Universal crossover in the three-channel charge Kondo model at high-transparency

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Main questions

- What kind of physics emerges in the three-channel Kondo model (3CK)?
- Why is the Non-Perturbative Functional Renormalization Group (NPFRG) relevant?

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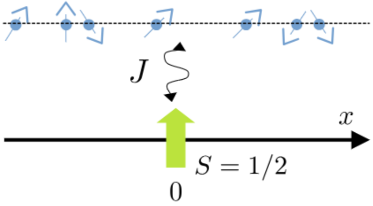
Kondo model: phenomenology

$\Rightarrow$



with Hamiltonian

$$H = i v_F \sum_{\sigma=\uparrow,\downarrow} \int_{-\infty}^{+\infty} dx\, \psi_\sigma^\dagger \partial_x \psi_\sigma + v_F J \boldsymbol{s}(0) \cdot \boldsymbol{S}, \qquad \boldsymbol{s}(0) = \frac{1}{2} \boldsymbol{\psi}_\sigma^\dagger(0) \boldsymbol{\sigma}_{\sigma\sigma'} \boldsymbol{\psi}_{\sigma'}(0)$$

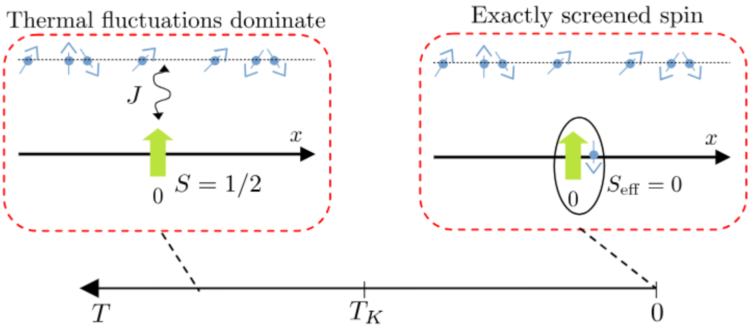
where $J \geq 0$. It describes a static spin-$1/2$ in an electron gas.

Kondo model: phenomenology

The Hamiltonian of the Kondo model is:

$$H = i v_F \sum_{\sigma=\uparrow,\downarrow} \int_{-\infty}^{+\infty} dx\, \psi_\sigma^\dagger \partial_x \psi_\sigma + v_F J \boldsymbol{s}(0) \cdot \boldsymbol{S}$$

where $J \geq 0$. It describes a static spin-$1/2$ in an electron gas.



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Experimental set-up

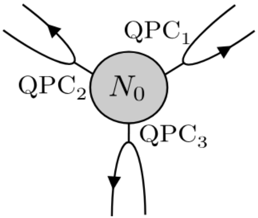
A charge version of the Kondo effect can be realized in mesoscopic devices:

- Metallic island with charging energy E_C and gate voltage $N_0 = 1/2$

$$H_C = E_C(\hat{N} - N_0)^2$$

$$\Rightarrow \text{pseudo-spin } 1/2$$

- Chiral quantum Hall edges $\Rightarrow$ one-dimensional electronic leads
- Quantum point contacts (QPCs) with **transparency** $0 < \tau < 1$



- ○ $\tau \ll 1$: only a few electrons can tunnel $\Rightarrow$ Exact mapping on the Kondo model
- ○ $\tau \simeq 1$: high-transparency regime $\Rightarrow$ The mapping on the usual Kondo model fails

High-transparency regime: an effective action

A $(0+1)$D effective action for the metallic island located at $x=0$ can be derived by:

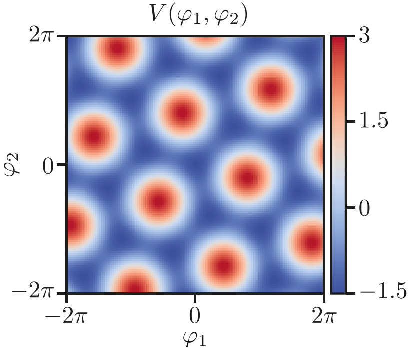
- Bosonizing the 1D leads $\rightarrow$ Three bosonic fields $\phi_{1,2,3}(x,\tau)$
- Integrating the leads $\rightarrow$ $\tilde{\phi}_{1,2,3}(\tau) \equiv \phi_{1,2,3}(x=0,\tau)$
- Integrating the center of mass of the fields, which is gapped $\rightarrow$ $\varphi_{1,2}(\tau)$

$$\boxed{S[\boldsymbol{\varphi}] = \frac{1}{\pi K}\sum_{\omega_n} |\omega_n||\boldsymbol{\varphi}(i\omega_n)|^2 - r\sum_{a=1}^{3}\int d\tau \cos\left(2\boldsymbol{G}_a\cdot\boldsymbol{\varphi}(\tau) - \frac{\pi}{3}\right).}$$

High-transparency regime and QBM

Particle in a 2-dimensional periodic potential with a honeycomb lattice of minima and subject to dissipation:

$$\boxed{S[\boldsymbol{\varphi}] = \frac{1}{\pi K}\sum_{\omega_n}|\omega_n||\boldsymbol{\varphi}(i\omega_n)|^2 - r\sum_{a=1}^{3}\int d\tau\,\cos\left(2\boldsymbol{G}_a\cdot\boldsymbol{\varphi}(\tau)-\frac{\pi}{3}\right).}$$

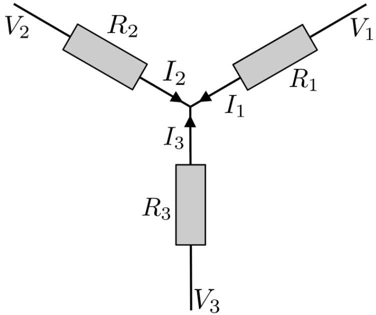


- $\boldsymbol{\varphi} = (\varphi_1, \varphi_2)$: 2D vector, coordinate of the quantum Brownian particle.
- Luttinger parameter K: interactions in the 1d leads. Mapping on Kondo at $K = 1$.
- Supplemented by UV cutoff D (electron bandwidth).

Conductance

To identify the Kondo fixed point, experimentalists measure the conductance matrix G_{ij} defined by

$$I_j = \sum_k G_{jk} V_k.$$

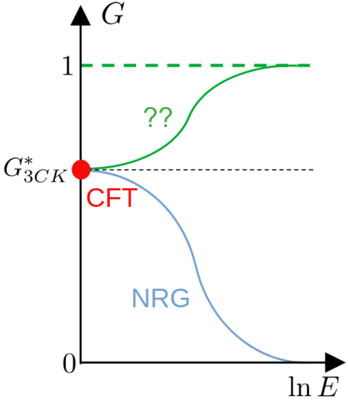


In the symmetric case,

$$G_{ij}(\omega) = G(\omega)\left(\delta_{ij} - \frac{1}{3}\right)$$

where $G(\omega)$ can be understood as the renormalized transparency of the QPCs.

Different approaches for different regimes

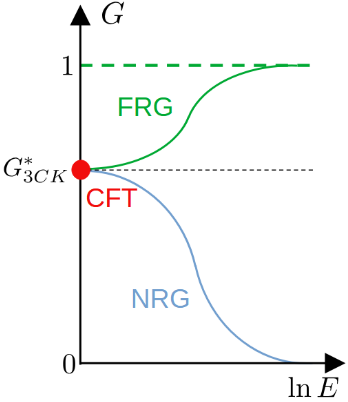


(i) Cross-over in the low transparency regime solved with numerical renormalization group (NRG)

(ii) Kondo fixed point at zero-energy described by a conformal field theory (CFT): $G^*_{\mathrm{3CK}} = 2\sin^2\left(\frac{\pi}{5}\right)$

(iii) No theoretical approach to describe the cross-over in the high transparency regime

Different approaches for different regimes



In the high transparency regime:

- Bosonic action
- Similar to the boundary sine-Gordon model [Daviet, Dupuis 2023]
- Non-perturbative fixed point

$\Rightarrow$ FRG

$$G(\omega, T=0) = \frac{2}{\pi K}\mathrm{Re}\left\{|\omega|\langle\boldsymbol{\varphi}(i\omega)\cdot\boldsymbol{\varphi}(-i\omega)\rangle\Big|_{i\omega\to\omega+i0^+}\right\} \Rightarrow \textbf{BMW scheme}$$

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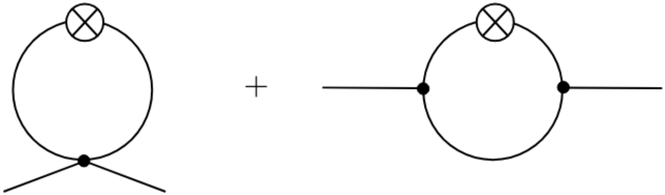
The flow equation of $\Gamma_k^{(n)}$ involves $\Gamma_k^{(n+1)}$, $\Gamma_k^{(n+2)}$. The BMW approximation closes this infinite hierarchy.

$$\partial_t \Gamma_k^{(2)}(i\omega, \phi) =$$



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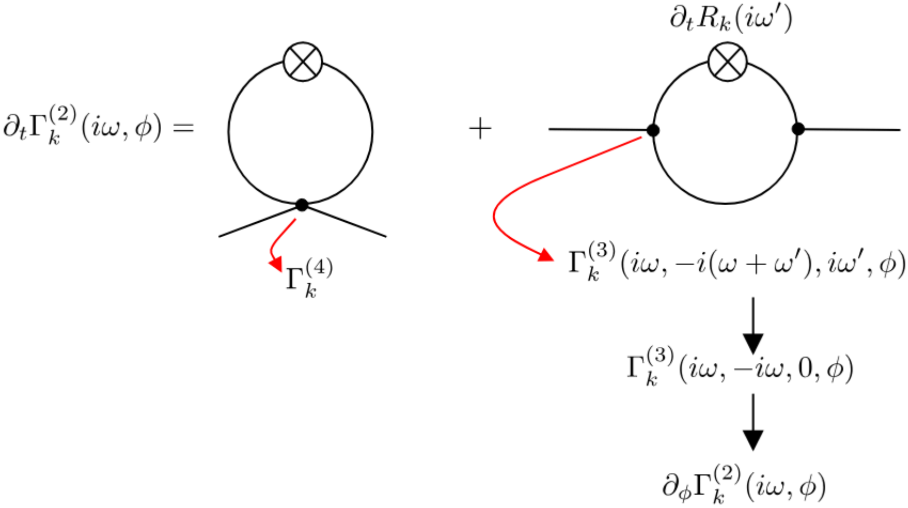

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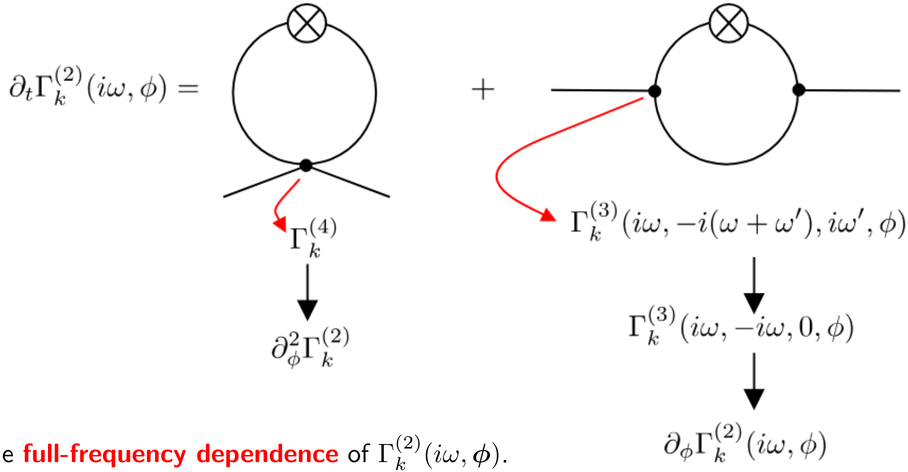


$\Rightarrow$ Keep the **full-frequency dependence** of $\Gamma_k^{(2)}(i\omega, \phi)$.

We parametrize the effective action and the 1PI 2-point vertex in uniform field:

$$\frac{1}{\beta}\,\Gamma_k[\boldsymbol{\phi}]|_{\phi\,\text{unif.}} = U_k(\boldsymbol{\phi})$$

$$\Gamma^{(2)}_{k,ij}(i\omega_n, \boldsymbol{\phi}) = \frac{|\omega_n|}{\pi K}\delta_{ij} + \Delta_{k,ij}(i\omega_n, \boldsymbol{\phi}) + \partial_{\phi_i}\partial_{\phi_j}U_k(\boldsymbol{\phi}),$$

where U_k is the effective potential and Δ_k the 'self-energy', with initial conditions:

$$U_{k_{\text{in}}}(\boldsymbol{\phi}) = -r\sum_{a=1}^{3}\int d\tau\,\cos\left(2\boldsymbol{G}_a\cdot\boldsymbol{\phi}(\tau) - \frac{\pi}{3}\right)$$

$$\Delta_{k_{\text{in}}}(i\omega_n, \boldsymbol{\phi}) = 0$$

Δ_k and U_k are periodic $\Rightarrow$ Expand in Fourier series and keep 35×35 harmonics

Numerics: 6-core parallelization, simulations lasting $\sim$2 days

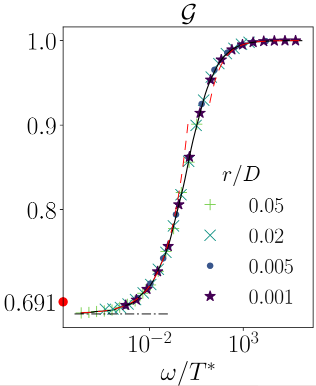
Universal conductance at $T = 0$ (BMW)

In the limit $|\omega| \ll D$, the conductance exhibits a universal scaling behavior:

$$G(\omega, T=0) \underset{|\omega|\ll D}{=} \mathcal{G}\left(\frac{\omega}{T^*}\right).$$

At low frequency

$$\mathcal{G}(\omega/T^*) \underset{|\omega|\ll T^*}{=} G^*_{3CK} + \alpha|\omega/T^*|^\nu.$$



	LPA	BMW	Exact [Yi,2002]
$G^*_{3\mathrm{CK}}$	$-$	0.677	$2\sin^2(\pi/5) \simeq 0.691$
ν	0.455	0.408	$2/5$

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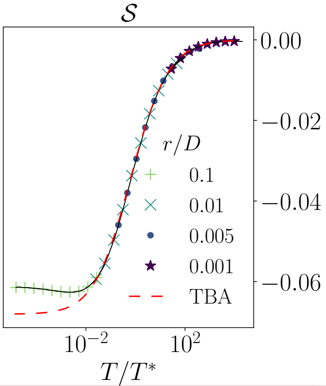
Universal entropy cross-over (LPA and BMW)

The FRG approach gives access to $\Delta S(T) = S(T) - S(T)\big|_{r=0}$. Like the conductance, it exhibits a universal scaling behavior

$$\Delta S(T) \underset{T \ll D}{=} \mathcal{S}\left(\frac{T}{T^*}\right)$$

At zero temperature, it takes a value characteristic of Fibonnacci anyons.



	LPA	BMW	Exact (TBA) [Affleck et al.,2001]
$\mathcal{S}(0)$	-0.058	-0.062	$\ln\left(\frac{1+\sqrt{5}}{2\sqrt{3}}\right) \simeq -0.068$

Conclusion

NPFRG allows us to study the **high-transparency regime** of the 3CK model:

- Excellent quantitative agreement with known results ($G^*_{3CK}, \mathcal{S}(0), \nu$).
- Full universal crossover ($\mathcal{G}(\omega/T^*), \mathcal{S}(T/T^*)$).
- Extends naturally to interacting leads ($K \neq 1$).

$\Rightarrow$ Establishes FRG as a powerful nonperturbative tool for quantum impurity problems in regimes inaccessible to conventional approaches

This work is detailed in *Phys. Rev. Lett. 136, 066501*.

These results are confirmed in *arXiv:2607.07797*, in which we present a new and involved Monte Carlo approach to solve the multichannel Kondo model.

Thank you for your attention!

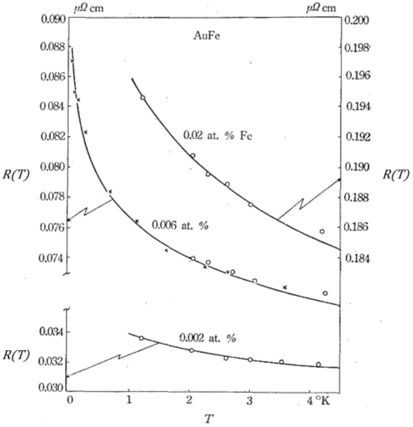


Figure: Comparison of experimental and theoretical $R(T)$ curves for AuFe alloys. [Kondo 1964]

$$\rho(T) = \rho_0 + c\ln(A/T) + aT^2 + bT^5 \tag{1}$$

NPFRG framework

$$\mathcal{Z}_k[J] = \int \mathcal{D}\varphi \, e^{-\mathcal{S}[\varphi] - \Delta\mathcal{S}_k[\varphi] + \int d\tau \, J\varphi}$$

$$\Delta\mathcal{S}_k[\varphi] = \int d\tau \, d\tau' \, \varphi(\tau) R_k(\tau - \tau') \varphi(\tau')$$

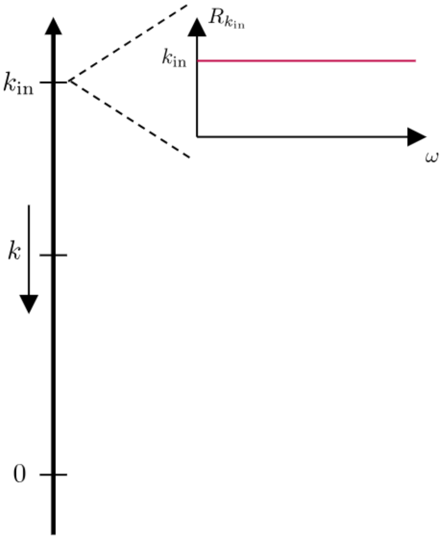
We define $\phi = \langle\varphi\rangle$ and the slightly modified Legendre transform:

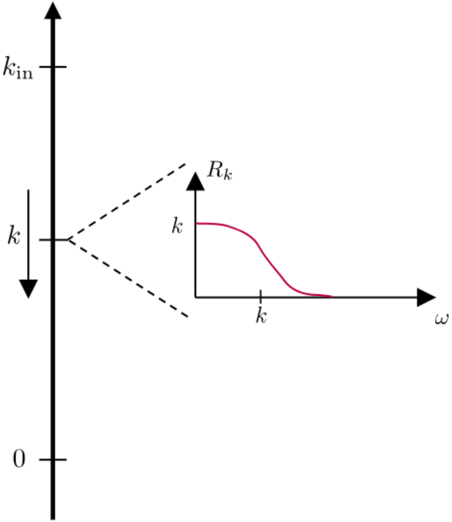
$$\Gamma_k[\phi] = -\ln \mathcal{Z}_k[J] - \Delta\mathcal{S}_k[\phi] + \int d\tau \, J\phi$$

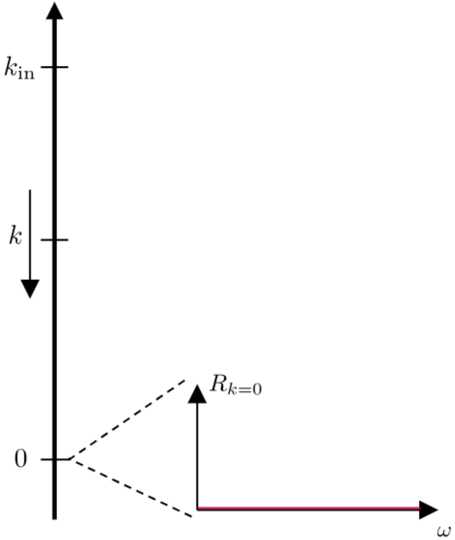
$$U_k(\phi) = \frac{1}{\beta} \, \Gamma_k[\phi]|_{\phi \, \mathrm{unif.}}$$

with the flow equation

$$\partial_k \Gamma_k = \frac{1}{2} \int_0^\beta d\tau \, \mathrm{Tr} \left[\partial_k R_k (\Gamma_k^{(2)} + R_k)^{-1} \right]$$


$R_{k_{\rm in}}$
$k_{\rm in}$
ω
$k_{\rm in}$
k
0


k_{in}
k
0
R_k
k
k
ω


k_{in}
k
0
$R_{k=0}$
ω

Approximation schemes

We parametrize the effective action and the 1PI 2-point vertex in uniform field:

$$\frac{1}{\beta}\,\Gamma_k[\phi]|_{\phi\,\mathrm{unif.}} = U_k(\phi)$$

$$\Gamma^{(2)}_{k,ij}(i\omega_n,\phi) = \frac{|\omega_n|}{\pi K}\delta_{ij} + \Delta_{k,ij}(i\omega_n,\phi) + \partial_{\phi_i}\partial_{\phi_j}U_k(\phi),$$

where U_k is the effective potential and Δ_k the self-energy. Initial conditions:

$$U_{k_{\mathrm{in}}}(\varphi_1,\varphi_2) = r_1\cos(\varphi_1 + \tfrac{\varphi_2}{\sqrt{3}} + \tfrac{2\pi N_g}{3}) + r_2\cos(\tfrac{2\varphi_2}{\sqrt{3}} - \tfrac{2\pi N_g}{3}) + r_3\cos(\varphi_1 - \tfrac{\varphi_2}{\sqrt{3}} - \tfrac{2\pi N_g}{3})$$

$$\Delta_{k_{\mathrm{in}}} = 0$$

Δ_k and U_k are periodic over the triangular lattice generated by:

$$G_1 = (1,-1/\sqrt{3}),\quad G_2 = (0,2/\sqrt{3}),\quad G_3 = (-1,-1/\sqrt{3}) = -G_1 - G_2$$

Local potential approximation

Neglect the renormalization of the self-energy: $\Delta_k = 0$.

$$\frac{1}{\beta}\,\Gamma_k[\phi]|_{\phi\,\mathrm{unif.}} = U_k(\phi)$$

$$\Gamma^{(2)}_{k,ij}(i\omega_n,\phi) = \frac{|\omega_n|}{\pi K}\delta_{ij} + \Delta_{k,ij}(i\omega_n,\phi) + \partial_{\phi_i}\partial_{\phi_j}U_k(\phi),$$

The Wetterich equation reads:

$$\partial_k U_k = \frac{1}{2}\int_0^\beta d\tau \,\mathrm{Tr}\left\{\partial_k R_k\left(\frac{|\omega_n|}{\pi K}\delta_{ij} + U_k'' + R_k)^{-1}\right)\right\}$$

Blaizot-Méndez–Galain-Wschebor (BMW) scheme

Differentiating functionally the Wetterich equation with respect to ϕ generates an infinite hierarchy of flow equations: $\partial_k \Gamma_k^{(n)}$ involves $\Gamma_k^{(n+1)}$ and $\Gamma_k^{(n+2)}$.

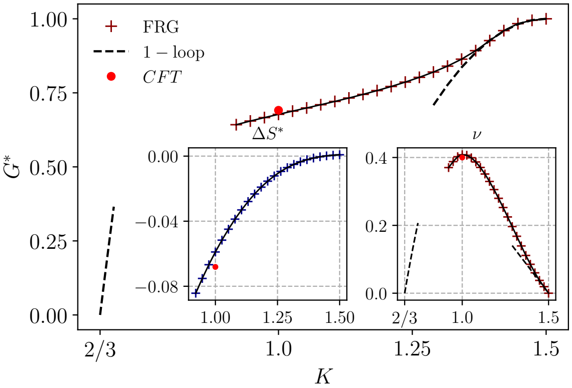
$$\begin{aligned}
\partial_t \Gamma^{(2)}_{k,ij}(i\omega_\nu, -i\omega_\nu; \boldsymbol{\phi}) = \frac{1}{2}\tilde{\partial}_t \sum_{\omega_n, i_1, i_2} G_{k,i_1 i_2}(i\omega_n)\Gamma^{(4)}_{k,iji_2i_1}(i\omega_\nu, -i\omega_\nu, i\omega_n, -i\omega_n) \\
-\tilde{\partial}_t \sum_{\omega_n, i_1, i_2, i_3, i_4} \Big[G_{k,i_1 i_2}(i\omega_n)\Gamma^{(3)}_{k,ii_2i_3}(i\omega_\nu, i\omega_n, -i\omega_{n+\nu}) \\
\times G_{k,i_3 i_4}(i\omega_{n+\nu})\Gamma^{(3)}_{k,ji_4i_1}(-i\omega_\nu, i\omega_{n+\nu}, -i\omega_n) \Big],
\end{aligned}$$

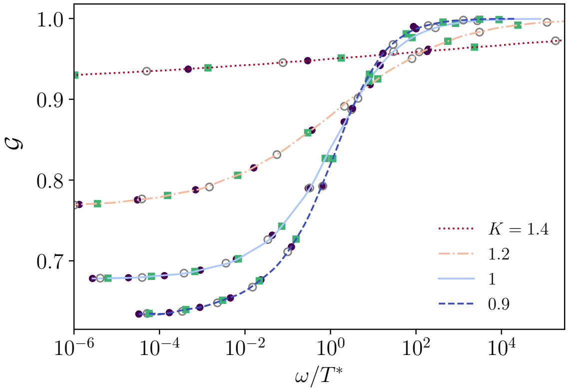
The BMW approximation closes this infinite hierarchy:

- Analyticity of $\Gamma^{(n)}$ at $\omega = 0$: put the internal loop frequency to zero in the vertices
- Use:

$$\Gamma^{(n+1)}_{k,i_1,\dots,i_n,i_{n+1}}(\omega_1, \dots, \omega_n, 0; \boldsymbol{\phi}) = \frac{\partial \Gamma_k^{(n)}(\omega_1, \dots, \omega_n; \boldsymbol{\phi})}{\partial \phi_{i_{n+1}}},$$

$\Rightarrow$ Keep the **full-frequency dependence** of the 1PI 2-point vertex.


FRG
1 − loop
CFT
G^*
K
ΔS^*
ν
1.00
0.75
0.50
0.25
0.00
2/3
1.0
1.25
1.5


$\mathcal{G}$
ω/T^*
$K = 1.4$
1.2
1
0.9