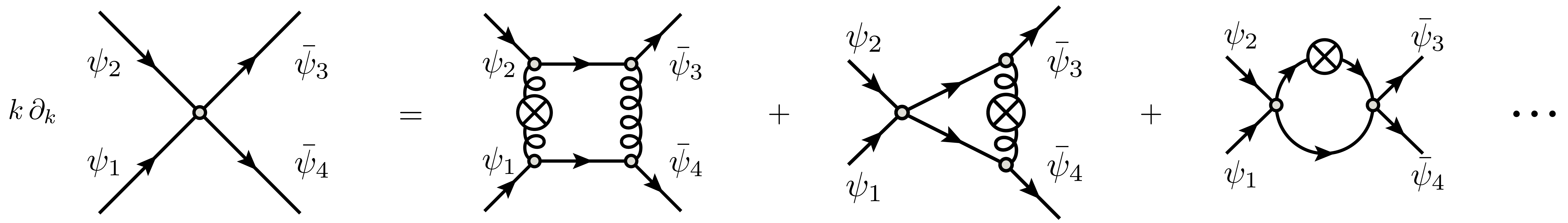
Technical steps on regularization:

- Callan-Symanzik type / masslike / momentum-independent regulators
- Preserve Osterwalder-Schrader axioms

Fluctuation field gauge variance and background independence are encoded in identities that are unmodified by the regulator presence!

Local four-fermion interactions

Truncation of Γ_k: **Fierz complete** set of local four-fermion interactions



$$k\,\partial_k \lambda_i = \quad \#\left(\frac{g}{k}\right)^4 + \quad \sum_{j=1}^{3} \#_j \left(\frac{g}{k}\right)^2 + \quad \sum_{j,l=1}^{3} \#_{jl}\, \lambda_j\, \lambda_l$$

Flow equation can be used to determine a Fierz complete basis

[procedure similarly decribed in Braun, Geißel, Pawlowski, Sattler and Wink (2025) 'Juggling with tensor bases in functional approaches']

3 interaction terms needed

1. Write down over complete set of interactions
2. Take 4 functional derivatives of flow eq.
3. Project the equations / contract indices with projectors

$$M_{ij}\,(k\partial_k\lambda_j) = b_i$$

4. Reduce system until it becomes invertible

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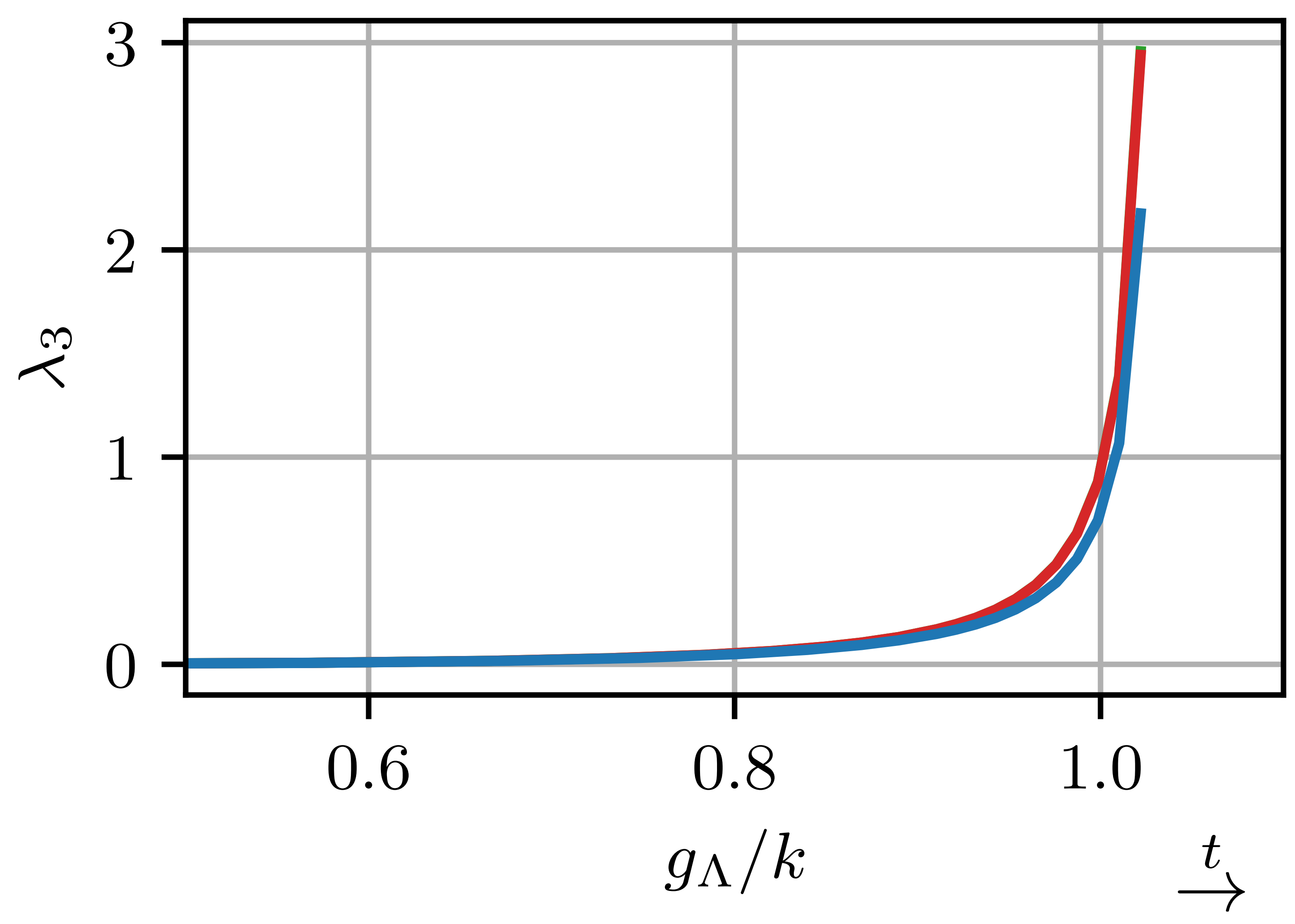
(Some) Results

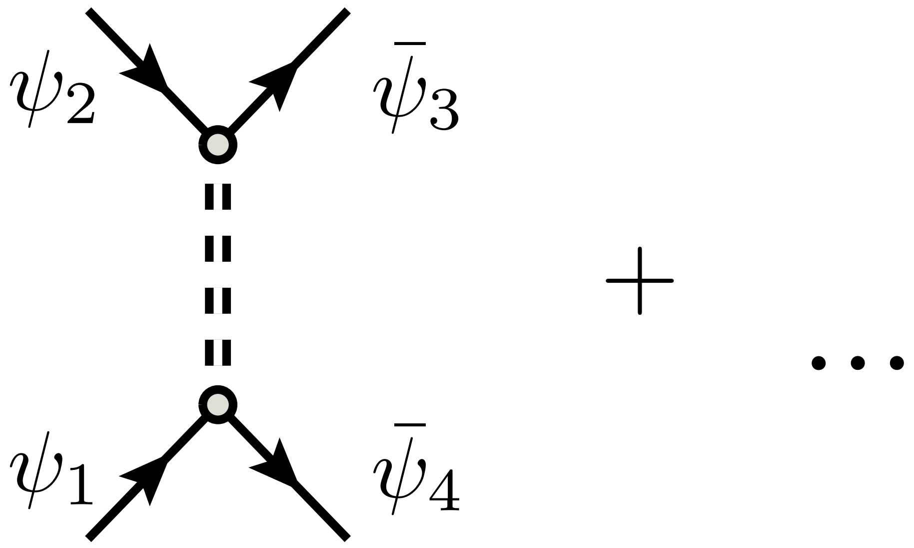
- Scale at which **divergence occurs given by g_Λ**
- Divergence of four-fermion coupling manifest as soon as non-zero (dimensionless couplings)



- Our paper: further discussions, e.g. on large N

[Cresswell-Hogg and Litim (2025) with agreements]

- Next: **resolve momentum dependence** of four-fermion vertex, include bound states



Non-relativistic QCD in 1+1 dimensions

EO, Steller, Floerchinger (work in progress)

Example for fully calculable two-body quark-antiquark bound state spectrum via a Hubbard-Stratonovich transformation

Non-relativistic action

$$S = \int_{\mathbb{R}^2} \mathrm{d}t\,\mathrm{d}x \Big[-\frac{1}{4g^2} F^z_{\mu\nu} F^{\mu\nu z} + \chi_c^\dagger\, \mathrm{i}\,(\partial_t - \mathrm{i}A_0^z T^z)\chi + \frac{1}{2m}\chi_c^\dagger (\partial_x - \mathrm{i}A_1^z T^z)^2 \chi$$
$$+ \tilde{\chi}_c^\dagger\, \mathrm{i}\,(\partial_t - \mathrm{i}A_0^z T^z)\tilde{\chi} + \frac{1}{2m}\tilde{\chi}_c^\dagger (\partial_x - \mathrm{i}A_1^z T^z)^2 \tilde{\chi}\Big]$$

EoM are type Schrödinger eqs.

- Quark field χ and anti-quarks $\tilde{\chi}$ transform under $U \in \mathrm{SU}(N)$ as

$$\chi \mapsto U\chi\,, \qquad \tilde{\chi} \mapsto U^* \tilde{\chi}$$

- Independent global $U(1)$ symmetry for quarks and anti-quarks

  → Time dependent $U(1)$ symmetry: add a potential to temporal derivative

- **Galilei invariance** of all terms in the action!

Gauge fixing and IR regularisation

Spatial axial gauge

$$A_1^z = 0$$

- Static gluons
- No gluon self-interactions

$$S = \int_{\mathbb{R}^2} \mathrm{d}t\,\mathrm{d}x \left[\frac{1}{2g^2} A^{0z}\,\partial_x^2 A_0^z + \left(\chi^\dagger T^z \chi - \tilde{\chi}^\dagger (T^z)^{\mathrm{T}} \tilde{\chi}\right) A^{0z} + \chi_c^\dagger \left(\mathrm{i}\,\partial_t + \frac{1}{2m}\partial_x^2\right)\chi^c + \tilde{\chi}_c^\dagger \left(\mathrm{i}\,\partial_t + \frac{1}{2m}\partial_x^2\right)\tilde{\chi}^c \right]$$

Galilei invariance preserved

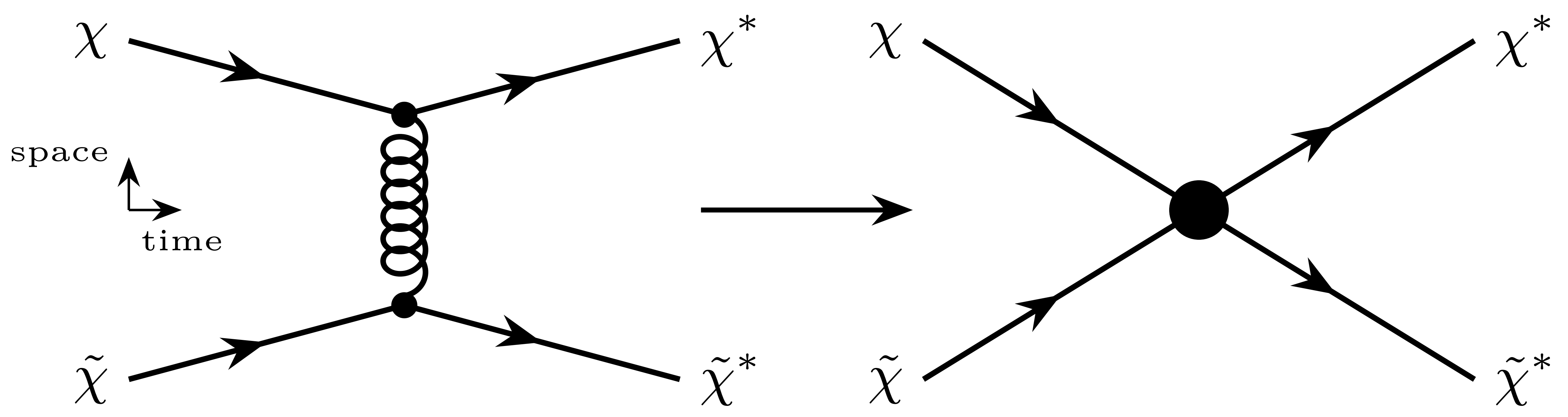
Adding a gluon mass term [c.f. regularization in Goldman, Nefediev and Wagebrunn '25]

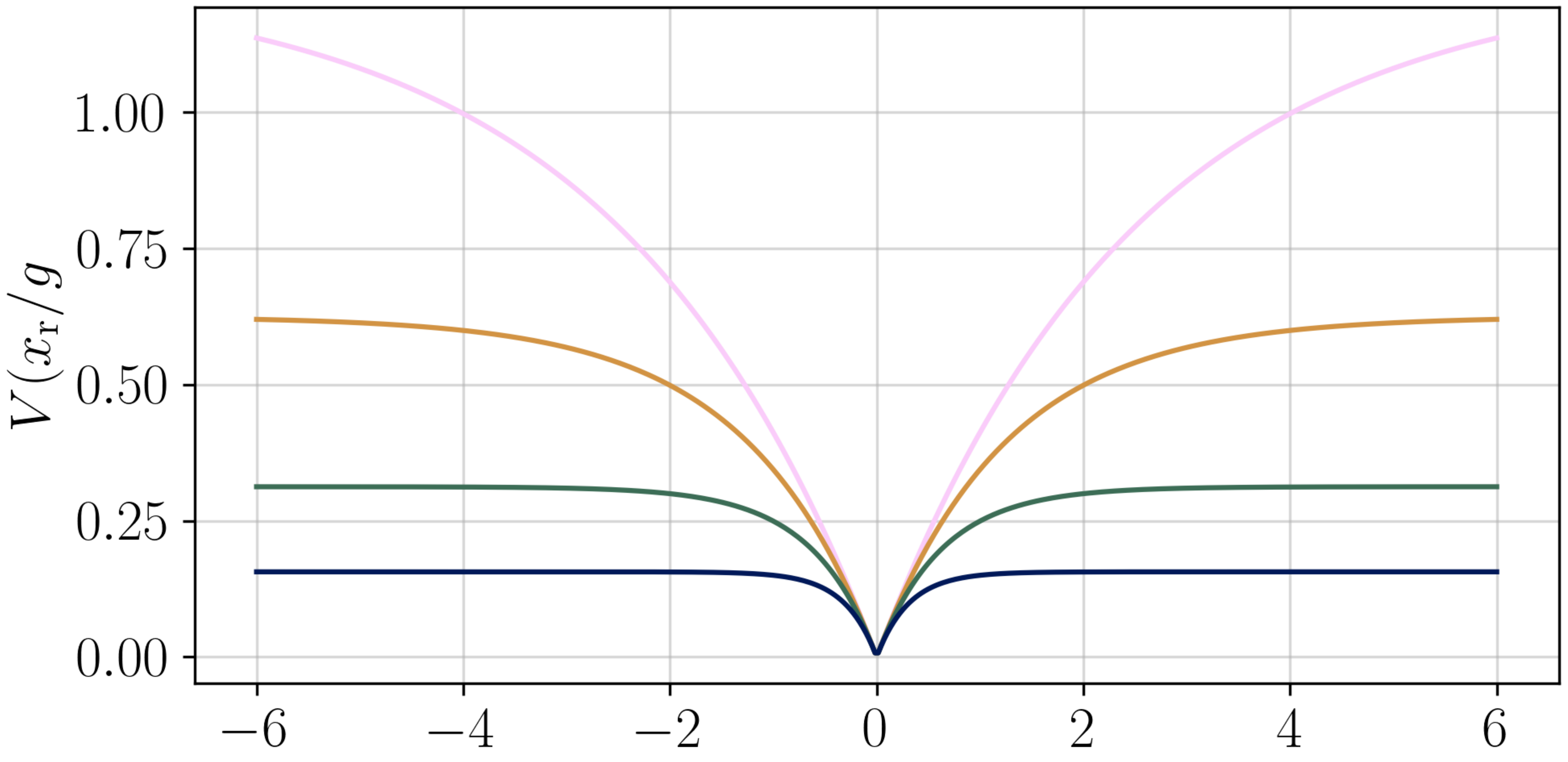
$$\int_{\mathbb{R}^2} \mathrm{d}t\,\mathrm{d}x \left(-\frac{1}{4g^2} F^z_{\mu\nu} F^{\mu\nu z} - \frac{m_{\mathrm{gl}}^2}{2g^2} A^z_\mu A^{\mu z} \right)$$

keeps Galilei invariance after gauge fixing

Integrate out gauge field

Transition to purely fermionic description [c.f. e.g. Alkhofer and Reinhardt '95]

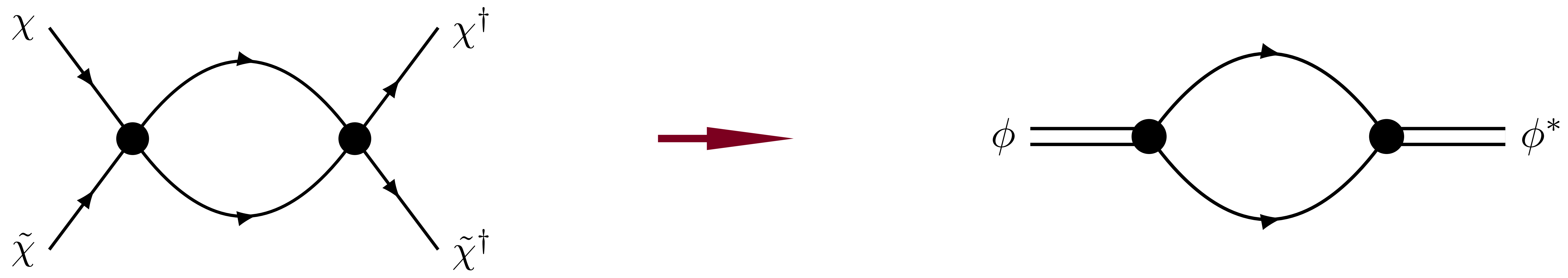




[Floerchinger '10]

Use FRG to connect field description and two-body Schrödinger equation

1. Introduce bi-local field via a HS transformation



2. Solve flow equation for propagator $G_\phi = P_\phi^{-1}$ of ϕ

$$\Gamma_k^{(\phi,2)} = \int_{E,p,q,q'} \phi^*(E,p,q)\, P_{\phi,k}(E,p,q,q')\, \phi(E,p,q')$$

Non-relativistic limit:

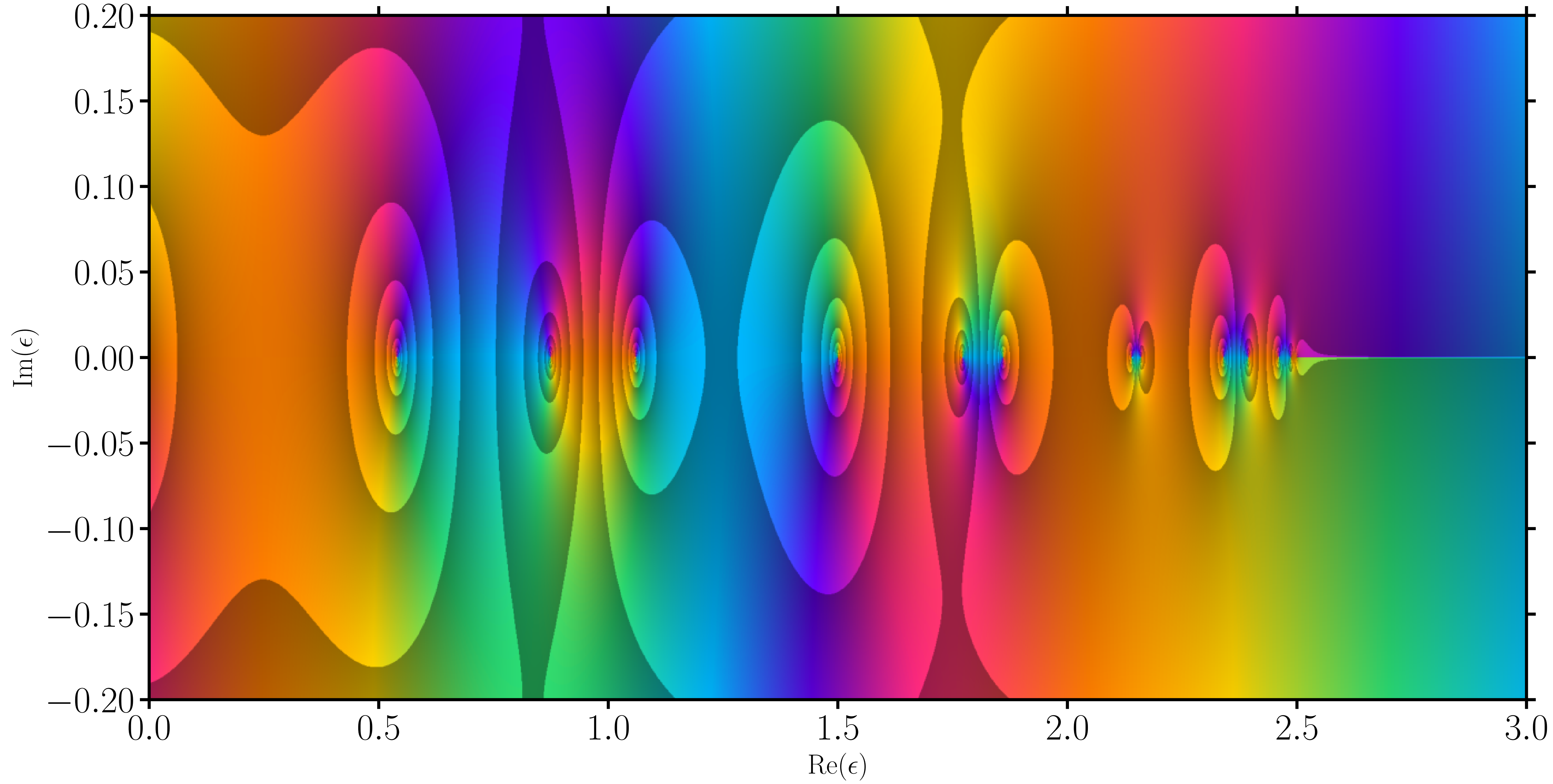
- Only non-tour diagrams
- N-body problems decouple

Spectrum

Poles of $G_\phi(E)$ are poles of $(E-H)^{-1}$ with

$$H = -\frac{1}{4m}\frac{\partial^2}{\partial x_c^2} - \frac{1}{m}\frac{\partial^2}{\partial x_r^2} + V_1(x_r)$$

→ Solve Schrödinger equation, Jost / scattering solutions



Full Solution

$$\phi^*(E,p,x_\mathrm{r})\,\bar{G}_\phi(E,p,x_\mathrm{r},x_\mathrm{r}')\,\phi(E,p,x_\mathrm{r}') = \sum_n \frac{\varphi_n^*(x_\mathrm{r})\,\varphi_n(x_\mathrm{r}')}{E-E_\mathrm{c}-E_n}$$
$$+\int_{4/9g^2/(2m_\mathrm{gl})}^{\infty}\frac{\mathrm{d}E_\mathrm{r}}{2\pi}\,\frac{1}{E-E_\mathrm{c}-E_\mathrm{r}}\sum_{i\in\{\mathrm{L,R}\}}\Big(\overline{\psi_i(E_\mathrm{r},x_\mathrm{r}')}\,\psi_i(E_\mathrm{r},x_\mathrm{r})\Theta(x_\mathrm{r}-x_\mathrm{r}')+\overline{\psi_i(E_\mathrm{r},x_\mathrm{r})}\,\psi_i(E_\mathrm{r},x_\mathrm{r}')\Theta(x_\mathrm{r}'-x_\mathrm{r})\Big)$$

- Can express all quantities explicitly and relate to Jost function/solution

IR limit:

- Linear potential
- Continuum removed from spectrum

UV limit:

- Delta potential
- One bound state of vanishing energy

Thank you! Questions?

Summary

- QCD in 1+1 with the FRG: minimal truncation in glue and quark sector but with Fierz complete set of local four-fermion interactions
    A. Discussed properties of Callen-Symanzik regulator
    B. Fierz completeness determined by flow equation
    C. Divergence of couplings signals formation of mesonic degrees of freedom

- Non-relativistic QCD in 1+1 as example for fully calculable two-body quark-antiquark bound state spectrum, e.g., via a Hubbard-Stratonovich transformation

*Back-up

Equations

$$S = \int \mathrm{d}^2 x \Big[\bar{\psi} \, (\gamma^\mu D_\mu + m) \, \psi + \frac{1}{2g^2} \, \mathrm{tr}(F_{\mu\nu} F^{\mu\nu}) + \frac{1}{\xi g^2} \, \mathrm{tr}\big((D_\mu[\bar{A}] \, a^\mu) \, (D_\nu[\bar{A}] \, a^\nu) \big) - 2 \, \mathrm{tr}\big(\bar{c} \, D^\mu[\bar{A}] \, D_\mu[A] \, c \big) \Big]$$

$$\Delta S_k = \int \mathrm{d}^2 x \Big[\bar{\psi} \, k \, \psi + \frac{1}{g^2} \, \mathrm{tr}(a_\mu \, k^2 \, a^\mu) + 2 \, \mathrm{tr}(\bar{c} \, k^2 c) \Big]$$

$$V_1(x_\mathrm{r}) = -\frac{4}{9} \, \frac{g^2}{2m_\mathrm{gl}} \left(1 - \mathrm{e}^{-m_\mathrm{gl} |x_\mathrm{r}|} \right)$$

$$S = \int_{t,x} \left[\chi^\dagger \left(\mathrm{i} \, \partial_t + \frac{1}{2m} \partial_x^2 \right) \chi + \tilde{\chi}^\dagger \left(\mathrm{i} \, \partial_t + \frac{1}{2m} \partial_x^2 \right) \tilde{\chi} \right] - \int_{t, x_\mathrm{c}, x_\mathrm{r}} \left[(\tilde{\chi}^\mathrm{T} \chi)^\dagger (t, x_\mathrm{c}, x_\mathrm{r}) \, V_1(x_\mathrm{r}) \, (\tilde{\chi}^\mathrm{T} \chi)(t, x_\mathrm{c}, x_\mathrm{r}) + \ldots \right]$$

$$Z = \int \mathcal{D}\chi^\dagger \, \mathcal{D}\chi \, \mathcal{D}\tilde{\chi}^\dagger \, \mathcal{D}\tilde{\chi} \, \mathrm{e}^{\mathrm{i} S[\chi, \tilde{\chi}, \chi^\dagger, \tilde{\chi}^\dagger]} \propto \int \mathcal{D}\chi^\dagger \, \mathcal{D}\chi \, \mathcal{D}\tilde{\chi}^\dagger \, \mathcal{D}\tilde{\chi} \, \mathcal{D}\phi^* \, \mathcal{D}\phi \, \mathrm{e}^{\mathrm{i}(S + S_\mathrm{pb})}$$

$$S_\mathrm{pb} = \int_{t, x_\mathrm{c}, x_\mathrm{r}} \left[\phi^*(t, x_\mathrm{c}, x_\mathrm{r}) - (\tilde{\chi}^\mathrm{T} \chi)^\dagger (t, x_\mathrm{c}, x_\mathrm{r}) \right] V_1(x_\mathrm{r}) \left[\phi(t, x_\mathrm{c}, x_\mathrm{r}) - (\tilde{\chi}^\mathrm{T} \chi)(t, x_\mathrm{c}, x_\mathrm{r}) \right]$$