Pion Distribution Amplitude from Functional QCD

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fqcd-collaboration.github.io

IsoQuant SFB1225

fQCD collaboration:
Braun, Chen, Fu, Gao, Geissel, Huang, Lu, Ihssen, Pawlowski, Rennecke, Sattler, Schallmo, Stoll, Tan, Töpfel, Turnwald, Wessely, Wen, Wink, Yin, Zheng, Zorbach

Based on:
Wei-jie Fu, CH, Jan M. Pawlowski, Yang-yang Tan, Li-jun Zhou, '*Four-quark scatterings in QCD III*', Phys.Rev.D 112 (2025) 5, 054047;
Dao-yu Zhang, CH, Wei-jie Fu, '*Quasiparton distributions of pions at large longitudinal momentum*', Phys.Rev.D 112 (2025) 7, 074001;
Lei Chang, Wei-jie Fu, CH, Jan M. Pawlowski, Yang-yang Tan, '*Pion Distribution Amplitudes from Functional QCD*', arXiv: 2607.04871;

Outline

- Introduction
- First principle Functional QCD
- Pion distribution amplitudes
- Summary and outlook

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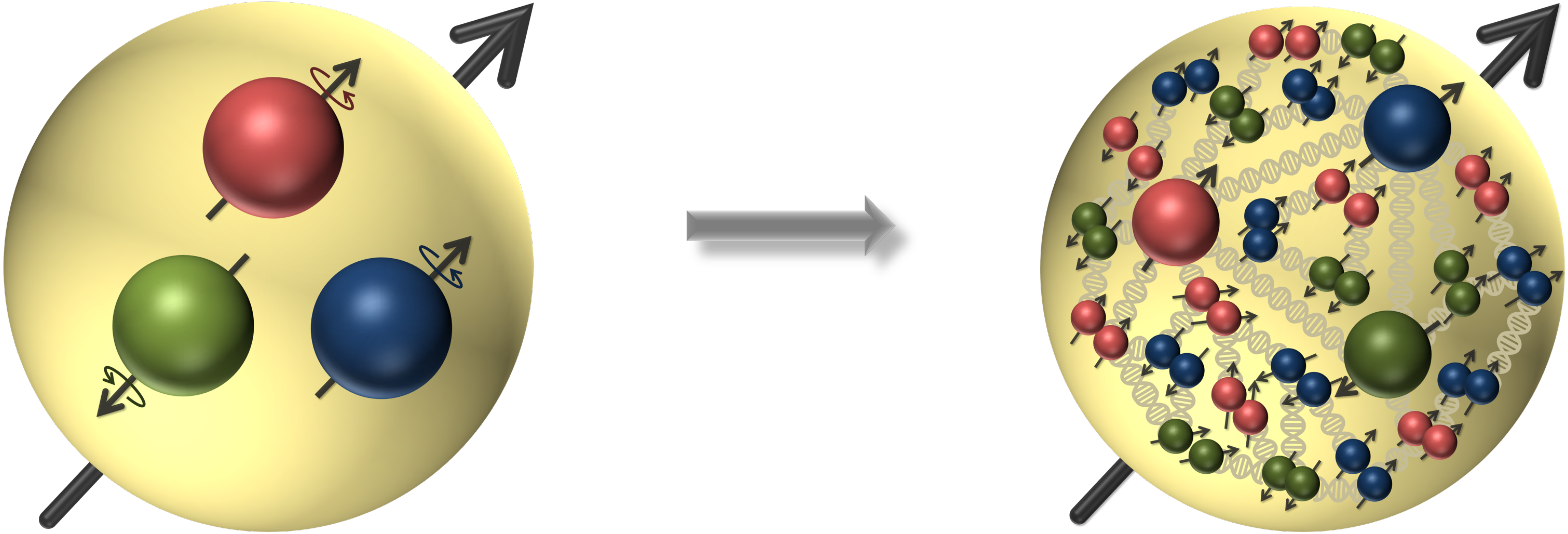
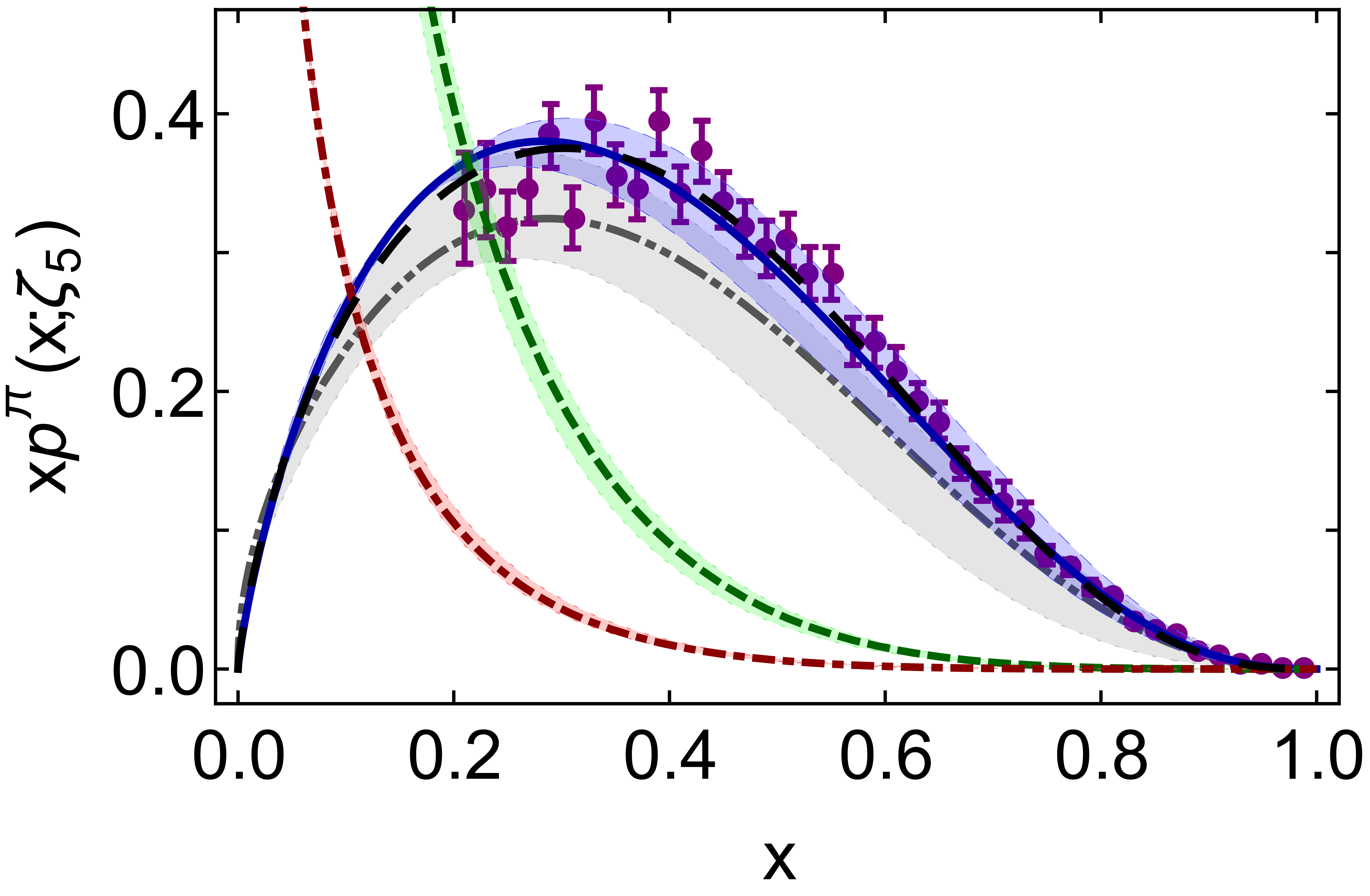
The physical picture of hadron structure



D. P. Anderle, V. Bertone, X. Cao et al., *Front.Phys.*(Beijing) 16 (2021) 6, 64701

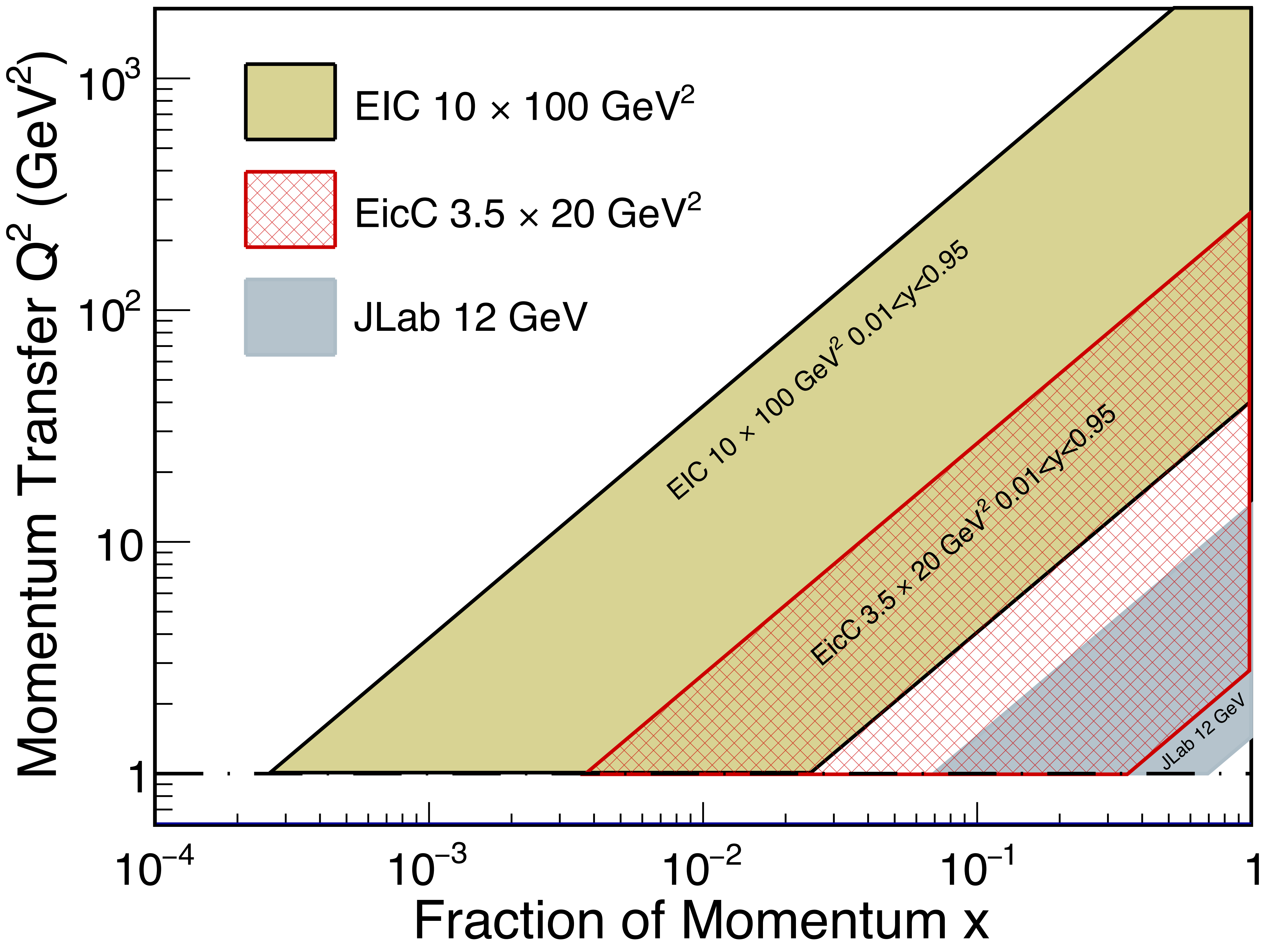
Prediction of hadron structure

Distribution function of pion



M. Ding, K. Raya, X. Cao et al., *Phys.Rev.D* 101 (2020) 5, 054014

Ongoing and proposed colliders



D. P. Anderle et al., *Front.Phys.*(Beijing) 16 (2021) 6, 64701

- Our goal: to understand and predict the hadron structure from first-principles functional QCD

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- Introduction
- First principle functional QCD
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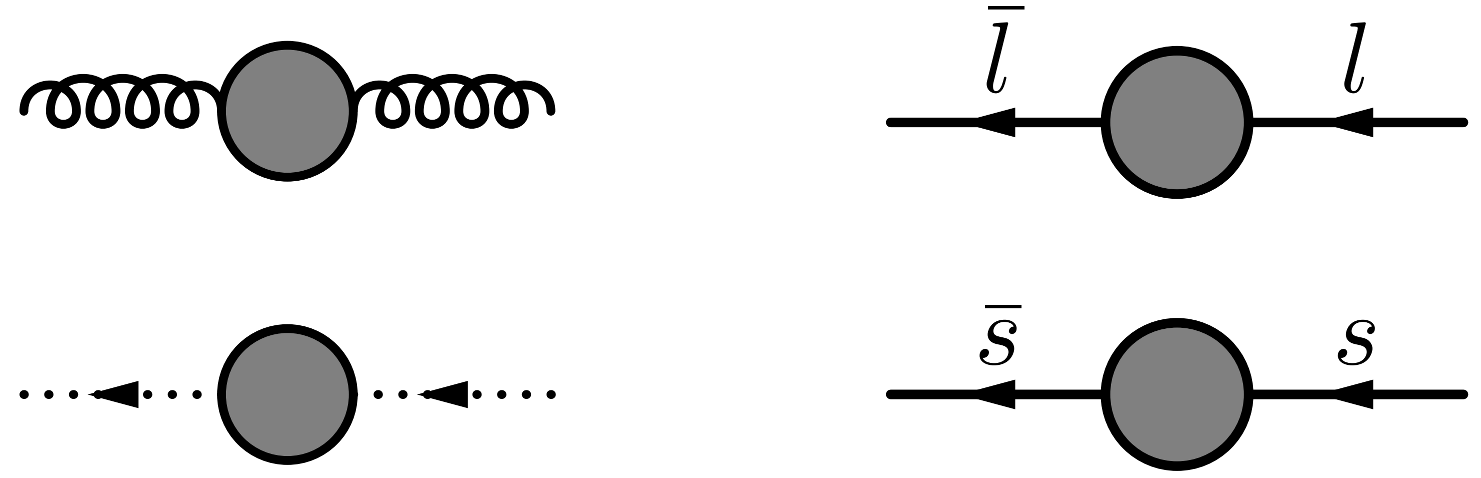
First principle functional QCD

Observables and Parameters

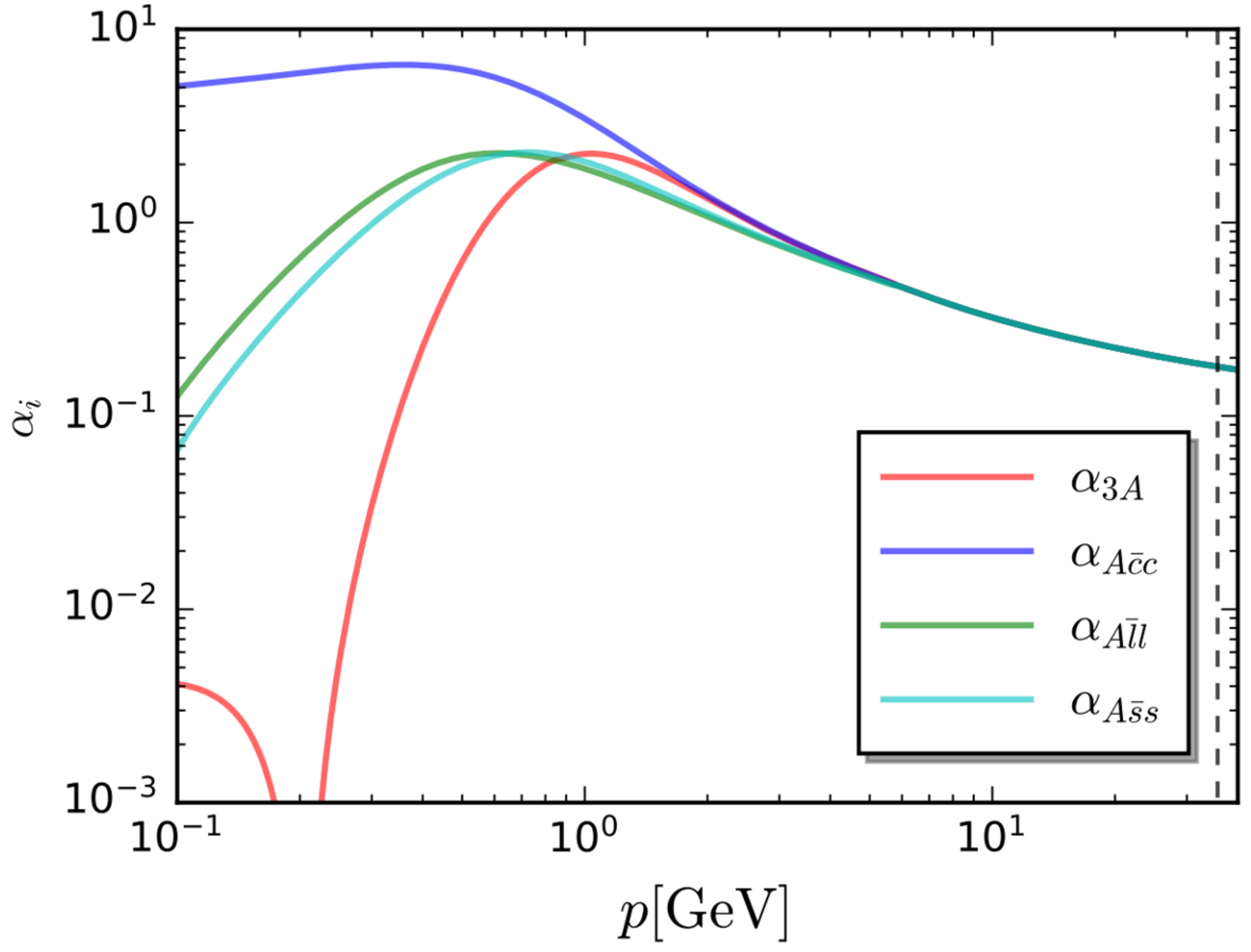
Observables	Value	Parameters	Value
m_π/f_π	137.0/93.0	m_l	2.1 MeV
m_K/f_π	494.0/93.0	m_s	55.9 MeV
M_l	344.5 MeV		
M_s	487.3 MeV		
m_σ	515.2 MeV		
f_K	114.1 MeV		

Fu, CH, Pawlowski, Tan, Zhou, PRD 112 (2025), 054047

Correlation functions

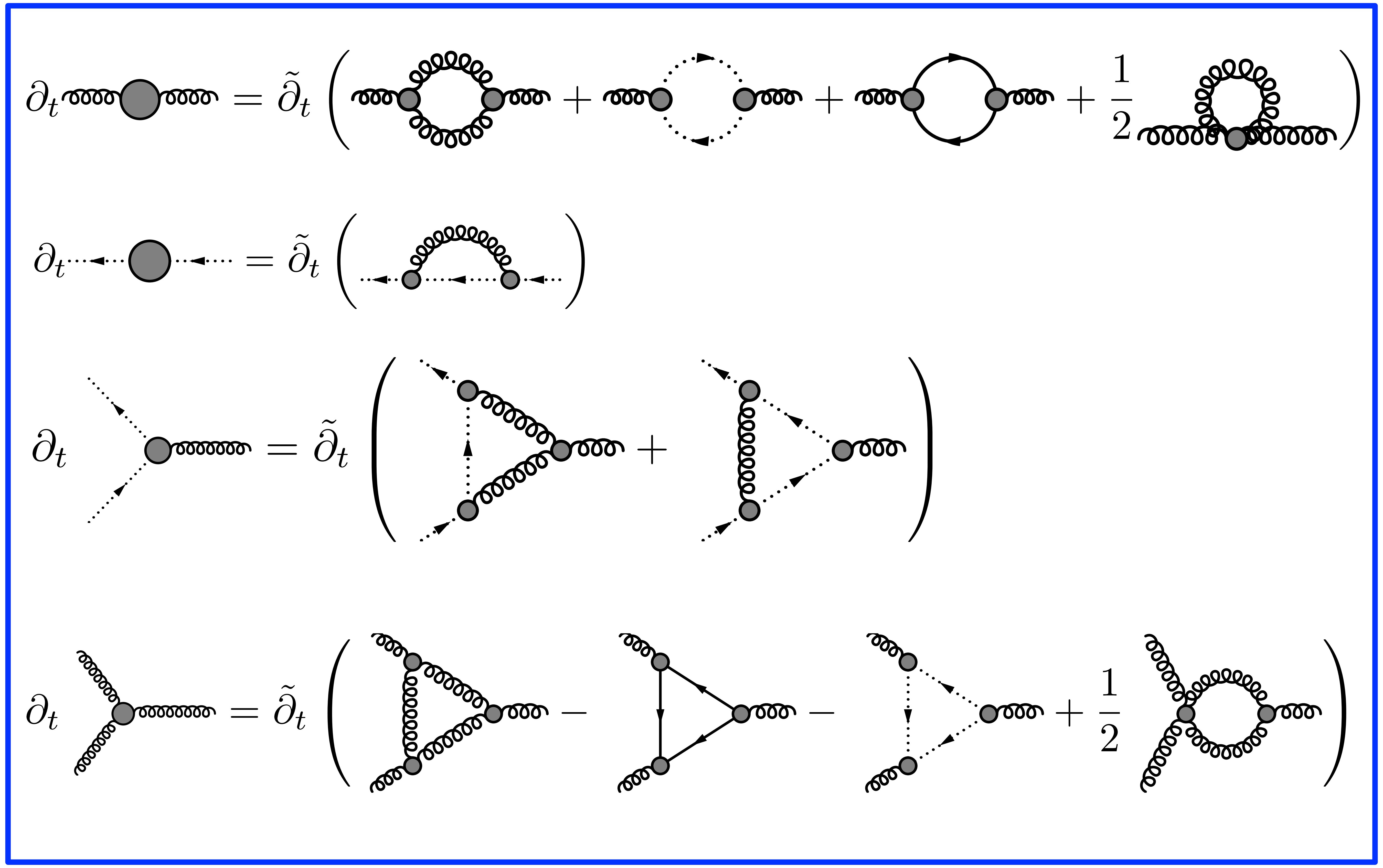


Strong couplings

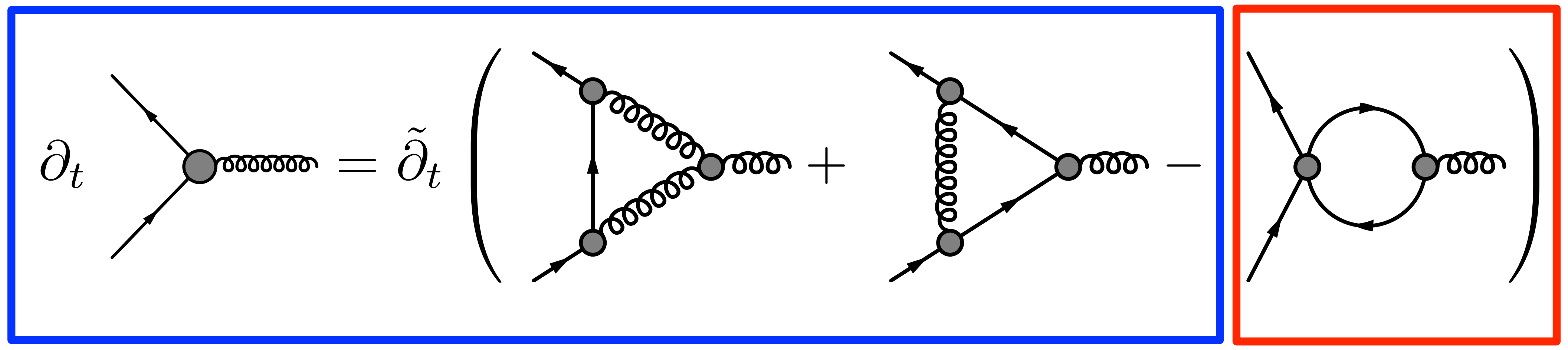


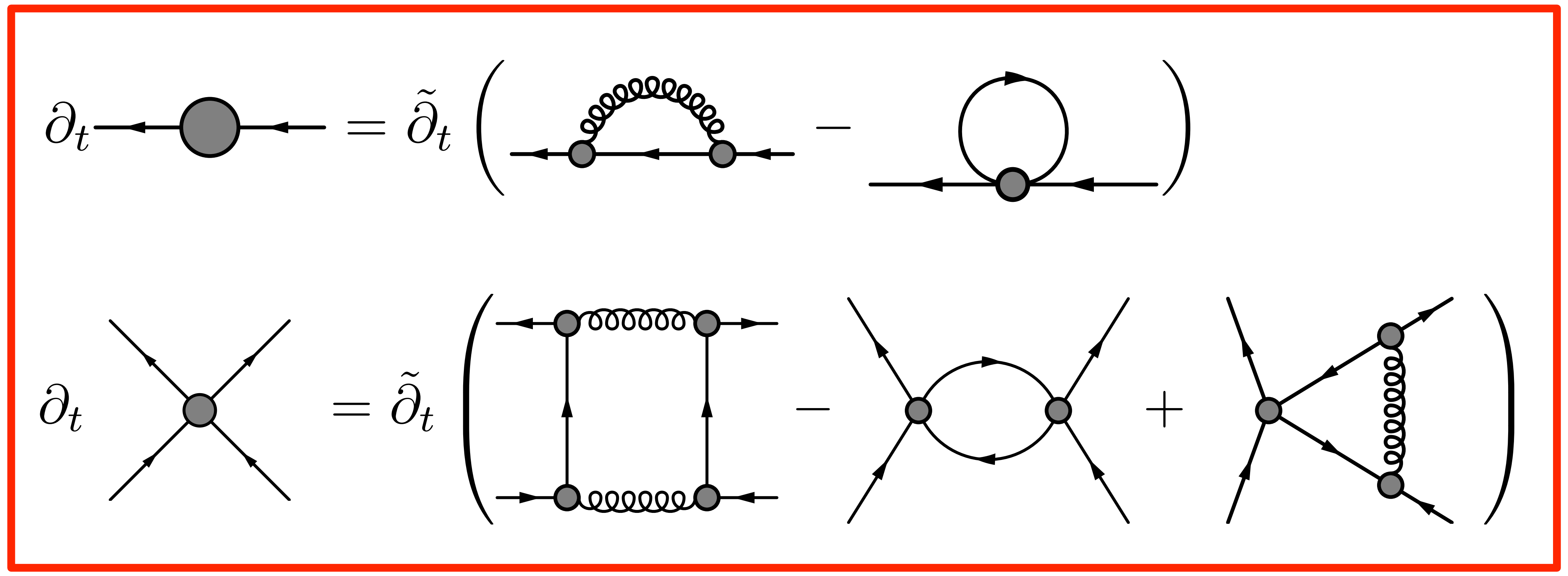
QCD flows with four-quark scattering

Glue sector



Matter sector

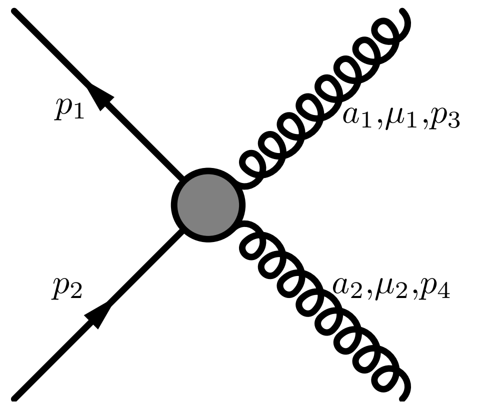




Symmetry point truncation

$$\lambda^{(n)}(p_1, p_2, \cdots, p_n) \approx \lambda^{(n)}(\bar{p}) \quad \bar{p} = \sqrt{1/n \sum_i^n p_i^2}$$

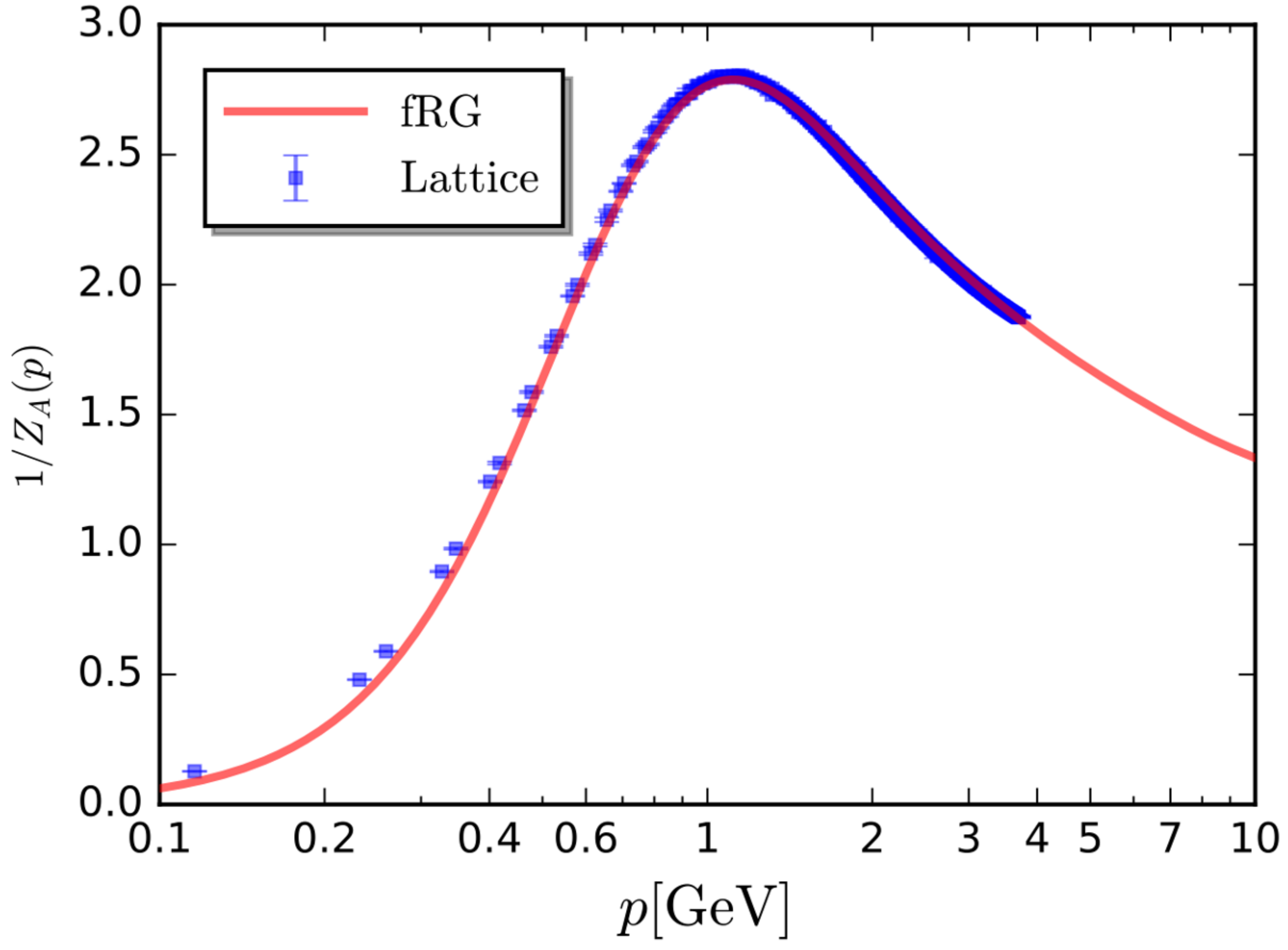
s,t,u-channel truncation

+ 

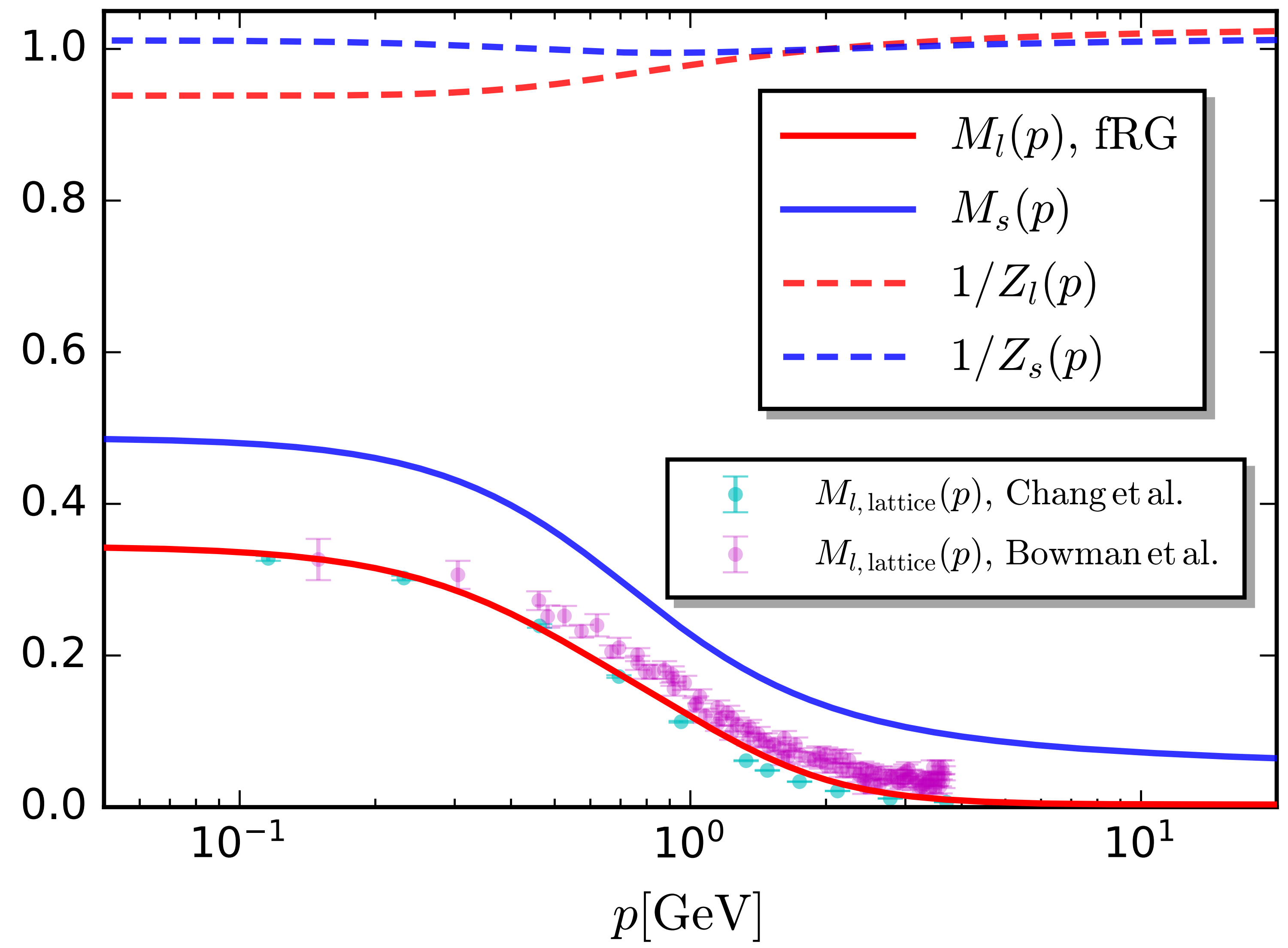
terms

QCD correlation functions

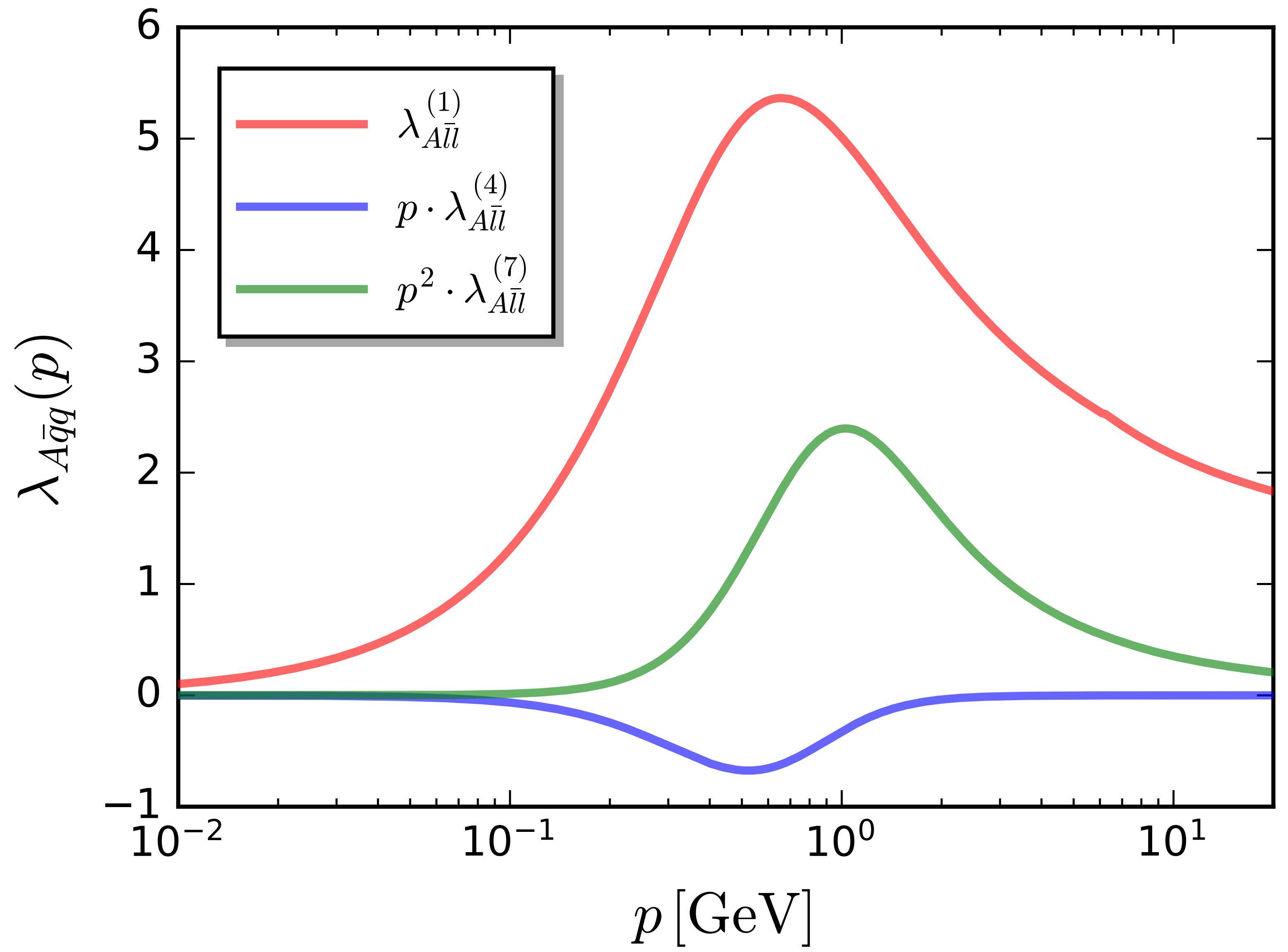
Gluon propagators



Quark propagators



Quark-gluon vertex

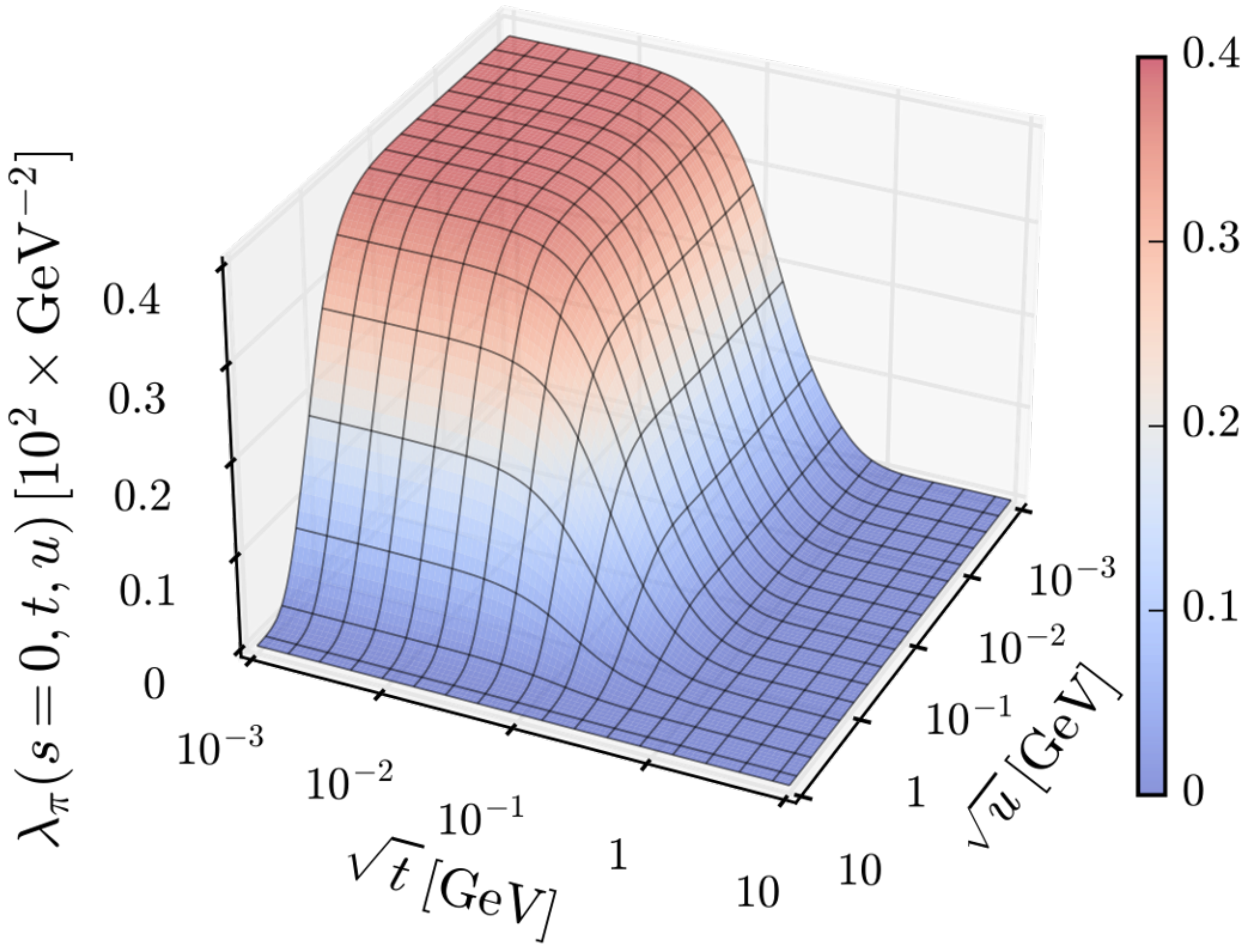


Lattice: Boucaud *et al.*, *PRD* 98 (2018) 114515, Chang *et al.*, *PRD* 104 (2021) 9, 094509, Bowman *et al.*, *PRD* 71 (2005) 054507

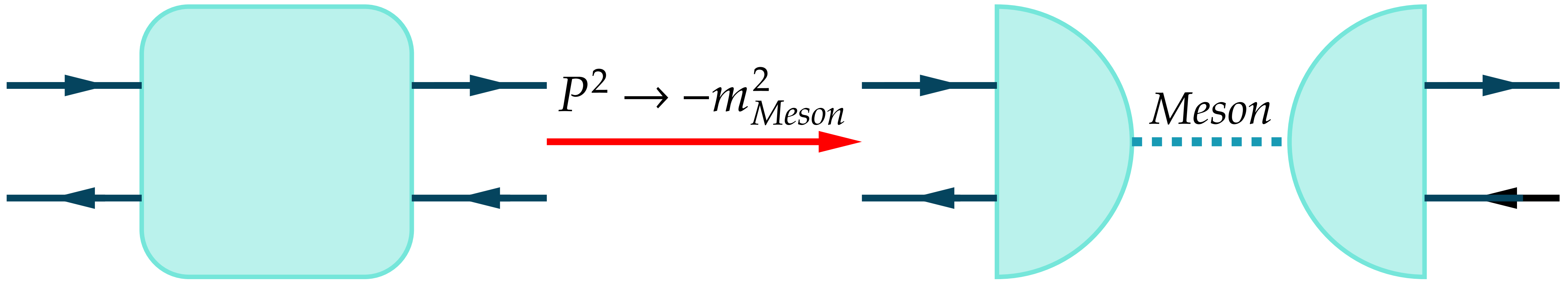
fRG: Fu, CH, Pawlowski, Tan, Zhou, *PRD* 112 (2025), 054047

Meson emergence from four-quark interaction

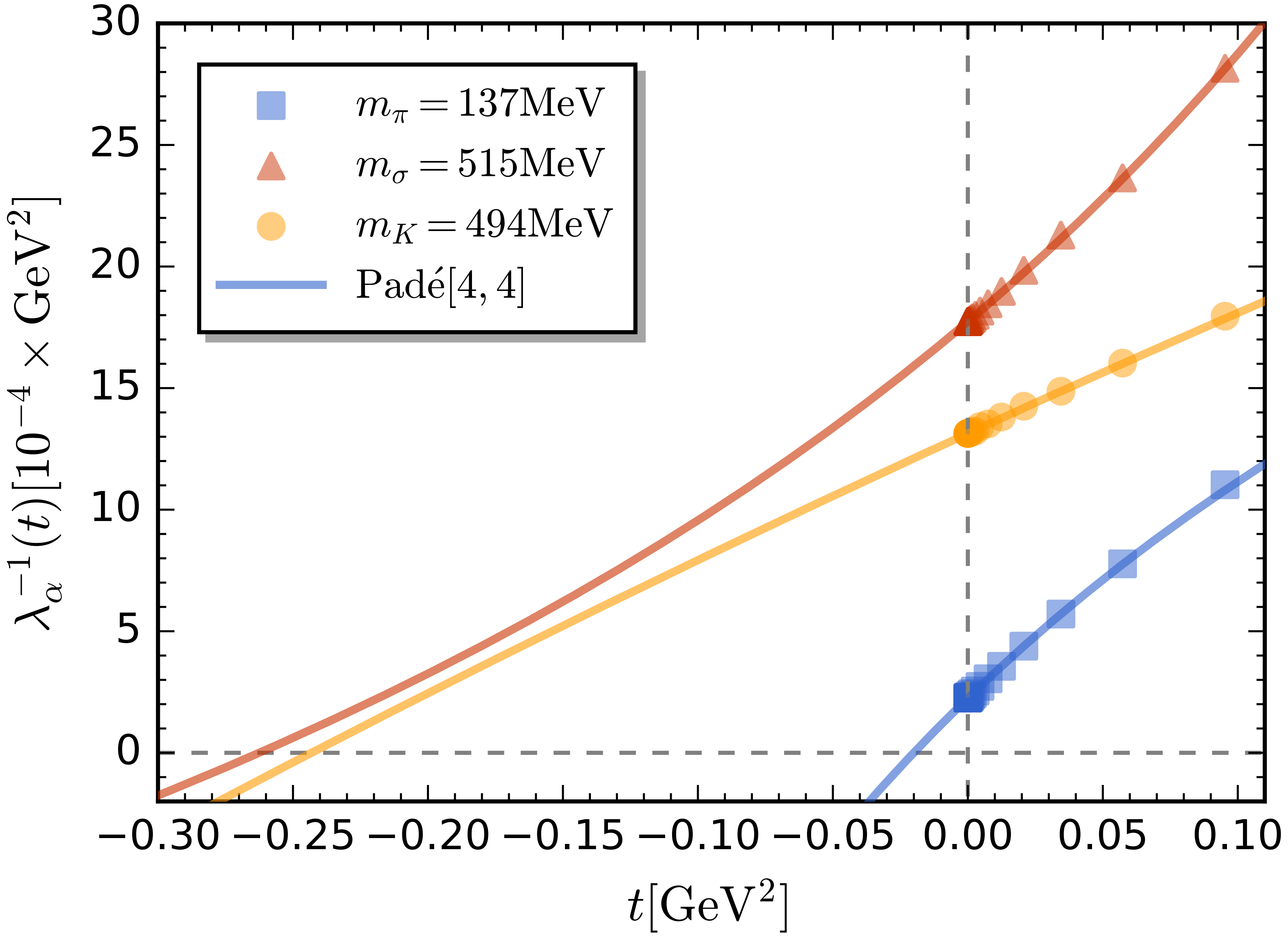
Pion Four-quark coupling



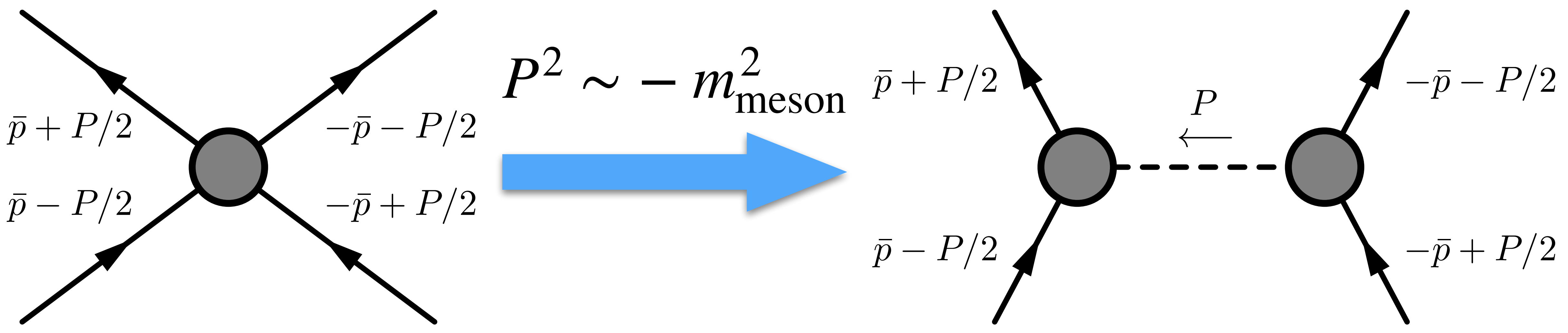
Fu, CH, Pawlowski, Tan, Zhou, Phys. Rev. D 112 (2025), 054047



Meson bound state



Pion Bethe-Salpeter amplitude

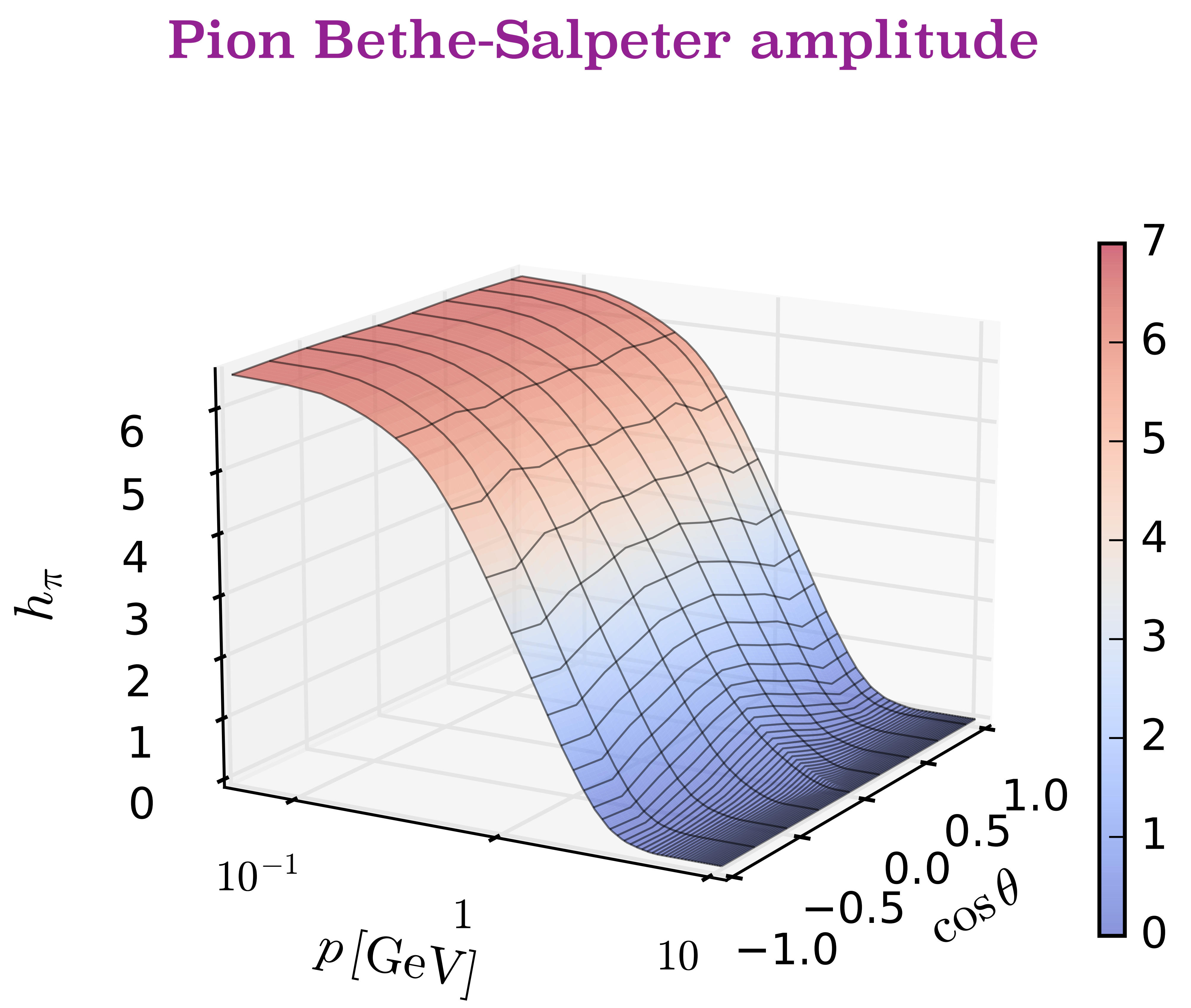


The 4-quark vertex near the on-shell momentum

$$\lambda_\pi(P^2, p, \cos\theta) \sim \frac{h_\pi^2(p, \cos\theta)}{P^2 + m_\pi^2}$$

with the BS amplitude

$$h_\pi(p, \cos\theta) = \lim_{P^2 \to -m_\pi^2} \left[\lambda_\pi(P^2, p, \cos\theta)\,(P^2 + m_\pi^2)\right]^{1/2}$$



Fu, CH, Pawlowski, Tan, Zhou, Phys. Rev. D 112 (2025), 054047

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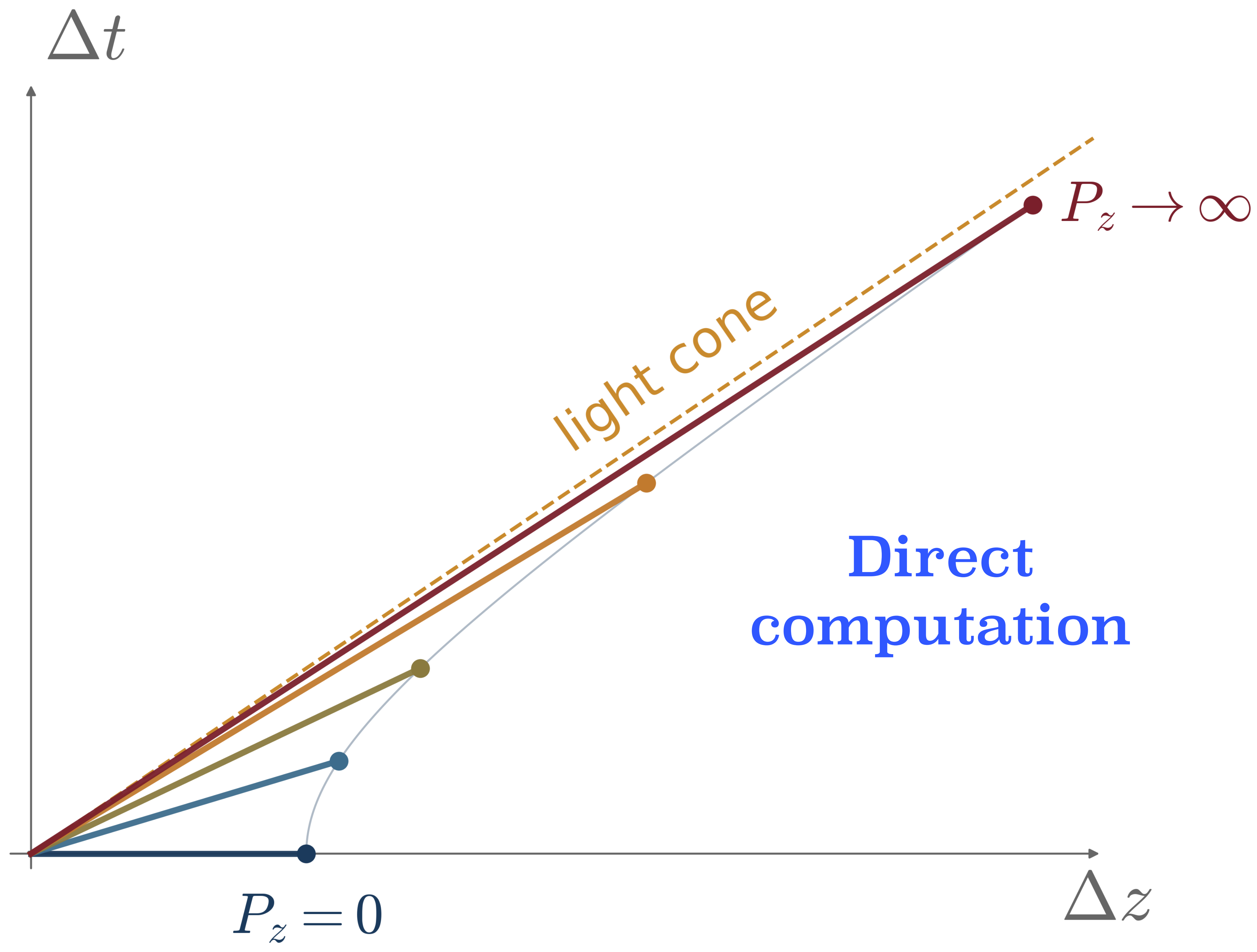
Large momentum effective theory meeting functional method

From quasi light-cone to light-cone (LC)



$$\varphi_\pi(x) = \phi_\pi(x, P_z \to \infty)$$

DA **quasi-DA**

Ji et al., Rev.Mod.Phys. 93 (2021) 3, 035005

Quasi parton distribution amplitude

$$\phi_\pi(x, P_z) = \frac{1}{f_\pi}\mathrm{Tr}_{\mathrm{CD}}\left[\int \frac{d^4k}{(2\pi)^4}\delta(\tilde{n}\cdot k_+ - x\tilde{n}\cdot P)\gamma_5\gamma\cdot\tilde{n}\chi_\pi(k;P)\right]$$

Quasi Bethe-Salpeter amplitude (unamputated):

$$\chi_\pi(k;P) = G_q(k_+)\Gamma(k;P)G_q(k_-)$$

with

$$\Gamma(k;P) = i\gamma_5 h_\pi(k;P)$$

Momentum

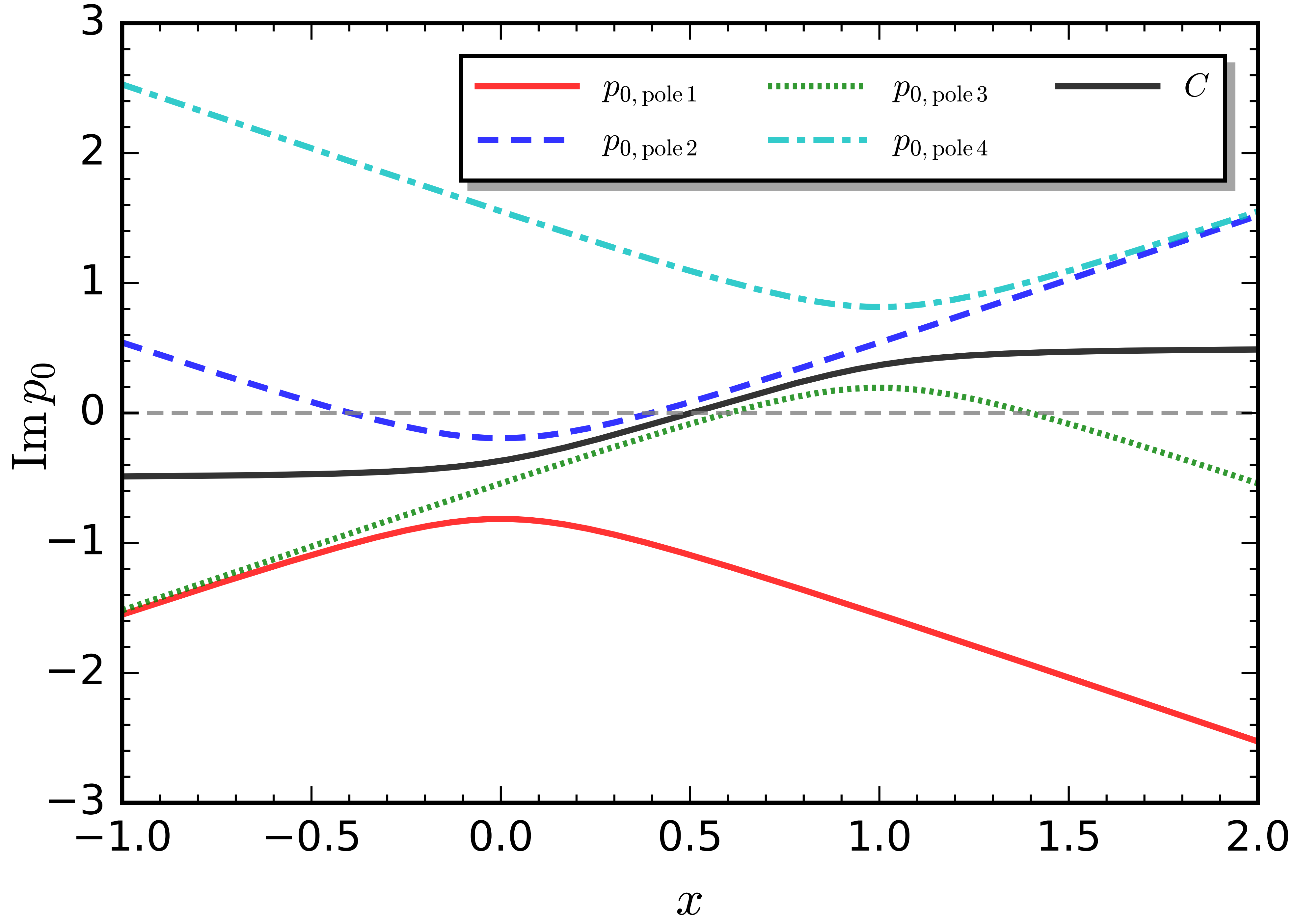
$$P = (im_\pi, 0,0,0) \quad \longrightarrow \quad P = (iE_\pi, P_z, 0,0)$$

LC **quasi-LC**

Zhang, CH, Fu, Phys.Rev.D 112 (2025) 7, 074001
Chang, Fu, CH, Pawlowski, Tan, arXiv: 2607.04871;

Contour integration

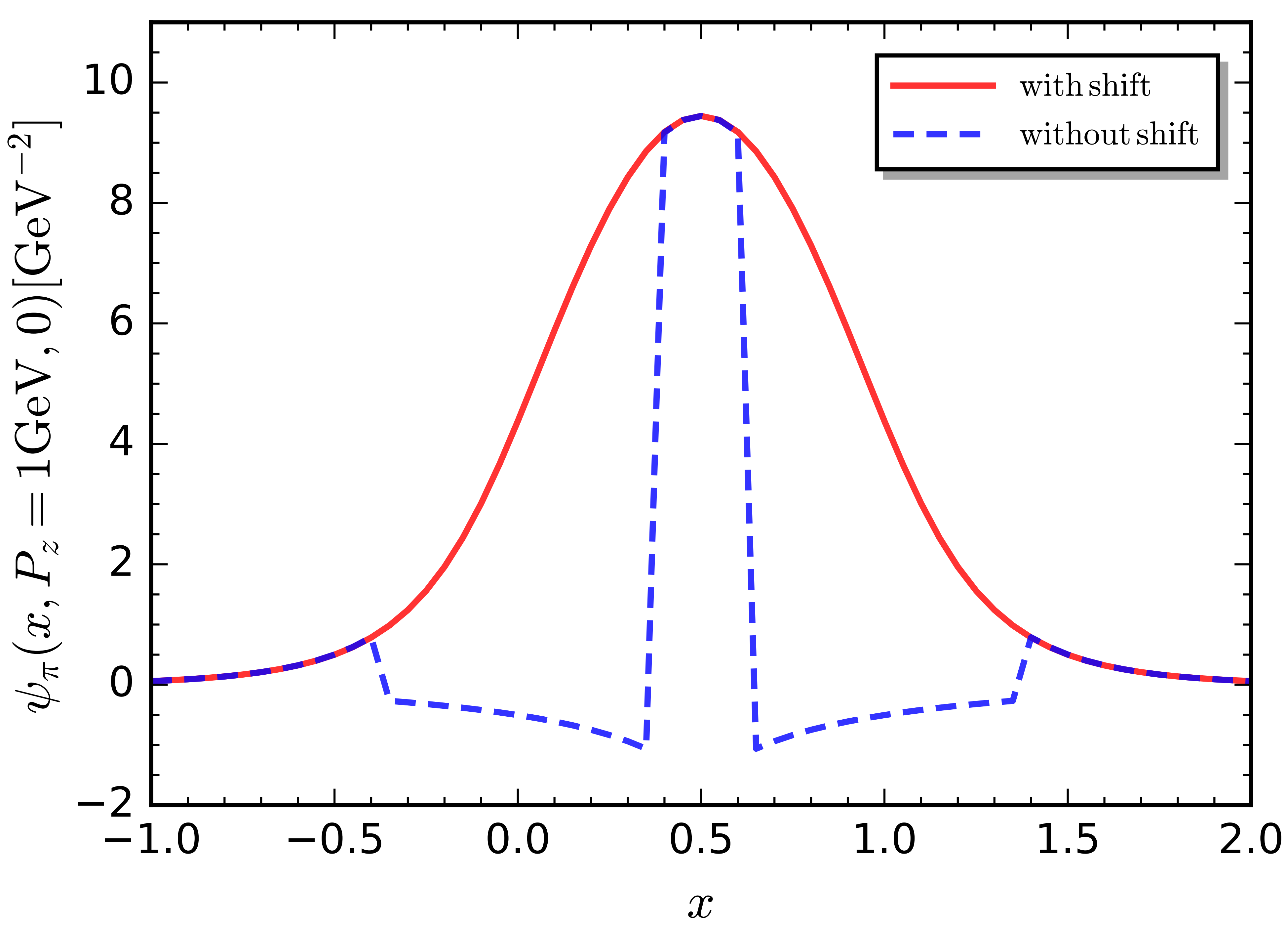
Pole structure of pion qPDA



$$\phi_\pi(x, P_z) \propto \int_p h_\pi(p; P) G_q(p + \frac{P}{2}) G_q(p - \frac{P}{2})$$

Zhang, CH, Fu, Phys.Rev.D 112 (2025) 7, 074001

quasi-DA (example) with/without shift

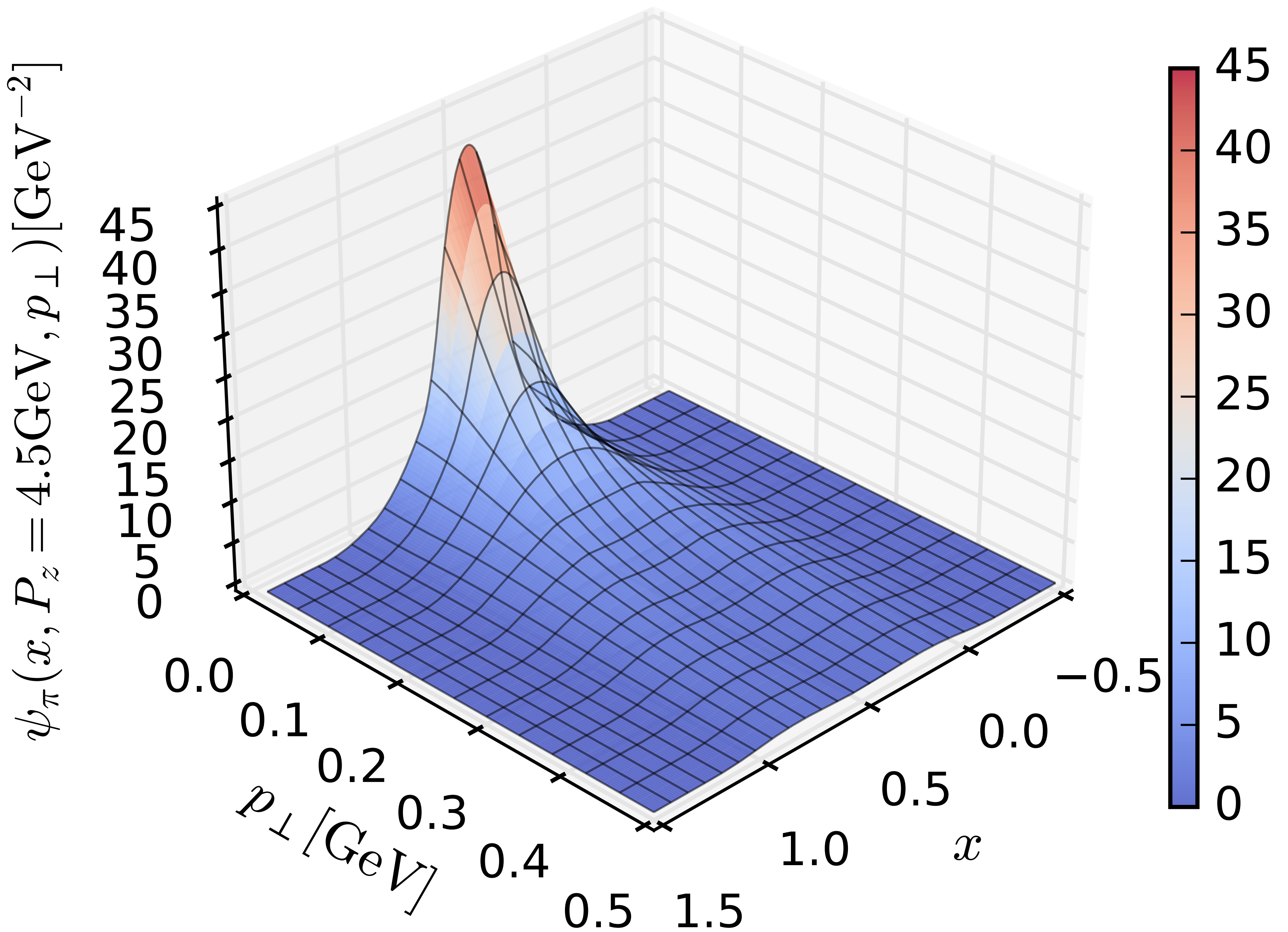


$$\int_{-\infty}^{\infty} dp_0 \to \int_{-\infty+\mathrm{i}C}^{\infty+\mathrm{i}C} dp_0 \qquad C = \frac{\mathrm{Im}\, p_{0,\text{pole}\,2} + \mathrm{Im}\, p_{0,\text{pole}\,3}}{2}$$

Integral shift

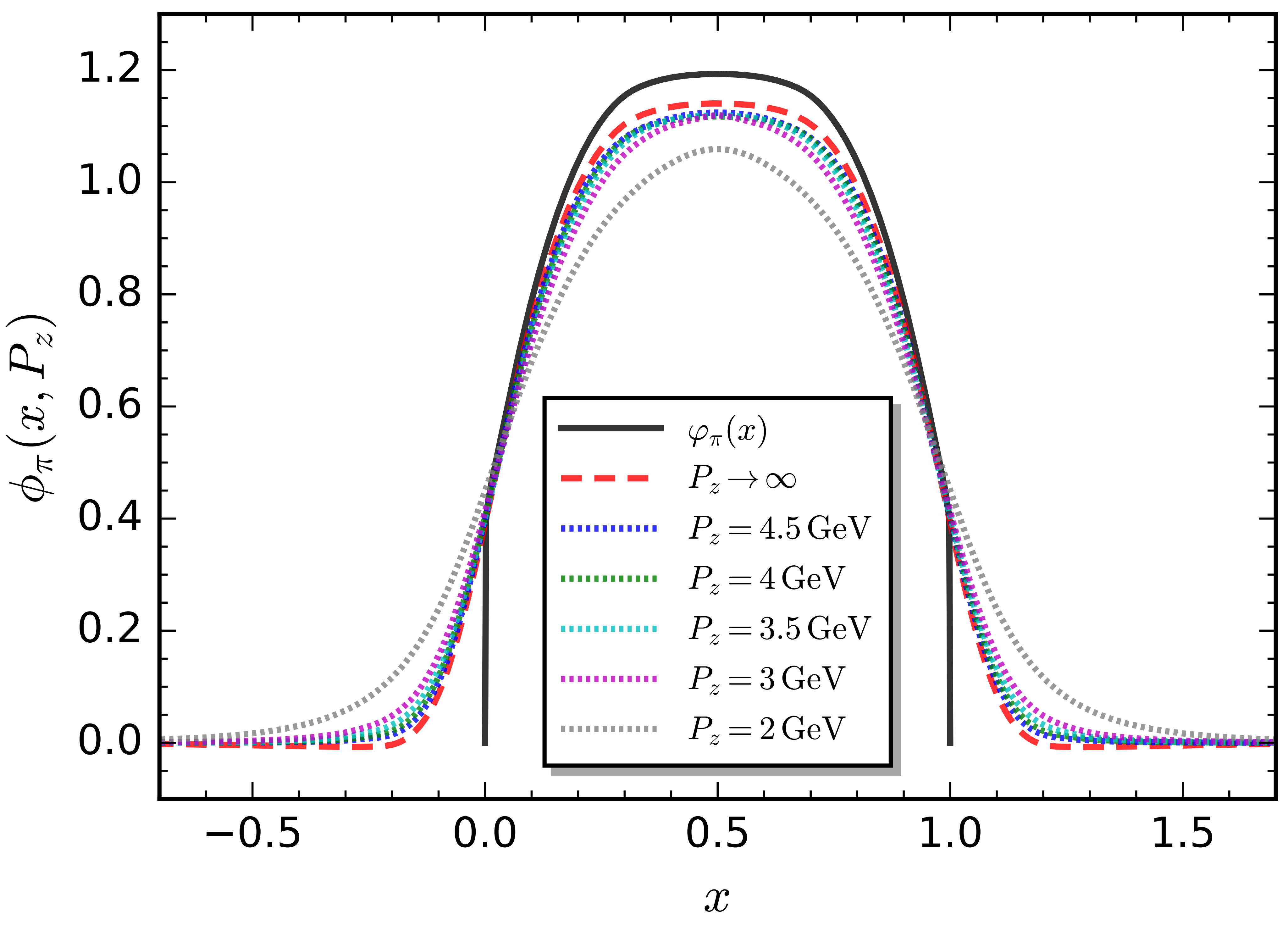
results in quasi light-cone

Pion quasi-wave function



$$\phi_\pi(x, P_z) = \frac{1}{16\pi^3} \int d^2p_\perp \psi_\pi(x, P_z, p_\perp)$$

Pion quasi-DA

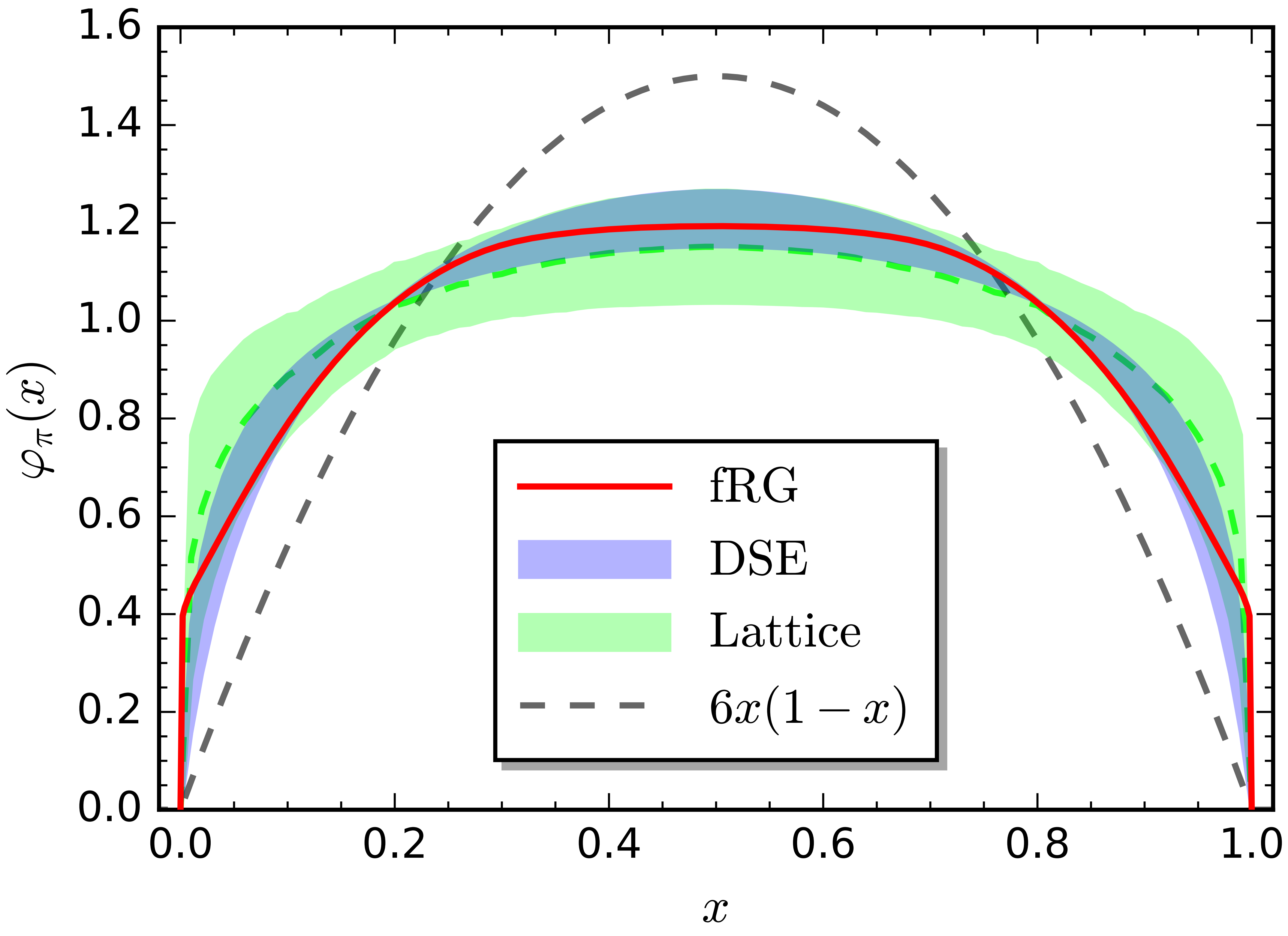


Saturation in $P_z \gtrsim 3.5$ GeV!

Chang, Fu, CH, Pawlowski, Tan, arXiv: 2607.04871;

Pion DA and its moments

Pion DA



$$\phi_\pi(x, P_z) = \varphi_\pi(x) + \frac{c_2(x)}{P_z^2} + \mathcal{O}\left(\frac{1}{P_z^4}\right)$$

Chang, Fu, CH, Pawlowski, Tan, arXiv: 2607.04871;

Moments of pion DA $\langle \xi^n \rangle \equiv \int_0^1 dx(2x-1)^n \phi_\pi(x)$

Method	$\langle\xi^2\rangle_\pi$	$\langle\xi^4\rangle_\pi$	$\langle\xi^6\rangle_\pi$
functional LaMET (This Work)	0.267	0.139	0.088
Lattice LaMET (LPC)[8]	0.300(41)	-	-
Lattice OPE (RQCD)[10]	$0.234^{+6}_{-6}(4)(4)(2)$	-	-
Lattice OPE (RBC and UKQCD)[9]	0.28(1)(2)	-	-
Sum Rule[11, 12]	0.271(13)	0.138(10)	0.087(6)
DSE/BSE [RL, DB][22]	0.280, 0.247	0.149, 0.121	0.097, 0.073

Lattice LaMET: J. Hua *et al.* (LPC), *PRL* 129 (2022) 132001.
DSE: C. Roberts *et al.*, *PPNP* 120 (2021) 103883; Chang *et al.* (LPC), *PRL* 110 (2013) 132001.
Lattice OPE: G. Bali *et al.* (RQCD), *JHEP* 08 (2019) 065; 11 (2020) 37.
Lattice OPE: R. Arthur *et al.* (RBC and UKQCD), *PRD* 83 (2011) 074505.
Sum rules: P. Ball *et al.*, *JHEP* 08 (2007) 090; T. Zhong *et al.*, *PRD* 104 (2021) 016021

Smaller than the lattice-LaMET, agree with other methods

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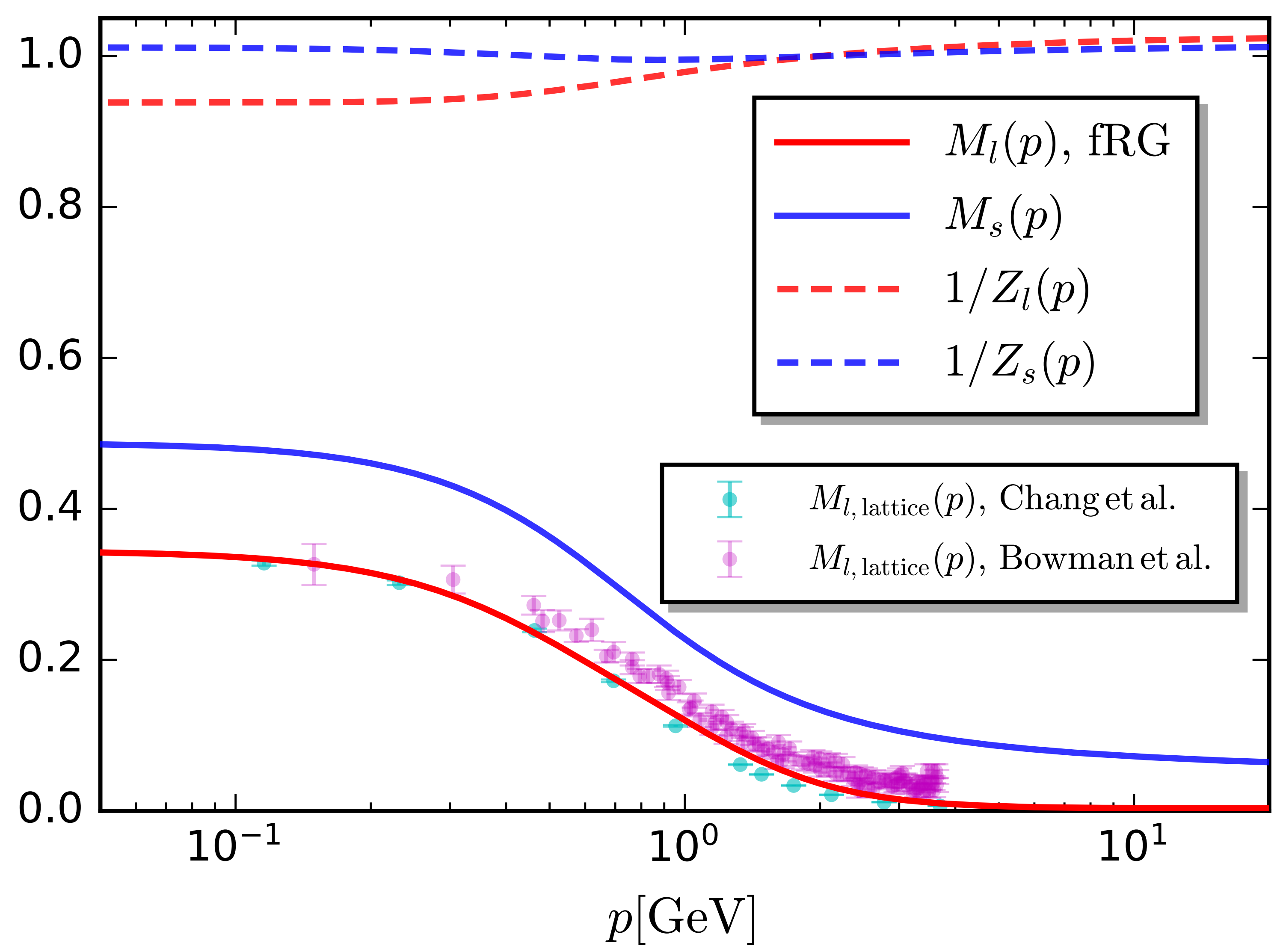
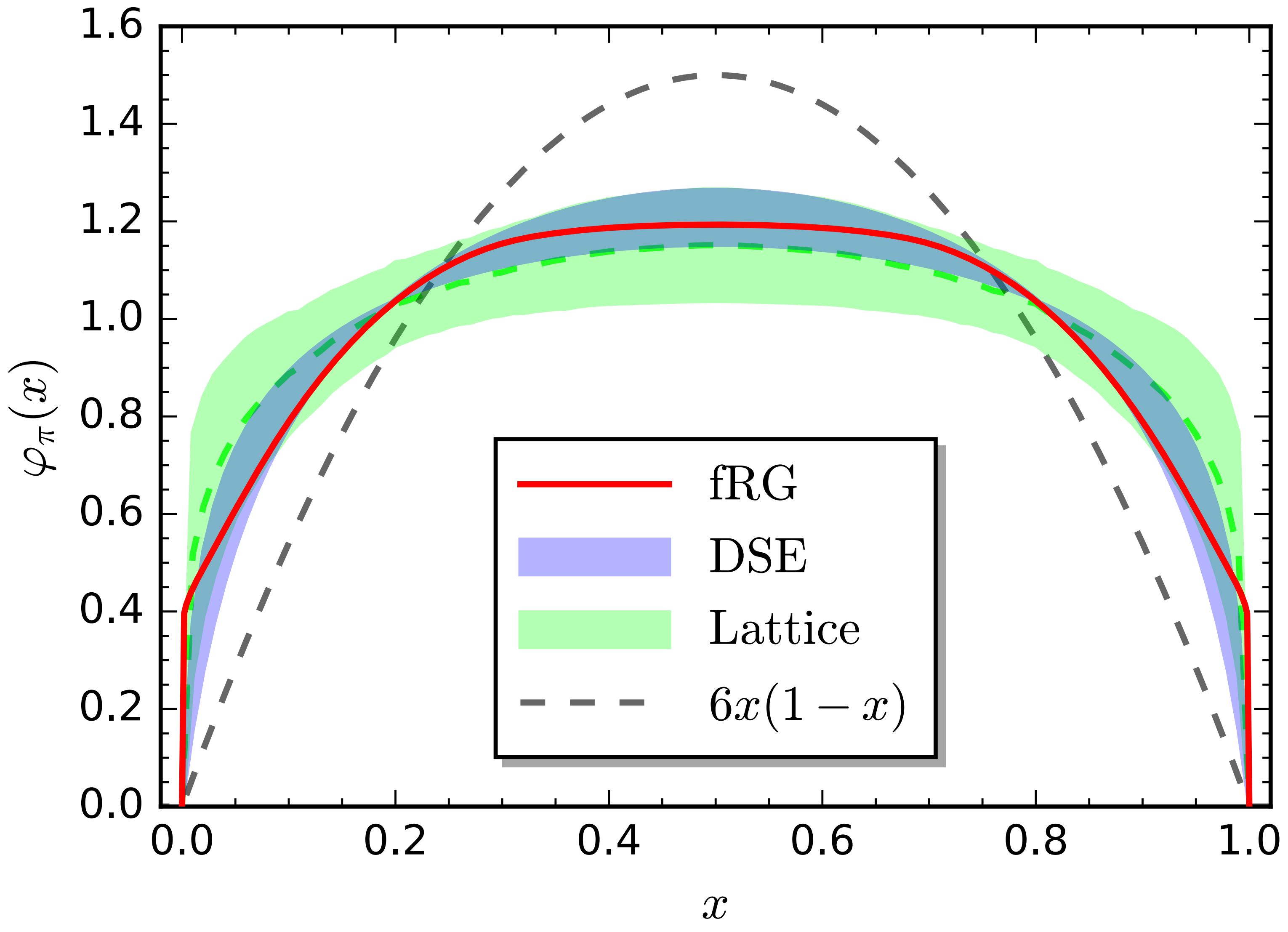
Summary and Outlook

- ★ The first-principle functional QCD framework is built.
- ★ The LaMET framework for hadron distribution functions in functional method is established.
- ★ The saturation of pion quasi-DA in $P_z \gtrsim 3.5$ GeV is investigated.
- ★ Explore functional QCD correlation functions in the complex plane.
- ★ Extend to more distribution functions (PDF, GPD, TMD, etc.) for more hadrons.





Thank you very much for your attentions!

Backup

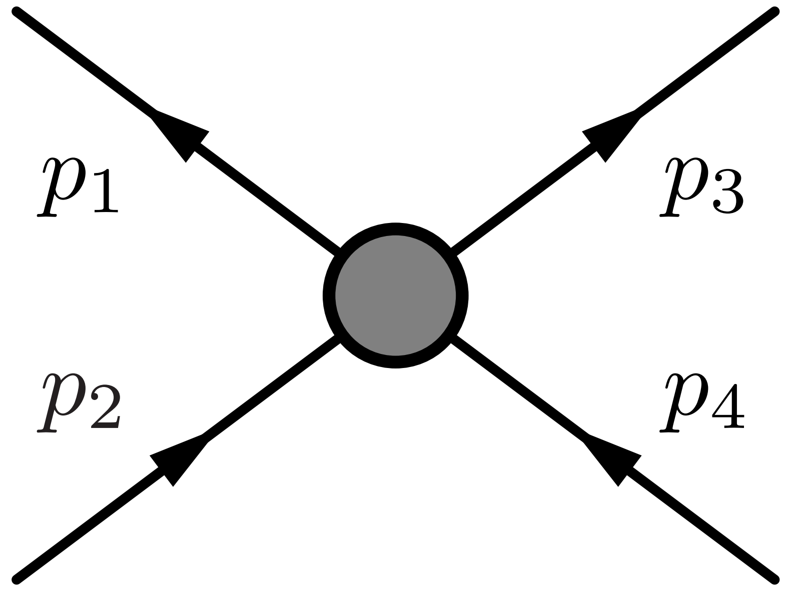
Four-quark vertices

4-quark effective action

$$\Gamma_{4q,k} = -\sum_{\alpha}\int_{\vec{p}} \lambda_\alpha(\vec{p})\mathcal{O}^{(\alpha)}_{ijlm}\bar{q}_i\left(p_1\right)q_j\left(p_2\right)\bar{q}_l\left(p_3\right)q_m\left(p_4\right)$$

4-quark vertex

$$\begin{aligned}
&\Gamma^{(4)}_{\bar{q}_i q_j \bar{q}_l q_m}\left(p_1,p_2,p_3,p_4\right)\\
\equiv&\frac{\delta}{\delta q_m\left(p_4\right)}\frac{\delta}{\delta \bar{q}_l\left(p_3\right)}\frac{\delta}{\delta q_j\left(p_2\right)}\frac{\delta}{\delta \bar{q}_i\left(p_1\right)}\Gamma_k[q,\bar{q}]\\
=&\sum_{\alpha}\left(\lambda_\alpha\left(p_1,p_2,p_3,p_4\right)\mathcal{O}^{(\alpha)}_{ijlm}-\lambda_\alpha\left(p_3,p_2,p_1,p_4\right)\mathcal{O}^{(\alpha)}_{ljim}\right)\\
&\times(-2)(2\pi)^4\delta^4\left(p_1+p_2+p_3+p_4\right)\\
=&\sum_{\alpha}\left(\lambda^+_\alpha\left(p_1,p_2,p_3,p_4\right)\mathcal{T}^{(\alpha-)}_{ijlm}+\lambda^-_\alpha\left(p_1,p_2,p_3,p_4\right)\mathcal{T}^{(\alpha^+)}_{ijlm}\right)\\
&\times(-4)(2\pi)^4\delta^4\left(p_1+p_2+p_3+p_4\right)
\end{aligned}$$



$$= -\Gamma^{(4)}_{\bar{q}_i q_j \bar{q}_l q_m}\left(p_1,p_2,p_3,p_4\right)$$

where

$$\lambda^+_\alpha\left(p_1,p_2,p_3,p_4\right)\equiv\frac{1}{2}\left[\lambda_\alpha\left(p_1,p_2,p_3,p_4\right)+\lambda_\alpha\left(p_3,p_2,p_1,p_4\right)\right]$$

$$\lambda^-_\alpha\left(p_1,p_2,p_3,p_4\right)\equiv\frac{1}{2}\left[\lambda_\alpha\left(p_1,p_2,p_3,p_4\right)-\lambda_\alpha\left(p_3,p_2,p_1,p_4\right)\right]$$

$$\mathcal{T}^{(\alpha^+)}_{ijlm}\equiv\left(\mathcal{O}^{(\alpha)}_{ijlm}+\mathcal{O}^{(\alpha)}_{ljim}\right)/2\quad \mathcal{T}^{(\alpha^-)}_{ijlm}\equiv\left(\mathcal{O}^{(\alpha)}_{ijlm}-\mathcal{O}^{(\alpha)}_{ljim}\right)/2$$

Symmetry

$$\begin{aligned}
\lambda^+_\alpha\left(p_1,p_2,p_3,p_4\right)&=\lambda^+_\alpha\left(p_3,p_2,p_1,p_4\right)\\
&=\lambda^+_\alpha\left(p_1,p_4,p_3,p_2\right)=\lambda^+_\alpha\left(p_3,p_4,p_1,p_2\right)
\end{aligned}$$

$$\begin{aligned}
\lambda^-_\alpha\left(p_1,p_2,p_3,p_4\right)&=-\lambda^-_\alpha\left(p_3,p_2,p_1,p_4\right)\\
&=-\lambda^-_\alpha\left(p_1,p_4,p_3,p_2\right)=\lambda^-_\alpha\left(p_3,p_4,p_1,p_2\right)
\end{aligned}$$

DA and Quasi-DA of pion

Bethe-Salpeter amplitude (unamputated):

$$\chi_\pi(k;P) = G_q(k_+)\Gamma(k;P)G_q(k_-)$$

with

$$\Gamma(k;P) = i\gamma_5 h_\pi(k;P)$$

and $P = (im_\pi, 0,0,0)$ and $k_\pm = k \pm P/2$

Parton distribution amplitude on light-front

$$\phi_\pi(x,P_z) = \frac{1}{f_\pi}\mathrm{Tr}_{\mathrm{CD}}\left[\int \frac{d^4k}{(2\pi)^4}\delta(n\cdot k_+ - xn\cdot P)\gamma_5\gamma\cdot n\chi_\pi(k;P)\right]$$

with $n = (i,1,0,0)$.

Momentum

$$k_\mu = \left(k_0, k_3, k_\perp\right),$$

$$k_{+\mu} = \left(k_0 + im_\pi/2, k_3, k_\perp\right),$$

$$k_{-\mu} = \left(k_0 - im_\pi/2, k_3, k_\perp\right)$$

Quasi Bethe-Salpeter amplitude (unamputated):

$$\chi_\pi(k;P) = G_q(k_+)\Gamma(k;P)G_q(k_-)$$

with

$$\Gamma(k;P) = i\gamma_5 h_\pi(k;P)$$

and $P = (iE_\pi, P_z, 0,0)$, $E_\pi = \sqrt{P_z^2 + m_\pi^2}$ and $k_\pm = k \pm P/2$

Quasi parton distribution amplitude

$$\phi_\pi(x,P_z) = \frac{1}{f_\pi}\mathrm{Tr}_{\mathrm{CD}}\left[\int \frac{d^4k}{(2\pi)^4}\delta(\tilde{n}\cdot k_+ - x\tilde{n}\cdot P)\gamma_5\gamma\cdot \tilde{n}\chi_\pi(k;P)\right]$$

with $\tilde{n} = (0,1,0,0)$.

Momentum

$$k_\mu = \left(k_0, k_3, k_\perp\right),$$

$$k_{+\mu} = \left(k_0 + iE_\pi/2, k_3 + xP_z/2, k_\perp\right),$$

$$k_{-\mu} = \left(k_0 - iE_\pi/2, k_3 - xP_z/2, k_\perp\right)$$

Zhang, CH, Fu, Phys.Rev.D 112 (2025) 7, 074001

Integration detail

$$\phi_\pi(x,P_z) = \frac{1}{f_\pi}\frac{4N_c}{(2\pi)^4}\int d^2k_\perp dk_0\, h_\pi(k;P)P_z\left[xM_q(k_-^2)+(1-x)M_q(k_+^2)\right]$$
$$\times\frac{1}{Z_q(k_+^2)Z_q(k_-^2)}\frac{1}{k_+^2+M_q^2(k_+^2)}\frac{1}{k_-^2+M_q^2(k_-^2)}$$

Poles of two quark propagators

$$k_{0,1} = i\left[-\sqrt{k_\perp^2+(xP_z)^2+M_q^2(k_+^2)}-\sqrt{P_z^2+m_\pi^2}/2\right],$$
$$k_{0,2} = i\left[\sqrt{k_\perp^2+(xP_z)^2+M_q^2(k_+^2)}-\sqrt{P_z^2+m_\pi^2}/2\right],$$
$$k_{0,3} = i\left[-\sqrt{k_\perp^2+(x-1)^2P_z^2+M_q^2(k_-^2)}+\sqrt{P_z^2+m_\pi^2}/2\right],$$
$$k_{0,4} = i\left[\sqrt{k_\perp^2+(x-1)^2P_z^2+M_q^2(k_-^2)}+\sqrt{P_z^2+m_\pi^2}/2\right]$$

- We use Taylor expansion to continue h_π, M_q, Z_q in the complex plane of k_0:

$$h_\pi(k^2,P^2,\cos\theta) = h_\pi(\bar{k}^2,P^2,\cos\theta)+\frac{\partial}{\partial k^2}h_\pi\Big|_{k^2=\bar{k}^2}k_0^2+\cdots$$

and

$$M_q(k_+^2) = M_q(\bar{k}_+^2)+\frac{\partial}{\partial k_+^2}M_q\Big|_{k_+^2=\bar{k}_+^2}(k_0+iE_\pi/2)^2+\cdots$$
$$M_q(k_-^2) = M_q(\bar{k}_-^2)+\frac{\partial}{\partial k_-^2}M_q\Big|_{k_-^2=\bar{k}_-^2}(k_0-iE_\pi/2)^2+\cdots$$

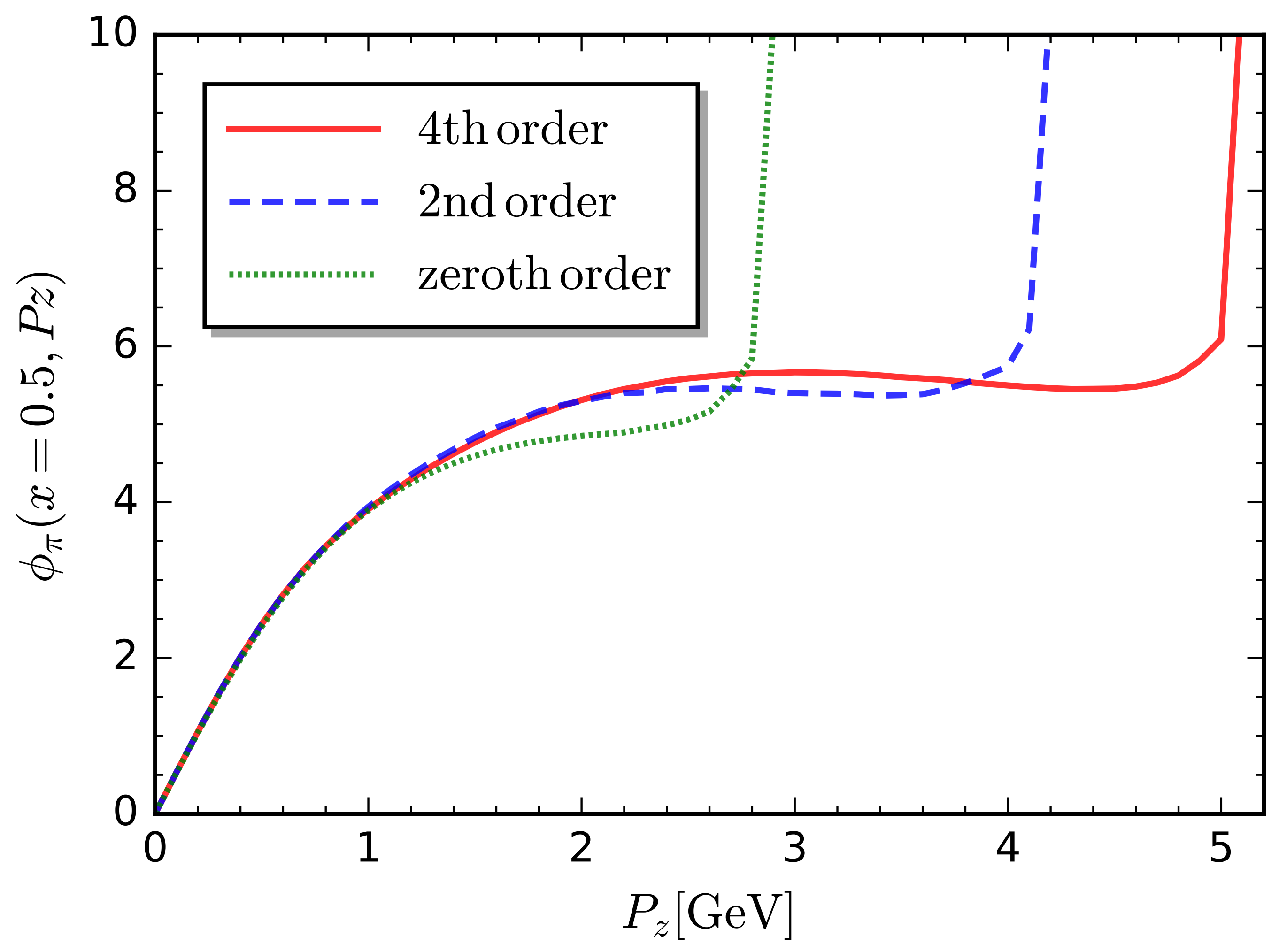
$$k^2 = \bar{k}^2+k_0^2$$
$$\bar{k}^2 = k_\perp^2+(x-1/2)^2P_z^2$$

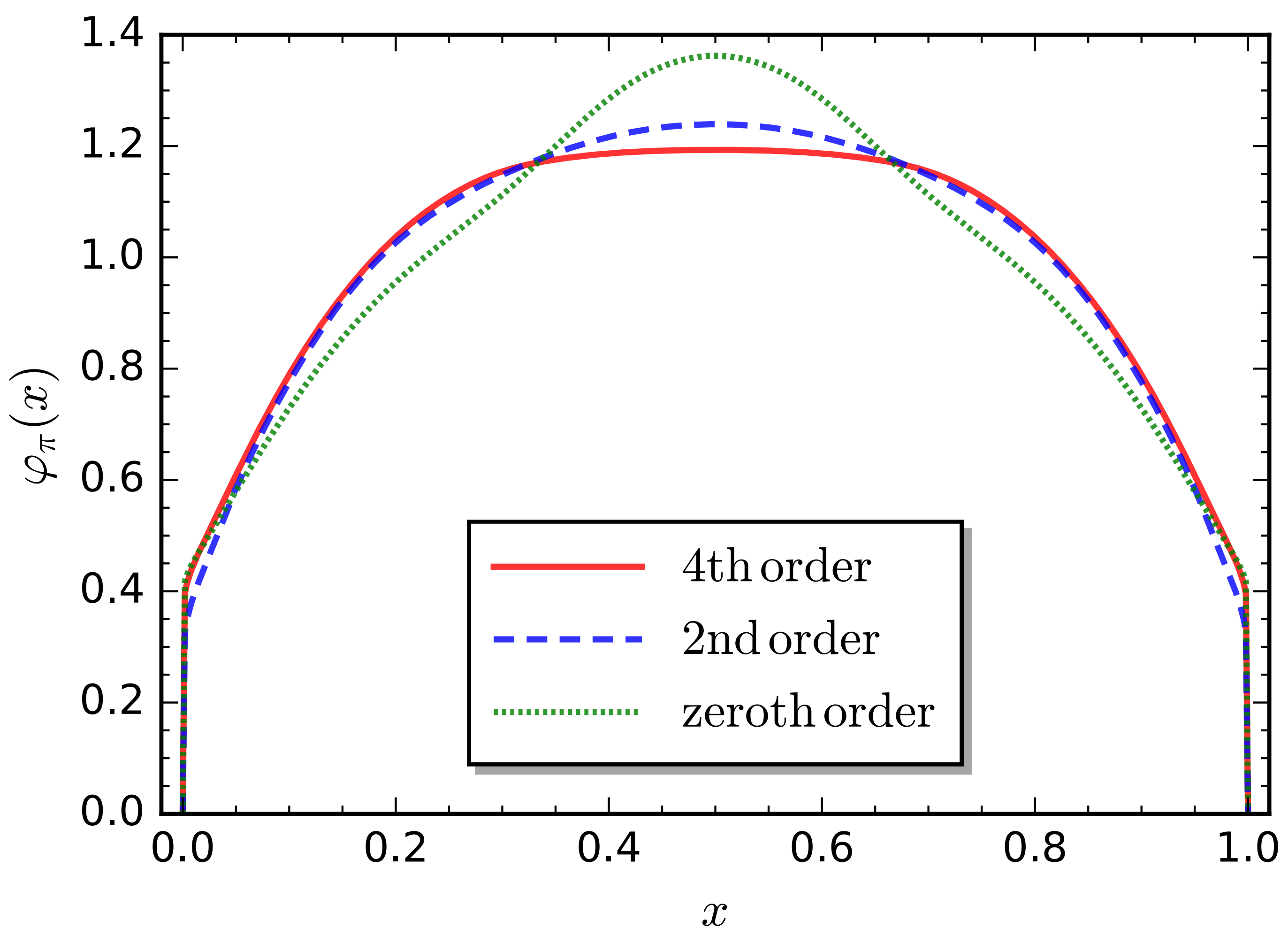
$$k_+^2 = \bar{k}_+^2+(k_0+iE_\pi/2)^2$$
$$\bar{k}_+^2 = k_\perp^2+x^2P_z^2$$

$$k_-^2 = \bar{k}_-^2+(k_0-iE_\pi/2)^2$$
$$\bar{k}_-^2 = k_\perp^2+(x-1)^2P_z^2$$

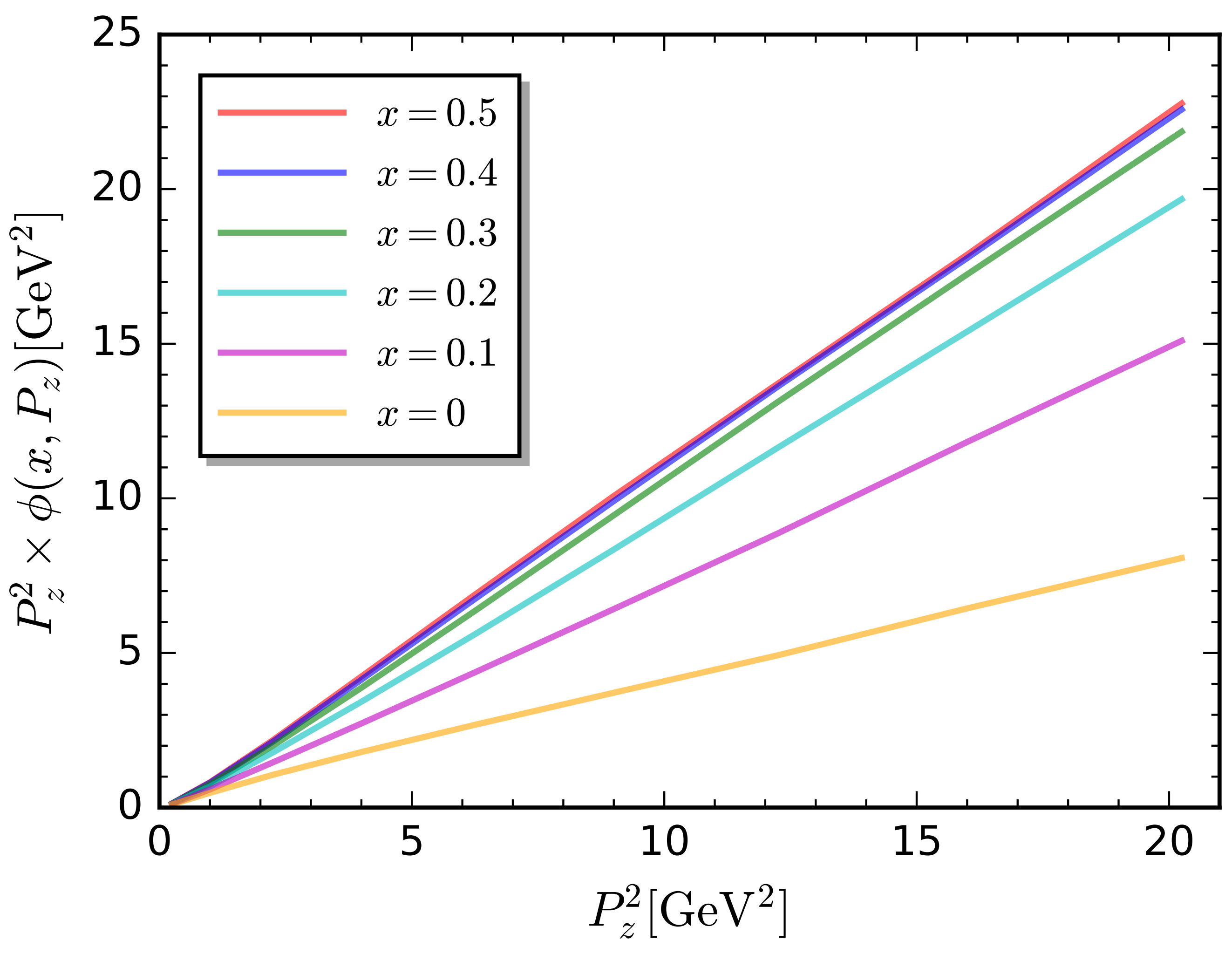
Numerical check for expansion and extrapolation

Expansion check





Extrapolation check



Chang, Fu, CH, Pawlowski, Tan, arXiv: 2607.04871;

$$\phi_\pi(x, P_z) = \phi_\pi(x, P_z \to \infty) + \frac{c_2(x)}{P_z^2} + \mathcal{O}\left(\frac{1}{P_z^4}\right)$$