Chiral Lagrangian as a proxy for ASQG

ERG 2026,
September 1-5, 2026, University of Sussex

Francesco Del Porro
CoG & NBIA

THE CENTER OF GRAVITY

In collaboration with:
F. Ferrarin and A. Platania

Outline

Asymptotic Safety for chiral theory: why and how

Two physically interesting cases: SU(2) and SU(3)

Chiral Lagrangian vs higher derivative gravity

Conclusions

Asymptotic Safety for chiral theory: why and how

Asymptotic Safety for chiral theory: why and how

Two physically interesting cases: SU(2) and SU(3)

Chiral Lagrangian vs higher derivative gravity

Conclusions

Asymptotic Safety in a nutshell

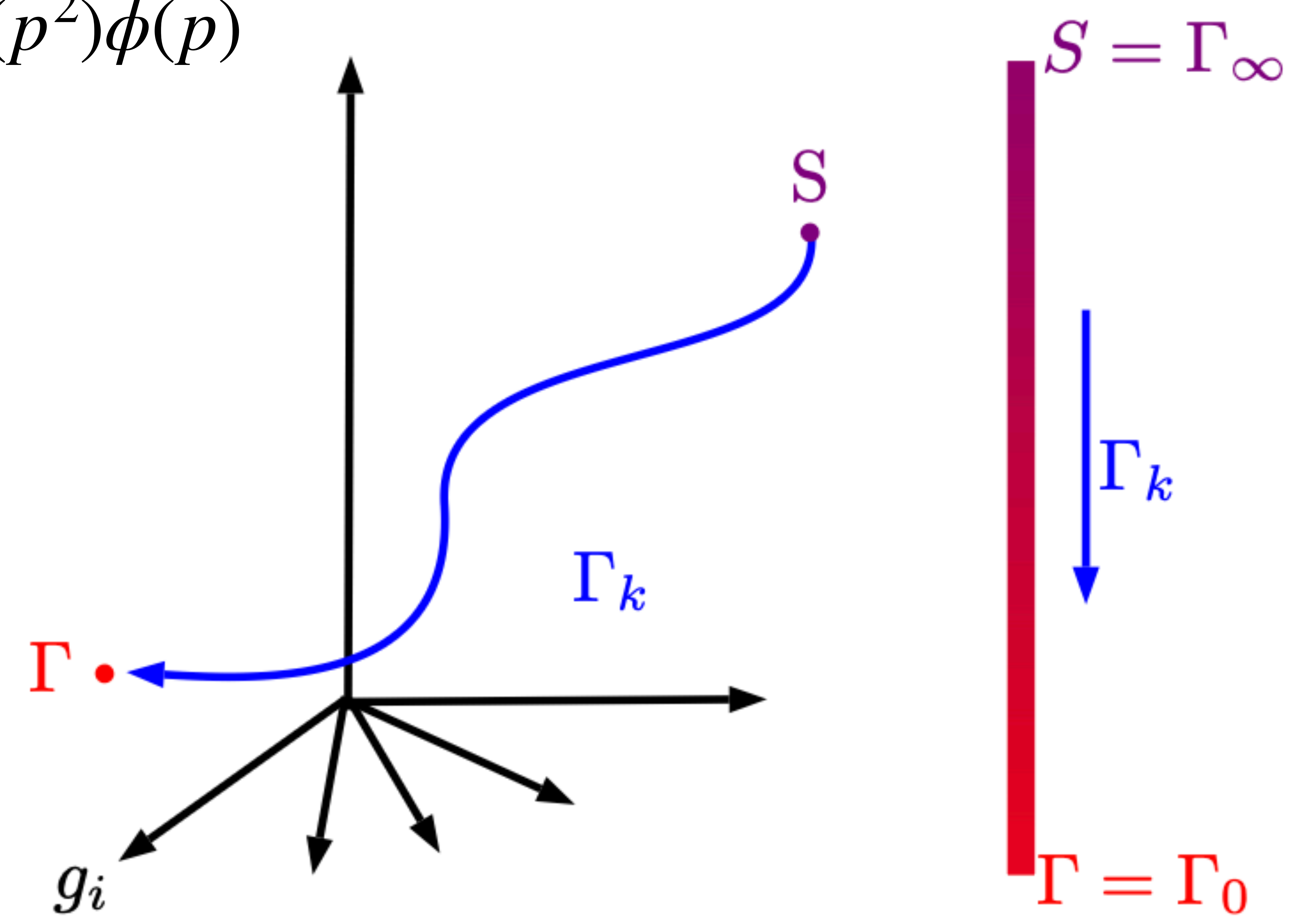
[S. Weinberg ’76]

$$Z_k[J] = \int D\phi \, e^{-S-\Delta S_k + \int J\phi} \qquad \Delta S_k = \frac{1}{2}\int \phi(-p)R_k(p^2)\phi(p)$$

$$\boxed{k\frac{d\Gamma_k[\phi]}{dk} = \frac{1}{2}\mathrm{Tr}\frac{k\partial_k R_k}{\Gamma_k^{(2)}[\phi] + R_k}}$$

$$\Gamma_k[\phi] = \sum_i g_i(k)\mathcal{O}_i[\phi] \qquad \beta_i(g_j) = \frac{dg_i(k)}{dk}$$

Fixed points are at $\{\beta_j(g_i) = 0\}$



[I. Basile+ **2412.08690**]

How generic is AS as a UV completion?

The presence of an UV fixed point has been computed in different truncations, signature, and field contents, such as:

Einstein-Hilbert truncation [M. Reuter '**96**, W. Souma '**99**], higher derivative gravity [N. Christiansen+ **1711.09259**, K. Falls+ **1801.00162**, B. Knorr **2104.11336**, Y. Kluth+ **2202.10436**, A. Baldazzi+ **2312.03831**], gravity-matter systems [P. Donà + **1311.2898**, A. Eichhorn+ **2212.07456**, J. Biemans + **1702.06539**, Á. Pastor-Gutiérrez + **2207.09817**], Lorentzian signature [E. Manrique+ **1102.5012**, J. Fehre+ **2111.13232**, E. D'Angelo+ **2202.07580**, J.M. Pawlowski+ **2507.22169**], non-gravitational systems [J. Braun + **1011.1456**, C. Cresswell-Hogg + **2603.19357**].

How generic is AS as a UV completion?

The presence of an UV fixed point has been computed in different truncations, signature, and field contents, such as:

Einstein-Hilbert truncation [M. Reuter '96, W. Souma '99], higher derivative gravity [N. Christiansen+ 1711.09259, K. Falls+ 1801.00162, B. Knorr 2104.11336, Y. Kluth+ 2202.10436, A. Baldazzi+ 2312.03831], gravity-matter systems [P. Donà + 1311.2898, A. Eichhorn+ 2212.07456, J. Biemans + 1702.06539, Á. Pastor-Gutiérrez + 2207.09817], Lorentzian signature [E. Manrique+ 1102.5012, J. Fehre+ 2111.13232, E. D'Angelo+ 2202.07580, J.M. Pawlowski+ 2507.22169], non-gravitational systems [J. Braun + 1011.1456, C. Cresswell-Hogg + 2603.19357].

In summary, asymptotic safety is by no means "exotic". Instead, interacting fixed points are very common in quantum field theories and numerous examples with interacting UV fixed points exist [A. Eichhorn 2606.21522].

Asymptotic Safety for chiral theories?

Chiral Lagrangians are effective theories for a spontaneously broken symmetry $G_L \times G_R \to G_V$, with G compact Lie group. If $U \in (G_L \times G_R)/G_V$, we have

$$S[U] = -\frac{1}{f^2}\int \mathrm{d}^4x \mathrm{Tr}[\partial_\mu U^\dagger \partial^\mu U] + \mathcal{O}(\partial^4)\,.$$

- The theory is (perturbatively) non-renormalizable.
- For $G = SU(N)$ and $N = 2,3$ it describes the effective dynamics of mesons in QCD.
- It shows indications of an interacting UV fixed point

[A. Codello+ **0810.0715**, R. Percacci+ **0910.0851**]

Asymptotic Safety for chiral theories?

Chiral Lagrangians can be described as non-linear sigma models,
writing $U = \exp\left[i\varphi^a T_a\right]$

$$S[\varphi] = \frac{1}{2f^2}\int \mathrm{d}^4x\, h_{ab}(\varphi)\partial_\mu\varphi^a\partial^\mu\varphi^b + \mathcal{O}(\partial^4)\,.$$

- The UV fixed point exists nonperturbatively at the leading truncation order in derivatives, and at one-loop for the next-to-leading ∂^4 .

[A. Codello+ **0810.0715**, R. Percacci+ **0910.0851**]

- Structurally, it is very similar to the gravitational Lagrangian as fields enter through $g^{-1}\partial g$.

Q: Can the NLSM be asymptotically safe?

Two physically interesting cases

Asymptotic Safety for chiral theory: why and how

Two physically interesting cases: SU(2) and SU(3)

Chiral Lagrangian vs higher derivative gravity

Conclusions

Two physically interesting cases: SU(2)

The structure of the NLO terms depends on the target manifold.

[S. Weinberg '99, R. Percacci+ 0910.0851].

For $G = SU(2) \simeq S^3$ (massless pions):

$$[T_a, T_b] = i\epsilon_{abc}T_c$$

$$R_{abcd} = \frac{R}{6}\left(h_{ac}h_{bd} - h_{ad}h_{bc}\right) \qquad R_{ab} = \frac{R}{3}h_{ab}$$

$$\mathscr{L}_{\mathrm{NLO}}[\varphi] = -\frac{1}{2f^2}\left(\frac{\ell_1}{2}(h_{ab}\partial_\mu\varphi^a\partial^\mu\varphi^b)^2 + \frac{\ell_2}{2}h_{ab}h_{cd}\partial_\mu\varphi^a\partial_\nu\varphi^b\partial^\mu\varphi^c\partial^\nu\varphi^d\right)$$

Beta functions

- The limit $\ell_1 = \ell_2 = 0$ recovers the LO fixed point [A. Codello+ 0810.0715]
- The system has a Gaussian fixed point, but no interacting fixed point!
- The absence of an UV fixed point is consistent with previous works [R. Flore+ 1207.4499]. Here without the operator $h_{ab} \Box \varphi^a \Box \varphi^b$

$$\beta_f = \frac{(64 + 170\ell_1 + 65\ell_2) f^3 - 1536 f \pi^2}{((16 + 34\ell_1 + 13\ell_2) f^2 - 1536\pi^2)}$$

$$\begin{aligned}
\beta_{\ell_1} = \frac{1}{7680\pi^2} \cdot \frac{1}{(1536\pi^2 - (16 + 34\ell_1 + 13\ell_2) f^2)} \\
\times \Big[7480\ell_1^3 f^4 + 12\ell_1^2 \Big((1040 + 703\ell_2) f^4 - 358400 f^2 \pi^2 \Big) \\
+ 2\ell_1 \Big(-23040(32 + 49\ell_2) f^2 \pi^2 + 11796480\pi^4 + f^4(-6560 + 5024\ell_2 + 1457\ell_2^2 - 87040\pi^2) \Big) \\
+ f^2 \Big(299\ell_2^3 f^2 + 48\ell_2^2 (37 f^2 - 3680\pi^2) - 81920\pi^2 (27 + f^2 - 96\pi^2) \\
- 160\ell_2 \big(3072\pi^2 + f^2(25 + 416\pi^2) \big) \Big) \Big]
\end{aligned}$$

$$\begin{aligned}
\beta_{\ell_2} = \frac{1}{3840\pi^2} \cdot \frac{1}{(1536\pi^2 - (16 + 34\ell_1 + 13\ell_2) f^2)} \\
\times \Big[136\ell_1^3 f^4 + 377\ell_2^3 f^4 + 20480 f^2 \pi^2 (-9 + 2 f^2 - 192\pi^2) \\
+ 8\ell_2^2 (311 f^4 - 90240 f^2 \pi^2) + 4\ell_1^2 ((712 + 183\ell_2) f^4 - 7680 f^2 \pi^2) \\
+ 40\ell_2 \Big(9216 f^2 \pi^2 + 294912\pi^4 + f^4(-33 + 832\pi^2) \Big) \\
+ 2\ell_1 \Big(-7680(-32 + 95\ell_2) f^2 \pi^2 + f^4(-840 + 2880\ell_2 + 623\ell_2^2 + 43520\pi^2) \Big) \Big]
\end{aligned}$$

Two physically interesting cases: SU(3)

For $G = SU(3)$ (massless mesons):

$$[T_a, T_b] = i f_{abc} T_c \qquad R_{abcd} = \frac{1}{4} f_{abe} f^{e}{}_{cd} \qquad R_{ab} = \frac{R}{8} h_{ab}$$

$$\mathscr{L}_{\rm NLO}[\varphi] = -\frac{1}{2f^2}\Bigg(\frac{\ell_1}{2}(h_{ab}\partial_\mu\varphi^a\partial^\mu\varphi^b)^2 + \frac{\ell_2}{2} h_{ab}h_{cd}\partial_\mu\varphi^a\partial_\nu\varphi^b\partial^\mu\varphi^c\partial^\nu\varphi^d + \frac{\ell_3}{2} R_{acbd}\partial_\mu\varphi^a\partial_\nu\varphi^b\partial^\mu\varphi^c\partial_\nu\varphi^d\Bigg)$$

The beta-function presents a similar structure as for $SU(2)$. No interacting fixed point found at this truncation order.

Chiral theory vs higher derivative gravity

Asymptotic Safety for chiral theory: why and how

Two physically interesting cases: SU(2) and SU(3)

Chiral Lagrangian vs higher derivative gravity

Conclusions

Chiral theory vs higher derivative gravity

The structure of the NLSM terms with four derivatives can be compared with the known case of higher derivative gravity

$$S_Q = \frac{1}{2G}\int d^4x\sqrt{g}\,\left(2\Lambda - R - L_1 R^2 + L_2 C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma}\right)\,.$$

- Higher derivative gravity has been analyzed nonperturbatively and shows an interacting UV fixed point. [D. Benedetti+ **0902.4630**]

- The main difference sits in Λ. Setting $\Lambda = 0$ in the beta-functions destroys the interacting fixed point. However, a volume term cannot be ignored in the flow equations [G. De Brito+ **2012.08904**]

- What about the comparison with Unimodular gravity?

Conclusions

Asymptotic Safety for chiral theory: why and how

Two physically interesting cases: SU(2) and SU(3)

Chiral Lagrangian vs higher derivative gravity

Conclusions

Conclusions

- Asymptotically safe UV completions are common, both in gravitational and non-gravitational systems. However, they are not a generic feature of every field theory

- The case of chiral Lagrangians is an instructive example. Although a their structure is very similar to gravity theories, the UV fixed point disappears at the four-derivative level

- This suggests that gravity may possess a special structure that enables an asymptotically safe UV completion. Further study of this parallelism is needed.

Conclusions

- Asymptotically safe UV completions are common, both in gravitational and non-gravitational systems. However, they are not a generic feature of every field theory

- The case of chiral Lagrangians is an instructive example. Although a their structure is very similar to gravity theories, the UV fixed point disappears at the four-derivative level

- This suggests that gravity may possess a special structure that enables an asymptotically safe UV completion. Further study of this parallelism is needed.

Thank you!