

The Hunting of Physics with Machine Learning through the (f)RG looking glass,

and what we found there

An Agony, in Eight Fits



Jan M. Pawłowski

Universität Heidelberg & ExtreMe Matter Institute

13th International Conference on the Exact Renormalization Group (ERG2026)

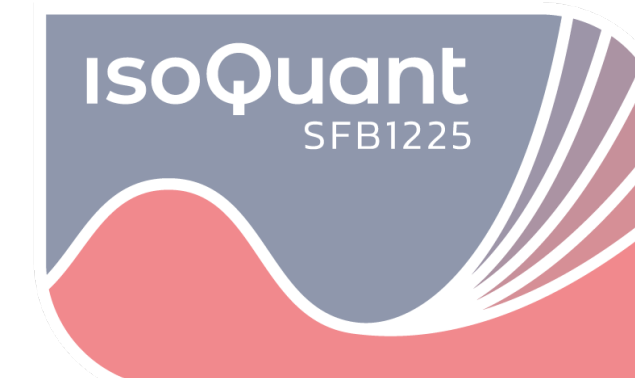
Brighton, 25th of September 2026



STRUCTURES
CLUSTER OF
EXCELLENCE

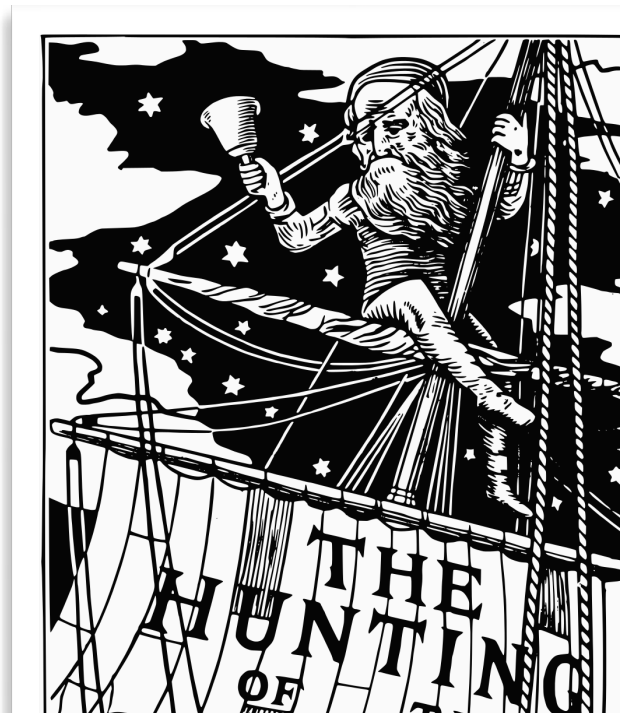


UNIVERSITÄT
HEIDELBERG
ZUKUNFT
SEIT 1386



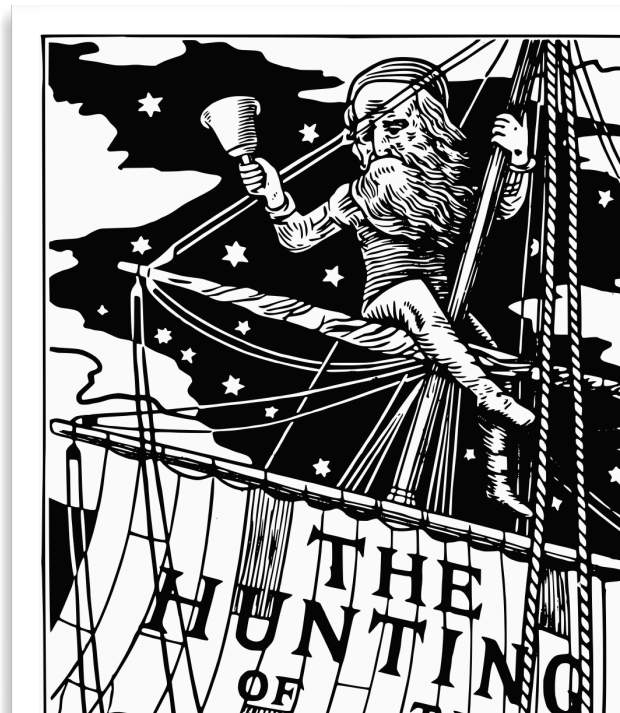
Outline

- **Fit the First: What?**
- **Fit the Second: Machine learning through the fRG looking glass**
- **Fit the Third to Fifth: What can Machine Learning do for us?**
- **Fit the Fifth to Seventh: The RG-Structure of Machine Learning**
- **Fit the Eighth: *The Vanishing***



Outline

- **Fit the First: What?**
- **Fit the Second: Machine learning through the fRG looking glass**
- **Fit the Third to Fifth: What can Machine Learning do for us?**
- **Fit the Fifth to Seventh: The RG-Structure of Machine Learning**



Fit the First: What?



Fit the First: What do we want, what should we definitely do and other what's?

ML as a tool

- **Coding**
- **Literature search**
- **Polishing**
- **Emulating readers**

-
-
-
-
-

ML for fRG tasks

- **'Importance sampling'**
- **Neural solvers**
- **Neural expansion schemes**
- **Optimisation**

-
-
-
-
-

The (f)RG structure of ML

- **ML as an RG**
- **Structure & Optimisation**
- **Fits**
- **Generative sampling**

Talk Friederike Ihssen

Saturday: 11:30 - 12:00

-
-
-
-
-

Agentic

4

Agentic

Fit the First: What do we want, what should we definitely do and other what's?

ML as a tool

- **Coding**
- **Literature search**
- **Polishing**
- **Emulating readers**

-
-
-
-
-

ML for fRG tasks

- 'Importance sampling'
- Neural solvers
- Neural expansion schemes
- Optimisation

-
-
-
-
-

The (f)RG structure of ML

- ML as an RG
- Structure & Optimisation
- Fits
- Generative sampling

Talk Friederike Ihssen

Saturday: 11:30 - 12:00

-
-
-
-
-

Agentic

4

Agentic

Fit the First: What?

‘Critical Review’ by ChatGPT

Renormalization Group and Machine Learning: A Critical Physics Review of Coarse-Graining, Continuous Transport, Generative Sampling, and RG Theories of Learning

Kai

Literature reviewed through 13 August 2026

Abstract

The renormalization group (RG) and machine learning (ML) now interact through several mathematically distinct programs: neural discovery of coarse variables; learned real-space transformations and coupling flows; probabilistic inverse RG and lattice super-resolution; reversible and wavelet multiscale maps; stochastic RG, stochastic quantization, score diffusion, and continuous transport; physics-informed renormalisation-group flows and physics-informed kernels (PIKs); neural solution or compression of functional and numerical RG equations; machine-learned fixed-point lattice actions; and RG theories of neural networks and learning curves. These subjects share equations more often than they share physical meaning. In particular, a continuity or Fokker–Planck equation may describe Wilsonian coarse-graining, relaxation to a stationary path-integral measure, a finite-time data-to-noise bridge, or a prescribed density transport without integrating out any degree of freedom. This review therefore asks what each paper actually defines, computes, and validates rather than accepting its preferred terminology.

The central standard is a five-part physics audit: the scale or transport map; the induced probability measure or effective action; behavior under composition; fixed points, operators, or universal data; and statistical exactness together with end-to-end scaling. A second claim ladder separates formal identities, numerical demonstrations, plausible extrapolations, and untested statements. The review gives particular attention to PIK/PIRG, to “diffusion models as stochastic quantization,” to genuinely RG-defined diffusion processes, and to recent diffusion samplers for scalar and gauge lattice theories. It concludes that PIK is a promising prescribed-action transport whose reported quantitative benchmark is still essentially zero-dimensional, whereas PIRG remains a genuine functional-RG program with scale-dependent fields. Score diffusion and stochastic quantization possess a precise SDE-level relation but are not the same physical process, and generic Gaussian noising is not by itself an inverse Wilsonian RG. Exactness depends on the correction mechanism: exact proposal densities, Metropolis-adjusted kernels, or valid importance weights, not on observable agreement or a small score loss. The revised corpus contains journal papers, conference proceedings, and influential online-only manuscripts through 13 August 2026, with separate source and claim audits. The final sections provide an evidence matrix, a research benchmark, and a plenary-talk narrative designed for physicists entering or surveying the field.

Keywords: renormalization group; machine learning; coarse-graining; stochastic RG; stochastic quantization; score diffusion; physics-informed kernels; inverse renormalization; functional RG; lattice field theory; neural-network field theory.

Contents

Executive synthesis	4
1 Scope, search method, and taxonomy	4
1.1 Literature protocol	4
1.2 Inclusion rule	5
1.3 Terminology that prevents category errors	5
2 What counts as a genuine RG claim?	5
2.1 The five-part physics audit	6
2.2 A claim ladder	6
2.3 Same PDE, different physics	6

3 Historical bridge: three traditions converge	7
3.1 From block spins to computable transformations	7
3.2 Quantum and tensor-network lineage	7
3.3 Why neural methods entered	7
3.4 What RG contributes back	7
4 Does deep learning implement RG?	7
4.1 Constructive correspondences	7
4.2 Why the statement is not generic	8
4.3 Current verdict	8
5 Information-theoretic learned coarse-graining	8
5.1 What the program established	8
5.2 Why this is one of the conceptually strongest branches	9
6 Learned RG maps, model-space flow, and RGFlow	9
6.1 Neural Monte Carlo RG	9
6.2 Optimizing the projector rather than assuming a block rule	9
6.3 Learned holographic maps	10
6.4 Machine-learning RG over model space	10
6.5 Bijective RGFlow for scalar fields	10
7 Inverse RG and neural super-resolution	11
7.1 What has been demonstrated	11
8 Reversible and exactly correctable generative RG	11
8.1 Why exactness is a separate axis from RG fidelity	12
8.2 Lattice field-theory context	12
9 From stochastic RG to diffusion and physics-informed kernels	12
9.1 Wavelets and optimal transport as controlled bridges	12
9.2 Five processes that are often conflated	13
9.3 Stochastic RG: when diffusion really is an RG	13
9.4 Stochastic quantization versus score diffusion	13
9.5 “Diffusion models as stochastic quantization”: what was actually done	14
9.6 Higher cumulants and the Gaussian endpoint	14
9.7 PIRG: a genuine functional-RG program with flowing fields	15
9.8 PIK: prescribe the action path, solve for the velocity	15
9.9 What the PIK paper demonstrates - and what it does not	16
9.10 Complex PIK and sign problems	16
9.11 PIK versus flow matching, stochastic interpolants, and NETS	16
10 Machine learning for functional RG equations	17
10.1 Latent neural ODE for the Hubbard vertex	17
10.2 Physics-informed solution of the flow	17
10.3 Standards for neural fRG	17
11 Quantum, numerical, similarity, and strong-disorder RG	18
11.1 Differentiable numerical RG	18
11.2 Operator learning as an RG generalization	18
11.3 Learning from RG output rather than learning the RG	18

11.4 Neural surrogates for similarity RG	18
11.5 Disordered real-time dynamics	18
11.6 Learning strong-disorder decimation	18
11.7 Broader lesson	19

12 Lattice field theory and gauge theory	19
12.1 Gauge equivariance as an enabling condition	19
12.2 Learning a fixed-point action	19
12.3 An exactness hierarchy for learned lattice samplers	20
12.4 Scalar-field diffusion: from proof of concept to failure diagnostics	20
12.5 Abelian gauge diffusion and topological modes	20
12.6 Non-Abelian and fermionic extensions in 2026	20
12.7 Generative and multiscale samplers beyond diffusion	21

13 Renormalization group as a theory of learning	21
13.1 Gaussian limits as free theories	21
13.2 Learning curves and learnable modes	22
13.3 Bayesian information shells	22
13.4 Beyond lazy/infinite-width limits	22

14 Boundary programs: networks, graph coarsening, invariance, and holography	22
14.1 Renormalization of complex networks	22
14.2 RG-informed invariance without an effective action	23
14.3 Holographic interpretation	23

15 Comparative evidence matrix and benchmark standard	23
15.1 A publication-level benchmark for learned RG and transport samplers	24

16 Annotated guide to representative papers	25
--	-----------

17 What happened in the literature, 2013–2026	27
17.1 2013–2017: analogy and exact correspondences	27
17.2 2018–2020: objectives, falsifiers, and stochastic RG	27
17.3 2021–2023: operational RG, transport, and solver ML	27
17.4 2024–2026: physics-informed paths, exact corrections, and skeptical scaling tests	28
17.5 Online-only work that materially affects the story	28

18 Open problems and a plenary/review-talk blueprint	28
18.1 Twelve open problems that can organize a research program	28
18.2 A flagship cross-domain benchmark	29
18.3 A fourteen-slide plenary narrative	29
18.4 How different audiences should use the literature	29
18.5 Conclusions	30

19 Bibliography and online-source policy	30
---	-----------

Do not assume that your colleagues
read your work more carefully than the AIs

Thanx to Kai Zhou (Shenzhen)

Fit the First: What?

ChatGPT Literature

MH, Metropolis-adjusted diffusion, and weighted transport. **Functional-RG readers** should read PIRG and PIK together but not identify them: PIRG changes coordinates within a scale-dependent effective theory, while PIK fixes an endpoint action path and solves its transport equation. **Machine-learning theorists** should compare PIK with flow matching/stochastic interpolants and NETS, focusing on sample access, path choice, residuals, and correction. **Speakers** should organize the talk by physical object and evidence level, not by architecture brand.

18.5 Conclusions

RG gives machine learning a disciplined answer to what information should survive a change of resolution. Machine learning gives RG expressive stochastic kernels, bidirectional maps, differentiable solvers, symmetry-aware function classes, and new ways to transport probability. The productive connection is not that all these methods are secretly the same. It is that their equations can be compared while their physical boundary conditions and correctness mechanisms remain explicit.

The current evidence supports a strong but bounded conclusion. Neural methods can learn or accelerate specific RG objects; score diffusion can be a powerful lattice proposal; Metropolis adjustment or valid weights can restore a target independently of model perfection; PIRG can reorganize functional flows; and PIK supplies an unusually transparent equation-level residual for a prescribed action transport. It does not support the universal claims that generic diffusion is inverse RG, that PIK has already solved high-dimensional lattice transport, that a broad topology histogram proves continuum ergodicity, or that independent neural draws eliminate critical slowing down without correction and cost accounting.

The mature standard is a closed chain of evidence: **define the physical transformation or transport; derive the evolving measure; identify what is removed and what is merely reparameterized; solve and certify the equation; restore or quantify statistical exactness; test the slow modes; and measure scaling at matched total cost.**

For researchers entering the topic. Read positive and skeptical papers in pairs: RSMI with its optimality analysis; RBM correspondences with direct countertests; generic diffusion with stochastic RG; diffusion-as-SQ with its own stated distinctions; PIK with flow matching and NETS; topology-transfer papers with exactness and scaling audits; and neural action fitting with actual Monte Carlo cutoff tests. That pairing gives a more reliable map of the field than a longer list of architectures.

19 Bibliography and online-source policy

Every bibliography entry below has an HTTPS DOI, arXiv identifier, publisher page, proceedings page, or institutional locator. Journal/accepted, conference/proceedings, online-only, and book status are distinguished in the accompanying source-audit CSV. Online-only manuscripts are included when they introduce a method or report results too recent for journal publication; they are not presented as peer reviewed. For the central PIK/PIRG/diffusion papers, the claim-audit CSV records the formal object, demonstrated system, exactness status, and statements that the evidence does not support.

- [1] L. P. Kadanoff. “Scaling laws for Ising models near T_c ”. In: *Physics Physique Fizika* 2 (1966), pp. 263–272. doi: 10.1103/PhysicsPhysiqueFizika.2.263. URL: <https://doi.org/10.1103/PhysicsPhysiqueFizika.2.263>.
- [2] K. G. Wilson and J. Kogut. “The renormalization group and the epsilon expansion”. In: *Physics Reports* 12 (1974), pp. 75–200. doi: 10.1016/0370-1573(74)90023-4. URL: [https://doi.org/10.1016/0370-1573\(74\)90023-4](https://doi.org/10.1016/0370-1573(74)90023-4).
- [3] K. G. Wilson. “The renormalization group: Critical phenomena and the Kondo problem”. In: *Reviews of Modern Physics* 47 (1975), pp. 773–840. doi: 10.1103/RevModPhys.47.773. URL: <https://doi.org/10.1103/RevModPhys.47.773>.
- [4] F. J. Wegner and A. Houghton. “Renormalization group equation for critical phenomena”. In: *Physical Review A* 8 (1973), pp. 401–412. doi: 10.1103/PhysRevA.8.401. URL: <https://doi.org/10.1103/PhysRevA.8.401>.
- [5] J. Polchinski. “Renormalization and effective Lagrangians”. In: *Nuclear Physics B* 231 (1984), pp. 269–295. doi: 10.1016/0550-3213(84)90287-6. URL: [https://doi.org/10.1016/0550-3213\(84\)90287-6](https://doi.org/10.1016/0550-3213(84)90287-6).
- [6] C. Wetterich. “Exact evolution equation for the effective potential”. In: *Physics Letters B* 301 (1993), pp. 90–94. doi: 10.1016/0370-2693(93)90726-X. URL: [https://doi.org/10.1016/0370-2693\(93\)90726-X](https://doi.org/10.1016/0370-2693(93)90726-X).
- [7] R. H. Swendsen. “Monte Carlo renormalization group”. In: *Physical Review Letters* 42 (1979), pp. 859–861. doi: 10.1103/PhysRevLett.42.859. URL: <https://doi.org/10.1103/PhysRevLett.42.859>.
- [8] R. H. Swendsen. “Optimization of real-space renormalization-group transformations”. In: *Physical Review Letters* 52 (1984), pp. 2321–2324. doi: 10.1103/PhysRevLett.52.2321. URL: <https://doi.org/10.1103/PhysRevLett.52.2321>.

- [9] D. Ron, R. H. Swendsen, and A. Brandt. “Inverse Monte Carlo renormalization group transformations for critical phenomena”. In: *Physical Review Letters* 89 (2002), p. 275701. doi: 10.1103/PhysRevLett.89.275701. URL: <https://doi.org/10.1103/PhysRevLett.89.275701>.
- [10] S. R. White. “Density matrix formulation for quantum renormalization groups”. In: *Physical Review Letters* 69 (1992), pp. 2863–2866. doi: 10.1103/PhysRevLett.69.2863. URL: <https://doi.org/10.1103/PhysRevLett.69.2863>.
- [11] G. Vidal. “Entanglement renormalization”. In: *Physical Review Letters* 99 (2007), p. 220405. doi: 10.1103/PhysRevLett.99.220405. URL: <https://doi.org/10.1103/PhysRevLett.99.220405>.
- [12] G. Evenbly and G. Vidal. “Tensor network renormalization”. In: *Physical Review Letters* 115 (2015), p. 180405. doi: 10.1103/PhysRevLett.115.180405. URL: <https://doi.org/10.1103/PhysRevLett.115.180405>.
- [13] R. Orús. “Tensor networks for complex quantum systems”. In: *Nature Reviews Physics* 1 (2019), pp. 538–550. doi: 10.1038/s42254-019-0086-7. URL: <https://doi.org/10.1038/s42254-019-0086-7>.
- [14] H. W. Lin, M. Tegmark, and D. Rolnick. “Why does deep and cheap learning work so well?” In: *Journal of Statistical Physics* 168 (2017), pp. 1223–1247. doi: 10.1007/s10955-017-1836-5. URL: <https://doi.org/10.1007/s10955-017-1836-5>.
- [15] C. Bény. *Deep learning and the renormalization group*. 2013. arXiv: 1301.3124 [quant-ph]. URL: <https://arxiv.org/abs/1301.3124>.
- [16] P. Mehta and D. J. Schwab. *An exact mapping between the variational renormalization group and deep learning*. 2014. arXiv: 1410.3831 [stat.ML]. URL: <https://arxiv.org/abs/1410.3831>.
- [17] S.-H. Li and L. Wang. “Neural network renormalization group”. In: *Physical Review Letters* 121 (2018), p. 260601. doi: 10.1103/PhysRevLett.121.260601. URL: <https://doi.org/10.1103/PhysRevLett.121.260601>.
- [18] S. Iso, S. Shiba, and S. Yokoo. “Scale-invariant feature extraction of neural network and renormalization group flow”. In: *Physical Review E* 97 (2018), p. 053304. doi: 10.1103/PhysRevE.97.053304. URL: <https://doi.org/10.1103/PhysRevE.97.053304>.
- [19] R. de Mello Koch, A. de Mello Koch, N. Kastanos, and L. Cheng. “Short-sighted deep learning”. In: *Physical Review E* 102 (2020), p. 013307. doi: 10.1103/PhysRevE.102.013307. URL: <https://doi.org/10.1103/PhysRevE.102.013307>.
- [20] R. Veiga and R. Vicente. *Restricted Boltzmann machine flows and the critical temperature of Ising models*. 2020. arXiv: 2006.10176 [cond-mat.stat-mech]. URL: <https://arxiv.org/abs/2006.10176>.
- [21] K. T. Taylor. *A deep dive into the connections between the renormalization group and deep learning in the Ising model*. 2023. arXiv: 2308.11075 [cond-mat.stat-mech]. URL: <https://arxiv.org/abs/2308.11075>.
- [22] M. Koch-Janusz and Z. Ringel. “Mutual information, neural networks and the renormalization group”. In: *Nature Physics* 14 (2018), pp. 578–582. doi: 10.1038/s41567-018-0081-4. URL: <https://doi.org/10.1038/s41567-018-0081-4>.
- [23] P. M. Lenggenhager, D. E. Gökmen, Z. Ringel, S. D. Huber, and M. Koch-Janusz. “Optimal renormalization group transformation from information theory”. In: *Physical Review X* 10 (2020), p. 011037. doi: 10.1103/PhysRevX.10.011037. URL: <https://doi.org/10.1103/PhysRevX.10.011037>.
- [24] D. E. Gökmen, Z. Ringel, S. D. Huber, and M. Koch-Janusz. “Statistical physics through the lens of real-space mutual information”. In: *Physical Review Letters* 127 (2021), p. 240603. doi: 10.1103/PhysRevLett.127.240603. URL: <https://doi.org/10.1103/PhysRevLett.127.240603>.
- [25] D. E. Gökmen, Z. Ringel, S. D. Huber, and M. Koch-Janusz. “Symmetries and phase diagrams with real-space mutual information neural estimation”. In: *Physical Review E* 104 (2021), p. 064106. doi: 10.1103/PhysRevE.104.064106. URL: <https://doi.org/10.1103/PhysRevE.104.064106>.
- [26] J.-H. Chung and Y.-J. Kao. “Neural Monte Carlo renormalization group”. In: *Physical Review Research* 3 (2021), p. 023230. doi: 10.1103/PhysRevResearch.3.023230. URL: <https://doi.org/10.1103/PhysRevResearch.3.023230>.
- [27] A. G. Kline and S. E. Palmer. “Gaussian information bottleneck and the non-perturbative renormalization group”. In: *New Journal of Physics* 24 (2022), p. 033007. doi: 10.1088/1367-2630/ac395d. URL: <https://doi.org/10.1088/1367-2630/ac395d>.
- [28] P. Pessoa and A. Caticha. “Exact renormalization groups as a form of entropic dynamics”. In: *Entropy* 20 (2018), p. 25. doi: 10.3390/e20010025. URL: <https://doi.org/10.3390/e20010025>.
- [29] D. S. Berman, M. S. Klinger, and A. G. Stapleton. “Bayesian renormalization”. In: *Machine Learning: Science and Technology* 4 (2023), p. 045011. doi: 10.1088/2632-2153/ad0102. URL: <https://doi.org/10.1088/2632-2153/ad0102>.
- [30] J.-H. Chung and Y.-J. Kao. *Optimal real-space renormalization-group transformations with artificial neural networks*. Also presented at the NeurIPS 2019 Workshop on Machine Learning and the Physical Sciences. 2019. arXiv: 1912.09005 [cond-mat.stat-mech]. URL: <https://arxiv.org/abs/1912.09005>.
- [31] H.-Y. Hu, S.-H. Li, L. Wang, and Y.-Z. You. “Machine learning holographic mapping by neural network renormalization group”. In: *Physical Review Research* 2 (2020), p. 023369. doi: 10.1103/PhysRevResearch.2.023369. arXiv: 1903.00804 [cond-mat.stat-mech]. URL: <https://doi.org/10.1103/PhysRevResearch.2.023369>.

- [51] M. Gerdes, P. de Haan, R. Bondesan, and M. C. N. Cheng. “Nonperturbative trivializing flows for lattice gauge theories”. In: *Physical Review D* 112 (2025), p. 094516. doi: 10.1103/31d5-hvp6. URL: <https://doi.org/10.1103/31d5-hvp6>.
- [52] T. Marchand, M. Ozawa, G. Biroli, and S. Mallat. “Multiscale data-driven energy estimation and generation”. In: *Physical Review X* 13 (2023). The arXiv version circulated as *Wavelet Conditional Renormalization Group*., p. 041038. doi: 10.1103/PhysRevX.13.041038. URL: <https://doi.org/10.1103/PhysRevX.13.041038>.
- [53] J. Cotler and S. Rezchikov. “Renormalization group flow as optimal transport”. In: *Physical Review D* 108 (2023), p. 025003. doi: 10.1103/PhysRevD.108.025003. URL: <https://doi.org/10.1103/PhysRevD.108.025003>.
- [54] A. Carosso. “Stochastic renormalization group and gradient flow”. In: *Journal of High Energy Physics* 2020 (2020), p. 172. doi: 10.1007/JHEP01(2020)172. arXiv: 1904.13057. URL: [https://doi.org/10.1007/JHEP01\(2020\)172](https://doi.org/10.1007/JHEP01(2020)172).
- [55] A. Carosso, A. Hasenfratz, and E. T. Neil. *Stochastic renormalization group and gradient flow in scalar field theory*. 2020. arXiv: 1912.01766 [hep-lat]. URL: <https://arxiv.org/abs/1912.01766>.
- [56] J. Cotler and S. Rezchikov. *Renormalizing diffusion models*. 2023. arXiv: 2308.12355 [hep-th]. URL: <https://arxiv.org/abs/2308.12355>.
- [57] A. Sheshmani, Y.-Z. You, B. Buyukates, A. Ziahashabi, and S. Avestimehr. “Renormalization group flow, optimal transport, and diffusion-based generative model”. In: *Physical Review E* 111 (2025), p. 015304. doi: 10.1103/PhysRevE.111.015304. arXiv: 2402.17090 [cs.LG]. URL: <https://doi.org/10.1103/PhysRevE.111.015304>.
- [58] K. Masuki and Y. Ashida. *Generative diffusion model with inverse renormalization group flows*. 2025. arXiv: 2501.09064 [cond-mat.stat-mech]. URL: <https://arxiv.org/abs/2501.09064>.
- [59] G. Parisi and Y.-S. Wu. “Perturbation theory without gauge fixing”. In: *Scientia Sinica* 24 (1981), pp. 483–496. URL: <https://inspirehep.net/literature/10432>.
- [60] P. H. Damgaard and H. Hüffel. “Stochastic quantization”. In: *Physics Reports* 152 (1987), pp. 227–398. doi: 10.1016/0370-1573(87)90144-X. URL: [https://doi.org/10.1016/0370-1573\(87\)90144-X](https://doi.org/10.1016/0370-1573(87)90144-X).
- [61] Y. Song, J. Sohl-Dickstein, D. P. Kingma, A. Kumar, S. Ermon, and B. Poole. “Score-based generative modeling through stochastic differential equations”. In: *International Conference on Learning Representations*. 2021. arXiv: 2011.13456. URL: <https://arxiv.org/abs/2011.13456>.
- [62] L. Wang, G. Aarts, and K. Zhou. “Diffusion models as stochastic quantization in lattice field theory”. In: *Journal of High Energy Physics* 2024 (2024), p. 060. doi: 10.1007/JHEP05(2024)060. arXiv: 2309.17082. URL: [https://doi.org/10.1007/JHEP05\(2024\)060](https://doi.org/10.1007/JHEP05(2024)060).
- [63] L. Wang, G. Aarts, and K. Zhou. “Generative diffusion models for lattice field theory”. In: *NeurIPS 2023 Workshop on Machine Learning and the Physical Sciences*. 2023. arXiv: 2311.03578. URL: <https://arxiv.org/abs/2311.03578>.
- [64] K. Fukushima and S. Kamata. “Stochastic quantization and diffusion models”. In: *Journal of the Physical Society of Japan* 94 (2025), p. 031010. doi: 10.7566/JPSJ.94.031010. arXiv: 2411.11297. URL: <https://doi.org/10.7566/JPSJ.94.031010>.
- [65] G. Aarts, D. E. Habibi, L. Wang, and K. Zhou. *On learning higher-order cumulants in diffusion models*. 2024. arXiv: 2410.21212. URL: <https://arxiv.org/abs/2410.21212>.
- [66] J. M. Pawłowski. “Aspects of the functional renormalisation group”. In: *Annals of Physics* 322 (2007), pp. 2831–2915. doi: 10.1016/j.aop.2007.01.007. arXiv: hep-th/0512261. URL: <https://doi.org/10.1016/j.aop.2007.01.007>.
- [67] F. Ihssen and J. M. Pawłowski. “Functional flows for complex effective actions”. In: *SciPost Physics* 15 (2023), p. 074. doi: 10.21468/SciPostPhys.15.2.074. arXiv: 2207.10057. URL: <https://doi.org/10.21468/SciPostPhys.15.2.074>.
- [68] F. Ihssen and J. M. Pawłowski. “Physics-informed renormalisation group flows”. In: *Annals of Physics* 481 (2025), p. 170177. doi: 10.1016/j.aop.2025.170177. arXiv: 2409.13679. URL: <https://doi.org/10.1016/j.aop.2025.170177>.
- [69] F. Ihssen and J. M. Pawłowski. “Physics-informed gauge theories”. In: *Physical Review D* 112 (2025), p. 105005. doi: 10.1103/jhq-d-cjzk. arXiv: 2503.22638. URL: <https://arxiv.org/abs/2503.22638>.
- [70] F. Ihssen and J. M. Pawłowski. “Physics-informed operator flows and observables”. In: *Physical Review D* 113 (2026), p. 076003. doi: 10.1103/zy96-jg9g. arXiv: 2507.13011. URL: <https://doi.org/10.1103/zy96-jg9g>.
- [71] A. Bonanno, F. Ihssen, and J. M. Pawłowski. “Tunneling with physics-informed RG flows in the anharmonic oscillator”. In: *SciPost Physics Core* 9 (2026), p. 005. doi: 10.21468/SciPostPhysCore.9.1.005. arXiv: 2504.03437. URL: <https://doi.org/10.21468/SciPostPhysCore.9.1.005>.
- [72] F. J. Wegner. “Some invariance properties of the renormalization group”. In: *Journal of Physics C: Solid State Physics* 7 (1974), pp. 2098–2108. doi: 10.1088/0022-3719/7/12/004. URL: <https://doi.org/10.1088/0022-3719/7/12/004>.
- [73] F. Ihssen, R. Kapust, and J. M. Pawłowski. *Generative sampling with physics-informed kernels*. 2025. doi: 10.48550/arXiv.2510.26678. arXiv: 2510.26678 [hep-lat]. URL: <https://arxiv.org/abs/2510.26678>.
- [74] F. Ihssen, R. Kapust, and J. M. Pawłowski. *Solving sign problems with physics-informed kernels*. 2026. doi: 10.48550/arXiv.2603.03159. arXiv: 2603.03159 [hep-lat]. URL: <https://arxiv.org/abs/2603.03159>.

- [32] W. Hou and Y.-Z. You. “Machine learning renormalization group for statistical physics”. In: *Machine Learning: Science and Technology* 4 (2023), p. 045010. doi: 10.1088/2632-2153/ad0101. URL: <https://doi.org/10.1088/2632-2153/ad0101>.
- [33] D. Bachtis, G. Aarts, and B. Lucini. “Adding machine learning within Hamiltonians: Renormalization group transformations, symmetry breaking and restoration”. In: *Physical Review Research* 3 (2021), p. 013134. doi: 10.1103/PhysRevResearch.3.013134. URL: <https://doi.org/10.1103/PhysRevResearch.3.013134>.
- [34] Y. Zhao, M. M. Fogler, and Y.-Z. You. “Application of deep neural networks for computing the renormalization group flow of the two-dimensional ϕ^4 field theory”. In: *Machine Learning: Science and Technology* 7 (2026), p. 025064. doi: 10.1088/2632-2153/ae493b. URL: <https://doi.org/10.1088/2632-2153/ae493b>.
- [35] H.-Y. Hu, D. Wu, Y.-Z. You, B. Olshausen, and Y. Chen. *RG-Flow: A hierarchical and explainable flow model based on renormalization group and sparse prior*. 2020. arXiv: 2010.00029 [cs.LG]. URL: <https://arxiv.org/abs/2010.00029>.
- [36] D. Bachtis. “Generating configurations of increasing lattice size with machine learning and the inverse renormalization group”. In: *Proceedings of the European Network for Particle Physics, Lattice Field Theory and Extreme Computing (EuroPLEX2023)*. Vol. 451. 2024, p. 001. doi: 10.22323/1.451.0001. URL: <https://doi.org/10.22323/1.451.0001>.
- [37] S. Efthymiou, K. S. D. Beach, and R. G. Melko. “Super-resolving the Ising model with convolutional neural networks”. In: *Physical Review B* 99 (2019), p. 075113. doi: 10.1103/PhysRevB.99.075113. URL: <https://doi.org/10.1103/PhysRevB.99.075113>.
- [38] K. Shiina, H. Mori, Y. Tomita, H. K. Lee, and Y. Okabe. “Inverse renormalization group based on image super-resolution using deep convolutional networks”. In: *Scientific Reports* 11 (2021), p. 9617. doi: 10.1038/s41598-021-88605-w. URL: <https://doi.org/10.1038/s41598-021-88605-w>.
- [39] D. Bachtis, G. Aarts, F. D. Renzo, and B. Lucini. “Inverse renormalization group in quantum field theory”. In: *Physical Review Letters* 128 (2022), p. 081603. doi: 10.1103/PhysRevLett.128.081603. URL: <https://doi.org/10.1103/PhysRevLett.128.081603>.
- [40] D. Bachtis. “Inverse renormalization group in the three-dimensional Edwards-Anderson spin glass”. In: *Physical Review B* 110 (2024), p. L140202. doi: 10.1103/PhysRevB.110.L140202. URL: <https://doi.org/10.1103/PhysRevB.110.L140202>.
- [41] A. Rançon, U. Rançon, T. Ivek, and I. Balog. “Dreaming up scale invariance via inverse renormalization group”. In: *Physical Review E* 113 (2026), p. 055302. doi: 10.1103/3njc-5tlx. URL: <https://doi.org/10.1103/3njc-5tlx>.
- [42] M. Bauer, R. Kapust, J. M. Pawłowski, and F. L. Temmen. “Super-resolving normalising flows for lattice field theories”. In: *SciPost Physics* 19 (2025), p. 077. doi: 10.21468/SciPostPhys.19.3.077. URL: <https://doi.org/10.21468/SciPostPhys.19.3.077>.
- [43] A. Singha, J. Kauffmann, E. Cellini, K. Jansen, and S. Nakajima. *Scalable Generative Sampling and Multilevel Estimation for Lattice Field Theories Near Criticality*. 2026. doi: 10.48550/arXiv.2604.10209. arXiv: 2604.10209 [hep-lat]. URL: <https://arxiv.org/abs/2604.10209>.
- [44] M. S. Albergo, G. Kanwar, and P. E. Shanahan. “Flow-based generative models for Markov chain Monte Carlo in lattice field theory”. In: *Physical Review D* 100 (2019), p. 034515. doi: 10.1103/PhysRevD.100.034515. URL: <https://doi.org/10.1103/PhysRevD.100.034515>.
- [45] D. Boyda, G. Kanwar, S. Racanière, D. J. Rezende, M. S. Albergo, K. Cranmer, D. C. Hackett, and P. E. Shanahan. “Sampling using SU(N) gauge equivariant flows”. In: *Physical Review D* 103 (2021), p. 074504. doi: 10.1103/PhysRevD.103.074504. URL: <https://doi.org/10.1103/PhysRevD.103.074504>.
- [46] M. S. Albergo, G. Kanwar, S. Racanière, D. J. Rezende, J. M. Urban, D. Boyda, K. Cranmer, D. C. Hackett, and P. E. Shanahan. “Flow-based sampling for fermionic lattice field theories”. In: *Physical Review D* 104 (2021), p. 114507. doi: 10.1103/PhysRevD.104.114507. URL: <https://doi.org/10.1103/PhysRevD.104.114507>.
- [47] M. S. Albergo, D. Boyda, D. C. Hackett, G. Kanwar, K. Cranmer, S. Racanière, D. J. Rezende, and P. E. Shanahan. *Introduction to normalizing flows for lattice field theory*. 2021. arXiv: 2101.08176 [hep-lat]. URL: <https://arxiv.org/abs/2101.08176>.
- [48] M. Caselle, E. Cellini, A. Nada, and M. Panero. “Stochastic normalizing flows for lattice field theory”. In: *Proceedings of the 39th International Symposium on Lattice Field Theory*. Vol. 430. 2023, p. 005. doi: 10.22323/1.430.0005. URL: <https://doi.org/10.22323/1.430.0005>.
- [49] R. Abbott, M. S. Albergo, D. Boyda, D. C. Hackett, G. Kanwar, F. Romero-López, P. E. Shanahan, and J. M. Urban. “Multiscale normalizing flows for gauge theories”. In: *Proceedings of the 40th International Symposium on Lattice Field Theory*. Vol. 453. 2024, p. 035. doi: 10.22323/1.453.0035. URL: <https://doi.org/10.22323/1.453.0035>.
- [50] S. Bacchio, P. Kessel, S. Schaefer, and L. Vailt. “Learning trivializing gradient flows for lattice gauge theories”. In: *Physical Review D* 107 (2023), p. L051504. doi: 10.1103/PhysRevD.107.L051504. URL: <https://doi.org/10.1103/PhysRevD.107.L051504>.

Fit the First: What?

ChatGPT Literature

[51] M. Gerdes, P. de Haan, R. Bondesan, and M. C. N. Cheng. “Nonperturbative trivializing flows for lattice gauge theories”. In: *Physical Review D* 112 (2025), p. 094516. doi: 10.1103/31d5-hvp6. URL: <https://doi.org/10.1103/31d5-hvp6>.

[52] T. Marchand, M. Ozawa, G. Biroli, and S. Mallat. “Multiscale data-driven energy estimation and generation”. In: *Physical Review X* 13 (2023). The arXiv version circulated as *Wavelet Conditional Renormalization Group*, p. 041038. doi: 10.1103/PhysRevX.13.041038. URL: <https://doi.org/10.1103/PhysRevX.13.041038>.

[53] J. Cotler and S. Rezhikov. “Renormalization group flow as optimal transport”. In: *Physical Review D* 108 (2023), p. 025003. doi: 10.1103/PhysRevD.108.025003. URL: <https://doi.org/10.1103/PhysRevD.108.025003>.

[54] A. Carosso. “Stochastic renormalization group and gradient flow”. In: *Journal of High Energy Physics* 2020 (2020), p. 172. doi: 10.1007/JHEP01(2020)172. arXiv: 1904.13057. URL: [https://doi.org/10.1007/JHEP01\(2020\)172](https://doi.org/10.1007/JHEP01(2020)172).

[55] A. Carosso, A. Hasenfratz, and E. T. Neil. *Stochastic renormalization group and gradient flow in scalar field theory*. 2020. arXiv: 1912.01766 [hep-lat]. URL: <https://arxiv.org/abs/1912.01766>.

[56] J. Cotler and S. Rezhikov. *Renormalizing diffusion models*. 2023. arXiv: 2308.12355 [hep-th]. URL: <https://arxiv.org/abs/2308.12355>.

[57] A. Sheshmani, Y.-Z. You, B. Buyukates, A. Ziahashabi, and S. Avestimehr. “Renormalization group flow, optimal transport, and diffusion-based generative model”. In: *Physical Review E* 111 (2025), p. 015304. doi: 10.1103/PhysRevE.111.015304. arXiv: 2402.17090 [cs.LG]. URL: <https://doi.org/10.1103/PhysRevE.111.015304>.

[58] K. Masuki and Y. Ashida. *Generative diffusion model with inverse renormalization group flows*. 2025. arXiv: 2501.09064 [cond-mat.stat-mech]. URL: <https://arxiv.org/abs/2501.09064>.

[59] G. Parisi and Y.-S. Wu. “Perturbation theory without gauge fixing”. In: *Scientia Sinica* 24 (1981), pp. 483–496. URL: <https://inspirehep.net/literature/10432>.

[60] P. H. Damgaard and H. Hüffel. “Stochastic quantization”. In: *Physics Reports* 152 (1987), pp. 227–398. doi: 10.1016/0370-1573(87)90144-X. URL: [https://doi.org/10.1016/0370-1573\(87\)90144-X](https://doi.org/10.1016/0370-1573(87)90144-X).

[61] Y. Song, J. Sohl-Dickstein, D. P. Kingma, A. Kumar, S. Ermon, and B. Poole. “Score-based generative modeling through stochastic differential equations”. In: *International Conference on Learning Representations*. 2021. arXiv: 2011.13456. URL: <https://arxiv.org/abs/2011.13456>.

[62] L. Wang, G. Aarts, and K. Zhou. “Diffusion models as stochastic quantization in lattice field theory”. In: *Journal of High Energy Physics* 2024 (2024), p. 060. doi: 10.1007/JHEP05(2024)060. arXiv: 2309.17082. URL: [https://doi.org/10.1007/JHEP05\(2024\)060](https://doi.org/10.1007/JHEP05(2024)060).

[63] L. Wang, G. Aarts, and K. Zhou. “Generative diffusion models for lattice field theory”. In: *NeurIPS 2023 Workshop on Machine Learning and the Physical Sciences*. 2023. arXiv: 2311.03578. URL: <https://arxiv.org/abs/2311.03578>.

[64] K. Fukushima and S. Kamata. “Stochastic quantization and diffusion models”. In: *Journal of the Physical Society of Japan* 94 (2025), p. 031010. doi: 10.7566/JPSJ.94.031010. arXiv: 2411.11297. URL: <https://doi.org/10.7566/JPSJ.94.031010>.

[65] G. Aarts, D. E. Habibi, L. Wang, and K. Zhou. *On learning higher-order cumulants in diffusion models*. 2024. arXiv: 2410.21212. URL: <https://arxiv.org/abs/2410.21212>.

[66] J. M. Pawłowski. “Aspects of the functional renormalisation group”. In: *Annals of Physics* 322 (2007), pp. 2831–2915. doi: 10.1016/j.aop.2007.01.007. arXiv: hep-th/0512261. URL: <https://doi.org/10.1016/j.aop.2007.01.007>.

[67] F. Ihssen and J. M. Pawłowski. “Functional flows for complex effective actions”. In: *SciPost Physics* 15 (2023), p. 074. doi: 10.21468/SciPostPhys.15.2.074. arXiv: 2207.10057. URL: <https://doi.org/10.21468/SciPostPhys.15.2.074>.

[68] F. Ihssen and J. M. Pawłowski. “Physics-informed renormalisation group flows”. In: *Annals of Physics* 481 (2025), p. 170177. doi: 10.1016/j.aop.2025.170177. arXiv: 2409.13679. URL: <https://doi.org/10.1016/j.aop.2025.170177>.

[69] F. Ihssen and J. M. Pawłowski. “Physics-informed gauge theories”. In: *Physical Review D* 112 (2025), p. 105005. doi: 10.1103/physrevd.112.105005. arXiv: 2503.22638. URL: <https://arxiv.org/abs/2503.22638>.

[70] F. Ihssen and J. M. Pawłowski. “Physics-informed operator flows and observables”. In: *Physical Review D* 113 (2026), p. 076003. doi: 10.1103/physrevd.113.076003. arXiv: 2507.13011. URL: <https://doi.org/10.1103/physrevd.113.076003>.

[71] A. Bonanno, F. Ihssen, and J. M. Pawłowski. “Tunneling with physics-informed RG flows in the anharmonic oscillator”. In: *SciPost Physics Core* 9 (2026), p. 005. doi: 10.21468/SciPostPhysCore.9.1.005. arXiv: 2504.03437. URL: <https://doi.org/10.21468/SciPostPhysCore.9.1.005>.

[72] F. J. Wegner. “Some invariance properties of the renormalization group”. In: *Journal of Physics C: Solid State Physics* 7 (1974), pp. 2098–2108. doi: 10.1088/0022-3719/7/12/004. URL: <https://doi.org/10.1088/0022-3719/7/12/004>.

[73] F. Ihssen, R. Kapust, and J. M. Pawłowski. *Generative sampling with physics-informed kernels*. 2025. doi: 10.48550/arXiv.2510.26678. arXiv: 2510.26678 [hep-lat]. URL: <https://arxiv.org/abs/2510.26678>.

[74] F. Ihssen, R. Kapust, and J. M. Pawłowski. *Solving sign problems with physics-informed kernels*. 2026. doi: 10.48550/arXiv.2603.03159. arXiv: 2603.03159 [hep-lat]. URL: <https://arxiv.org/abs/2603.03159>.

[75] Y. Lipman, R. T. Q. Chen, H. Ben-Hamu, M. Nickel, and M. Le. “Flow matching for generative modeling”. In: *International Conference on Learning Representations*. 2023. arXiv: 2210.02747. URL: <https://arxiv.org/abs/2210.02747>.

[76] M. S. Albergo and E. Vanden-Eijnden. “Building normalizing flows with stochastic interpolants”. In: *International Conference on Learning Representations*. 2023. arXiv: 2209.15571. URL: <https://arxiv.org/abs/2209.15571>.

[77] C. Jarzynski. “Nonequilibrium equality for free energy differences”. In: *Physical Review Letters* 78 (1997), pp. 2690–2693. doi: 10.1103/PhysRevLett.78.2690. URL: <https://doi.org/10.1103/PhysRevLett.78.2690>.

[78] R. M. Neal. “Annealed importance sampling”. In: *Statistics and Computing* 11 (2001), pp. 125–139. doi: 10.1023/A:1008923215028. URL: <https://doi.org/10.1023/A:1008923215028>.

[79] M. S. Albergo and E. Vanden-Eijnden. “NETS: A non-equilibrium transport sampler”. In: *Proceedings of the 42nd International Conference on Machine Learning*. Vol. 267. Proceedings of Machine Learning Research. 2025, pp. 1026–1055. arXiv: 2410.02711. URL: <https://proceedings.mlr.press/v267/albergo25a.html>.

[80] W. Metzner, M. Salmhofer, C. Honerkamp, V. Meden, and K. Schönhammer. “Functional renormalization group approach to correlated fermion systems”. In: *Reviews of Modern Physics* 84 (2012), pp. 299–352. doi: 10.1103/RevModPhys.84.299. URL: <https://doi.org/10.1103/RevModPhys.84.299>.

[81] D. D. Sante, M. Medvidović, A. Toschi, G. Sangiovanni, C. Franchini, A. M. Sengupta, and A. J. Millis. “Deep learning the functional renormalization group”. In: *Physical Review Letters* 129 (2022), p. 136402. doi: 10.1103/PhysRevLett.129.136402. URL: <https://doi.org/10.1103/PhysRevLett.129.136402>.

[82] J. Zang, M. Medvidović, D. Kiese, D. D. Sante, A. M. Sengupta, and A. J. Millis. “Machine learning-based compression of quantum many body physics: PCA and autoencoder representation of the vertex function”. In: *Machine Learning: Science and Technology* 5.4 (2024), p. 045076. doi: 10.1088/2632-2153/ad9f20. arXiv: 2403.15372 [cond-mat.str-el]. URL: <https://doi.org/10.1088/2632-2153/ad9f20>.

[83] T. Yokota. “Physics-informed neural networks for solving functional renormalization group on a lattice”. In: *Physical Review B* 109 (2024), p. 214205. doi: 10.1103/PhysRevB.109.214205. URL: <https://doi.org/10.1103/PhysRevB.109.214205>.

[84] Y.-y. Tan, W.-j. Fu, L. He, and L. Wang. “Solving functional renormalization group equations with neural networks”. In: *Physical Review D* (2026). Accepted 24 July 2026. doi: 10.1103/PhysRevD.109.214205. URL: <https://doi.org/10.1103/PhysRevD.109.214205>.

[85] J. B. Rigo and A. K. Mitchell. “Automatic differentiable numerical renormalization group”. In: *Physical Review Research* 4 (2022), p. 013227. doi: 10.1103/PhysRevResearch.4.013227. URL: <https://doi.org/10.1103/PhysRevResearch.4.013227>.

[86] J. B. Rigo and A. K. Mitchell. “Unsupervised learning of effective quantum impurity models”. In: *Physical Review Research* 6 (2024), p. 043044. doi: 10.1103/PhysRevResearch.6.043044. arXiv: 2401.13600 [cond-mat.str-el]. URL: <https://doi.org/10.1103/PhysRevResearch.6.043044>.

[87] X.-Z. Luo, D. Luo, and R. G. Melko. *Operator learning renormalization group*. 2024. arXiv: 2403.03199 [quant-ph]. URL: <https://arxiv.org/abs/2403.03199>.

[88] C. Miles, M. R. Carbone, E. J. Sturm, D. Lu, A. Weichselbaum, K. Barros, and R. M. Konik. “Machine learning of Kondo physics using variational autoencoders and symbolic regression”. In: *Physical Review B* 104 (2021), p. 235111. doi: 10.1103/PhysRevB.104.235111. URL: <https://doi.org/10.1103/PhysRevB.104.235111>.

[89] S. Yoshida. “IMSRG-Net: A machine-learning-based solver for the in-medium similarity renormalization group method”. In: *Physical Review C* 108 (2023), p. 044303. doi: 10.1103/PhysRevC.108.044303. URL: <https://doi.org/10.1103/PhysRevC.108.044303>.

[90] H. Burauf and M. Heyl. “Unitary long-time evolution with quantum renormalization groups and artificial neural networks”. In: *Physical Review Letters* 127 (2021), p. 050601. doi: 10.1103/PhysRevLett.127.050601. URL: <https://doi.org/10.1103/PhysRevLett.127.050601>.

[91] A. Ustyuzhanin, J. Vahedi, and S. Kettemann. *Machine learning the strong disorder renormalization group method for disordered quantum spin chains*. Online preprint; no journal version was confirmed by the review cutoff. 2026. arXiv: 2603.05164 [cond-mat.dis-nn]. URL: <https://arxiv.org/abs/2603.05164>.

[92] N. Maskara, M. Buchhold, M. Endres, and E. van Nieuwenburg. “Learning algorithm reflecting universal scaling behavior near phase transitions”. In: *Physical Review Research* 4 (2022), p. L022032. doi: 10.1103/PhysRevResearch.4.L022032. URL: <https://doi.org/10.1103/PhysRevResearch.4.L022032>.

[93] M. Favoni, A. Ipp, D. I. Müller, and D. Schuh. “Lattice gauge equivariant convolutional neural networks”. In: *Physical Review Letters* 128 (2022), p. 032003. doi: 10.1103/PhysRevLett.128.032003. URL: <https://doi.org/10.1103/PhysRevLett.128.032003>.

[94] J. Aronsson, D. I. Müller, and D. Schuh. *Geometrical aspects of lattice gauge equivariant convolutional neural networks*. 2023. arXiv: 2303.11448 [hep-lat]. URL: <https://arxiv.org/abs/2303.11448>.

[95] K. Holland, A. Ipp, D. I. Müller, and U. Wenger. “Fixed point actions from convolutional neural networks”. In: *Proceedings of the 40th International Symposium on Lattice Field Theory*. Vol. 453. 2024, p. 038. doi: 10.22323/1.453.0038. URL: <https://doi.org/10.22323/1.453.0038>.

[96] K. Holland, A. Ipp, D. I. Müller, and U. Wenger. “Machine learning a fixed point action for SU(3) gauge theory with a gauge equivariant convolutional neural network”. In: *Physical Review D* 110 (2024), p. 074502. doi: 10.1103/PhysRevD.110.074502. URL: <https://doi.org/10.1103/PhysRevD.110.074502>.

[97] U. Wenger. *Machine learning for four-dimensional SU(3) lattice gauge theories*. 2026. arXiv: 2604.12416 [hep-lat]. URL: <https://arxiv.org/abs/2604.12416>.

[98] K. Holland, A. Ipp, D. I. Müller, and U. Wenger. “Machine-learned renormalization-group-improved gauge actions and classically perfect gradient flows”. In: *Physical Review Letters* 136 (2026), p. 031901. doi: 10.1103/k41k-2pnc. URL: <https://doi.org/10.1103/k41k-2pnc>.

[99] Y.-y. Tan, G. Aarts, D. E. Habibi, B. Lucini, and L. Wang. *Diffusion models for sampling near criticality in lattice field theories*. 2026. arXiv: 2607.08505. URL: <https://arxiv.org/abs/2607.08505>.

[100] L. M. D. Bono, G. Biroli, P. Charbonneau, and M. Gabrié. *The critical slowing down in diffusion models*. 2026. arXiv: 2605.12597. URL: <https://arxiv.org/abs/2605.12597>.

[101] Q. Zhu, G. Aarts, W. Wang, K. Zhou, and L. Wang. “Diffusion models for lattice gauge field simulations”. In: *NeurIPS 2024 Workshop on Machine Learning and the Physical Sciences*. 2024. arXiv: 2410.19602. URL: <https://arxiv.org/abs/2410.19602>.

[102] Q. Zhu, G. Aarts, W. Wang, K. Zhou, and L. Wang. “Physics-conditioned diffusion models for lattice gauge theory”. In: *Journal of High Energy Physics* 2026 (2026), p. 111. doi: 10.1007/JHEP03(2026)111. arXiv: 2502.05504. URL: [https://doi.org/10.1007/JHEP03\(2026\)111](https://doi.org/10.1007/JHEP03(2026)111).

[103] G. Aarts, D. E. Habibi, A. Ipp, D. I. Müller, T. R. Ranner, L. Wang, W. Wang, and Q. Zhu. *Generalizable equivariant diffusion models for non-Abelian lattice gauge theory*. 2026. arXiv: 2601.19552. URL: <https://arxiv.org/abs/2601.19552>.

[104] H. Alharazin, J. Y. Panteleva, and B.-D. Sun. “Diffusion models for SU(2) lattice gauge theory in two dimensions”. In: *Physical Review D* (2026). Accepted 8 July 2026; version of record pending at the review cutoff. doi: 10.1103/n3bh-gs98. arXiv: 2602.09045. URL: <https://doi.org/10.1103/n3bh-gs98>.

[105] J. Komijani, M. K. Marinkovic, and L. Turgut. *Diffusion model for SU(N) gauge theories*. 2026. arXiv: 2605.06134. URL: <https://arxiv.org/abs/2605.06134>.

[106] O. Vega and A. X. El-Khadra. *Sampling the Schwinger model with gauge-equivariant diffusion*. 2026. arXiv: 2606.27481. URL: <https://arxiv.org/abs/2606.27481>.

[107] G. Aarts, D. E. Habibi, L. Wang, and K. Zhou. “Combining complex Langevin dynamics with score-based and energy-based diffusion models”. In: *Journal of High Energy Physics* 2025 (2025), p. 160. doi: 10.1007/JHEP12(2025)160. arXiv: 2510.01328. URL: [https://doi.org/10.1007/JHEP12\(2025\)160](https://doi.org/10.1007/JHEP12(2025)160).

[108] K. A. Nicoli, C. J. Anders, L. Funcke, K. Jansen, S. Nakajima, and P. Kessel. “NeuLat: a toolbox for neural samplers in lattice field theories”. In: *Proceedings of the 40th International Symposium on Lattice Field Theory*. Vol. 453. 2024, p. 286. doi: 10.22323/1.453.0286. URL: <https://doi.org/10.22323/1.453.0286>.

[109] B. Poole, S. Lahiri, M. Raghu, J. Sohl-Dickstein, and S. Ganguli. “Exponential expressivity in deep neural networks through transient chaos”. In: *Advances in Neural Information Processing Systems*. Vol. 29. 2016. eprint: 1606.05340. URL: <https://arxiv.org/abs/1606.05340>.

[110] S. S. Schoenholz, J. Gilmer, S. Ganguli, and J. Sohl-Dickstein. “Deep information propagation”. In: *International Conference on Learning Representations*. 2017. eprint: 1611.01232. URL: <https://arxiv.org/abs/1611.01232>.

[111] E. Dyer and G. Gur-Ari. “Asymptotics of wide networks from Feynman diagrams”. In: *International Conference on Learning Representations*. 2020. eprint: 1909.11304. URL: <https://arxiv.org/abs/1909.11304>.

[112] J. Lee, Y. Bahri, R. Novak, S. S. Schoenholz, J. Pennington, and J. Sohl-Dickstein. “Deep neural networks as Gaussian processes”. In: *International Conference on Learning Representations*. 2018. eprint: 1711.00165. URL: <https://arxiv.org/abs/1711.00165>.

[113] A. G. de G. Matthews, M. Rowland, J. Hron, R. E. Turner, and Z. Ghahramani. “Gaussian process behaviour in wide deep neural networks”. In: *International Conference on Learning Representations*. 2018. eprint: 1804.11271. URL: <https://arxiv.org/abs/1804.11271>.

[114] A. Jacot, F. Gabriel, and C. Hongler. “Neural tangent kernel: Convergence and generalization in neural networks”. In: *Advances in Neural Information Processing Systems*. Vol. 31. 2018. eprint: 1806.07572. URL: <https://arxiv.org/abs/1806.07572>.

[115] Y. Bahri, J. Kadmon, J. Pennington, S. S. Schoenholz, J. Sohl-Dickstein, and S. Ganguli. “Statistical mechanics of deep learning”. In: *Annual Review of Condensed Matter Physics* 11 (2020), pp. 501–528. doi: 10.1146/annurev-conmatphys-031119-050745. URL: <https://doi.org/10.1146/annurev-conmatphys-031119-050745>.

[116] D. A. Roberts, S. Yaida, and B. Hanin. *The Principles of Deep Learning Theory*. Cambridge University Press, 2022. doi: 10.1017/9781009023405. URL: <https://doi.org/10.1017/9781009023405>.

[117] J. Halverson, A. Maiti, and K. Stoner. “Neural networks and quantum field theory”. In: *Machine Learning: Science and Technology* 2 (2021), p. 035002. doi: 10.1088/2632-2153/abeca3. URL: <https://doi.org/10.1088/2632-2153/abeca3>.

[118] H. Erbin, V. Lahoche, and D. O. Samary. *Nonperturbative renormalization for the neural network-QFT correspondence*. 2021. arXiv: 2108.01403 [hep-th]. URL: <https://arxiv.org/abs/2108.01403>.

[119] H. Erbin, V. Lahoche, and D. O. Samary. *Renormalization in the neural network-quantum field theory correspondence*. 2022. arXiv: 2212.11811 [hep-th]. URL: <https://arxiv.org/abs/2212.11811>.

[120] O. Cohen, O. Malka, and Z. Ringel. “Learning curves for deep neural networks: A Gaussian field theory perspective”. In: *Physical Review Research* 3 (2021), p. 023034. doi: 10.1103/PhysRevResearch.3.023034. URL: <https://doi.org/10.1103/PhysRevResearch.3.023034>.

[121] J. N. Howard, R. Jefferson, A. Maiti, and Z. Ringel. “Wilsonian renormalization of neural network Gaussian processes”. In: *Machine Learning: Science and Technology* 6 (2025), p. 025038. doi: 10.1088/2632-2153/adc8fc. URL: <https://doi.org/10.1088/2632-2153/adc8fc>.

[122] J. N. Howard, M. S. Klinger, A. Maiti, and A. G. Stapleton. “Bayesian RG flow in neural network field theories”. In: *SciPost Physics Core* 8 (2025), p. 027. doi: 10.21468/SciPostPhysCore.8.1.027. URL: <https://doi.org/10.21468/SciPostPhysCore.8.1.027>.

[123] R. Aiudi, R. Pacelli, P. Baglioni, A. Vezzani, R. Burioni, and P. Rotondo. “Local kernel renormalization as a mechanism for feature learning in overparametrized convolutional neural networks”. In: *Nature Communications* 16 (2025), p. 568. doi: 10.1038/s41467-024-55229-3. URL: <https://doi.org/10.1038/s41467-024-55229-3>.

[124] C. Merger, A. René, K. Fischer, P. Bouss, S. Nestler, D. Dahmen, C. Honerkamp, and M. Helias. “Learning interacting theories from data”. In: *Physical Review X* 13 (2023), p. 041033. doi: 10.1103/PhysRevX.13.041033. URL: <https://doi.org/10.1103/PhysRevX.13.041033>.

[125] D. Lee, H.-S. Lee, and J. Yi. “Dynamic neuron approach to deep neural networks: Decoupling neurons for renormalization group analysis”. In: *Physical Review Research* 7 (2025), p. 023276. doi: 10.1103/PhysRevResearch.7.023276. URL: <https://doi.org/10.1103/PhysRevResearch.7.023276>.

[126] G. Peraza Coppola, M. Helias, and Z. Ringel. *Renormalization group for deep neural networks: Universality of learning and scaling laws*. Version 2, 2026. 2026. arXiv: 2510.25553 [cond-mat.stat-mech]. URL: <https://arxiv.org/abs/2510.25553>.

[127] P. Villegas, T. Gili, G. Caldarelli, and A. Gabrielli. “Laplacian renormalization group for heterogeneous networks”. In: *Nature Physics* 19 (2023), pp. 445–450. doi: 10.1038/s41567-022-01866-8. URL: <https://doi.org/10.1038/s41567-022-01866-8>.

[128] G. Caldarelli, A. Gabrielli, T. Gili, and P. Villegas. “Laplacian renormalization group: An introduction to heterogeneous coarse-graining”. In: *Journal of Statistical Mechanics: Theory and Experiment* (2024), p. 084002. doi: 10.1088/1742-5468/ad57b1. arXiv: 2406.02337 [physics.soc-ph]. URL: <https://doi.org/10.1088/1742-5468/ad57b1>.

[129] A. Gabrielli, D. Garlaschelli, S. P. Patil, and M. A. Serrano. “Network renormalization”. In: *Nature Reviews Physics* 7 (2025), pp. 203–219. doi: 10.1038/s42254-025-00817-5. URL: <https://doi.org/10.1038/s42254-025-00817-5>.

[130] Z. Hao, R. Kansal, A. Gandrakota, C. Sun, J. Ngadiuba, J. Duarte, and M. Spiropulu. *RINO: Renormalization group invariance with no labels*. NeurIPS 2025 Workshop on Machine Learning and the Physical Sciences, spotlight. 2025. arXiv: 2509.07486 [hep-ex]. URL: <https://arxiv.org/abs/2509.07486>.

Do not assume that your colleagues
read your work more carefully than the AIs

7

Thanx to Kai Zhou (Shenzhen)

Coding & tasks

Moating away

Goal: refactor the QCD_current.nb mathematica file.

Context: I want to move away from one big mathematica notebook to a set of multiple .wl files spread over several folders. This is because notebooks carry to much metadata, making it hard for ai to read and pollutes git commits. Use /home/konrad/PhD/private_gits/QCD_current_NumTracer/Mathematica as inspiration. Keep it as easy to understand as possible, do not introduce small functions for everything. Prioritise readability.

The new project should behave identically to the current QCD_current.nb file, to be extended using the moat rotation. Especially, do not switch to the numtracer backend. Include also a testing suite.

Limitations: For now, restrict yourself to planning. Do not edit any files yet. Keep QCD_current.nb as legacy files.



Konrad Kockler

DiFFRG

Preparing PIKs for Bachelors

Hi I have provided you with tex files in the context directory. Please read them.

The LatticePIK.tex file and ComplexPIK.tex provide a background. The HubbardStratonovichPIK.tex describes the project I want you to do. Please first familiarise yourself with the project. Then summarise the HubbardPIK project to me.

I am especially interested in the possibilities a Hubbard Stratonovich transformation gives me in simplifying solution (methods) to the Wegner equation. By this I mean using the described operator splitting from the notes or a way to absorb long-ranged correlations in auxiliary fields.

I want to hand out the project described in HubbardStratonovichPIK.tex as a Bachelor's project. Therefore, I want you to make first tests and set up a small coding framework in order to test the viability of the idea.

In order to save tokens and gain some speed please also use sub-agents.

In order to access the quality of the project, do not immediately consult the internet but rather think about it yourself and discuss with me.

Please do not start any of these tasks immediately but first make a plan and create a progress.md file.

Show less

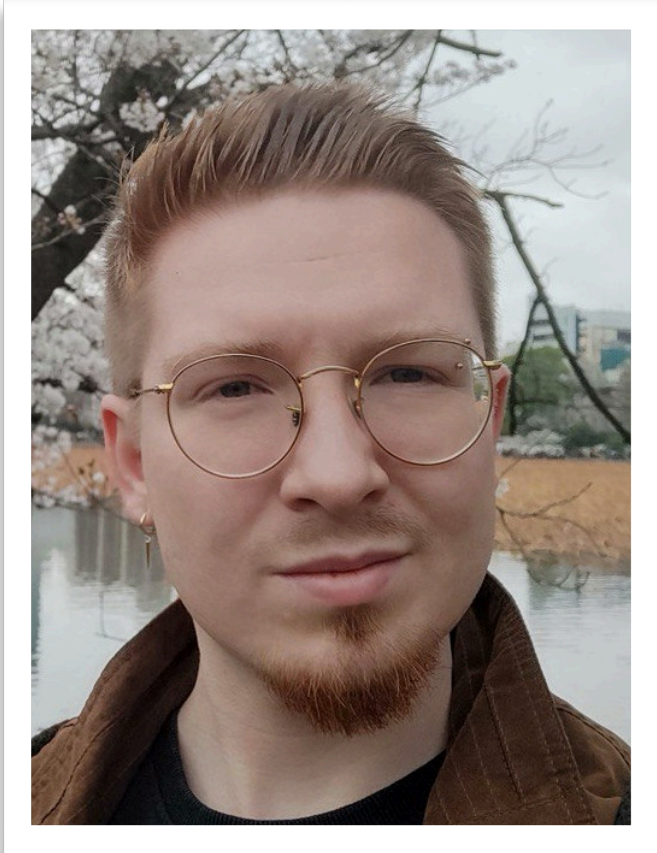
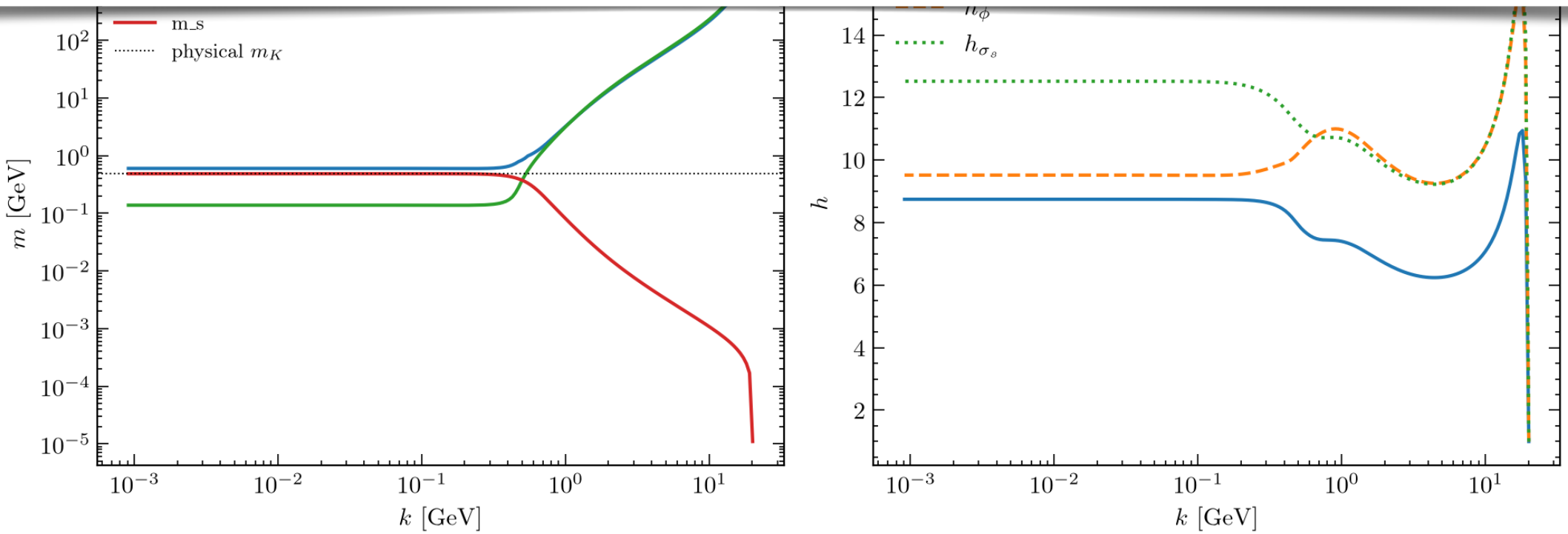


Renzo Kapust

PIKs

Emergent QCD

```
> Copy `2408.08413/` to `with_strange/`, add dynamical hadronisation of the strange quark as in
`finite_T/YMQ_Mesons_4F_FullMom/`, with a Taylor potential for  $\sigma_s$  run the way
`vacuum/QCD_Nf2/with_mesons_taylor/` does.
Add a kaon - dynamically hadronise it, give it a taylor potential again (third order) and we'll see what
comes out.
```



Franz Sattler

DiFFRG

Lennart Klebl: *Towards First-Principles fRG for Quantum Materials*

Tuesday 10:00 AM - 10:30 AM

In this talk, I introduce the first open-source functional renormalization group (FRG) code for condensed matter systems: divERGe. The development of flexible and accessible computational frameworks is becoming increasingly important as modern condensed matter problems continue to grow in complexity, spanning from minimal theoretical models to realistic descriptions of quantum materials. divERGe is designed to bridge this gap by providing a versatile platform for implementing and applying FRG approaches across a broad range of systems. We employ deep neural networks to represent the field derivative of the scale-dependent effective potential in the functional renormalization group (fRG) framework for nonperturbative quantum field theory. By embedding the fRG flow equations directly into the loss function, the network parameters are determined so as to provide a continuous and differentiable representation of the scale- and field-dependent effective potential without relying on precomputed training data. Focusing on the $O(N)$ scalar field theory within the local potential approximation at finite temperature, we demonstrate that this neural network representation accurately captures the renormalization group flow across symmetric, broken, and critical regimes. A key ingredient is a decomposition of the representation into an analytically known large- N contribution and a learned finite- N correction, which efficiently mitigates numerical stiffness associated with convexity restoration in the broken phase. The physics-driven solutions show excellent agreement with established finite-difference and discontinuous Galerkin methods. We further apply the same strategy to the Wilson-Fisher fixed point equation in three dimensions, illustrating that neural network representations provide a unified framework for both scale-dependent flows and fixed-point problems. Our results indicate that physics-driven deep learning offers a robust and flexible numerical tool for functional renormalization group studies.

divERGe

Franz Richard Sattler: *Preconden. and inhom. instabilities in high-density QCD*

Thursday 2026, 9:20 AM - 9:40 AM

QCD at large densities exhibits a moat regime in the scalar-pseudoscalar sector. The resolution of its dynamics is pivotal to access the onset of new phases, including the potential critical endpoint of QCD. I report on a first self-consistent analysis of this regime, using the first-principles functional renormalisation group approach to QCD. Mapping out the moat regime, we find a potential inhomogeneous instability at baryon chemical potential MeV on the chiral crossover line. I discuss the stability of this instability together with the closely connected phenomenon of precondensation, which can be probed in a natural manner using the fRG.

DiFfRG

Fit the Second: Machine learning through the fRG looking glass



Quantum field theory & the functional RG in a nutshell

Classical action $S(\phi)$

Simple example: ϕ real scalar field with $S(\phi) = \int d^d x \left\{ \frac{1}{2} \phi (-\partial_\mu^2 + m_\phi^2) \phi + \frac{\lambda_\phi}{8} \phi^4 \right\} + \log \mathcal{N}_\phi$

Expectation values $\langle \mathcal{O}(\hat{\phi}) \rangle = \int \prod_x d\hat{\phi}(x) p(\hat{\phi}) \mathcal{O}(\hat{\phi})$

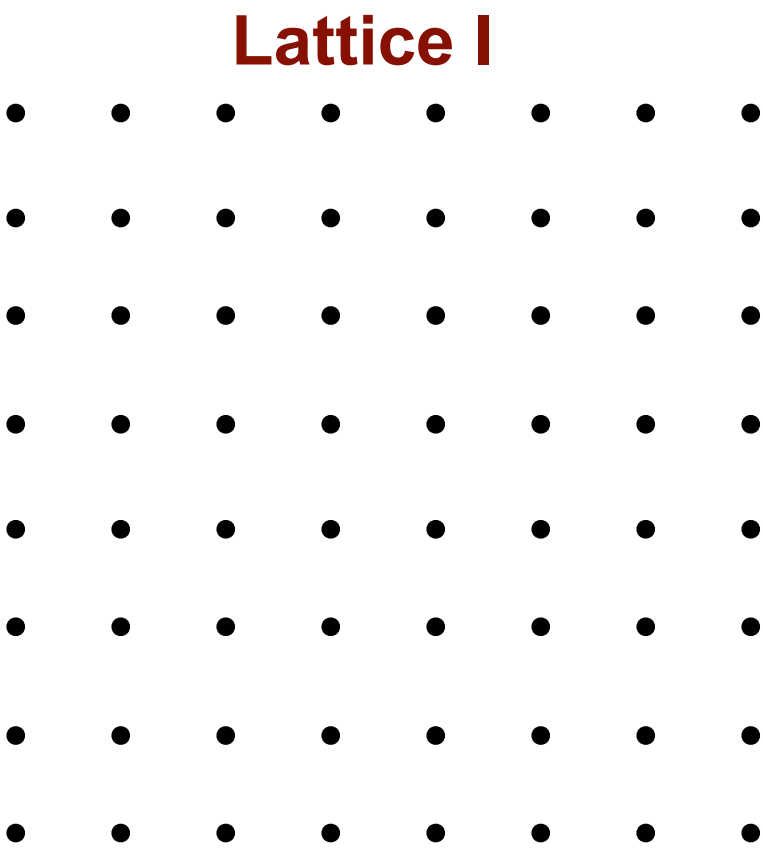
Euclidean measure $p(\hat{\phi}) = e^{-S(\hat{\phi})}$

Quantum field theory & the RG in a nutshell

Classical action $S(\phi)$

Simple example: ϕ real scalar field with $S(\phi) = \int d^d x \left\{ \frac{1}{2} \phi \left(-\partial_\mu^2 + m_\phi^2 \right) \phi + \frac{\lambda_\phi}{8} \phi^4 \right\} + \log \mathcal{N}_\phi$

Linear block-spinning renormalisation group

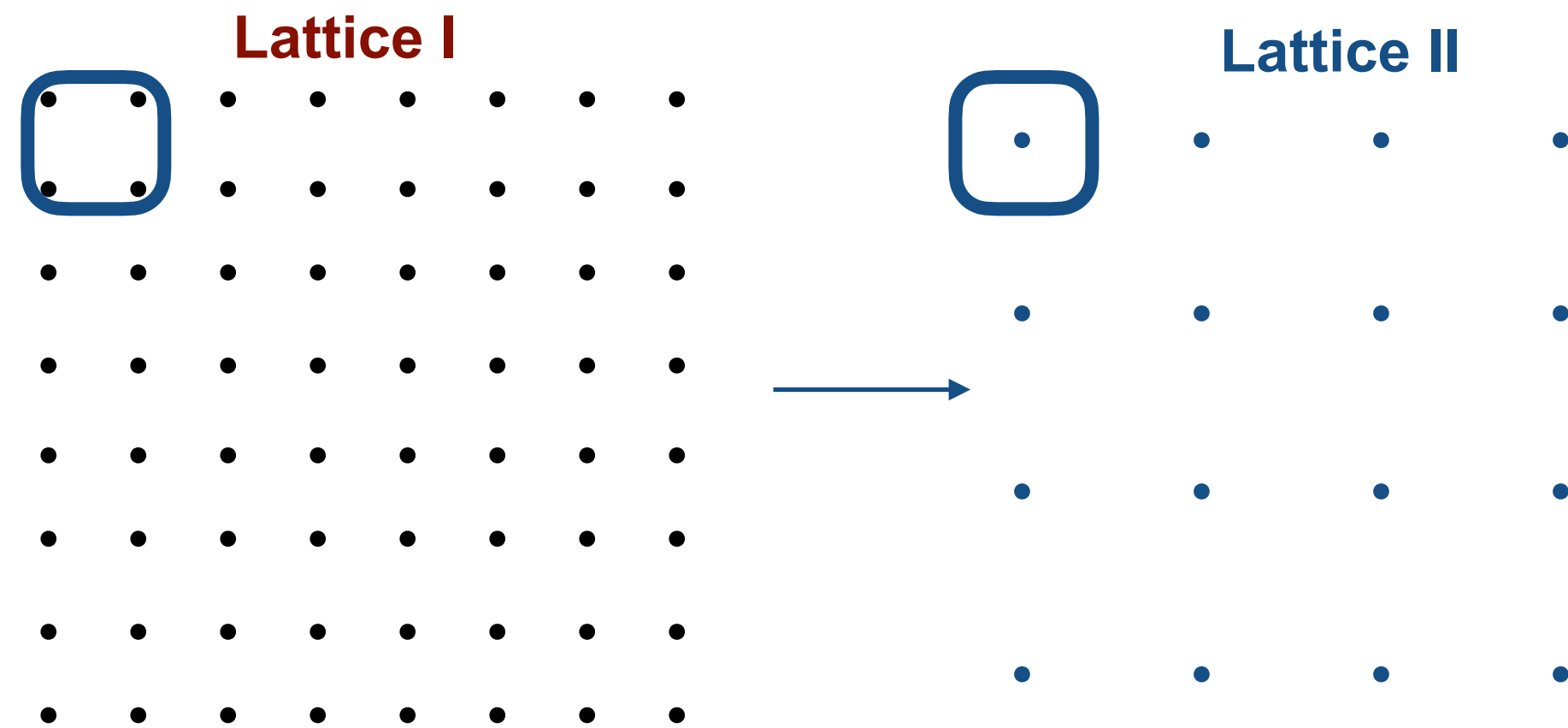


Quantum field theory & the RG in a nutshell

Classical action $S(\phi)$

Simple example: ϕ real scalar field with $S(\phi) = \int d^d x \left\{ \frac{1}{2} \phi (-\partial_\mu^2 + m_\phi^2) \phi + \frac{\lambda_\phi}{8} \phi^4 \right\} + \log \mathcal{N}_\phi$

Linear block-spinning renormalisation group

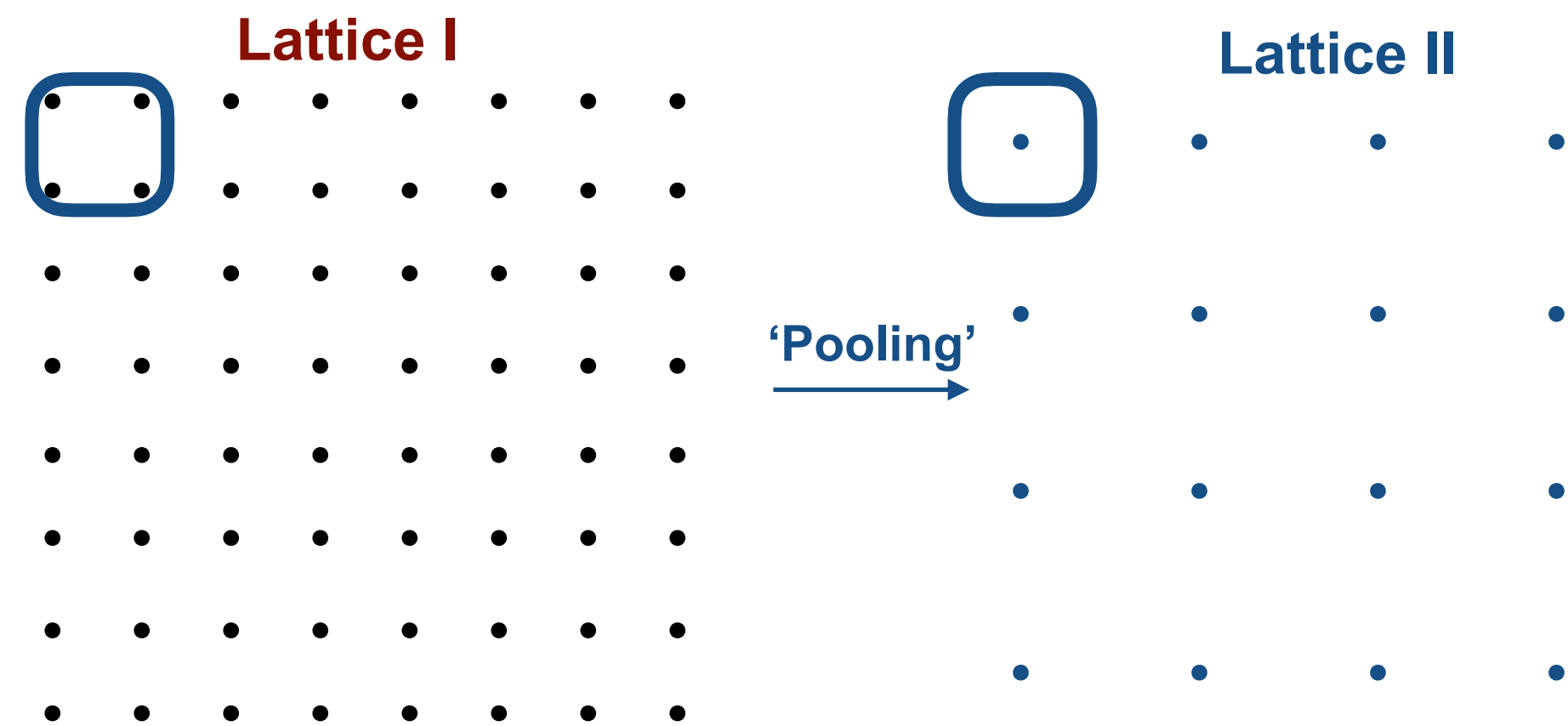


Quantum field theory & the RG in a nutshell

Classical action $S(\phi)$

Simple example: ϕ real scalar field with $S(\phi) = \int d^d x \left\{ \frac{1}{2} \phi (-\partial_\mu^2 + m_\phi^2) \phi + \frac{\lambda_\phi}{8} \phi^4 \right\} + \log \mathcal{N}_\phi$

Linear block-spinning renormalisation group

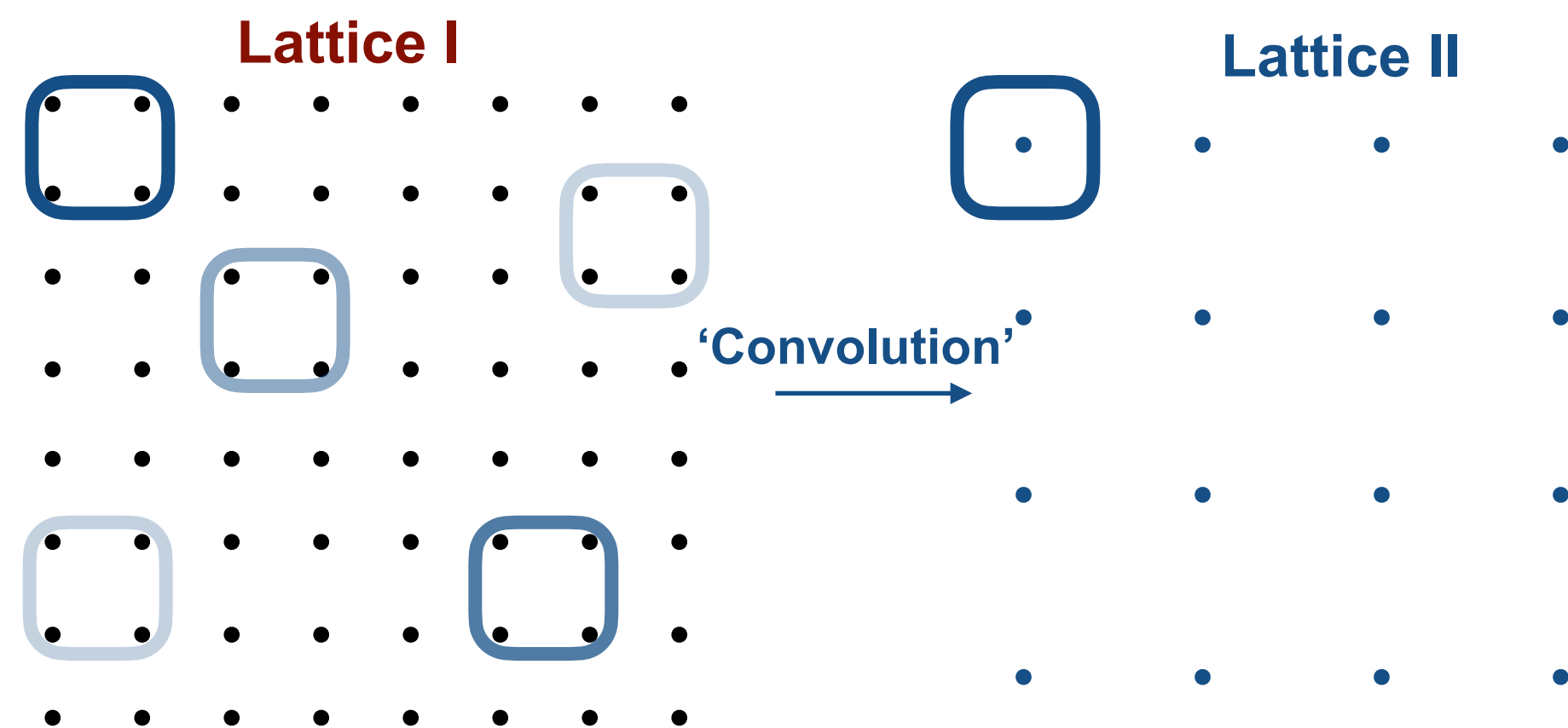


Quantum field theory & the RG in a nutshell

Classical action $S(\phi)$

Simple example: ϕ real scalar field with $S(\phi) = \int d^d x \left\{ \frac{1}{2} \phi (-\partial_\mu^2 + m_\phi^2) \phi + \frac{\lambda_\phi}{8} \phi^4 \right\} + \log \mathcal{N}_\phi$

Linear block-spinning renormalisation group

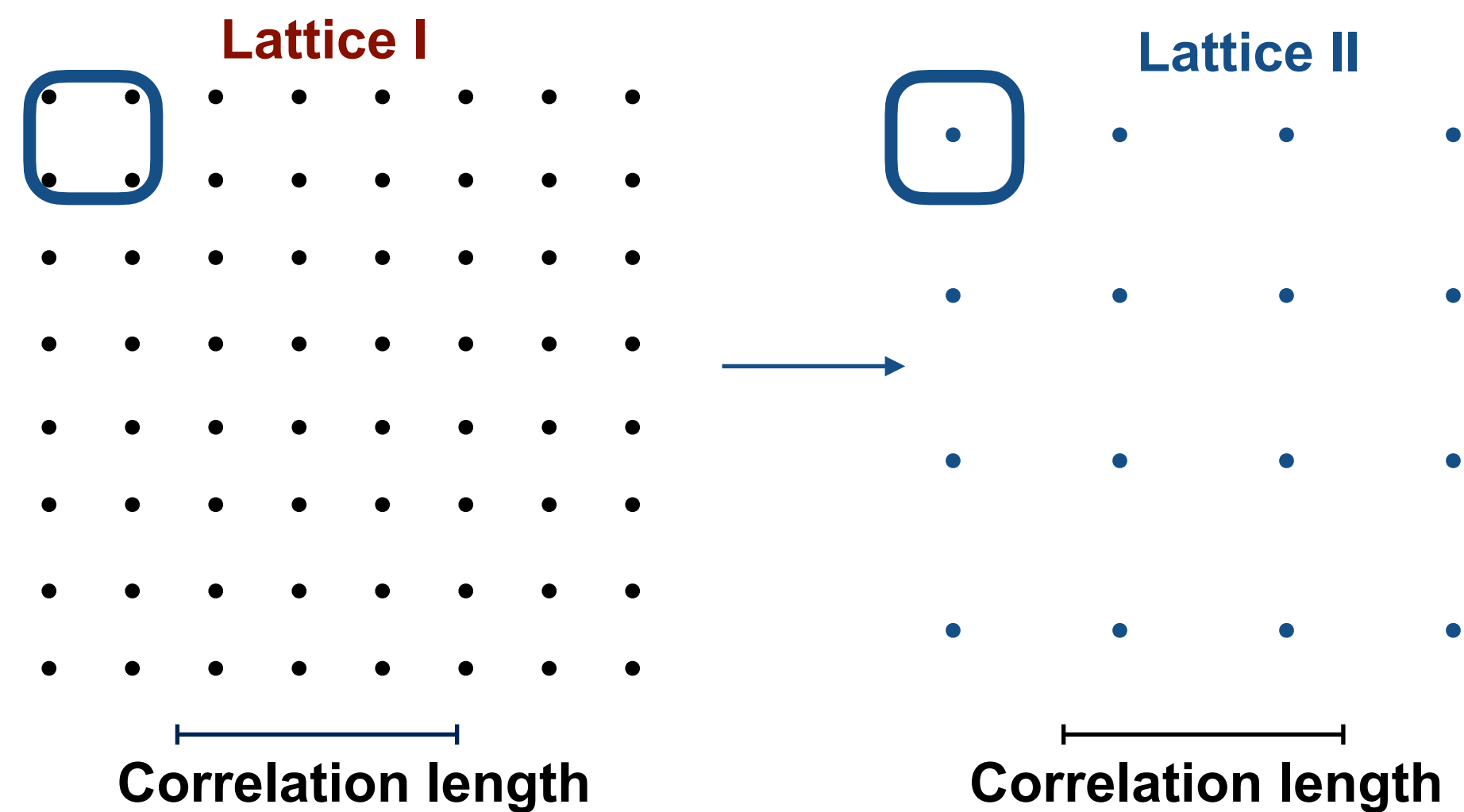


Quantum field theory & the RG in a nutshell

Classical action $S(\phi)$

Simple example: ϕ real scalar field with $S(\phi) = \int d^d x \left\{ \frac{1}{2} \phi (-\partial_\mu^2 + m_\phi^2) \phi + \frac{\lambda_\phi}{8} \phi^4 \right\} + \log \mathcal{N}_\phi$

Linear block-spinning renormalisation group

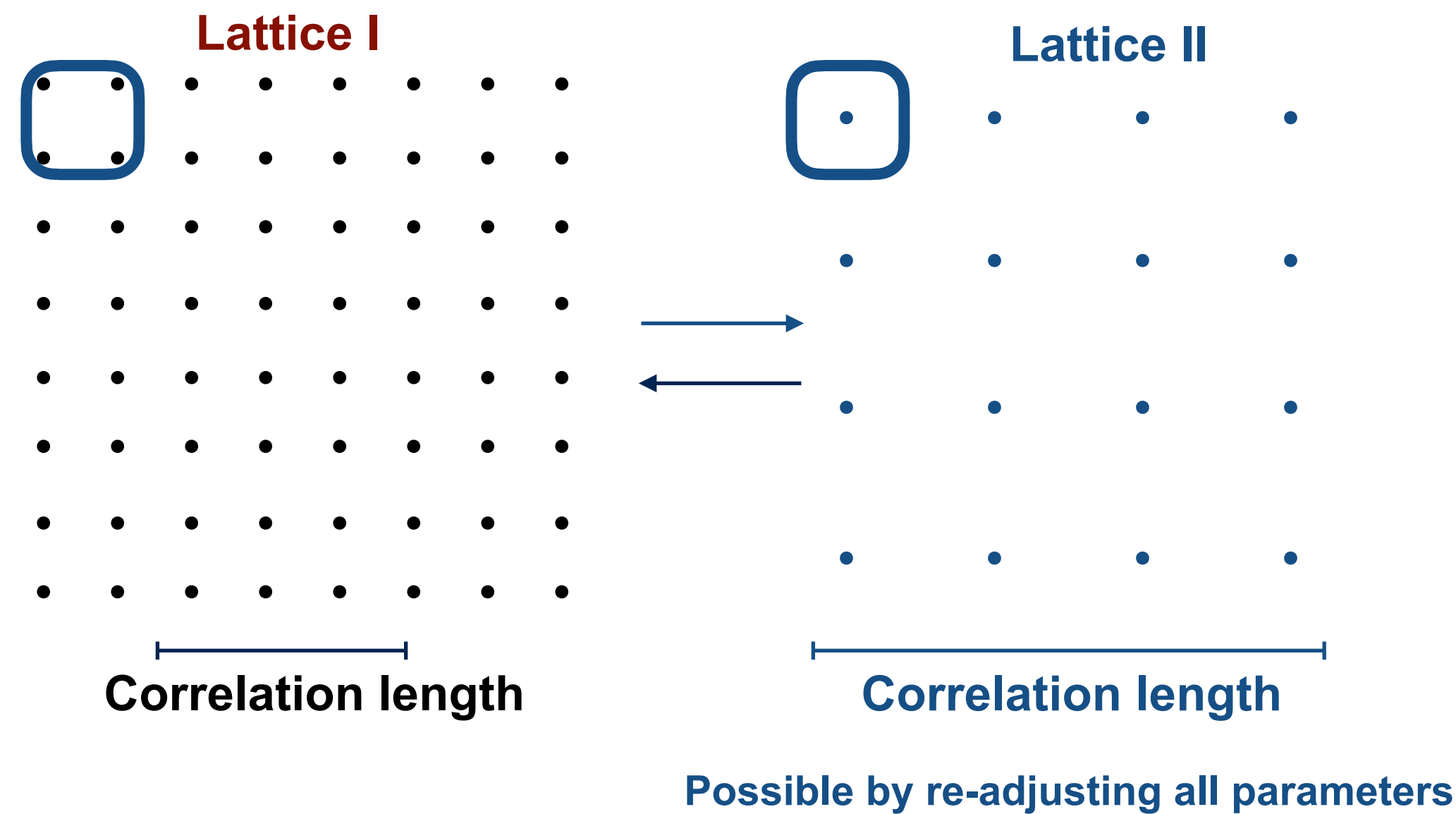


Quantum field theory & the RG in a nutshell

Classical action $S(\phi)$

Simple example: ϕ real scalar field with $S(\phi) = \int d^d x \left\{ \frac{1}{2} \phi (-\partial_\mu^2 + m_\phi^2) \phi + \frac{\lambda_\phi}{8} \phi^4 \right\} + \log \mathcal{N}_\phi$

Linear block-spinning renormalisation group

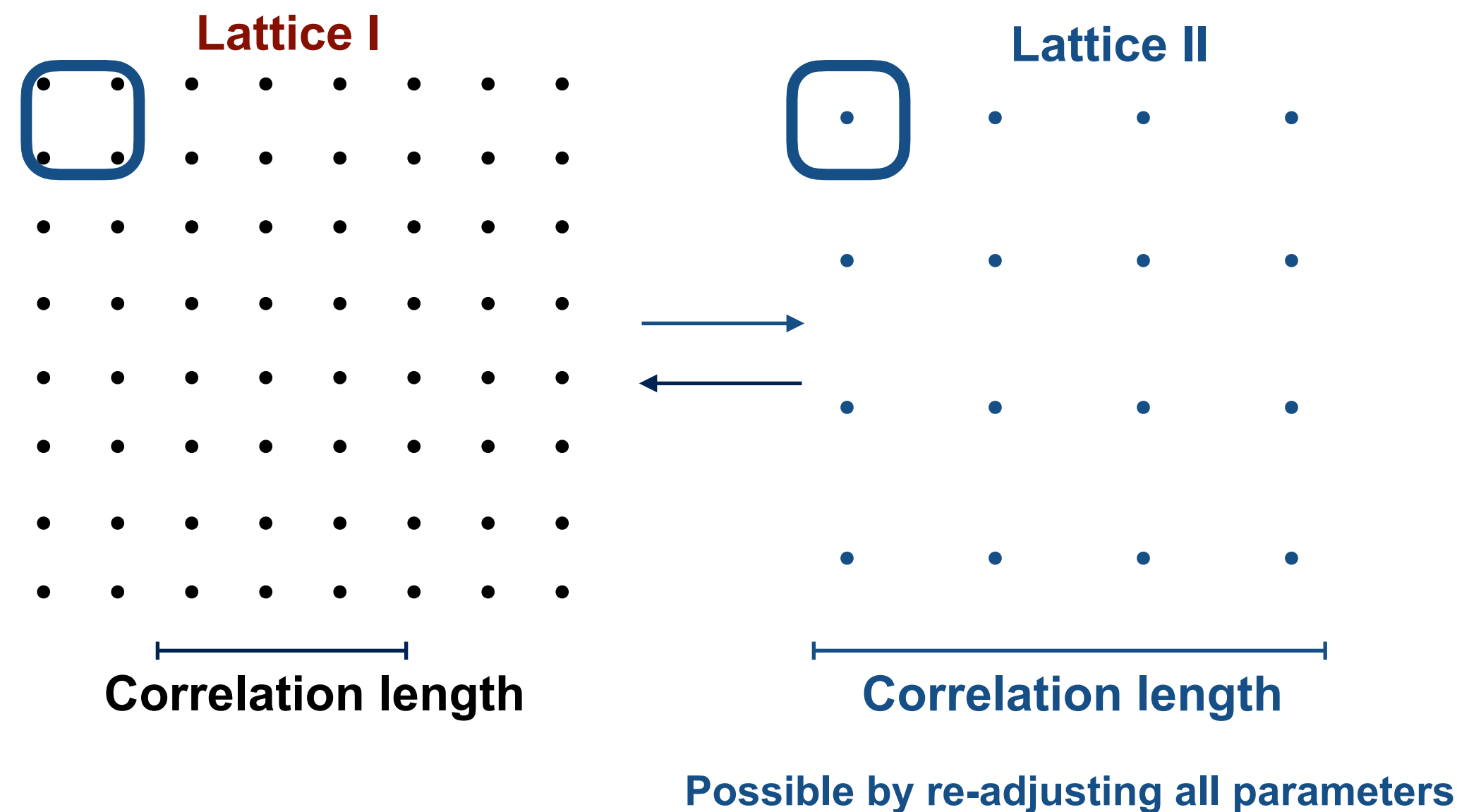


Quantum field theory & the RG in a nutshell

Classical action $S(\phi)$

Simple example: ϕ real scalar field with $S(\phi) = \int d^d x \left\{ \frac{1}{2} \phi (-\partial_\mu^2 + m_\phi^2) \phi + \frac{\lambda_\phi}{8} \phi^4 \right\} + \log \mathcal{N}_\phi$

Linear block-spinning renormalisation group



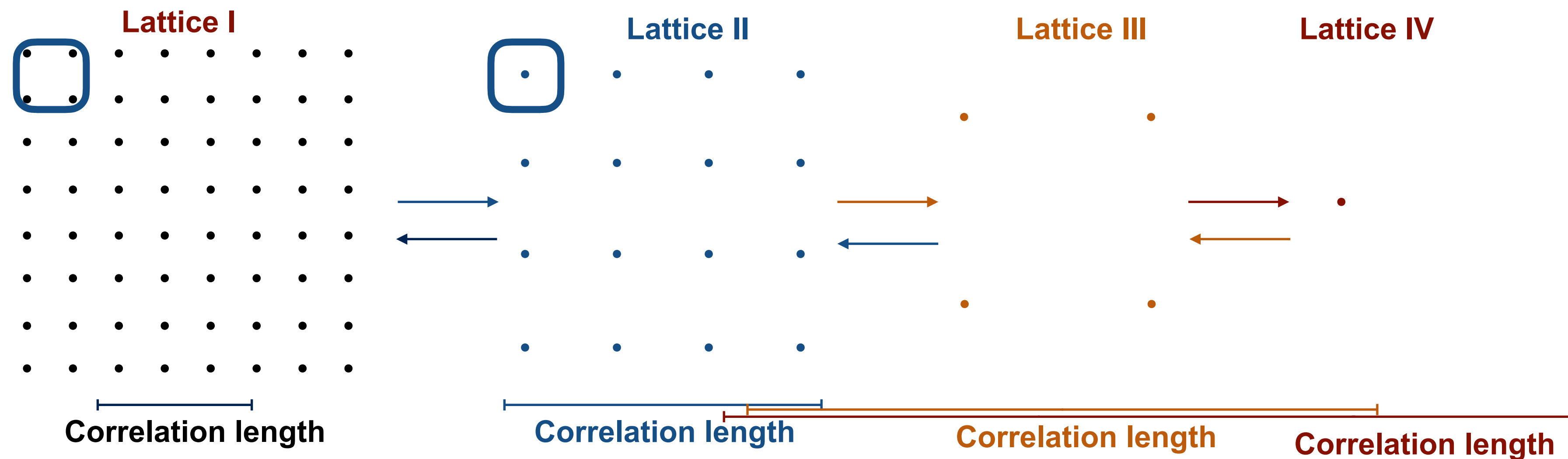
→ Finite lattices: RG step involves loss of information (hypothesis of RG entropy that increases)

Quantum field theory & the RG in a nutshell

Classical action $S(\phi)$

Simple example: ϕ real scalar field with $S(\phi) = \int d^d x \left\{ \frac{1}{2} \phi (-\partial_\mu^2 + m_\phi^2) \phi + \frac{\lambda_\phi}{8} \phi^4 \right\} + \log \mathcal{N}_\phi$

Linear block-spinning renormalisation group



Possible by re-adjusting all parameters

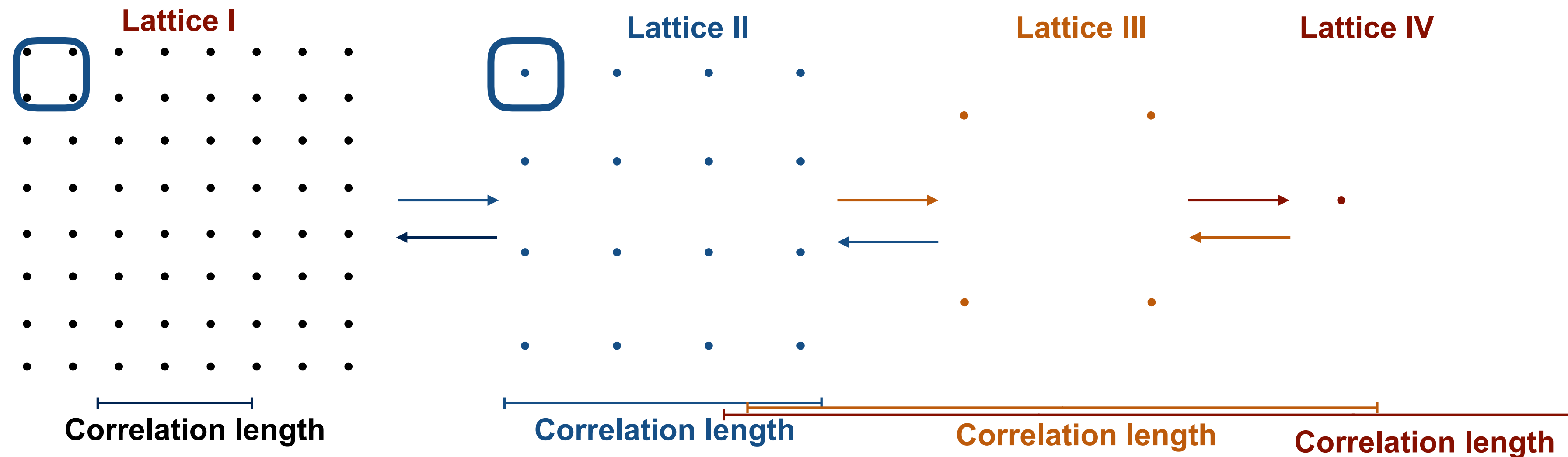
→ Finite lattices: RG step involves loss of information (hypothesis of RG entropy that increases)

Quantum field theory & the RG in a nutshell

Classical action $S(\phi)$

Simple example: ϕ real scalar field with $S(\phi) = \int d^d x \left\{ \frac{1}{2} \phi (-\partial_\mu^2 + m_\phi^2) \phi + \frac{\lambda_\phi}{8} \phi^4 \right\} + \log \mathcal{N}_\phi$

Linear block-spinning renormalisation group



Possible by re-adjusting all parameters

← Full inversion only possible for lattice (system) with infinite extend/infinite # of degrees of freedom

Inverse RG

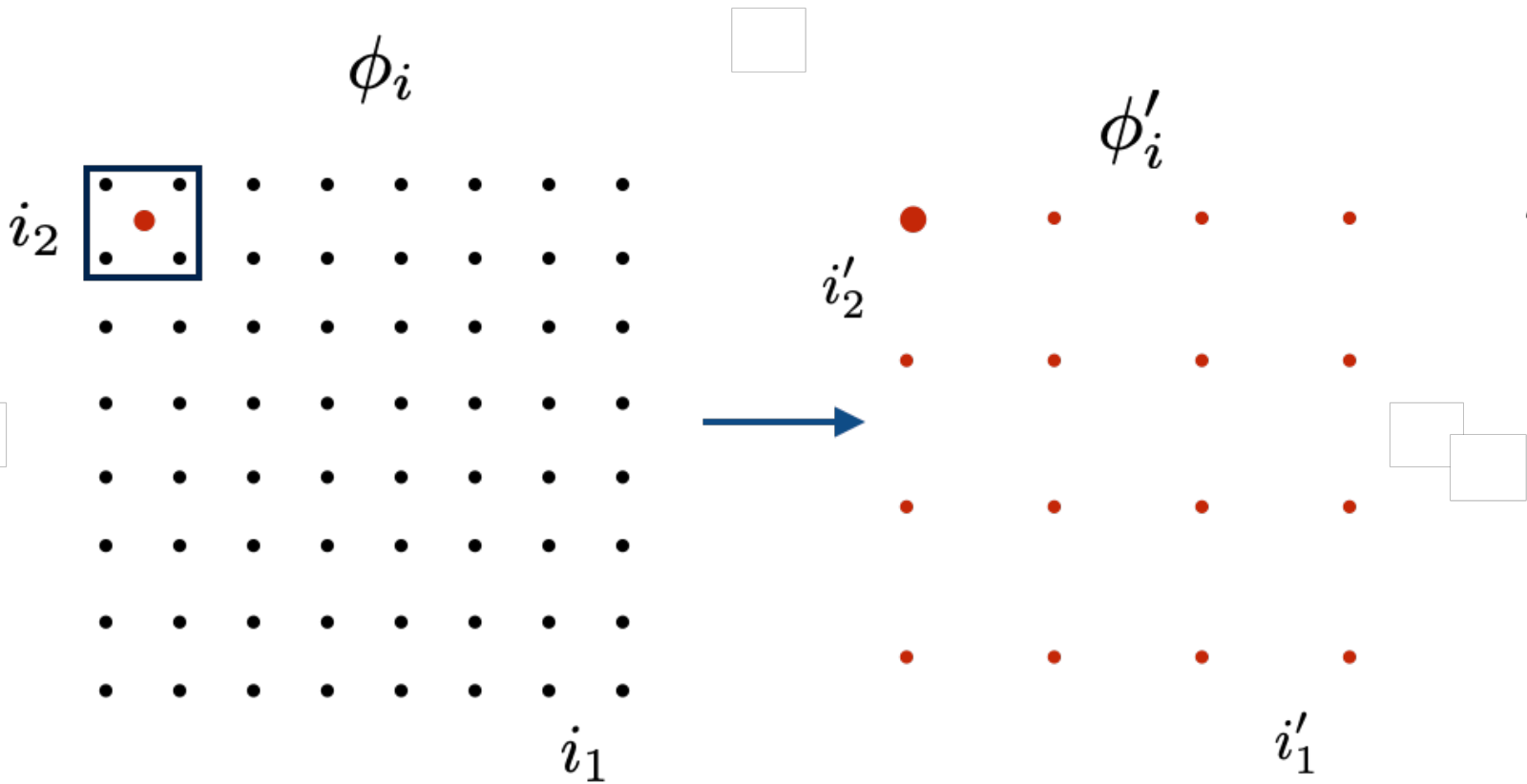
Quantum field theory & the RG in a nutshell

Classical action $S(\phi)$

Simple example: ϕ real scalar field with $S(\phi) = \int d^d x \left\{ \frac{1}{2} \phi \left(-\partial_\mu^2 + m_\phi^2 \right) \phi + \frac{\lambda_\phi}{8} \phi^4 \right\} + \log \mathcal{N}_\phi$

ML-spinning renormalisation group

Pooling = scale transformation

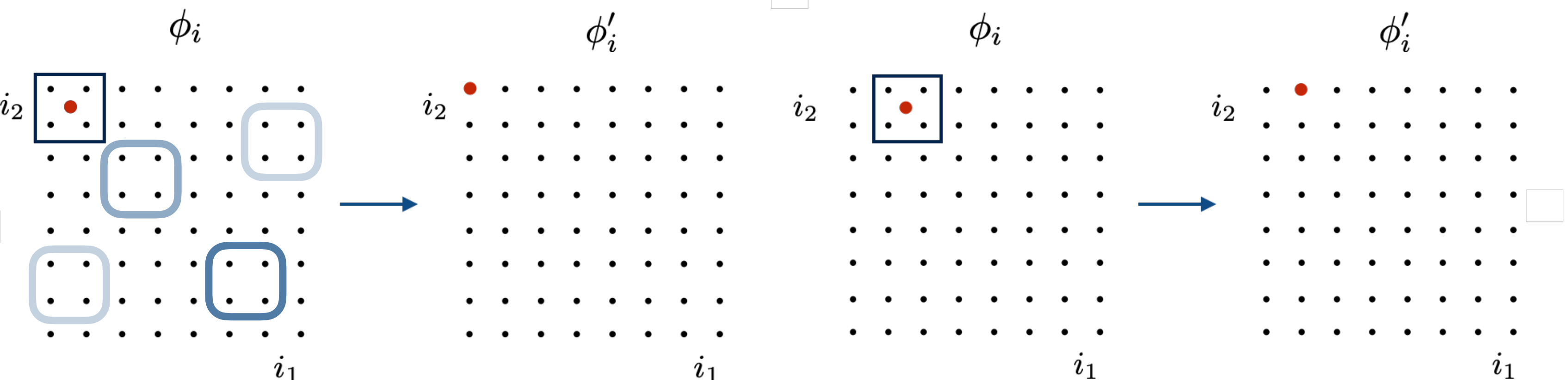


Lattice I

Lattice II

$$\phi'_{i'} = \frac{1}{4} \sum_{i \in \mathcal{C}_{(1,4)}} \phi_i$$

Convolution = linear reparametrisation



$$A \left(W_{(1,8)j} \phi_j + b_j \right)$$

$$A \left(W_{(2,8)j} \phi_j + b_j \right)$$

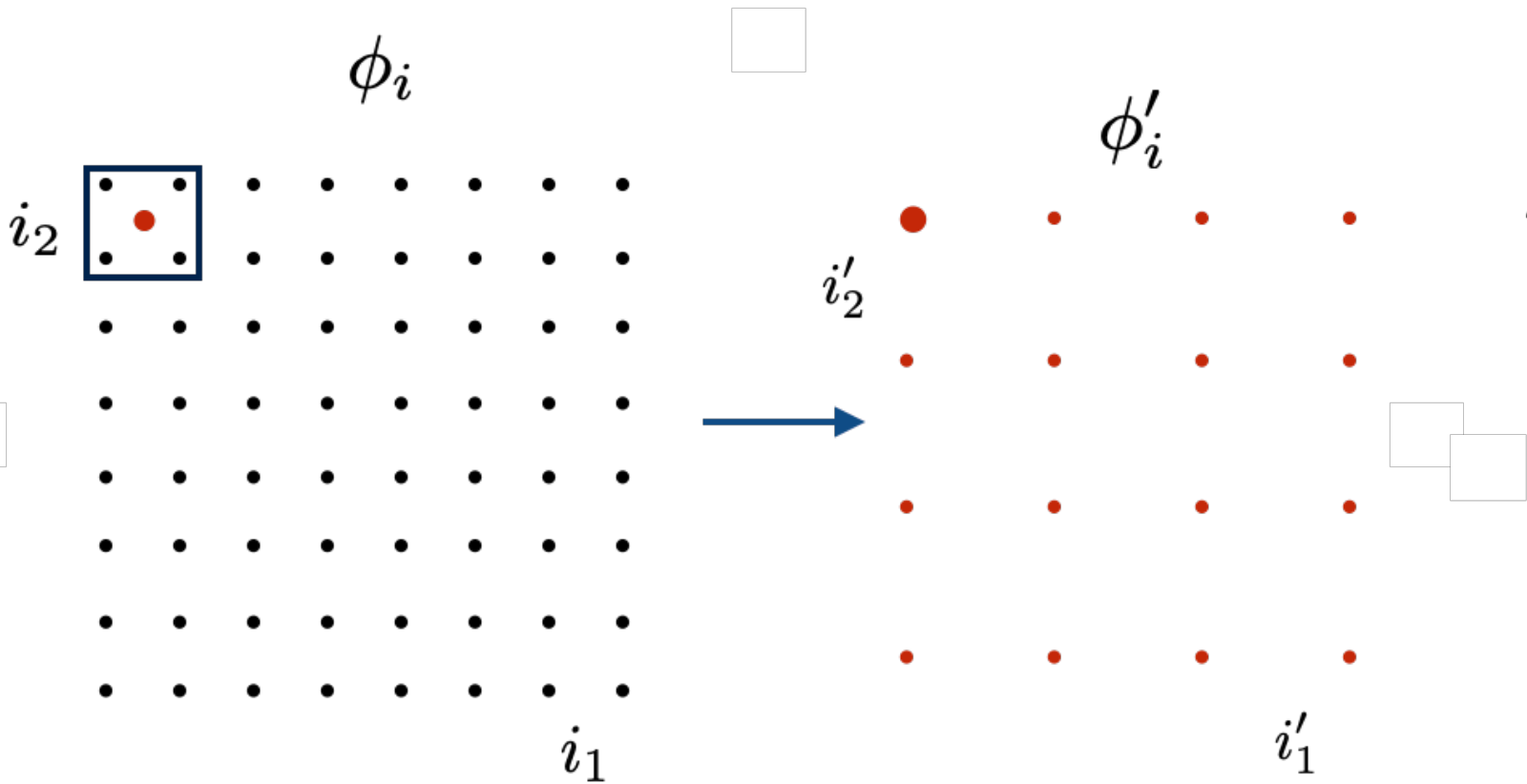
Quantum field theory & the RG in a nutshell

Classical action $S(\phi)$

Simple example: ϕ real scalar field with $S(\phi) = \int d^d x \left\{ \frac{1}{2} \phi \left(-\partial_\mu^2 + m_\phi^2 \right) \phi + \frac{\lambda_\phi}{8} \phi^4 \right\} + \log \mathcal{N}_\phi$

ML-spinning renormalisation group

Pooling = scale transformation

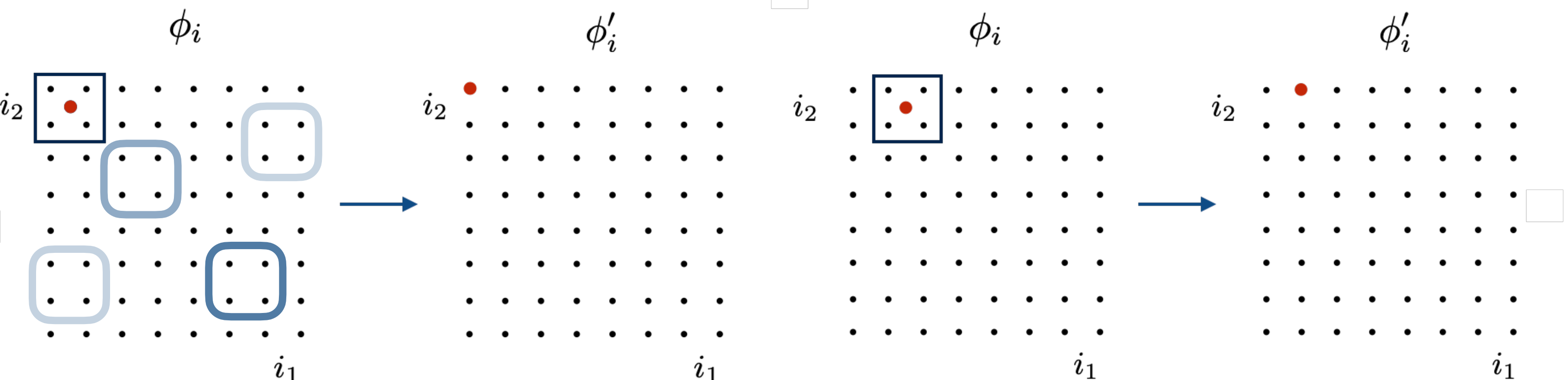


Lattice I

Lattice II

$$\phi'_{i'} = \frac{1}{4} \sum_{i \in \mathcal{C}_{(1,4)}} \phi_i$$

Convolution = linear reparametrisation



$$A(W_{(1,8)j} \phi_j + b_j)$$

$$A(W_{(2,8)j} \phi_j + b_j)$$

Activation function: linear $\rightarrow$ non-linear

Quantum field theory & the functional RG in a nutshell

Jump forward to: Fit the Third to Fifth: What can Machine Learning do for us?

Classical action $S(\phi)$

Simple example: ϕ real scalar field with $S(\phi) = \int d^d x \left\{ \frac{1}{2} \phi (-\partial_\mu^2 + m_\phi^2) \phi + \frac{\lambda_\phi}{8} \phi^4 \right\} + \log \mathcal{N}_\phi$

RG: General rescalings & reparametrisations via general differential map $\dot{\phi}(\phi)$

Wegner equation

$$\partial_t p_t(\phi) = - \int_x \frac{\delta}{\delta \phi(x)} \left(\dot{\phi}(\phi) p_t(\phi) \right)$$

Wegner, J. Phys. C 7 (1974) 2098

flow of distribution aka optimal transport

Generalised flow equation

$$\left(\partial_t + \int_x \dot{\phi} \frac{\delta}{\delta \phi} \right) \Gamma_\phi = \frac{1}{2} \text{Tr} \left[G_\phi \left(\partial_t + 2 \frac{\delta \dot{\phi}}{\delta \phi} \right) R_\phi \right]$$

JMP, *Annals Phys.* 322 (2007) 2831

flow of target action aka rate function

RG-time: t
 $\longleftrightarrow$
Legendre transform

Locality

Quantum field theory & the functional RG in a nutshell

Classical action $S(\phi)$

Simple example: ϕ real scalar field with $S(\phi) = \int d^d x \left\{ \frac{1}{2} \phi (-\partial_\mu^2 + m_\phi^2) \phi + \frac{\lambda_\phi}{8} \phi^4 \right\} + \log \mathcal{N}_\phi$

Continuous block-spinning aka integrating out momentum shells



Generating functionals $F[\phi]$

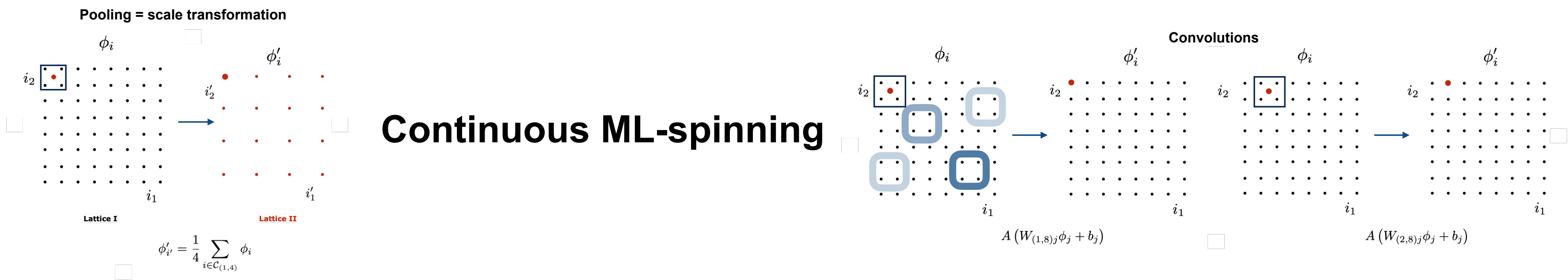
Probability measure

$$F[\phi] = p[\phi]$$

Quantum field theory & the functional RG in a nutshell

Classical action $S(\phi)$

Simple example: ϕ real scalar field with $S(\phi) = \int d^d x \left\{ \frac{1}{2} \phi \left(-\partial_\mu^2 + m_\phi^2 \right) \phi + \frac{\lambda_\phi}{8} \phi^4 \right\} + \log \mathcal{N}_\phi$



Generating functionals $F[\phi]$

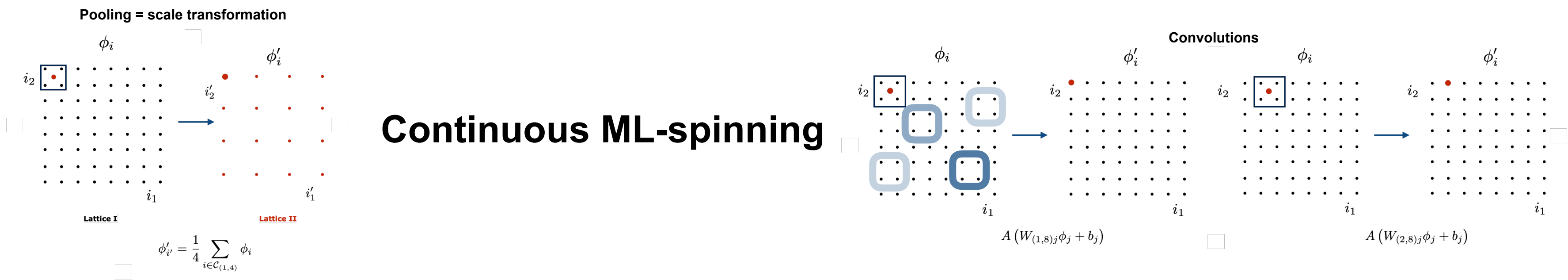
Probability measure

$$F[\phi] = p[\phi]$$

Quantum field theory & the functional RG in a nutshell

Classical action $S(\phi)$

Simple example: ϕ real scalar field with $S(\phi) = \int d^d x \left\{ \frac{1}{2} \phi \left(-\partial_\mu^2 + m_\phi^2 \right) \phi + \frac{\lambda_\phi}{8} \phi^4 \right\} + \log \mathcal{N}_\phi$



Generating functionals $F[\phi]$

Probability measure

$$F[\phi] = p[\phi]$$

Wilson effective action = log likelihood

$$F[\phi] \simeq \log p[\phi] = S_{\text{eff}}[\phi]$$

Special case: Polchinski flow '83

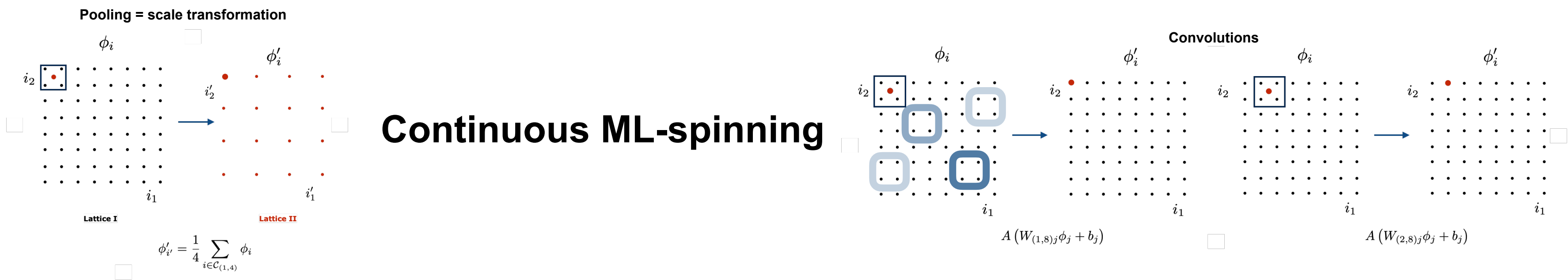
$$\dot{\phi} \propto \mathcal{C}_t \frac{\delta S_{\text{eff}}}{\delta \phi}$$

Wegner flow '74

Quantum field theory & the functional RG in a nutshell

Classical action $S(\phi)$

Simple example: ϕ real scalar field with $S(\phi) = \int d^d x \left\{ \frac{1}{2} \phi \left(-\partial_\mu^2 + m_\phi^2 \right) \phi + \frac{\lambda_\phi}{8} \phi^4 \right\} + \log \mathcal{N}_\phi$



Generating functionals $F[\phi]$

Probability measure

$$F[\phi] = p[\phi]$$

Wilson effective action = log likelihood

$$F[\phi] \simeq \log p[\phi] = S_{\text{eff}}[\phi]$$

1PI effective action = rate function

$$F[\phi] \simeq \mathcal{L}(\log p[\phi]) = \Gamma[\phi]$$

Special case: Polchinski flow '83

$$\dot{\phi} \propto \mathcal{C}_t \frac{\delta S_{\text{eff}}}{\delta \phi}$$

Special case: Wetterich flow '93

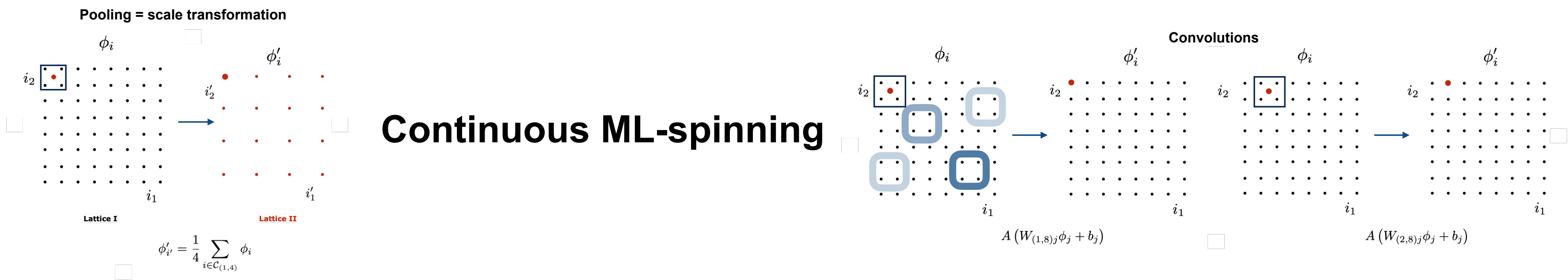
Wegner flow '74

Generalised flow equation '05

Quantum field theory & the functional RG in a nutshell

Classical action $S(\phi)$

Simple example: ϕ real scalar field with $S(\phi) = \int d^d x \left\{ \frac{1}{2} \phi \left(-\partial_\mu^2 + m_\phi^2 \right) \phi + \frac{\lambda_\phi}{8} \phi^4 \right\} + \log \mathcal{N}_\phi$



Generating functionals $F[\phi]$

Probability measure

$$F[\phi] = p[\phi]$$

Wilson effective action = log likelihood

$$F[\phi] \simeq \log p[\phi] = S_{\text{eff}}[\phi]$$

1PI effective action = rate function

$$F[\phi] \simeq \mathcal{L}(\log p[\phi]) = \Gamma[\phi]$$

Special case: Polchinski flow '83

$$\dot{\phi} \propto \mathcal{C}_t \frac{\delta S_{\text{eff}}}{\delta \phi}$$

Legendre transform

Special case: Wetterich flow '93

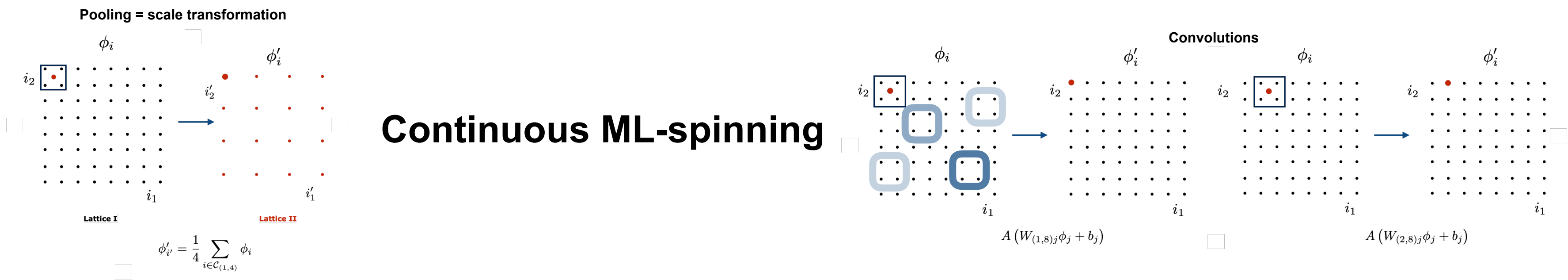
Wegner flow '74

Generalised flow equation '05

Quantum field theory & the functional RG in a nutshell

Classical action $S(\phi)$

Simple example: ϕ real scalar field with $S(\phi) = \int d^d x \left\{ \frac{1}{2} \phi \left(-\partial_\mu^2 + m_\phi^2 \right) \phi + \frac{\lambda_\phi}{8} \phi^4 \right\} + \log \mathcal{N}_\phi$



perfect for sampling

Probability measure

$$F[\phi] = p[\phi]$$

Generating functionals $F[\phi]$

Wilson effective action = log likelihood

$$F[\phi] \simeq \log p[\phi] = S_{\text{eff}}[\phi]$$

Least redundant

perfect for optimisation

1PI effective action = rate function

$$F[\phi] \simeq \mathcal{L}(\log p[\phi]) = \Gamma[\phi]$$

Special case: Polchinski flow '83

Special case: Wetterich flow '93

$$\dot{\phi} \propto \mathcal{C}_t \frac{\delta S_{\text{eff}}}{\delta \phi}$$

Legendre transform

Wegner flow '74

Generalised flow equation '05

$$\partial_t p_t[\phi] = - \int_x \frac{\delta}{\delta \phi(x)} \left(\dot{\phi}[\phi] p_t[\phi] \right) \longleftrightarrow \left(\partial_t + \int_x \dot{\phi} \frac{\delta}{\delta \phi} \right) \Gamma_t[\phi] = \frac{1}{2} \text{Tr } G_t[\phi] \partial_t R_t + \text{Tr } G_t[\phi] \frac{\delta \dot{\phi}}{\delta \phi} R_t$$

Legendre transform

Wegner flow

Wegner, *J. Phys. C* **7** (1974) 2098

Generalised flow equation

JMP, *Annals Phys.* **322** (2007) 2831

$$G_t[\phi](p, q) = \langle \hat{\phi}(p) \hat{\phi}(q) \rangle_c = \frac{1}{\frac{\delta^2 \Gamma_t[\phi]}{\delta \phi^2} + R_t}(p, q)$$

$$\partial_t p_t[\phi] = - \int_x \frac{\delta}{\delta \phi(x)} \left(\dot{\phi}[\phi] p_t[\phi] \right)$$

Wegner flow

Wegner, J. Phys. C 7 (1974) 2098



Legendre transform

$$\left(\partial_t \right)$$

Wetterich equation $\dot{\phi} = 0$

Wetterich, PLB 301 (1993) 90

$$\Gamma_t[\phi] = \frac{1}{2} \text{Tr} G_t[\phi] \partial_t R_t$$

Generalised flow equation

JMP, *Annals Phys.* 322 (2007) 2831

$$G_t[\phi](p, q) = \langle \hat{\phi}(p) \hat{\phi}(q) \rangle_c = \frac{1}{\frac{\delta^2 \Gamma_t[\phi]}{\delta \phi^2} + R_t}(p, q)$$

$$\partial_t p_t[\phi] = - \int_x \frac{\delta}{\delta \phi(x)} \left(\dot{\phi}[\phi] p_t[\phi] \right)$$



$$\left(\partial_t + \int_x \dot{\phi} \frac{\delta}{\delta \phi} \right) \Gamma_t[\phi] = \frac{1}{2} \text{Tr } G_t[\phi] \partial_t R_t + \text{Tr } G_t[\phi] \frac{\delta \dot{\phi}}{\delta \phi} R_t$$

Wegner flow

Wegner, J. Phys. C 7 (1974) 2098

Generalised flow equation

JMP, *Annals Phys.* 322 (2007) 2831

Wegner flow or Wegner-Morris flow

Wegner, J. Phys. C 7 (1974) 2098

Morris, hep-th/9810104 (gauge theories, lecture ERG 1998)

Latorre, Morris, JHEP 11 (2000) 004

⋮

Rosten, Phys.Rept. 511 (2012) 177

Generalised 1PI flows

Gies, Wetterich, PRD 65 (2002) 065001 (bilinear composites)

JMP, *Annals Phys.* 322 (2007) 2831 (generalised flow equation)

⋮

$$\partial_t p_t[\phi] = - \int_x \frac{\delta}{\delta \phi(x)} \left(\dot{\phi}[\phi] p_t[\phi] \right)$$

Wegner flow

Wegner, J. Phys. C 7 (1974) 2098



Legendre transform

$$\left(\partial_t + \int_x \dot{\phi} \frac{\delta}{\delta \phi} \right) \Gamma_t[\phi] = \frac{1}{2} \text{Tr } G_t[\phi] \partial_t R_t + \text{Tr } G_t[\phi] \frac{\delta \dot{\phi}}{\delta \phi} R_t$$

Generalised flow equation

JMP, *Annals Phys.* 322 (2007) 2831

Wegner flow or Wegner-Morris flow

Wegner, J. Phys. C 7 (1974) 2098

Morris, hep-th/9810104 (gauge theories, lecture ERG 1998)

Latorre, Morris, JHEP 11 (2000) 004

⋮

Rosten, Phys.Rept. 511 (2012) 177

Generalised 1PI flows

Gies, Wetterich, PRD 65 (2002) 065001 (bilinear composites)

JMP, *Annals Phys.* 322 (2007) 2831 (generalised flow equation)

⋮

Novel conceptual developments based on general flows

Essential RG: Baldazzi, Falls, *SciPost Phys.* 13 (2022) 085, ... (generalised flow eq. with essential/inessential operators) **fixed theory**

Fit the Third to Fifth: What can Machine Learning do for us?

Yang-yang Tan: *Solving Functional Renormalization Group Equations with Neural Networks* Friday 2:20 PM - 2:40 PM

We employ deep neural networks to represent the field derivative of the scale-dependent effective potential in the functional renormalization group (fRG) framework for nonperturbative quantum field theory. By embedding the fRG flow equations directly into the loss function, the network parameters are determined so as to provide a continuous and differentiable representation of the scale- and field-dependent effective potential without relying on precomputed training data. Focusing on the $O(N)$ scalar field theory within the local potential approximation at finite temperature, we demonstrate that this neural network representation accurately captures the renormalization group flow across symmetric, broken, and critical regimes. A key ingredient is a decomposition of the representation into an analytically known large- N contribution and a learned finite- N correction, which efficiently mitigates numerical stiffness associated with convexity restoration in the broken phase. The physics-driven solutions show excellent agreement with established finite-difference and discontinuous Galerkin methods. We further apply the same strategy to the Wilson-Fisher fixed point equation in three dimensions, illustrating that neural network representations provide a unified framework for both scale-dependent flows and fixed-point problems. Our results indicate that physics-driven deep learning offers a robust and flexible numerical tool for functional renormalization group studies.

Takeru Yokota: *Physics-informed neural networks for solving FRG*

Friday 2026, 2:00 PM - 2:20 PM

Although the formulation of the FRG is exact, practical calculations must be carried out approximately. This is because solvers for functional differential equations (FDEs), such as the Wetterich equation, have not yet been established. Meanwhile, physics-informed neural networks (PINNs) have recently gained attention as an efficient method for solving high-dimensional partial differential equations. Since FDEs are essentially high-dimensional differential equations, PINNs are expected to serve as useful solvers for the FRG. In this talk, I will discuss the application of PINNs to the FRG. In particular, I will present numerical applications to low-dimensional scalar models to demonstrate their applicability. In our approach, the effective action is represented by a neural network, and I will discuss network architectures that help ensure convexity, which is important for describing phase transitions. This talk is based on Refs. [1,2].

Constructing UV-critical Hypersurfaces
using Machine Learning ^[1]

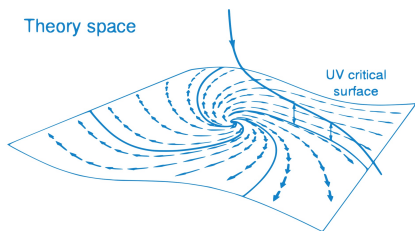
Stijn Hennissen
Radboud University - IMAPP
Nijmegen, The Netherlands

BACKGROUND

UV complete RG trajectories with a fixed point span its UV-critical surface in theory space.

Constructing this surface allows one to find the full effective action at $k=0$.

Mapping this surface is usually done using shooting methods. Algorithmically this is random search. Thus its computational performance can be improved on by modern algorithms.



ALGORITHM

Embed the UV-critical surface in theory space through a generating functions $\{F\}$, that vanish on the UV-critical surface.

The generating functions are flow invariants ^[2]. They are the solutions to a decoupled set of linear PDEs involving the beta-functions.

Introduce the Flow Invariant Neural Network (FINN) to solve these PDEs for the generating function.

Recover UV-critical surface where FINN vanishes.

RESULTS

Applied to a scalar-tensor theory of gravity ^[3] with couplings $\lambda, G, v_2, v_4, f_2, f_4$. The coupling λ decouples and G functions as RG time, so becomes 0 in the IR. We then find v and f_4 as functions of v_2 and f_2 at $k=0$ surfaces. We match existing results to 1% accuracy (comparing to shooting method).

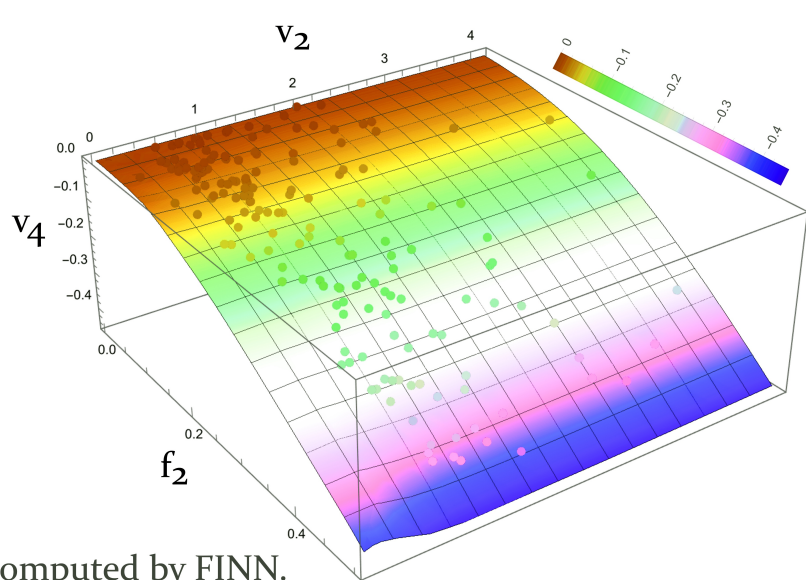
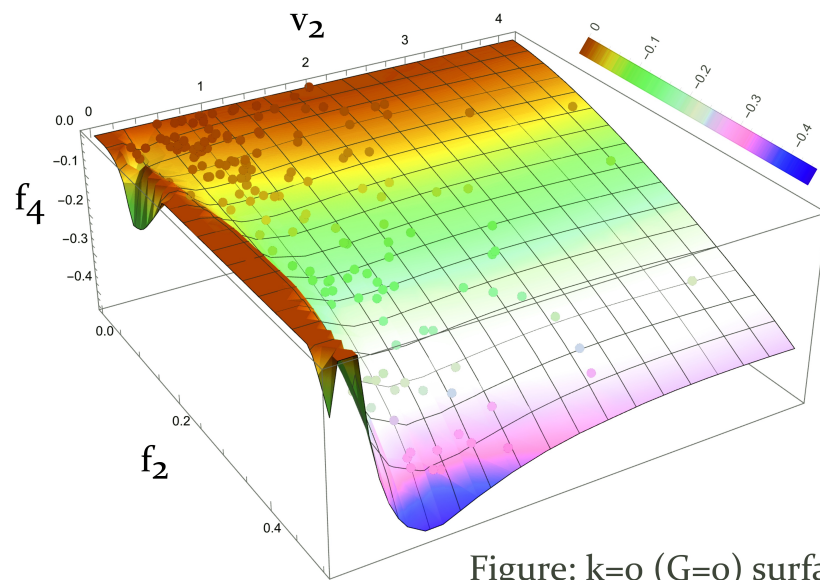


Figure: $k=0$ ($G=0$) surfaces computed by FINN. Dots are shooting method results and match to 1%

CONCLUSIONS

The decoupled nature of the PDEs allows split neural network structure, so less training parameters.

Expanding beta functions as Taylor series allows faster numerical and symbolic operation.

One can narrow down theory space domain by sparse probing using shooting method.

Even without careful analysis of neural network structure the method obtains remarkably accurate results in fast computation times (order of minutes).

OUTLOOK

The problem lends itself to more sophisticated neural architectures, leveraging the renormalization concept in an adaptive/dynamic network sturcture.

Dynamic sampling of the theory space domain should supercharge the general applicability of the method.

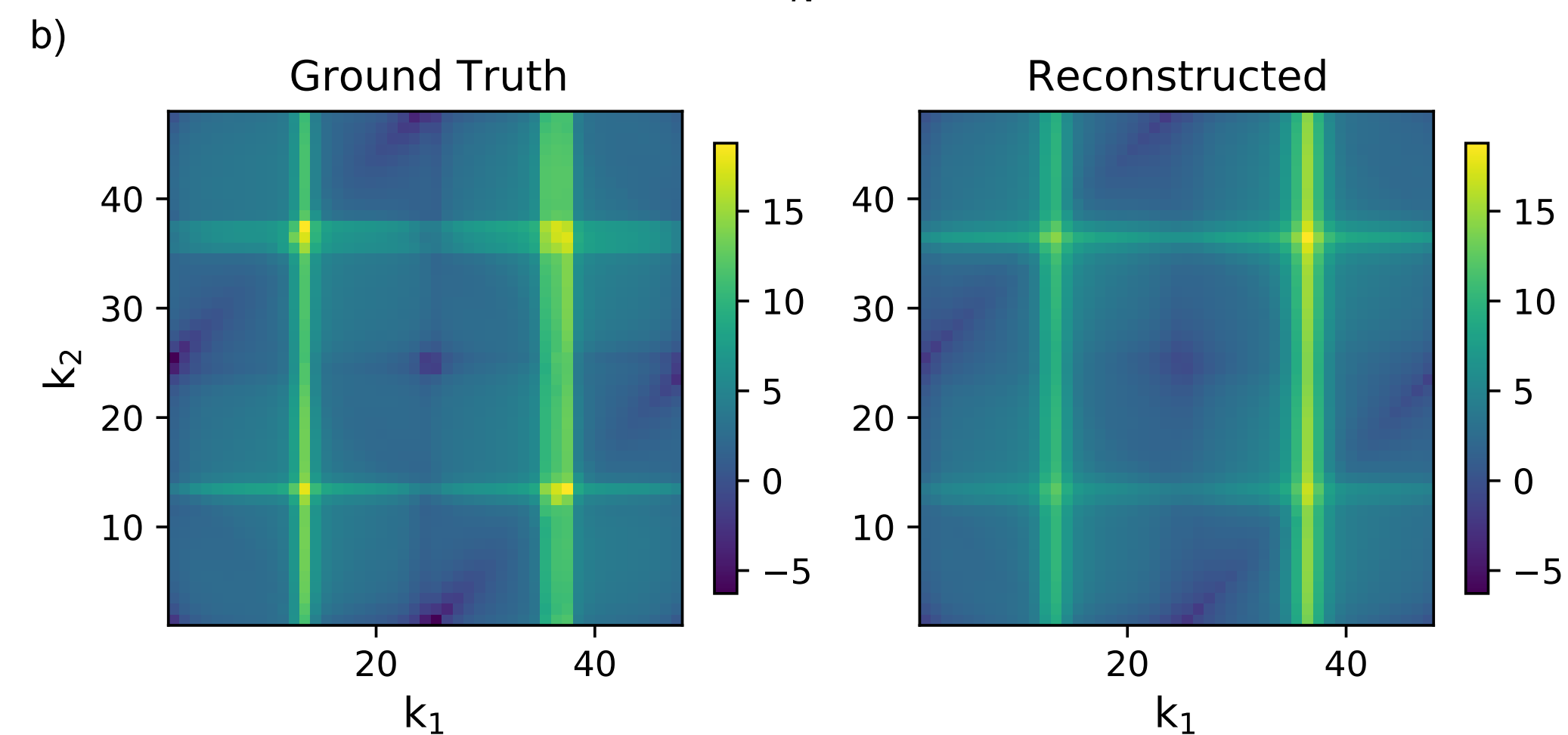
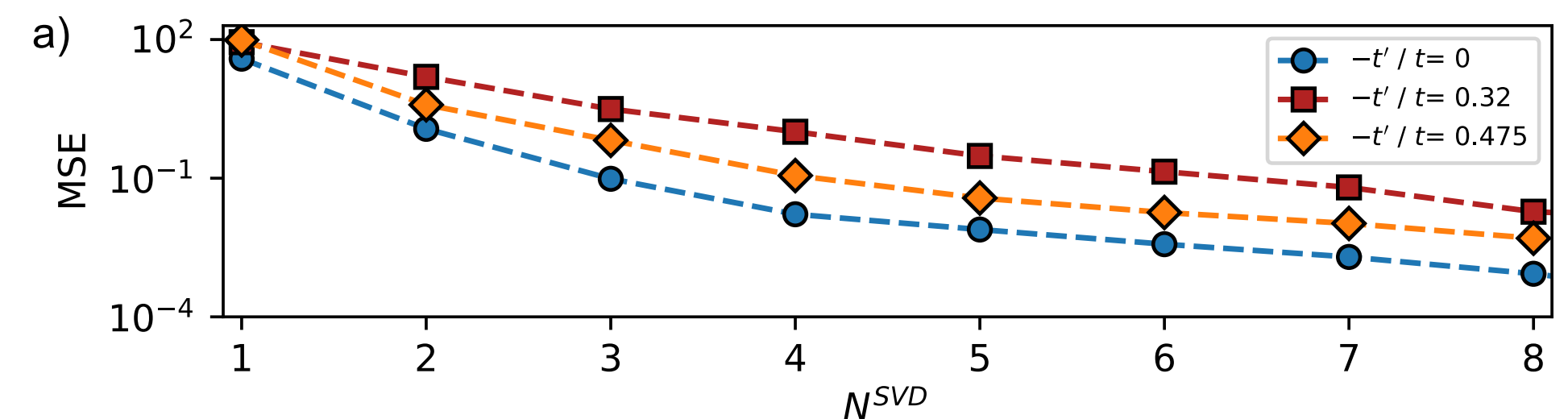
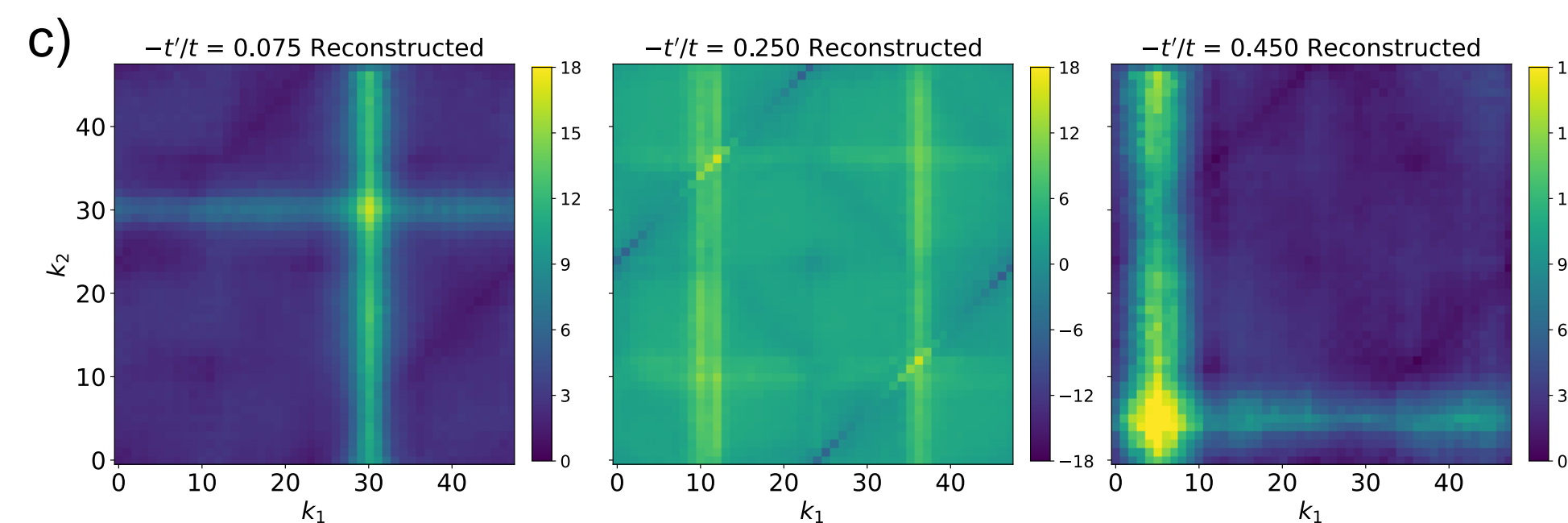
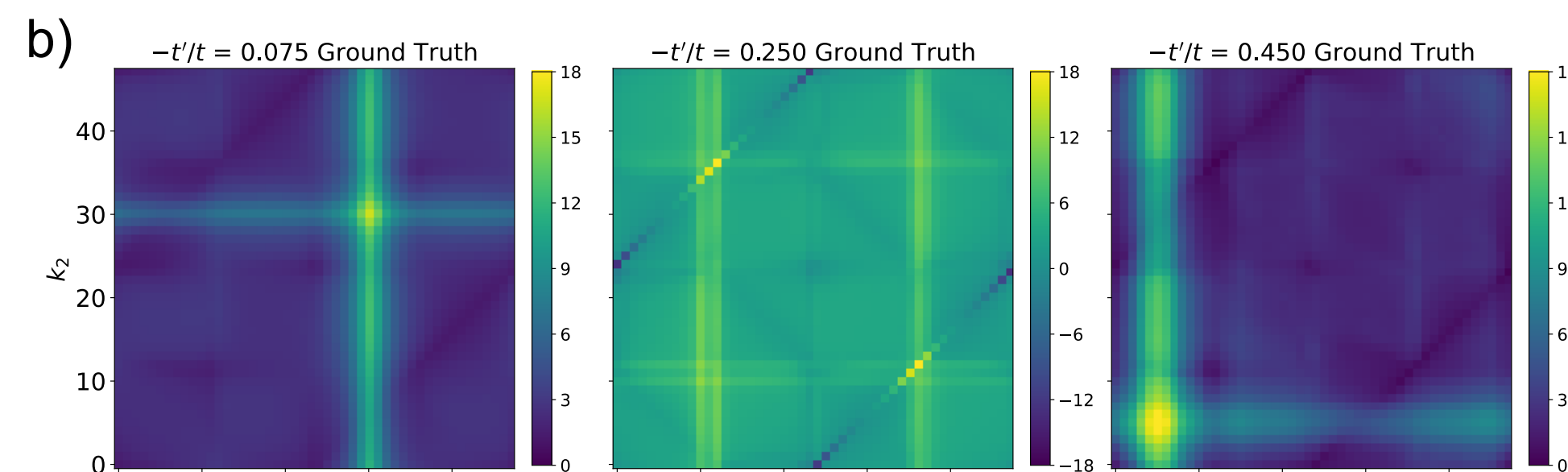
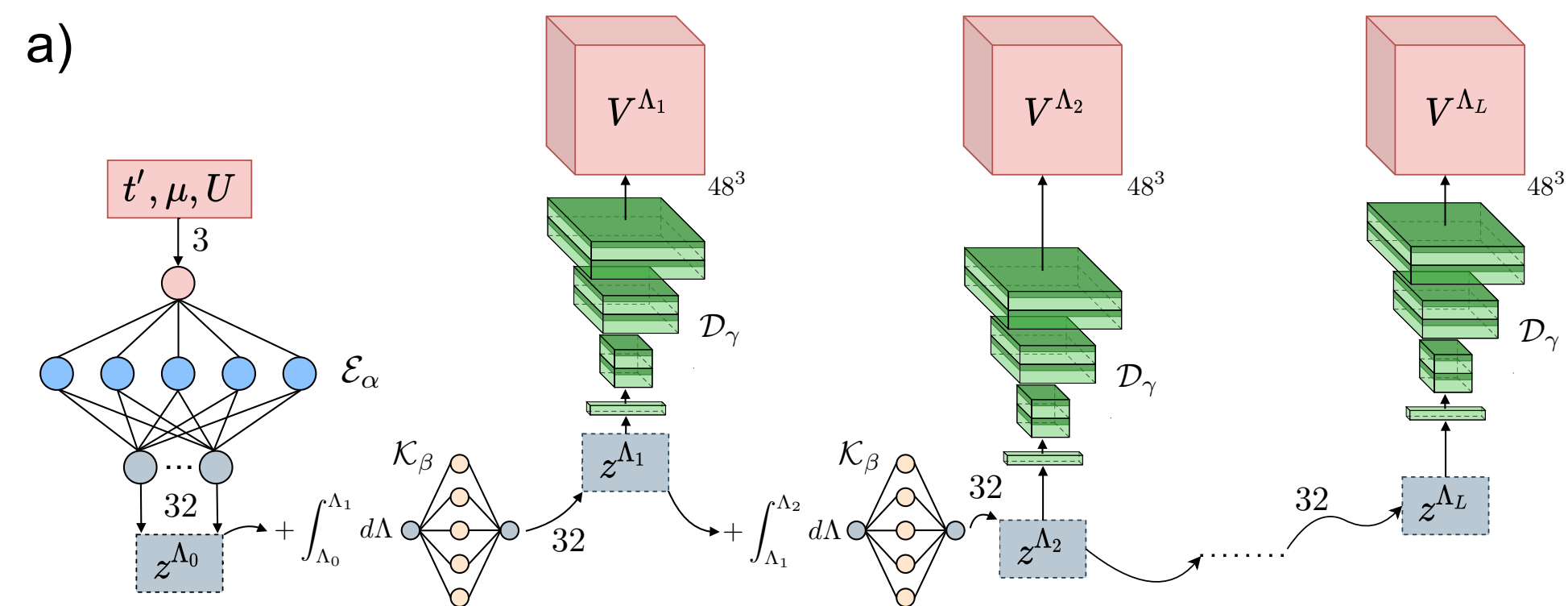
- In terms of physics, one expects this method to shine when applied to models with many couplings:
- Asymptotically safe (Beyond the) Standard Model
 - Asymptotically safe gravity-matter systems
 - Condensed matter applications
 - Tensor/Group field theories

In further generalization, this method could be used to systematically judge potential theories on their physical relevance in terms of renormalizability.

Bibliography:
[1] S. Hennissen, F. Saueressig, A. Silva: in preparation.
[2] F. Saueressig, A. Silva: arXiv:2403.08541
[3] A. Silva: arXiv:2406.10170

Example: Learning importance ordering of vertices

Deep Learning the Functional Renormalization Group



Fit the Fifth to Seventh: The RG-Structure of Machine Learning



Lingxiao Wang: *Diffusion Models as Stochastic Quantisation*

Wednesday 4:40 PM - 5:10 PM

Generative diffusion models — the engine behind much of modern image and video synthesis — turn out to admit a transparent physical reading: the forward process is a Langevin diffusion that washes out structure, while the reverse, score-driven process reconstructs configurations from noise, realising a stochastic quantisation à la Parisi–Wu with the learned score playing the role of the drift derived from an effective action.

Building on this correspondence, I will present our recent results from the DM-QFT collaboration on generative sampling for lattice field theory: scalar theories in two and three dimensions including their critical regions, group-preserving diffusion for $U(N)/SU(N)$ gauge fields, and ongoing progress. I will emphasise Expandability, Exactness and Efficiency, and close with the road ahead QCD.

Yuto Ashida: *Exact RG and generative diffusion model*

Friday 11:00 AM - 11:30 AM

In recent years, the rapid advancement of generative AI, particularly diffusion models, has led to the adoption of new data-driven methods in widely diverse fields—from high-quality image generation to drug discovery and material design. In this talk, I will discuss their theoretical similarities to the theory of the exact renormalization group widely used in statistical and high-energy physics. Specifically, I will introduce an approach based on RG concepts that aim to capture the hierarchical structure of data more efficiently.

Riccardo Finotello: *A fRG Approach to Anomaly Detection via Dimensional Phase Transition* Friday 3:20 PM - 3:40 PM

Detecting signals within nearly continuous spectra remains a fundamental challenge in statistical inference, particularly in high-noise regimes where standard random matrix theory approaches, such as the Baik-Ben Arous-Péché (BBP) transition, frequently fail. We introduce a methodology that bridges statistical mechanics and statistical inference. By treating the empirical spectrum as an effective equilibrium field theory, we establish a formal mapping between anomaly detection and the Functional Renormalisation Group (FRG) flow of non-equilibrium systems using a stochastic field framework, wherein the noise-to-signal ratio functions mathematically as a physical temperature. Instead of treating signals as isolated statistical outliers, the FRG approach models them as ordered domains emerging within a thermalised background of fluctuations. This dynamic induces a structural deformation of the spectral geometry. By using the scale-dependent canonical dimension of this geometry as a highly sensitive order parameter, we demonstrate how the system undergoes a sharp dimensional phase transition. Expanding upon fundamental formulations, our results show that this transition directly correlates with stochastic ergodicity breaking, a spontaneous symmetry breaking in the effective potential, and a measurable deviation of eigenvector statistics from the universal Porter-Thomas distribution. Consequently, this enables signal resolution at signal-to-noise thresholds significantly below standard limits, even when the underlying data distribution closely follows the Marchenko-Pastur law. Furthermore, we validate the framework on realistic datasets, including those typical in computer vision, and critical phenomena, such as the determination of the phase transition temperature in physical systems. The FRG thus provides a rigorous, physics-grounded technique for extracting extensive-rank signals in complex datasets.

The resolution of strongly correlated systems is a computationally challenging task, which can be simplified enormously by choosing a formulation in terms of the appropriate degrees of freedom. Here, we distinguish between transformations of the fundamental degrees of freedom in the path integral, and transformations of mean fields.

Both are incorporated naturally in the renormalisation group approach and appear as dynamic, infinitesimal changes of the field representation with changing scales. A historically very prominent use case is dynamic rebosonisation, which implements the Hubbard-Stratonovich transformation throughout the RG flow, but recent investigations are studying a wider field of applications. In my talk I will discuss the point of view of the physics-informed RG (PIRG) [1,2], which considers both the flow of a generating functional and the flow of the field on equal footing. This pair must fulfill a generalised flow equation and an additional constraint equation that can be chosen to suit a specific purpose. Firstly, I we consider the PIRG for microscopic field-transformations [3]. Here, they bridge the gap to generative sampling architectures in lattice field theory. Specifically, the optimal transport nature of the RG allows to address long standing problems such as the sign-problem in real-time lattice simulations [4]. Secondly, I consider generalised flows of the effective action. Here the PIRG perspective allows to formulate novel optimisation procedures for expansion schemes [5,6], or the restoration of gauge symmetries which are explicitly broken by the RG flow [7,8].

Generative PIRGs

encode Normalising Flows

Diffusion Models

Stochastic Quantisation (real and complex Langevin)

Lefschetz thimbles ('you may seek it with thimbles—and seek it with care')

■
■
■

See also talk Massimiliano Gubinelli,

Wednesday: 4:10 PM - 4:40 PM

Kanta Masuki, Yuto Ashida



Kanta Masuki and Yuto Ashida
The University of Tokyo

arXiv:2501.09064
arXiv:2608.23696



Generative Modeling with Inverse Renormalization Group Flows

1. Diffusion model and flow matching

- Forward diffusion: convert data to noise

$$\begin{aligned} & \text{Langvin diffusion} \\ & \frac{d\phi_t}{dt} = -f(t)\phi_t + g(t)dw \\ & \partial_t p_t = \sum_i \frac{1}{2} \frac{\partial^2 (g_i^2 p_t)}{\partial \phi_i^2} + \sum_i \frac{\delta (f_i p_t)}{\delta \phi_i} \end{aligned}$$

- Generative flow: reverse the probability path with SDE/ODE

- Flow depends on p_{data} through the score $s_t(\phi) = \partial \ln p_t / \partial \phi$
- With the learned score s_t , generative flow can create data from $\phi_1 \sim p_{GS}$

$$\begin{aligned} & \text{SDE: } d\phi_t = (f_t \phi_t + g_t^2 s_t(\phi_t)) dt + g_t dw \\ & \text{ODE: } d\phi_t = \left(f_t \phi_t + \frac{g_t^2}{2} s_t(\phi_t) \right) dt \end{aligned}$$

Challenges in generative modeling

- Noise schedule is essential for quality and efficiency but is determined heuristically.
- Iterative evaluation of NN demands a huge computational cost: $O(D^2)$ for $\phi \in \mathbb{R}^D$.

3. RG diffusion model (RGDM), RG flow matching (RGFM)

- Probability path based on the exact RG

Polchinski RG flow with cutoff K_Λ

$$S_\Lambda^{\text{RG}}(\phi) = \frac{1}{2} \int_k K_\Lambda(k) k^2 |\phi_k|^2 + U_\Lambda(\phi)$$

$$\partial_\Lambda U_\Lambda = -\frac{1}{2} \int_k k^{-2} \partial_\Lambda K_\Lambda \left(\frac{\delta^2 U_\Lambda}{\delta \phi_k \delta \phi_{-k}} - \frac{\delta U_\Lambda}{\delta \phi_k} \frac{\delta U_\Lambda}{\delta \phi_{-k}} \right)$$

RG flow for probability distribution

$$\text{rescaling: } S_\Lambda(\phi) = S_{K_\Lambda}^{\text{RG}}(\sqrt{K_\Lambda} \phi) = \frac{1}{2} \int (\nabla \phi_x)^2 + U_\Lambda(\sqrt{K_\Lambda} \phi) \quad \text{keep GS part fixed}$$

flow equation for $p_t(\phi) \propto e^{-S_t(\phi)}$, $\Lambda(t) = \Lambda_{UV} e^{-t/\tau}$:

$$\partial_t p_t = -\frac{1}{2} \int_k \left[k^{-2} \frac{\partial_\Lambda K_\Lambda}{K_\Lambda} \frac{\delta^2 p_t}{\delta \phi_k \delta \phi_{-k}} + \frac{\delta}{\delta \phi_k} \left(\frac{\partial_\Lambda K_\Lambda}{K_\Lambda} \phi_k p_t \right) \right] \quad \text{diffusion eq.}$$

RG diffusion: multiscale diffusion

$$p_t(\phi) = \int d\phi_0 p_t(\phi|\phi_0) p_{\text{data}}(\phi_0)$$

$$p_t(\phi|\phi_0) = \prod_k \mathcal{N}(\phi_k; \sqrt{\alpha_{tk}} \phi_{0k}, \tilde{\beta}_{tk})$$

$$\alpha_{tk} = K_\Lambda(k^2), \tilde{\beta}_{tk} = k^{-2}(1 - K_\Lambda(k^2))$$

Main Result 1

- Based on the exact RG flow, we construct RGDM/RGFM, which create data from coarse to fine scales.
- Once the RG cutoff K_Λ is fixed, all the noise schedules are naturally determined by the RG flow.

5. Scalable local generative modeling with RGFM

- Local-patch generative modeling

Global vs local velocity field

$$\frac{\partial_\Lambda \phi_{\text{data}}}{\partial \Lambda} = v(\phi, \Lambda)$$

Patchwise training with patch size l

$$\frac{\partial_\Lambda \phi_{\text{data}}}{\partial \Lambda} = v^l(\phi, \Lambda)$$

- Approximate velocity field in FM by a local velocity field
- Scalable computational cost for applying NN globally: $O((L/l)^d) \times O(l^{2d}) = O(L^d l^d)$

- Locality during the RG diffusion

Site-decimation rescaling

$$p_t(\phi) = p_{t/2}(\phi/2) p_{\text{data}}(\phi/2)$$

Locality during the probability flow

$$p_t(\phi) = p_{t/2}(\phi/2)$$

Main Result 2

- We theoretically prove that, for data with good locality, p_t is quasi-local with length scale $\propto t^{-1}$.
- Site-decimation of the RG retains the length scale to $O(l^d)$ along the flow.
- ODE can be accurately approximated with local patches of linear size $l = O(\ln L)$.

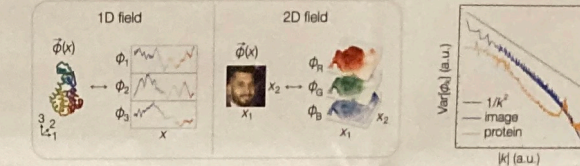
2. Multiscale data generation

- Multiscale structure of natural data

- Short- to long- distance length scale fluctuations
- Empirical scaling law $\text{Var}[\phi_k] \sim k^{-2}$ reminiscent of a field theory
- Data ansatz with locality/conditional-locality.

$$p_{\text{data}}(\phi) \propto e^{-S_{\text{loc}}(\phi)}, S_{\text{loc}}(\phi) = \frac{1}{2} \int d^d x (\nabla \phi_x)^2 + U_{\text{loc}}(\phi), \quad \text{local}$$

$$p_{\text{data}}(\phi) \propto \int dz p(z) p_{\text{loc}}(\phi|z), \ln p_{\text{loc}}(\phi|z) = \frac{1}{2} \int d^d x (\nabla \phi_x)^2 + U_{\text{loc},z}(\phi) \quad \text{conditionally local}$$



- Renormalization Group (RG)-based generative modeling

- Create data from coarse to fine (macro to micro) structures
- Forward diffusion is implemented by the RG
- Generative flow is the inverse RG



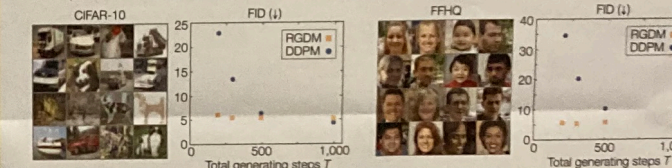
4. Multiscale data generation with RGDM

generation $\phi_{t-1k} = \frac{1}{\sqrt{\alpha_{tk}}} \left(\phi_{tk} - \frac{\beta_{tk}}{\alpha_{tk}} \xi(\phi_t, t) \right) + \sqrt{\beta_{tk}} \epsilon_k$

$$\text{cost function } L_\theta = \sum_t \lambda_t \mathbb{E}_{\phi_t \sim p_{\text{data}}} \mathbb{E}_{\epsilon_t \sim \mathcal{N}(0, I)} \left[\left\| \xi(\phi_t, t) - \sqrt{\beta_{tk}} \epsilon \right\|^2 \right] \quad \text{NN learns colored noise}$$

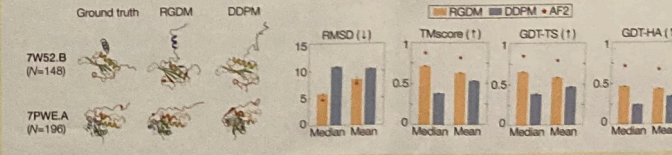
Application to image generation

- Image generation at resolution 32x32 (CIFAR10) and 64x64 (FFHQ)
- Comparison with the conventional white-noise diffusion model (DDPM)



Application to protein structure prediction

- Generative flow is conditioned by amino-acid sequence



RG-based noise schedule enables efficient/high-quality data generation.

Numerical experiments

Local model: 1D Ising



Conditionally local model

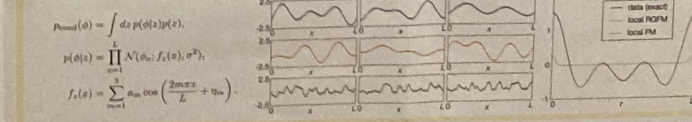
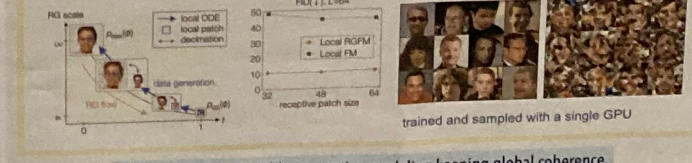


Image generation



RGFM enables scalable generative modeling keeping global coherence.

Conclusion

RG-based generative modeling enables multiscale data generation with scalable computational cost!

Generative architectures & their problems



'... and now for something
completely different...'

OOD problem 1: *Curse of limited information* in training data (extrapolation problem)

OOD problem 2: *Curse of dimensionality*

General set-up and some ideas/problems

Data sets

$$\{\varphi_i\}$$

Experimental data

Sampling data

General data sets

⋮

Probability distributions

$$p(\varphi)$$

Known distributions ← in physics

Unknown distributions

action



$$e^{-S[\varphi]}$$



log likelihood

General set-up and some ideas/problems

Data sets

$$\{\varphi_i\}$$

Experimental data

Sampling data

General data sets

⋮

Probability distributions

$$p(\varphi)$$

Known distributions ← in physics

Unknown distributions

action
↓

$$e^{-S[\varphi]}$$

↑

log likelihood

Tasks

Known distributions: How to sample efficiently from the distribution?

Unknown distributions: How to learn the distribution from the data set?

and all in between

General set-up and some ideas/problems

Data sets

$$\{\varphi_i\}$$

Probability distributions

$$p(\varphi)$$

What can possibly go wrong?

Out Of Domain problems!

General set-up and some ideas/problems

Data sets

$$\{\varphi_i\}$$

Probability distributions

$$p(\varphi)$$

What can possibly go wrong?

Out Of Domain problems!

OOD problem 1: *Curse of limited information* in training data (extrapolation problem)

$$|\{\varphi_i\}| \longrightarrow |\{\mathcal{C}_n\}| \lesssim N(|\{\varphi_i\}|)$$

$$\mathcal{C}_n = \langle \varphi_{i_1} \cdots \varphi_{i_n} \rangle_{\text{irr}}$$

Irreducible part of nth moment

General set-up and some ideas/problems

Data sets

$$\{\varphi_i\}$$

Probability distributions

$$p(\varphi)$$

What can possibly go wrong?

Out Of Domain problems!

OOD problem 1: *Curse of limited information* in training data (extrapolation problem)

$$|\{\varphi_i\}| \longrightarrow |\{\mathcal{C}_n\}| \lesssim N(|\{\varphi_i\}|)$$

$$\mathcal{C}_n = \langle \varphi_{i_1} \cdots \varphi_{i_n} \rangle_{\text{irr}}$$

Irreducible part of nth moment

$$p(\varphi) \neq p_{\mathcal{C}_n}(\varphi)$$

Architecture *has to* encode structural information about *all* $\mathcal{C}_n$

General set-up and some ideas/problems

Data sets

$$\{\varphi_i\}$$

Probability distributions

$$p(\varphi)$$

What can possibly go wrong?

Out Of Domain problems!

OOD problem 1: *Curse of limited information* in training data (extrapolation problem)

Why does it not always go wrong?

aka why are generative networks *without structural information* successful!

General set-up and some ideas/problems

Data sets

$$\{\varphi_i\}$$

Probability distributions

$$p(\varphi)$$

What can possibly go wrong?

Out Of Domain problems!

OOD problem 1: *Curse of limited information* in training data (extrapolation problem)

Why does it not always go wrong?

Probability distribution approximately ‘finite’

$$p(\varphi) \approx p_{C_n}(\varphi)$$

General set-up and some ideas/problems

Data sets

$$\{\varphi_i\}$$

Probability distributions

$$p(\varphi)$$

What can possibly go wrong?

Out Of Domain problems!

OOD problem 1: *Curse of limited information* in training data (extrapolation problem)

Why does it not always go wrong?

Probability distribution approximately ‘finite’

No use of higher $\mathcal{C}_n$

$$p(\varphi) \approx p_{\mathcal{C}_n}(\varphi)$$

General set-up and some ideas/problems

Data sets

$$\{\varphi_i\}$$

Probability distributions

$$p(\varphi)$$

What can possibly go wrong?

Out Of Domain problems!

OOD problem 1: *Curse of limited information* in training data (extrapolation problem)

Why does it not always go wrong?

Probability distribution approximately ‘finite’

$$p(\varphi) \approx p_{\mathcal{C}_n}(\varphi)$$

No use of higher $\mathcal{C}_n$

$$p(\varphi) \approx p_{\mathcal{C}_n}(\varphi)$$

General set-up and some ideas/problems

Data sets

$$\{\varphi_i\}$$

Probability distributions

$$p(\varphi)$$

What can possibly go wrong?

Out Of Domain problems!

OOD problem 1: *Curse of limited information* in training data (extrapolation problem)

Why does it not always go wrong?

Probability distribution approximately ‘finite’

$$p(\varphi) \approx p_{\mathcal{C}_n}(\varphi)$$

No use of higher $\mathcal{C}_n$

$$p(\varphi) \approx p_{\mathcal{C}_n}(\varphi)$$

Afterburner

Final sampling step with $p(\varphi)$

Typically done in lattice architectures (and MC codes for LHC)

critical slowing down

General set-up and some ideas/problems

Data sets

$$\{\varphi_i\}$$

Probability distributions

$$p(\varphi)$$

What can possibly go wrong?

Out Of Domain problems!

OOD problem 1: *Curse of limited information* in training data (extrapolation problem)

Why does it not always go wrong?

Probability distribution approximately ‘finite’

$$p(\varphi) \approx p_{\mathcal{C}_n}(\varphi)$$

No use of higher $\mathcal{C}_n$

$$p(\varphi) \approx p_{\mathcal{C}_n}(\varphi)$$

Afterburner

...

Mircacles

Final sampling step with $p(\varphi)$

Typically done in lattice architectures (and MC codes for LHC)

critical slowing down

General set-up and some ideas/problems

Data sets

$$\{\varphi_i\}$$

Probability distributions

$$p(\varphi)$$

What can possibly go wrong?

Out Of Domain problems!

OOD problem 2: *Curse of dimensionality*

Generative sampling in large dimensional data spaces requires interpolations of high dimensional functions

interpolations in high dimensions are in the same problem class as extrapolations

General set-up and some ideas/problems

Data sets

$$\{\varphi_i\}$$

Probability distributions

$$p(\varphi)$$

What can possibly go wrong?

Out Of Domain problems!

OOD problem 2: *Curse of dimensionality*

Generative sampling in large dimensional data spaces requires interpolations of high dimensional functions

interpolations in high dimensions are in the same problem class as extrapolations

Architecture *has to* encode structural information about the interpolations

General set-up and some ideas/problems

Data sets

$$\{\varphi_i\}$$

Probability distributions

$$p(\varphi)$$

Architectures (Arc) with analytic access to local and global structures

Analytic access to reparametrisations

$$p(\varphi) \rightarrow \tilde{p}(\phi(\varphi))$$

with

$$\langle \varphi_{i_1} \cdots \varphi_{i_n} \rangle_p = \langle \varphi_{i_1} \cdots \varphi_{i_n} \rangle_{\tilde{p}}$$

Analytic access to extrapolations

$$\mathcal{C}_{N+m} = \mathcal{C}_{N+m}(\{\mathcal{C}_{n < N}\}, \text{Arc})$$

Scale transformations

Symmetries

⋮

General set-up and some ideas/problems

Data sets

$$\{\varphi_i\}$$

Probability distributions

$$p(\varphi)$$

Architectures (Arc) with analytic access to local and global structures

Analytic access to reparametrisations

$$p(\varphi) \rightarrow \tilde{p}(\phi(\varphi))$$

with

$$\langle \varphi_{i_1} \cdots \varphi_{i_n} \rangle_p = \langle \varphi_{i_1} \cdots \varphi_{i_n} \rangle_{\tilde{p}}$$

Analytic access to extrapolations

$$\mathcal{C}_{N+m} = \mathcal{C}_{N+m}(\{\mathcal{C}_{n < N}\}, \text{Arc})$$

Scale transformations

Symmetries

⋮

Renormalisation Group

General set-up and some ideas/problems

Data sets

$$\{\varphi_i\}$$

Probability distributions

$$p(\varphi)$$

Architectures (Arc) with analytic access to local and global structures

Analytic access to reparametrisations

$$p(\varphi) \rightarrow \tilde{p}(\phi(\varphi))$$

with

$$\langle \varphi_{i_1} \cdots \varphi_{i_n} \rangle_p = \langle \varphi_{i_1} \cdots \varphi_{i_n} \rangle_{\tilde{p}}$$

Analytic access to extrapolations

$$\mathcal{C}_{N+m} = \mathcal{C}_{N+m}(\{\mathcal{C}_{n < N}\}, \text{Arc})$$

Scale transformations

Symmetries

⋮

Renormalisation Group

Generative RGitectures

Example: Renormalization group flow as optimal transport

Cotler, Rezchikov, PRD 108 (2023) 2, 025003

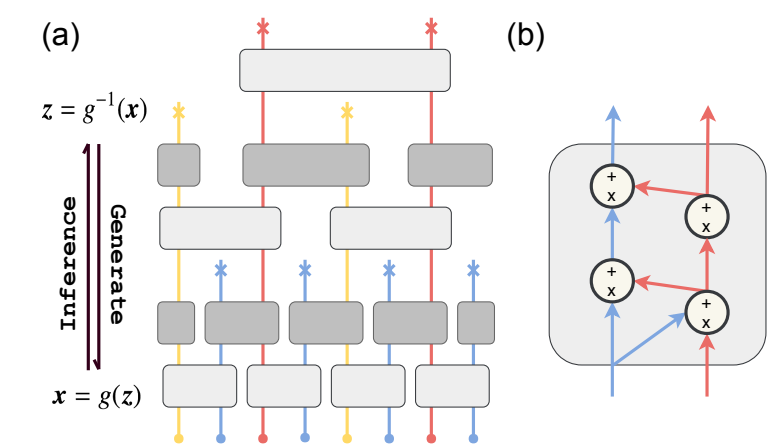
$$\partial_t p_t(\phi) = - \int_x \frac{\delta}{\delta \phi(x)} \left(\dot{\phi}(\phi) p_t(\phi) \right)$$

Wegner, J. Phys. C 7 (1974) 2098

See also talk Massimiliano Gubinelli,
Wednesday: 4:10 PM - 4:40 PM

Example: the inverse renormalisation group

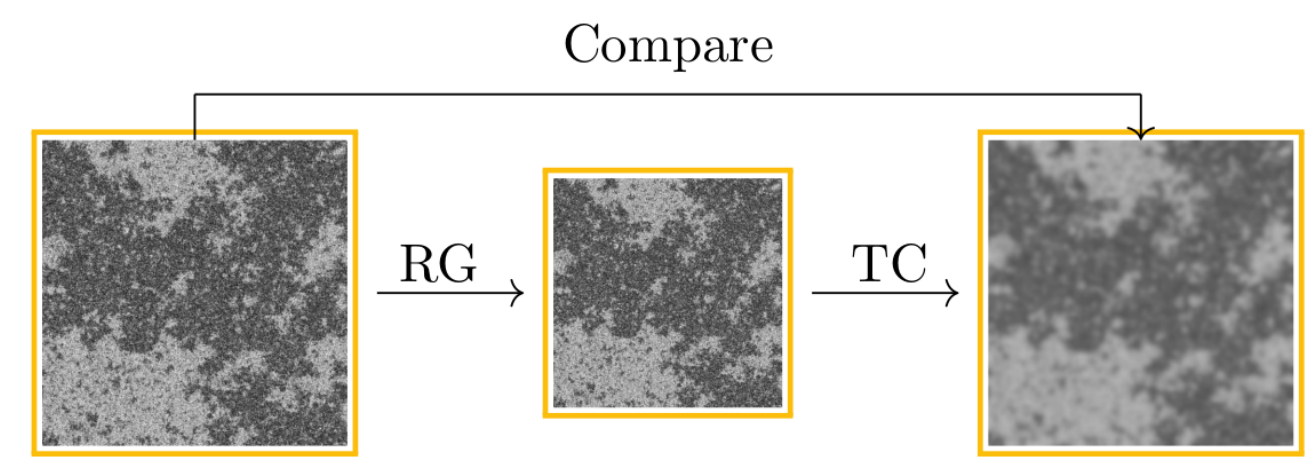
Neural Network Renormalization Group



Li, Wang, PRL 121 (2018) 26, 260601

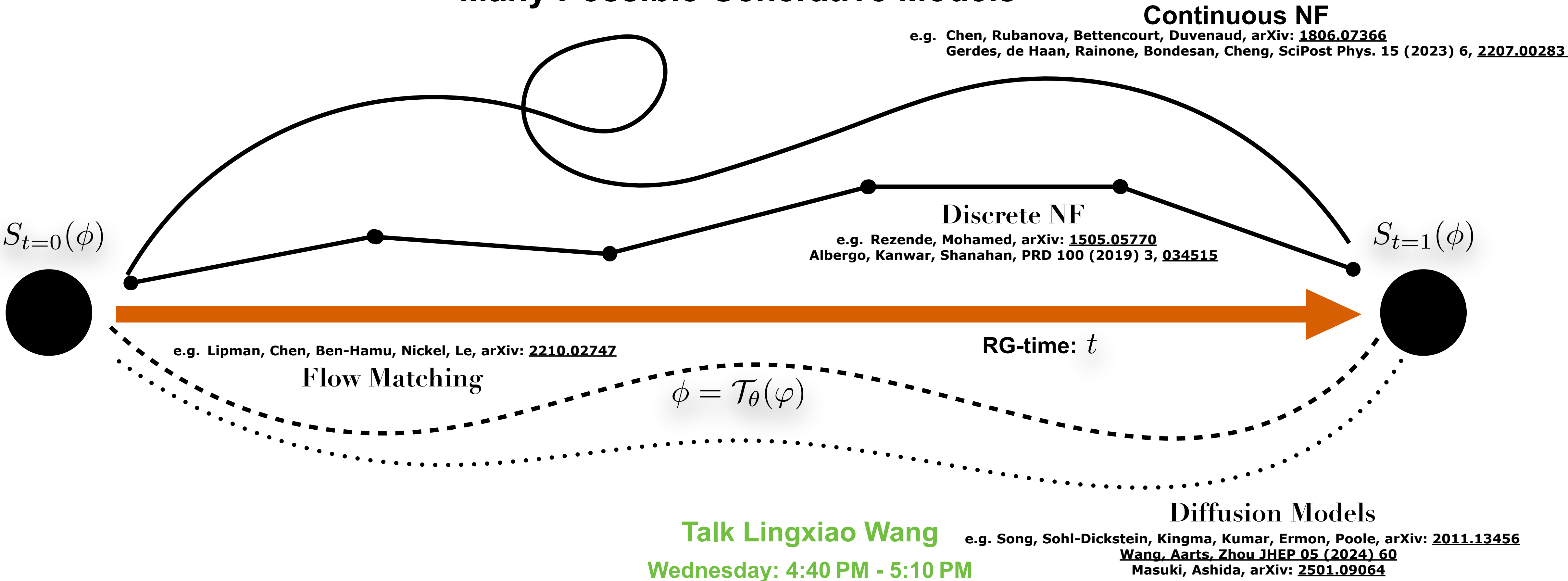
Inverse Renormalization Group in Quantum Field Theory

Bachtis, Aarts, Di Renzo, Lucini, PRL 128 (2022) 8, 081603



Sampling with RGitectures in lattice field theories

Many Possible Generative Models

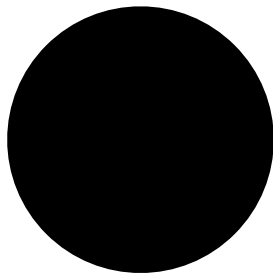


Talk Lingxiao Wang
Wednesday: 4:40 PM - 5:10 PM

Learn the transformation between the two distributions from data

Sampling with RGitectures in lattice field theories

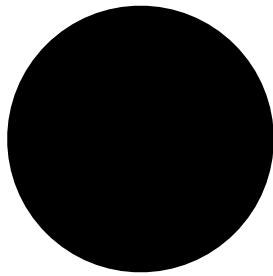
Continuum



$$S[\phi] = \int d^d x \left[\frac{1}{2} \phi(x) \left(-\partial_\mu^2 + M^2 \right) \phi(x) + \frac{\lambda}{4} \phi^4(x) \right]$$

Observables $\langle \mathcal{O} \rangle = \int D\phi p(\phi) \mathcal{O}(\phi)$ $D\phi = \prod_{x \in \Lambda} d\phi_x$

Jump forward to: Generative RGitectures with PIKs in lattice field theories

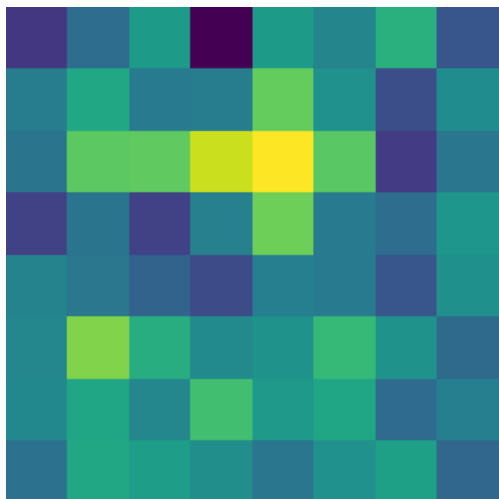
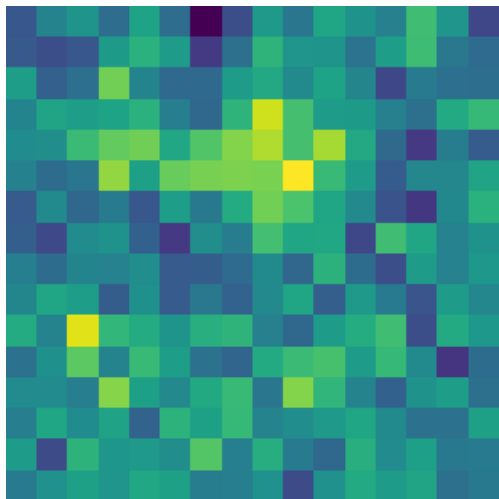
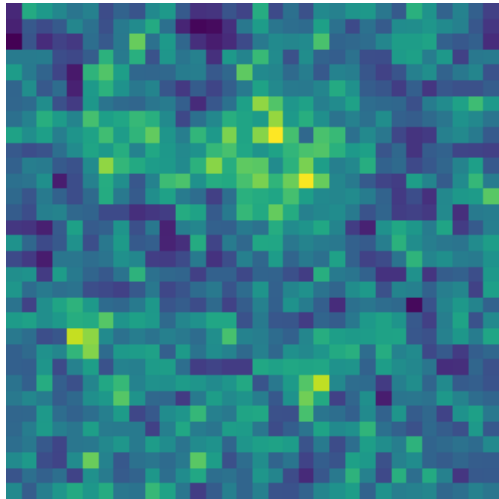
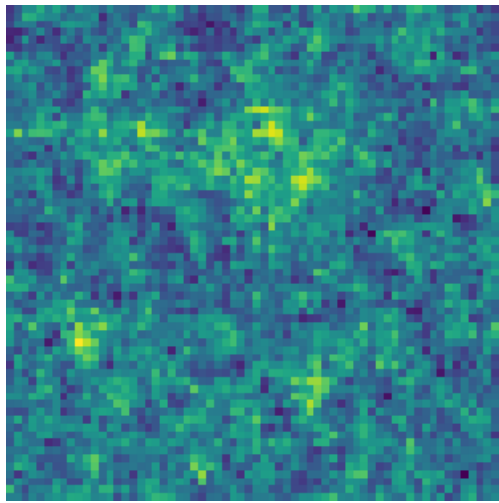


Lattice

$$S(\phi) = \sum_{x \in \Lambda} \left[(1 - 2\lambda') \phi_x^2 + \lambda' \phi_x^4 - 2\kappa \sum_{\mu} \phi_x \phi_{x+\mu} \right] + \log \mathcal{N}$$

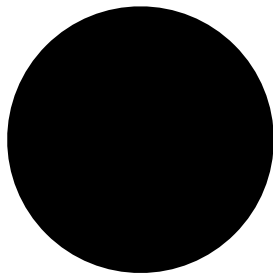
Coarse Graining — RG steps

Fine Graining — Inverse RG steps



Sampling with RGitectures in lattice field theories

Continuum

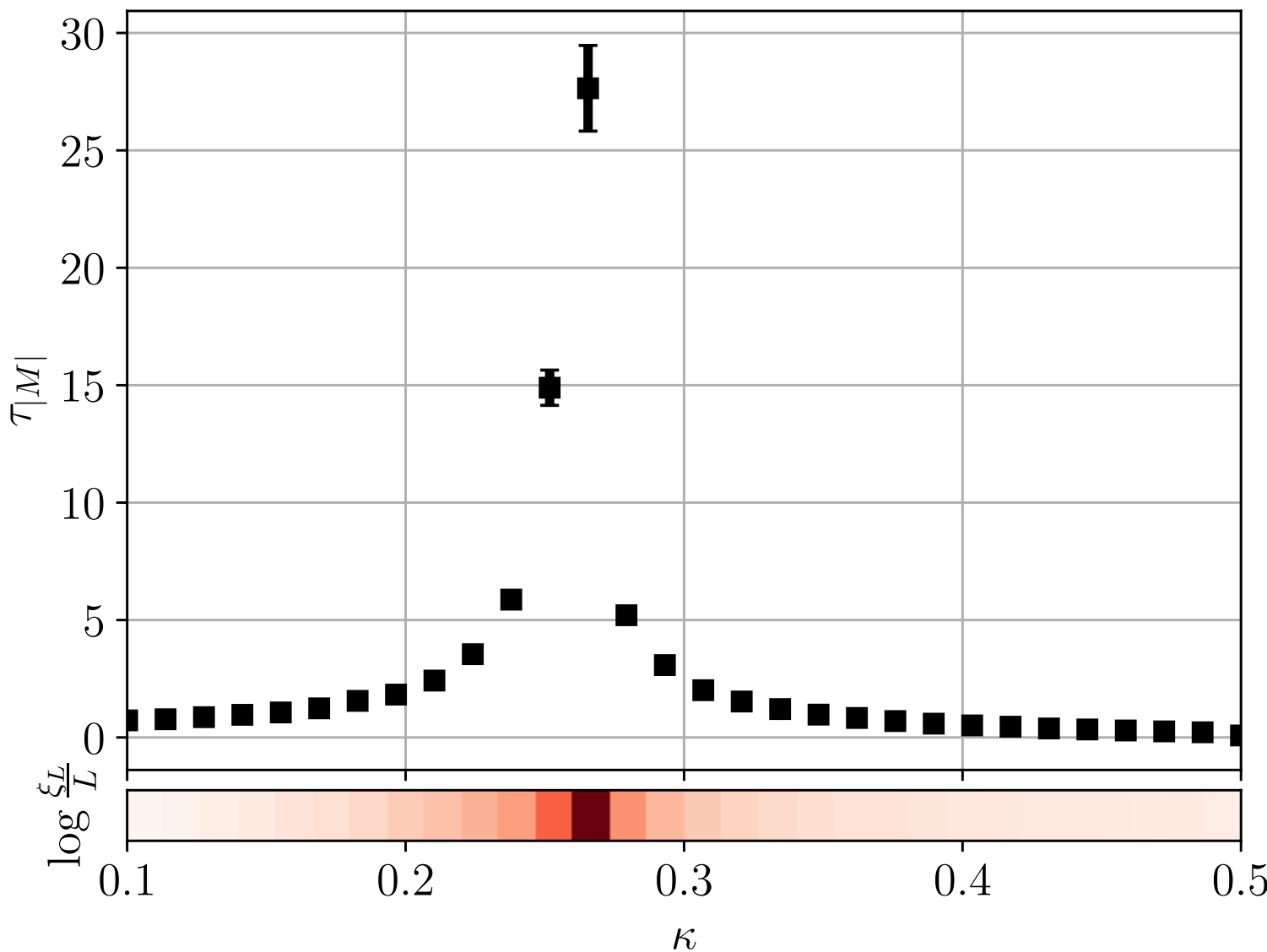


$S[\phi]$

Observables $\langle \mathcal{O} \rangle = \int D\phi p(\phi) \mathcal{O}(\phi)$

$\phi^{(0)} \longrightarrow \phi^{(1)} \longrightarrow \dots \longrightarrow \phi^{(N-1)} \longrightarrow \phi^{(N)}$

Monte Carlo Methods



Correlation length

$\xi = \xi_L a_L$

Autocorrelation time

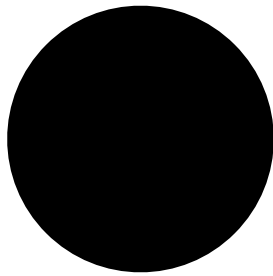
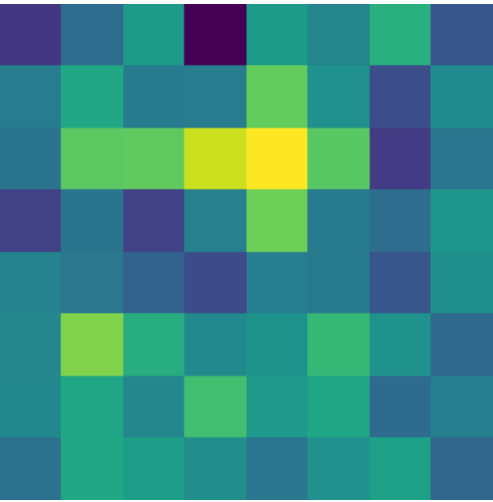
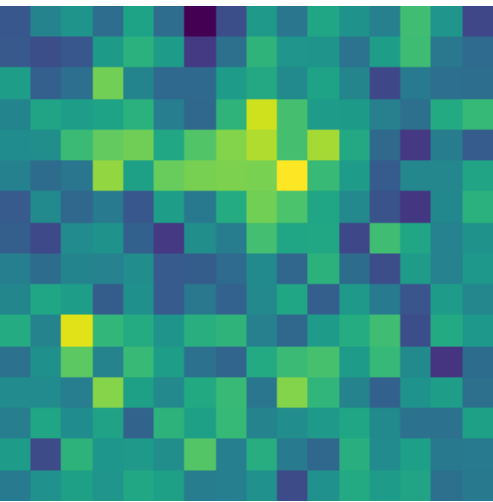
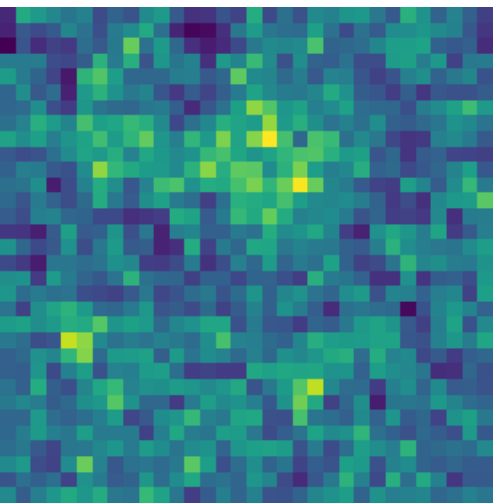
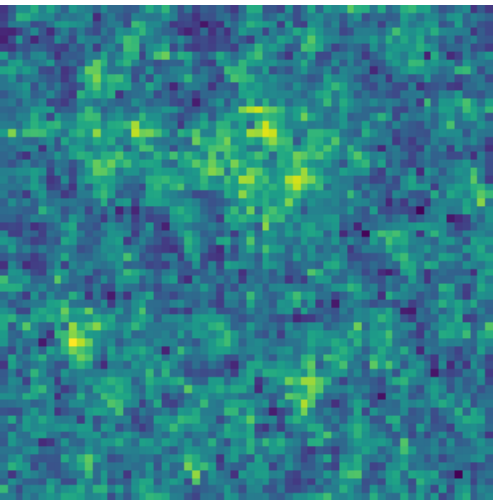
$\tau_{\mathcal{O}}$

Critical slowing down

$\tau_{\mathcal{O}} \sim \xi_L^{z_{\mathcal{O}}}$
Wolff Nucl.Phys.B. 17 (1990)
similar for topological freezing

Coarse Graining — RG steps

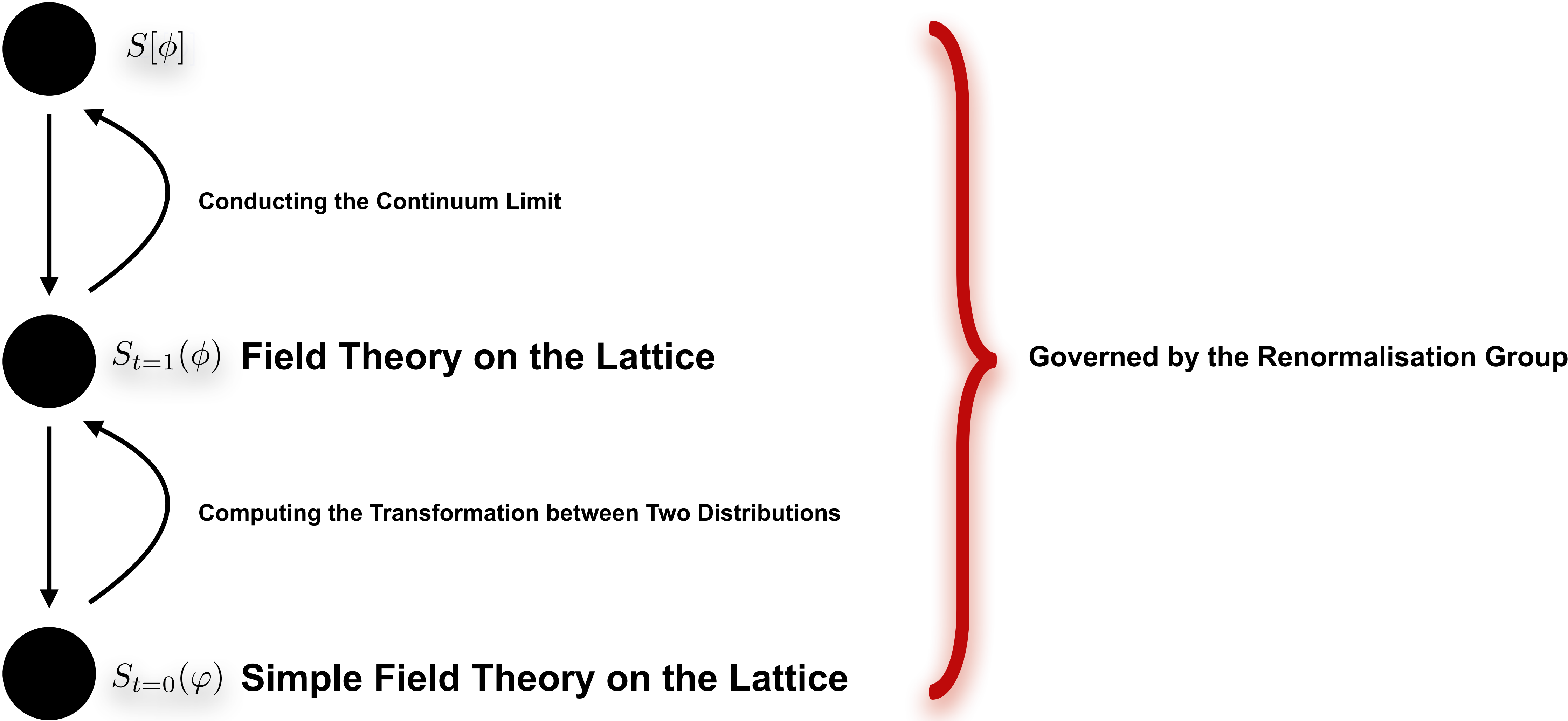
Fine Graining — Inverse RG steps



$S(\phi)$

Lattice

Sampling with RGitectures in lattice field theories (no PIKs)



Sampling with RGitectures in lattice field theories

Normalising Flow

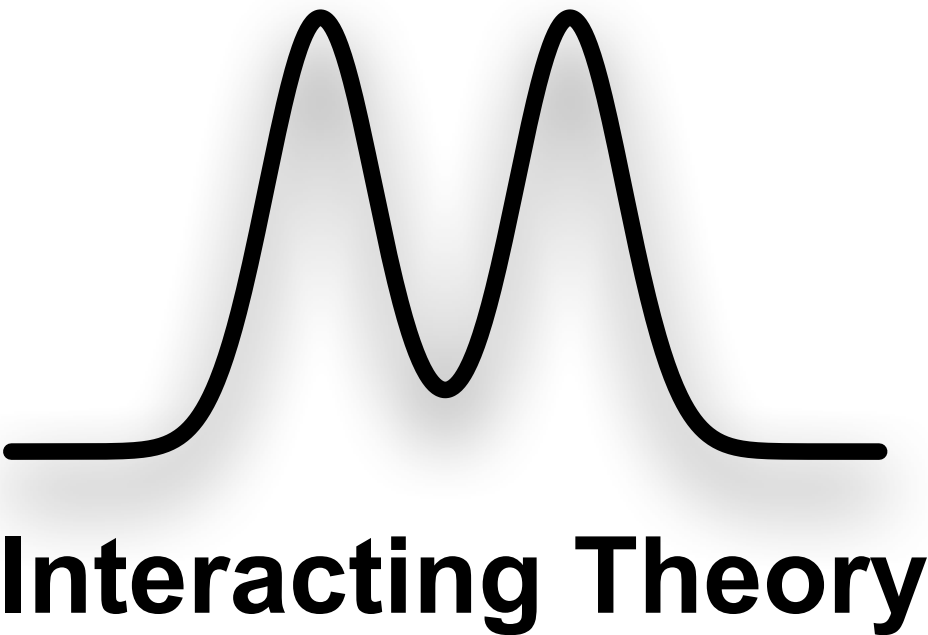
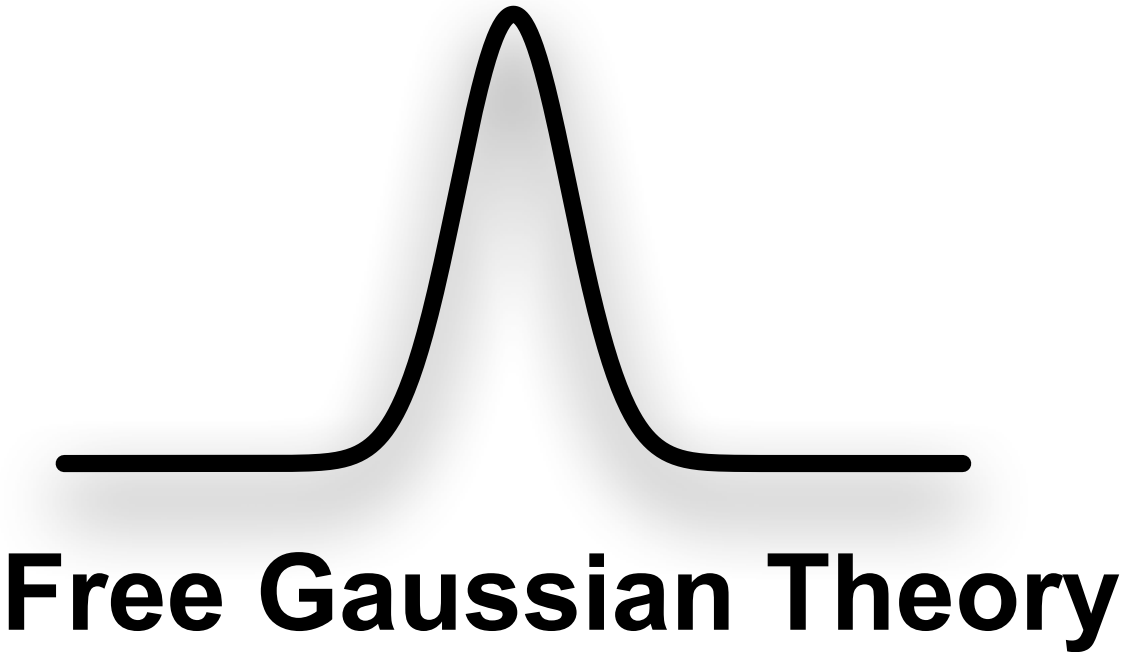


$$\phi = \mathcal{T}_{\theta}(\varphi)$$

θ : (Neural Network) Parameters

$$\tilde{p}_{t=1}(\phi) = p_{t=0}(\varphi) |\det J_{\mathcal{T}}(\varphi)|^{-1}$$

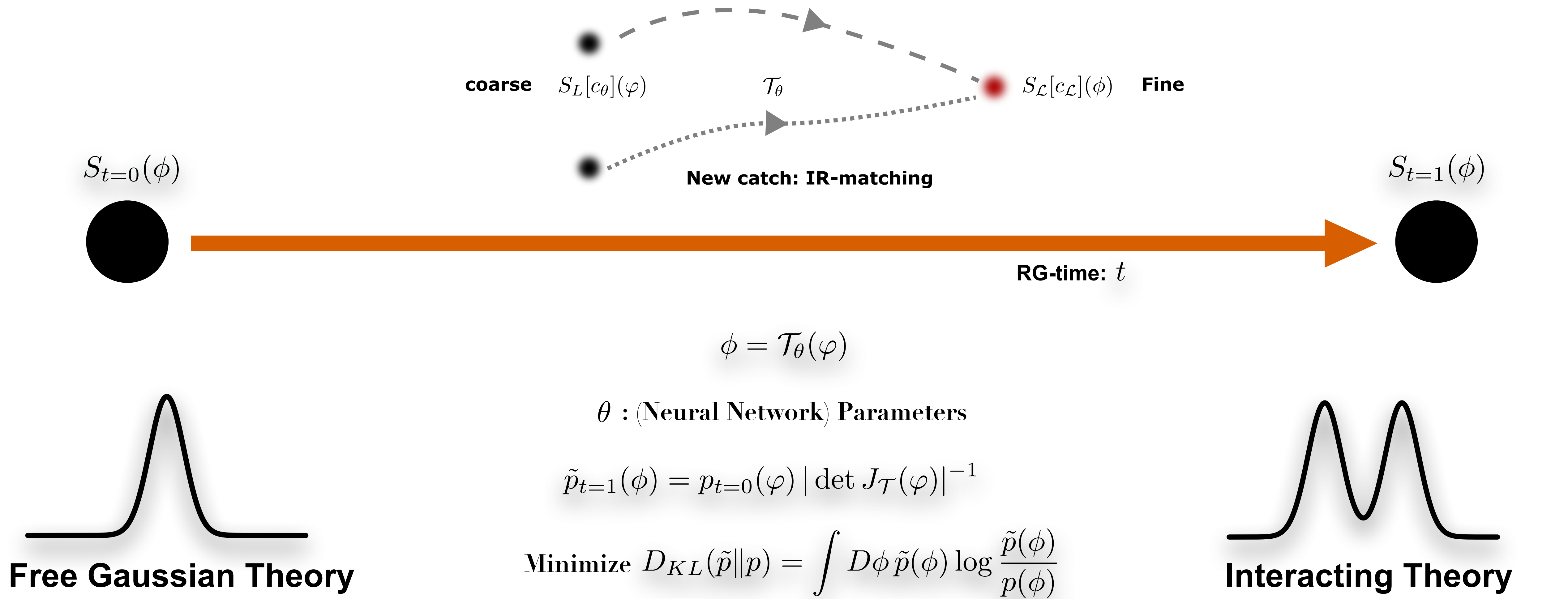
Minimize $D_{KL}(\tilde{p}||p) = \int D\phi \tilde{p}(\phi) \log \frac{\tilde{p}(\phi)}{p(\phi)}$



e.g. [Lüscher Commun.Math.Phys. 293 \(2010\)](#) Rezende, Mohamed, arXiv: [1505.05770](#)

Sampling with RGitectures in lattice field theories

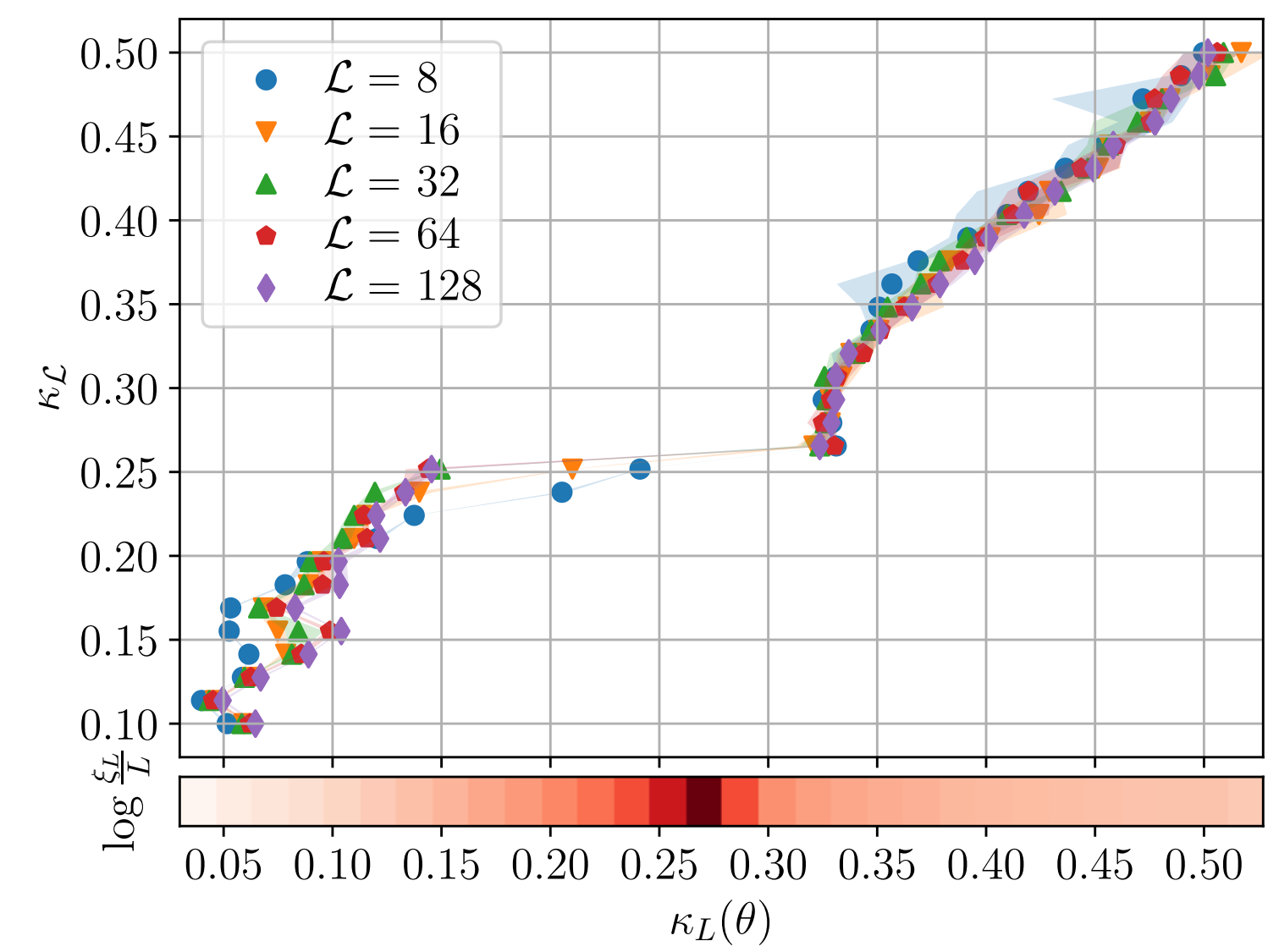
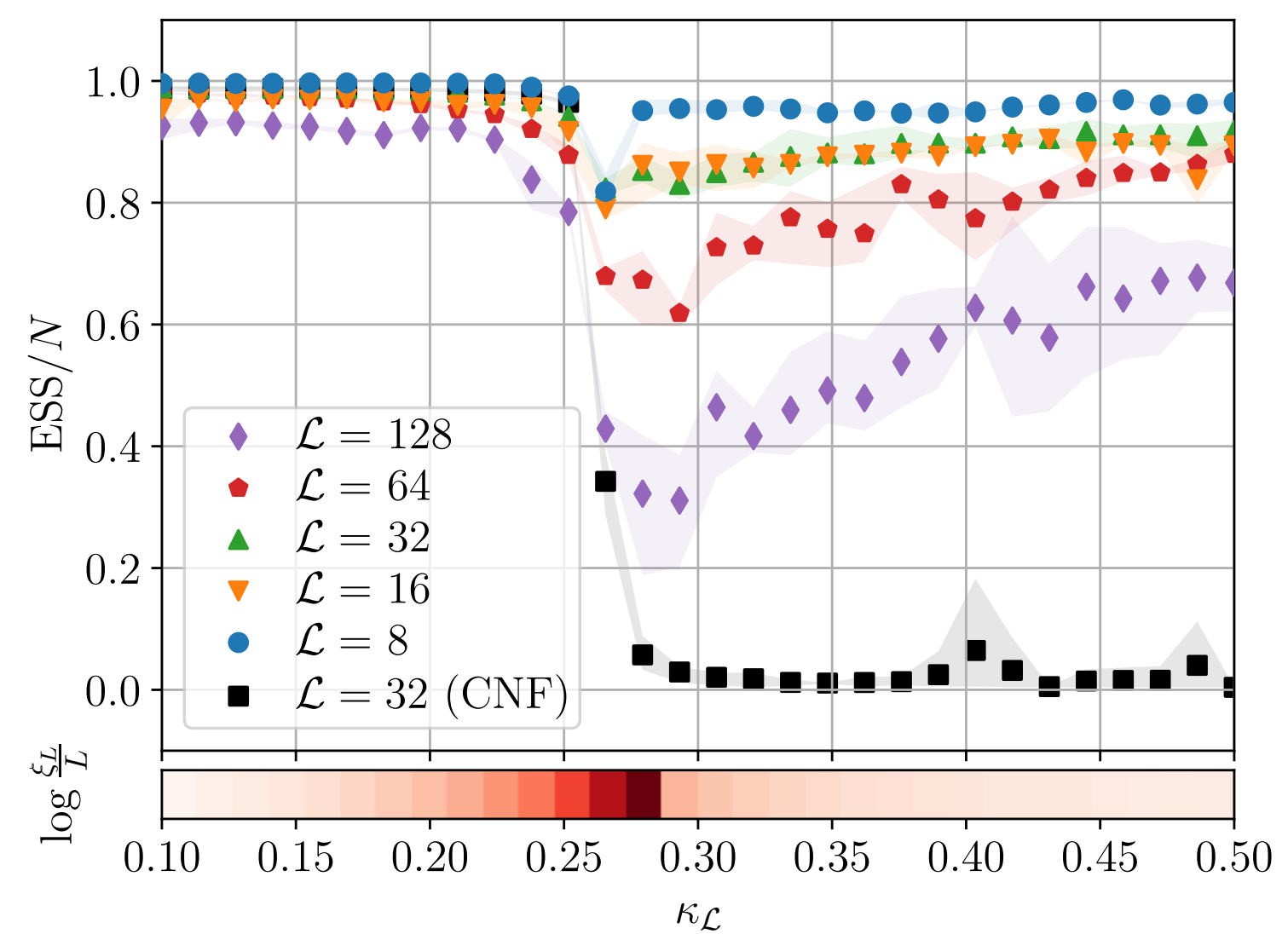
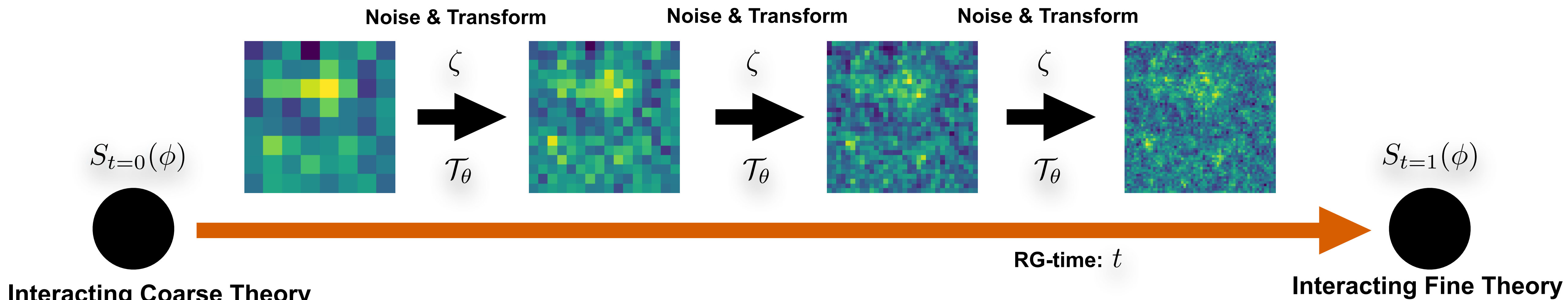
Super-resolving Normalising Flow with IR-matching



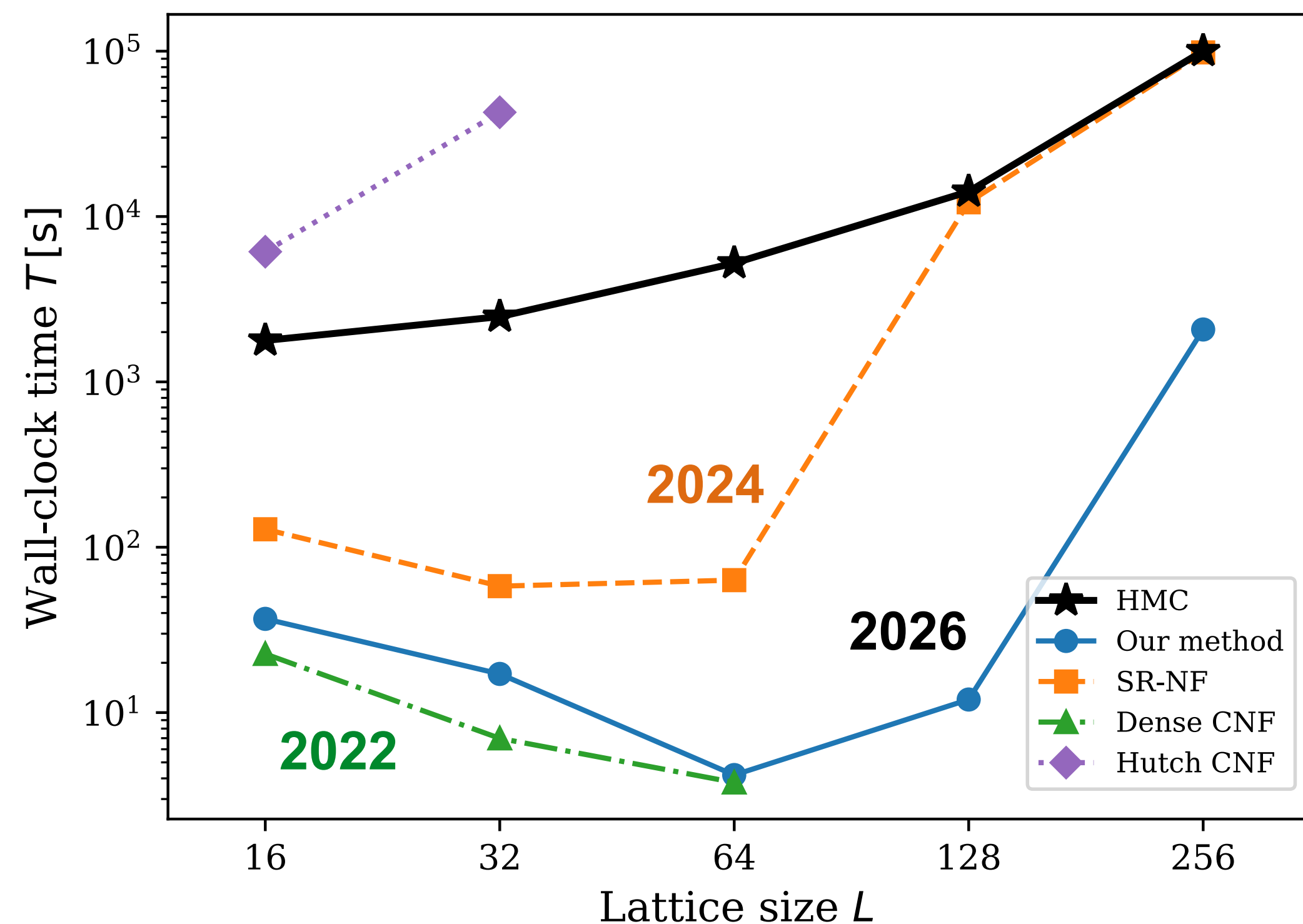
e.g. [Lüscher Commun.Math.Phys. 293 \(2010\)](#) Rezende, Mohamed, arXiv: [1505.05770](#)

Sampling with RGitectures in lattice field theories

Super-resolving Normalising Flow with IR-matching



Scalable Generative Sampling and Multilevel Estimation for Lattice Field Theories Near Criticality



- *Our Multilevel method*: The multilevel coarse-to-fine sampler introduced in Sec. III, evaluated at $L \in \{16, 32, 64, 128, 256\}$ (see Appendix B 1).
- *Super Resolving CNF* [64]: the same hierarchical structure but with full-lattice CNF upsampling and without coarse-site preservation, evaluated at $L \in \{16, 32, 64, 128, 256\}$ (see Appendix B 2).
- *Dense CNF* [30]: A single CNF on the target distribution with a Gaussian prior, evaluated at $L \in \{16, 32, 64\}$; training becomes prohibitive beyond $L = 64$ (see Appendix B 3).
- *Hutch CNF*: The Hutch CNF uses multiple CNN layers with the Hutchinson stochastic trace estimator [72] estimating the divergence stochastically, evaluated at $L \in \{16, 32\}$ (see Appendix B 4).

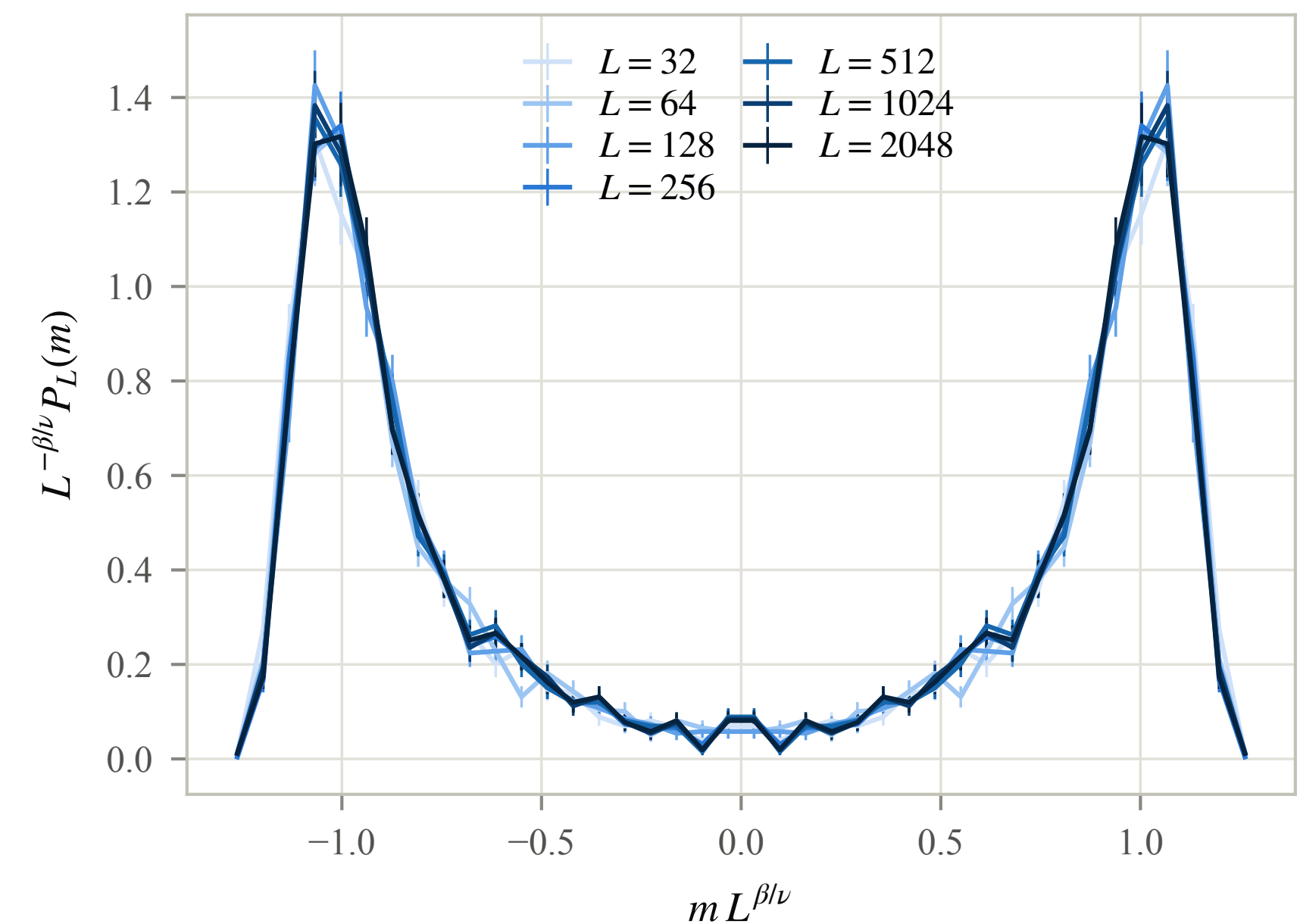
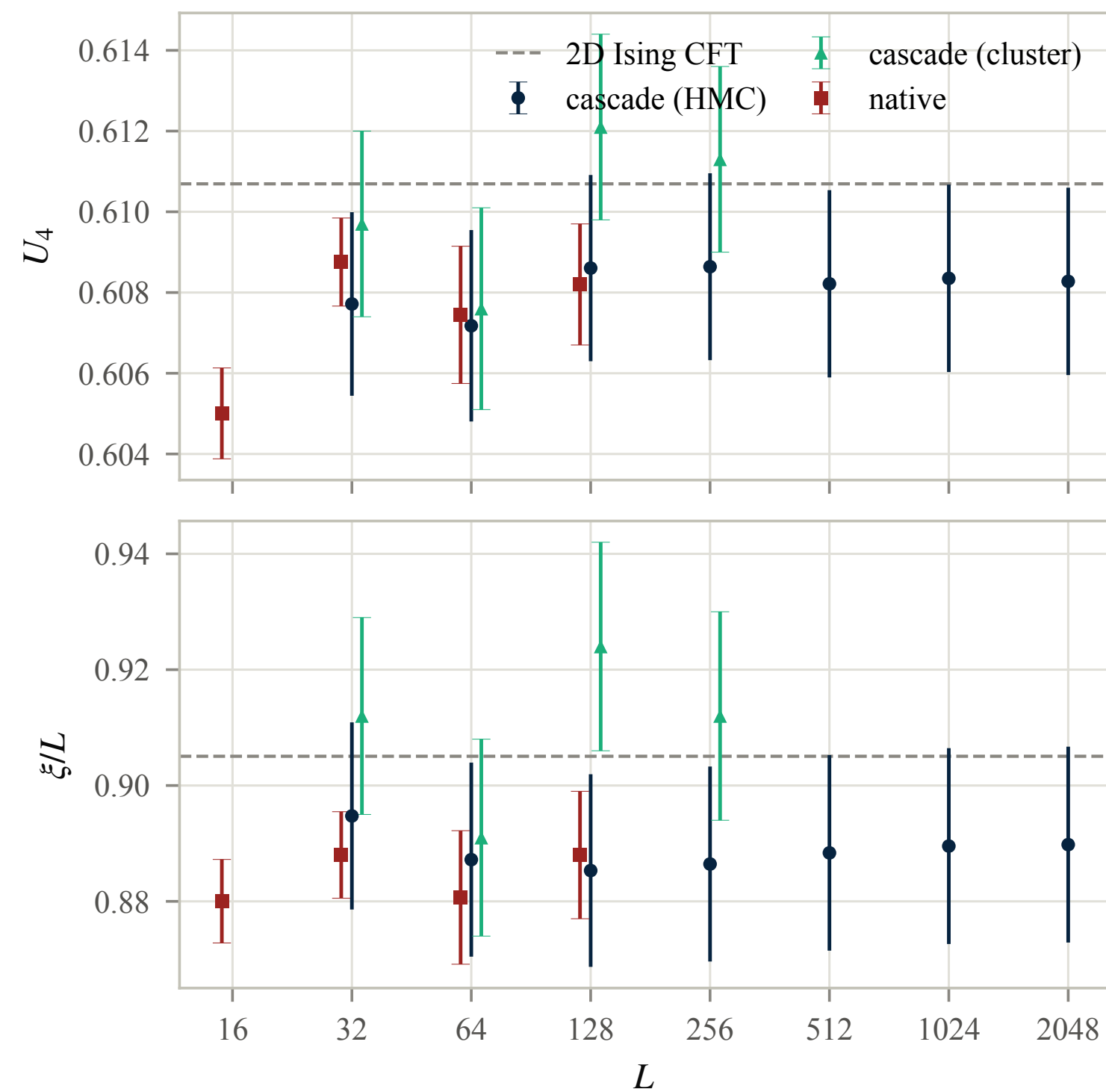
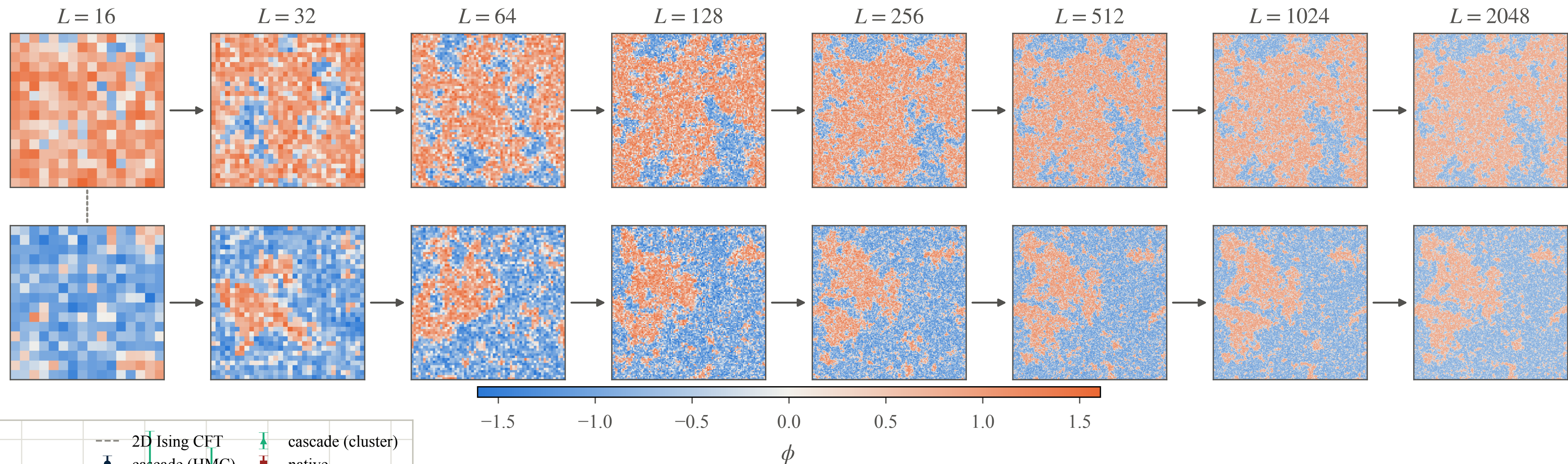
TABLE I. Integrated autocorrelation time $\tau_{\text{int}}(|m|)$ at $\kappa_c = 0.2705$, $\lambda = 0.022$. HMC: Wolff method on 2×10^6 -step chains ($L \leq 128$) and 3×10^6 steps ($L = 256$). Generative methods: IMH chains of 5×10^4 steps, 2×10^3 burn-in. The corresponding importance-sampling efficiency ESS/N for all methods is reported in Fig. 7 (Appendix B 7).

Method	$L = 16$	$L = 32$	$L = 64$	$L = 128$	$L = 256$
HMC	64.6 ± 1.3	228 ± 9	1370 ± 120	4930 ± 850	13265 ± 4321
Our method	0.72 ± 0.02	0.80 ± 0.02	1.16 ± 0.03	3.73 ± 0.15	302 ± 105
SR-NF	0.86 ± 0.01	1.46 ± 0.01	6.96 ± 0.12	161.4 ± 13.0	— ^a
Dense CNF	0.70 ± 0.02	0.76 ± 0.02	0.95 ± 0.03	—	—
Hutch CNF	25.7 ± 2.6	55.4 ± 8.3	—	—	—

^a The independence Metropolis–Hastings chain for SR-NF at $L = 256$ has an acceptance rate below 0.2% and sometimes gets stuck in a single configuration, so the τ_{int} estimate unreliable; we therefore omit it.

Recent developments

Renormalization-guided cascade upscaling for lattice field generation

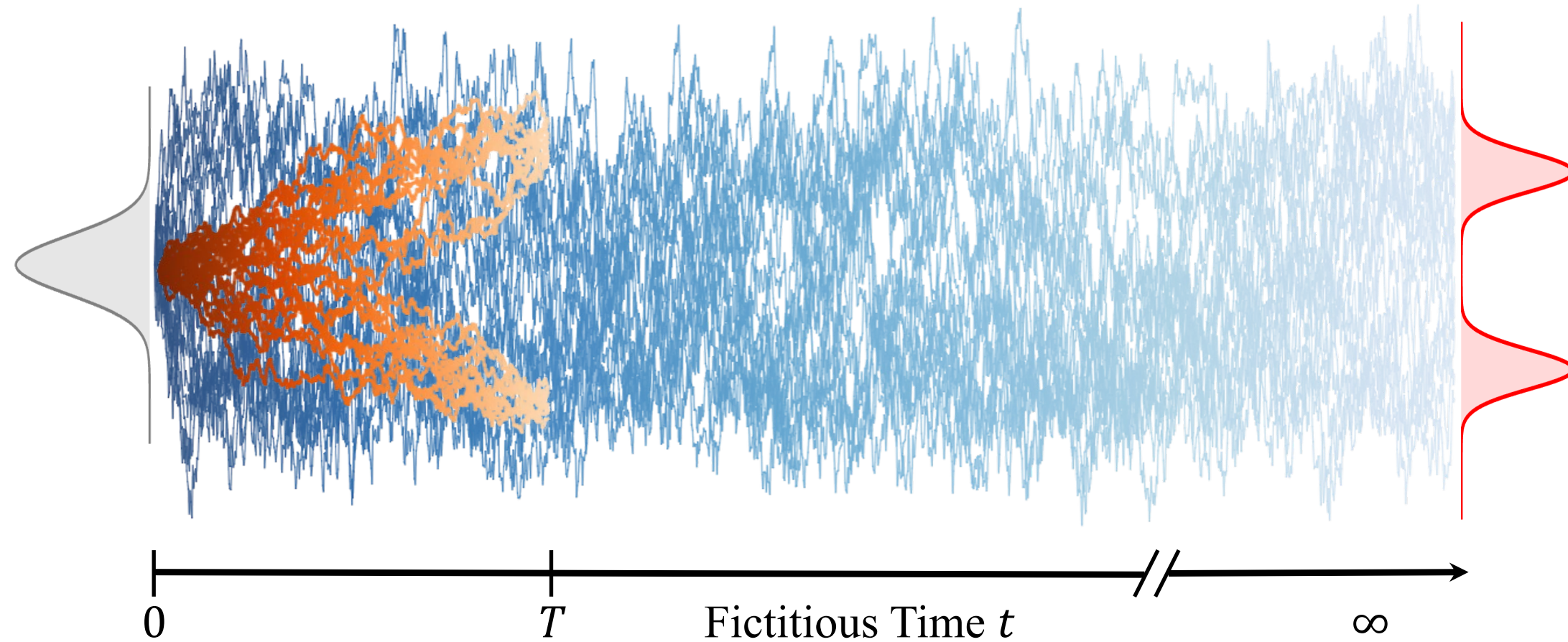


Diffusion Models in Lattice Field Theories

Stochastic Quantization as Optimal Control

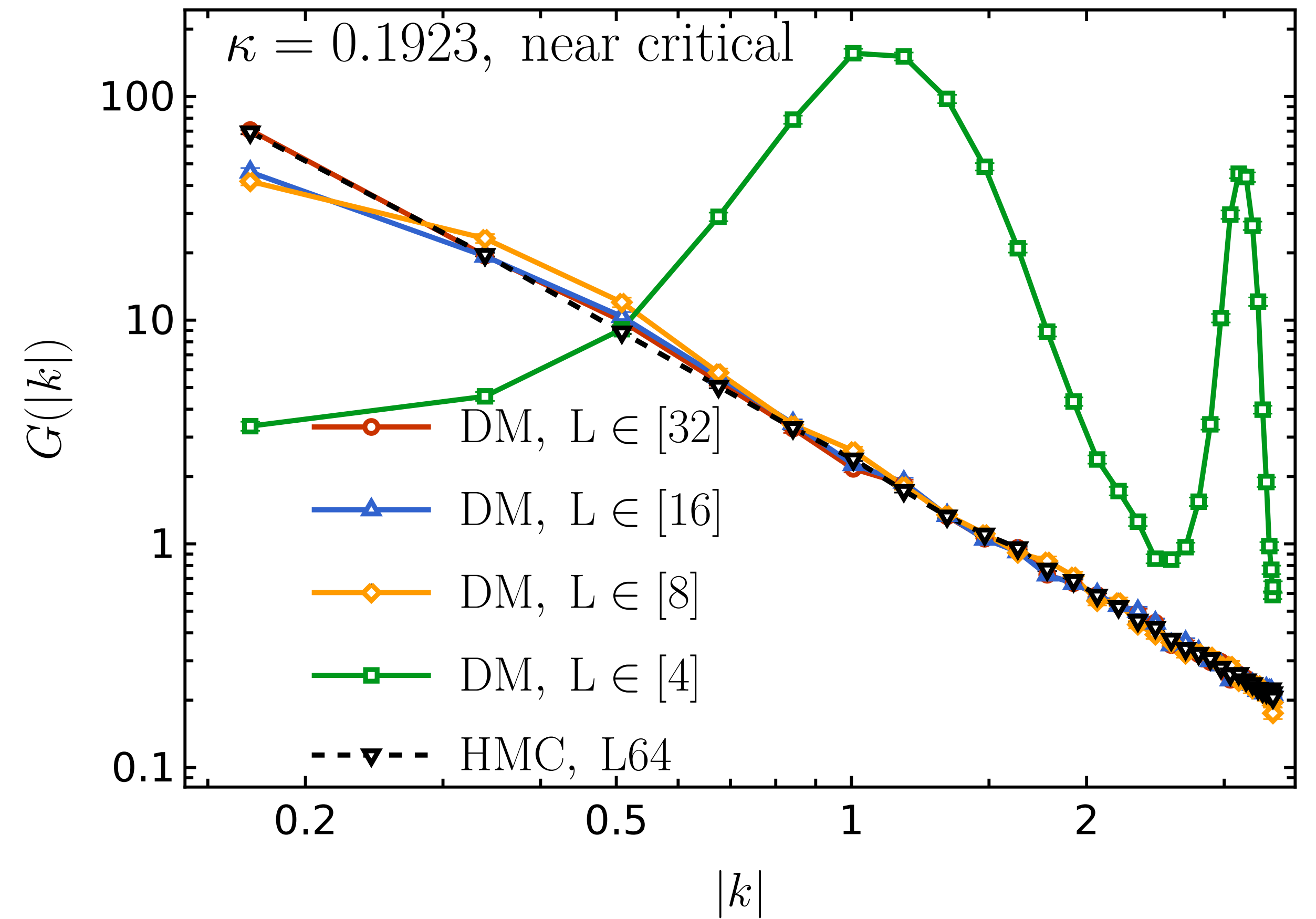
$$d\phi_t = u_\theta(\phi_t, t) dt + \eta$$

$$d\phi_t = -\nabla_\phi S[\phi] dt + \eta$$



Wang, [arXiv:2607.21436](#)

Diffusion Models for Sampling Near Criticality in Lattice Field Theories



Tan, Aarts, Habibi, Lucini, Wang, [arxiv:2607.08505](#)

Physics-informed renormalisation group

PIRG & PIKs

Recent developments

$$\partial_t p_t(\phi) = - \int_x \frac{\delta}{\delta \phi(x)} \left(\dot{\phi}(\phi) p_t(\phi) \right)$$

Wegner, J. Phys. C 7 (1974) 2098

$$\left(\partial_t + \int_x \dot{\phi} \frac{\delta}{\delta \phi} \right) \Gamma_\phi = \frac{1}{2} \text{Tr} \left[G_\phi \left(\partial_t + 2 \frac{\delta \dot{\phi}}{\delta \phi} \right) R_\phi \right]$$

JMP, *Annals Phys.* 322 (2007) 2831

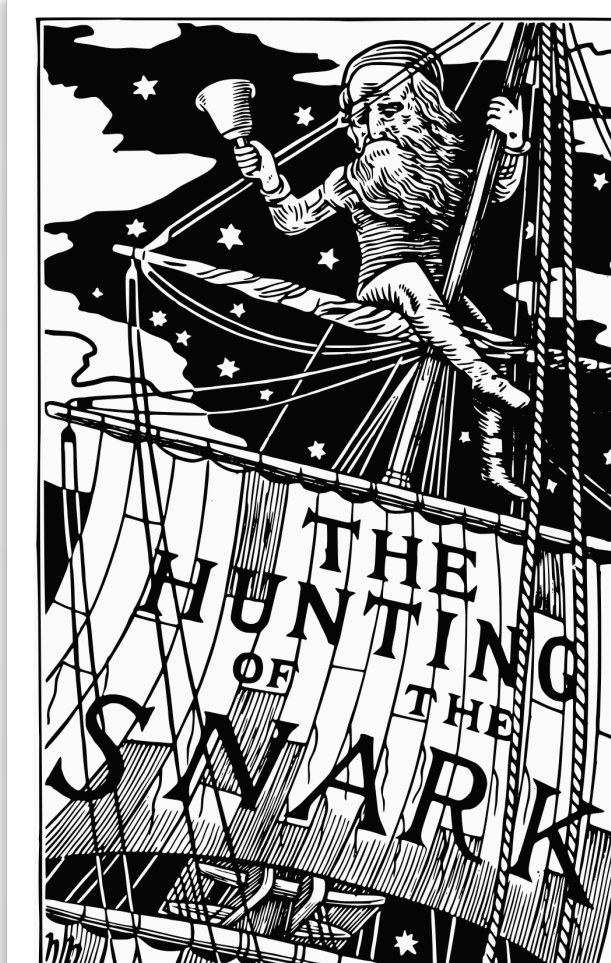
Generative RGitectures

Ihssen, Kapust, JMP

PIRGs for the effective action

Talk Friederike Ihssen

Saturday: 11:30 - 12:00

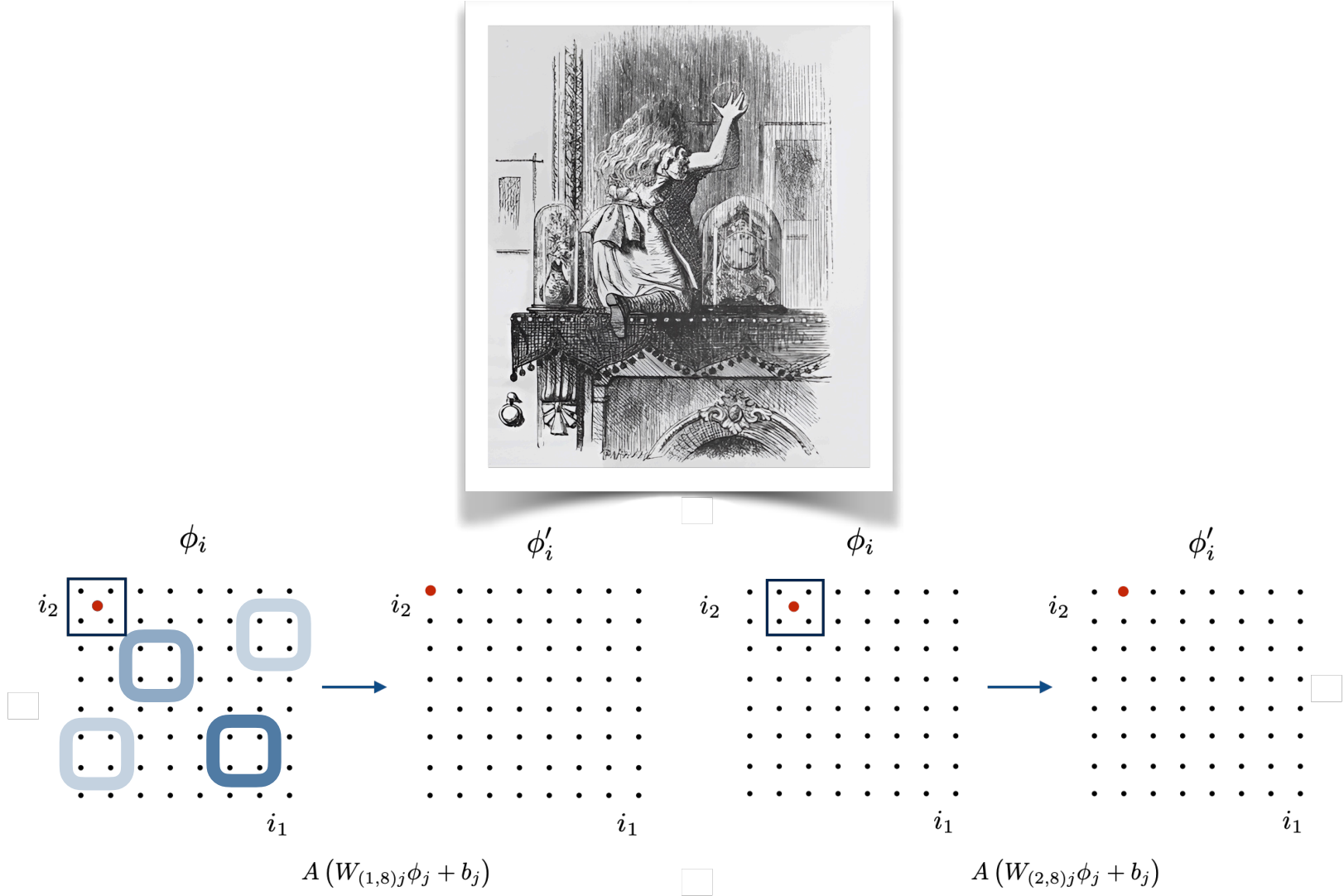


Fit the Eighth: *The Vanishing*

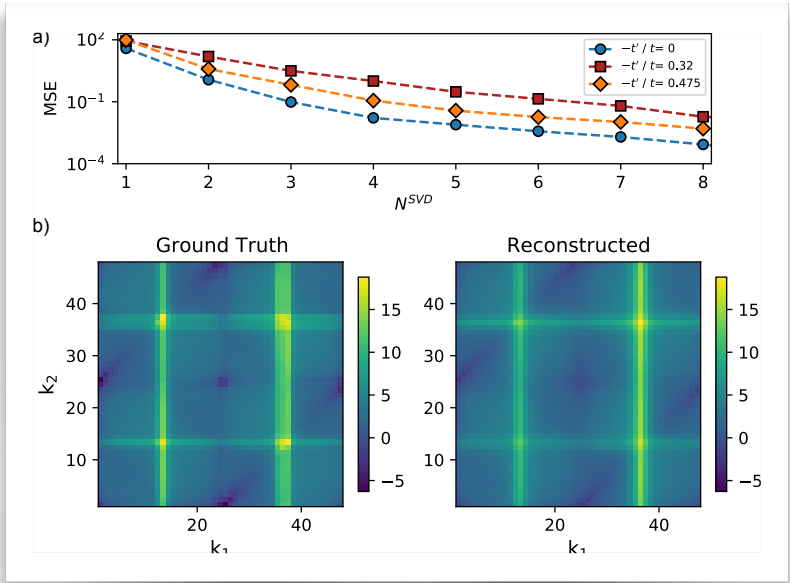
- Fit the First: What?

Fit the First: What do we want, what should we definitely do and other what's?		
ML as a tool	ML for fRG tasks	The (f)RG structure of ML
• Coding • Literature search • Polishing • Emulating readers • ...	• 'Importance sampling' • Neural solvers • Neural expansion schemes • Optimisation • ...	• ML as an RG • Structure & Optimisation • Fits • Generative sampling
Agentic	4	Agentic

- Fit the Second: Machine learning through the fRG looking glass



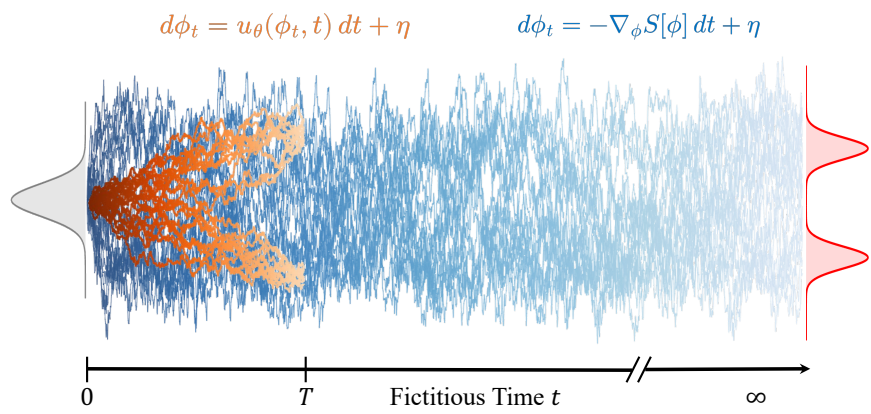
- Fit the Third to Fifth: What can Machine Learning do for us?



- Fit the Fifth to Seventh: The RG-Structure of Machine Learning

Generative Functional RG: PIRGs & PIK

Talk Friederike Ihssen Saturday: 11:30 - 12:00

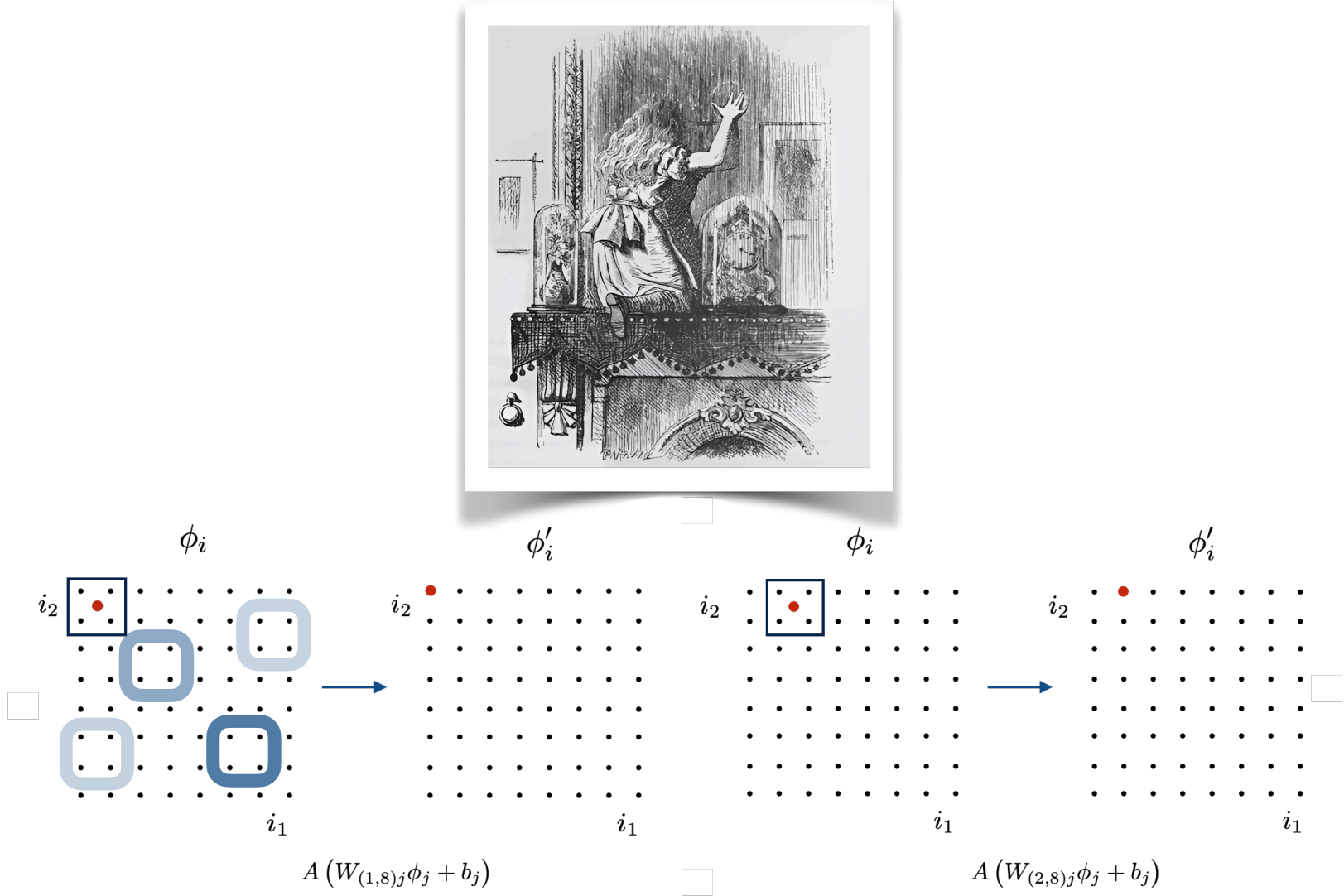


Fit the Eighth:

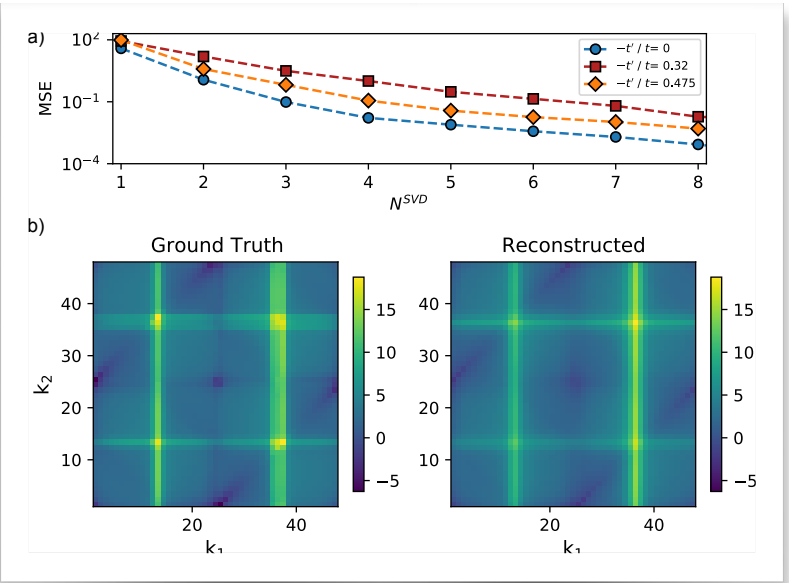
Fit the First: What?

Fit the First: What do we want, what should we definitely do and other what's?		
ML as a tool	ML for fRG tasks	The (f)RG structure of ML
• Coding • Literature search • Polishing • Emulating readers • ...	• 'Importance sampling' • Neural solvers • Neural expansion schemes • Optimisation • ...	• ML as an RG • Structure & Optimisation • Fits • Generative sampling Talk Friederike Ihssen Saturday: 11:30 - 12:00
Agentic	4	Agentic

Fit the Second: Machine learning through the fRG looking glass



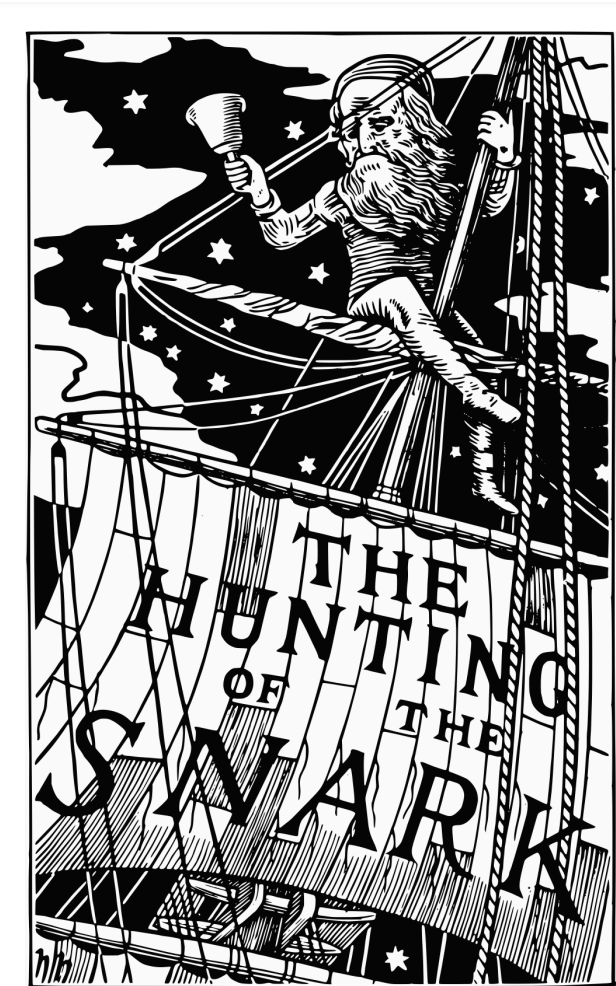
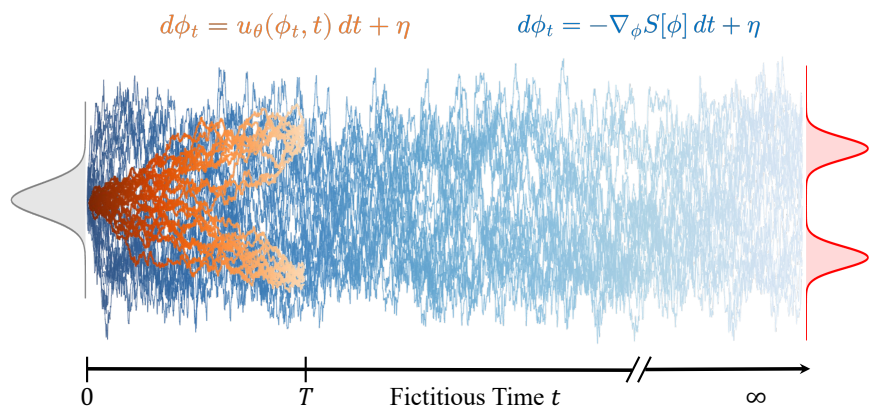
Fit the Third to Fifth: What can Machine Learning do for us?



Fit the Fifth to Seventh: The RG-Structure of Machine Learning

Generative Functional RG: PIRGs & PIK

Talk Friederike Ihssen Saturday: 11:30 - 12:00





The end

