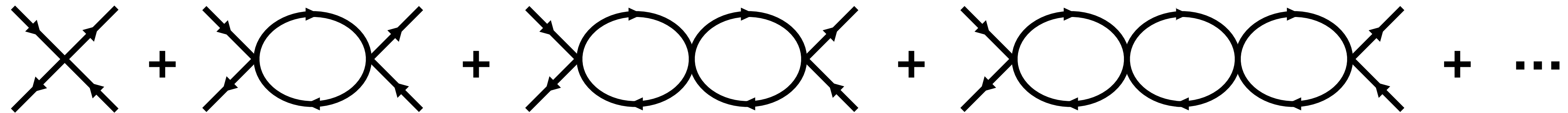


Renormalisable four-fermion interactions in four dimensions



Charlie Cresswell-Hogg

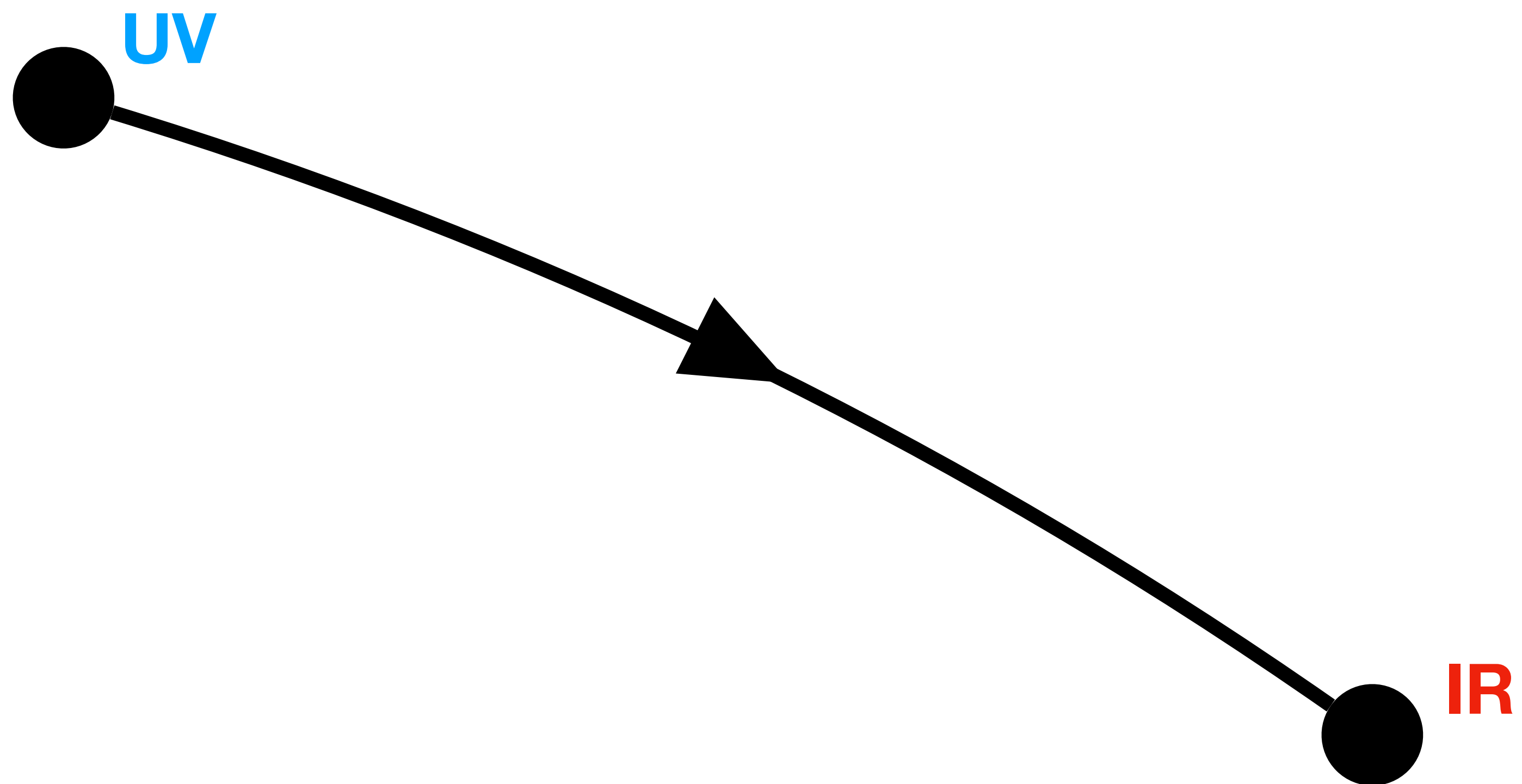
TU Dortmund / University of Sussex

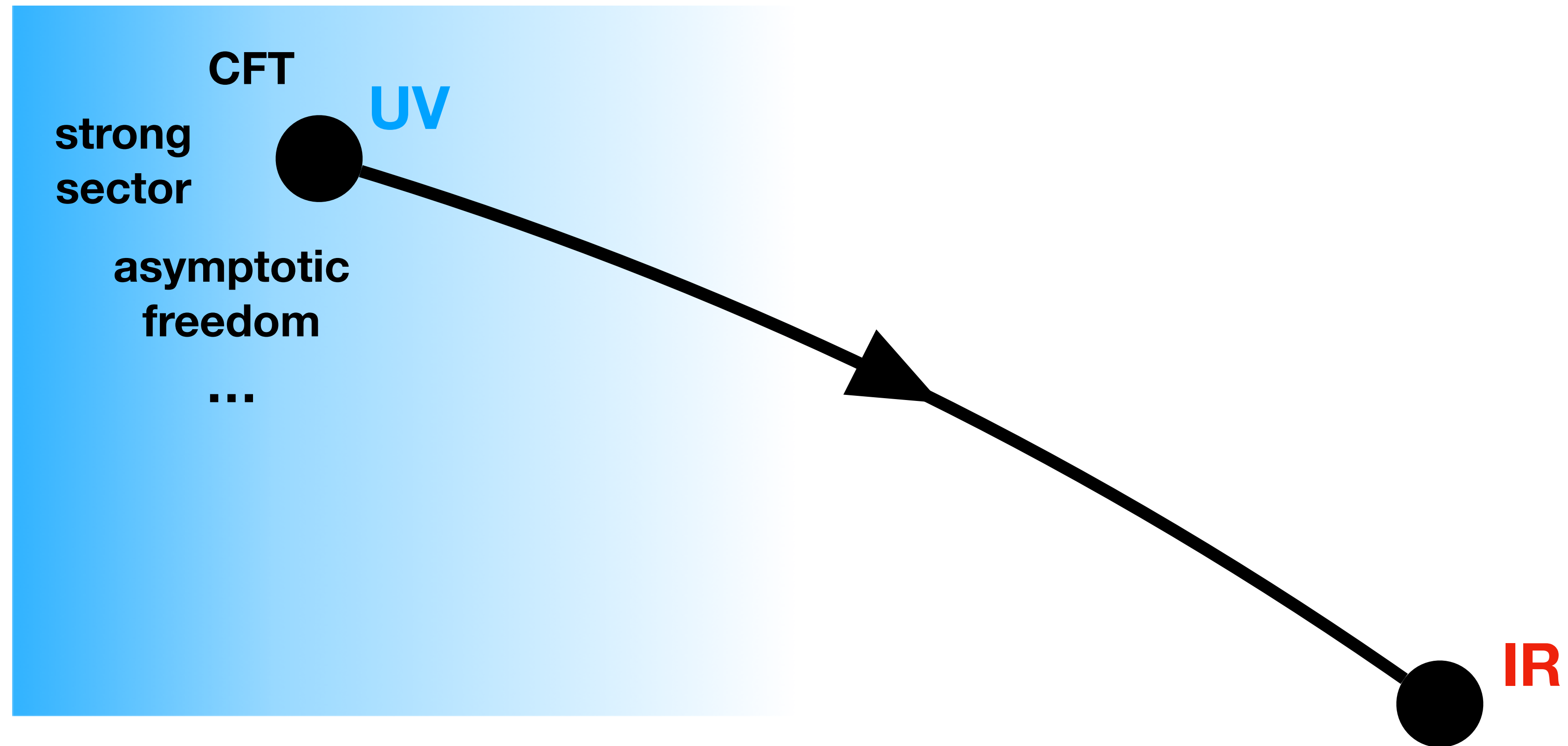
based on 2603.19357 with Daniel Litim

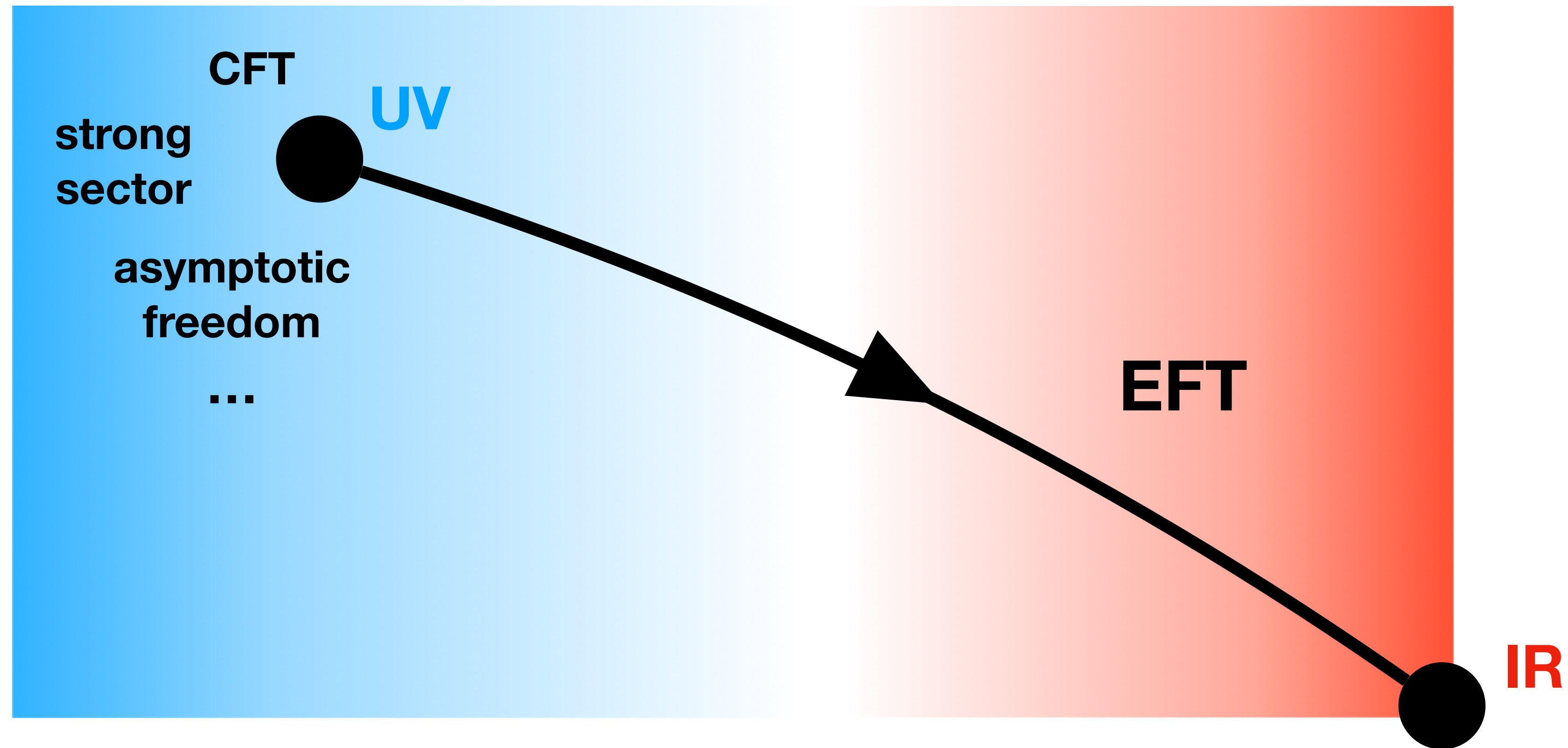
ERG 2026 @ Sussex

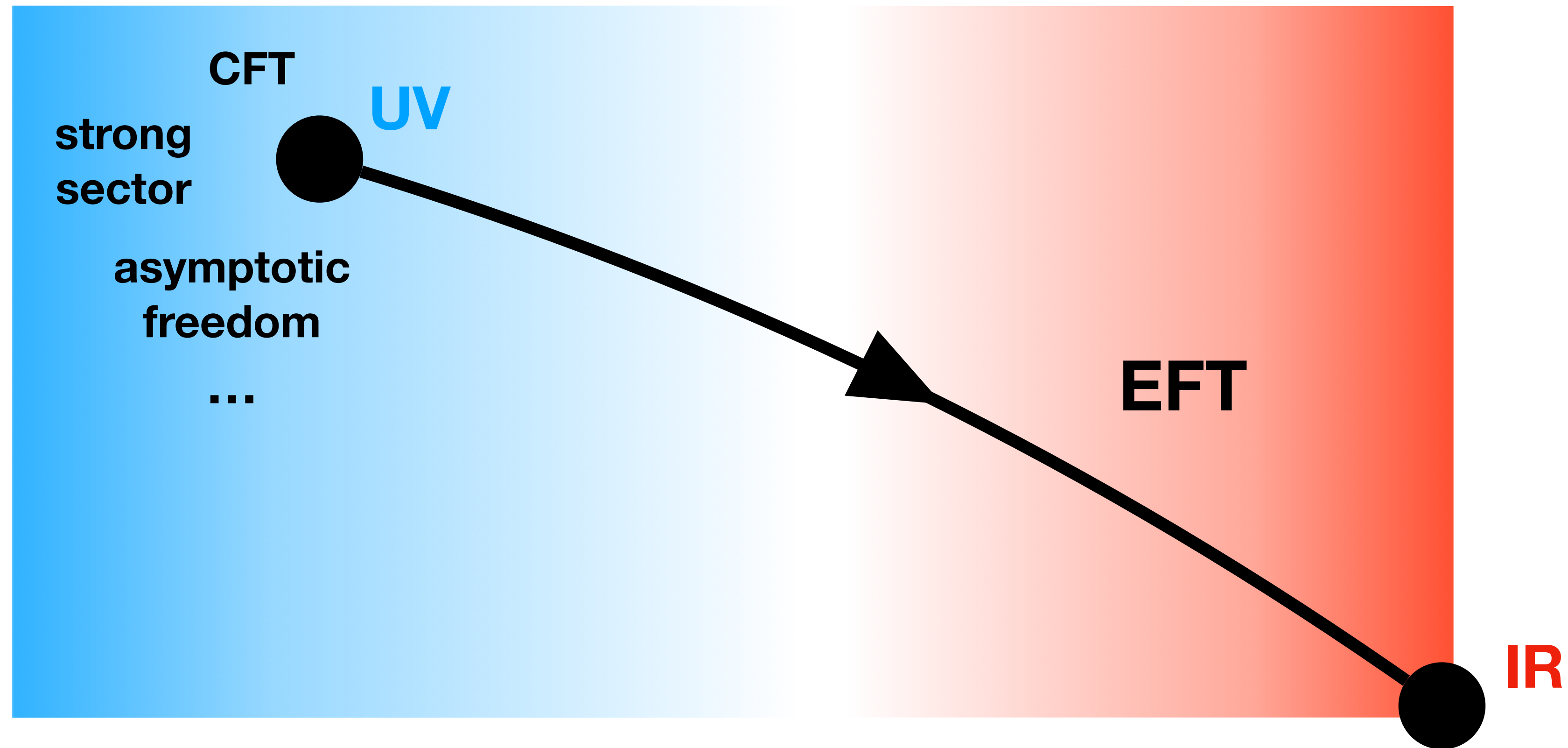
US
University of Sussex

tu technische universität
dortmund

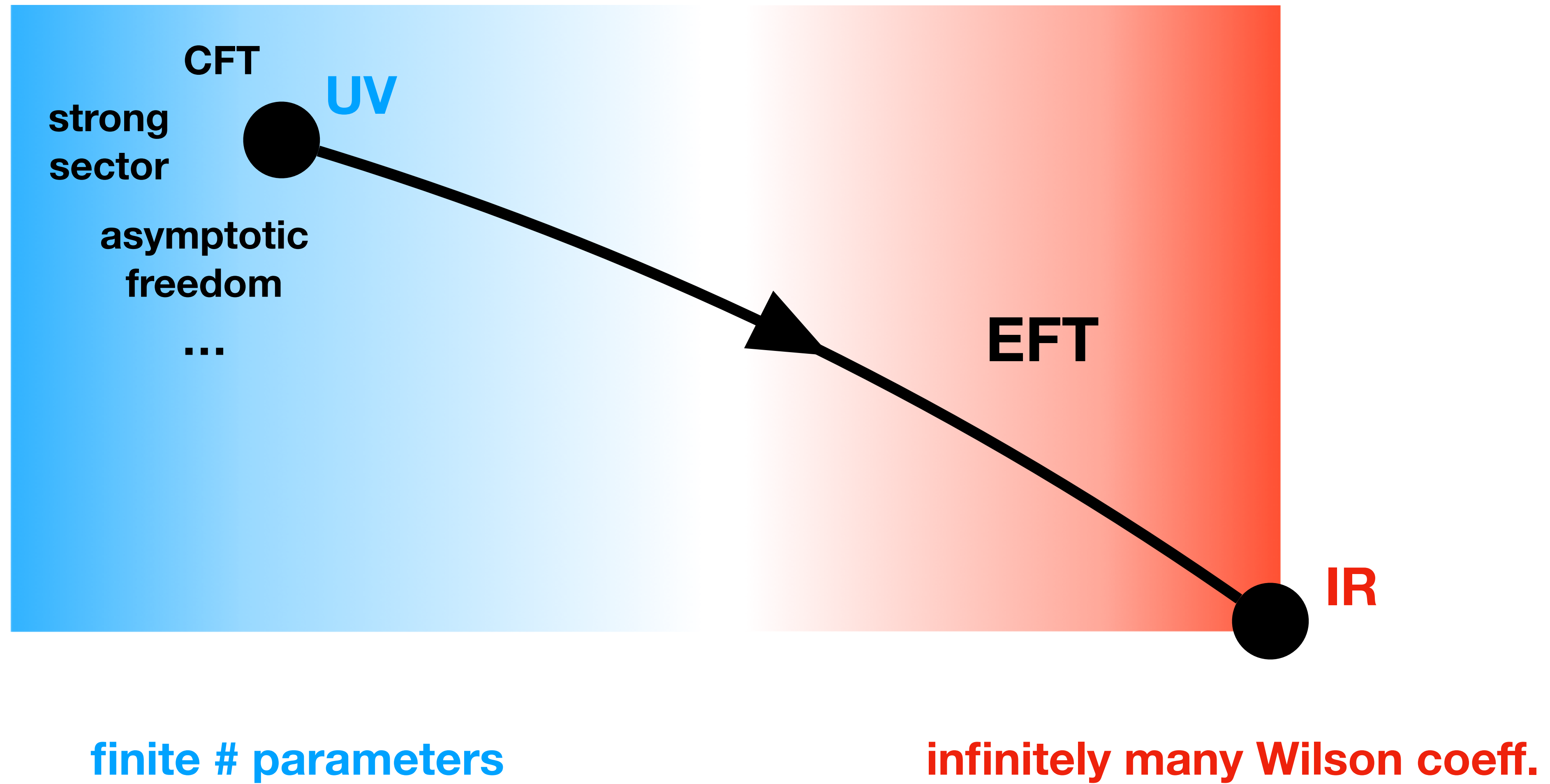


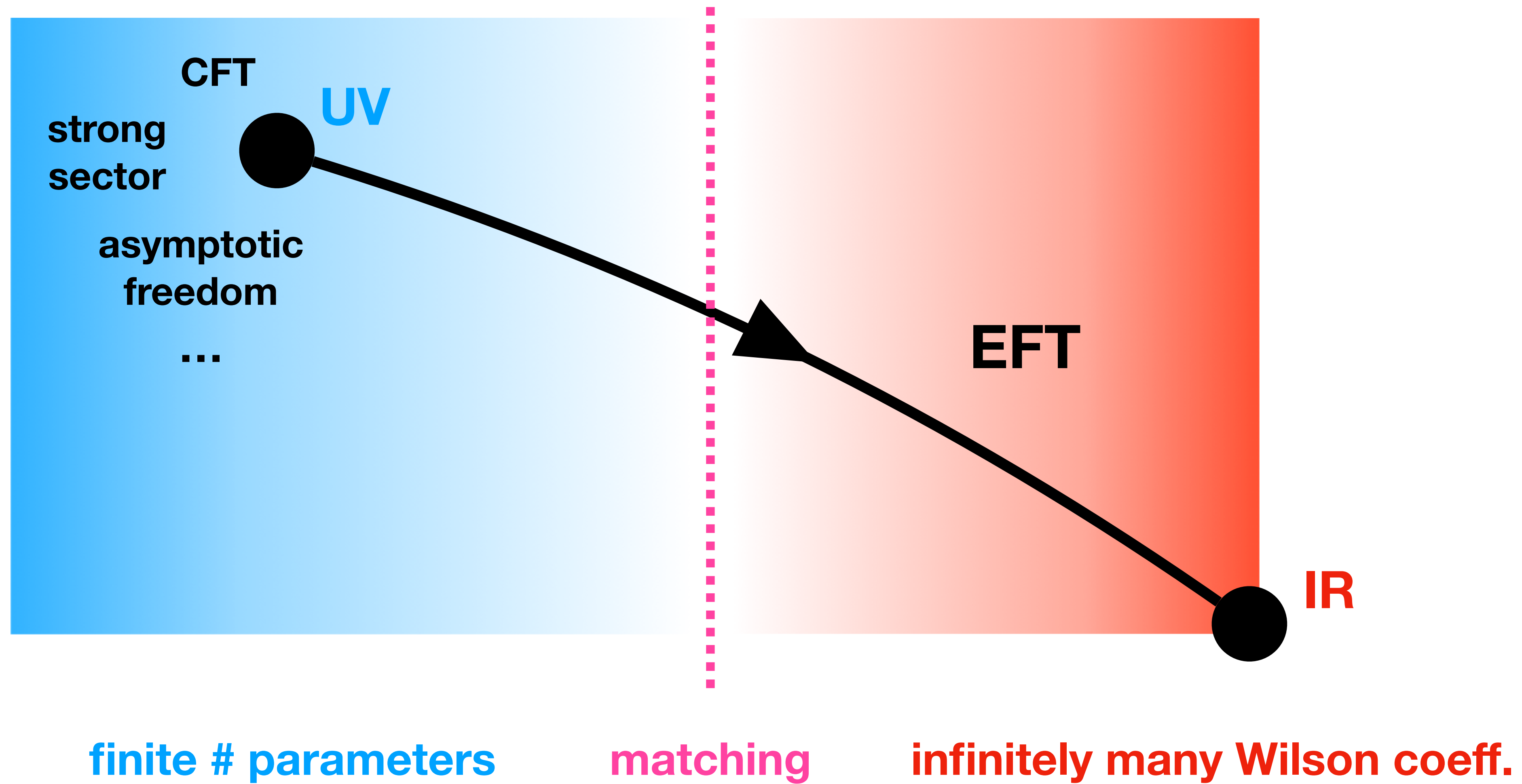


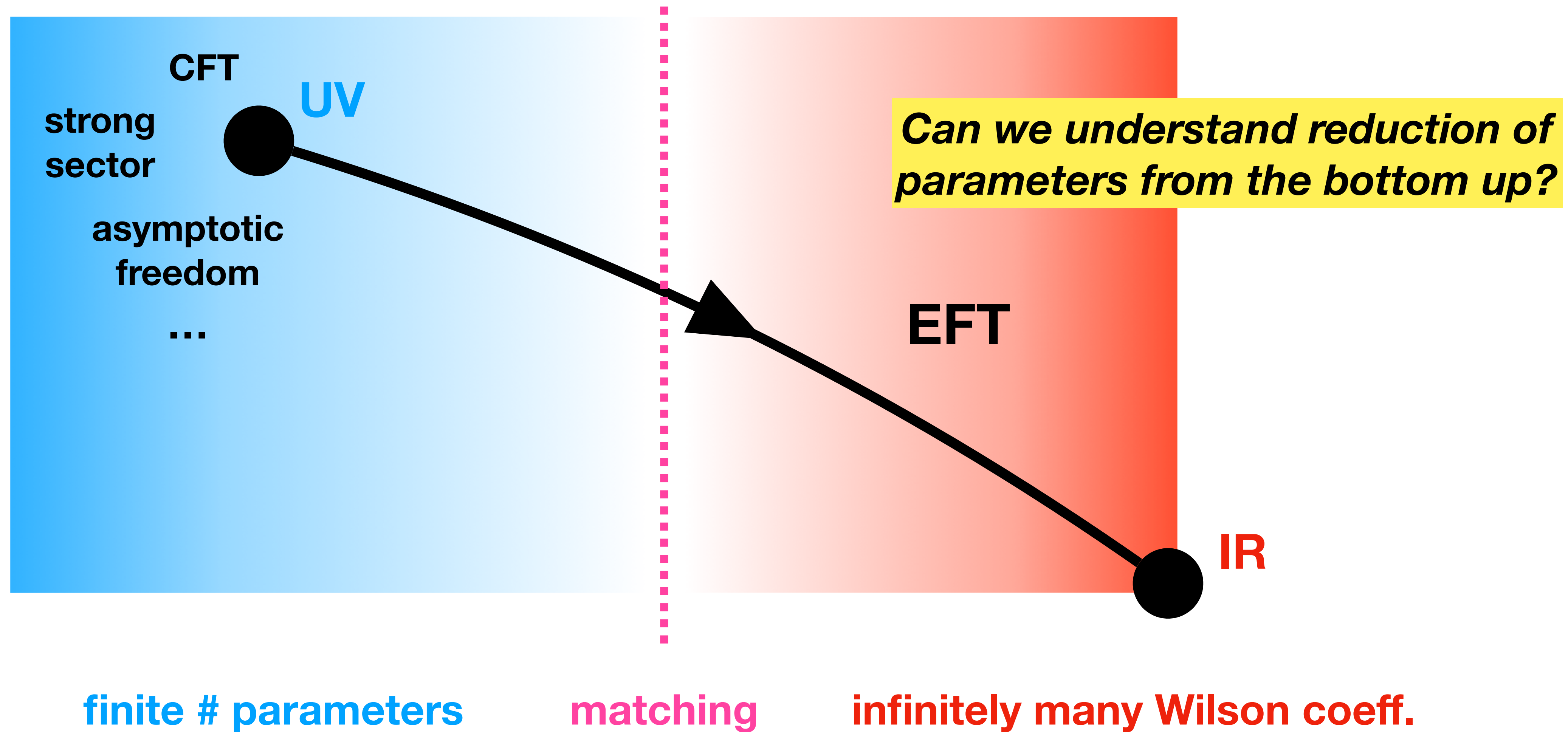




finite # parameters







A toy EFT

$$\bar{\psi}_a \not{\partial} \psi_a - \frac{1}{2N} G (\bar{\psi}_a \psi_a)^2 + \dots$$

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2+1 dimensions: $1/N$ renormalisable & conformal at high energy

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Wilson '73

Gross, Neveu '74

Parisi '75

Gawędzki, Kupiainen '85

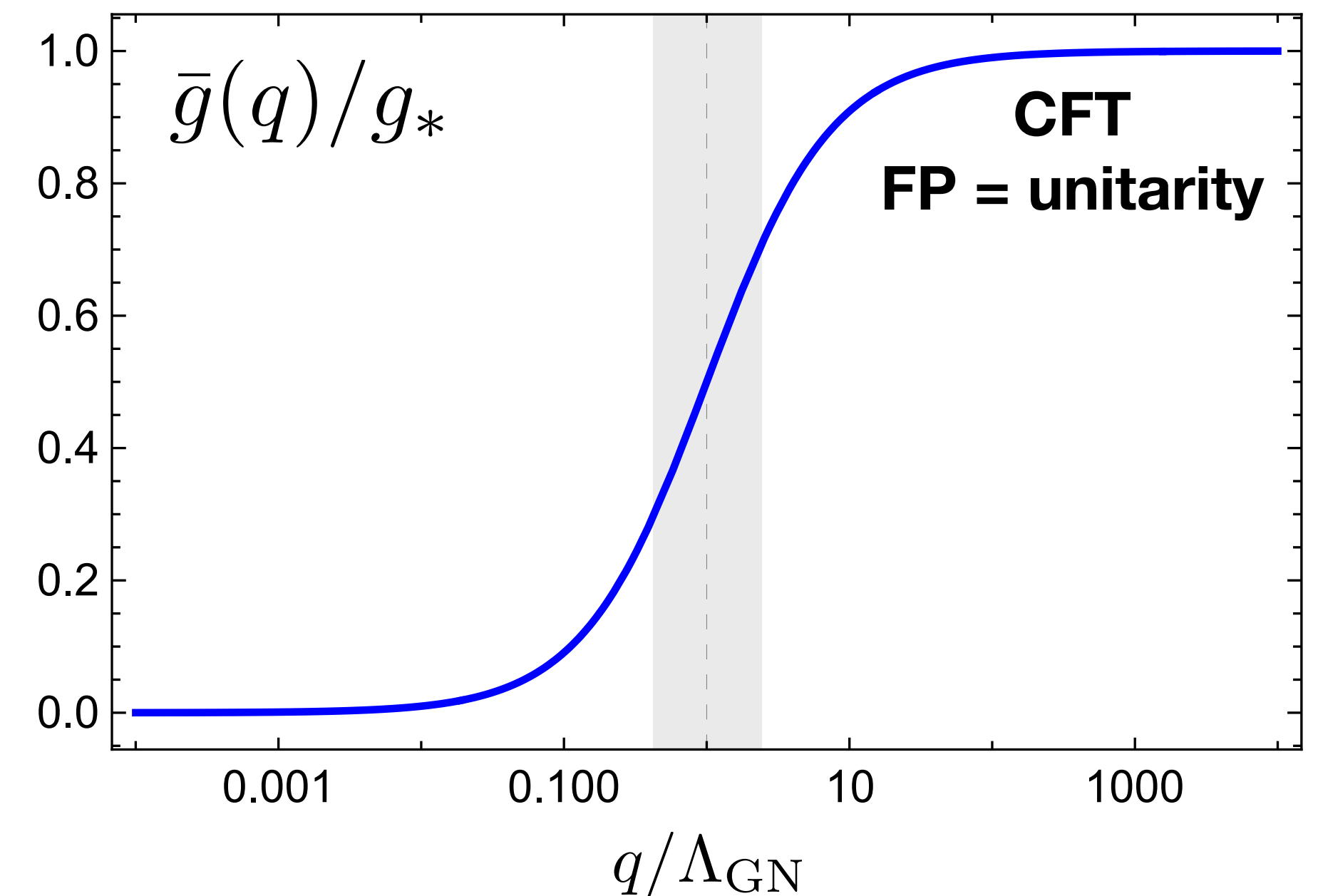
TIP! Rosenstein, Warr, Park '88 (PRL)

Rosenstein, Warr, Park '91 (Phys.Rept.)

Gat, Kovner, Rosenstein, Warr '90

de Calan, Faria Da Veiga, Magnen, Seneor '91

+ many more



A toy EFT

$$\bar{\psi}_a \not{\partial} \psi_a - \frac{1}{2N} G (\bar{\psi}_a \psi_a)^2 + \dots$$

3+1 dimensions: $1/N$ renormalisable & *near*-conformal at high energy

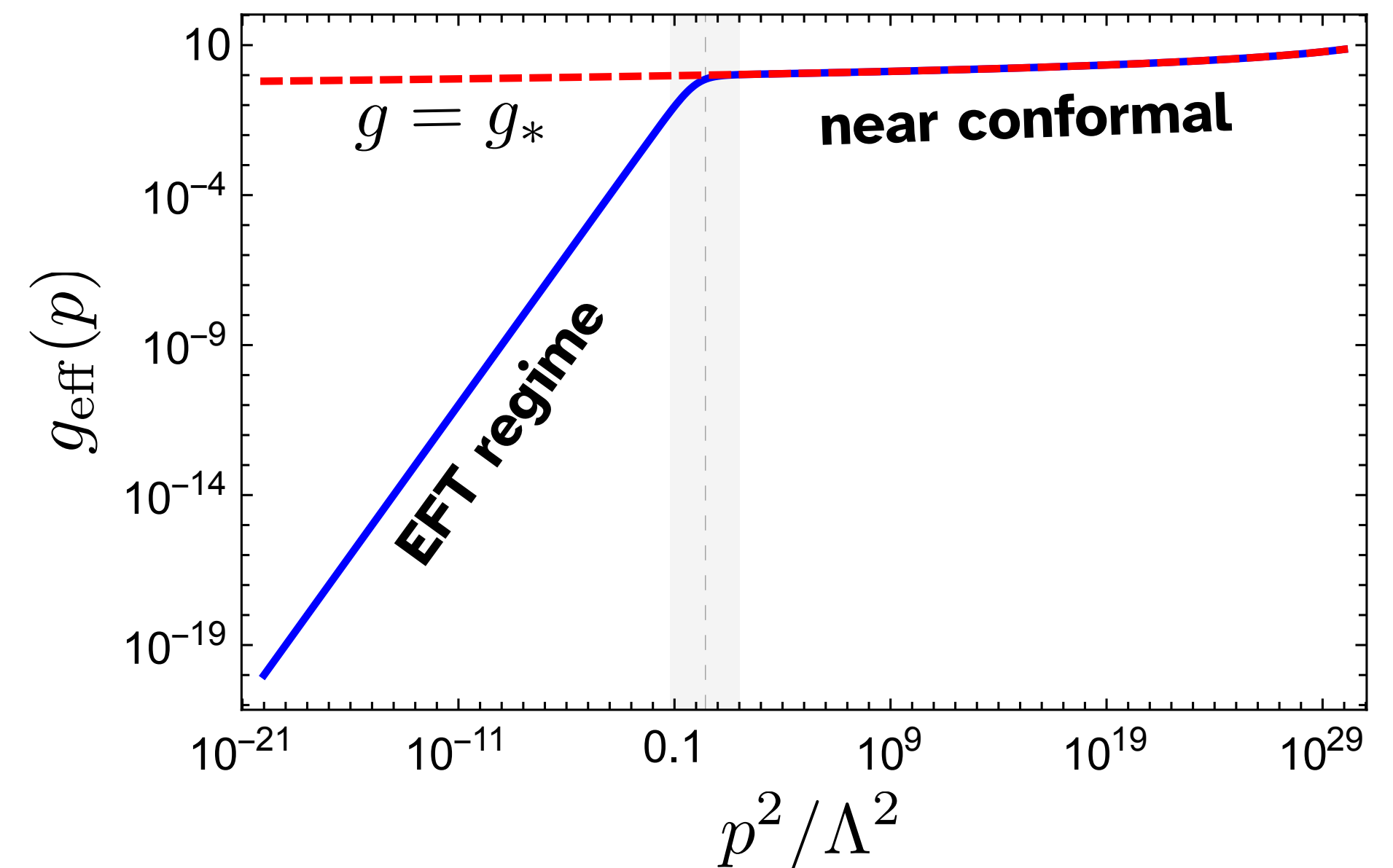
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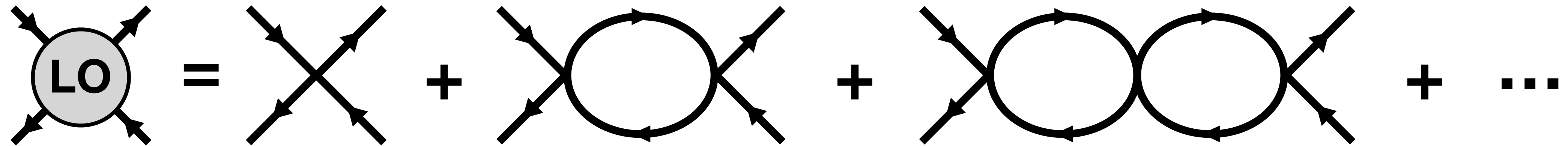
Formal renormalisability in $1/N$ dictates:

- tower of higher-derivative operators
- eight-fermion interactions (but no higher)
- only three fundamental parameters



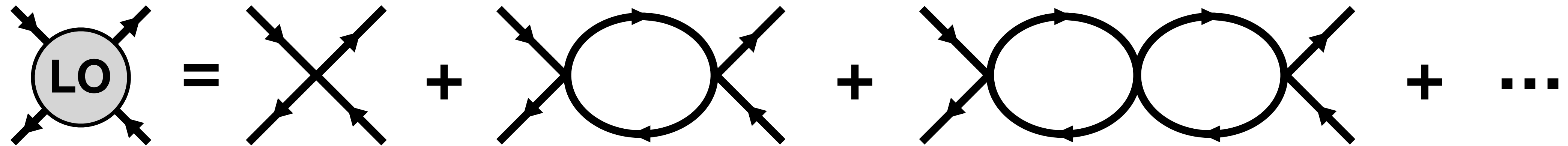
Large- N diagrammatics

Four-point function:

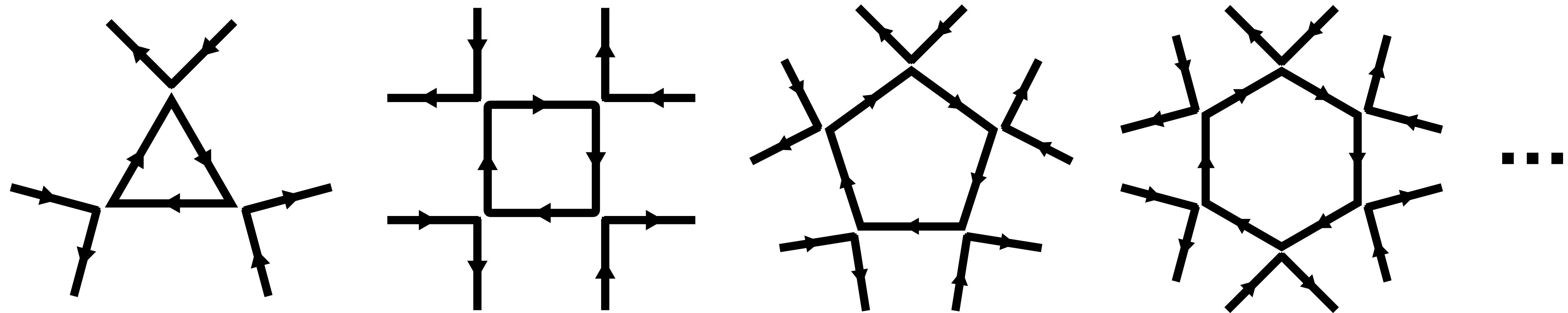


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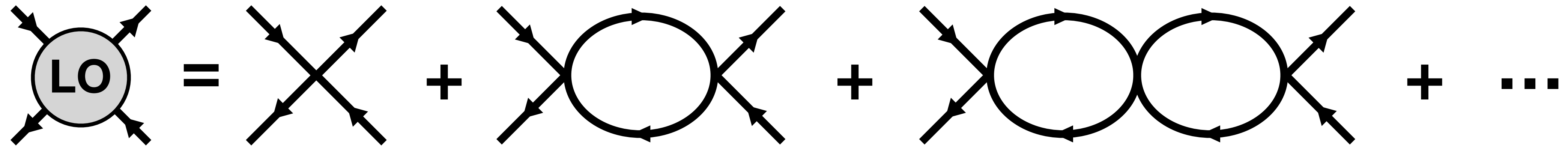


Higher-point functions:

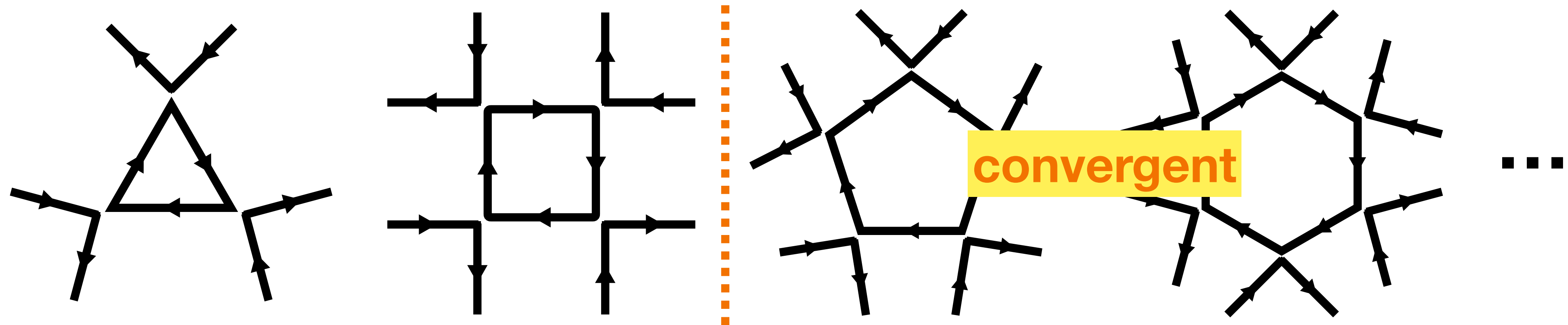


Large- N diagrammatics

Four-point function:

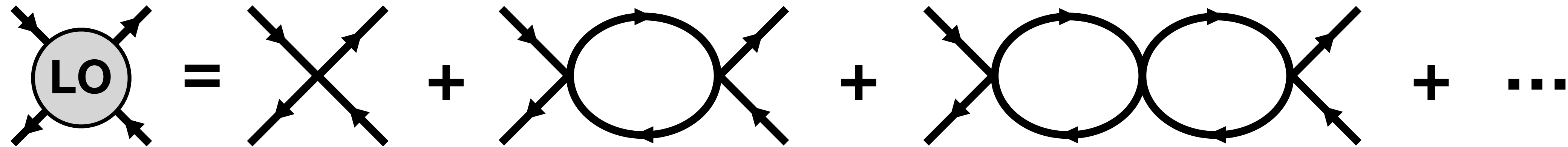


Higher-point functions:

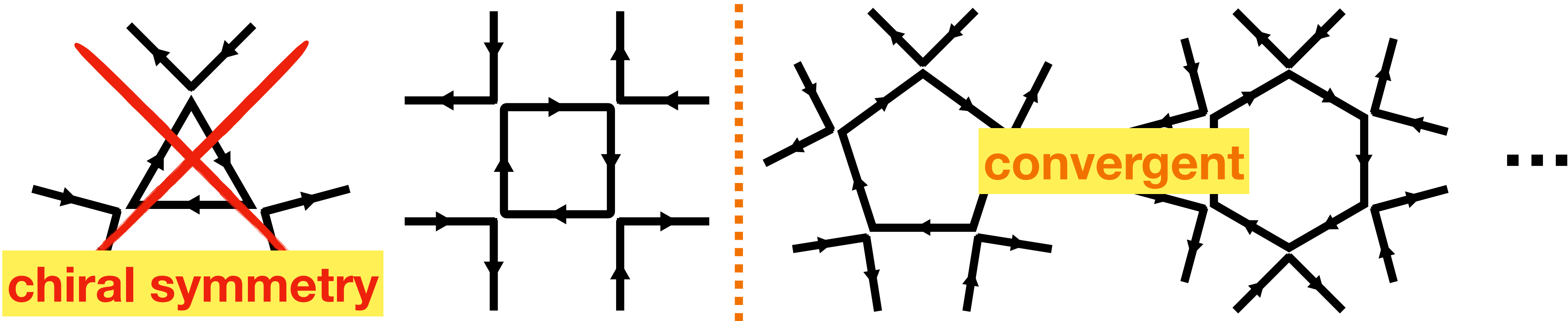


Large- N diagrammatics

Four-point function:

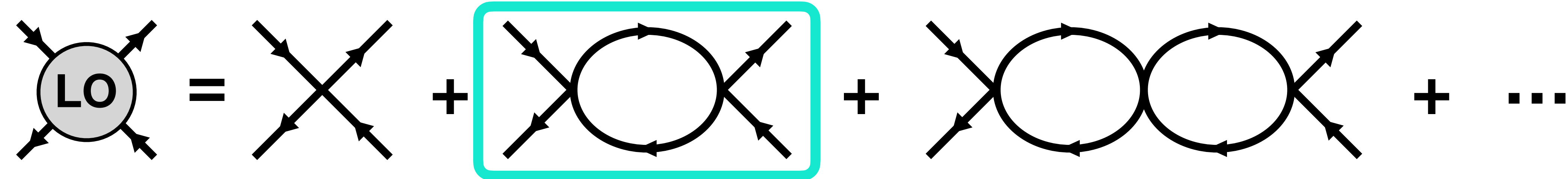


Higher-point functions:



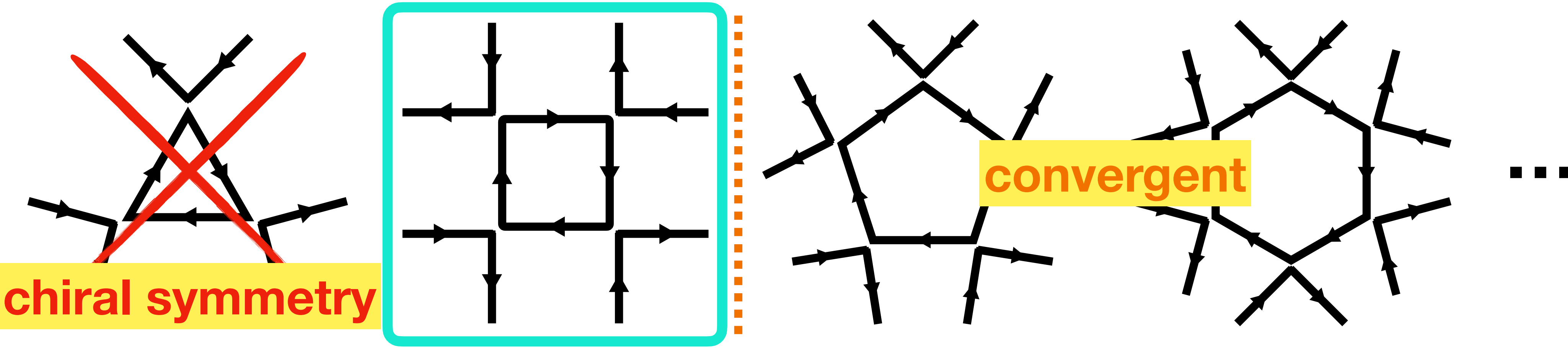
Large- N diagrammatics

Four-point function:



divergent
subdiagrams

Higher-point functions:



Four-point function

$$\text{LO} = \text{tree} + \text{1-loop} + \text{2-loops} + \dots \rightarrow \frac{N^{-1}}{G^{-1} + \text{1-loop}}$$

$$\text{1-loop} \sim \Lambda^2 + p^2 \ln \Lambda \quad \text{infinite tower of higher-derivative divergences}$$

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$$F(p^2)^2 \times \text{1-loop}$$

vertices factorise @ LO

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$$F(p^2) = \frac{1}{Z p^2 + G^{-1}} \rightarrow \text{LO} \sim \frac{N^{-1}}{Z p^2 + G^{-1} + \text{1-loop}}$$

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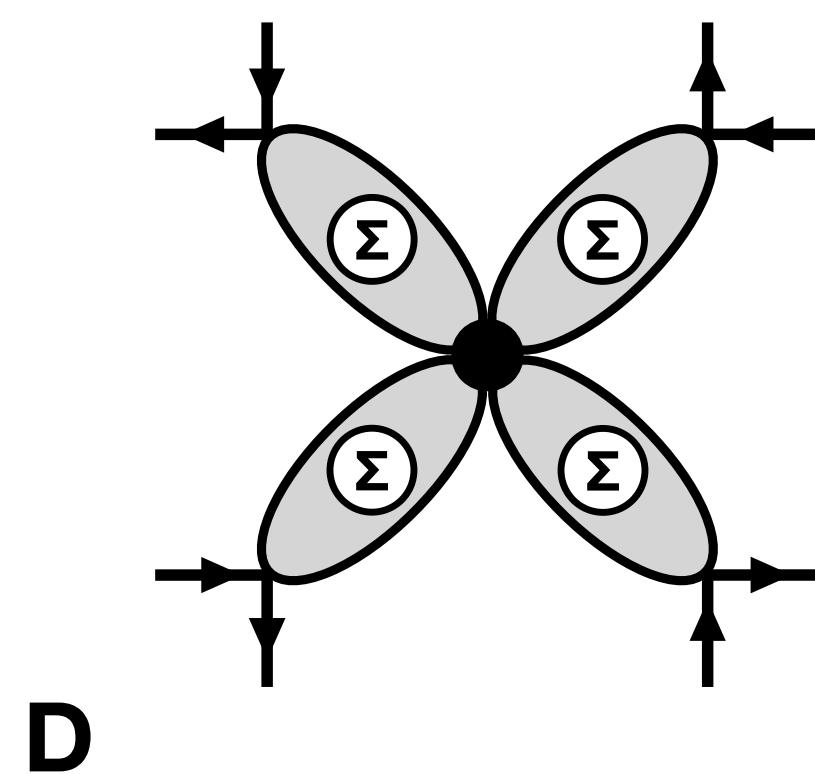
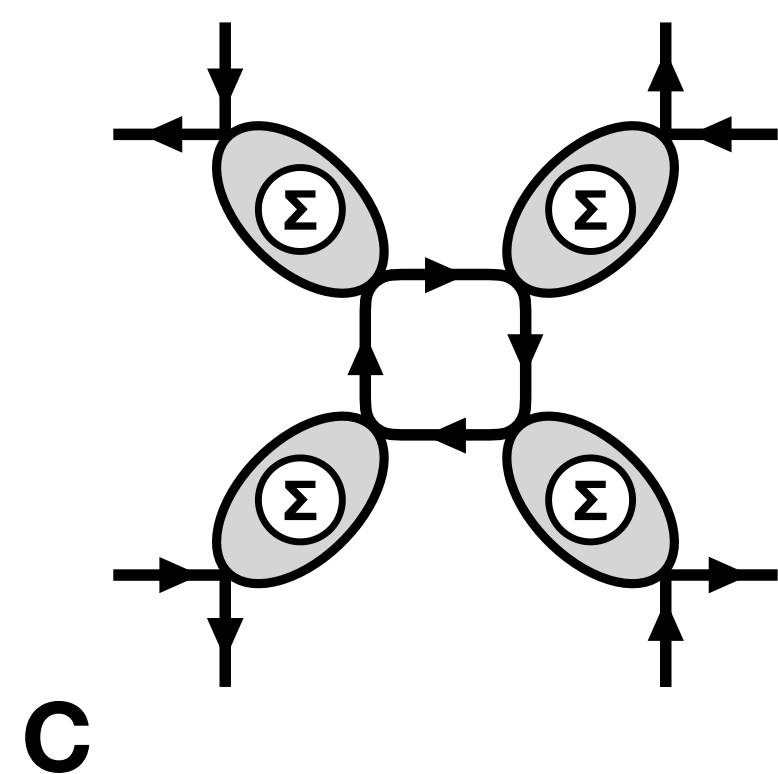
subtract infinitely many higher-derivative divergences with one additional parameter

Eight-point function

The diagram shows an equation for the eight-point function. On the left is a shaded oval with a central circle containing the Greek letter Σ . Two external lines enter from the left, and two exit to the right. The rightmost vertex is marked with a circle containing an 'X'. This is followed by an equals sign, then a series of terms separated by plus signs. The first term is a vertex with two incoming lines from the left and one outgoing line to the right, marked with a circle containing an 'X'. The second term is a loop diagram with two incoming lines from the left and one outgoing line to the right, marked with a circle containing an 'X'. The third term is a more complex loop diagram with two incoming lines from the left and one outgoing line to the right, marked with a circle containing an 'X'. The equation ends with an ellipsis (...).

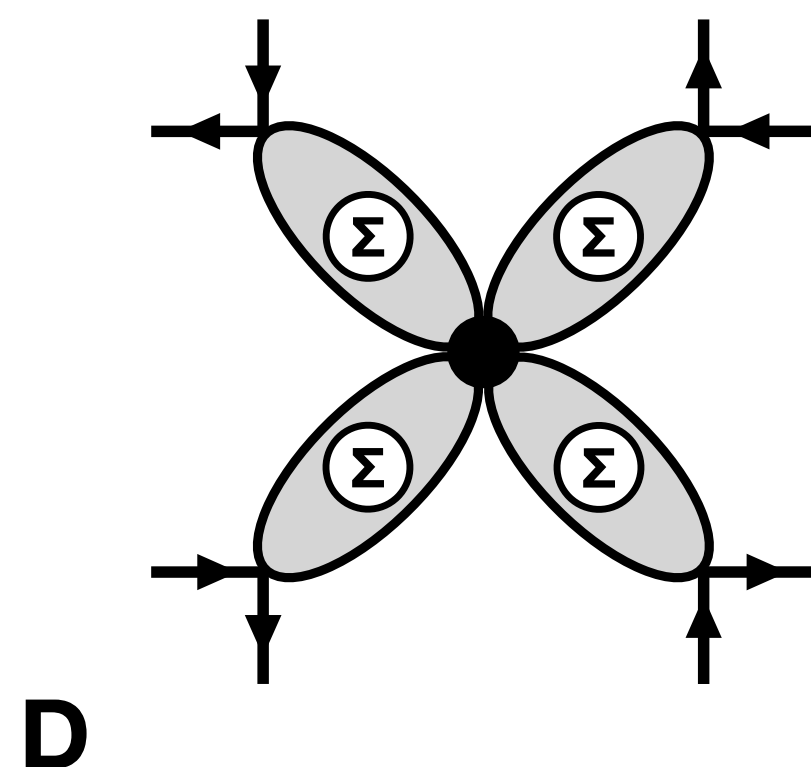
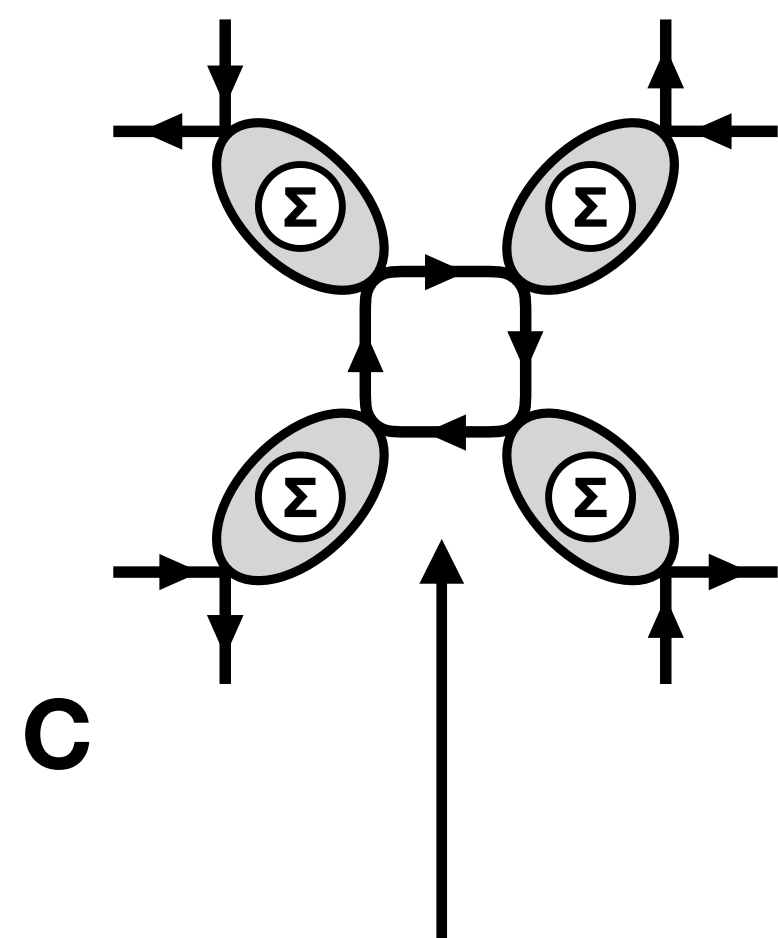
Eight-point function

$$\text{Diagram} = \text{Diagram}_1 + \text{Diagram}_2 + \text{Diagram}_3 + \dots$$



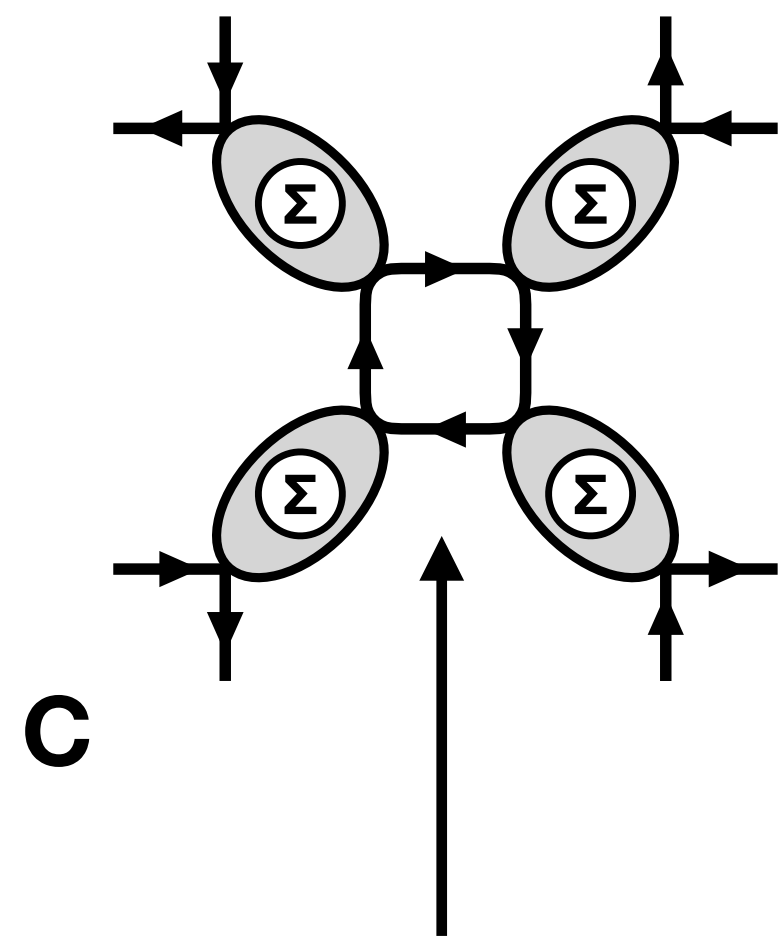
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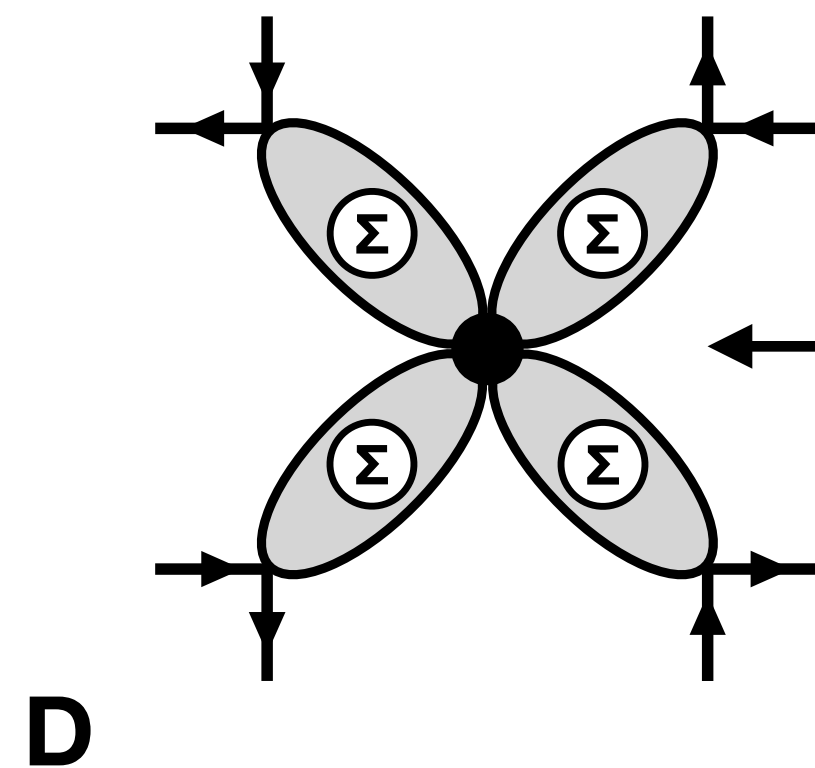
Eight-point function

A diagrammatic equation showing the expansion of a self-energy loop. On the left, a fermion line with two external arrows enters a shaded oval labeled Σ with a cross on its right side. This is equal to a sum of three terms: 1) a fermion line with a cross on it, 2) a fermion line entering a loop with a shaded Σ loop on the right, and 3) a fermion line entering a chain of two shaded Σ loops on the right. The sequence ends with an ellipsis.



C

$\sim \ln \Lambda$

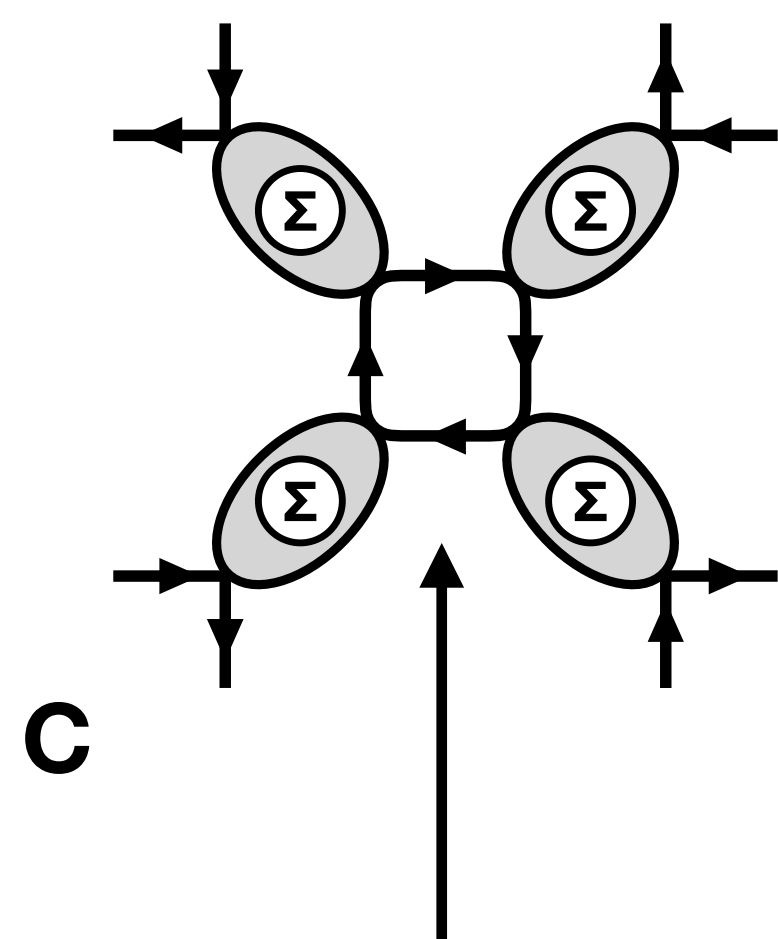


D

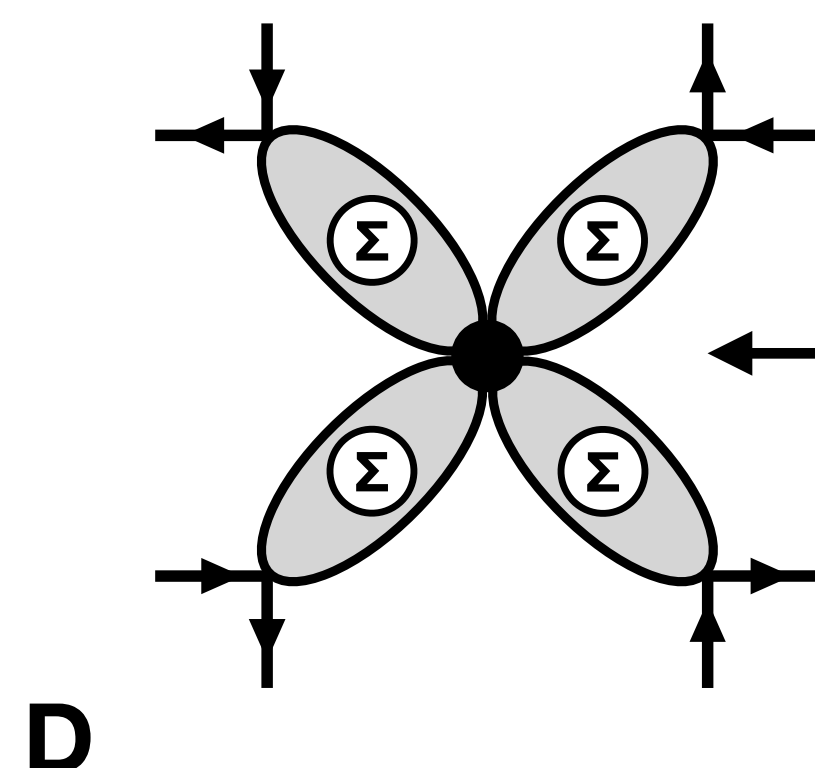
$$\frac{\lambda}{4!N^3} \left[F(-\partial^2) (\bar{\psi}_a \psi_a) \right]^4$$

Eight-point function

$$\text{Diagram with } \Sigma \text{ and } \otimes = \text{Diagram with } \otimes + \text{Diagram with bubble} + \text{Diagram with two bubbles} + \dots$$

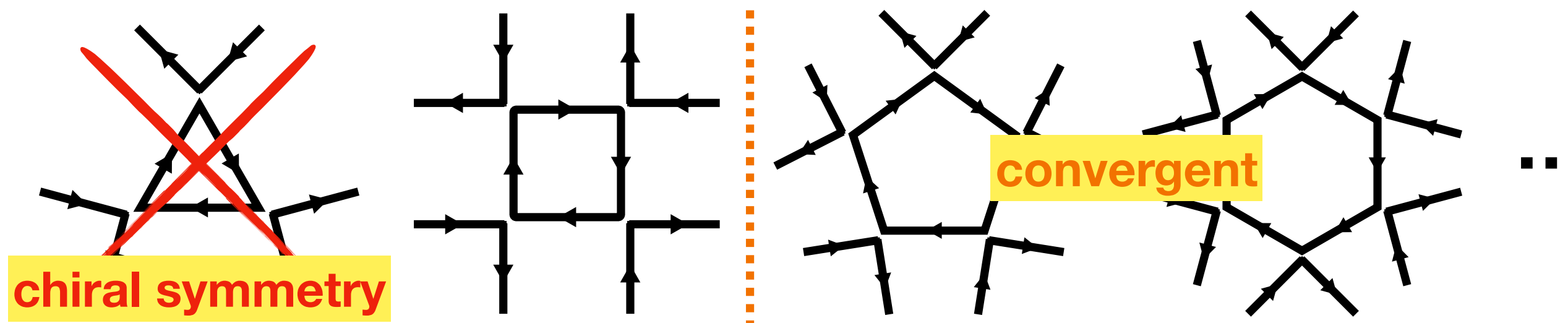


$\sim \ln \Lambda$



$$\frac{\lambda}{4!N^3} \left[F(-\partial^2) (\bar{\psi}_a \psi_a) \right]^4$$

Everything @ LO = products of bubble sums with:



>> **THEORY RENORMALISED.** <<

Renormalisable four-fermion theory

$$\bar{\psi}_a \not{\partial} \psi_a - \frac{1}{2N} (\bar{\psi}_a \psi_a) F(-\partial^2) (\bar{\psi}_b \psi_b) + \frac{\lambda}{4!N^3} [F(-\partial^2) (\bar{\psi}_a \psi_a)]^4$$

$$F(p^2) = \frac{1}{Zp^2 + G^{-1}} \approx G \quad \text{for} \quad Zp^2 \ll G^{-1}$$

- **Predictive:** 3 parameters Z , G , λ
- **Tower** of higher derivative $4F + 8F$
- **NO higher** ($10F$, $12F$, ...)

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holds order-by-order in $1/N$.

cf. 3D: *Park, Rosenstein, Warr, Phys.Rept. 1991*

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- **NO** higher ($10F$, $12F$, ...)

Renormalised couplings:

$$g(\mu) = \mu^2 G(\mu)$$

$$Z(\mu) \quad \lambda(\mu)$$

holds order-by-order in $1/N$.

cf. 3D: *Park, Rosenstein, Warr, Phys.Rept. 1991*

Running couplings

non-universal:

$$\beta_g(g) = 2g \left(1 - \frac{g}{g_*} \right)$$

UV fixed point

e.g. MOM scheme: $g_* = 8\pi^2$

universal:

$$\beta_Z = -\frac{1}{4\pi^2} \quad \beta_\lambda = -\frac{3}{\pi^2}$$

logarithmic running

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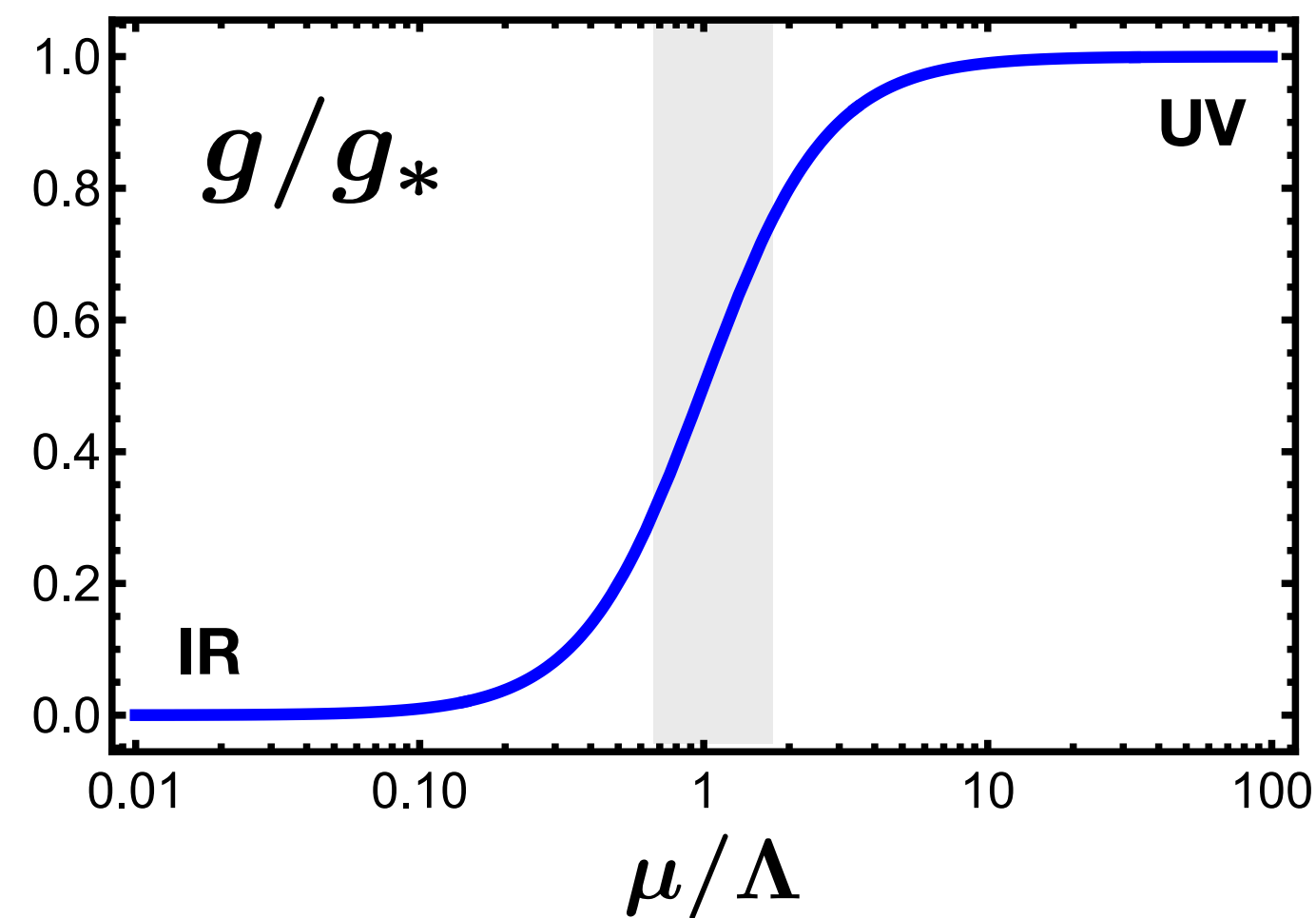
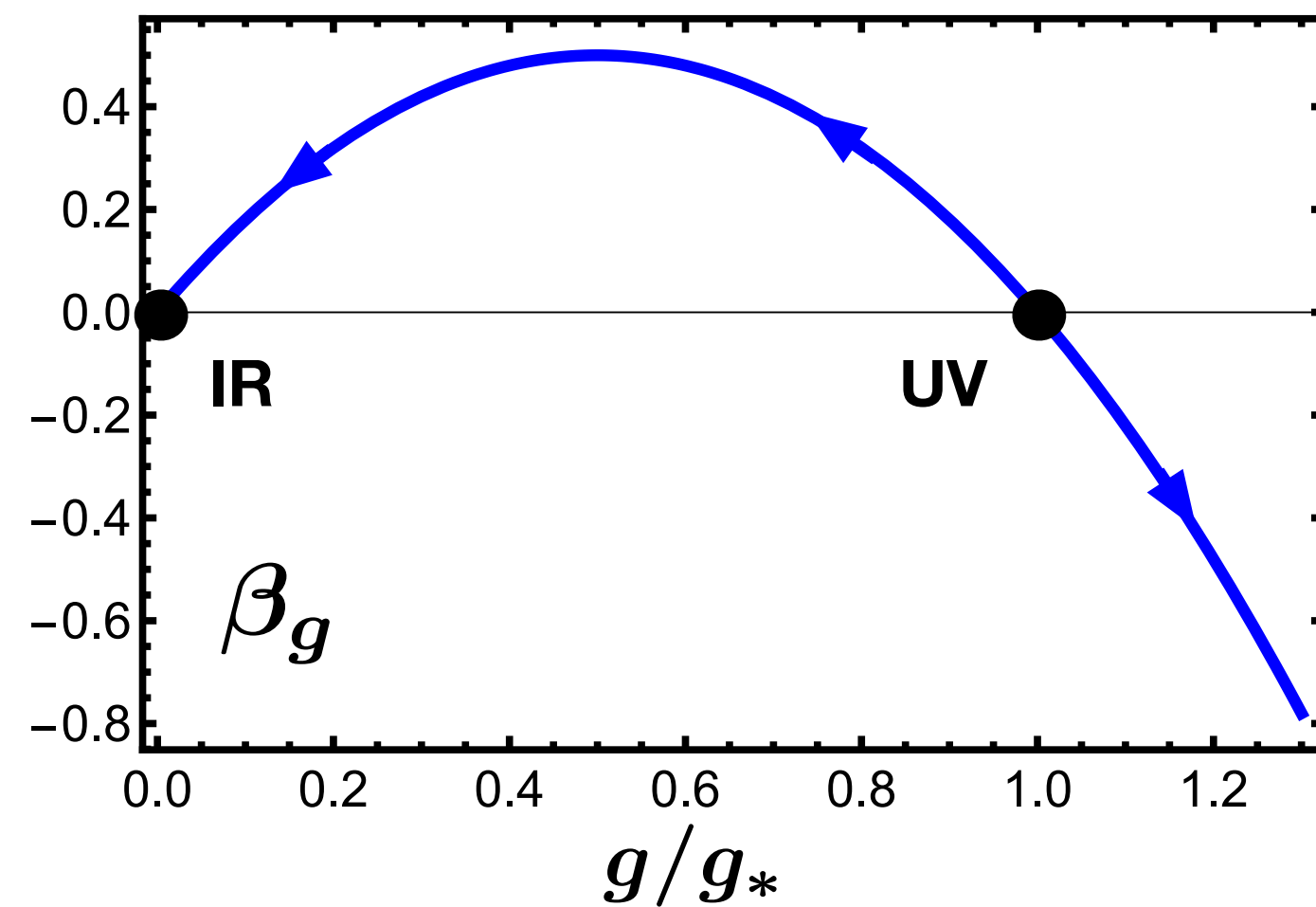
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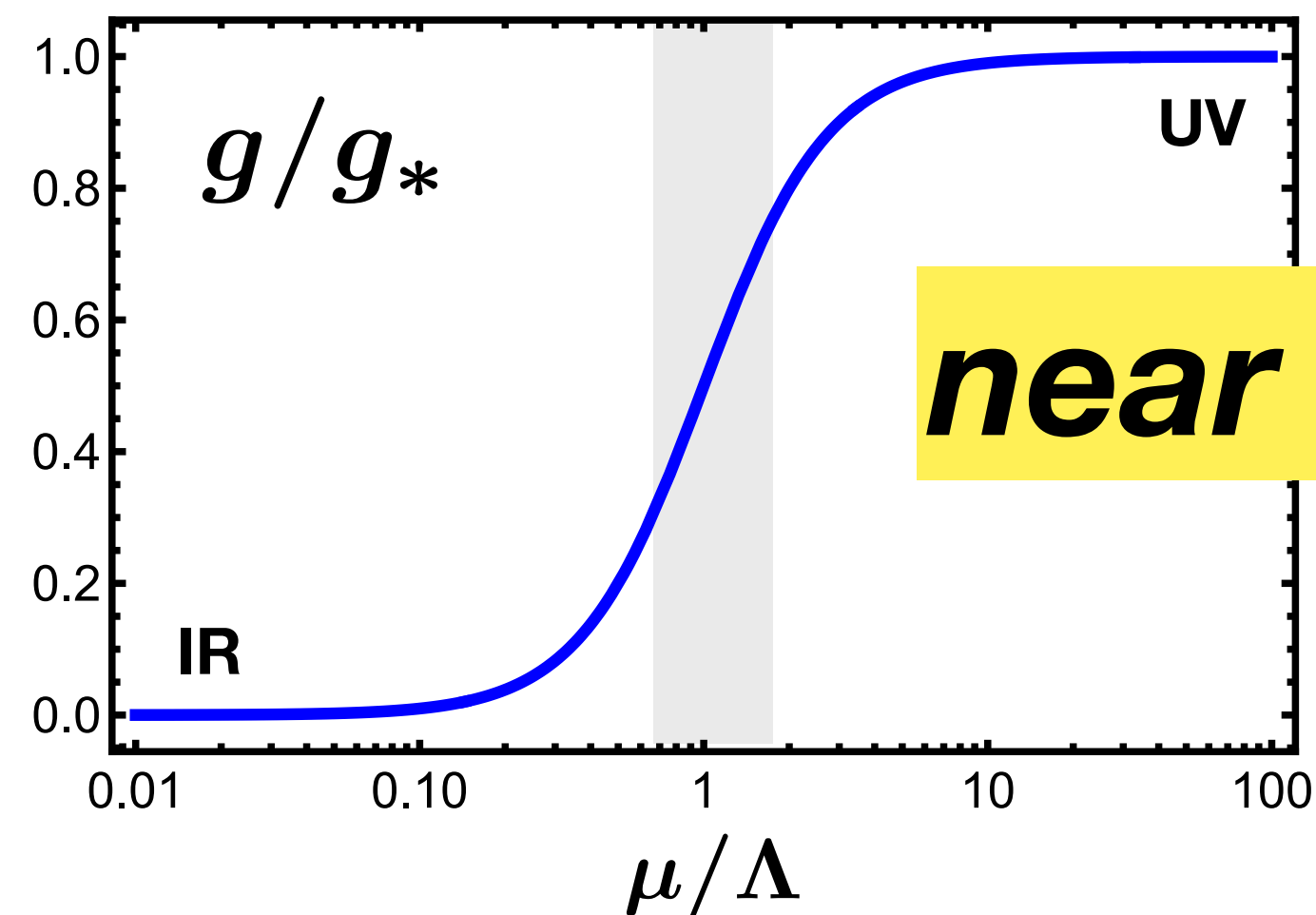
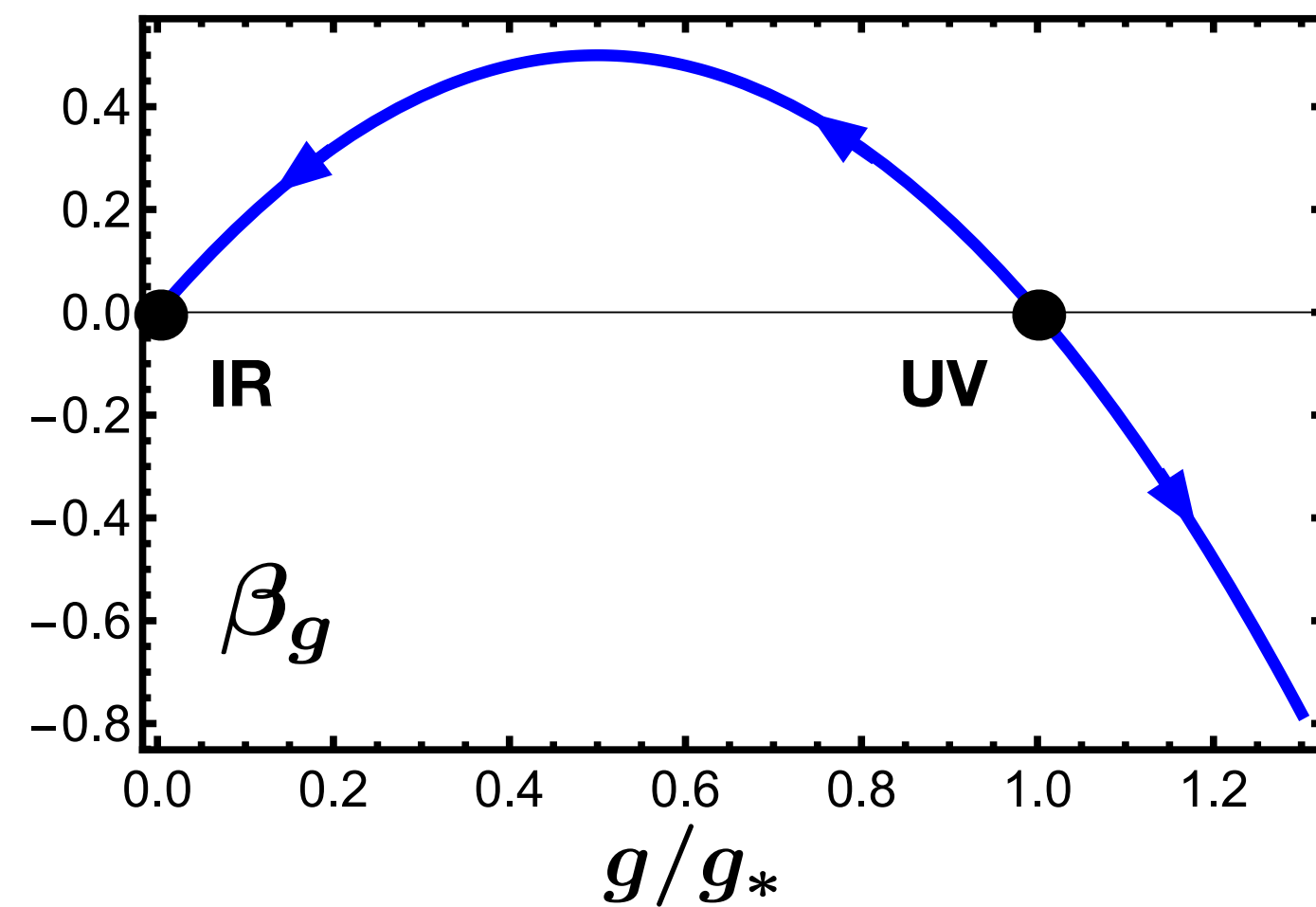
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near conformal in UV

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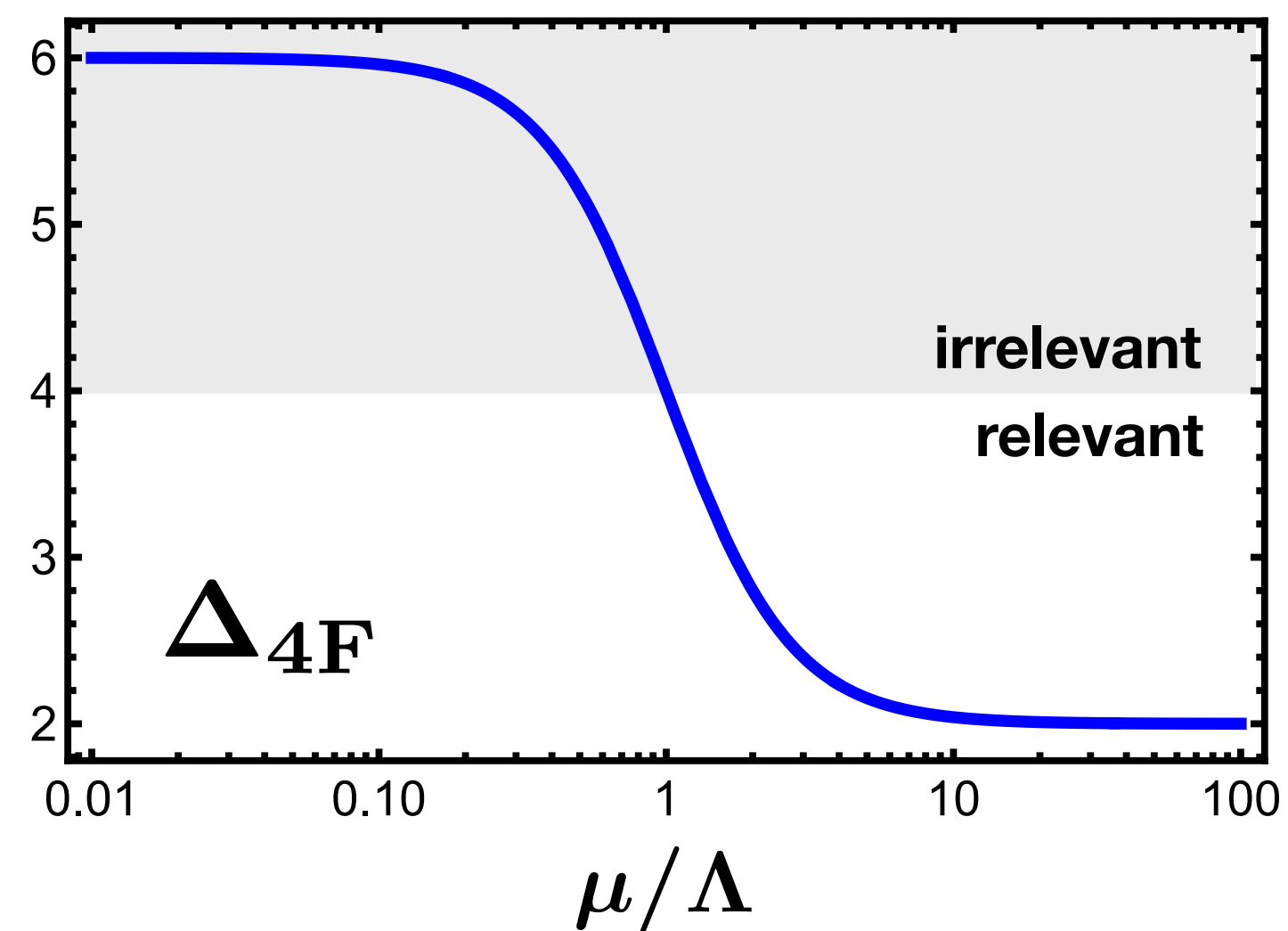
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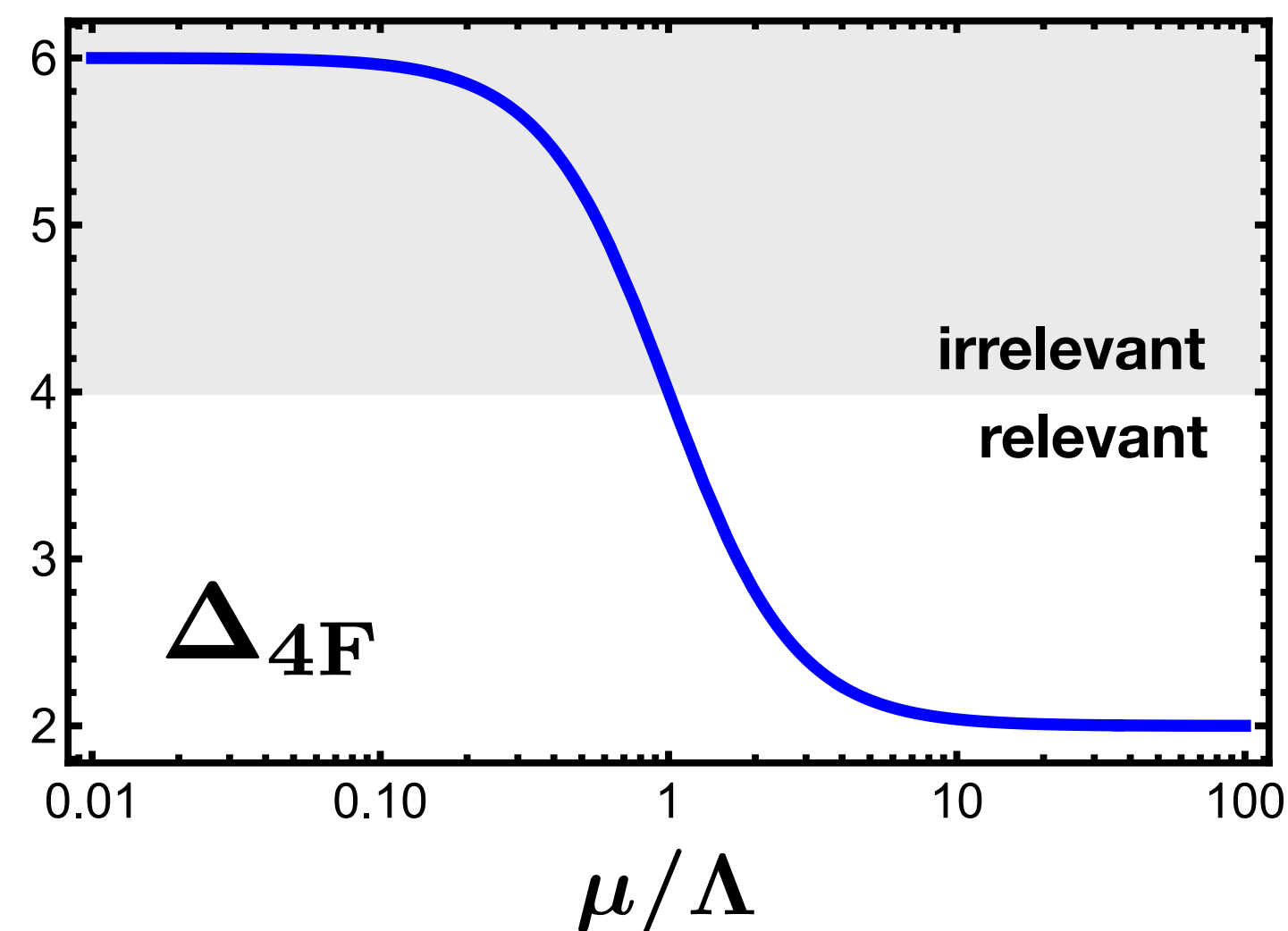
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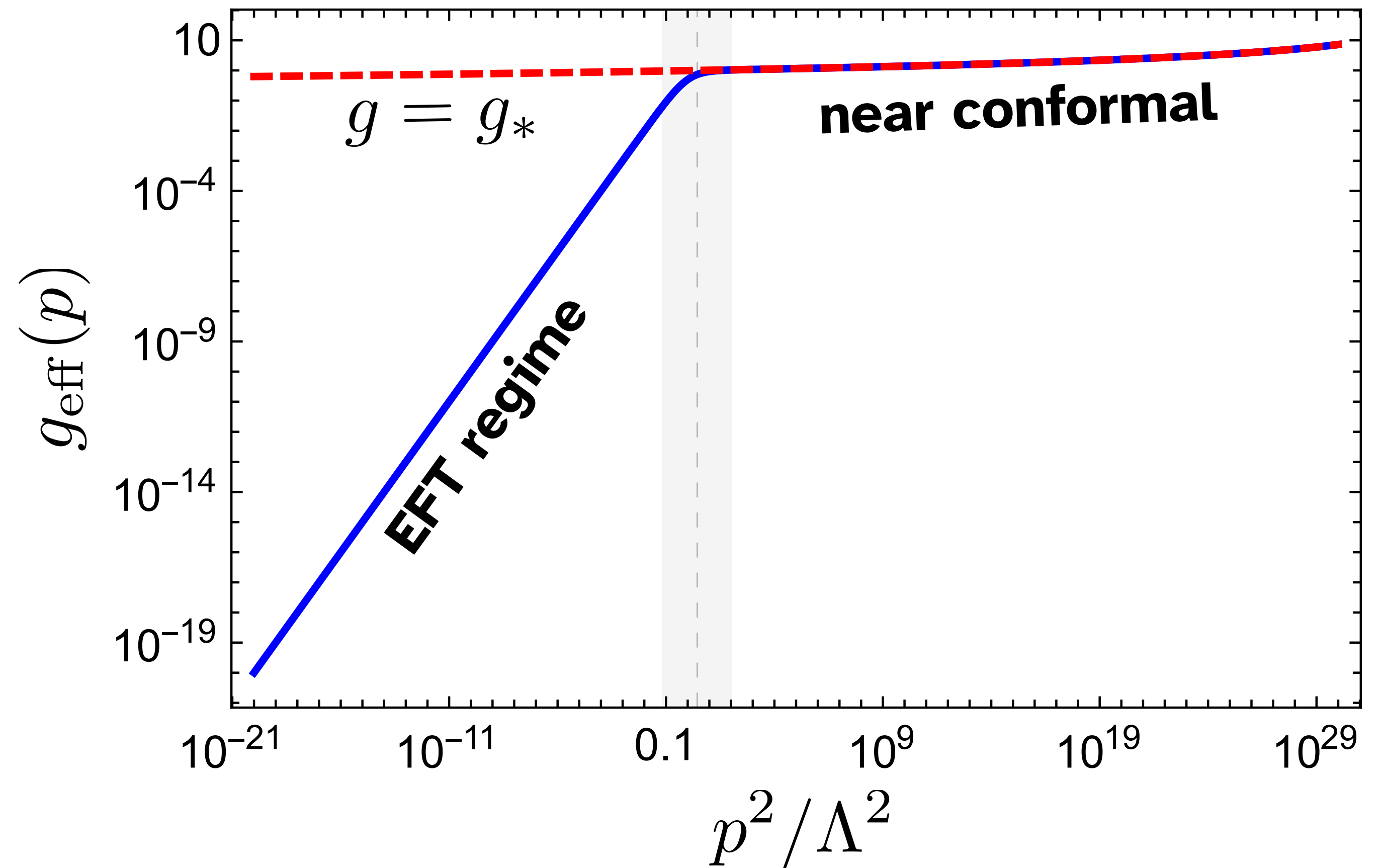
large anomalous dimensions:

dim-6 becomes dim-2

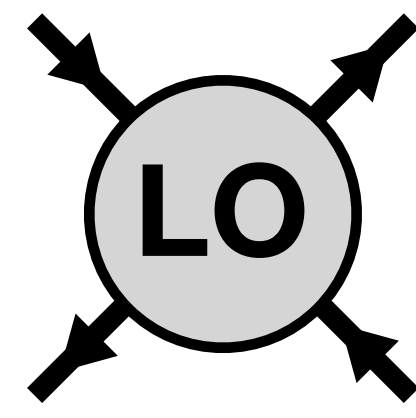
EFT power counting breaks

Near conformality & the EFT crossover

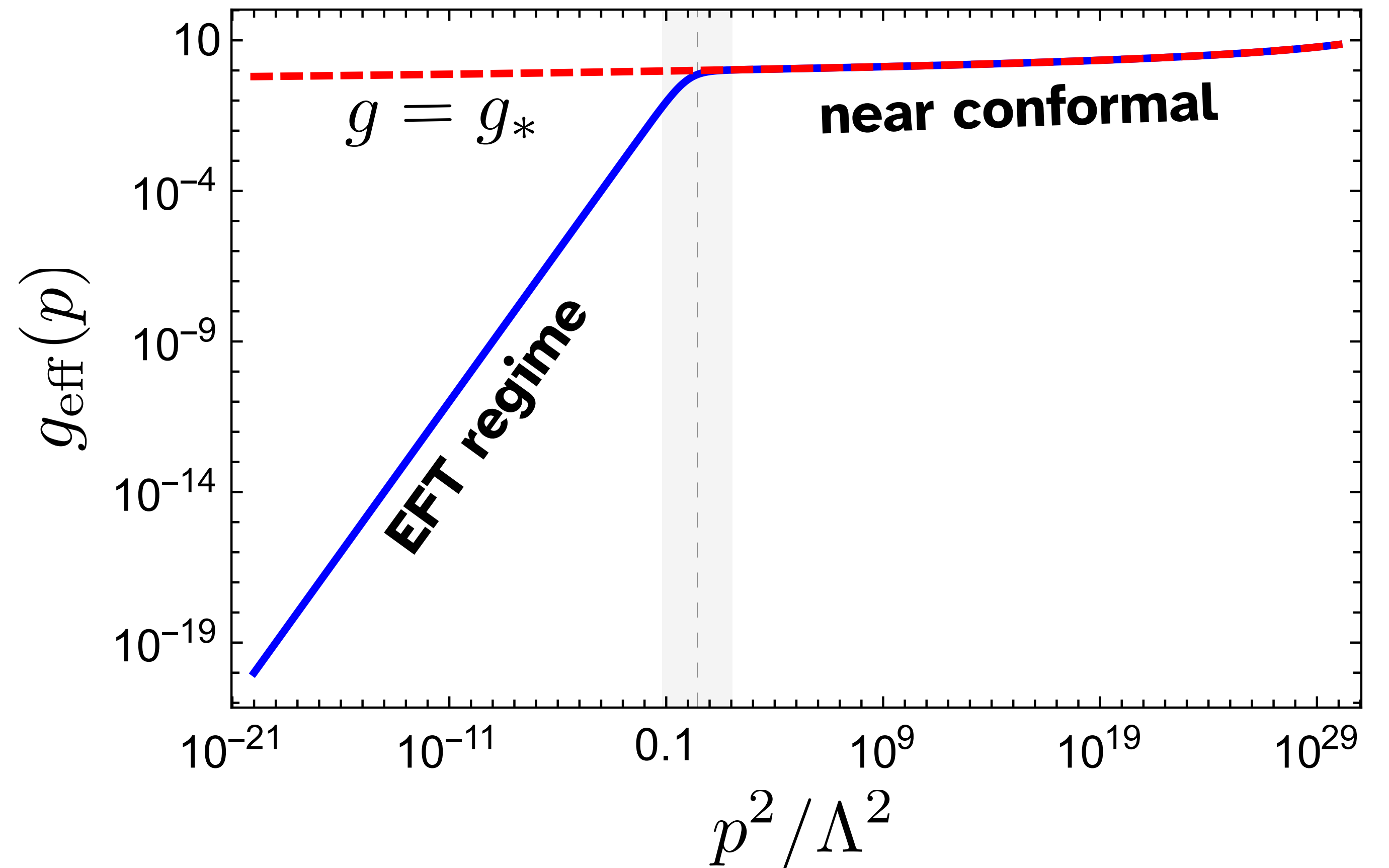
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Near conformality & the EFT crossover

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$$g_{\text{eff}}(p) = \frac{1}{Z(p) + g^{-1}(p)}$$

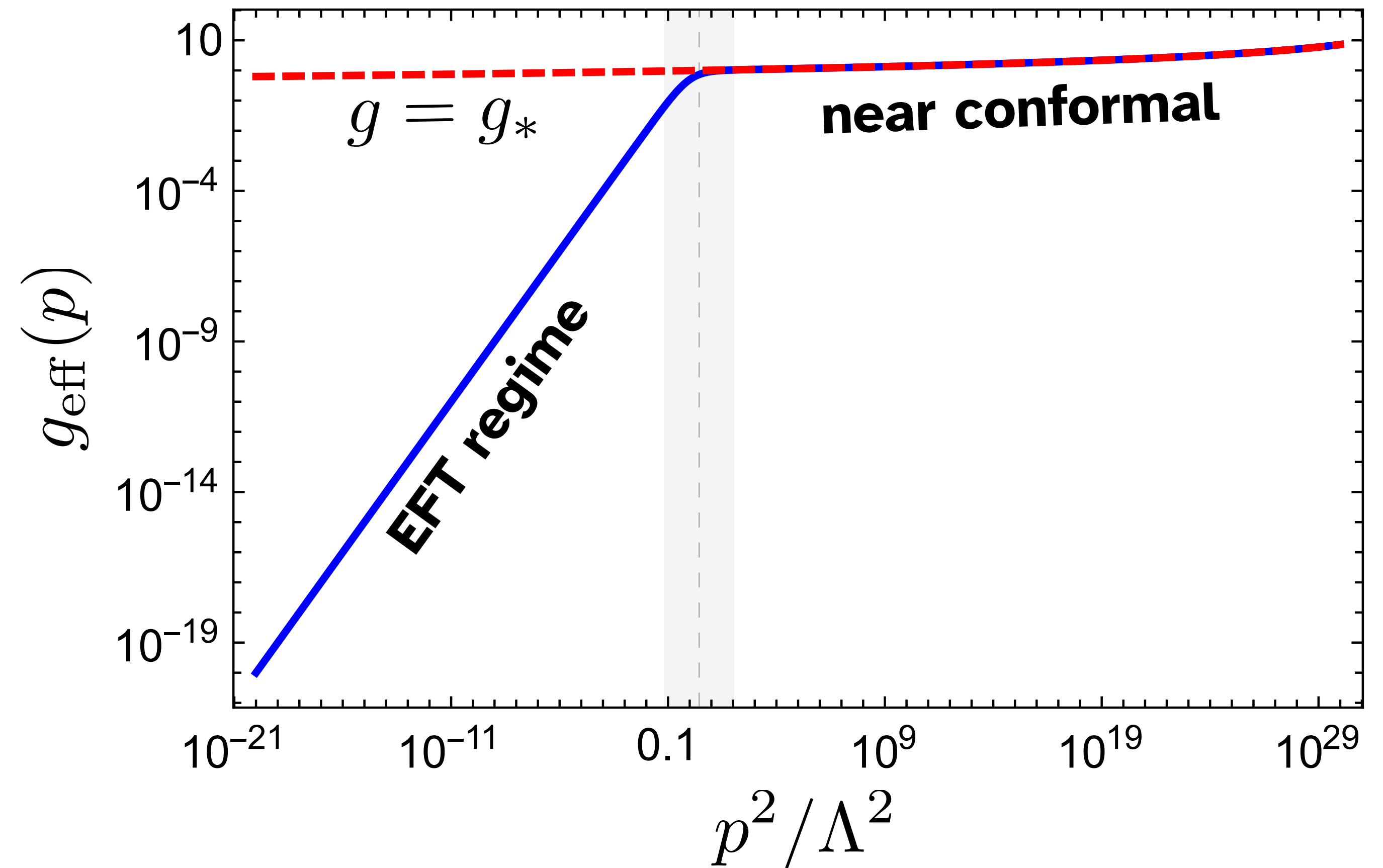


Near conformality & the EFT crossover

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$$g_{\text{eff}}(p) = \frac{1}{Z(p) + g^{-1}(p)}$$

EFT scale marks crossover:
onset of strong coupling, higher
derivative terms become important



Perturbative dual description

Functional Legendre transform maps to Yukawa theory at infinite N:

$$\bar{\psi}_a (i \not{\partial} - y \phi) \psi_a + \frac{1}{2} (\partial \phi)^2 - \frac{1}{2} m^2 \phi^2 - \frac{1}{4!} \lambda_\phi \phi^4$$

e.g. Weinberg hep-th/9706042
(see also Zinn-Justin '91; Hasenfratz+ '91)

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Parameter relations preserved under RG flow:

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CCH, Litim, *in prep.*

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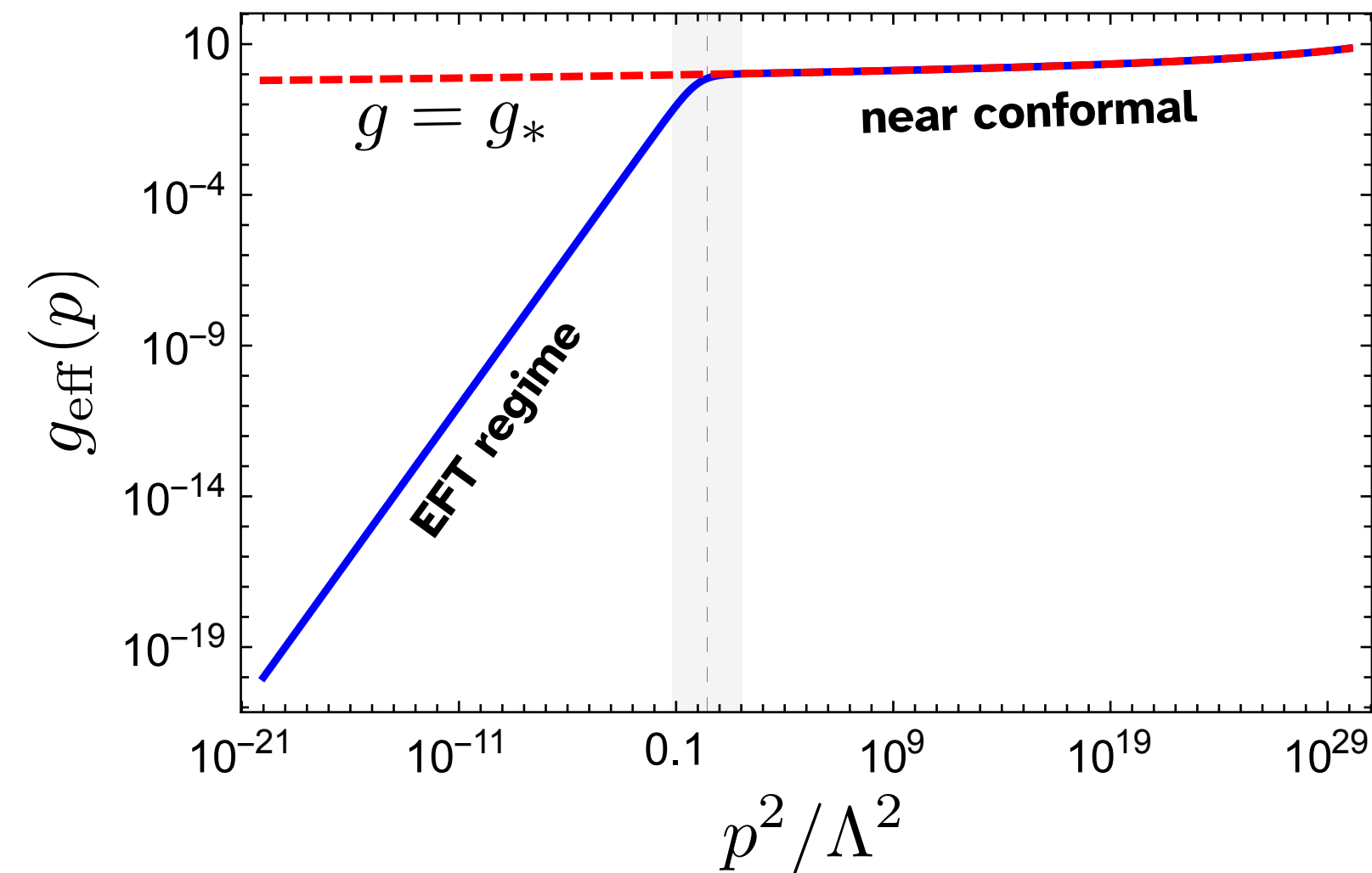
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CCH, Litim, *in prep.*

Four-fermion fixed point = massless separatrix in Yukawa theory.

$$\mu^2 \frac{y^2(\mu)}{m^2(\mu)} = g_*$$

Importance of RG scheme



$$\beta_g(g) = 2g \left(1 - \frac{g}{g_*} \right)$$

Physics is universal, e.g. exact amplitudes, but the beta function is not.

- g^* is a **scheme dependent** constant
- **MS** corresponds to limit g^* to infinity
- Not a good choice for **approximate calculations** at intermediate scales
- Convergence improved by **non-minimal schemes** (WIP + Yannick Kluth @ Toronto)

Summary

Four-fermion interactions can UV-complete themselves

- Tower of higher derivative 4F + 8F operators, but only **3 parameters**
- **Near scale invariance** in UV with relevant 4F + marginal 8F
- Strong coupling sets in near EFT scale — scheme choice important

Further aspects

- **Landscape of theories:** symmetries, mom. dependence, ... , SMEFT, ...
- **Unitarity restoration:** (partial) RG fixed points in EFT $\iff$ UV completions?
- **New perspective** on 4F fixed points in truncated FRG approximations