From Bose glass to Many-Body Localization in a one-dimensional disordered Bose gas

Exact Renormalization Group 2026

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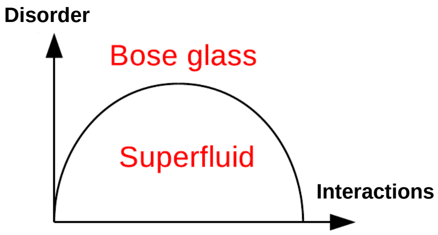
Introduction : The Bose Glass

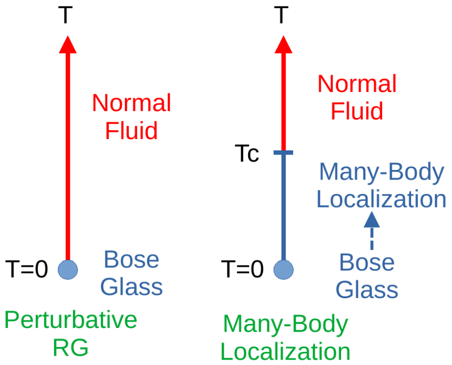
- $T = 0$ quantum phase transition towards localised phase, the **Bose Glass**. First studied in 1D [Giamarchi and Schulz 1988], existence of BG established in $d > 1$ soon after [Fisher and al. 1989].
- **Gapless** and **compressible** phase, but also **insulating** and with vanishing superfluid stiffness.
- Many studies since then, among which an **FRG approach** to the 1D Bose Glass more recently [Dupuis and Daviet 2020].

  Our goal = **Extending this work to finite temperatures**



Finite Temperature

- **Perturbative RG** predicts a crossover to a Normal Fluid phase [Glatz and Nattermann 2004]. → 1^{st} **scenario**
- [Michal, Aleiner, Altshuler, Shlyapnikov 2016] suggests there is **finite-temperature insulating phase** above the Bose Glass. → 2^{nd} **scenario**
- This is very close to original proposal of **Many-body Localization (MBL)** [Gornyi and al. 2005, Basko and al. 2006] for fermions, which generalizes Anderson localization to the interacting case.



Many-Body Localization (MBL)

- Coupling an Anderson insulator to a **delocalized bath** (e.g. phonons) induces a **finite conductivity**.
- In its first incarnation, MBL asks whether same is true if one adds **interactions** between particles, i.e. if **system can acts as its own bath**.
- Answer turns out to be negative at low enough temperatures [Gornyi and al. 2005, Basko and al. 2006], inducing a **finite-temperature phase transition**. In that case strong enough disorder **impedes transport and thermalization**.
- Later on questions started revolving around the behavior of the whole spectrum, i.e. **infinite-temperature behavior** → not our concern for today.
- → We aim at understanding **which scenario is the correct one**, using FRG.

Model

- Generic boson Hamiltonian + disordered potential

$$\hat{\mathcal{H}} = \int_x \frac{1}{2m} \nabla \hat{\psi}^\dagger \nabla \hat{\psi} + g \hat{\psi}^\dagger \hat{\psi}^\dagger \hat{\psi} \hat{\psi} + \int_x V(x) \hat{\psi}^\dagger \hat{\psi}$$

- Bosonized and replicated action: replicas allow to obtain typical (averaged over the disorder) values of observables

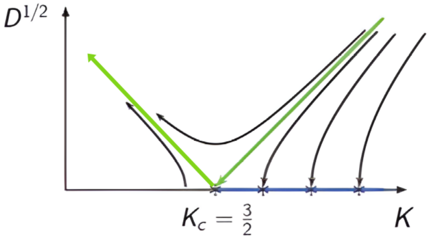
$$\overline{\mathcal{Z}^n} = \int \prod_a \mathcal{D}\varphi_a \, e^{-S[\{\varphi_a\}]}$$

$$S[\{\varphi_a\}] = \sum_a \int_{x,\tau} \frac{v}{2\pi K} \left\{ (\partial_x \varphi_a)^2 + \frac{(\partial_\tau \varphi_a)^2}{v^2} \right\} - D \sum_{a,b} \int_{x,\tau,\tau'} \cos(2\varphi_a(x,\tau) - 2\varphi_b(x,\tau'))$$

- $v =$ speed of sound, $K =$ Luttinger parameter $\propto 1/$interactions and $D =$ disorder strength.

Methods: The Nonperturbative Renormalization Group (NPRG)

- 1D transition was studied using **perturbative, one-loop RG computation** and found to lie in **BKT universality class**. [Giamarchi and Schulz 1988]



- Accessing Bose Glass phase requires **nonperturbative** and **functional** approach, where basic objects are "coupling functions":

$$D\cos(2(\varphi_a - \varphi_b)) \xrightarrow{RG} V_k(\varphi_a - \varphi_b)$$

- It is known that a functional treatment is necessary to correctly account for the effect of **metastable states** in disordered systems [D.S. Fisher 1986, Le Doussal, Wiese and Chauve 2004, Tarjus and Tissier 2008].

Methods: The Nonperturbative Renormalization Group (NPRG)

- Initial action:

$$S[\{\varphi_a\}] = \sum_a \int_{x,\tau} \frac{v}{2\pi K}\left\{(\partial_x \varphi_a)^2 + \frac{(\partial_\tau \varphi_a)^2}{v^2}\right\} - D \sum_{a,b} \int_{x,\tau,\tau'} \cos(2\varphi_a(x,\tau) - 2\varphi_b(x,\tau'))$$

- **DE2 ansatz** for the replicated effective action:

$$\Gamma_k[\{\phi_a\}] = \sum_a \int_{x,\tau} \frac{v_k}{2\pi K_k}\left((\partial_x \phi_a)^2 + \frac{(\partial_\tau \phi_a)^2}{v_k^2}\right)$$

$$-\frac{1}{2}\sum_{a,b}\int_{x,\tau,\tau'} V_k(\phi_a - \phi_b) + \frac{1}{2} W_{2,k}(\phi_a - \phi_b)\big(\partial_x(\phi_a - \phi_b)\big)^2 + \frac{1}{2} W_{3,k}(\phi_a - \phi_b)\big((\partial_\tau \phi_a)^2 + (\partial_{\tau'} \phi_b)^2\big)$$

- **Key quantities**: renormalized Luttinger parameter $K_k \sim \hbar$, and $\tilde{\Delta}_k(u) = -\tilde{V}_k''(u)$. Its first harmonic $\tilde{\Delta}_{1,k}$ defines the renormalized strength of the disorder $\sim D$.
- Until further notice **we ignore the purple term**.

Quantum-classical crossover

- Dimensionless, 1-replica action:

$$\frac{1}{2\pi K_k}\int_{\tilde{x},\tilde{\tau}}(\partial_{\tilde{x}}\phi_a)^2+(\partial_{\tilde{\tau}}\phi_a)^2=\frac{1}{2\pi K_k}\sum_n\int_{\tilde{q}}(\tilde{q}^2+\tilde{\omega}_n^2)|\phi_a(\tilde{q},i\tilde{\omega}_n)|^2$$

  with $\tilde{\omega}_n=2\pi\tilde{T}_k$ dimensionless Matsubara frequencies, $\tilde{T}_k$ = dimensionless temperature.

- As $\tilde{T}_k\sim 1$ contributions of the $\tilde{\omega}_{n\neq 0}$ start being **heavily suppressed** by regulator. Ultimately **only** $\tilde{\omega}_0=0$ **remains**, and the field does not depend on $\tilde{\tau}$ anymore.

  $\boxed{\longrightarrow \textbf{Quantum-classical crossover.}}$

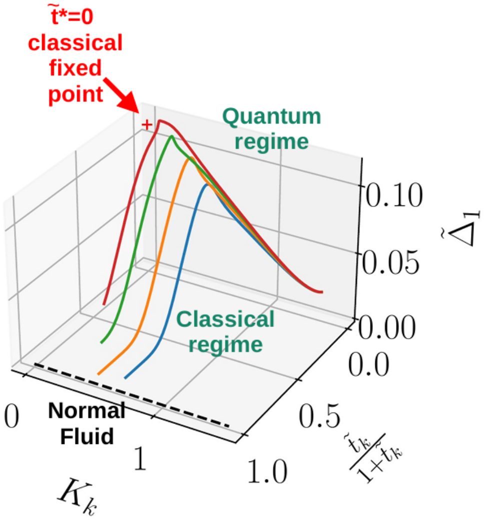
- Action becomes:

$$\frac{1}{2\pi K_k}\int_{\tilde{x},\tilde{\tau}}(\partial_{\tilde{x}}\phi_a)^2+(\partial_{\tilde{\tau}}\phi_a)^2\longrightarrow\frac{1}{2\pi K_k\tilde{T}_k}\int_{\tilde{x}}(\partial_{\tilde{x}}\phi_a)^2$$

- Aside from Luttinger parameter K_k, variance of the disorder $\tilde{\Delta}_{1,k}$, we'll use **classical (dimensionless) temperature** $\tilde{t}_k=K_k\tilde{T}_k$ to describe flow of the system.

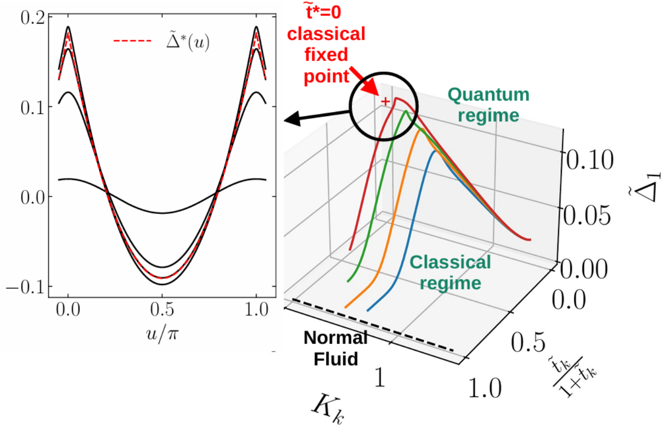
Results: Normal Fluid phase

- First disorder $\tilde{\Delta}_{1,k}$ is **relevant** and K_k **diminishes**.
- **Quantum-classical crossover** at $\tilde{T}_k \sim 1$.
- K_k saturates at large scales and coupling functions, i.e. $\tilde{\Delta}_{1,k}$, are strongly irrelevant: **Normal fluid phase (NF)**, where $\tilde{t}_k \to \infty$, **in agreement with perturbative computations**.
- As T is lowered, system spends more and more time close to **localized** $\tilde{t}^* = 0$ **fixed point** in the classical regime.



Results: Intermediate glassy physics

- This fixed point presents **cusps** at special values.
- These encode the presence of **many low-lying metastable states**.
  $\rightarrow$**Transient, glassy regime**.
- Depending on the initial temperature, this regime can be **purely classical (CGNF)**, or **first quantum and then classical (QGNF)**.
- This fixed point is also the one reached by system at $T = 0$.

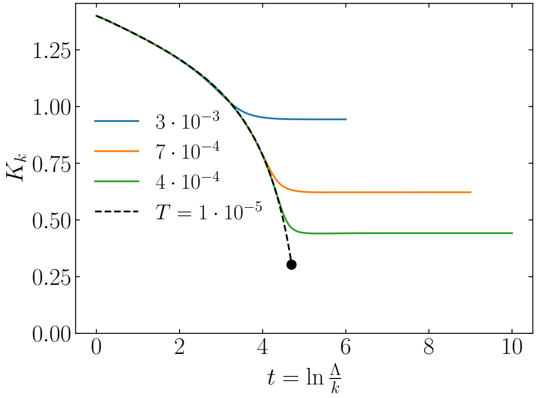


Results: Low-temperature singularities

- We now consider the effect of the purple term

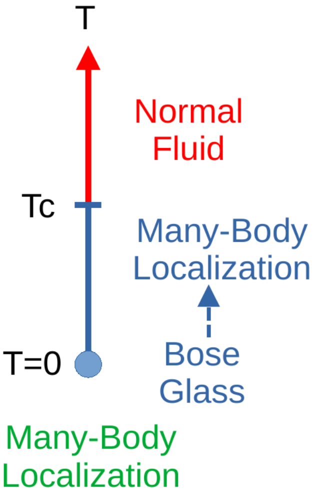
$$\Gamma_k[\{\phi_a\}] = \text{Previous terms} - \frac{1}{2}\sum_{a,b}\int_{x,\tau,\tau'}\frac{1}{2}W_{3,k}(\phi_a-\phi_b)\left((\partial_\tau\phi_a)^2+(\partial_{\tau'}\phi_b)^2\right)$$

- At high enough temperatures **phenomenology is identical**.
- For $T < T_c$ flow of K_k ends with a **singularity** at finite length scale $1/k_c$, with $K_c > 0$.
- Coupling functions either **deform abruptly** or **diverge** at the same point, pointing again at the **localized** nature of corresponding phase.



Many-Body Localization (MBL)

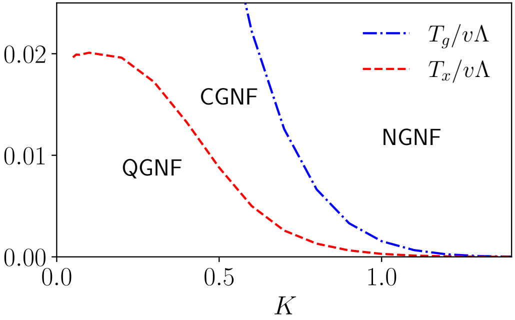
- Numerical stability of singularities and our inability to obtain meaningful results from other nonperturbative methods makes us conjecture they have a **physical meaning**.
- In MBL scenario strong enough disorder **impedes thermalization** [Michal, Aleiner, Altshuler, Shlyapnikov 2016].
- If this interpretation is correct, the singularities reflect the **breakdown of equilibrium Matsubara formalism**.
- Under this interpretation, the **second ansatz would realize the MBL scenario**.



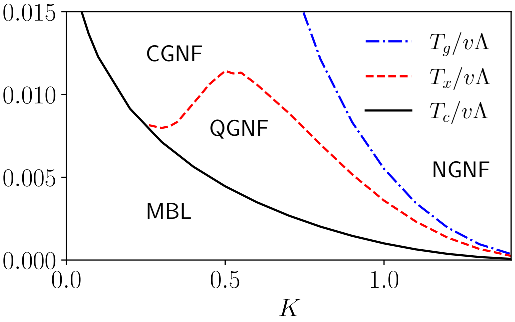
Results: Phase diagrams

- One can define **length scales** to measure the proximity of system to glassy fixed point, and occurrence of quantum-classical crossover.
- Comparing them yields **crossover temperatures** T_x and T_g separating the various regimes we introduced, and thus **phase diagrams** in $K - T$ plane at fixed dimensionless disorder.

Ansatz 1 : No phase transition



Ansatz 2 : Phase transition (?)



Conclusion

- At high enough temperatures the **perturbative picture is always valid**. A **zero-temperature localized fixed point** controls the behavior of the system on a wide range of length scales, and **several crossovers take place**.
- At lower temperatures, the most complete truncation of the effective action yields **robust singularities** in the flow equations.
- A possible explanation for the latter is to invoke the **MBL scenario for 1D bosons**, where strong enough **disorder prevents the system from thermalizing**.
- Further work is needed to assess the validity of this conjecture.

Thank you for your attention !