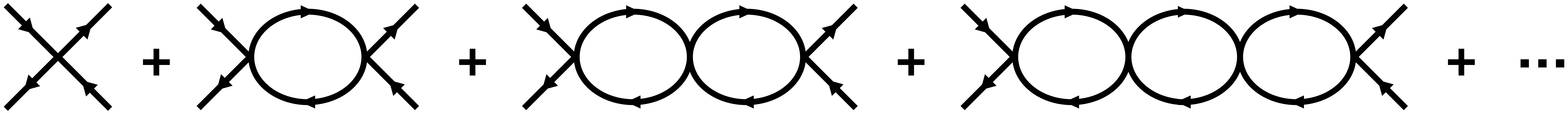
Renormalisable four-fermion interactions in four dimensions

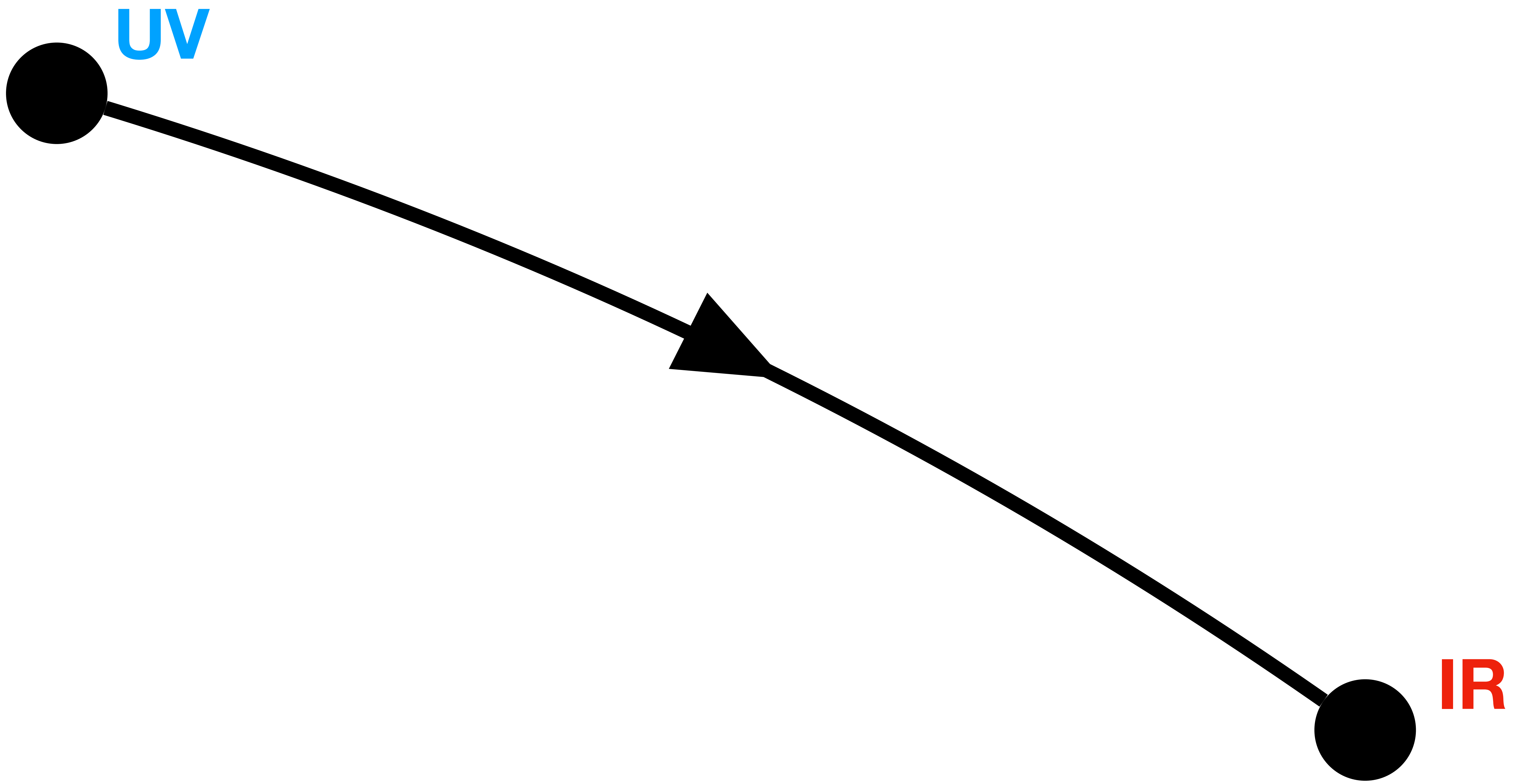


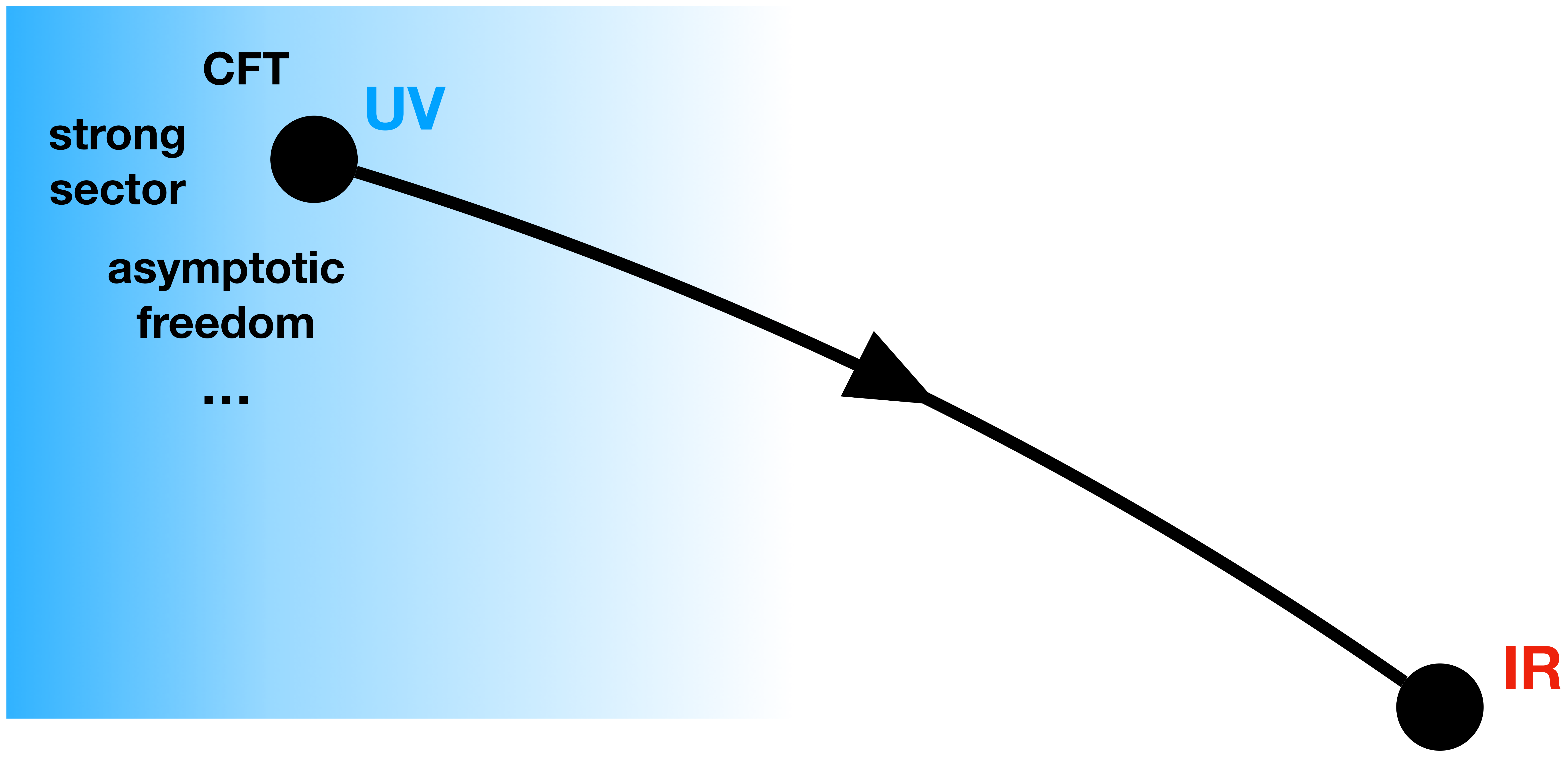
Charlie Cresswell-Hogg
TU Dortmund / University of Sussex

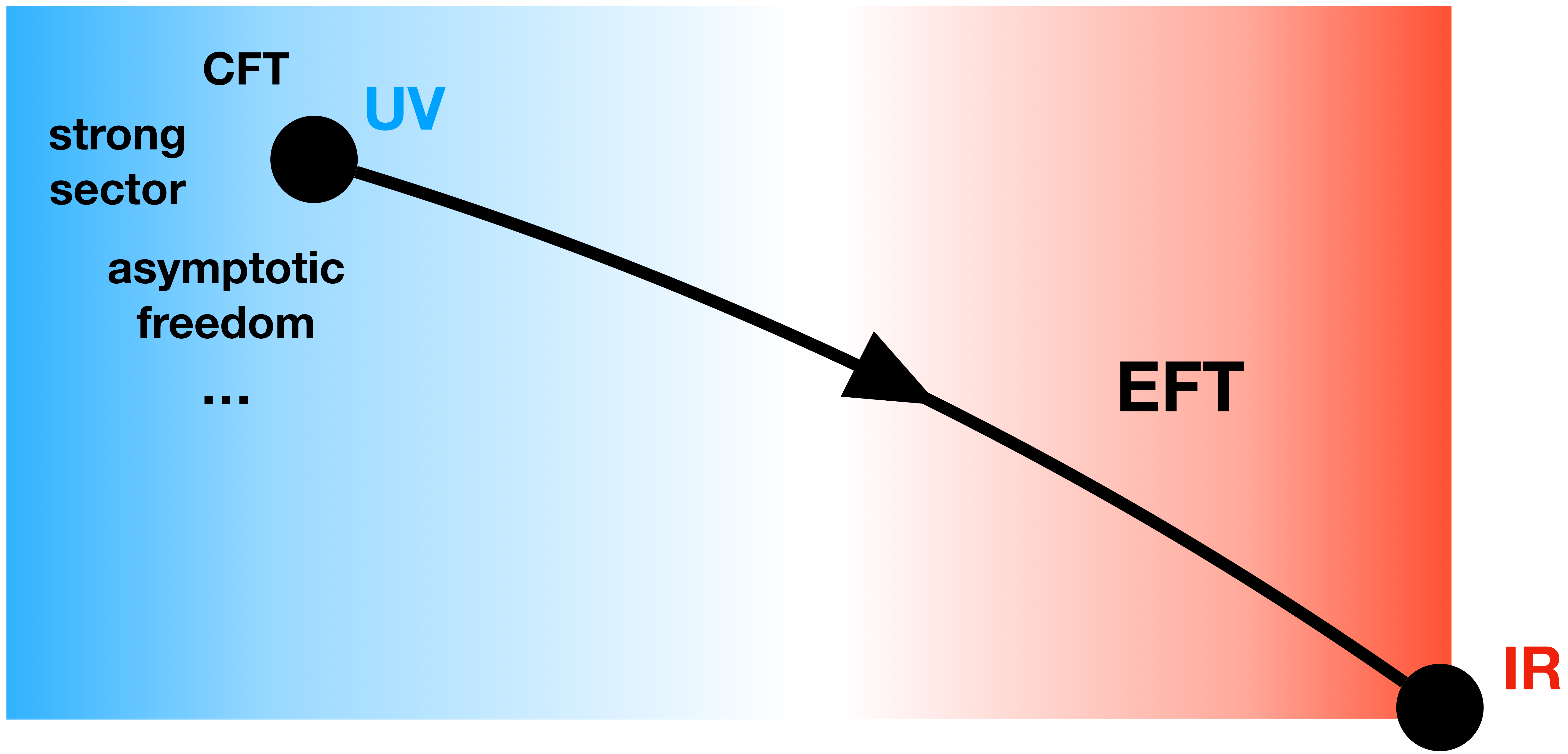
based on ***2603.19357*** *with Daniel Litim*

ERG 2026 @ Sussex

University of Sussex
technische universität dortmund


UV
IR


CFT
UV
strong
sector
asymptotic
freedom
...
IR



CFT
UV
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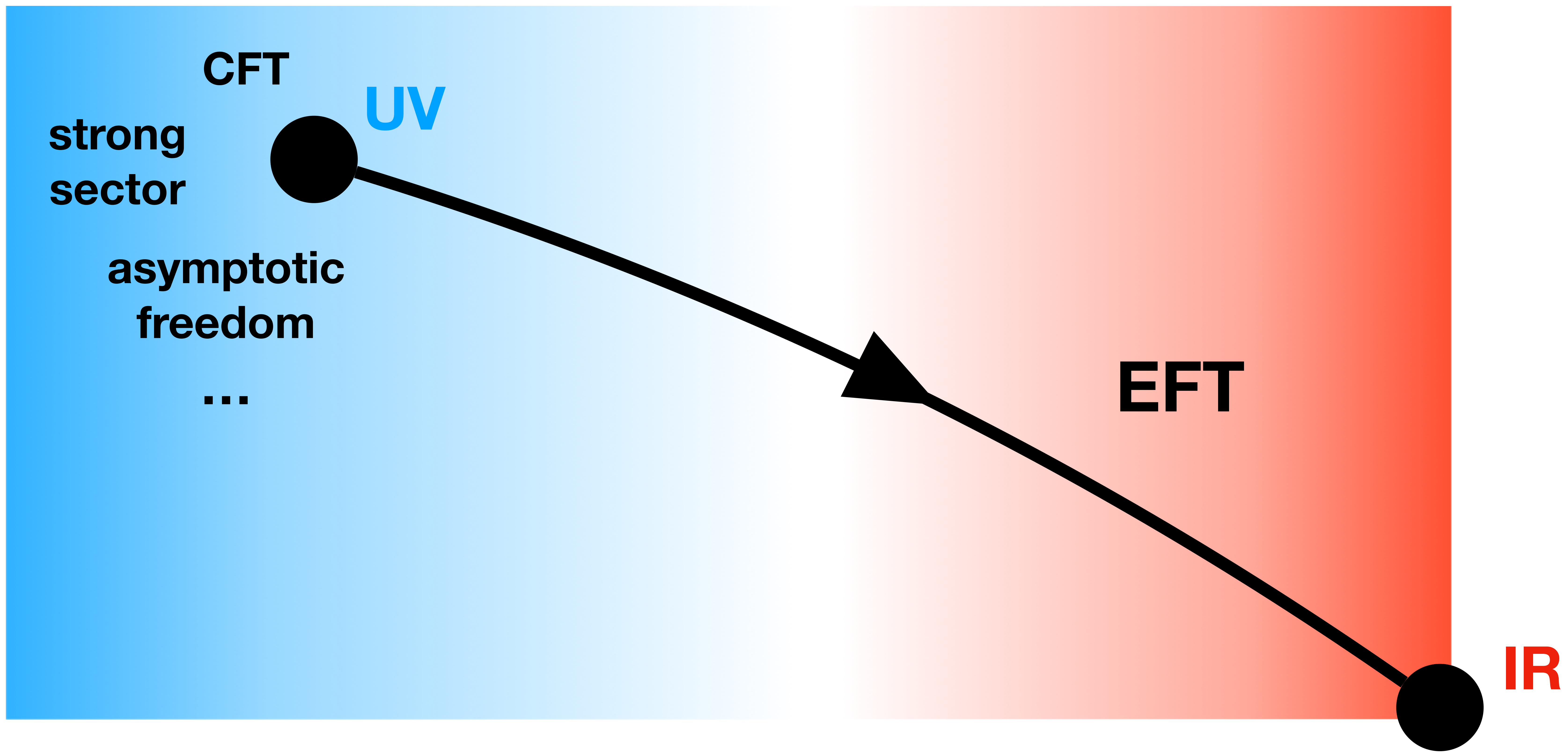
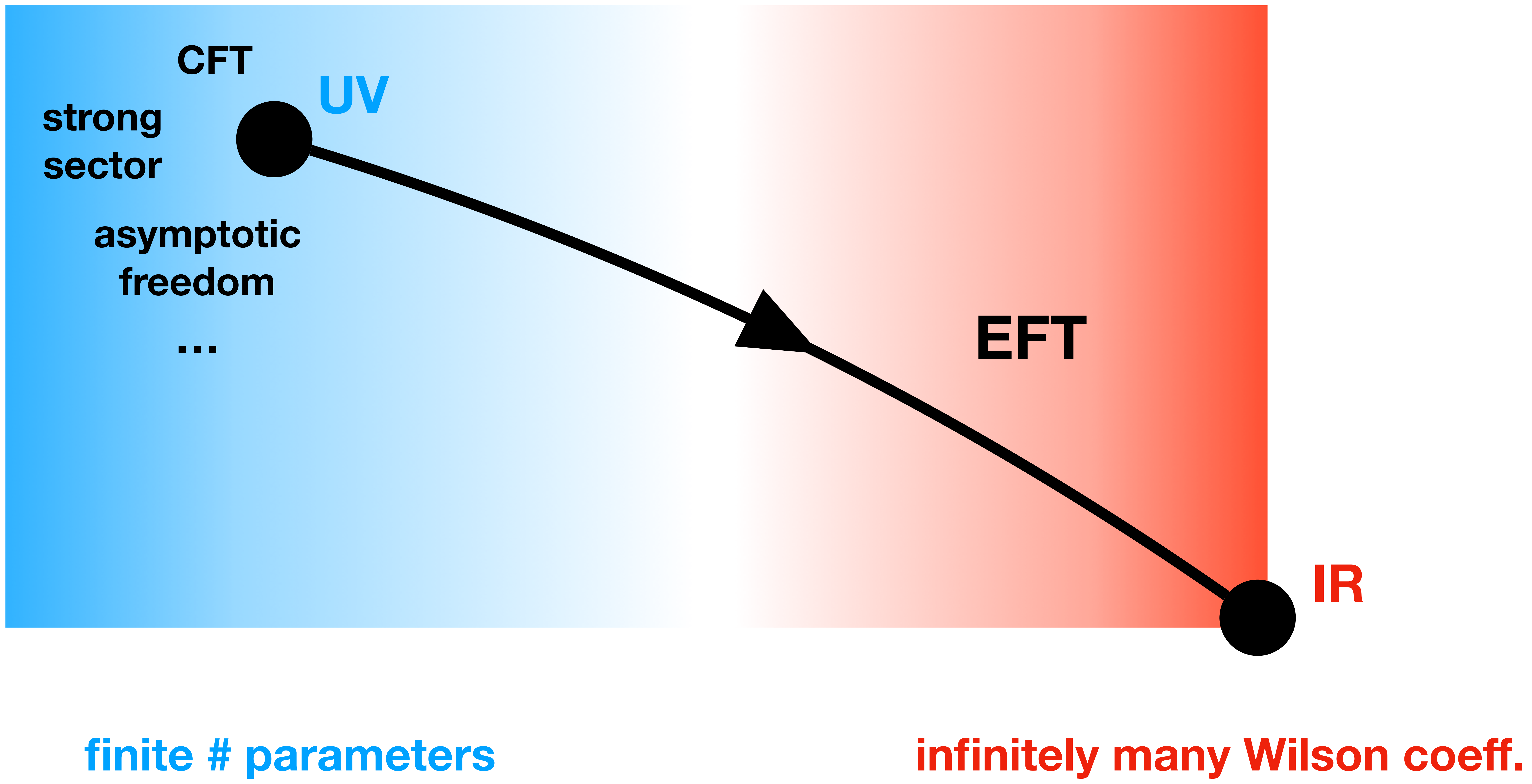
...
EFT
IR



finite # parameters


CFT
UV
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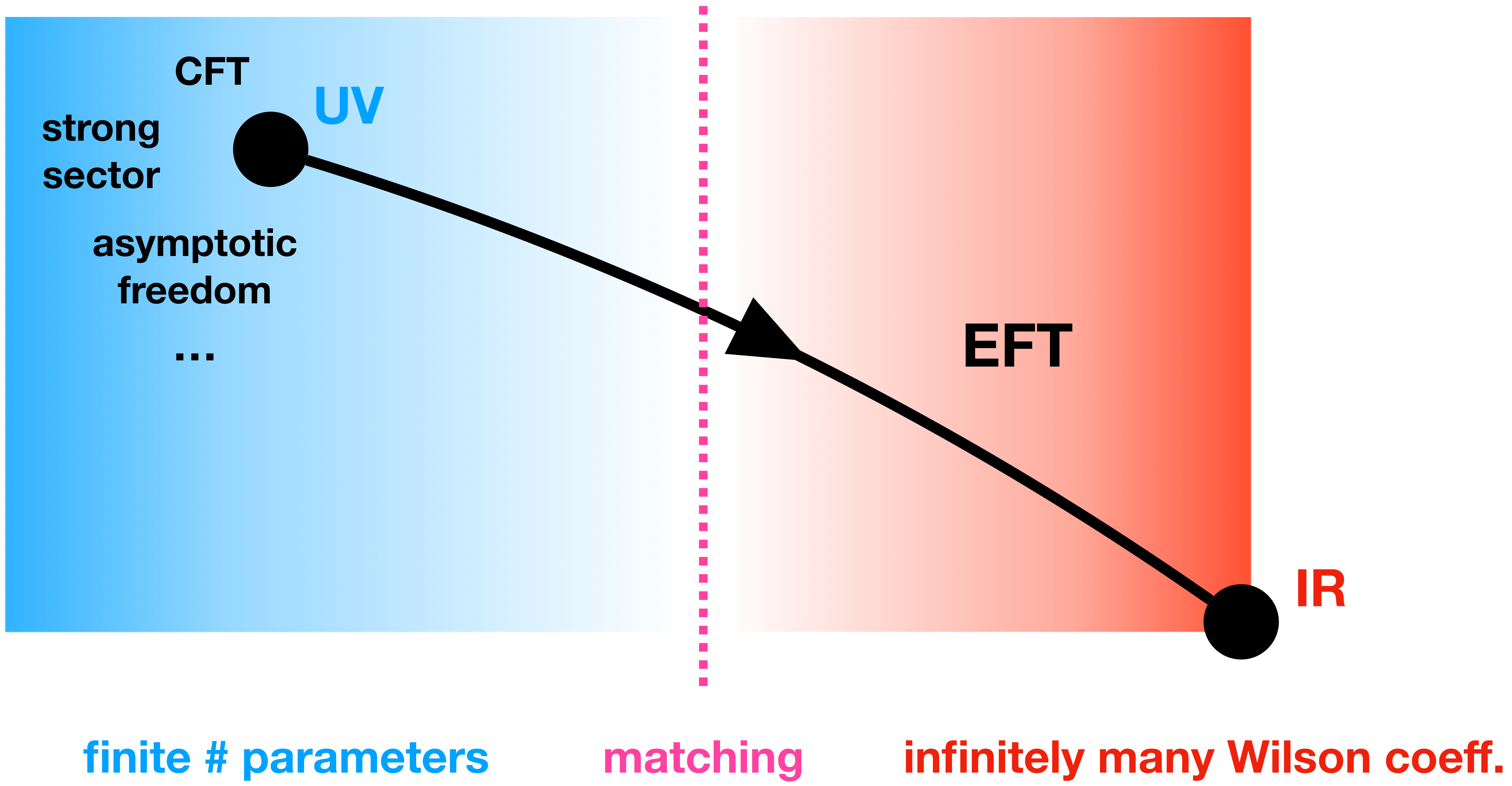
finite # parameters
infinitely many Wilson coeff.


CFT
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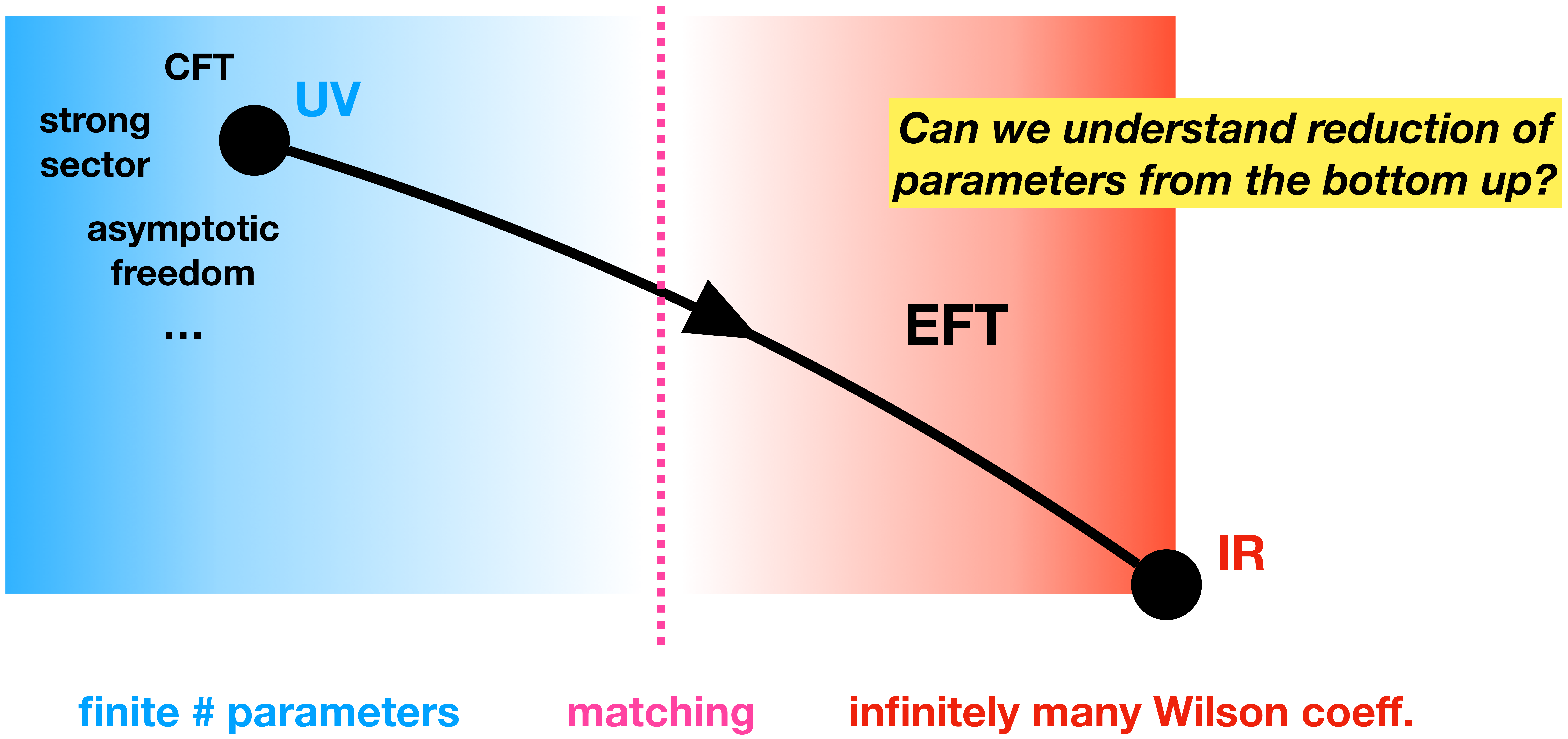
CFT
UV
strong
sector
asymptotic
freedom
...
Can we understand reduction of
parameters from the bottom up?
EFT
IR
finite # parameters
matching
infinitely many Wilson coeff.

A toy EFT

$$\bar{\psi}_a \partial\!\!\!/ \psi_a - \frac{1}{2N} G (\bar{\psi}_a \psi_a)^2 + \cdots$$

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2+1 dimensions: 1/*N* renormalisable & conformal at high energy

A toy EFT

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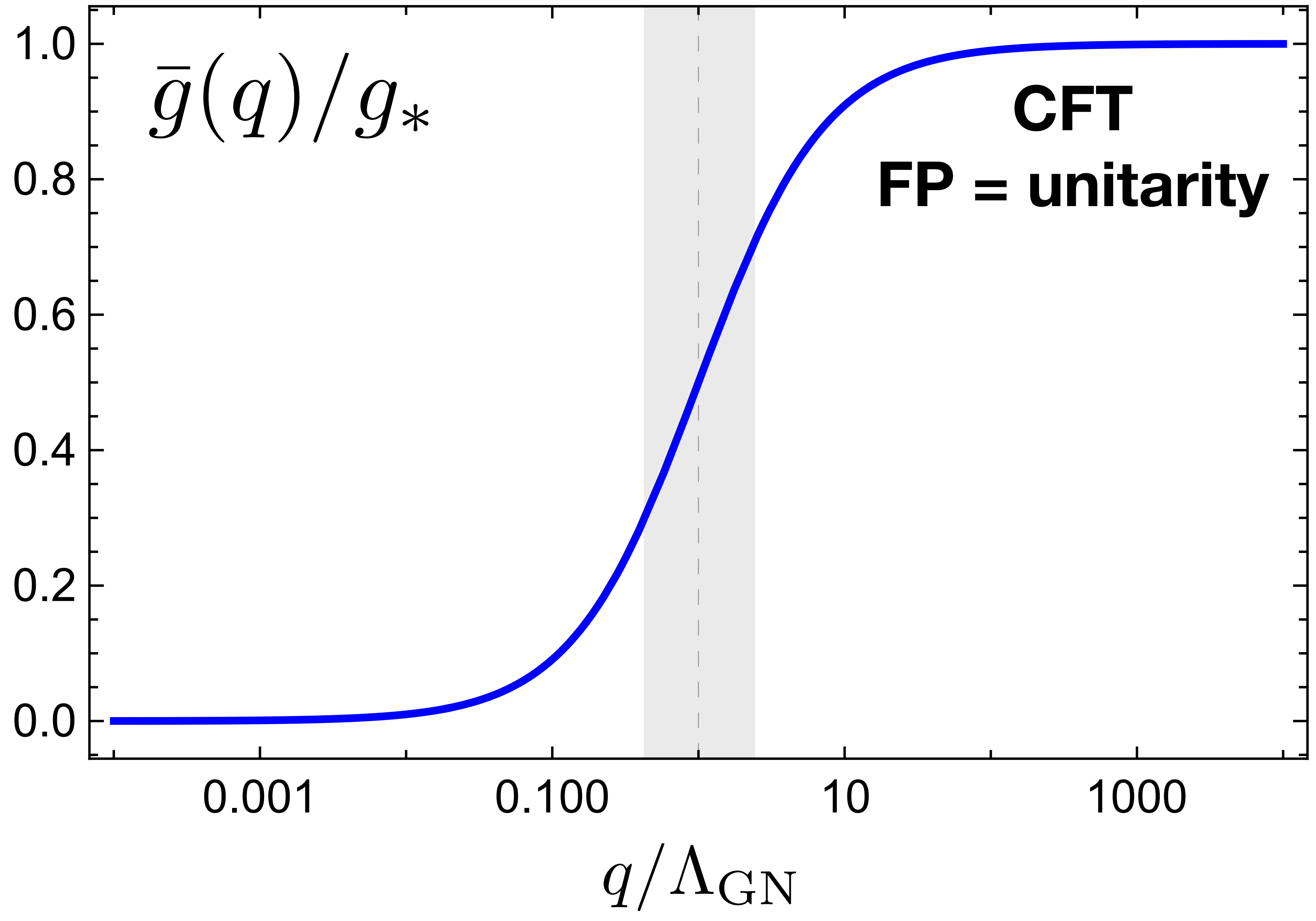
2+1 dimensions: 1/*N* renormalisable & conformal at high energy

Wilson ’73
Gross, Neveu ’74
Parisi ’75
Gawędzki, Kupiainen ’85
TIP! **Rosenstein, Warr, Park ’88 (PRL)**
Rosenstein, Warr, Park ’91 (Phys.Rept.)
Gat, Kovner, Rosenstein, Warr ’90
de Calan, Faria Da Veiga, Magnen, Seneor ’91
+ many more



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A toy EFT

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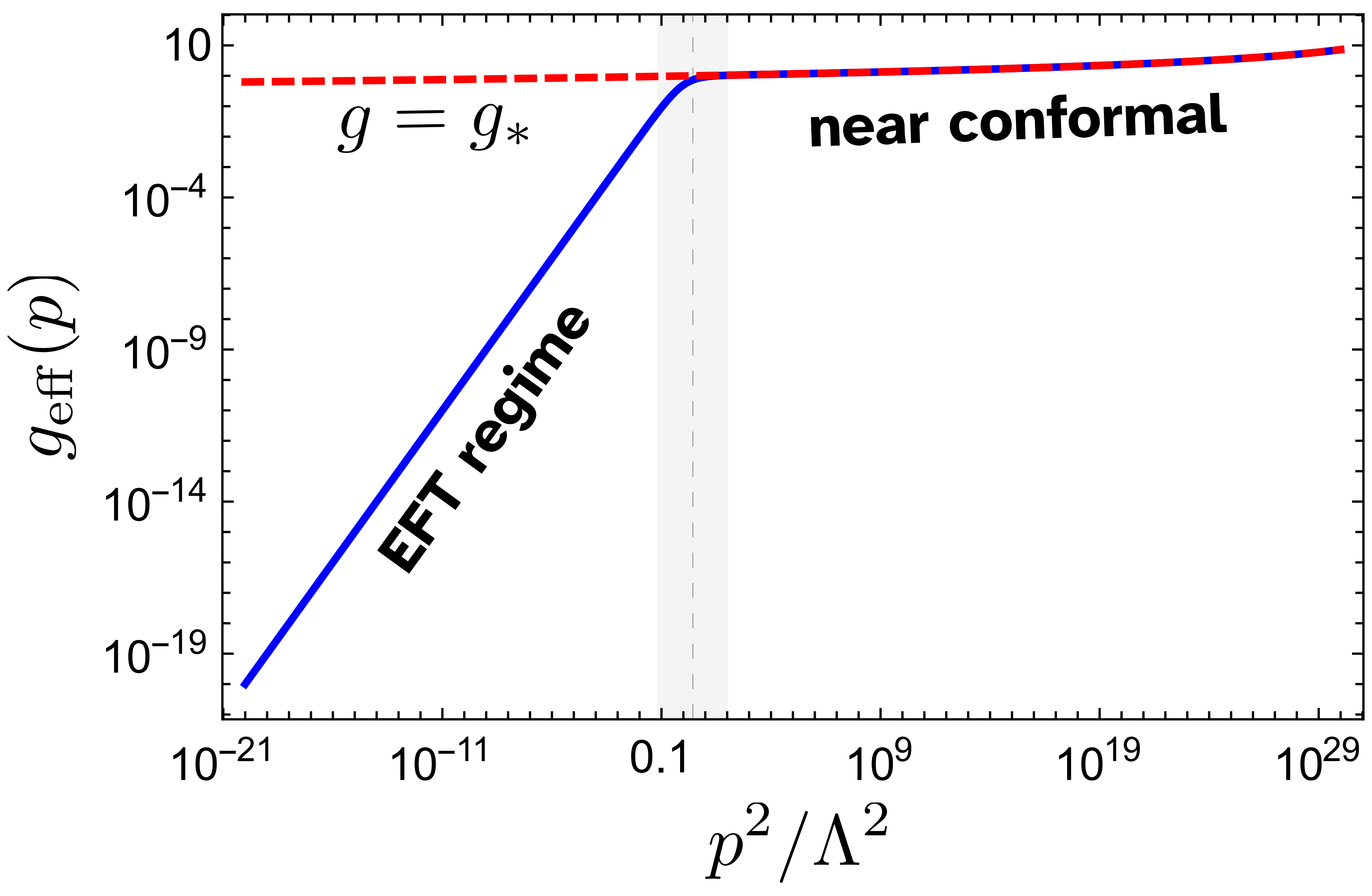
3+1 dimensions: $1/N$ renormalisable & *near*-conformal at high energy

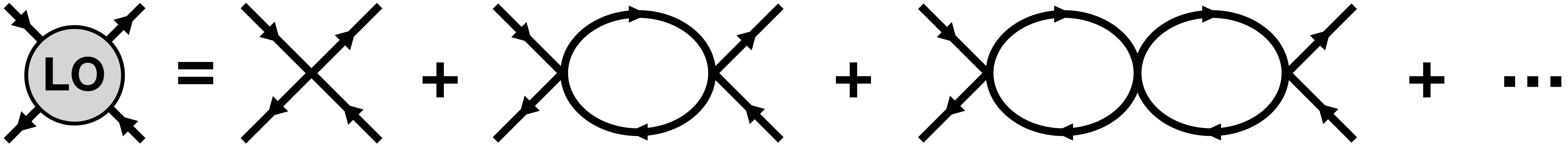
Formal renormalisability in 1/N dictates:

- tower of higher-derivative operators
- eight-fermion interactions (but no higher)
- only three fundamental parameters



Large-*N* diagrammatics

Four-point function:



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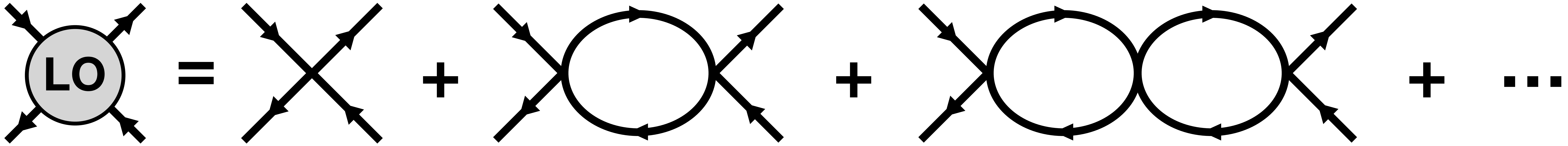
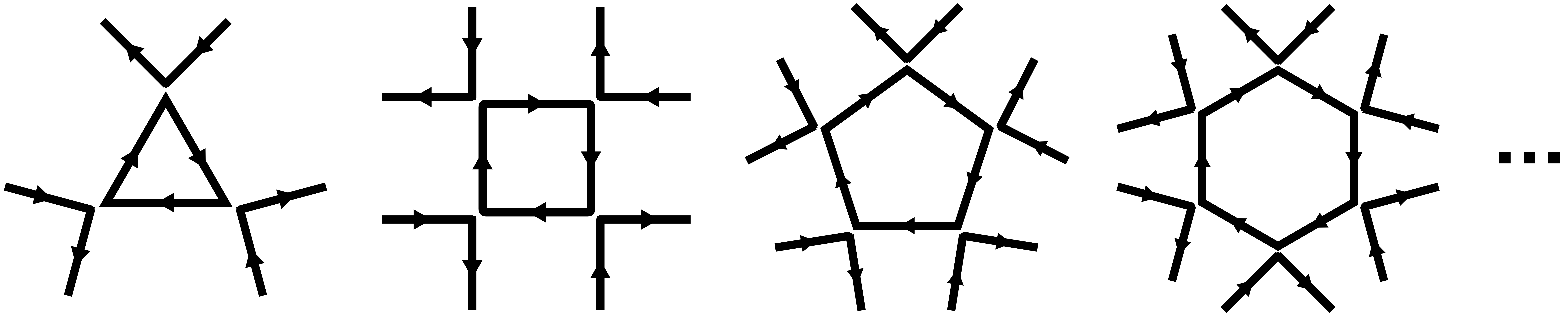


Higher-point functions:



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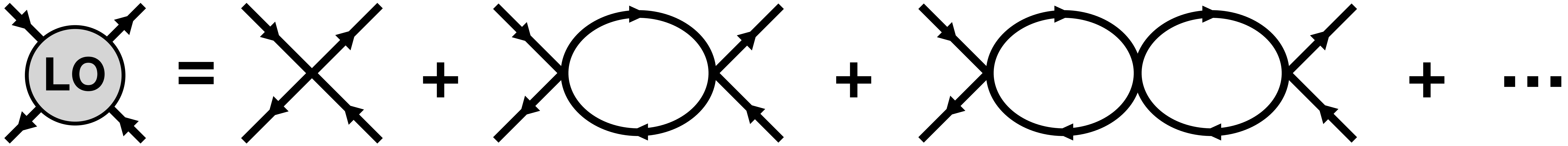

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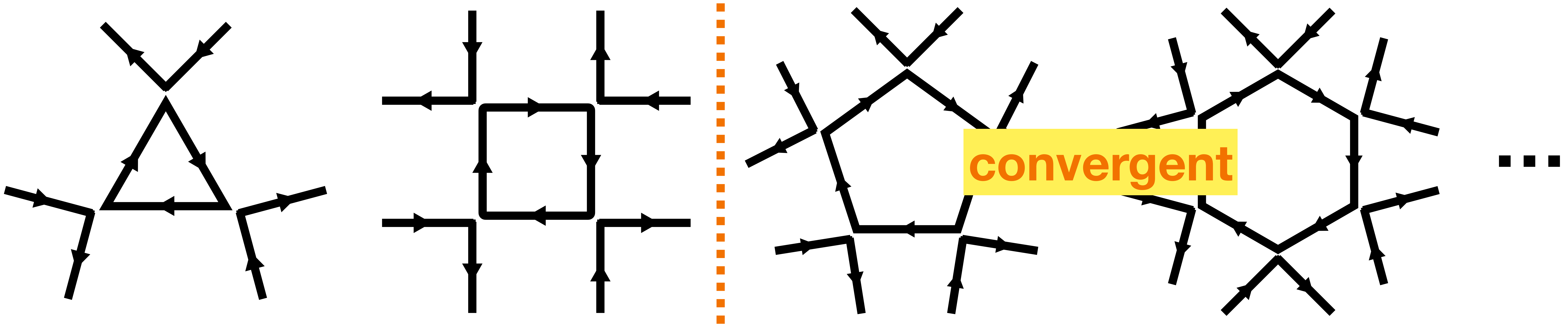

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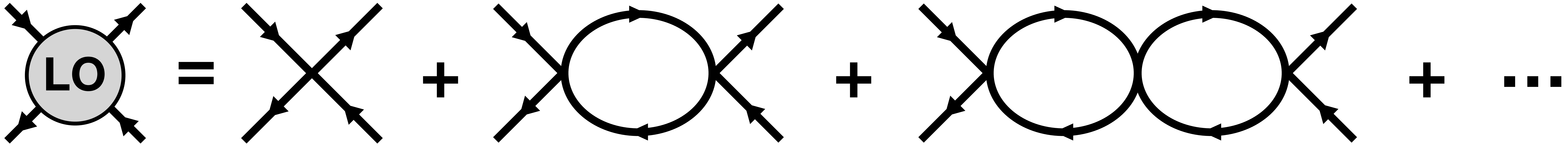

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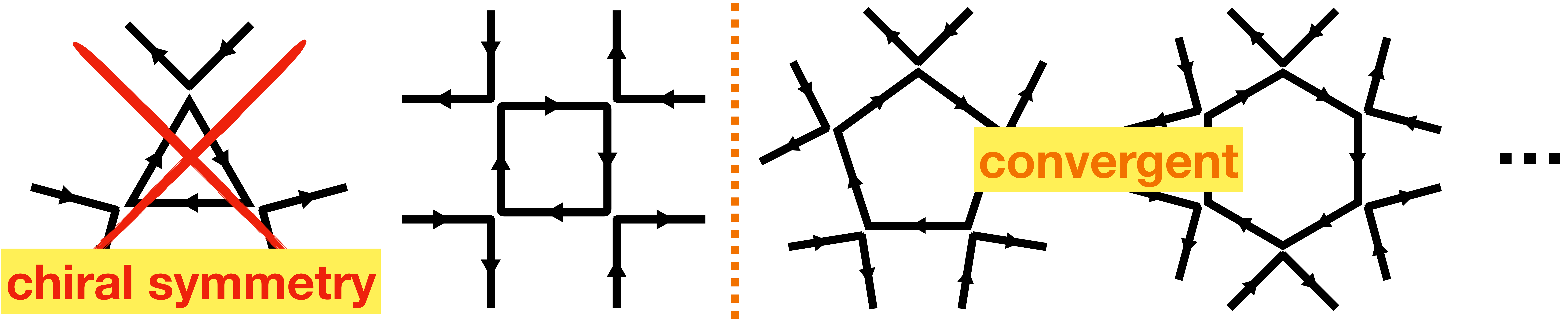

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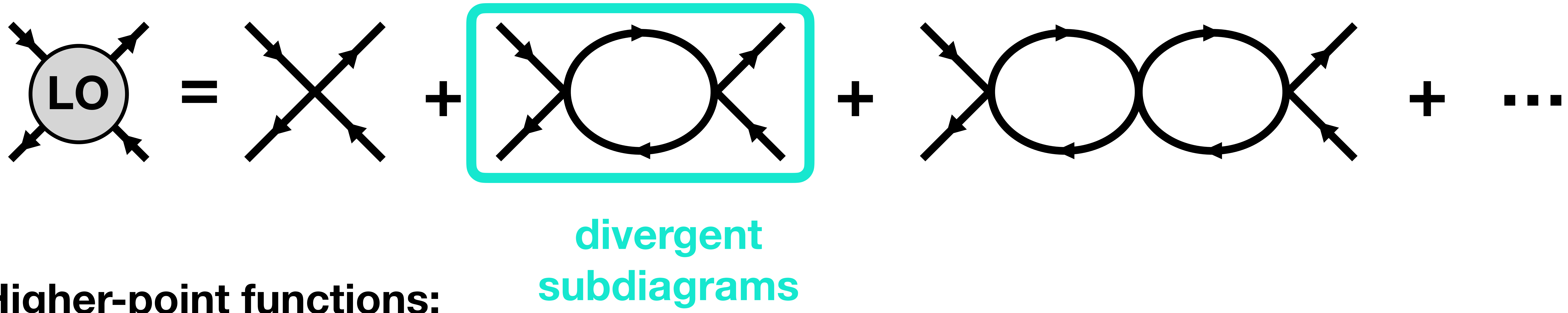

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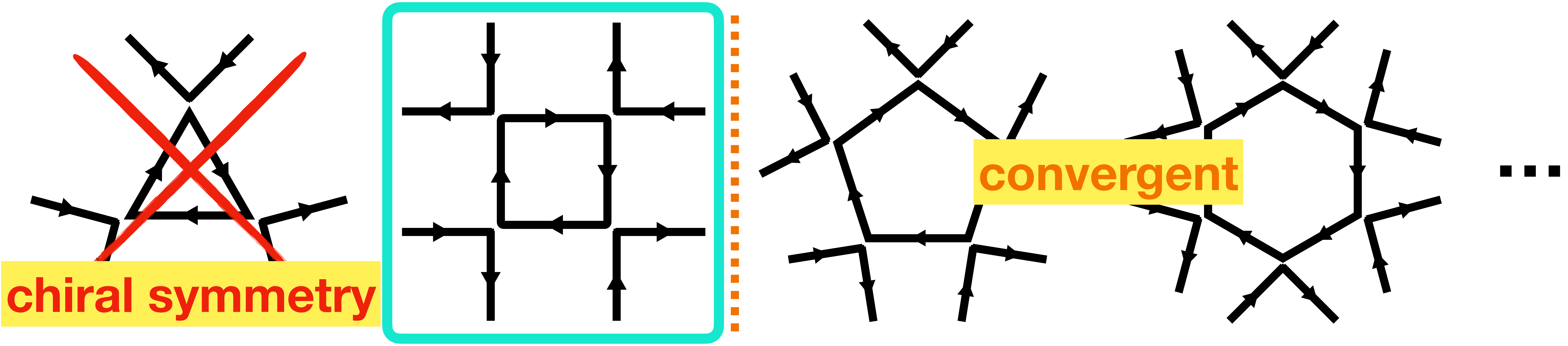
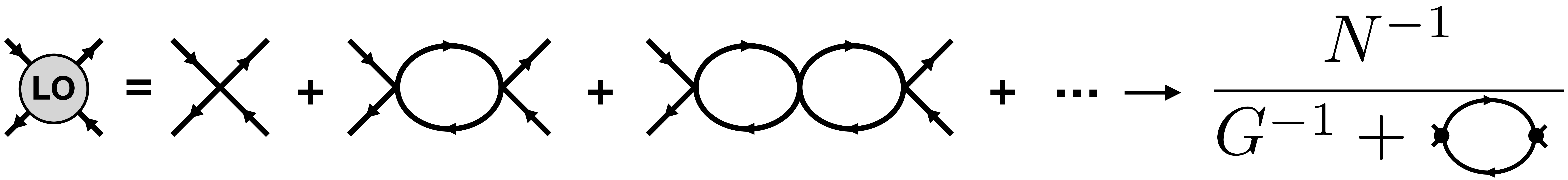
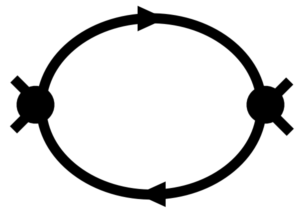


Four-point function



 $\sim \Lambda^2 + p^2 \ln \Lambda$ **infinite tower of higher-derivative divergences**

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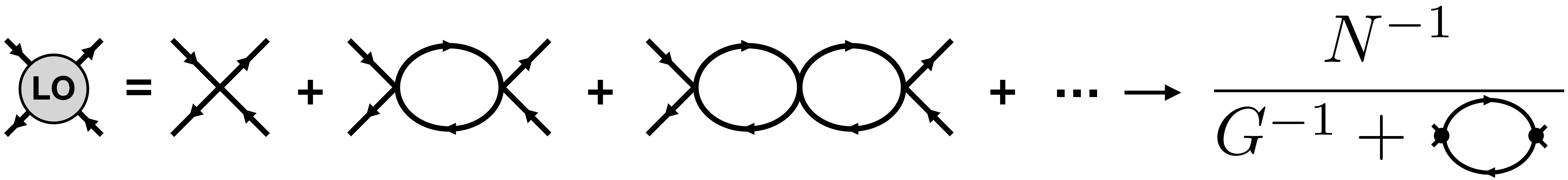

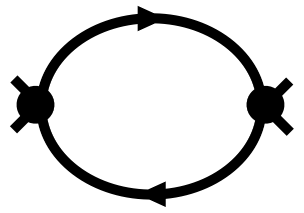 $\sim \Lambda^2 + p^2 \ln \Lambda$ **infinite tower of higher-derivative divergences**

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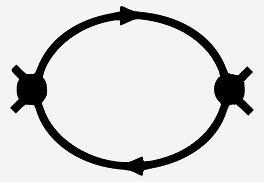
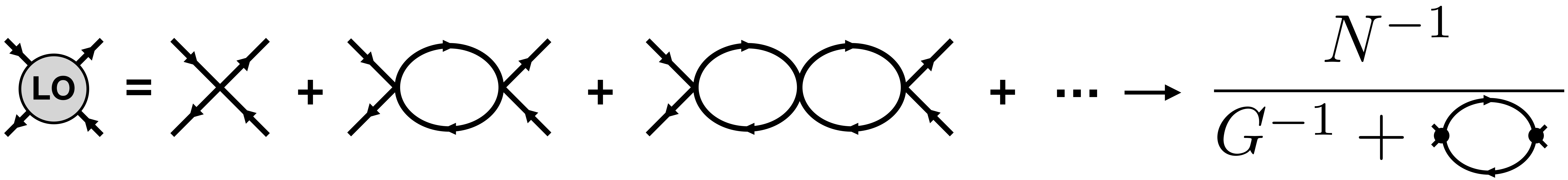
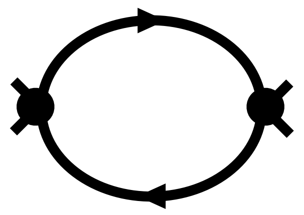
$F(p^2)^2 \times$ 

vertices factorise @ LO

Four-point function



 $\sim \Lambda^2 + p^2 \ln \Lambda$ **infinite tower of higher-derivative divergences**

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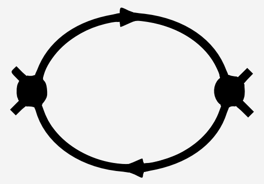
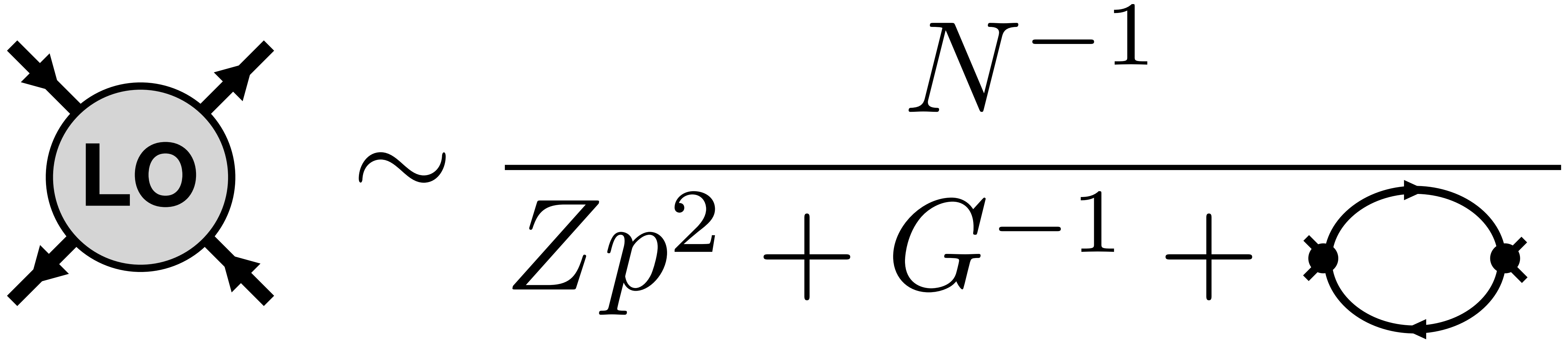
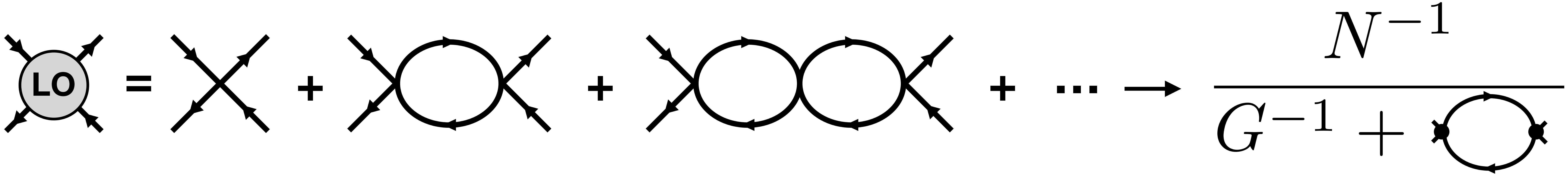
$F(p^2)^2 \times$ 

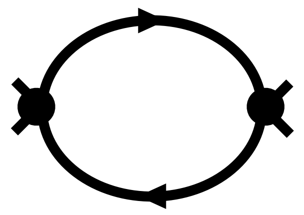
vertices factorise @ LO

$$F(p^2) = \frac{1}{Zp^2 + G^{-1}} \;\rightarrow$$



Four-point function



 $\sim \Lambda^2 + p^2 \ln \Lambda$ **infinite tower of higher-derivative divergences**

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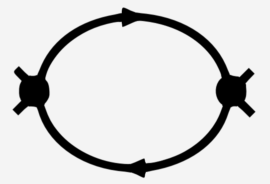
$F(p^2)^2 \times$ 

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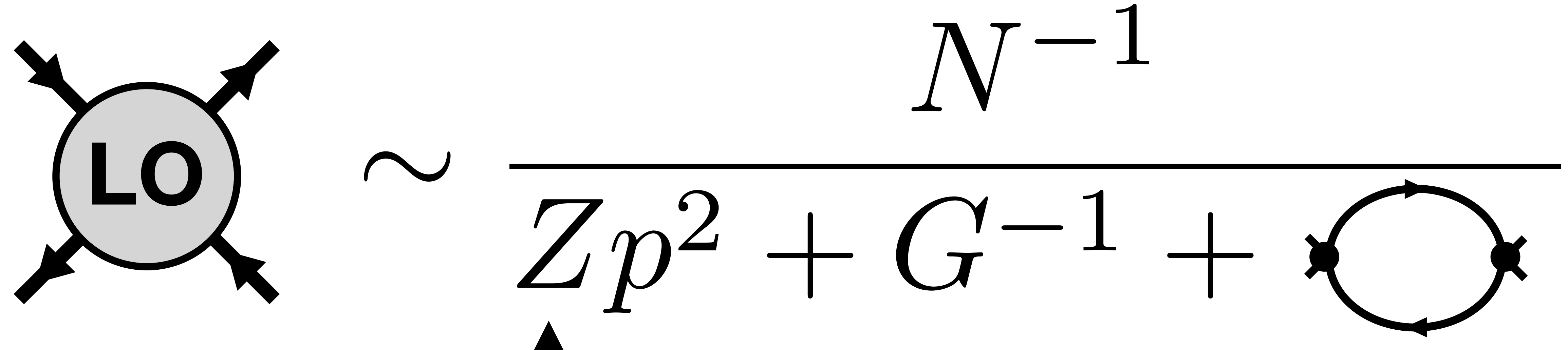
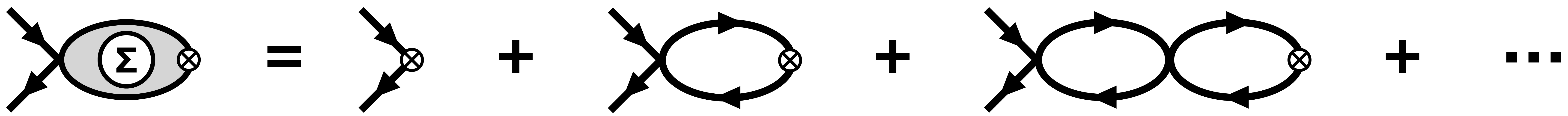


subtract infinitely many higher-derivative divergences with one additional parameter

Eight-point function



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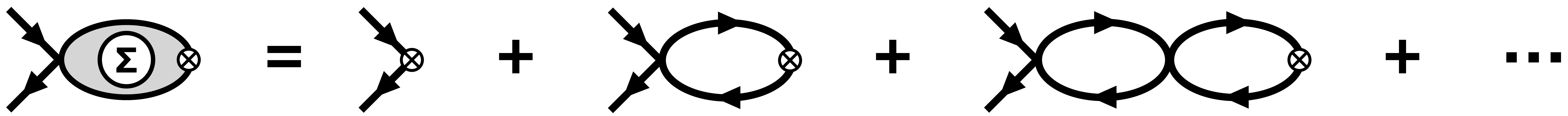
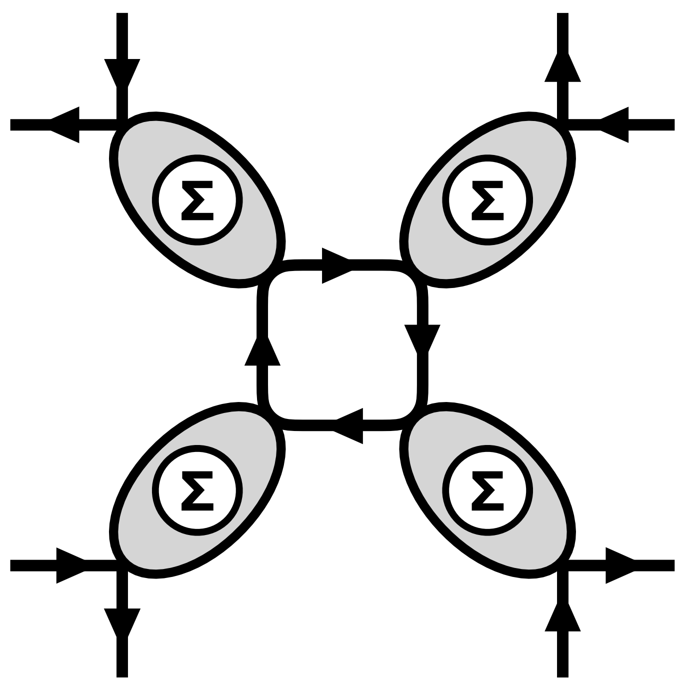
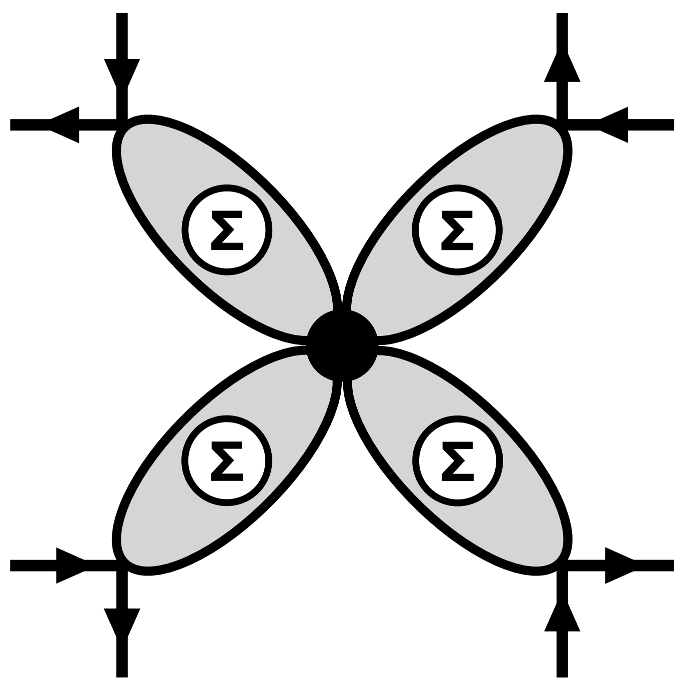


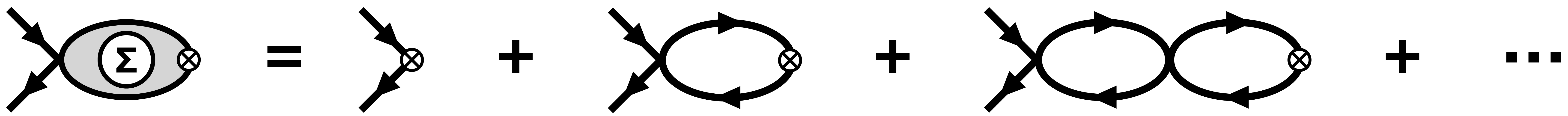


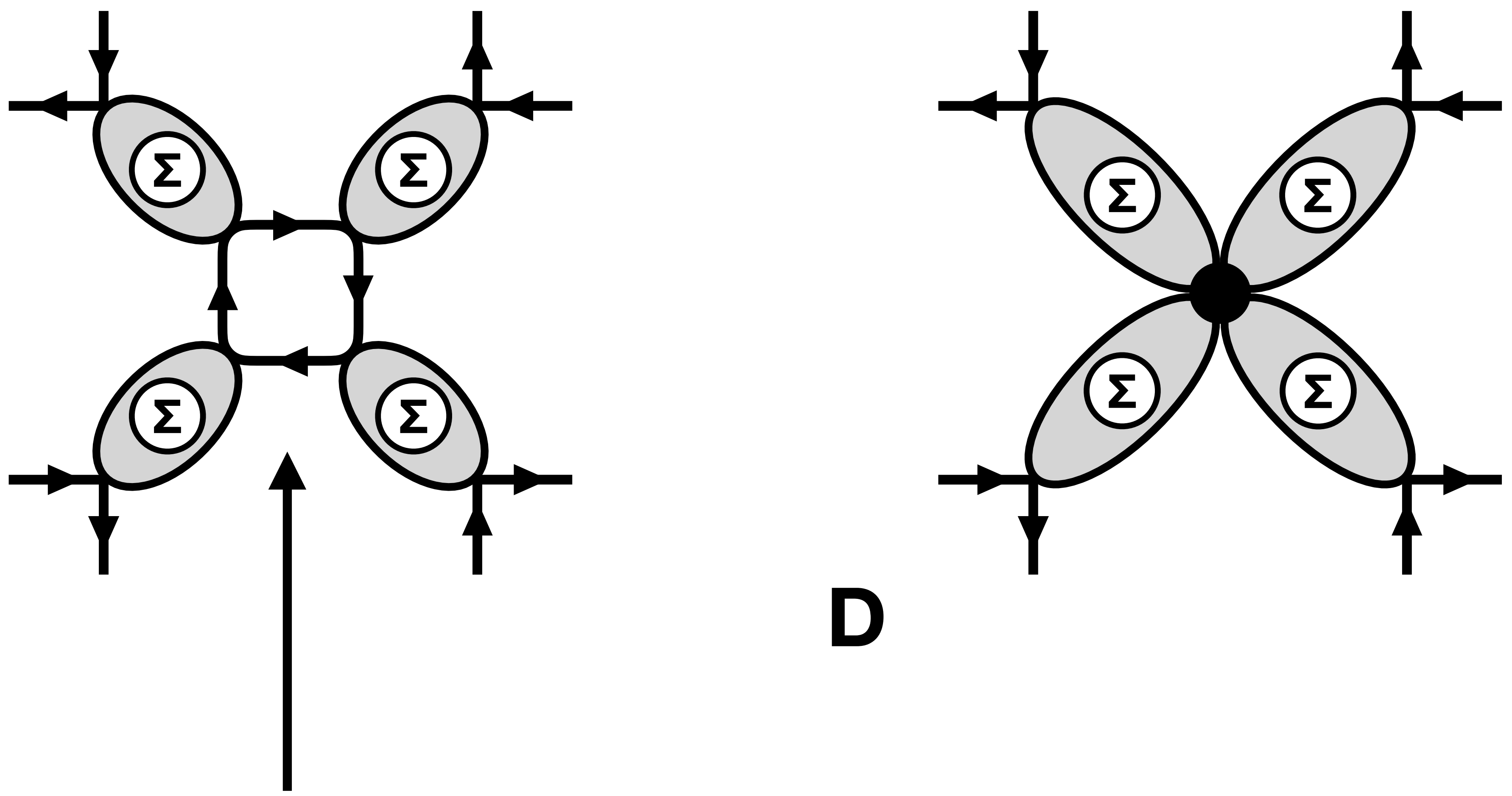
C



D

Eight-point function

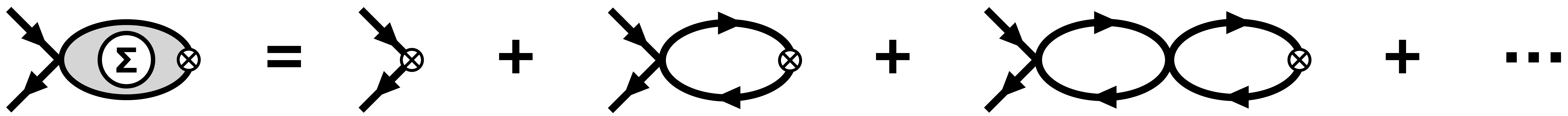


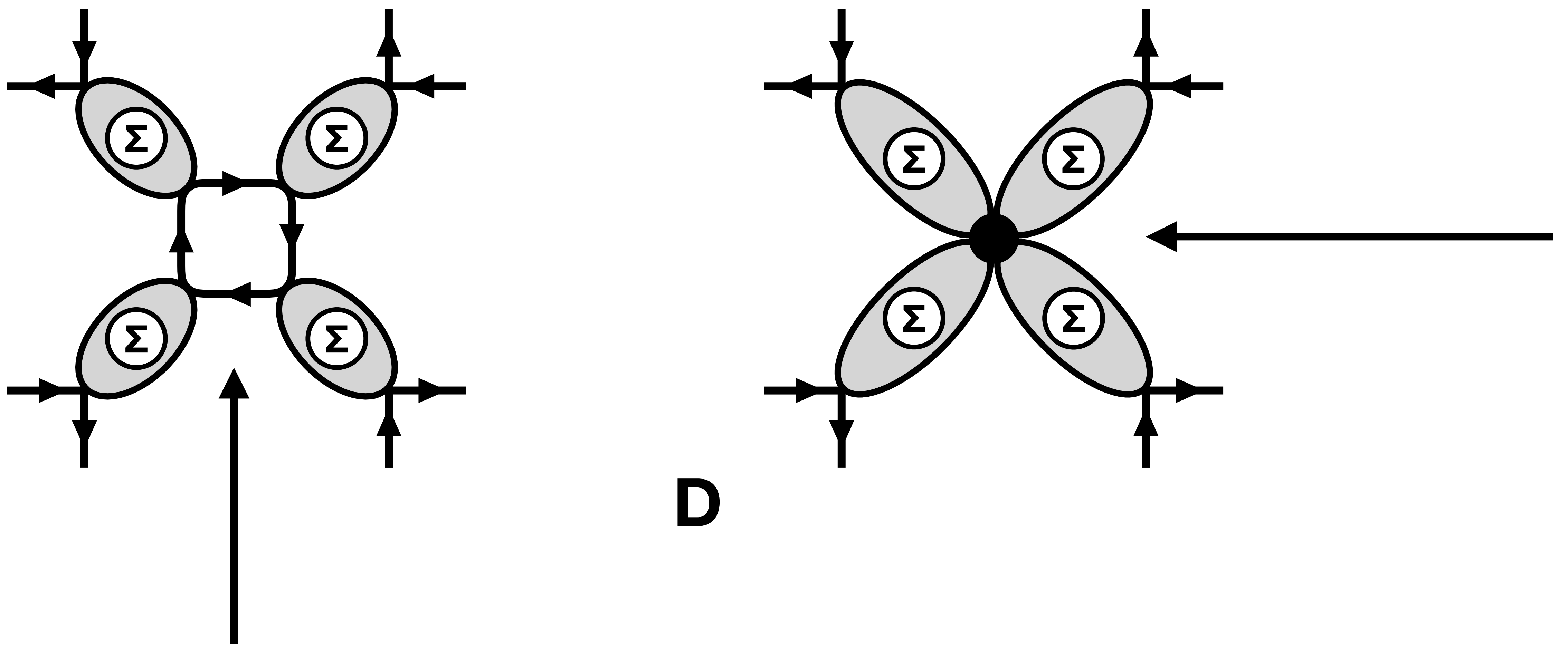


C **D**

$$\sim \ln \Lambda$$

Eight-point function





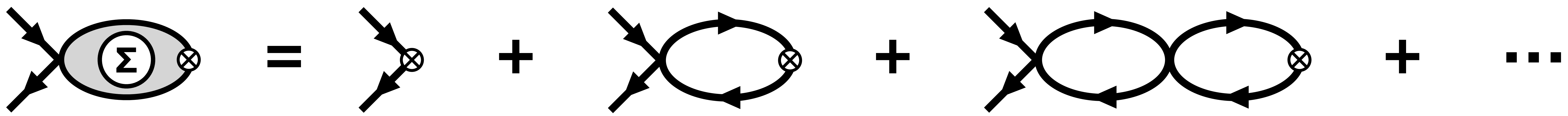
C

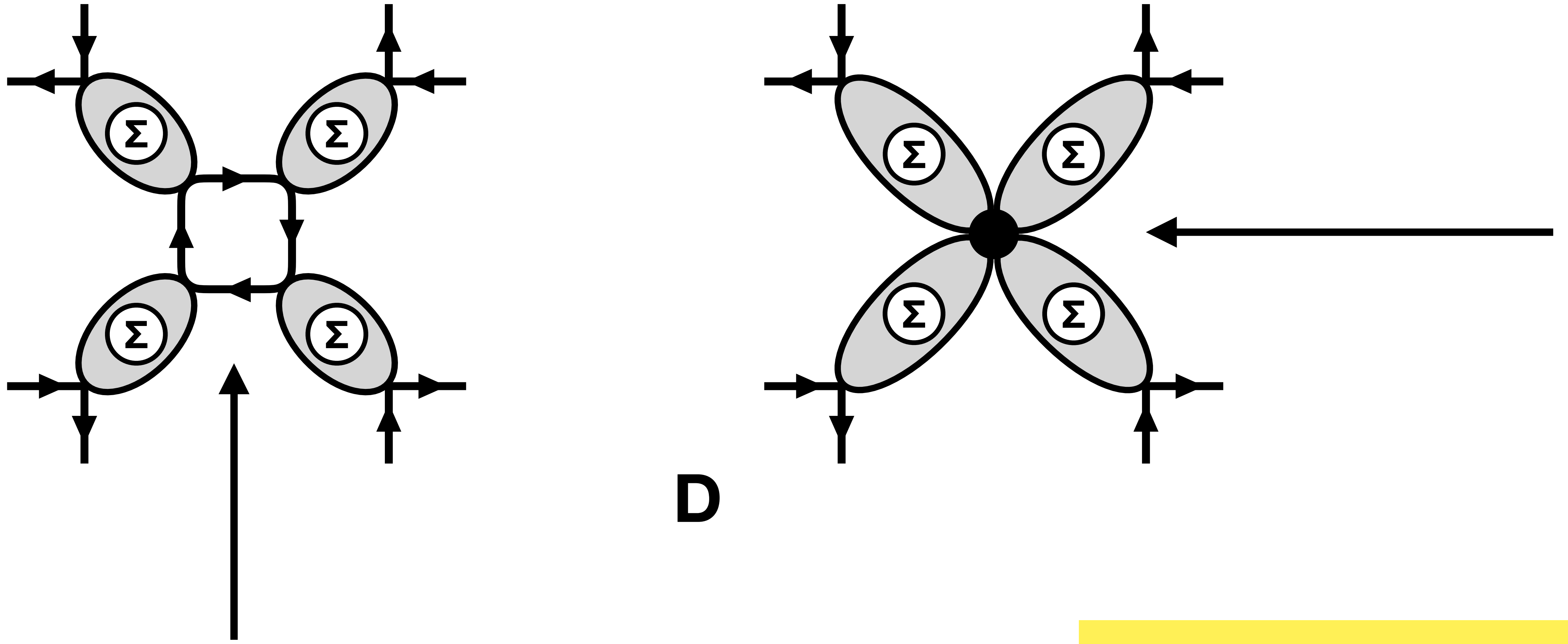
D

$$\frac{\lambda}{4!N^3}\left[F(-\partial^2)(\bar{\psi}_a\psi_a)\right]^4$$

$$\sim \ln \Lambda$$

Eight-point function





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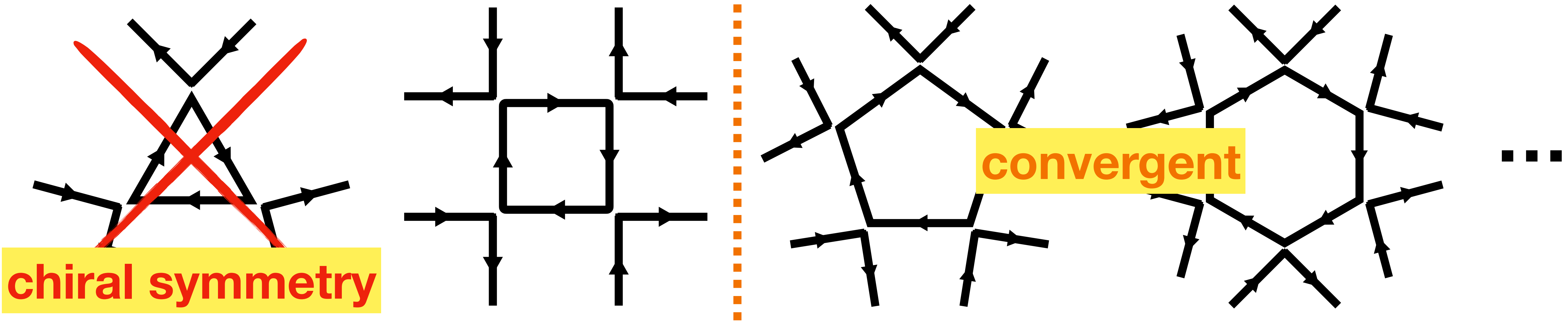
Everything @ LO = products of bubble sums with:



>> THEORY RENORMALISED. <<

Renormalisable four-fermion theory

$$\bar{\psi}_a \not{\partial} \psi_a - \frac{1}{2N}(\bar{\psi}_a\psi_a)F(-\partial^2)(\bar{\psi}_b\psi_b) + \frac{\lambda}{4!N^3}\left[F(-\partial^2)(\bar{\psi}_a\psi_a)\right]^4$$

$$F(p^2) = \frac{1}{Zp^2 + G^{-1}} \approx G \quad \text{for} \quad Zp^2 \ll G^{-1}$$

- **Predictive:** 3 parameters Z, G, λ
- **Tower** of higher derivative 4F + 8F
- **NO higher** (10F, 12F, …)

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cf. 3D: *Park, Rosenstein, Warr, Phys.Rept. 1991*

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Renormalised couplings:

$$g(\mu) = \mu^2\, G(\mu)$$

$$Z(\mu) \qquad \lambda(\mu)$$

Running couplings

non-universal:

$$\beta_g(g) = 2g\left(1 - \frac{g}{g_*}\right)$$

$$\underbrace{\hphantom{\beta_g(g) = 2g\left(1 - \frac{g}{g_*}\right)}}_{\textbf{UV fixed point}}$$

e.g. MOM scheme: $g_* = 8\pi^2$

universal:

$$\beta_Z = -\frac{1}{4\pi^2} \qquad \beta_\lambda = -\frac{3}{\pi^2}$$

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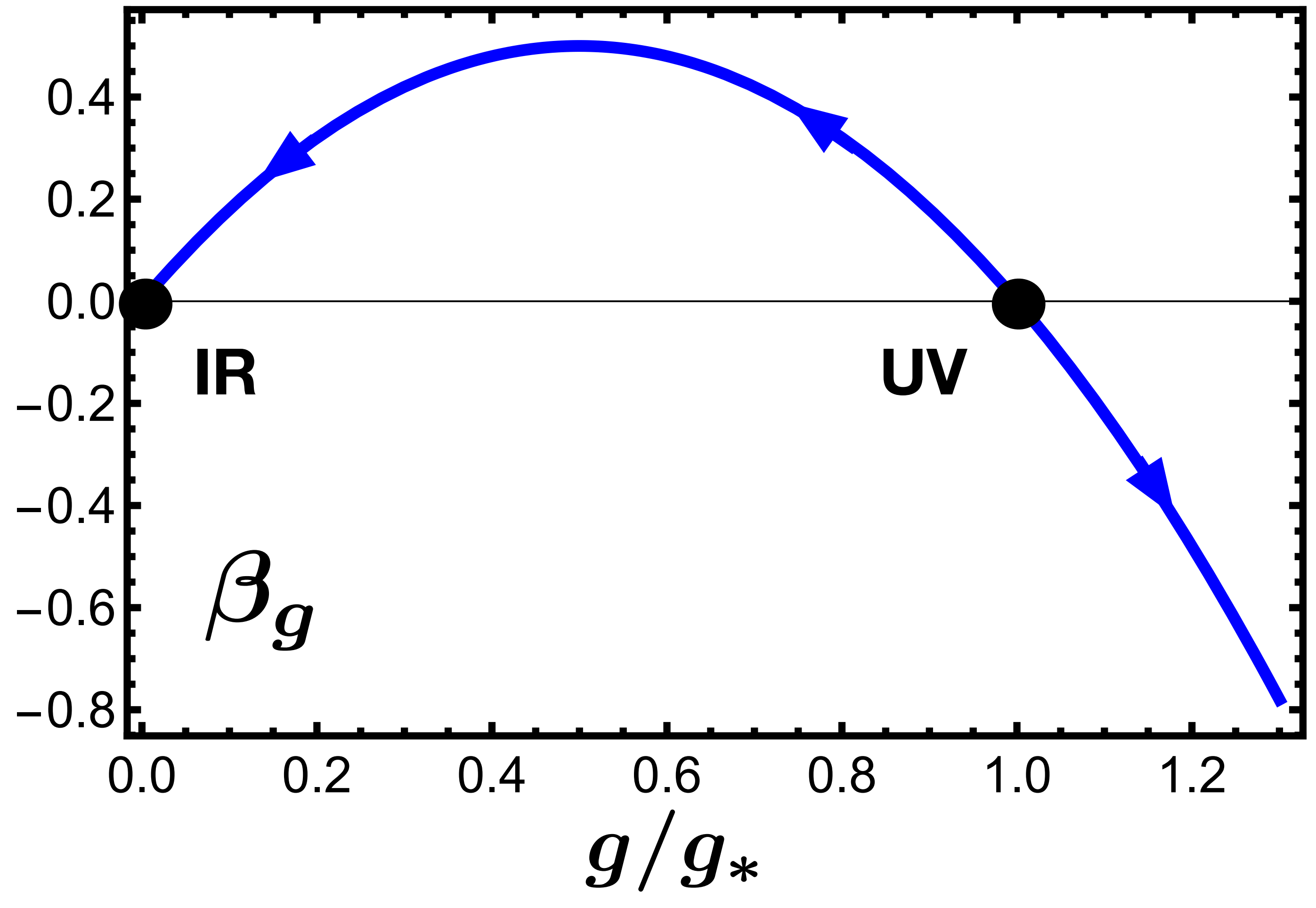

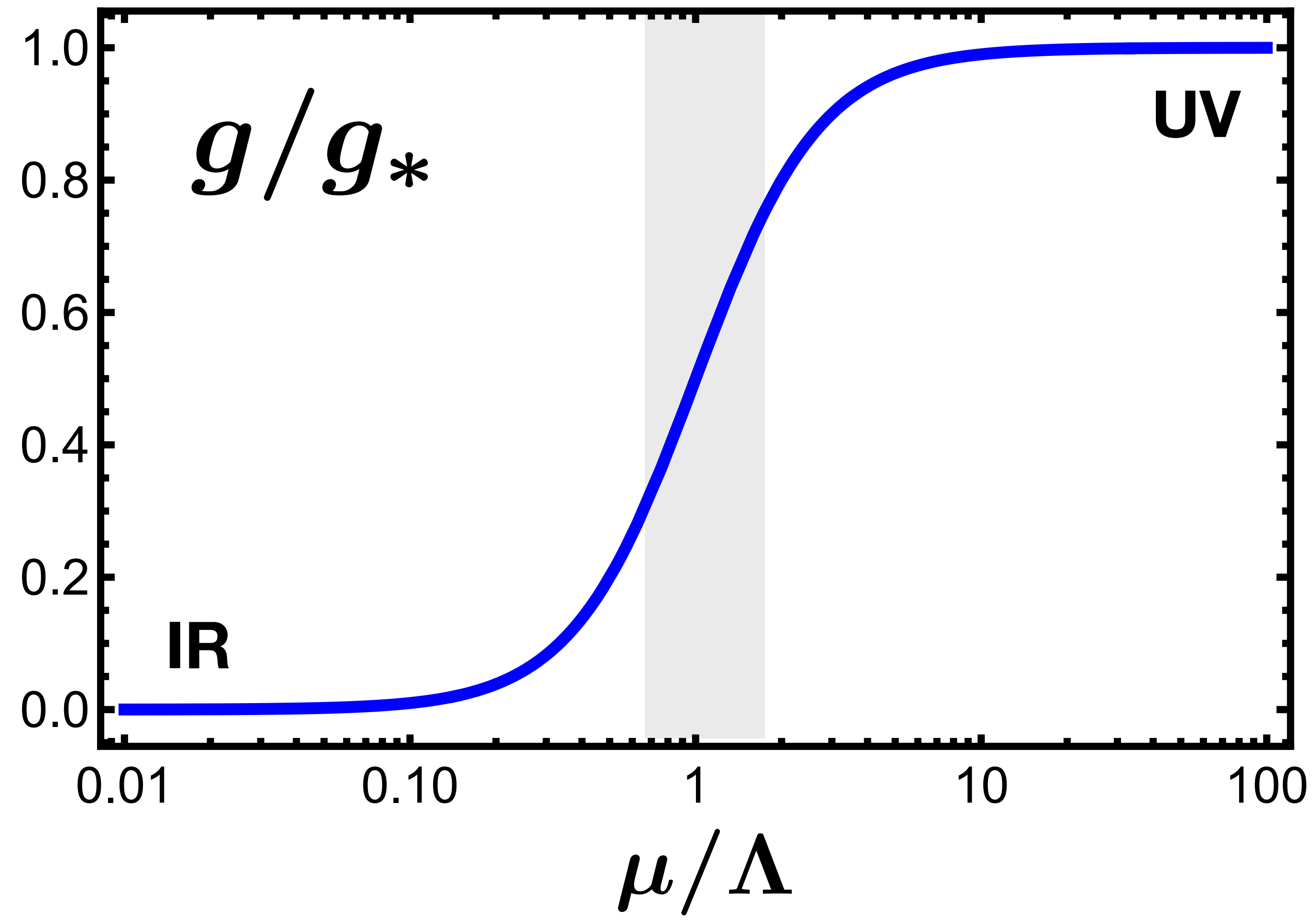

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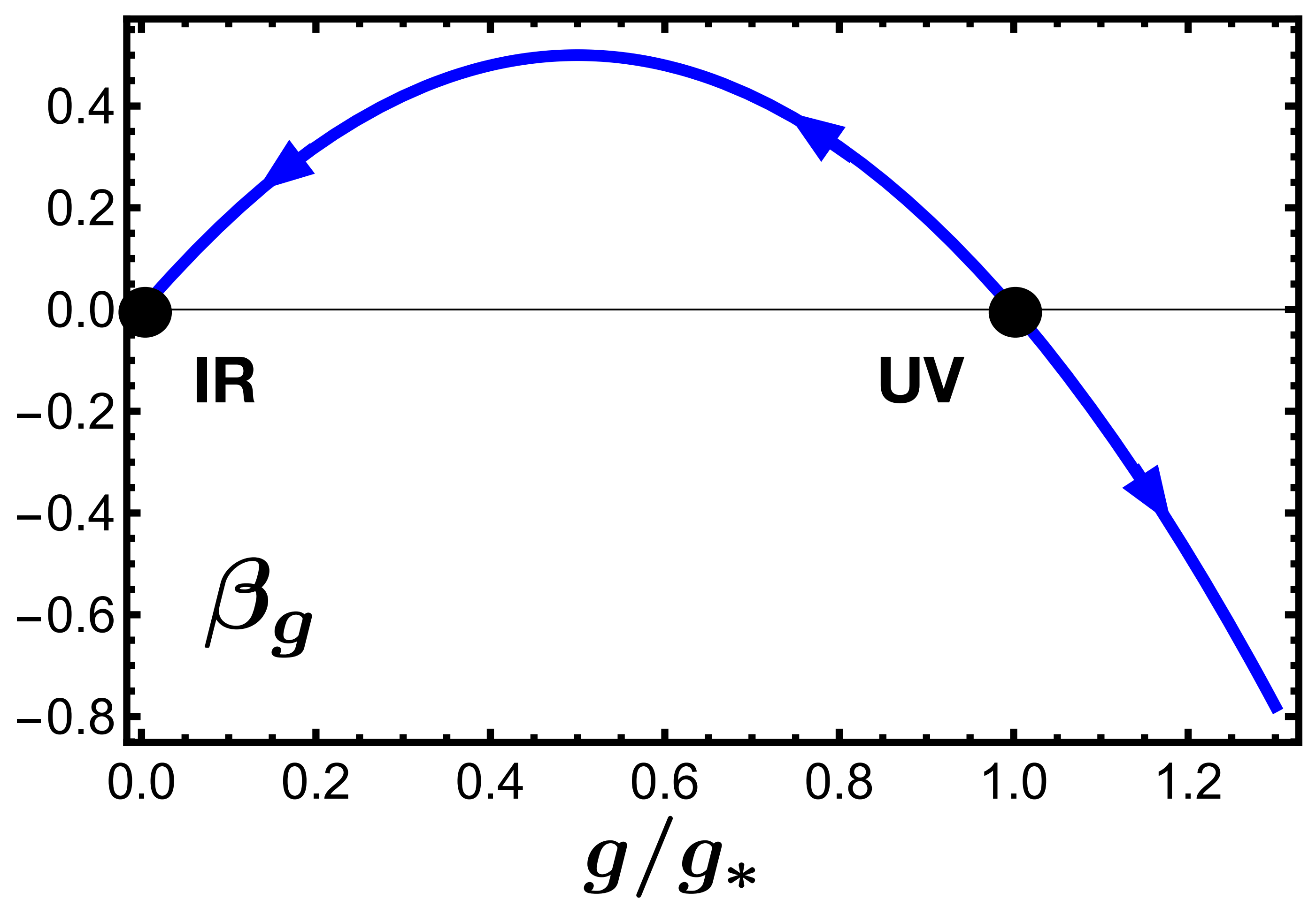

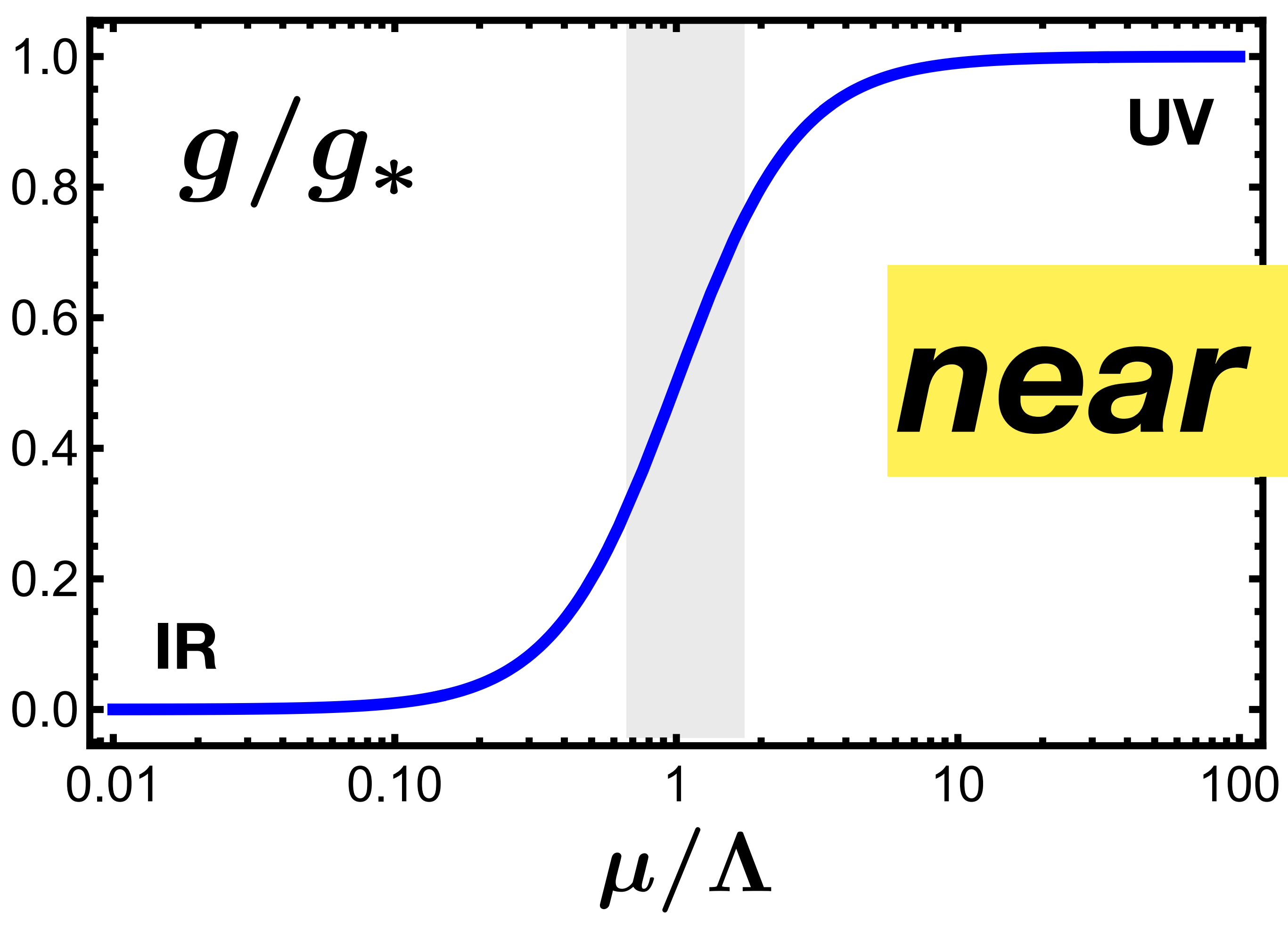

near **conformal in UV**

Running couplings

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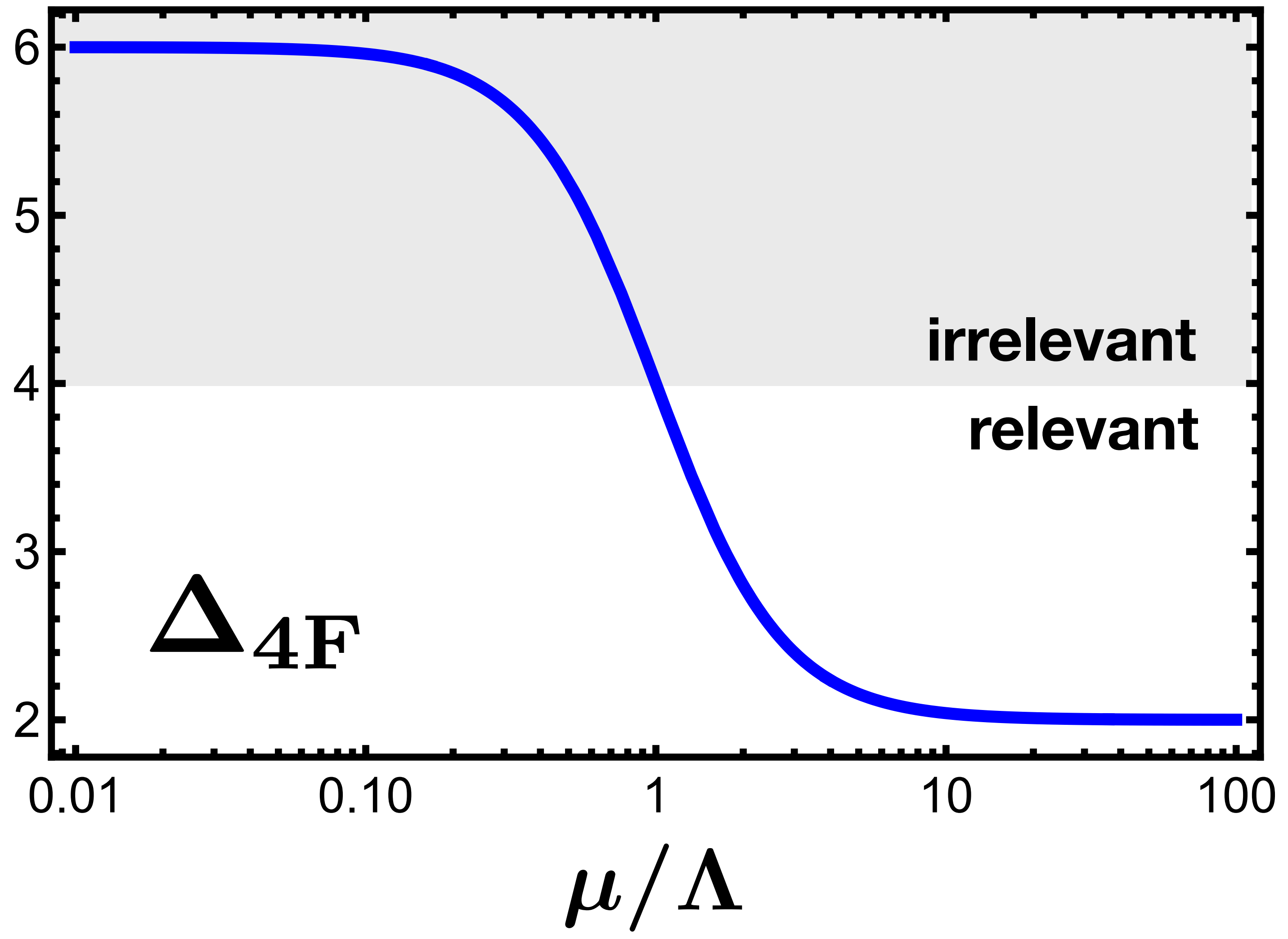

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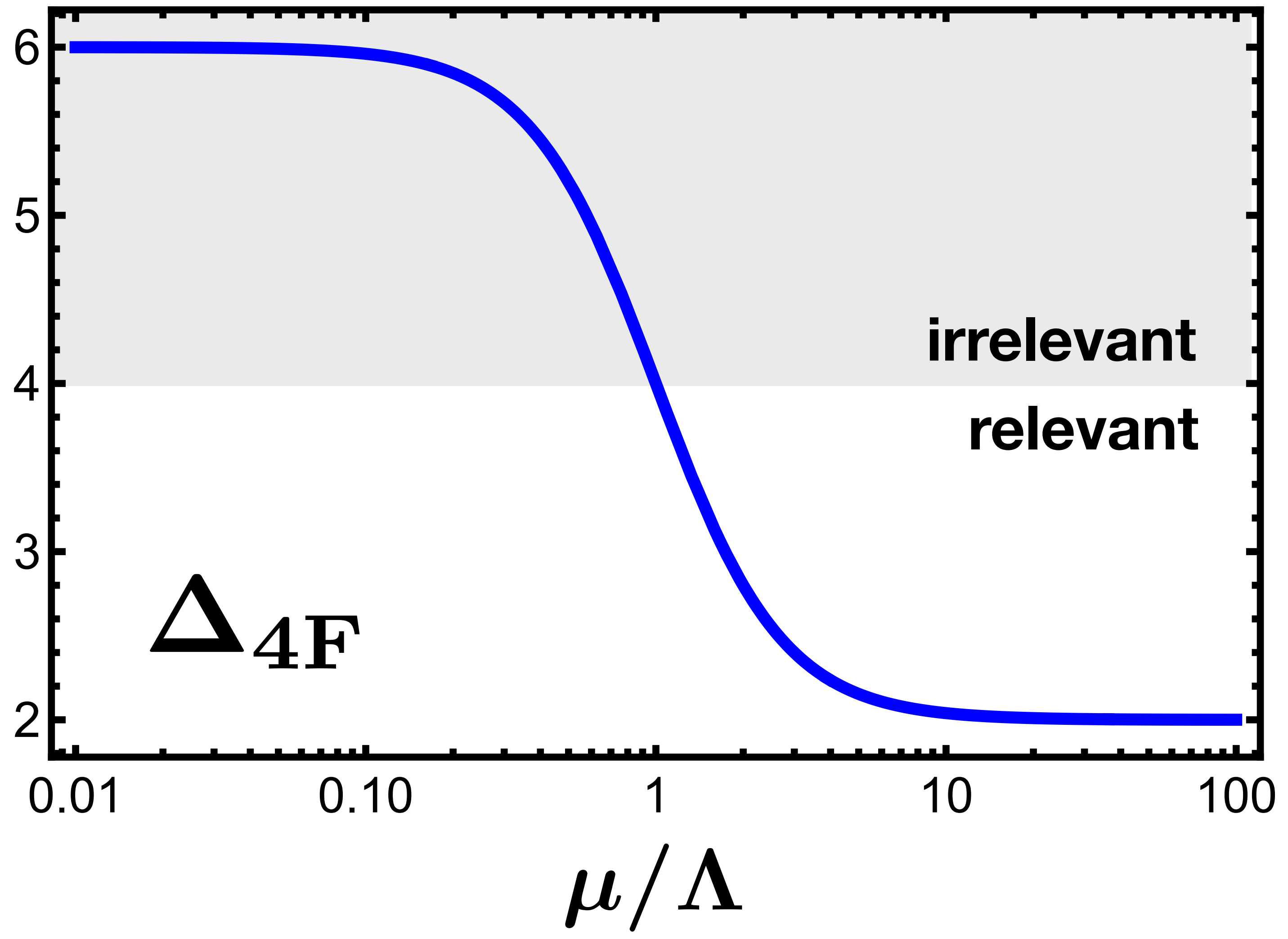
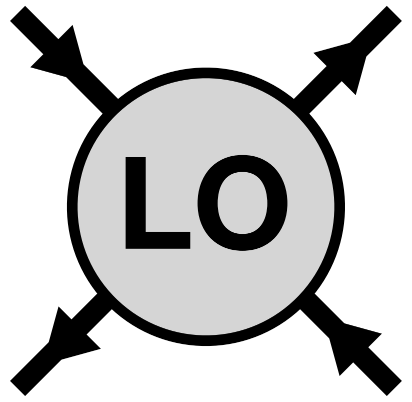


large anomalous dimensions:

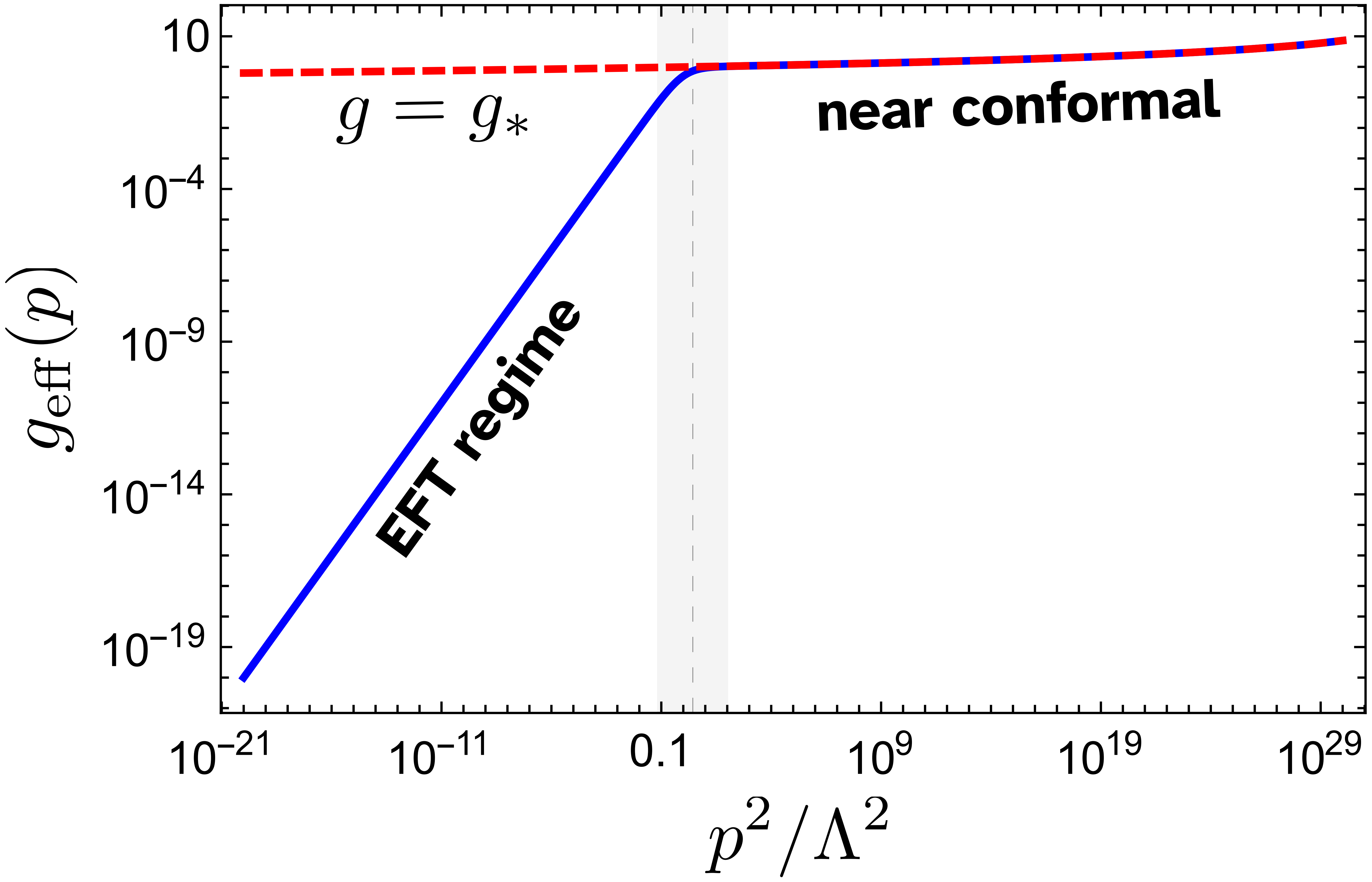
dim-6 becomes dim-2

EFT power counting breaks

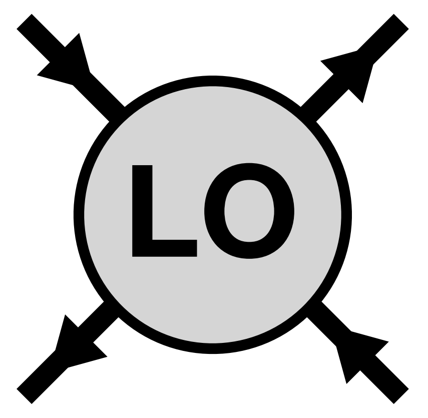
Near conformality & the EFT crossover



$$\sim \frac{N^{-1}}{p^2} g_{\rm eff}(p)$$



Near conformality & the EFT crossover



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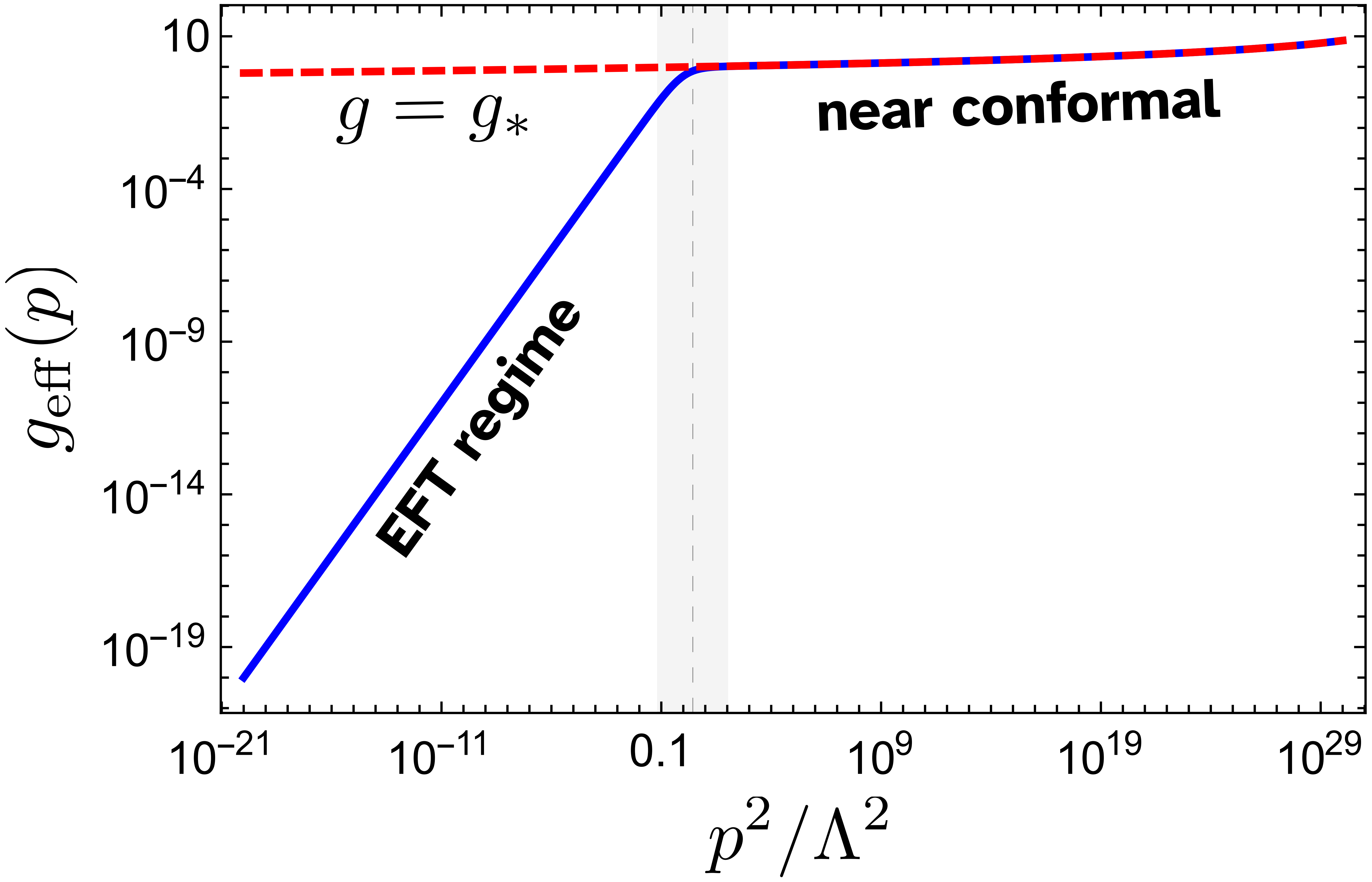
$$\boxed{g_{\text{eff}}(p) = \frac{1}{Z(p) + g^{-1}(p)}}$$



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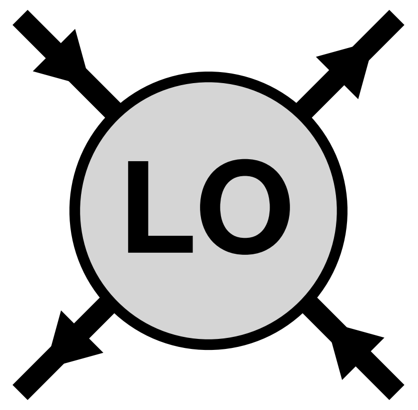

$$\sim \frac{N^{-1}}{p^2}\, g_{\text{eff}}(p)$$

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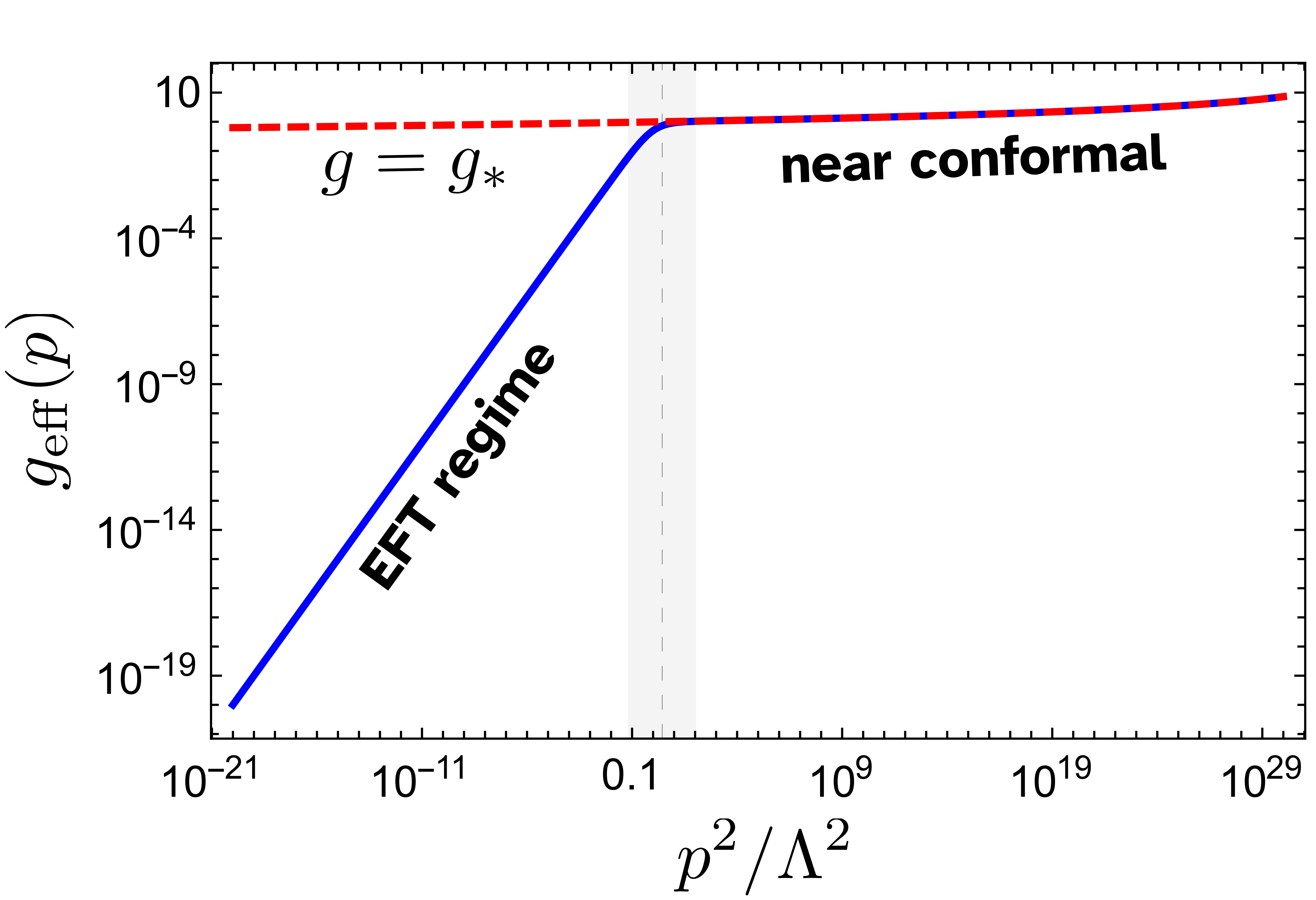
EFT scale marks crossover:
onset of strong coupling, higher derivative terms become important



Perturbative dual description

Functional Legendre transform maps to Yukawa theory at infinite N:

$$\bar{\psi}_a(i\partial\!\!\!/ - y\phi)\psi_a + \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{1}{4!}\lambda_\phi\phi^4$$

e.g. Weinberg hep-th/9706042

(see also Zinn-Justin ’91; Hasenfratz+ ’91)

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$$G = \frac{y^2}{m^2} \qquad Z = \frac{1}{y^2} \qquad \lambda = \frac{\lambda_\phi}{y^4}$$

CCH, Litim, *in prep*.

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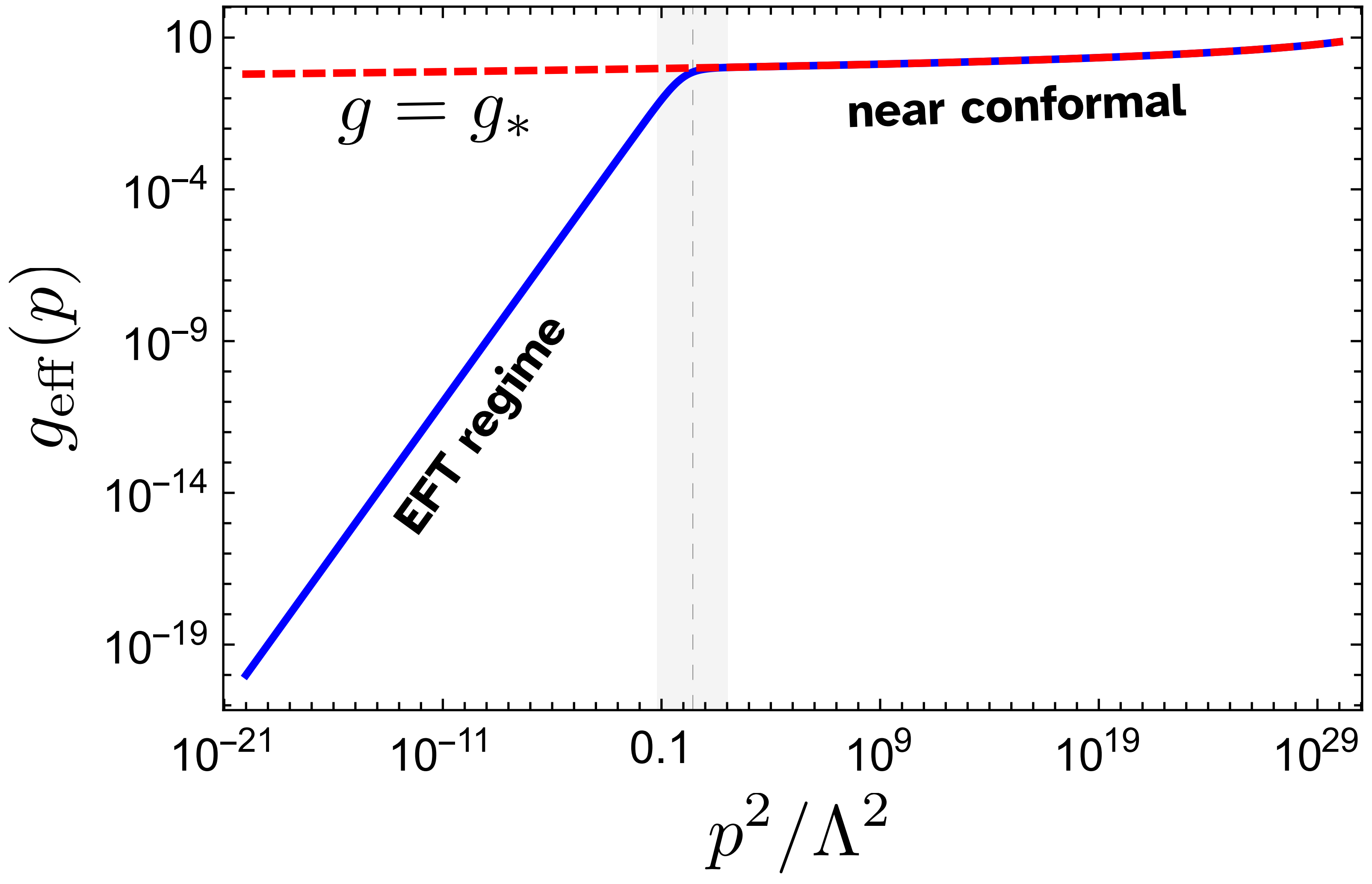
Four-fermion fixed point = massless separatrix in Yukawa theory.

$$\mu^2 \frac{y^2(\mu)}{m^2(\mu)} = g_*$$

Importance of RG scheme



$$\beta_g(g) = 2g\left(1 - \frac{g}{g_*}\right)$$

Physics is universal, e.g. exact amplitudes, but the beta function is not.

- g^* is a **scheme dependent** constant
- **MS** corresponds to limit ***g* to infinity***
- Not a good choice for **approximate calculations** at intermediate scales
- Convergence improved by **non-minimal schemes** (WIP + Yannick Kluth @ Toronto)

Summary

Four-fermion interactions can UV-complete themselves

- Tower of higher derivative 4F + 8F operators, but only **3 parameters**
- **Near scale invariance** in UV with relevant 4F + marginal 8F
- Strong coupling sets in near EFT scale — scheme choice important

Further aspects

- **Landscape of theories:** symmetries, mom. dependence, … , SMEFT, …
- **Unitarity restoration:** (partial) RG fixed points in EFT $\Longleftrightarrow$ UV completions?
- **New perspective** on 4F fixed points in truncated FRG approximations