Asymptotically safe quantum gravity: functional and lattice perspectives

ERG 2026, University of Sussex

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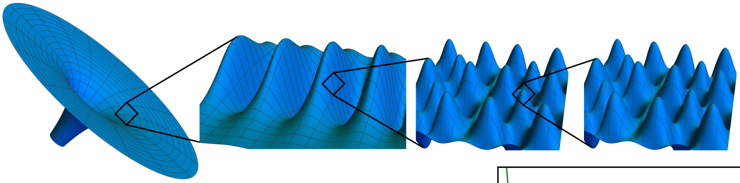
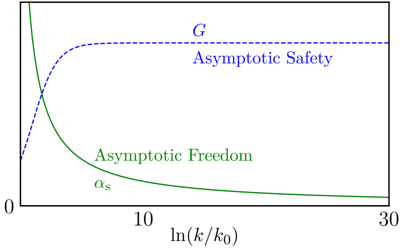
Asymptotically Safe Quantum Gravity



- Perturbative quantum gravity:
  loss of predictivity
- Key idea of asymptotic safety:
  Quantum realization of
  scale symmetry
  - imposes infinitely many conditions on theory



Asymptotic freedom:
free fixed point, $\alpha_{s,*} = 0$
Asymptotic safety:
interacting fixed point, $G_* \neq 0$

Motivation: Exploring AS with functional and lattice methods

Perturbative methods: see talk by Y. Kluth

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- Lattice methods:

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  - extract scale dependence of couplings/operators
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  - extract lattice correlators, order parameters, ...
  - AS: continuous phase transition

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  - finite size effects, extrapolations, "guessing" of relevant parameters, ...

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Use different methods in concerted fashion to extract evidence for and physical features of asymptotically safe quantum gravity.

Lattice quantum gravity via dynamical triangulations

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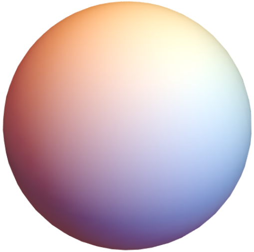

- Discretization of spacetime in terms of triangulations

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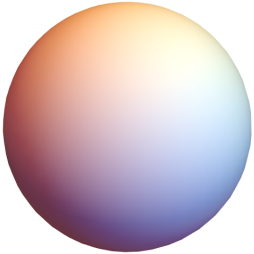
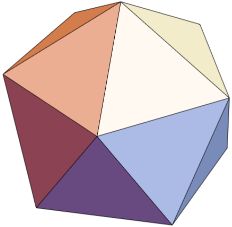


$$\Rightarrow$$



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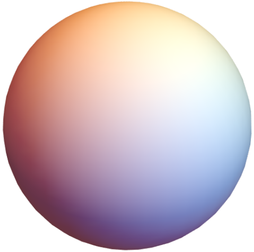

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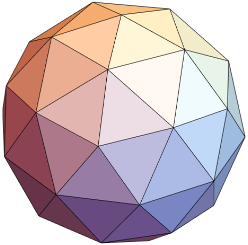

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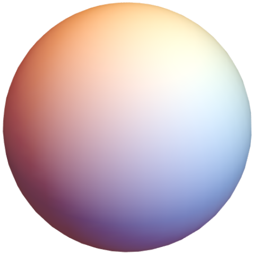

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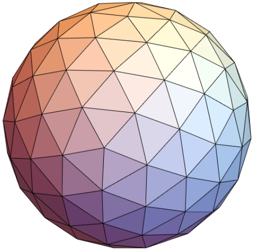

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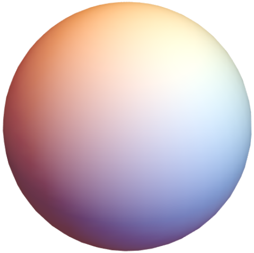

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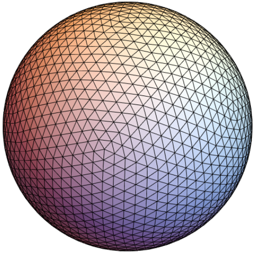

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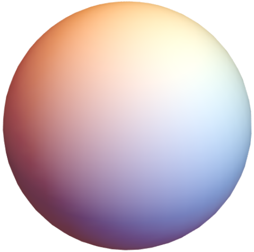

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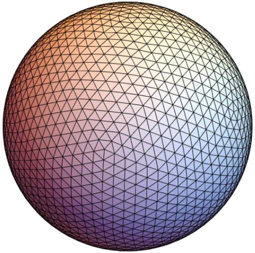

- Discretization of spacetime in terms of triangulations

  [Ambjørn and Jurkiewicz, 1992], [Agishtein and Migdal, 1992], . . .

$$\int \mathcal{D}g \, e^{-S[g]} \to \sum_{\mathcal{T}} \frac{1}{C_{\mathcal{T}}} \, e^{-S_{\mathrm{ER}}}$$

with Euclidean Einstein-Regge action $S_{\mathrm{ER}} = -\kappa_{d-2} N_{d-2} + \kappa_d N_d$ [Regge, 1961]

The Einstein-Regge action

- starting point: Einstein-Hilbert action

$$S_{\mathrm{EH}} = -\frac{1}{16\pi G_{\mathrm{N}}} \int \mathrm{d}^4 x \sqrt{g}\, (R - 2\Lambda)$$

The Einstein-Regge action

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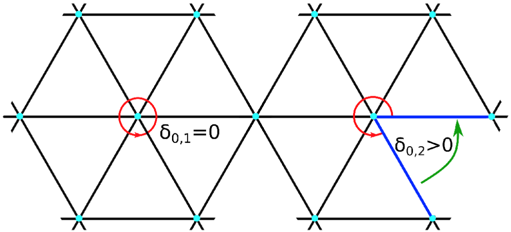
- Discretize volume term:

$$\int \mathrm{d}^4x \sqrt{g} \to \sum_{s_4} V_{s_4}$$

The Einstein-Regge action



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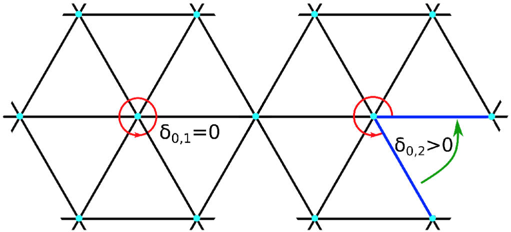

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$$\int \mathrm{d}^4x \sqrt{g}\, R \to \sum_{s_2} V_{s_2} \delta_{s_2}$$

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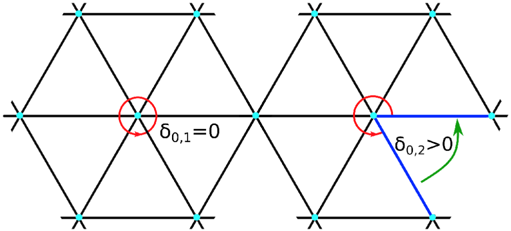

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- Einstein-Regge action: [Regge, 1961]

$$S_{\mathrm{ER}} = -\frac{1}{8\pi G_{\mathrm{N}}} \left(\sum_{s_2} V_{s_2} \delta_{s_2} - \Lambda \sum_{s_4} V_{s_4} \right)$$

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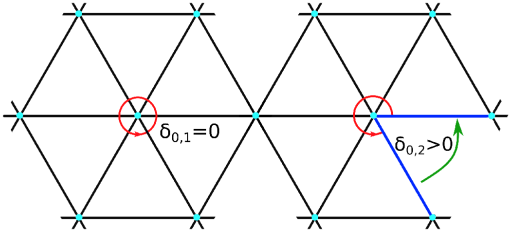

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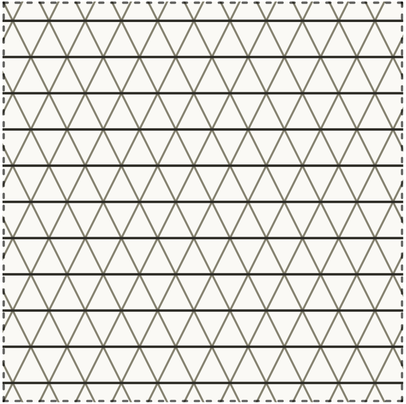
$$S_{\mathrm{ER}} = -\frac{1}{8\pi G_{\mathrm{N}}}\left(\sum_{s_2} V_{s_2}\delta_{s_2} - \Lambda\sum_{s_4} V_{s_4}\right) = \kappa_4\,N_4 - \kappa_2\,N_2$$

Dynamical Triangulations

- Discretization of spacetime in terms of triangulations

$$\int \mathcal{D}g \, e^{-S[g]} \to \sum_{\mathcal{T}} \frac{1}{C_{\mathcal{T}}} \qquad\qquad e^{-S_{\mathrm{ER}}}$$

- Perform sum via Monte Carlo methods



[Credits: J. van der Duin]

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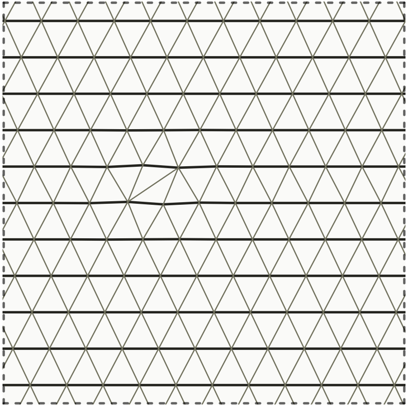

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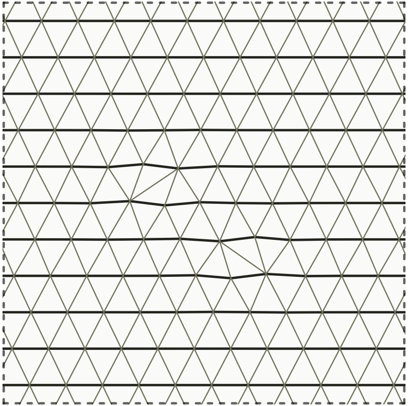

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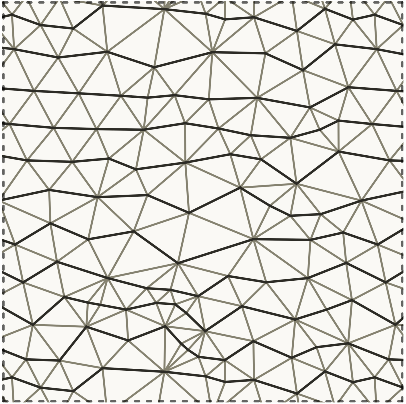
  - Tune κ_4 automatically



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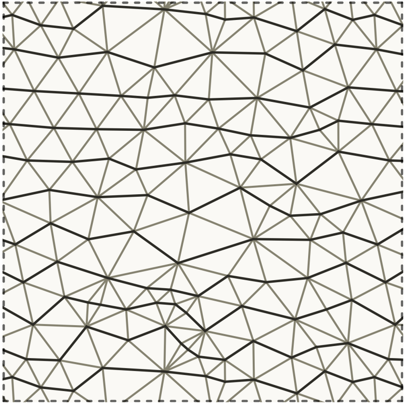
  - Pachner moves: change volume
  - Tune κ_4 automatically
  - Search for continuous phase transition



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Lattice quantum gravity in $d = 4$

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- in $d = 4$: no physical phase, no indications for higher-order transition in κ_2-κ_4 - space

[Ambjørn, Jain, Jurkiewicz, Kristjansen, 1993], [Bakker, Smit, 1994]
[Ambjørn, Jurkiewicz, '95], [Bialas, Burda, Krzywicki, Petersson, '96] . . .

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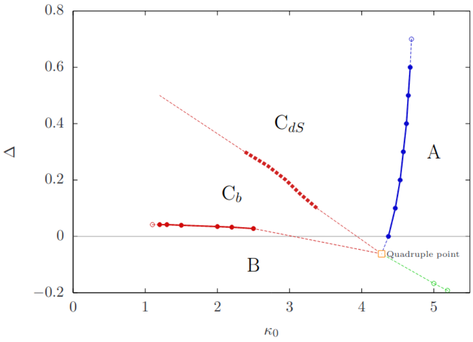
  [Ambjørn, Loll, 1998], [Ambjørn, Jurkiewicz, Loll, 2000], . . .
  RG on the lattice: [Ambjørn, Gizbert-Studnicki, Goerlich, Nemeth; 2024]



Taken from [Loll, 2020]

Lattice quantum gravity in $d = 4$

- Discretization of spacetime in terms of triangulations

$$\int \mathcal{D}g\, e^{-S[g]} \to \sum_{\mathcal{T}} \frac{1}{C_{\mathcal{T}}} \left[\prod_{j=1}^{N_2} \mathcal{O}(t_j)^{\beta} \right] e^{-S_{\mathrm{ER}}}$$

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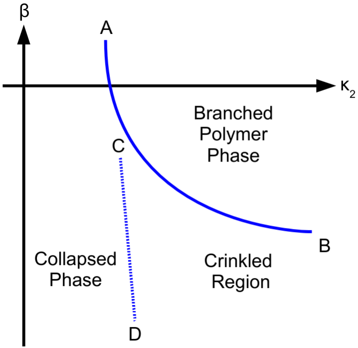
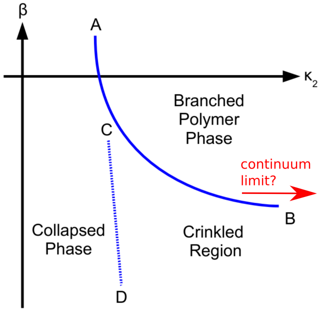
- EDT: include additional operator
  [Bruegmann, Marinari, 1993], [Bilke, Burda, Krzywicki, Petersson, Tabaczek, Thorleifsson, 1998], [Laiho, Coumbe, 2011], [Ambjørn, Glaser Goerlich, Jurkiewicz, 2013], . . .



[Ambjorn, Glaser, Goerlich, Jurkiewicz, 2013]
[Coumbe, Laiho, 2014]

Phase diagram of EDT

- Idea:
  follow AB-line towards large κ_2 and negative β



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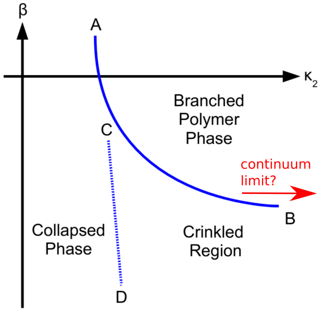

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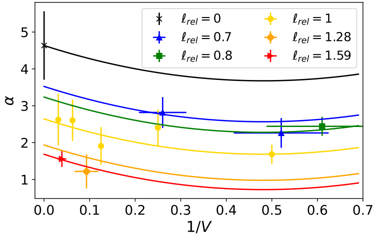

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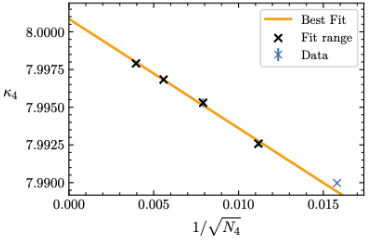

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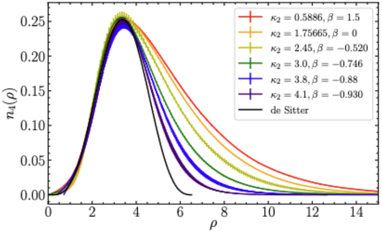

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- Key open questions:
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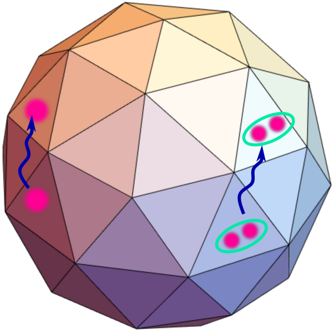
    - Four-dimensional geometries?
    - Semi-classical limit?
    - Physical volume profiles?
  - Is there a continuum limit?
    - Decreasing lattice spacings?

κ_2	G_a	a_{rel}	ℓ_{rel}	χ^2/d.o.f	p-value
2.45	0.477(27)(57)	1.125(92)	1.45(12)	0.61	0.61
3.0	0.604(38)(35)	1.00	1.00	0.47	0.71
3.4	0.805(81)(50)	0.866(67)	0.618(49)	0.39	0.76
3.8	0.98(23)(06)	0.78(10)	0.386(51)	1.55	0.20

Newtonian binding energy from EDT

- Use matter as probe for gravity



Newtonian binding energy from EDT

- Use matter as probe for gravity
- Study scalar fields on EDT (quenched approximation)

$$S_{\mathrm{M}}^{\mathrm{lattice}} = \sum_{x,y} \phi_x \, L_{xy} \, \phi_y \, ,$$
$$\text{with} \quad L_{xy} = (D_x + m_0^2)\delta_{xy} - A_{xy} \, .$$

Newtonian binding energy from EDT

- Use matter as probe for gravity
- Study scalar fields on EDT (quenched approximation)
- Extract binding energy E_b and renormalized mass m from one-and two-particle two-point correlators [de Bakker, Smit, 1996]

$$
\begin{aligned}
G_s(r) &\sim L^{-1}\,, \\
&\propto \quad \frac{e^{-m\,r}}{r^p} \quad \text{(for large } r\text{)}\,. \\
G_s^{(2)}(r) &\sim \left(L^{-1}\right)^2\,, \\
&\propto \quad \frac{e^{-M\,r}}{r^q} \quad \text{(for large } r\text{)}\,.
\end{aligned}
$$

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- Fit $E_\mathrm{b} = A\, m^\alpha$
- Continuum, non-relativistic limit: $E_\mathrm{b} = \frac{G\, m^5}{4}$ (in $d = 4$)

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$$G_s^{(2)}(r) \sim \left(L^{-1}\right)^2\,, \quad \propto \quad \frac{e^{-M\,r}}{r^q} \quad \text{(for large } r\text{)}\,.$$

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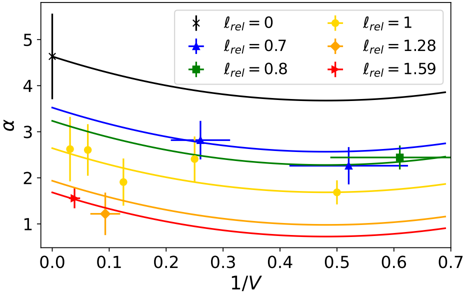

[Dai, Laiho, MS, Unmuth-Yockey; 2021]

- EDT fit: $\alpha = 4.6 \pm 0.9$

$\Rightarrow\ d = 3.9 \pm 0.2$

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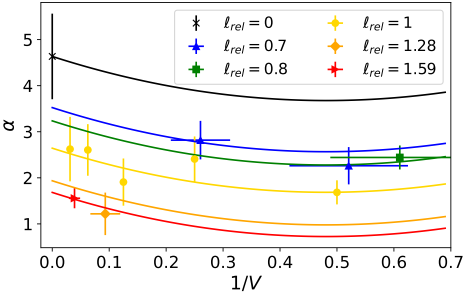
- Study scalar fields on EDT (quenched approximation)
- Extract binding energy E_{b} and renormalized mass m from one-and two-particle two-point correlators [de Bakker, Smit, 1996]
- Fit $E_{\mathrm{b}} = A\, m^{\alpha}$
- Continuum, non-relativistic limit: $E_{\mathrm{b}} = \frac{G\, m^5}{4}$ (in $d = 4$)



[Dai, Laiho, MS, Unmuth-Yockey; 2021]

- EDT fit: $\alpha = 4.6 \pm 0.9$

$\Rightarrow\ d = 3.9 \pm 0.2$

EDT might feature well-behaved non-relativistic limit! [Dai, Laiho, MS, Unmuth-Yockey; 2021]

The Saddle point approximation in EDT II

- Saddle point approximation:

  [Ambjorn, Goerlich, Jurkiewicz, Loll, 2012]

$$\langle N_4 \rangle \simeq \frac{A^2}{4(\kappa_4 - \kappa_4^c)^2} \Rightarrow A \sim |\kappa_4 - \kappa_4^c| \sqrt{N_4}\,.$$

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$$Z(\kappa_2, \, \kappa_4) \approx \exp\left(\frac{A^2(\kappa_2)}{4(\kappa_4 - \kappa_4^c)} \right) = \exp\left(\frac{3\pi}{G\,\Lambda} \right)$$

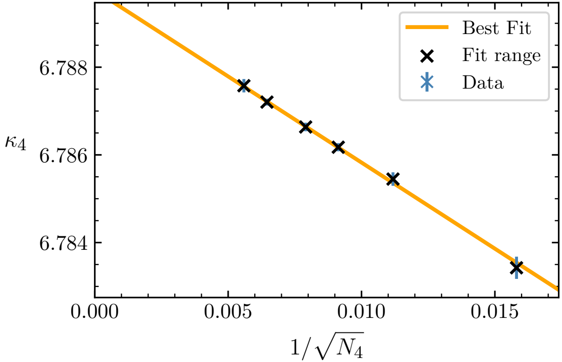
  Assumption: Continuum is dominated by de Sitter instanton [Hawking, Moss, 1987]

  Extract G from lattice data:

$$\frac{G}{\ell_{\mathrm{fid}}^2} \sim \left(\frac{a}{\ell}\right)^2 \frac{\ell_{\mathrm{rel}}^2}{|s|} \, , \quad s : \text{slope of } \kappa_4(N_4) \, ,$$

Numerical result: finite volume scaling

Example at $\beta = -0.6$, $\kappa_2 = 2.245$:



Extract slope s: $\kappa_4(N_4) = A + s\frac{1}{\sqrt{N_4}}$

Fit result: $s = -0.393 \pm 0.022$

Recovery of semi-classical scaling!

[Bassler, Laiho, MS, Unmuth-Yockey; 2021]

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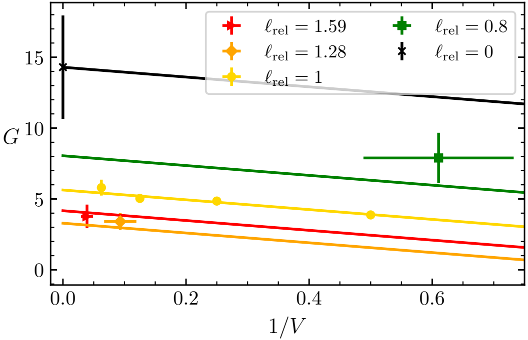

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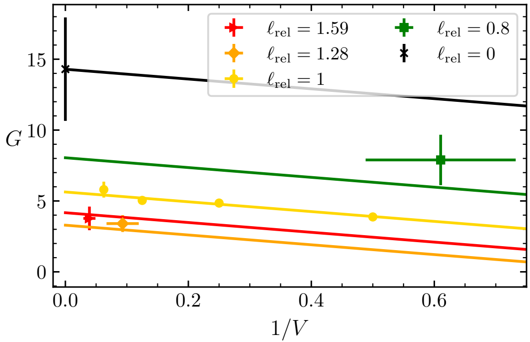

Continuum, infinite volume limit: $G/\ell_{\rm fid}^2 = 14.3 \pm 3.6$

$$\Rightarrow \ell_{\rm fid} \sim 0.26\, \ell_{\rm Pl}$$

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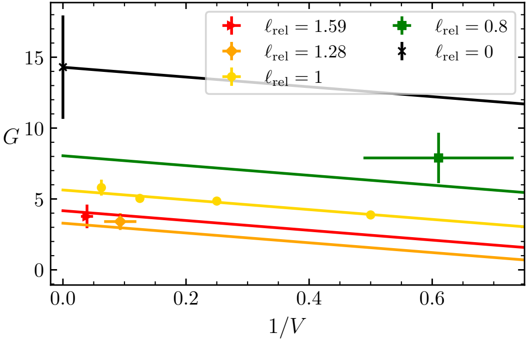

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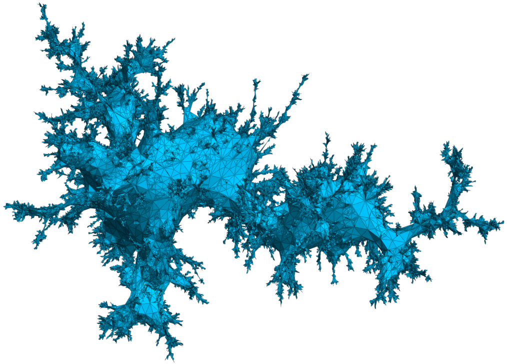
To date: $\ell_{\rm SF} \sim 0.057\, \ell_{\rm Pl}$; $a_{\rm SF} \sim 1.3\, \ell_{\rm Pl}$

[Dai, Freeman, Laiho, MS, Unmuth-Yockey; 2025]

Towards de Sitter geometries in EDT

- Shelling function $n_4(\rho)$: counts number of four-simplices at geodesic distance τ away from source-simplex.



Taken from [van der Duin, Loll, MS, Silva; 2025]

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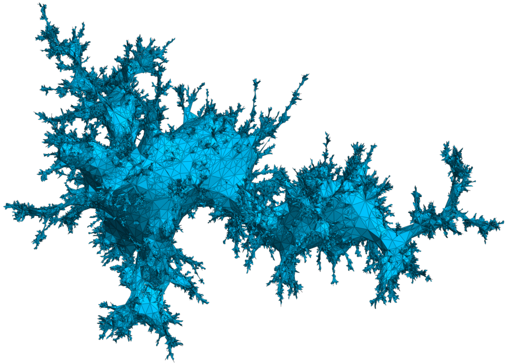

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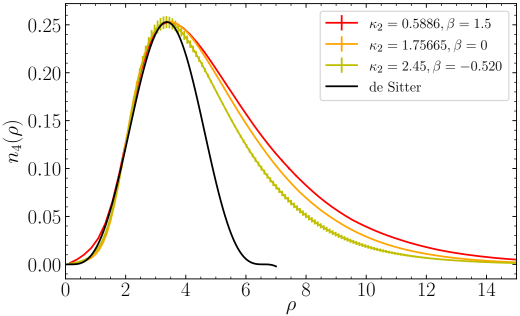

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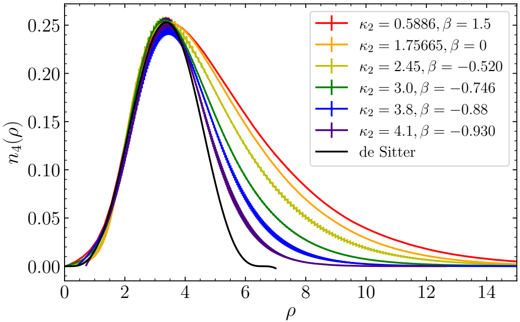
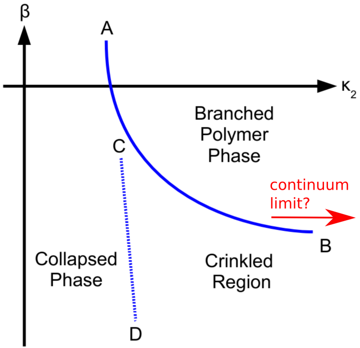


[Dai, Freeman, Laiho, MS, Unmuth-Yockey; 2023, 2024]

Lattice volume profiles: approximate de Sitter profile better for larger κ_2

Phase diagram of EDT

- Idea:
  follow AB-line towards large κ_2 and negative β
- Key open questions:
  - Is there a physical phase?
    - Four-dimensional geometries
    - Semi-classical limit
    - Physical volume profiles
  - Is there a continuum limit?
    - Decreasing lattice spacings



[Ambjorn, Glaser, Goerlich, Jurkiewicz, 2013]
[Coumbe, Laiho, 2014]

(Tentative) evidence for continuum limit and physical phase in $4d$ EDT.

Motivation: Exploring AS with functional and lattice methods

- Functional methods:
  - extract scale dependence of couplings/operators
  - AS: $k\partial_k \vec{g} = 0$
  - semi-analytical treatment, inclusion of matter, connection to low-energy physics, ...
  - need approximations on dynamics, regulator, gauge fixing, ...
- Lattice methods:
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Concrete example: critical exponent of scalar mass

Θ_{m^2} from CDT

Based on [van der Duin, Loll, MS; WIP]

- Use quenched approximation
- Invert L to compute propagator

$$S_{\mathrm{M}}^{\mathrm{lattice}} = \sum_{x,y} \phi_x \, L_{xy} \, \phi_y \, ,$$

$$\text{with} \quad L_{xy} = (D_x + m_0^2)\delta_{xy} - A_{xy} \, .$$

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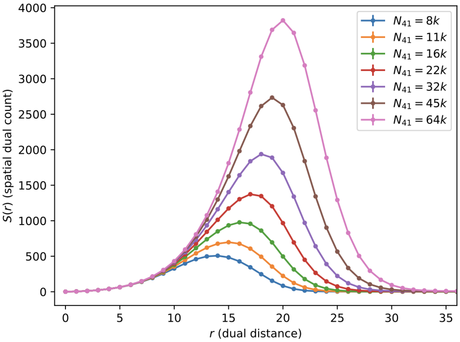

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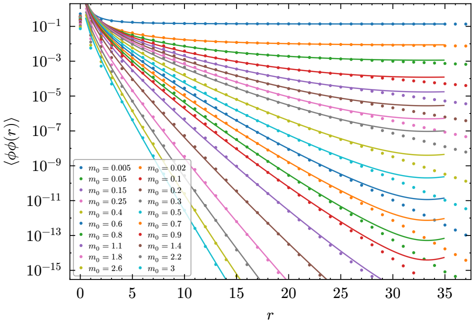

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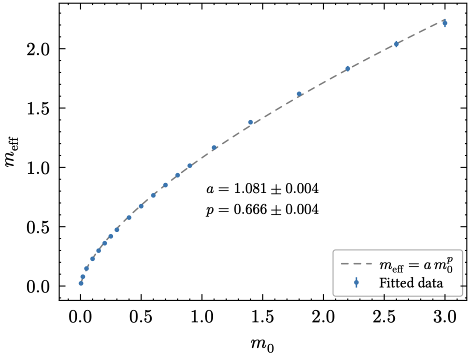
- Extract Θ_{m^2} from (volume) scaling of $m_{\text{eff}}(m_0)$
  [Jha, Laiho, Unmuth-Yockey; 2018]



Θ_{m^2} from FRG

Based on [Gyftopoulos, Portas-Chiesa, Reichert, MS; WIP]

- Θ_{m^2} computed extensively
  see e.g., [Narain, Percacci; 2009], [Narain, Rahmede; 2010], [Percacci, Vacca; 2015], [Labus, Percacci, Vacca; 2015], [Pawlowski, Reichert, Wetterich, Yamada; 2018], [Wetterich, Yamada; 2019], [Pastor-Gutierrez, Pawlowski, Reichert; 2023], . . .
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- These include: $R\phi^2$, $R^2\phi^2$, $C^2\phi^2$, $(\phi\Box\phi)\phi^2$, $F^2\phi^2$, $(\bar{\psi}\not{D}\psi)\phi^2$
- Results (preliminary):
  $\Theta_{m^2} = 0.4$ ($N_\mathrm{S} = 1$); $\Theta_{m^2} = 2.04$ (SM matter)
  $R\phi^2$ is irrelevant
- Ahead: stability analysis, estimate of systematic uncertainties

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