Non-Fermi-liquid metals and phase reconstruction: an FRG perspective

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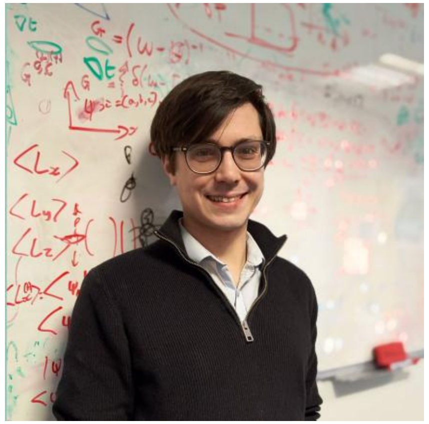
Acknowledgments

Sam Ridgway
former PhD student

Matt Trott
former PhD student



Luke Rhodes
EPSRC Fellow, St Andrews

Maria Ramirez
former MPhys student

Thomas Sheerin
former PhD student, now postdoc at St Andrews – **talk at 15.20 today**

The Scottish Doctoral Training Centre in Condensed Matter Physics

CM-CDT

An EPSRC Centre for Doctoral Training in Condensed Matter Physics

UKRI Engineering and Physical Sciences Research Council

Outline

1. Background: metals close to uniform order (magnetic or nematic).
2. Our work on quantum critical metals:
   a) a functional renormalisation group analysis, revealing a new non-Fermi-liquid fixed point;
   b) a better soft-cutoff fRG treatment, revealing a family of such fixed points;
   c) an analysis of what happens if the boson is a gauge boson.
3. A teaser for Thomas’s talk: phase competition near a higher-order Van Hove point.
4. Summary and outlook.

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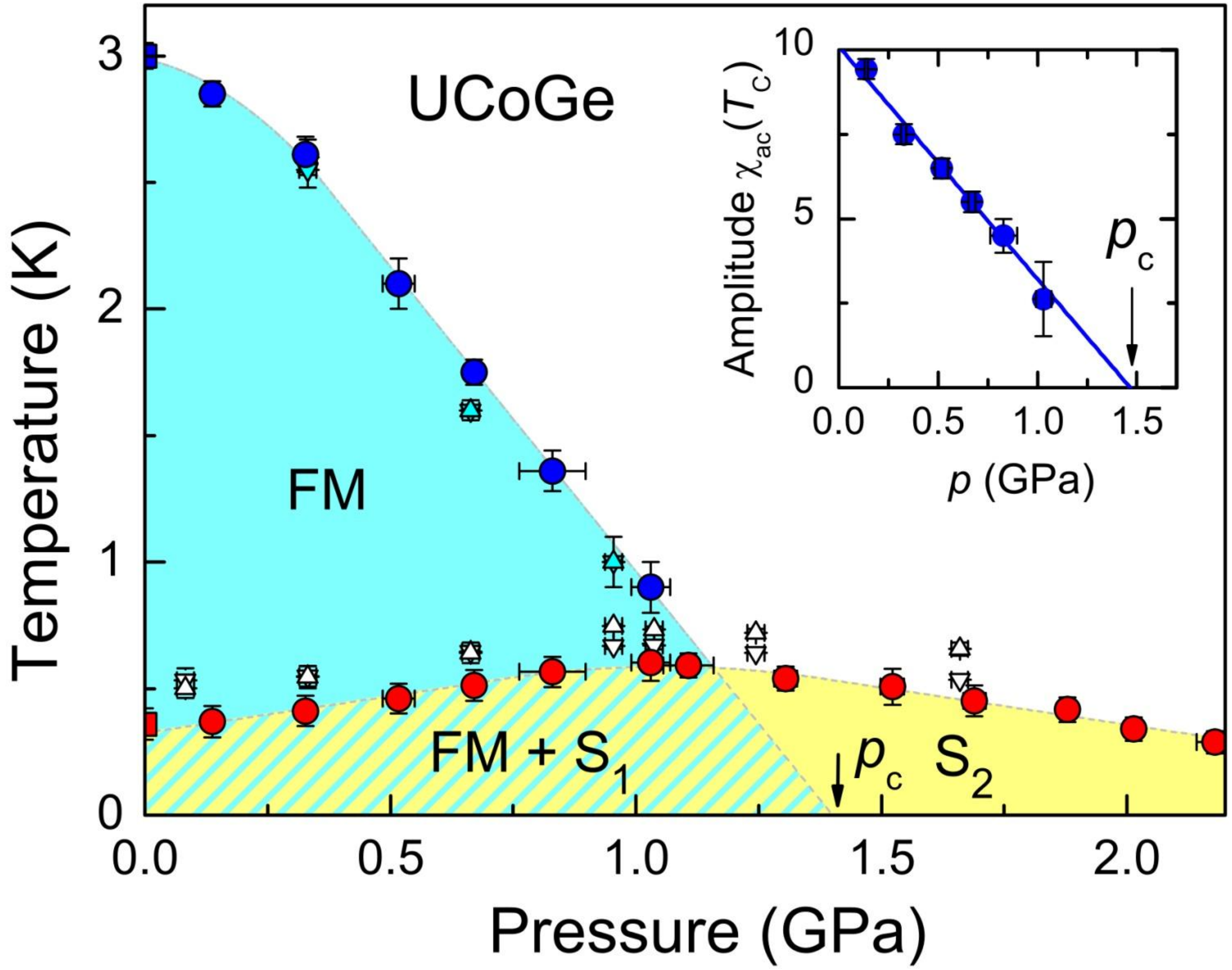
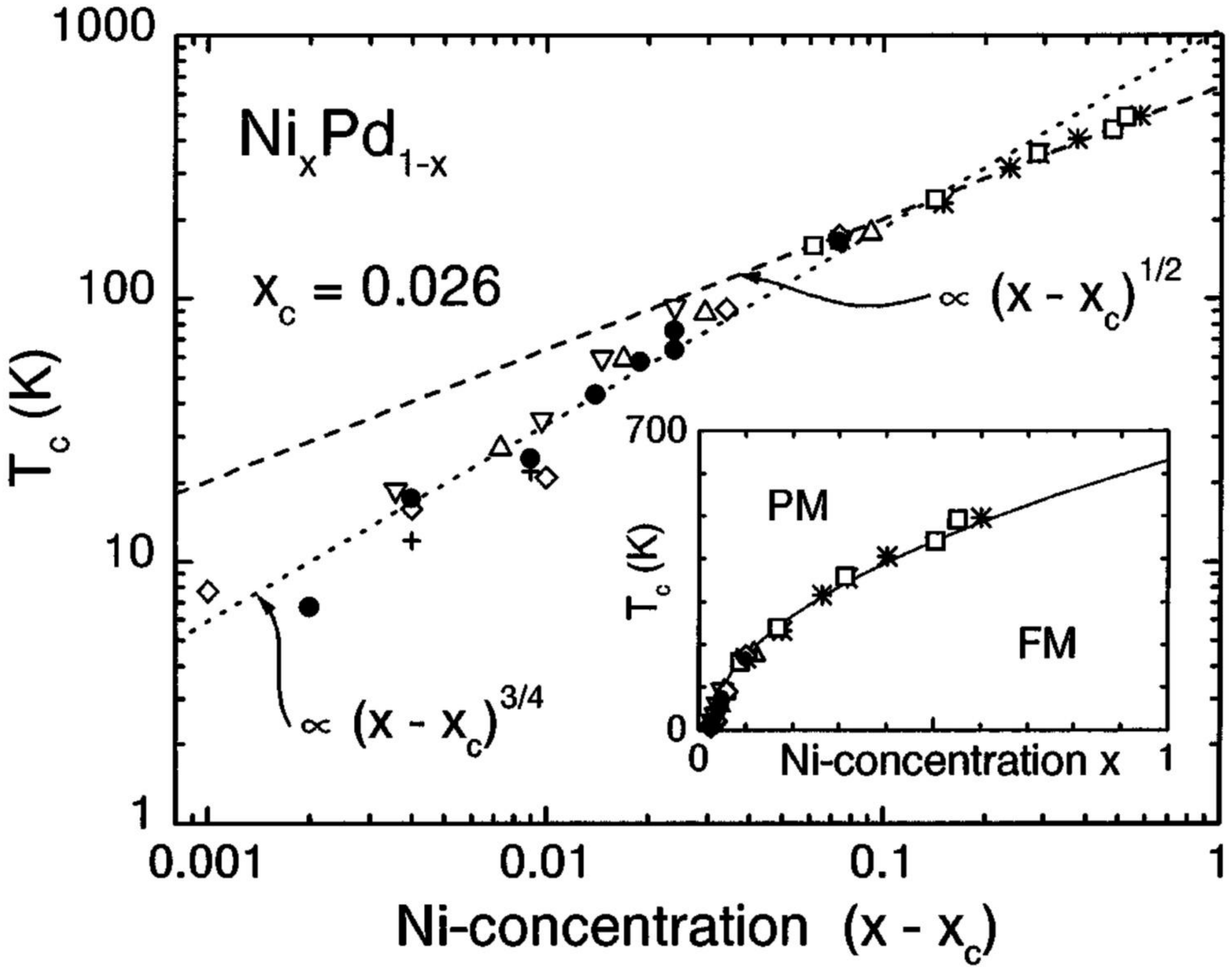
Phase diagram of UCoGe



E. Slooten, T. Naka, A. Gasparini, Y. K. Huang, and A. De Visser, *Phys. Rev. Lett.* **103**, 097003 (2009).

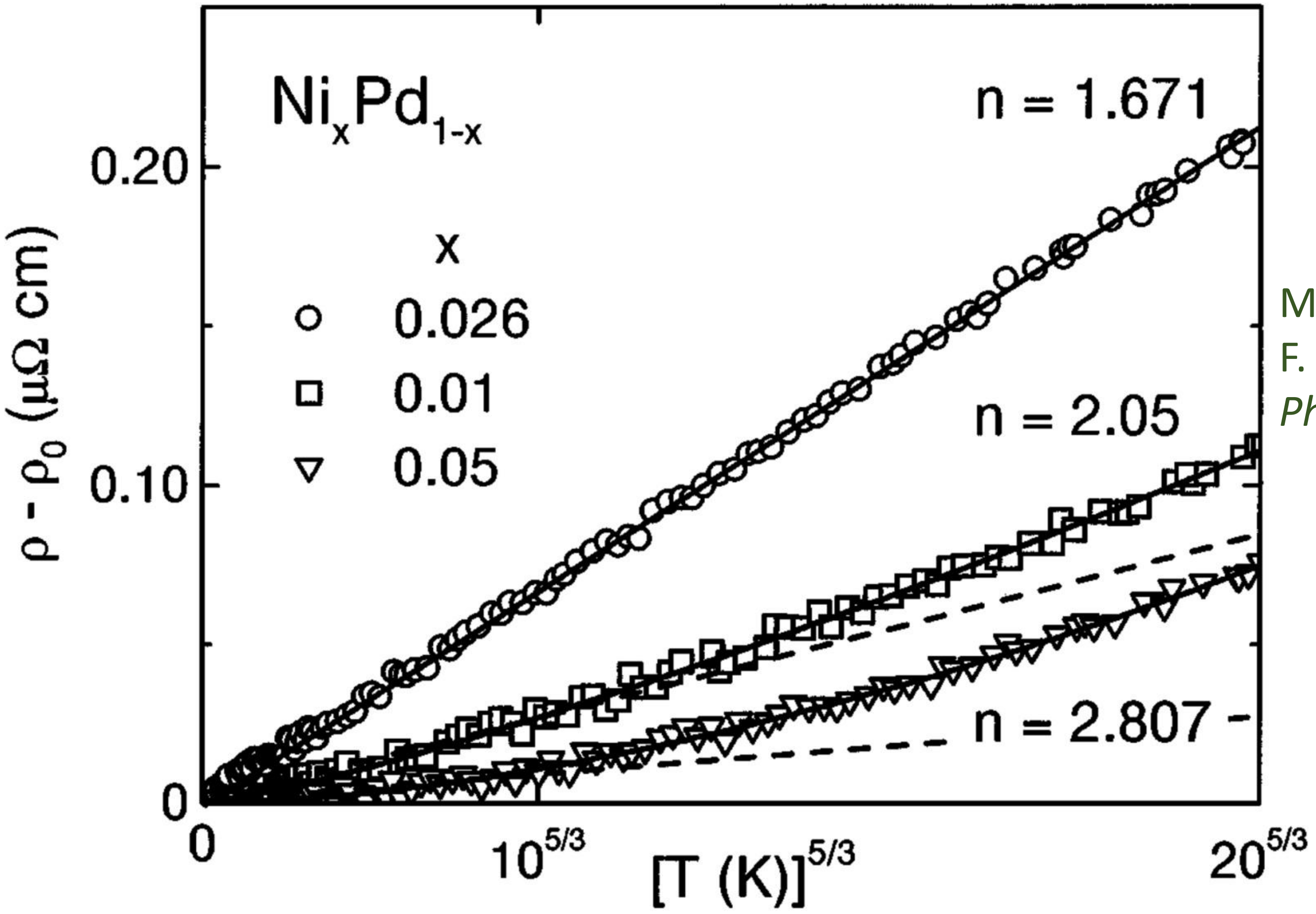
1. Background: metals close to uniform order

Phase diagram of Ni_xPd_{1-x}



M. Nicklas, M. Brando, G. Knebel, F. Mayr, W. Trinkl, and A. Loidl, *Phys. Rev. Lett.* **82**, 4268 (1999).

Anomalous resistivity power-law



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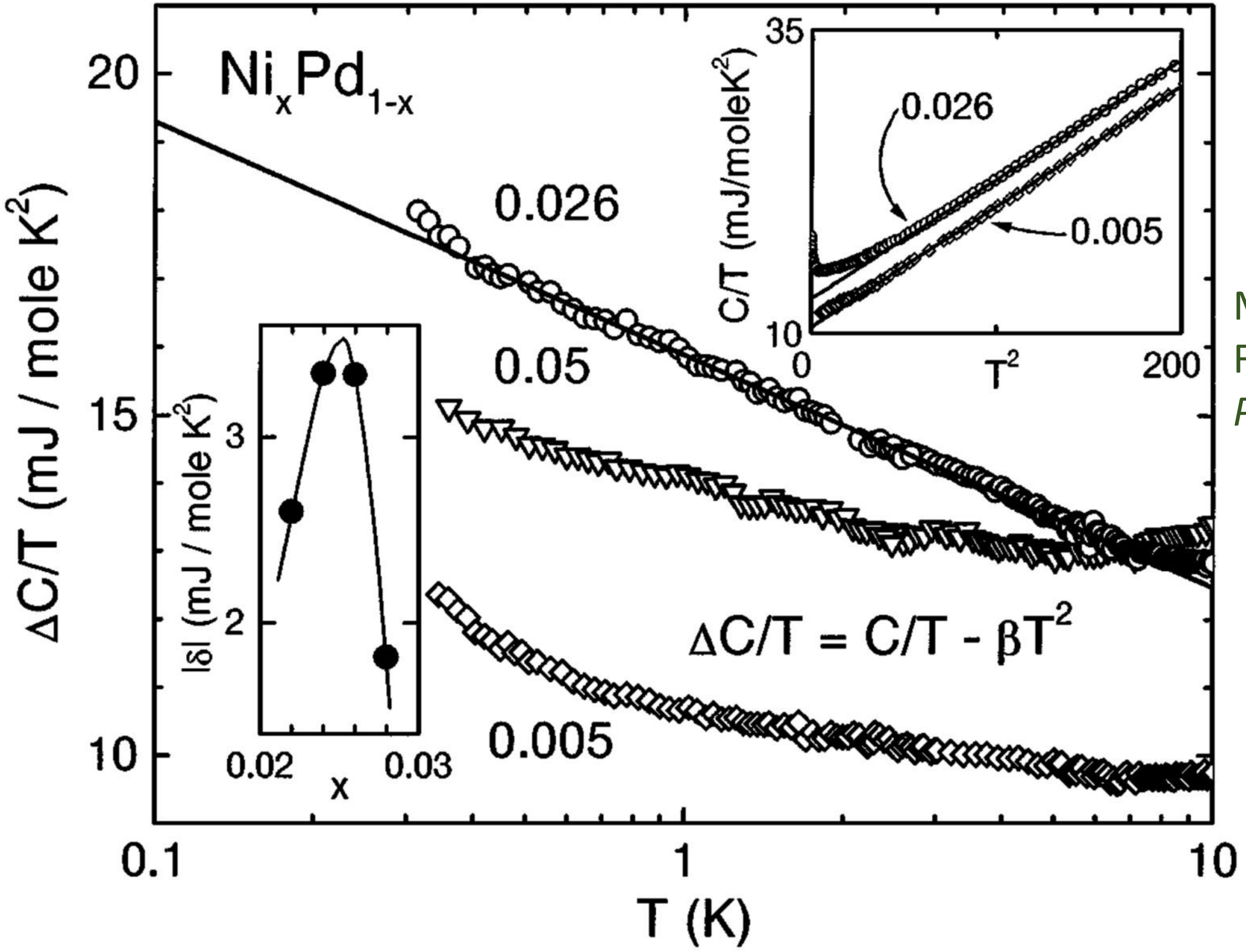
Anomalous specific heat



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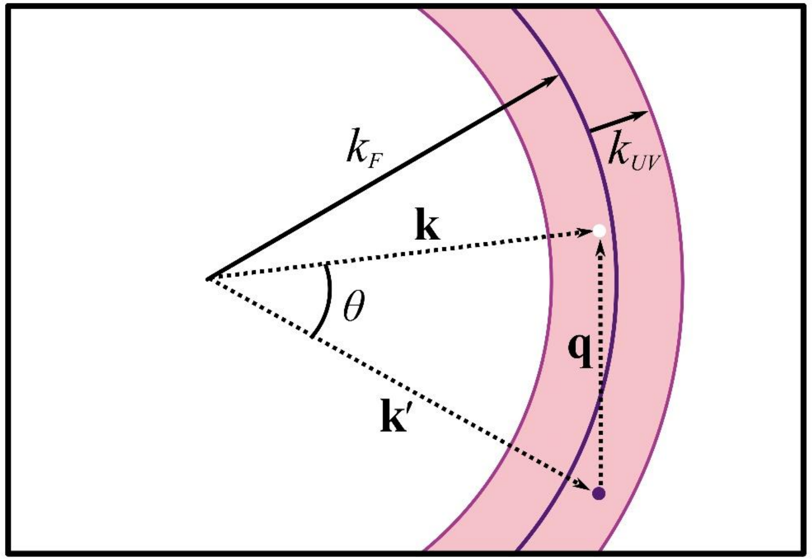
Fermion-boson model; FRG equation

$$S = -\int_{\omega\mathbf{k}} \bar{\psi}^{\alpha}_{\omega\mathbf{k}} \left(i\omega - \tilde{k}\right) \psi^{\alpha}_{\omega\mathbf{k}} + \frac{1}{2}\int_{\Omega\mathbf{q}} (|\mathbf{q}|^2 + m^2)\boldsymbol{\phi}^*_{\Omega\mathbf{q}} \cdot \boldsymbol{\phi}_{\Omega\mathbf{q}}$$

$$+ g \int_{\omega\mathbf{k}} \int_{\Omega\mathbf{q}} \bar{\psi}^{\alpha}_{\omega+\Omega,\mathbf{k}+\mathbf{q}} \boldsymbol{\sigma}^{\alpha\beta} \psi^{\beta}_{\omega\mathbf{k}} \cdot \boldsymbol{\phi}_{\Omega\mathbf{q}} \,.$$

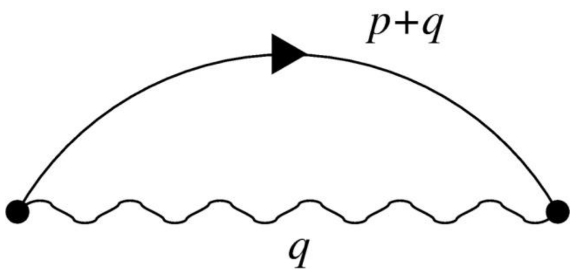


$$\frac{\mathrm{d}}{\mathrm{d}\Lambda}\Gamma^{\Lambda}_{R} = \frac{1}{2}\mathrm{STr}\left(\partial_{\Lambda} R^{\Lambda}\left[\Gamma^{(2)\Lambda}_{R} + R^{\Lambda}\right]^{-1}\right)$$

Equations for self-energies

$$\Sigma_f^\Lambda(k) = (Z_f^\Lambda - 1) i k_0 - (A_f^\Lambda - 1)\tilde{k}$$

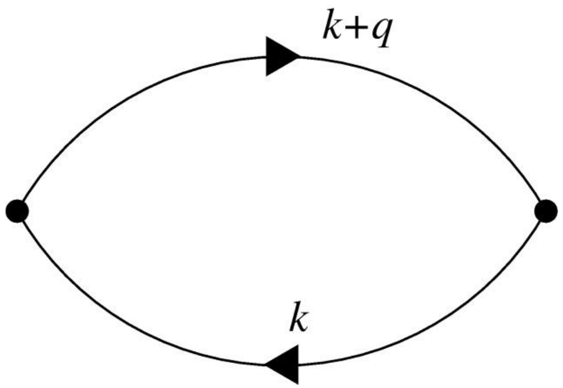
$$\partial_\Lambda \Sigma_f^\Lambda(k) = (g^\Lambda)^2 \int_q^R G_f^\Lambda(k+q) D_b^\Lambda(q)$$



$$\Sigma_b^\Lambda(q) = Z_b^\Lambda \frac{|q_0|}{|\mathbf{q}|}$$

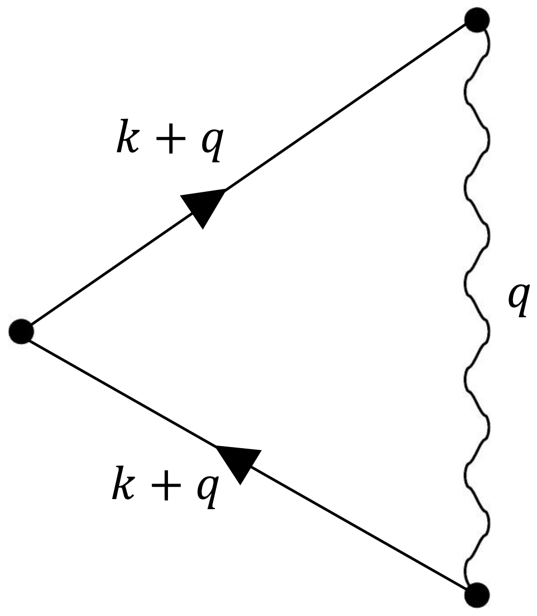
$$\Pi_{ph}^\Lambda(q) = (g^\Lambda)^2 \int_k G_f^\Lambda(k+q) G_f^\Lambda(k)$$

$$Z_b^\Lambda = -\lim_{q_0 \to 0} \left(\frac{\Pi^\Lambda(q_0, \mathbf{q}) - \Pi^\Lambda(0, \mathbf{0})}{|q_0|/|\mathbf{q}|} \right) \Bigg|_{|\mathbf{q}|=\Lambda}$$



Flow of Yukawa coupling

$$\partial_\Lambda g^\Lambda = (g^\Lambda)^3 \int_q^R (G_f^\Lambda(k+q))^2 D_q^\Lambda(q)\Big|_{\tilde{k}=0,k_0=0}$$



Closed flow equation for coupling

$$-\Lambda\partial_\Lambda \tilde{g}^\Lambda = \left(\tfrac{1}{2} - \eta_{\text{yuk}} - \tfrac{1}{2}\eta_f^Z - \tfrac{1}{2}\eta_f^A\right)\tilde{g}^\Lambda$$

$$\eta_f^Z = -\lim_{\Lambda\to 0}\left(\frac{d\ln \tilde{Z}_f^\Lambda}{d\ln\Lambda}\right)$$

$$\eta_f^A = -\lim_{\Lambda\to 0}\left(\frac{d\ln \tilde{A}_f^\Lambda}{d\ln\Lambda}\right)$$

$$\eta_{\text{yuk}} = \lim_{\Lambda\to 0}\left(\frac{d\ln \tilde{g}^\Lambda}{d\ln\Lambda}\right)$$

A new non-Fermi-liquid fixed point

$$\eta_\omega = -3\eta_g = \frac{3}{5}, \qquad \eta_k = 0, \qquad \tilde{g} = \frac{\pi}{\sqrt{5}}$$

$$\boxed{z_b = \frac{13}{5} \qquad z_f = \frac{13}{10} = \frac{z_b}{2}}$$

$$\left(D(\Omega, \boldsymbol{q})\right)^{-1} \sim \frac{|\Omega|}{|\boldsymbol{q}|^{3/5}} + \boldsymbol{q}^2$$

$$(G(\omega, \boldsymbol{k}_F))^{-1} \sim \omega^{1-(\eta_\omega/z_b)} \sim \omega^{10/13}$$

$$(G(0, \boldsymbol{k}))^{-1} \sim \tilde{k}^{1-(\eta_k/2)} \sim \tilde{k}$$

Experimental consequences

- Specific heat,

$$\frac{C}{T} \sim T^{-3/13}.$$

- Damping rate of magnetic fluctuations (paramagnons),

$$\Gamma(\boldsymbol{q}) \sim |\boldsymbol{q}|^{3/5}.$$

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A model that starts with undamped bosons

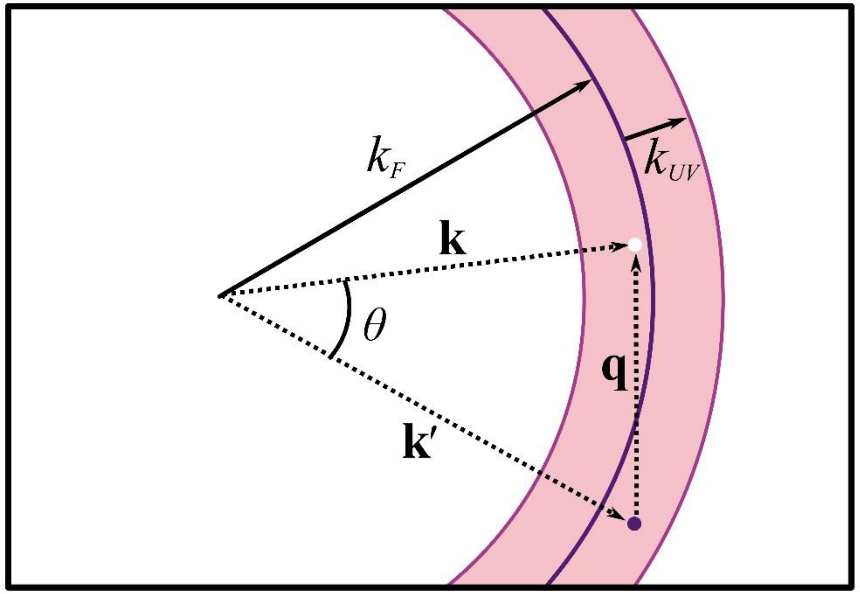
$$\Gamma_f = \sum_{\mu=\uparrow\downarrow} \int_k \overline{\psi}_{k,\mu} [i\,\mathcal{A}_\tau\,\omega - \mathcal{A}_x\,\ell] \psi_{k,\mu},$$

$$\Gamma_b = \frac{1}{2} \sum_a \int_q \phi^a_q [\mathcal{B}_\tau\,\Omega^2 + \mathcal{B}_x\,q^2 + \tilde{\delta}] \phi^a_{-q},$$

$$\Gamma_g = \sum_{a,\mu} \int_{k,q} \tilde{g}(k,q) \phi^a_q \overline{\psi}_{k+q,\mu} \psi_{k,\mu},$$

$$\Gamma_\lambda = \tilde{\lambda} \sum_{a,b} \int_{q_1,q_2,q_3} \phi^a_{q_1+q_3} \phi^a_{q_2-q_3} \phi^b_{q_1} \phi^b_{q_2}.$$

$N = k_F / k_{\mathrm{UV}}$



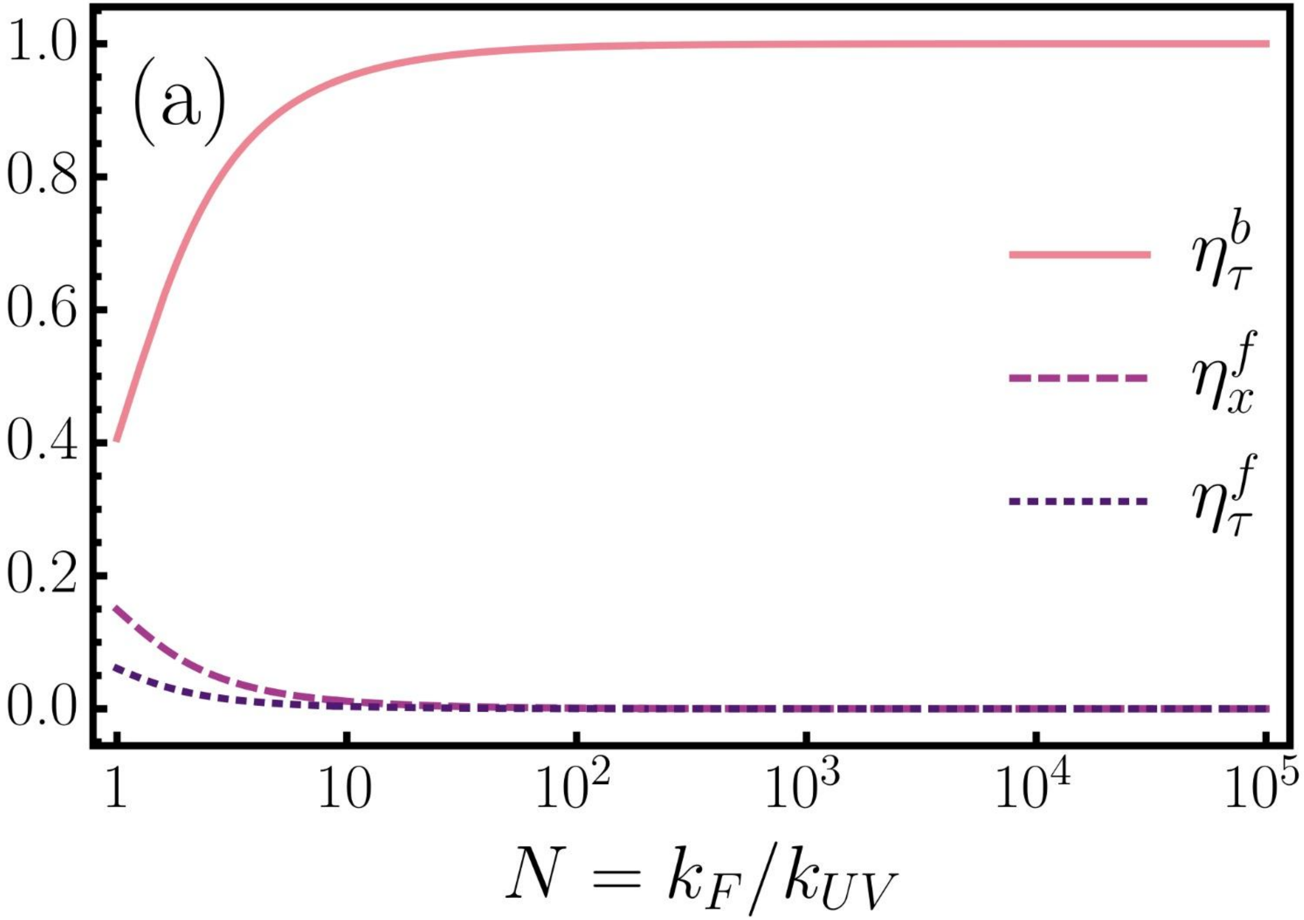
N_b flavours of boson

A soft frequency fermion regulator

$$R_f = [i\,\mathcal{A}_\tau\,\omega - \mathcal{A}_x\,\ell][\chi^{-1}(\omega,\Lambda) - 1],$$

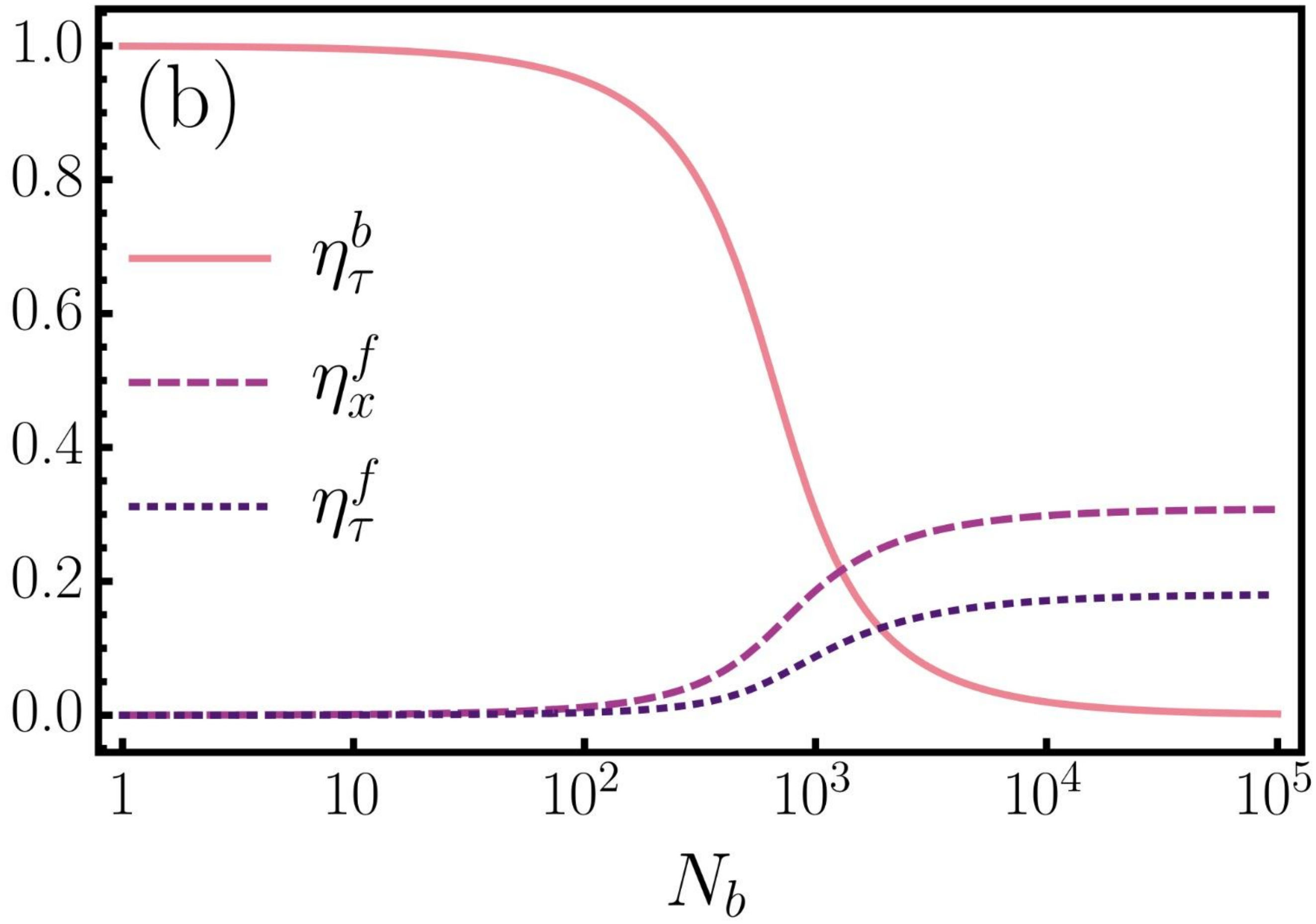
$$\chi(\omega,\Lambda) = \frac{\omega^2}{\omega^2 + \Lambda^2}$$

Results: $N_b = 1$ for a range of N



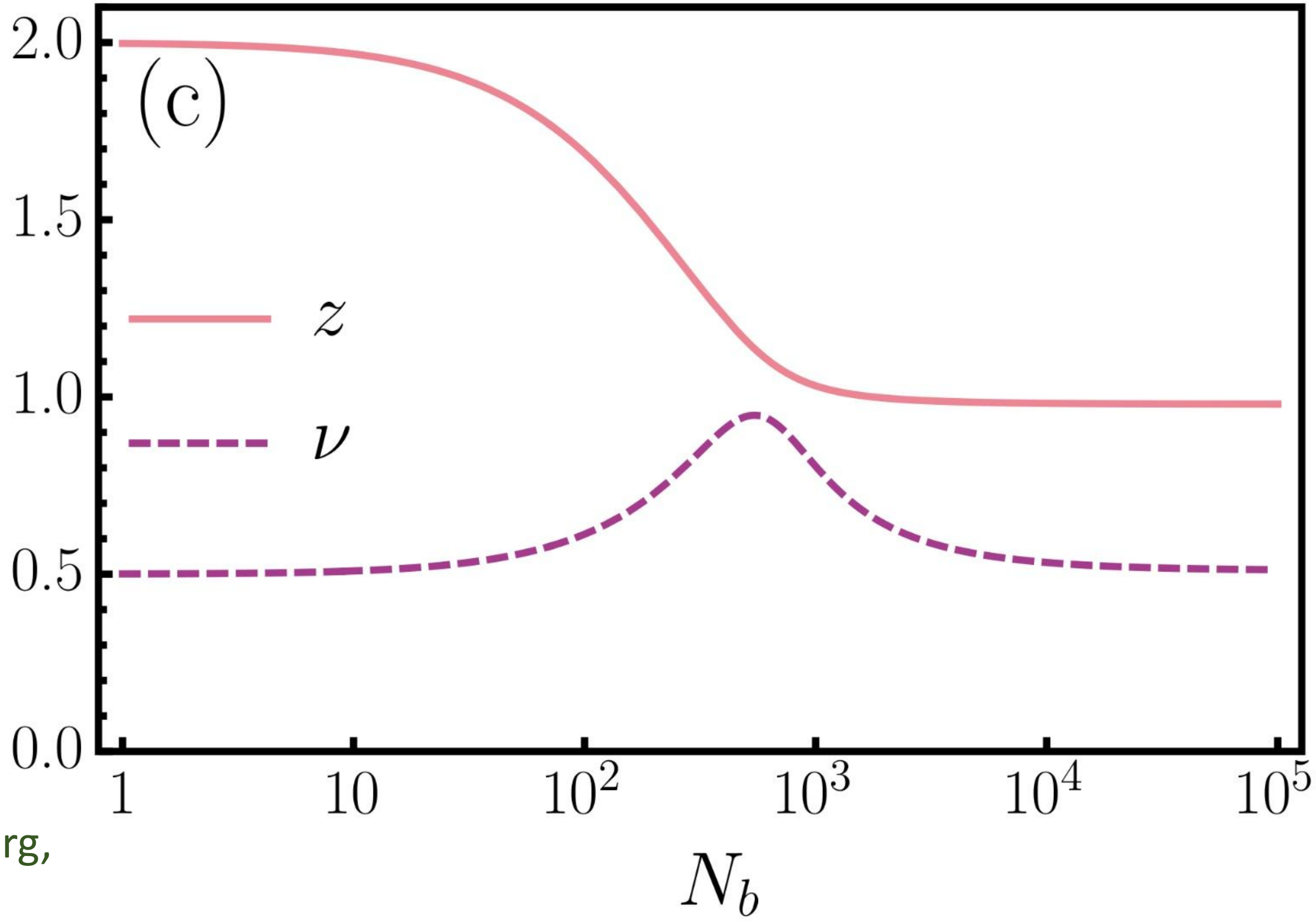
2(b). Functional RG with a soft frequency regulator for the nematic case

Results: $N = 10^3$ for a range of N_b



Results: $N = 10^3$, exponent crossover

N.B. – This matches the exponent observed in quantum Monte Carlo treatments of the nematic quantum critical point.



Y. Schattner, S. Lederer, S. A. Kivelson, and E. Berg, *Phys. Rev. X* **6**, 031028 (2016).

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Ansatz for low-energy effective action

$$\Gamma[\psi, \bar{\psi}, A] = -\int_k \sum_\sigma \bar{\psi}_\sigma(k)\left[G^{-1}(k) - R_f(k)\right]\psi_\sigma(k)$$

$$+\frac{1}{2}\int_q A_\mu(q)\left[(D^{-1})^{\mu\nu}(q) - R_A^{\mu\nu}(q)\right]A_\nu(-q)$$

$$-\int_{k,q}\sum_\sigma \Gamma^{(2,1)\mu}(k+q, k, q)\bar{\psi}_\sigma(k+q)$$

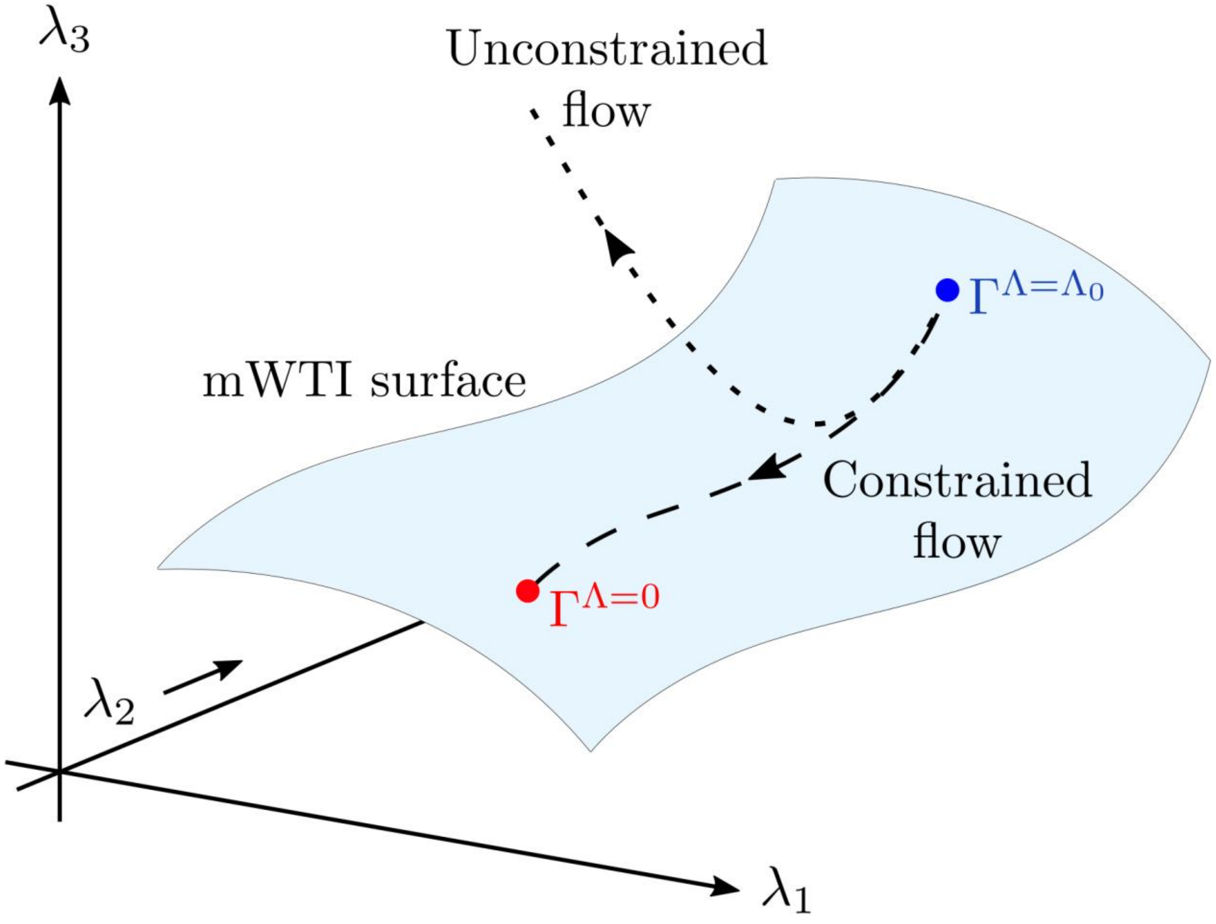
$$\times\, \psi_\sigma(k)A_\mu(q)$$

$$+\frac{1}{4!}\int_{q_1,q_2,q_3} \Gamma^{(0,4)\mu\nu\rho\sigma}(q_1, q_2, q_3, -q_1$$

$$-\, q_2 - q_3)$$

$$\times\, A_\mu(q_1)A_\nu(q_2)A_\rho(q_3)A_\sigma(-q_1 - q_2 - q_3).$$

Constrained vs. unconstrained flow



2(c). Functional RG with modified Ward-Takahashi identities for the case of a gauge boson

Fixed-point properties

$$Y = \frac{\mathcal{A}_{\mathbf{k}} k_{\rm UV}}{\mathcal{A}_\omega \Lambda}, \quad \zeta = \frac{\mathcal{A}_\omega \mathcal{B}_{\mathbf{A},\mathbf{q}}^{1/2}}{\mathcal{A}_{\mathbf{k}} \mathcal{B}_{\mathbf{A},\Omega}^{1/2}}, \quad \kappa = \sqrt{\frac{Ne^2}{8\pi \mathcal{B}_{\mathbf{A},\Omega}\Lambda}},$$

$$\delta = \frac{M_{\mathbf{A}}^2}{\mathcal{B}_{\mathbf{A},\Omega}\Lambda^2}, \quad g^2 = \frac{N\tilde{g}_{\mathbf{A}}^2}{8\pi \mathcal{A}_{\mathbf{k}}^2 \mathcal{B}_{\mathbf{A},\Omega}\Lambda}, \quad \lambda = \frac{\tilde{\lambda}}{8\pi \mathcal{B}_{\mathbf{A},\Omega}\mathcal{B}_{\mathbf{A},\mathbf{q}}\Lambda}.$$

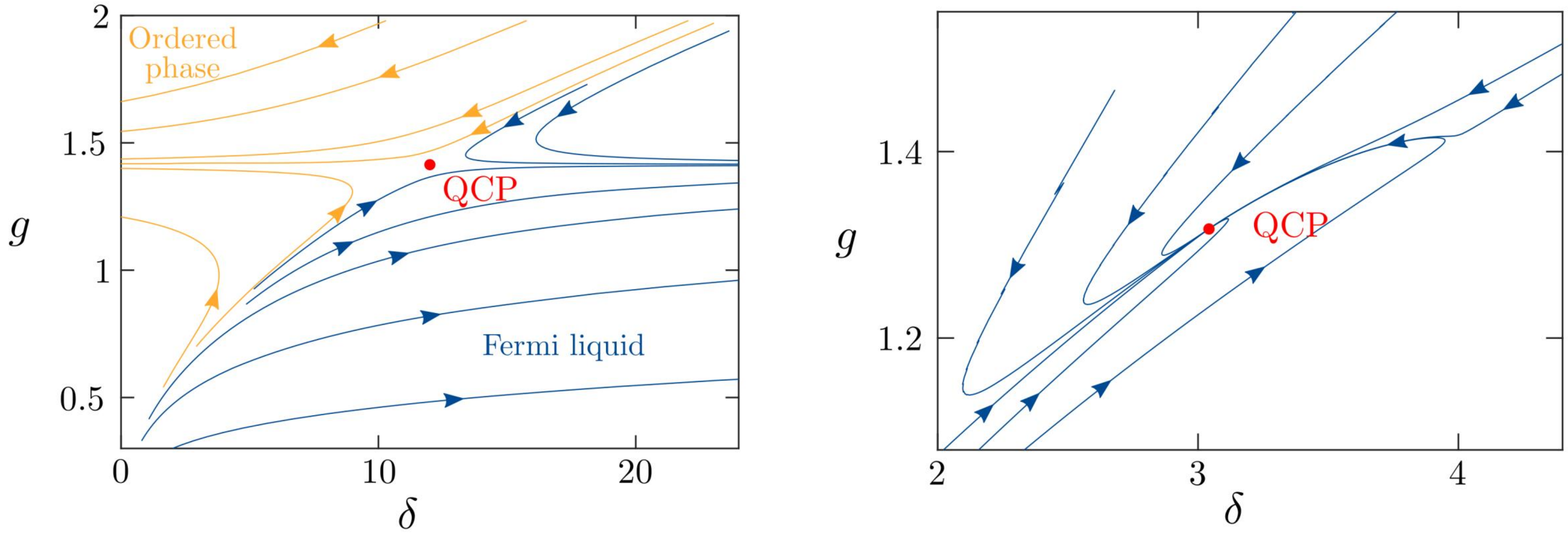
	Unconstrained	Constrained
η_ω	$\frac{1}{2} - 0.26\,\ell(N) + \cdots$	$\frac{1}{2} - 0.47\,\ell(N) + \cdots$
$\eta_{\mathbf{k}}$	$-0.21\,\ell(N) + \cdots$	$-0.47\,\ell(N) + \cdots$
$z_{\mathbf{A}}$	$2 - 0.19\,\ell(N) + \cdots$	2
ζ	$\frac{0.31}{N}[1 + 0.43\,\ell(N) + \cdots]$	$\frac{0.86}{N}[1 + 0.34\,\ell(N) + \cdots]$
κ	∞[a]	$\sqrt{2} - 0.44\,\ell(N) + \cdots$
δ	$12 - 3.4\,\ell(N) + \cdots$	$4 - 1.7\,\ell(N) + \cdots$
g	$\sqrt{2} - 0.14\,\ell(N) + \cdots$	$\sqrt{2} - 0.17\,\ell(N) + \cdots$
λ	$4\,\ell(N) + \cdots$	0

[a]Strictly this result holds at large but not infinite N. At $N = \infty$ in the unconstrained case, κ flows to some finite constant (generally not $\sqrt{2}$) that depends strongly on initial conditions.

$$N = k_F / k_{\rm UV}$$

$\ell(N)$ is $(\log N)/N$

Flow: unconstrained (left) vs constrained (right)



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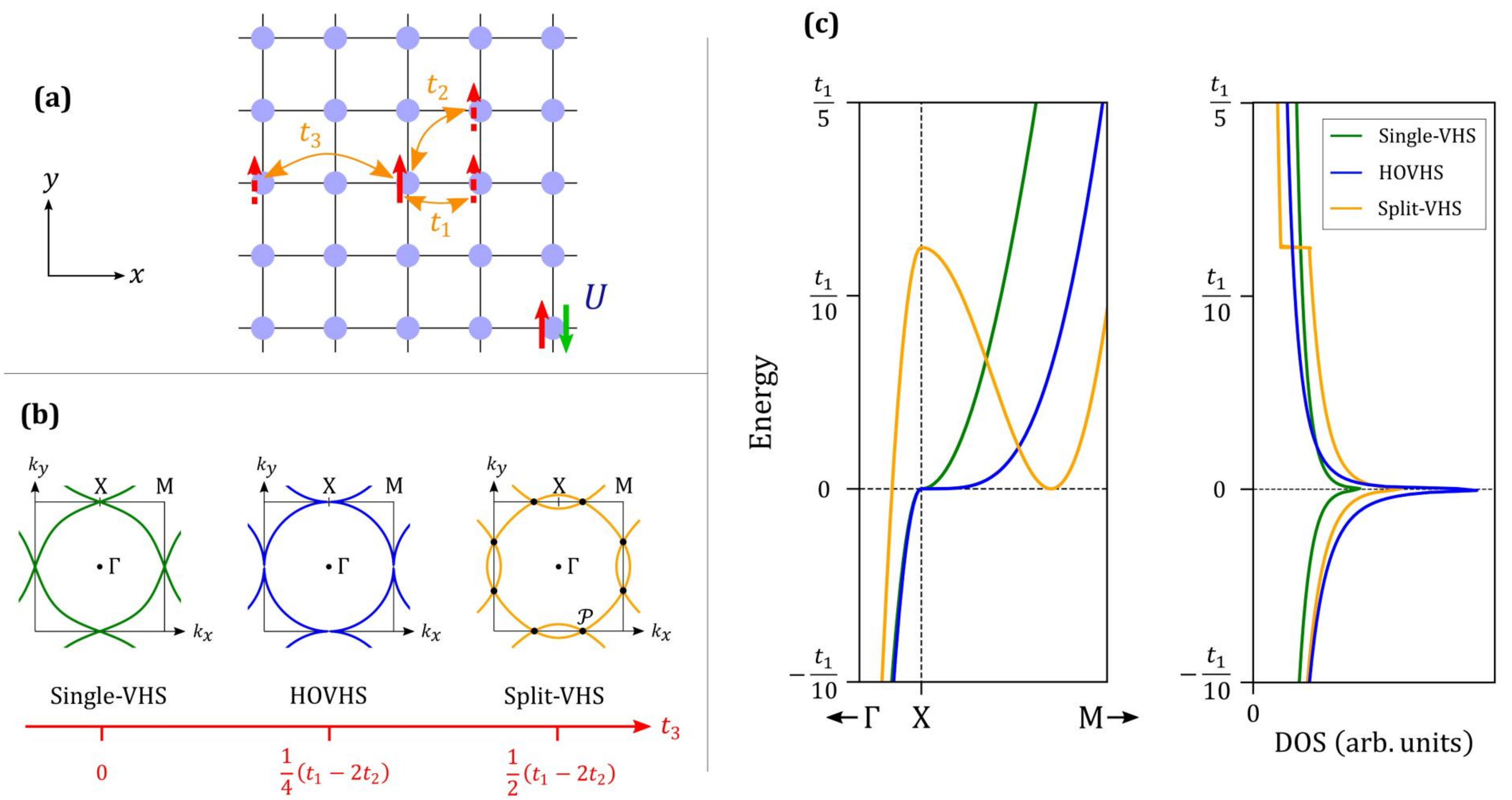
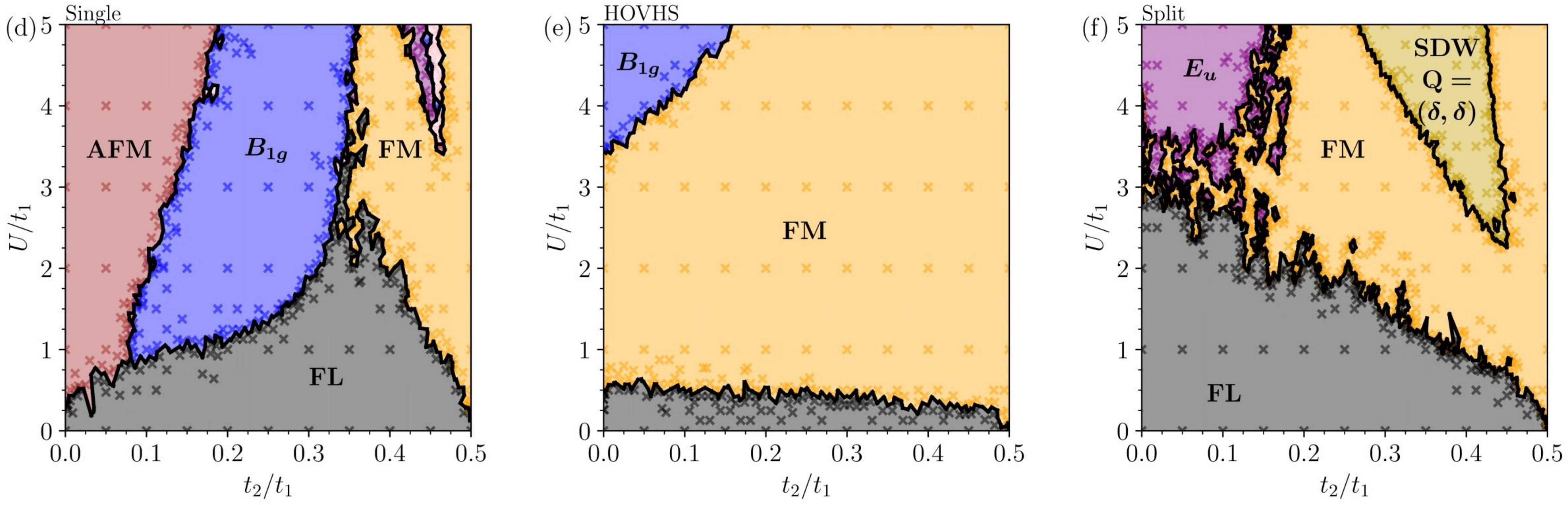
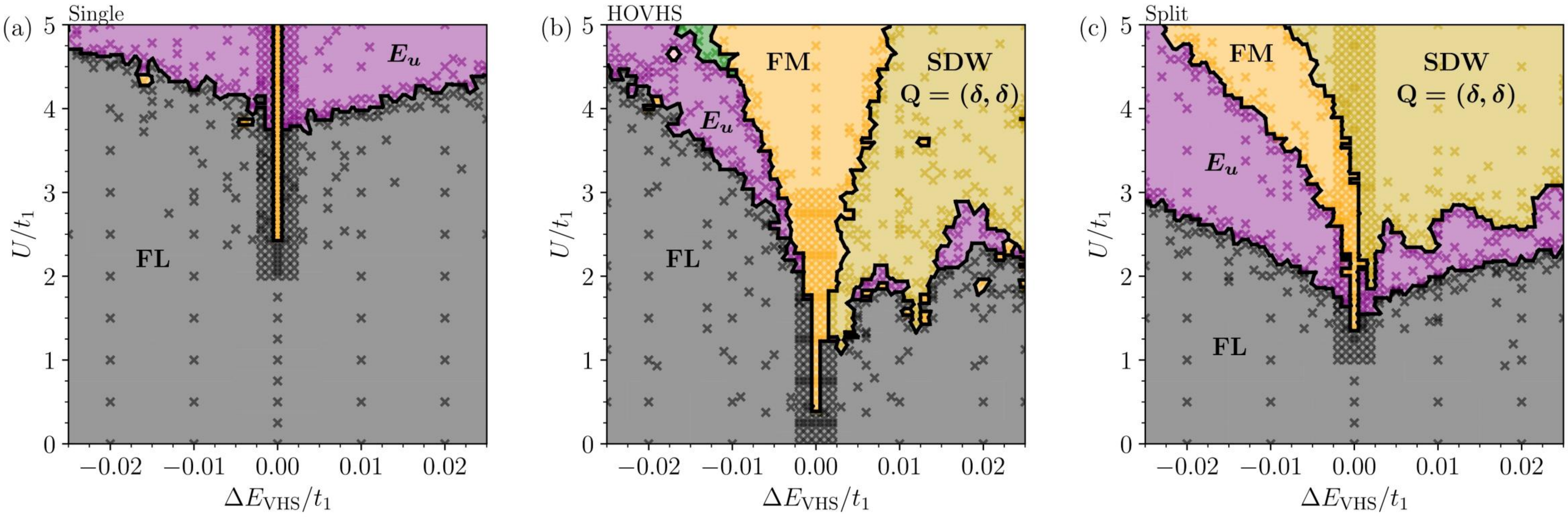
Lattice models and Van Hove singularities



Phase diagrams at Van Hove filling



Doping away from Van Hove filling



For more details, see the talk by Thomas Sheerin, 15.20 today, room 144

Summary

- Several FRG calculations for (2+1)-d quantum critical metals:
  - vector ferromagnet (fermion regulator replaced by self-consistency);
  - nematic (soft frequency regulator on fermions);
  - gauge-boson case (using modified Ward-Takahashi identities).
- Non-Fermi-liquid fixed points in all cases, but never the conventionally expected bosonic dynamical exponent $\mathrm{z} = 3$.
- Numerical (truncated unity) FRG for phase reconstruction near higher-order Van Hove points in the non-interacting dispersion relation – strong sensitivity to doping.