Landau damping and quantum criticality in Fermi systems from the Wilsonian RG perspective

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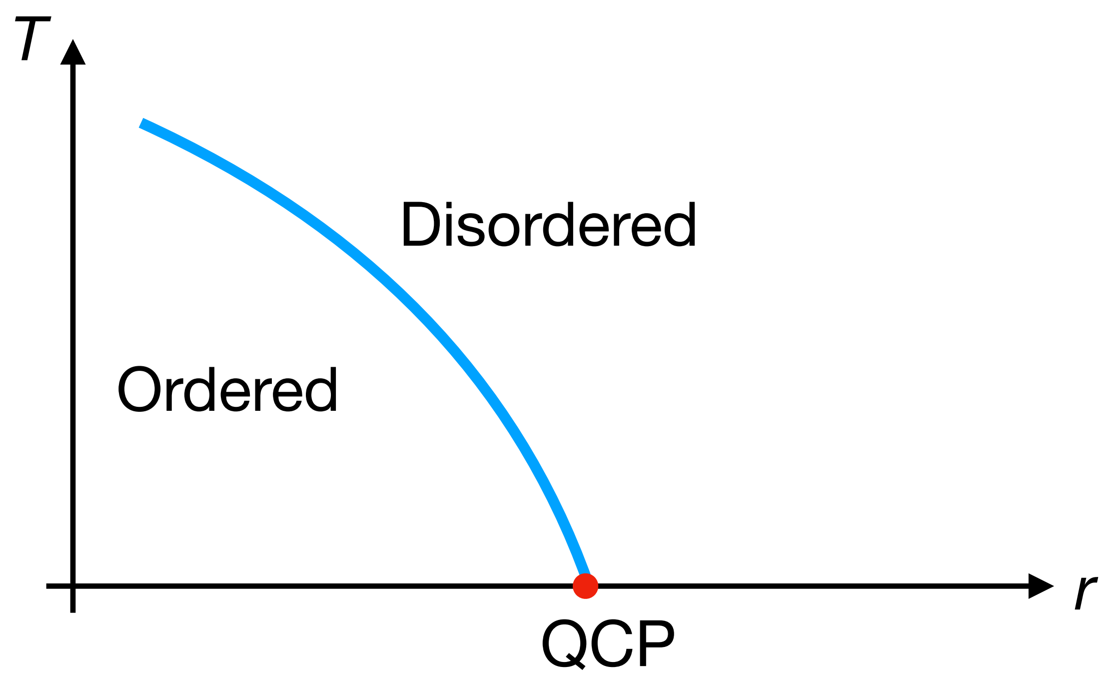
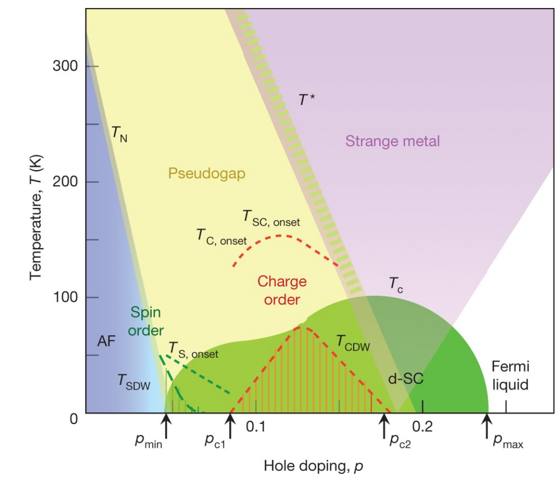
Quantum criticality in clean electronic systems

General setup:





Keimer *et. al.*, Nature **518**, 179 (2015)

At $T > 0$ critical singularities controlled by the classical (Wilson-Fisher) F-P

What is the **correct** low-energy action to describe the QCP ?

Hertz - Millis theory (1976, 1993)

$$\mathcal{Z} = \int \mathcal{D}[\bar{\psi}, \psi] e^{-\mathcal{S}_{mic}[\bar{\psi}, \psi]}$$

known dominant instability - order parameter ϕ

Hubbard - Stratonovich transformation

$$\mathcal{Z} = \int \mathcal{D}[\bar{\psi}, \psi, \phi] e^{-\mathcal{S}_{fb}[\bar{\psi}, \psi, \phi]}$$

(decoupled fermionic interactions)

integrate fermions out

$$\mathcal{Z} = \int \mathcal{D}\phi e^{-\mathcal{S}_b[\phi]}$$

(The problematic point !)

attempt an expansion of $\mathcal{S}_b[\phi]$ in powers of ϕ

and expansion of vertices in q $\qquad q := (q_0, \vec{q})$

$$\mathcal{S}_b[\phi] \longrightarrow \mathcal{S}_H[\phi]$$

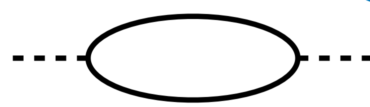
(Hertz action)

The problem with H-M:

(Focus on *d=2* and *Q=0* instabilities at *T=0*)

$$\mathcal{S}_b[\phi] \longrightarrow \mathcal{S}_b^{(2)}[\phi] + \mathcal{S}_b^{(4)}[\phi] + \dots$$

$$\mathcal{S}_b^{(2)}[\phi] = \int_q \phi_{-q}\,[1 - \chi_0(q)]\,\phi_q$$



Discontinuous function of q (at q=0)

$$S_H[\phi] = \int_q \phi_{-\vec{q},-q_0}\left[m^2 + Z\vec{q}^2 + A\,\frac{|q_0|}{|\vec{q}|}\right]\phi_{\vec{q},q_0} + u\int_x \phi(x)^4$$

$x := (\tau, \vec{x})$

Landau damping

singular quartic vertex
replaced by a local interaction coupling

form valid for
$|q_0|/|\vec{q}| \ll 1$

Problem more severe for interaction vertices.

The problem with H-M (ctd):

$$S_H[\phi] = \int_q \phi_{-\vec{q},-q_0} \left[m^2 + Z\vec{q}^2 + A \underbrace{\frac{|q_0|}{|\vec{q}|}}_{\text{Landau damping}} \right] \phi_{\vec{q},q_0} + u \int_x \phi(x)^4$$

highly singular quartic vertex
replaced by a local interaction

form valid for $|q_0|/|\vec{q}| \ll 1$

Remarks: Problem arises because of integrating out the soft (fermionic) modes

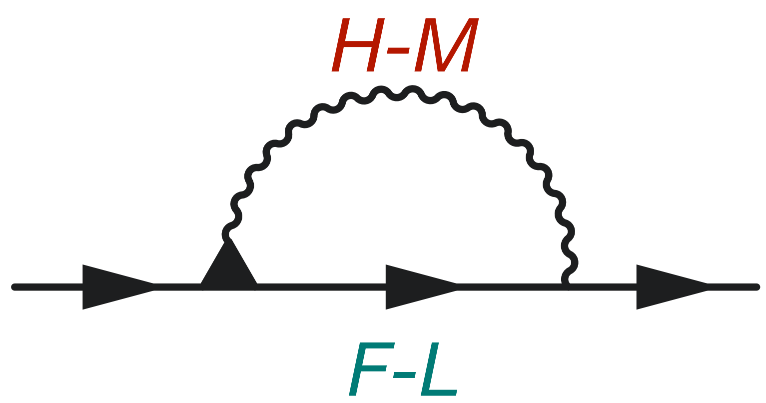
Landau damping generated from fermions at the FS

Necessity of keeping fermions well recognized in literature

Result of H-M: QCP governed by a classical-like F-P in effective dimensionality $D=d+z$, $z=3$.

Still many interesting and important predictions!

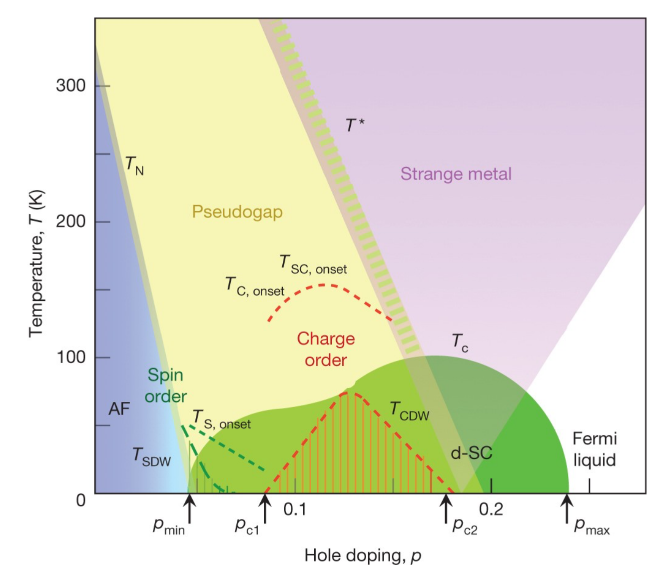
Fermion self-energy (one loop)



$$\Sigma \sim |k_0|^{2/3}$$

(at QCP)

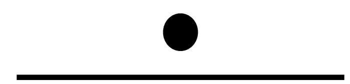
Non-Fermi liquid

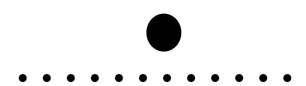


Keimer *et. al.*, Nature **518**, 179 (2015)

Earlier work on coupled f-b flows

(Mostly F-T RG)

 $= \ldots$

 $= \ldots$

(*Lee, Mandal, Metlitski, Mross, Sachdev, Holder, Metzner, Drukier, Kopietz, Fitzpatrick, Raghu …*)

Typically use the **fully dressed** (H-M) propagator to compute loop integrals

Wilsonian RG $\Longrightarrow$ Conventional Landau damping *cannot* appear until all fermions become integrated out completely.

What replaces it at finite scales?

Present focus:

Set up RG where the Bose propagator involves only contributions from integrating out high energy fermions.
Integrate out fermions along with bosons.

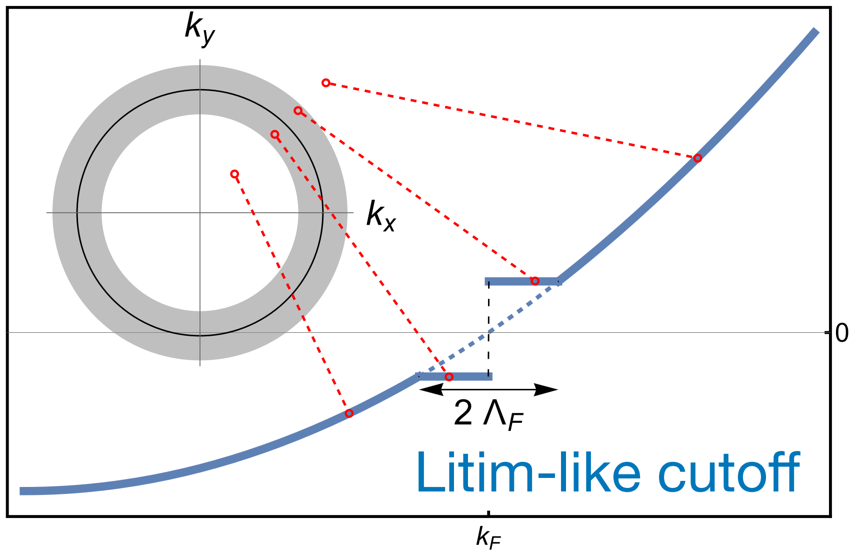
Use Wetterich framework.

Wilsonian RG, cutoffs

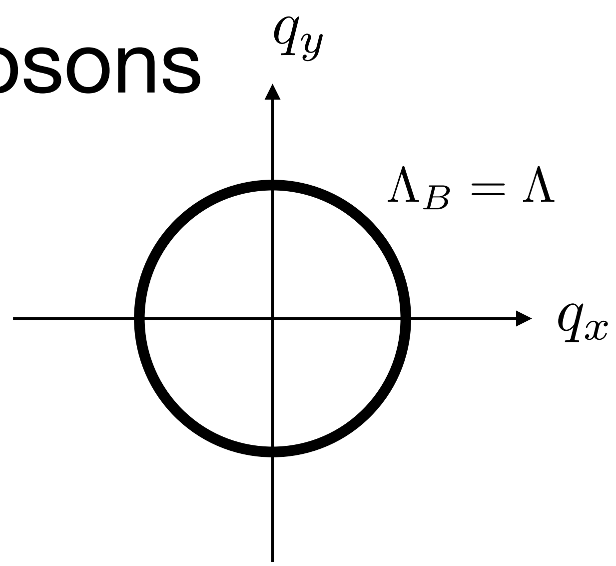
$$\mathcal{Z} = \int \mathcal{D}[\bar{\psi}, \psi, \phi] e^{-\mathcal{S}_{fb}[\bar{\psi},\psi,\phi]}$$

Cutoff scale(s):

Fermions



Bosons



H-M: spirit: $\Lambda_F \to 0$ first, and only then $\Lambda \to 0$

$$\Lambda_F = (\Lambda - \Lambda_0)\theta(\Lambda - \Lambda_0) \qquad \Lambda_0 > 0$$

Alternative: $\Lambda_F = \Lambda$

(fermions and bosons integrated out "in parallel")

(natural but deceiving)

$$\Lambda_F = \Lambda_F(\Lambda) \sim \Lambda^\gamma$$

In a truncated analysis computed observables **are sensitive** to the choice of γ

Basic questions:

How to choose γ in $\Lambda_F = \Lambda_F(\Lambda) \sim \Lambda^\gamma$

Value of z

Value of α in $G_f^{-1}(k_0, |\vec{k}| = k_f) \sim -i\,\mathrm{sgn}(k_0)|k_0|^\alpha$

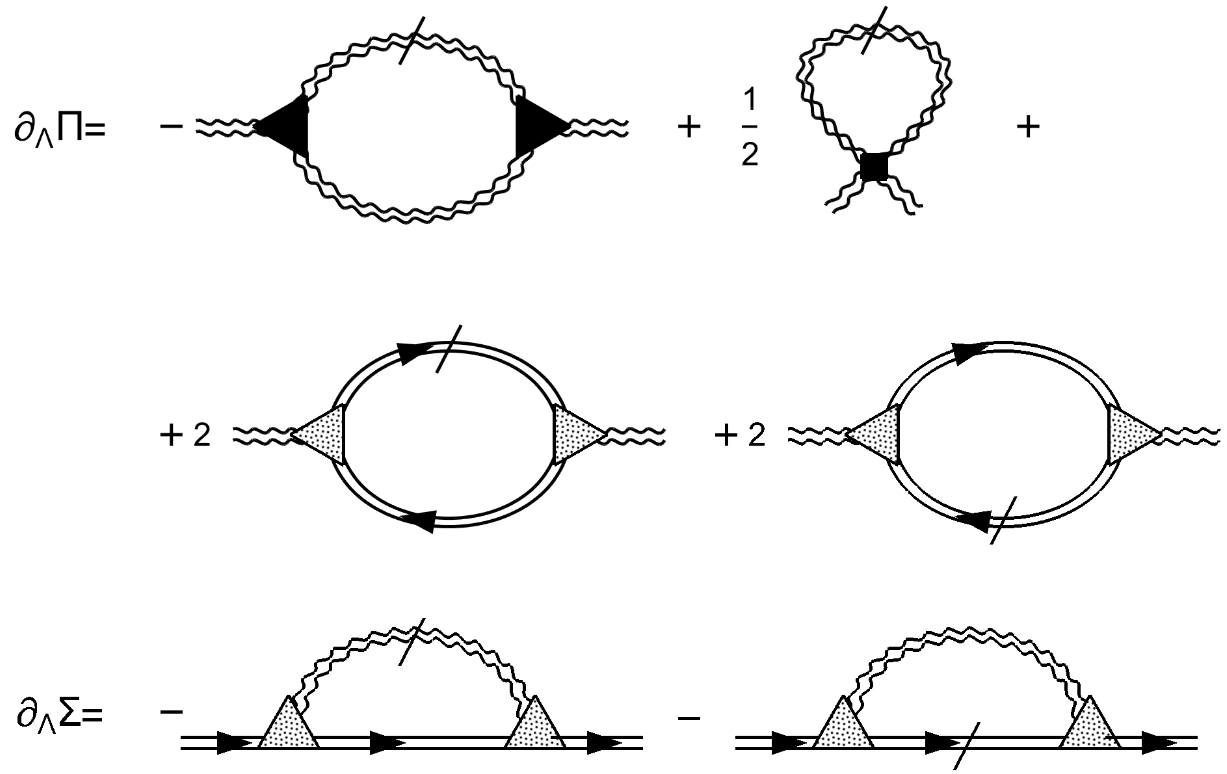
$z = 3$ broadly expected, robust until 4 loop in PT.

$$S_H[\phi] = \int_q \phi_{-\vec{q},-q_0}\left[m^2 + Z\vec{q}^2 + A\frac{|q_0|}{|\vec{q}|}\right]\phi_{\vec{q},q_0} + u\int_x \phi(x)^4$$

$z \approx 2$ seen in QMC simulations of fermionic QCPs with $\vec{Q} = 0$

Shattner *et al* PRX **6**, 031028 (2016)
Liu *et al* PRB **105**, L041111 (2022)

1PI (Wetterich) FRG, ”simplest” truncation



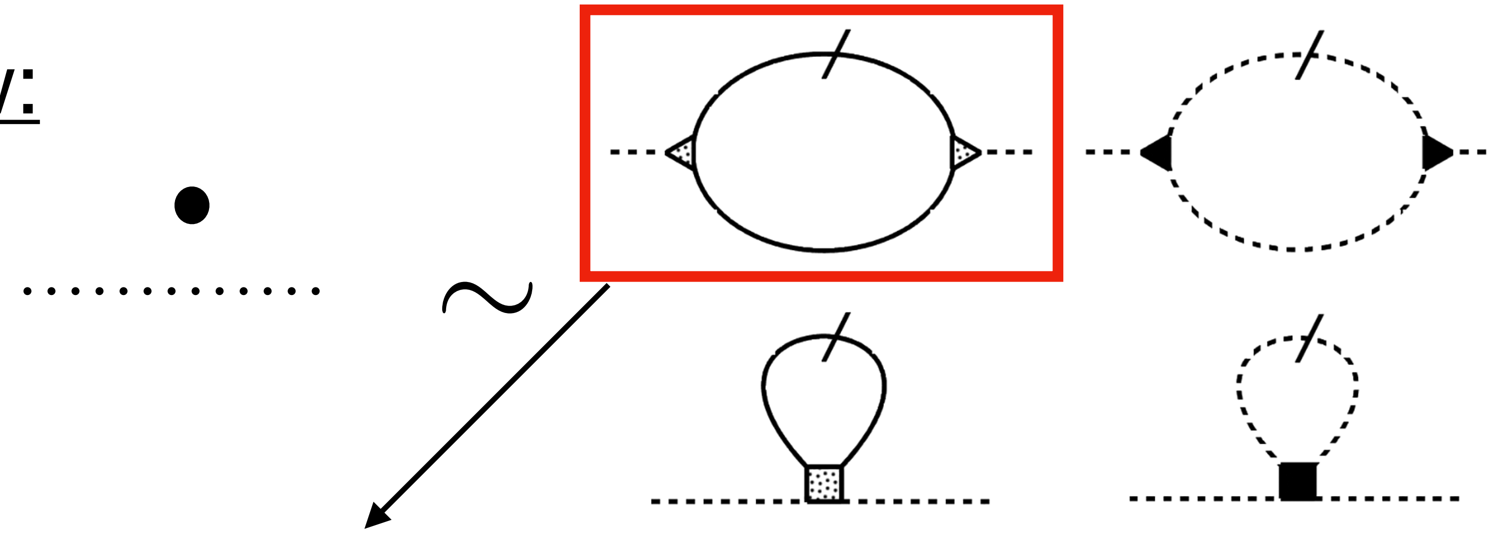
Drop Fermi self-energy in $\partial_\Lambda \Pi$

Non self-consistent

Drop flow of vertices involving fermionic legs

Always in contact with standard procedure

Boson propagator flow:



$$\mathcal{X}(q,\Lambda_F) = -2g^2 \int_k \partial_\Lambda R_f(\vec{k})\tilde{G}_0(k)^2(\tilde{G}_0(k+q)+\tilde{G}_0(k-q))$$

$$\tilde{G}_0(k)^{-1} = [-ik_0 + \xi_{\vec{k}} + R_f(\vec{k})].$$

$$B(\vec{q},q_0,\Lambda_F(\Lambda)) := \int_{\Lambda_u}^{\Lambda} d\Lambda' \mathcal{X}$$

boson propagator generated from integrating fermions down to the scale Λ

$$B(\vec{q},q_0,\Lambda_F) = B_<\theta(-|\vec{q}|+\Lambda_F) + B_>\theta(|\vec{q}|-\Lambda_F)$$

$$\boxed{B_< = -\mathcal{N}_< \frac{|\vec{q}|\Lambda_F}{q_0^2+4v_F^2\Lambda_F^2}}$$

New term

(Negative sign)

Recovers Landau damping

$$B_> \approx -\mathcal{N}_< \frac{\vec{q}^2}{q_0^2+4v_F^2\vec{q}^2}$$

$$+\mathcal{N}_> \frac{q_0}{|\vec{q}|}\left[\arctan\frac{2v_F|\vec{q}|}{q_0} - \arctan\frac{2v_F\Lambda_F}{q_0}\right]$$

+ analytical terms

Essential improvement over previous approaches:

use $$G_b = \frac{1}{m_b^2 + \vec{q}^{\,2} + B(q, \Lambda_F)}$$

instead of

$$G_{H-M} = \frac{1}{m_b^2 + \vec{q}^{\,2} + |q_0|/|\vec{q}|}$$

<u>Key point:</u>

$B(\vec{q}, q_0, \Lambda_F)$ has minimum at $(q_0, |\vec{q}|) = (0, Q_\Lambda)$

Ordering wavevector flows. $Q_\Lambda \sim \Lambda_F^{1/3}$ This must not be ignored.

<u>Aside note:</u> using frequency cutoff leads to $B(\vec{q}, q_0, \Lambda_F)$ with minimum at $q_0 > 0$

Fixing the dynamical exponent z

$$Q_\Lambda \sim \Lambda_F^{1/3} \qquad\qquad \epsilon_\Lambda := \Lambda_F / Q_\Lambda \ll 1$$

Simplified treatment

$$\downarrow$$

$$G_b \approx \frac{1}{\Delta m_b^2 + 3Q_\Lambda^2 + (|\vec{q}| - Q_\Lambda)^2 + \mathcal{A}\pi \frac{|q_0|}{v_f Q_\Lambda}} \qquad\qquad \frac{|q_0|}{v_f Q_\Lambda} \ll 1$$

(structure entirely different compared to H-M)

Require that each term scales as $\sim \Lambda^2$

$$\downarrow$$

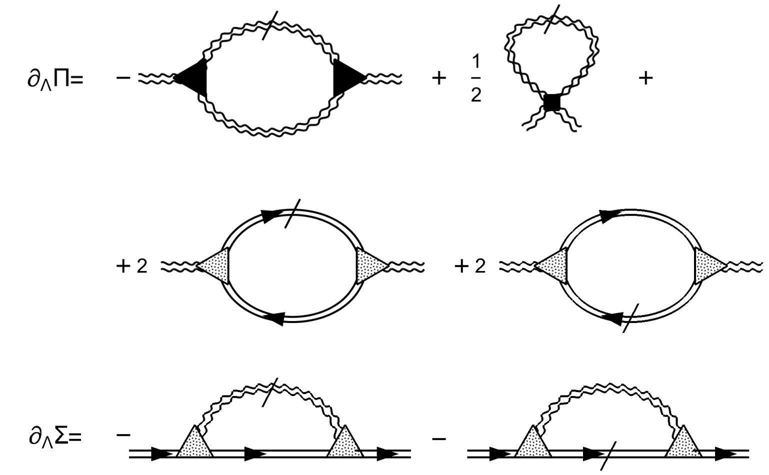
$$|q_0| \sim Q_\Lambda^3 \sim \Lambda^3 \sim \Lambda_f \longrightarrow z = 3$$

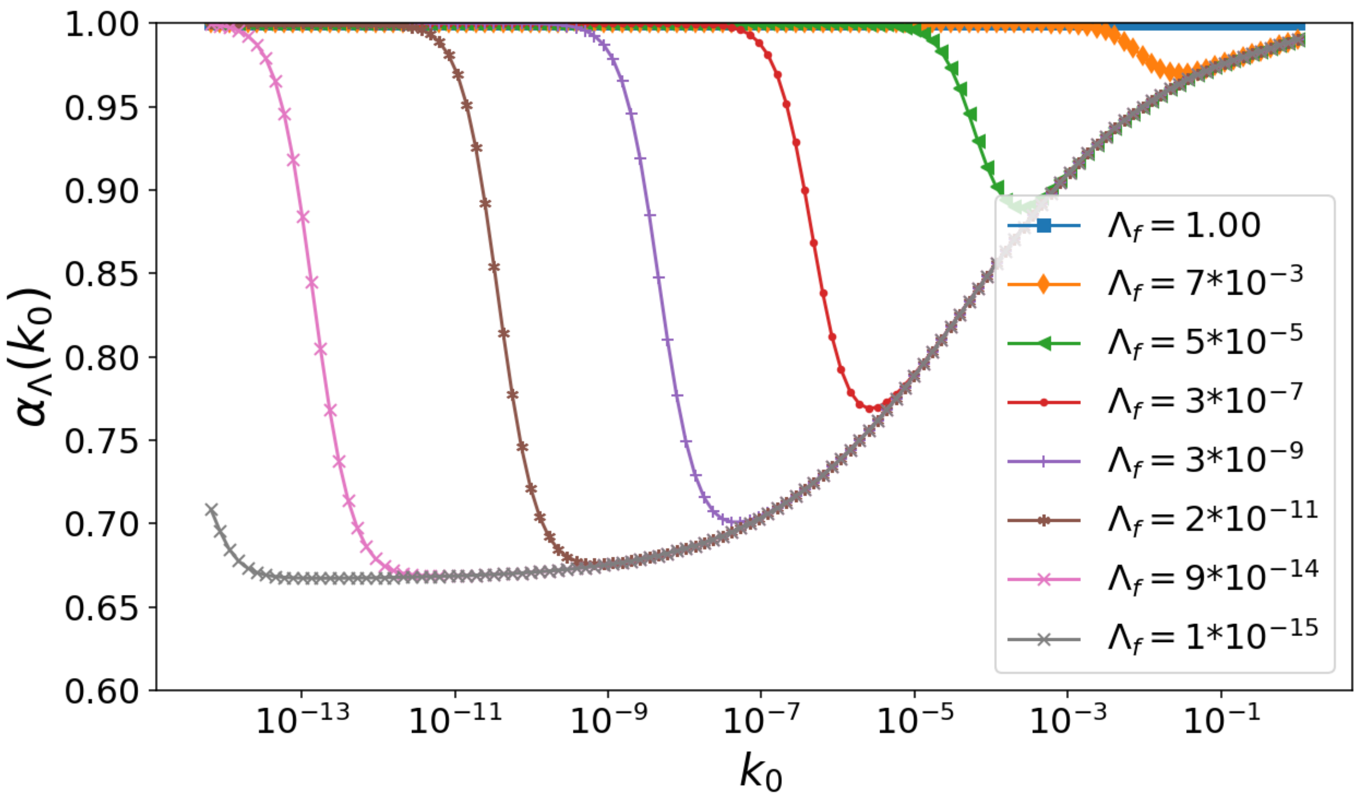
$$(\gamma = 1/3)$$

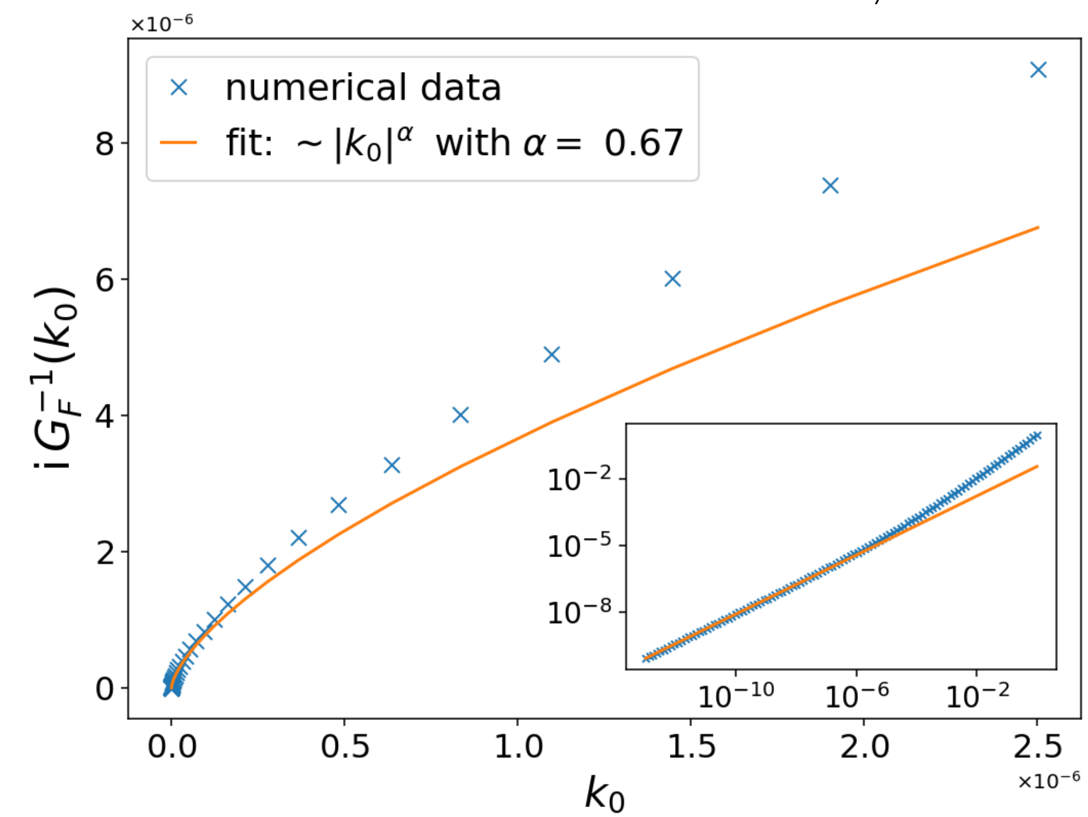
Fermi self-energy and the exponent α

$$\tilde{\Sigma}(k_0) := k_0 - \mathrm{Im}\left[\Sigma(k_0)\right]$$

$$\alpha_\Lambda(k_0) := \frac{\partial \ln\left(\tilde{\Sigma}(k_0)\right)}{\partial \ln(k_0)}$$







The ”simplest” non self-consistent truncation recovers key features of previous (RPA-type) approaches.

The problem of singular bosonic interactions avoided.

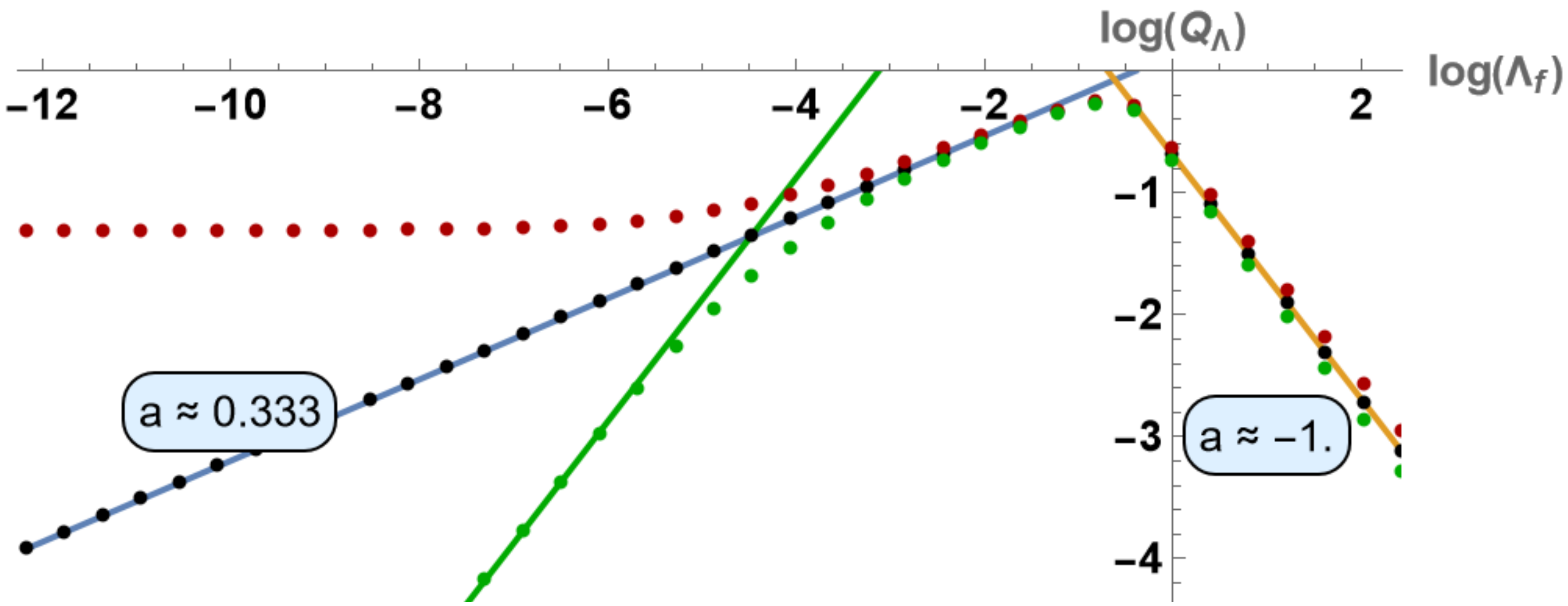
Account of the flow of Q_Λ indispensable.

Self-consistent treatment (overview/summary)

Inclusion of self energy (parametrized via Z-factors) and scaling of the Yukawa coupling.

No possibility of obtaining a fixed point with properties similar to RPA.

$$\lim_{\Lambda \to 0} Q_\Lambda > 0$$



$$\lim_{\Lambda\to 0} Q_\Lambda > 0$$

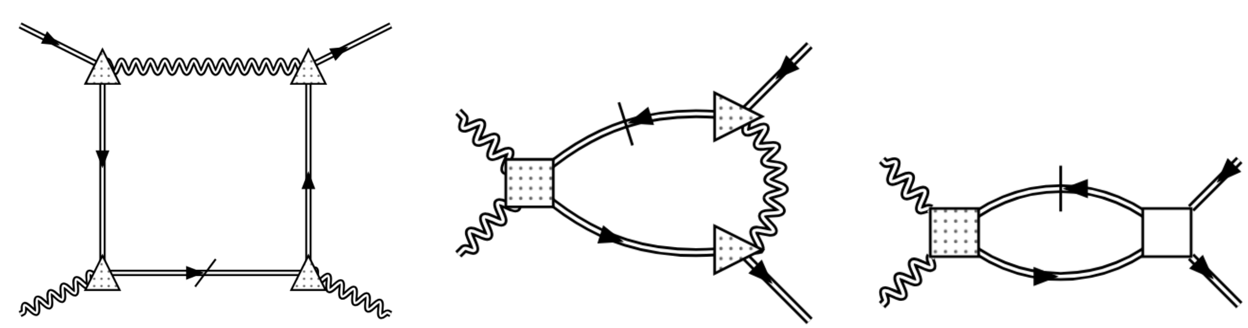
Instability towards a modulated phase?

[e.g. *Chubukov et. al.*, PRL **92**, 147003 (2004)]

Instability towards a first-order transition?

[e.g. Kirkpatrick and Belitz, PRB **113**, 214414 (2026)]

Substantially higher-level truncation necessary to correctly capture the QCP with Q=0?



Summary:

Simple non-self-consistent truncation $\longrightarrow$ RPA-type RG fixed point

(Account of the flow of Q_Λ indispensable.)

Relatively simple extensions $\longrightarrow$ instability of the RPA-type FP

$$\lim_{\Lambda \to 0} Q_\Lambda > 0$$

Modulated phase?
1-st order transition?
Necessity of higher-order truncation?