Scalar diquarks in the QCD vacuum

Ugo Mire

based on [Hosein Gholami, UM, Fabian Rennecke, Bernd-Jochen Schaefer, Shi Yin ; *2606.23772*]

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Diquarks & color superconductivity

Central object: (scalar) diquark

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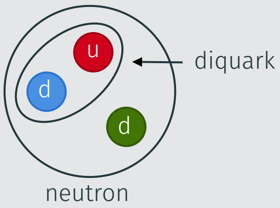
nucleon ($3q$) is well described in a quark-diquark picture (Δ + q)



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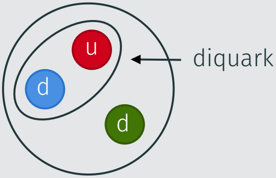
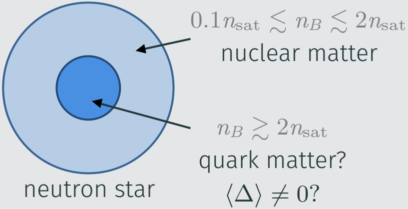


Astrophysics

Dense quark matter $\rightarrow$ diquark condensation $\langle\Delta\rangle \neq 0 \rightarrow$ color superconductivity

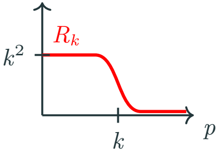


The functional renormalization group

Main idea: successively integrate out fluctuations in the spirit of Wilson's RG.

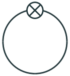
$$Z_k[J] = \int \mathcal{D}\Phi\, e^{-S[\Phi]-\Delta S_k[\Phi]-\int J\hat{\Phi}}$$

with $\quad \Delta S_k[\Phi] = \int_p \Phi(p) R_k(p) \Phi(-p)$



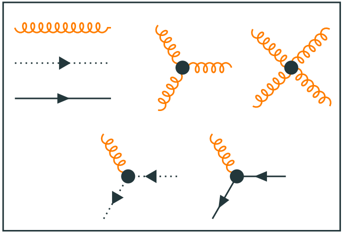
Formulated on the effective action: Γ_k

integrate from $k = \Lambda$ to $k = 0$ with

$$\partial_t \Gamma_k[\Phi] = \frac{1}{2} \mathrm{STr}\left[\left(\Gamma_k^{(2)}[\Phi] + R_k\right)^{-1} \partial_t R_k\right] = \frac{1}{2}$$ 

General strategy for QCD

quark-gluon sector



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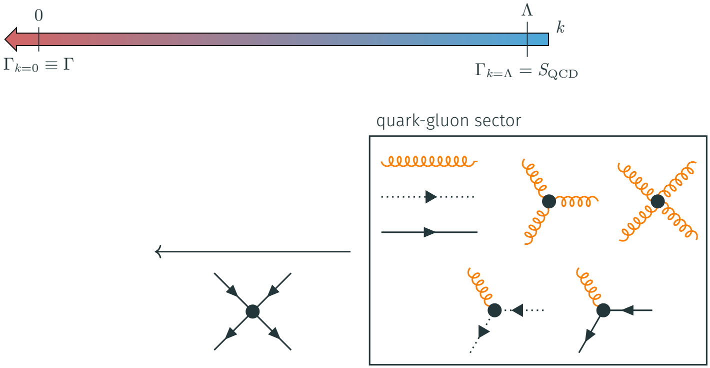

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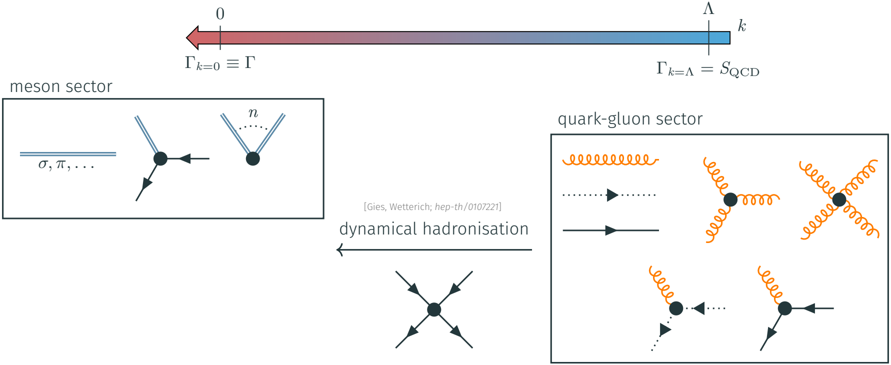

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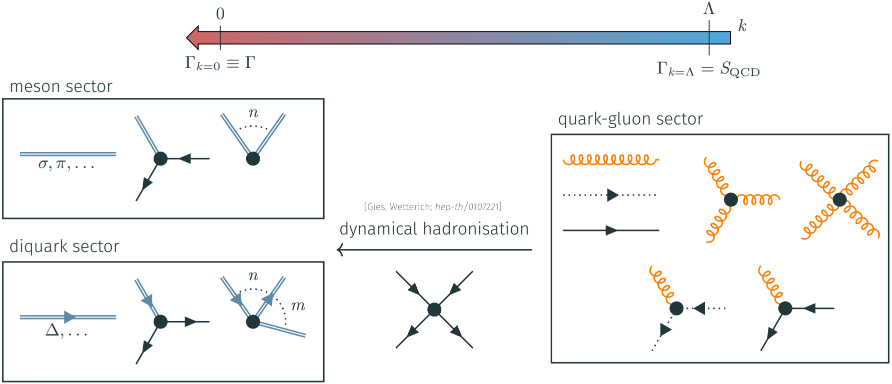

- $\langle \sigma \rangle \neq 0 \rightarrow$ dynamical chiral symmetry breaking
- continuously generate low-energy description of QCD $\rightarrow$ link to quark-meson-diquark model

Self-consistent QCD truncation

$$\Gamma_k[\Phi] = \Gamma_{\mathrm{glue},k}[A, \bar{c}, c] + \Gamma_{\mathrm{quark\text{-}gluon},k}[A, \bar{q}, q] + \Gamma_{\mathrm{qmd},k}[A, \bar{q}, q, \phi, \Delta^\dagger, \Delta] \ ,$$

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$\uparrow$

gauge sector
Landau gauge
only classical tensor
structure

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8 tensor structure

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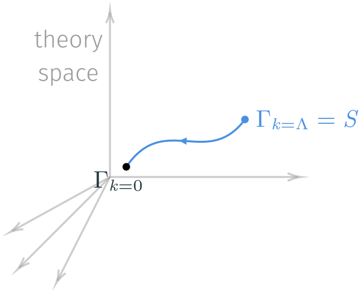
Key points

- Two free UV parameter: strong coupling + bare quark mass
- Self-consistent: all quantities computed from the same truncation
- Zero-momentum expansion: numerically inexpensive

Gauge sector

[Gies; *hep-ph/0611146*]

FRG for gauge theories:



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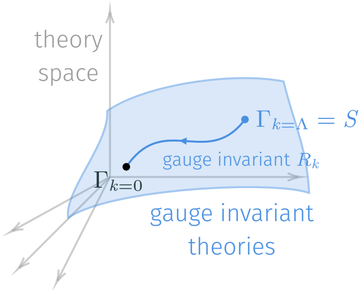

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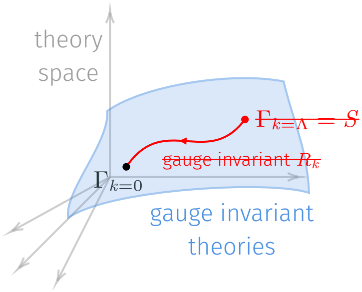

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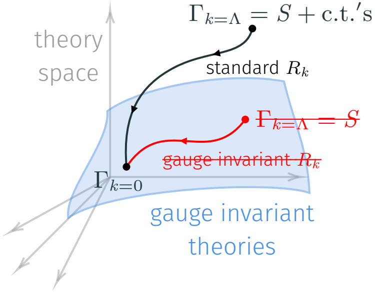

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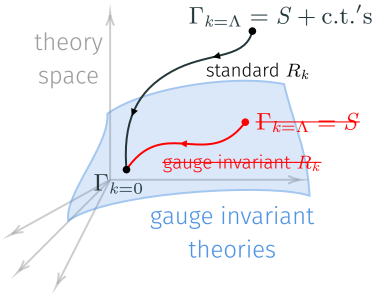

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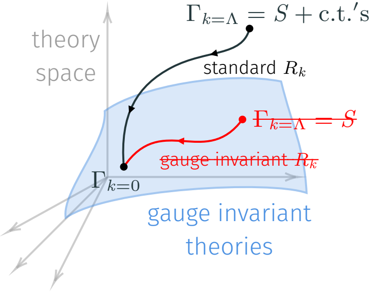
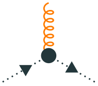
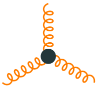
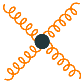


[Goertz, Pastor-Gutiérrez, Pawlowski; *2412.12254*]

$$\Gamma_{\text{glue},k}[A,\bar{c},c] = \frac{1}{2}\int_p A^a_\mu(p)\left[Z_{A,k}\left(p^2 + m^2_{\text{gap},k}\right)\Pi^{\text{T}}_{\mu\nu}(p) + \frac{1}{\xi}p^2\Pi^{\text{L}}_{\mu\nu}(p)\right]A^a_\nu(-p)$$

$+ Z_{c,k}$  $^{-1} + \lambda_{A\bar{c}c,k}$  $+ \lambda_{A^3,k}$  $+ \lambda_{A^4,k}$ 

$\uparrow$

ghosts – needed for gauge fixing

How to fix the UV gluon mass?

General idea

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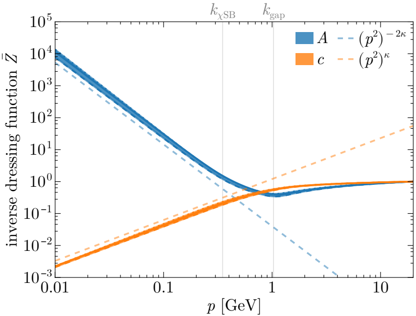
Proxy: tune to the scaling solution

$$\lim_{p\to 0} \bar{Z}_A \propto (p^2)^{-2\kappa} \qquad \lim_{p\to 0} \bar{Z}_c \propto (p^2)^{\kappa}$$

$\rightarrow$ uniquely fix UV mass gap



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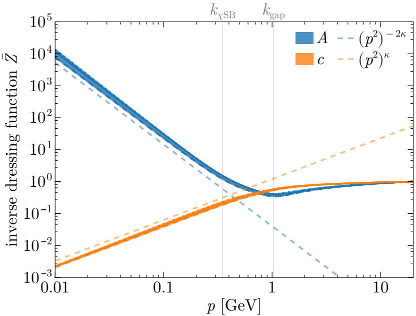
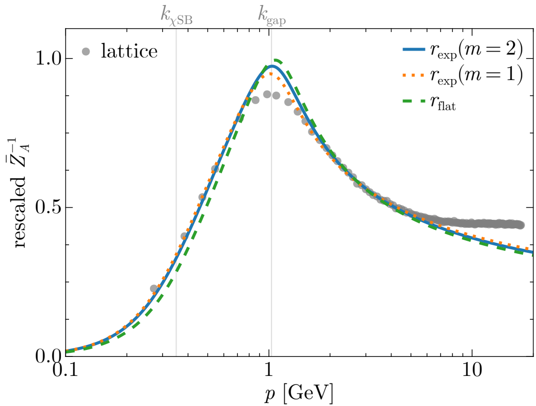
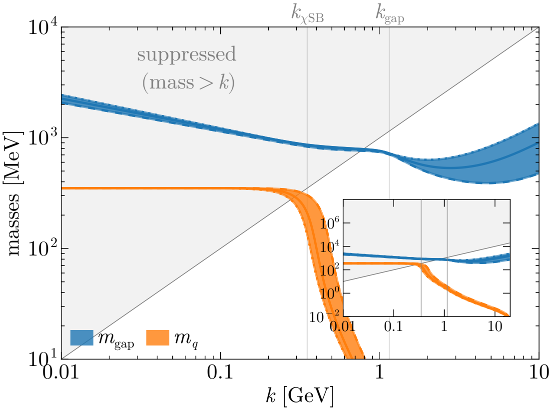


Scaling exponent: $\kappa = 0.63$–0.66 (compare to $\kappa \approx 0.595$ [Lerche, von Smekal; *hep-ph/0202194*])



$$G_{AA}(p) = \frac{1}{\bar{Z}_A(p)p^2} \qquad \text{and} \qquad \bar{Z}_A(p) = Z_{A,k=p}\Big(1 + \bar{m}^2_{\mathrm{gap},k=p}\Big)$$

Masses & successive decoupling



(band: minimal error estimate from regulator variation)

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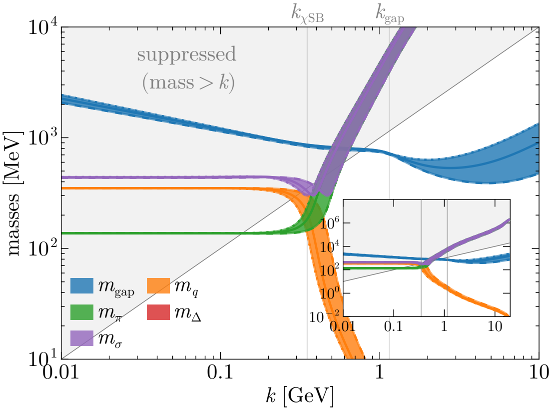

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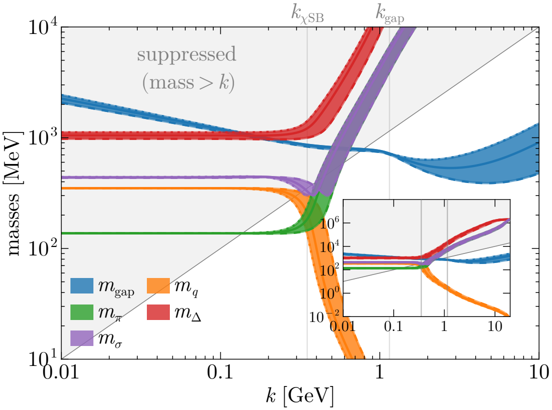

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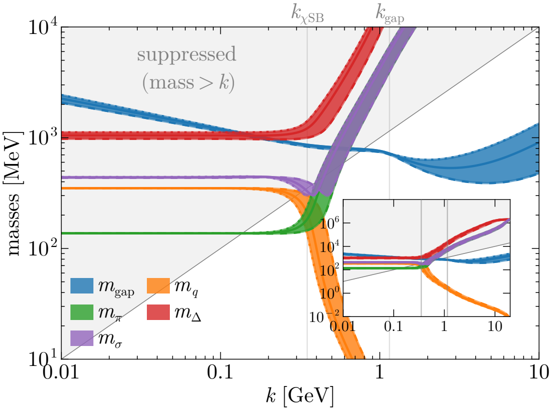

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Problem? Diquark curvature mass $m_\Delta \sim 1\,\mathrm{GeV} > 2m_q \sim 700\,\mathrm{MeV}$

Diquark Pole Mass

Curvature mass

$$G_\Delta^{-1}(p_0 = 0, \vec{p} = 0) = Z_\Delta m_{\Delta,\mathrm{curv}}^2$$

- easily accessible
- equal to pole mass for free theory

Pole mass

$$G_\Delta^{-1}(p_0 = i m_{\Delta,\mathrm{pole}}, \vec{p} = 0) = 0$$

- “physical” mass in experiment
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what we'd like to know!

Problem: we need information at **finite real-time momenta**

Solution: analytically continued FRG [Tripolt, Strodthoff, von Smekal, Wambach; *1311.0630*]

Analytically continued FRG

- Additional flow equation

$$\partial_t \Gamma^{(2),E}_{\Delta^\dagger\Delta}(p) = \eta_\Delta \Gamma^{(2),E}_{\Delta^\dagger\Delta}(p) + \mathrm{Flow}^{(2)}_{\Delta^\dagger\Delta}(p)$$

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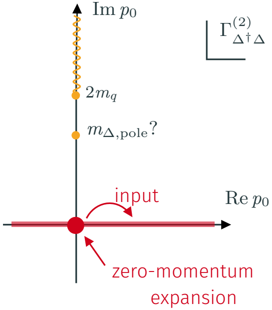

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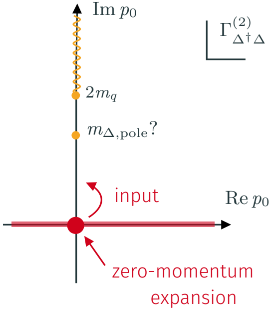

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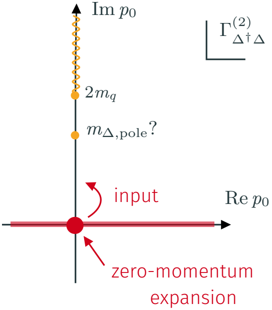
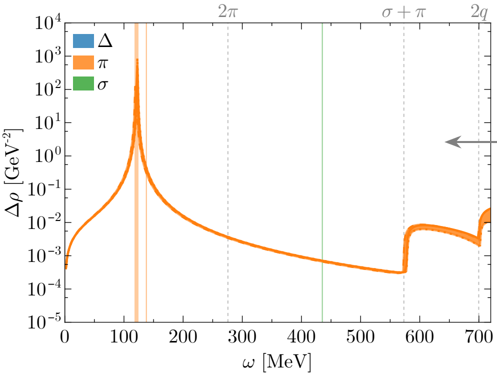
- Spectral function

$$\rho_\Delta(\omega) = -\frac{1}{\pi}\mathrm{Im}\left[\left(\Gamma^{(2),R}_{\Delta^\dagger\Delta}(\omega)\right)^{-1}\right]$$



Diquark Pole Mass



diquark single particle state

spectral function

(band: minimal error estimate from regulator variation)

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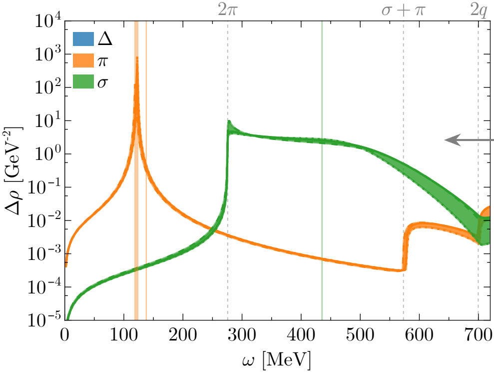

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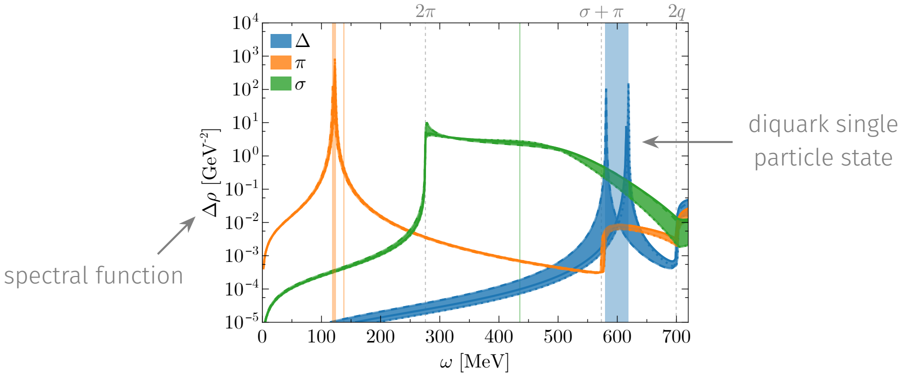

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Summary:

- pole mass: $m_{\Delta,\mathrm{pole}} \sim 0.6\,\mathrm{GeV} < 2m_q \to$ diquark bound state (consistent with lattice / DSE computations)

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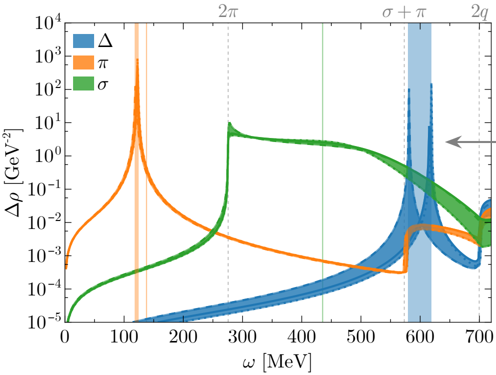

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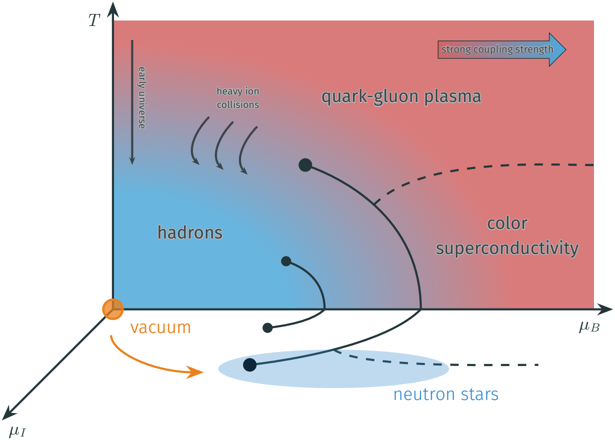
- curvature mass: $m_{\Delta,\mathrm{curv}} \sim 1\,\mathrm{GeV} \gg m_{\Delta,\mathrm{pole}} \sim 0.6\,\mathrm{GeV}$

Outlook: QCD constrained low-energy models I

see Bernd-Jochen Schaefer's talk (Thursday 10am)



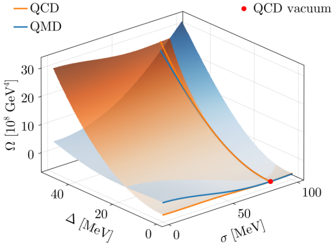
How to use these results for neutron star physics?

Outlook: QCD constrained low-energy models II

see Bernd-Jochen Schaefer's talk (Thursday 10am)

Correlations at vanishing momenta $\rightarrow$ can be fitted by a low-energy model:

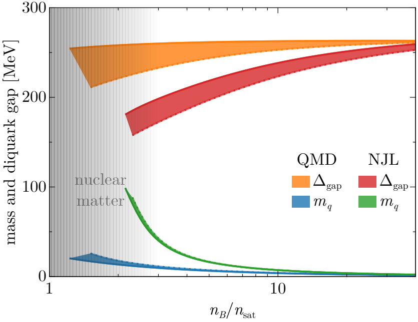
QCD (all momenta vanishing)		Low-energy model
$\langle \pi \bar{q} q \rangle \sim 4.3$	$\longrightarrow$	$\langle \pi \bar{q} q \rangle$
$\langle \Delta \bar{q} \bar{q} \rangle \sim 9.1$	$\longrightarrow$	$\langle \Delta \bar{q} \bar{q} \rangle$
$\langle \sigma\sigma \rangle \sim (438.1\,\mathrm{MeV})^2$	$\longrightarrow$	$\langle \sigma\sigma \rangle$
$\langle \Delta\Delta \rangle \sim (1049.2\,\mathrm{MeV})^2$	$\longrightarrow$	$\langle \Delta\Delta \rangle$
$\langle \sigma\sigma\Delta\Delta \rangle \sim 29.4$	$\longrightarrow$	$\langle \sigma\sigma\Delta\Delta \rangle$
$\dots$	$\longrightarrow$	$\dots$

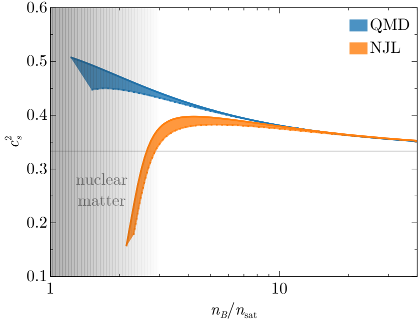


Outlook: QCD constrained low-energy models III

see Bernd-Jochen Schaefer's talk (Thursday 10am)

Parameter-free predictions for neutron star matter



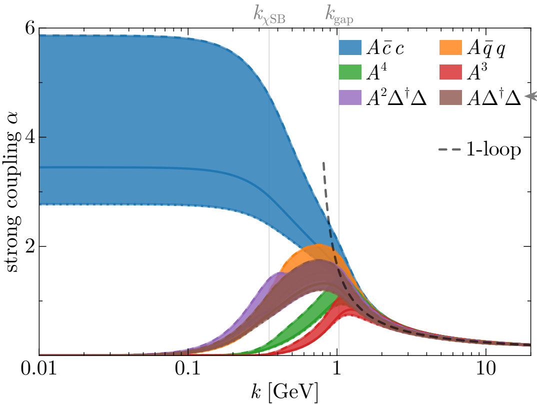


↑
high diquark gap $\Delta_{\text{gap}} \gtrsim 240\,\text{MeV}$

↑
speed of sound above the conformal limit → agree with Bayesian studies

Backup Slides

Flow of the strong coupling avatars

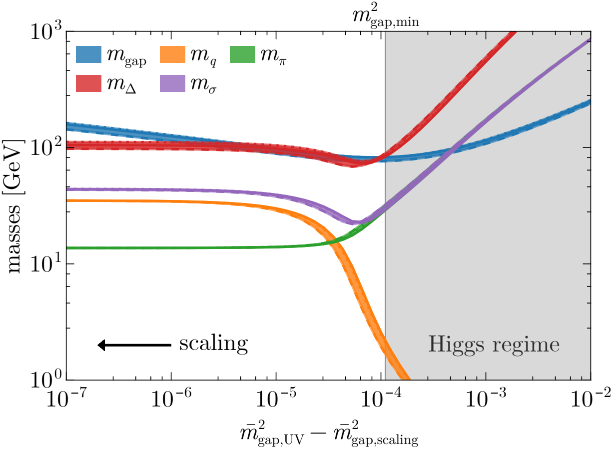


diquark-gluon strong couplings (diquark carry color charge)

(band: minimal error estimate from regulator variation)

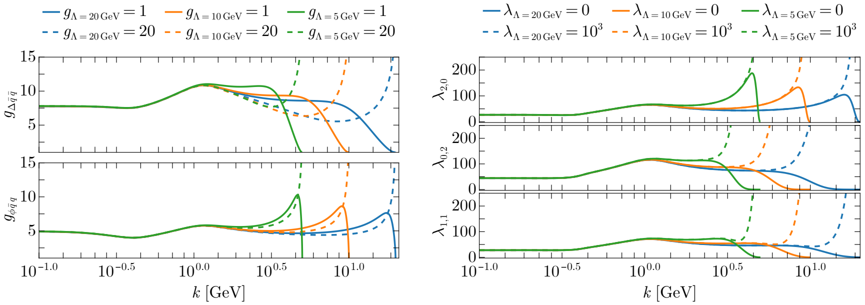
$$\alpha = \frac{g_s^2}{4\pi}$$

Invariance on initial conditions



- The results are invariant on the precise choice of $m^2_{\mathrm{gap,UV}} \rightarrow$ scaling and decoupling solutions

Invariance on initial conditions



- The results are independent on the UV value of the composite correlators