Functional renormalization group for extremely correlated electrons

Andreas Rückriegel

in collaboration with Jonas Arnold and Peter Kopietz

Institut für Theoretische Physik, Universität Frankfurt

ERG 2026, University of Sussex,

Brighton, September 4, 2026

GOETHE UNIVERSITÄT FRANKFURT AM MAIN

- Jonas Arnold, Peter Kopietz, and Andreas Rückriegel
  Phys. Rev. Lett. **137**, 096506 (2026)
- Jonas Arnold, Peter Kopietz, and Andreas Rückriegel
  Phys. Rev. B **114**, 055123 (2026)

Hubbard model at $U \to \infty$

- Hubbard model:

$$\mathcal{H} = \sum_{ij} \sum_{\sigma} t_{ij} c_{i\sigma}^{\dagger} c_{j\sigma} - \mu \sum_{i} \sum_{\sigma} n_{i\sigma} + U \sum_{i} n_{i\uparrow} n_{i\downarrow}.$$

- For on-site repulsion $U \gg |t|, |\mu|$: double occupancy forbidden,

$$\mathcal{H} \to \sum_{ij} \sum_{\sigma} t_{ij} h_{i\sigma}^{\dagger} h_{j\sigma} - \mu \sum_{i} \sum_{\sigma} h_{i\sigma}^{\dagger} h_{i\sigma}, \text{ for } U \to \infty.$$

$\Rightarrow$ Reduced Hilbert space, not perturbatively connected to weakly interacting Fermi liquid at small U.

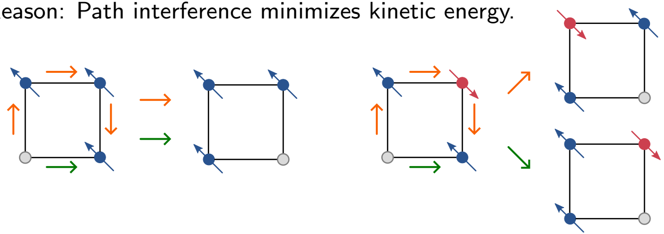
  - $h_{i\sigma}$ = projected Fermi operator ("holon creation operator").
  - Kinematic interactions via non-canonical anticommutation relations:

$$\left\{ h_{i\sigma}, h_{j\sigma'}^{\dagger} \right\} = \delta_{ij} \left[\delta_{\sigma\sigma'} \left(1 - h_{i\bar{\sigma}}^{\dagger} h_{i\bar{\sigma}} \right) + \delta_{\sigma\bar{\sigma}'} h_{i\bar{\sigma}}^{\dagger} h_{i\sigma} \right].$$

- Physics of t and t-J model believed to be relevant for high-T_C cuprates (pseudogap, Fermi arcs, ...), moiré materials, optical lattice experiments ...

What is known?

- The half-filled Hubbard model on square lattice has a unique ground state with $S = 0$ for all $U > 0$. [Lieb, 1989]
- But: Ground state with a single hole is ferromagnetic. [Nagaoka theorem, 1966]
  - Reason: Path interference minimizes kinetic energy.



- Variational wave functions, ED, VQMC, DMRG, DMFT, . . . :
  - Ferromagnetism may survive up to moderate doping.
  - Luttinger theorem may be broken.
  - Incoherent regime at large density.
  - ED on small clusters [Chiappe, PRB 1993]: Ferromagnetic ground state at large density has negligible overlap with $U = 0$ ground state.

Strong-coupling functional renormalization group (X-FRG)

X-FRG approach: Continuously deform hopping

$$t_{\boldsymbol{k}} \to t_{\Lambda,\boldsymbol{k}}, \quad \Lambda \in [0,1],$$

such that

1. $t_{\Lambda=0,\boldsymbol{k}} = 0$ (isolated Hubbard atoms),
2. $t_{\Lambda=1,\boldsymbol{k}} = t_{\boldsymbol{k}}$ (full interacting system).

$\Rightarrow$ Exact flow equation for the Λ-dependent generating functional of imaginary-time-ordered holon (or X-operator) correlation functions.

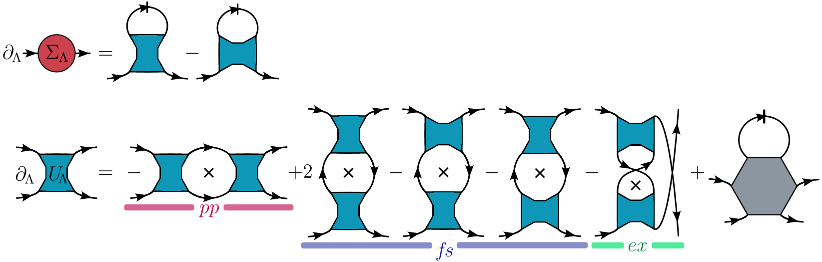
- Hilbert space projection taken into account exactly by Hubbard atom initial condition.
  - $\Rightarrow$ Non-trivial initial conditions with quantum dynamics.
- Standard fermionic FRG flow equations for hopping effects.
- FRG flow can be interpreted as flow of physical temperature T.

[**AR**, Arnold, Krämer, Kopietz, Phys. Rev. B **108**, 115104 (2023)]

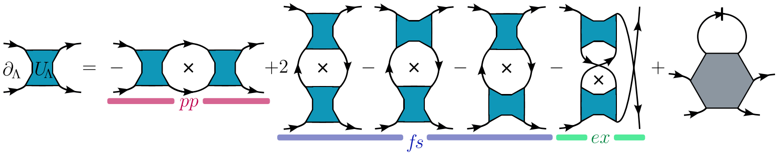
- Holon propagator in the t model:

$$G_\Lambda(\boldsymbol{k},\omega) = \frac{Z}{i\omega + \mu_0 - Z\left(t_{\Lambda,\boldsymbol{k}} - 2\delta\mu_\Lambda/3\right) - Z\Sigma_\Lambda(\boldsymbol{k},\omega)},$$

- Hubbard atom quasi-particle residue at $U \to \infty$: $Z = 1 - n/2$.
- $\delta\mu_\Lambda$ = chemical potential counter term; chosen such that $\partial_\Lambda n = 0$.
- Holon self-energy $\Sigma_\Lambda(\boldsymbol{k},\omega)$: beyond tree-level effects of hopping; generated by effective 2-body interaction vertex U_Λ:



Channel decomposition



- Channel decomposition: Split U_Λ into superconducting (S_Λ), magnetic (M_Λ), and charge (C_Λ) contributions:

$$U_\Lambda(K_1', K_2'; K_2, K_1) = V(\omega_2, \omega_1) - S_\Lambda(Q_{\mathrm{pp}}; \omega_2', \omega_1) + M_\Lambda(Q_{\mathrm{ex}}; \omega_2, \omega_1) + \frac{1}{2}\Big[M_\Lambda(Q_{\mathrm{fs}}; \omega_1, \omega_2) - C_\Lambda(Q_{\mathrm{fs}}; \omega_2, \omega_1)\Big].$$

- Additional frequency dependence necessary because of non-trivial Hubbard atom initial condition (Hilbert space projection):

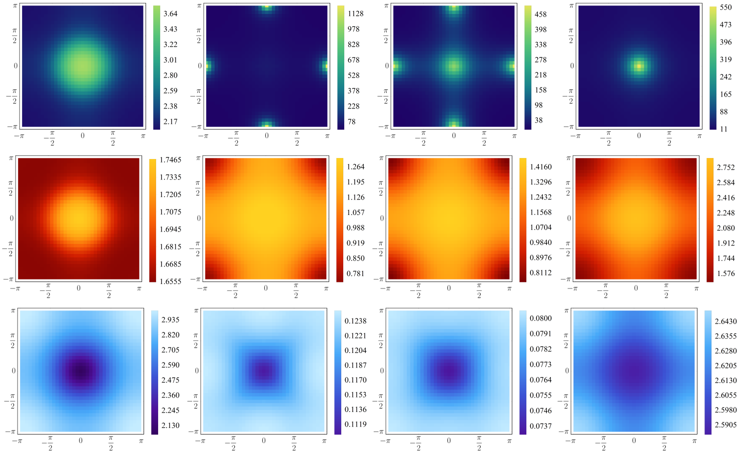
$$Z^2 V(\omega_2, \omega_1) = -G_0^{-1}(\omega_2) - G_0^{-1}(\omega_1), \quad S_0 = 0,$$
$$Z^2 M_0(\Omega; \omega_2, \omega_1) = \beta \delta_{\Omega,0} \frac{n}{2} G_0^{-1}(\omega_2) G_0^{-1}(\omega_1),$$
$$Z^2 C_0(\Omega; \omega_2, \omega_1) = \beta \delta_{\Omega,0} \frac{n}{2} (1 - n) G_0^{-1}(\omega_2) G_0^{-1}(\omega_1),$$

- If only most singular contributions kept: $\partial_\Lambda U_\Lambda$ analytically solvable.

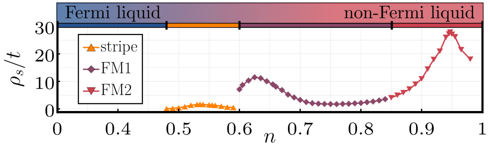
Channels & magnetic instabilities

$n = 0.2$	$n = 0.56$	$n = 0.59$	$n = 0.85$
$T = 0.14t$	$T = 0.13t$	$T = 0.16t$	$T = 0.65t$

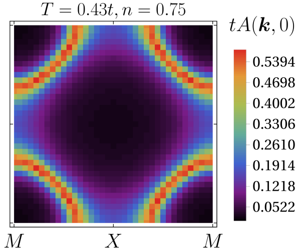
- Magnetic:
- Charge:
- Super-conducting:

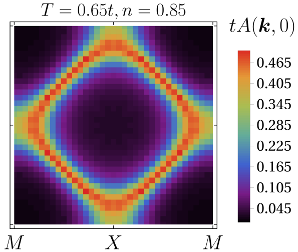


Phase diagram

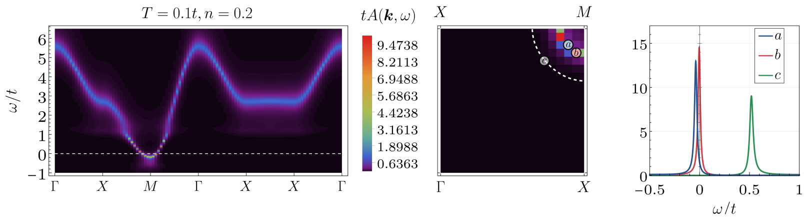


- Spin stiffness ρ_s shows two distinct ferromagnetic regimes (FM1 & FM2).
- Electronic spectrum: Lifshitz transition of the Fermi surface.
- Literature: FM phase boundaries group around $n \approx 0.6$ or $n \approx 0.85$, depending on method used.
  - Possibly partially polarized ground state in between.



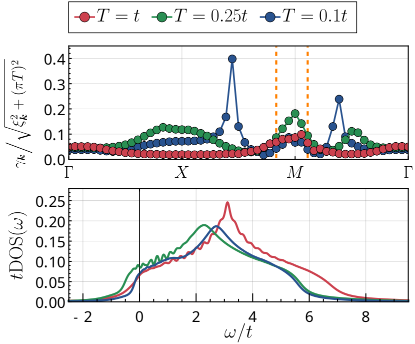


Low densities: extremely correlated Fermi liquid

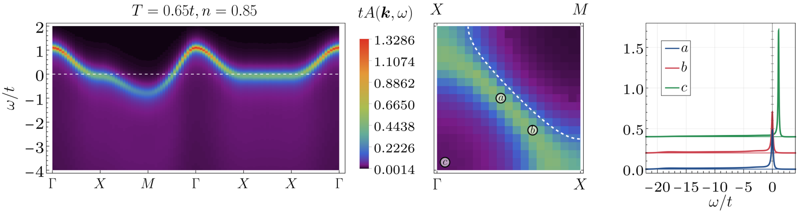


- Well-defined band, washed out away from Fermi level.
- Sharp modes at Fermi surface $\Rightarrow$ Fermi liquid.
- Luttinger's theorem violated.
- Slight ph-asymmetry.

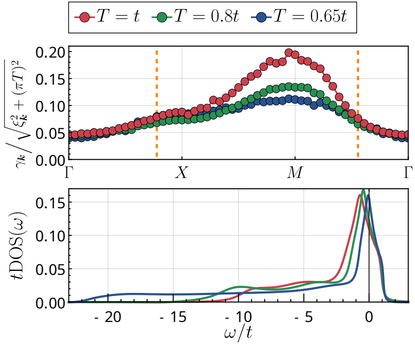
$\Rightarrow$ Shastry's extremely correlated Fermi liquid. [Shastry, 2010]



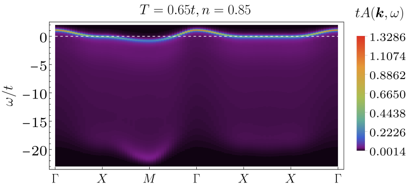
High densities: flat-band bad metal

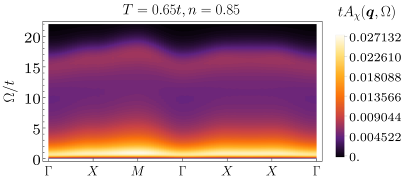


- Fermi energy close to van-Hove singularity for ferromagnetism.
- Narrow, incoherent band & large hole continuum.
- Fermi surface no longer singled out $\Rightarrow$ non-Fermi liquid.
- Damping $\gamma_{\boldsymbol{k}} \propto T$: bad metal.

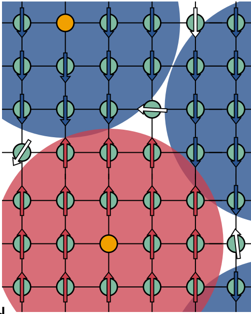


High densities: flat-band bad metal





- Narrow band with large hole continuum:
  - ⇒ Fingerprint of Nagaoka polarons.
  - For single hole: conjectured by Brinkman and Rice. [Brinkman, 1970]
- Reflected in magnetic spectrum $A_\chi(\boldsymbol{q}, \Omega)$:
  - No well-defined single-particle excitations.
  - Large polaron continuum.
- Polaron size limited by magnetic correlation length.
  - ⇒ Long-range hole dynamics, $\boldsymbol{k}$-dependence suppressed.



Conclusions & Outlook

- Strong-coupling functional renormalization group (X-FRG):
  - Can deal with non-canonical operators & projected Hilbert spaces.
  - Complementary to FRG for canonical fermions.
  - Unbiased approach to strongly correlated electrons.
- Outlook:
  - Different lattice geometries, next-nearest neighbor hopping, ...
  - Extension to t-J and full Hubbard models straightforward.

Further reading:

- Jonas Arnold, Peter Kopietz, and Andreas Rückriegel
  Phys. Rev. Lett. **137**, 096506 (2026)
- Jonas Arnold, Peter Kopietz, and Andreas Rückriegel
  Phys. Rev. B **114**, 055123 (2026)