GRAVITY FROM ENTROPY

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[Bianconi, G.,Gravity from entropy. Phys. Rev. D 111, 066001 *(2025)]*

GRAVITY AND STATISTICAL MECHANICS

Since the discovery
that black-hole have an entropy
and emit Hawking radiation,
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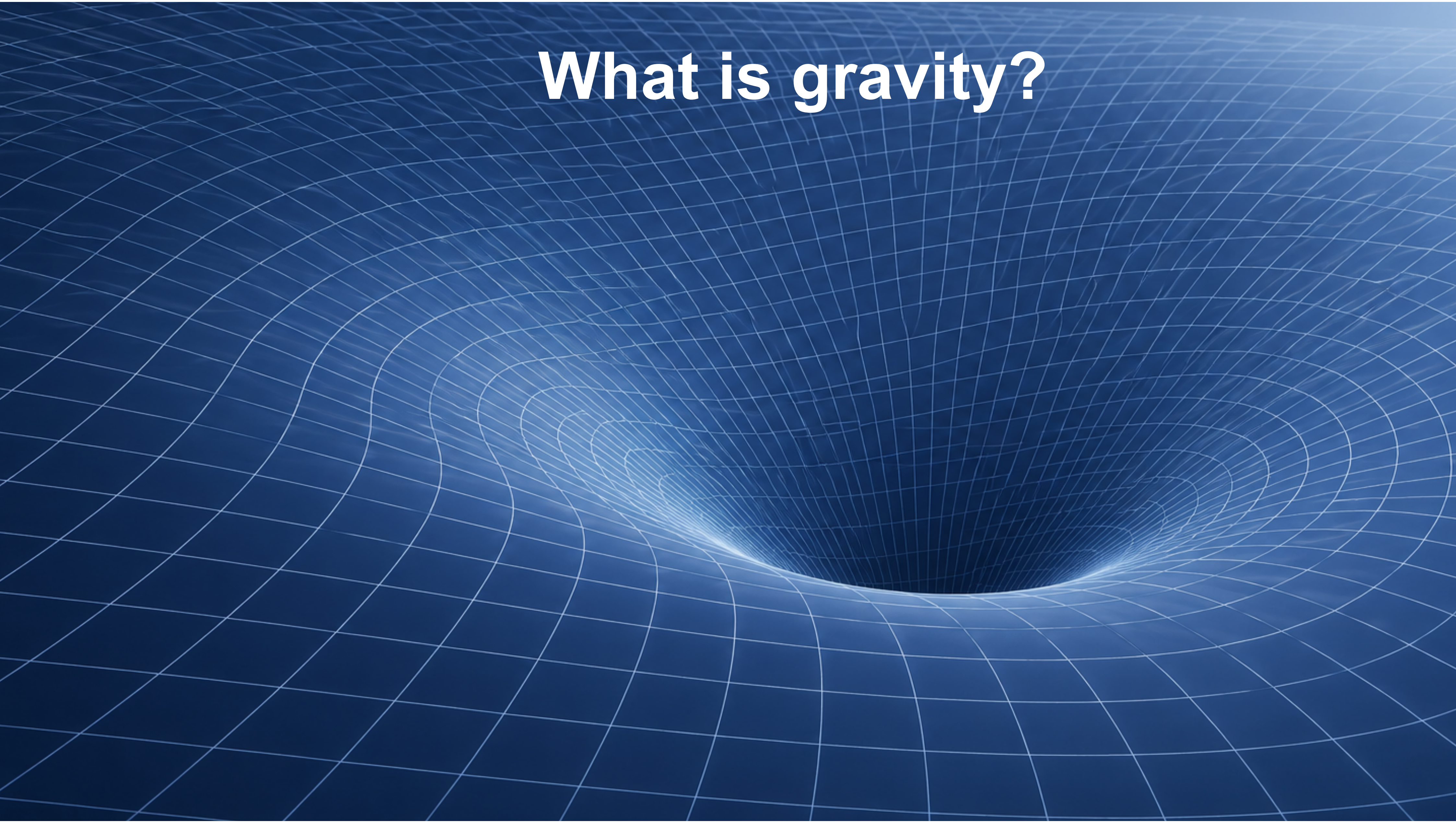
My belief is that
for building this theory we cannot start
from thermodynamic properties of black-holes and horizons
but we need a proper statistical mechanics approach

What is gravity?

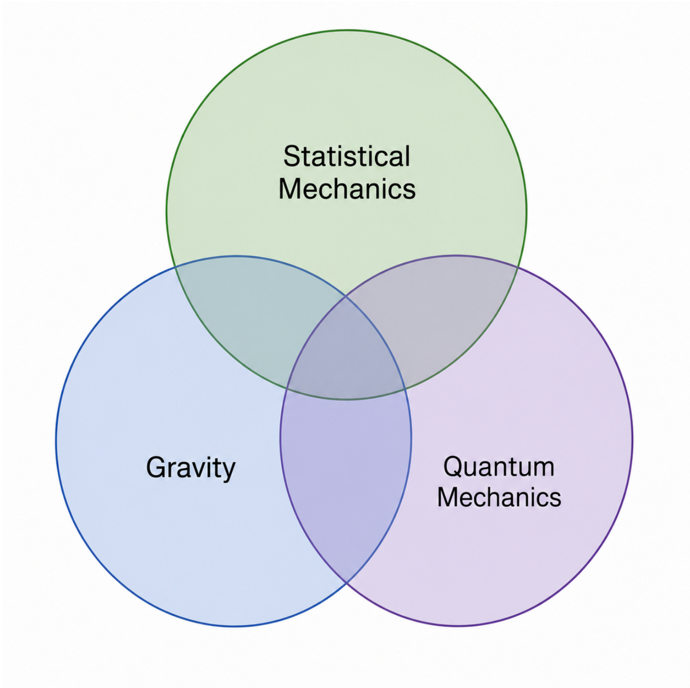


Gravity
is the information
encoded in the interplay
between geometry and matter fields

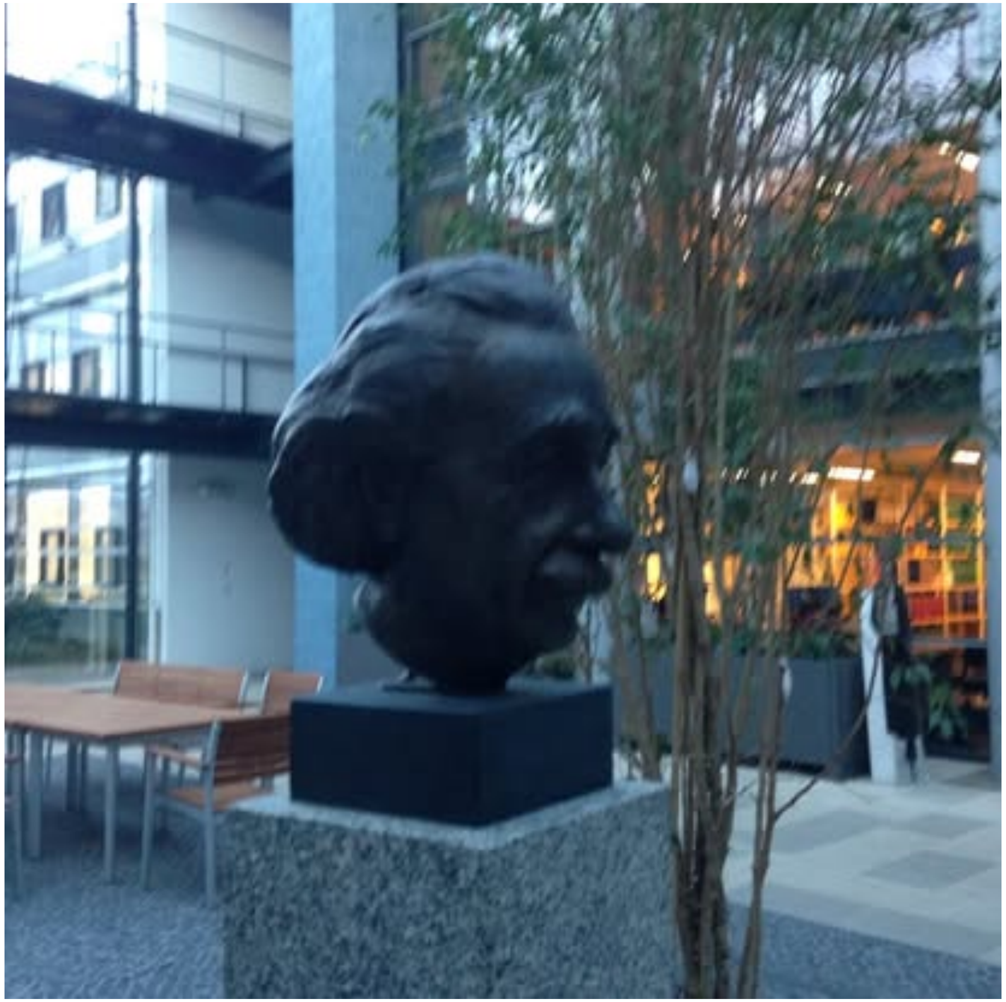
OUTLINE OF THE TALK

- Introduction of the GfE theory in a nutshell
- Discussion of the warm-up scenario
- General theory: Gravity from Entropy (GfE)
- The area law for the Schwarzschild metric and the thermodynamics of GfE cosmologies
- Conclusions

GRAVITY FROM ENTROPY IN A NUTSHELL



EINSTEIN GENERAL RELATIVITY



GENERAL RELATIVITY

Einstein-Hilbert action

Gravity is the result of the curvature of space time

$$\boxed{\mathscr{L}_G = \frac{1}{2\kappa} R}$$

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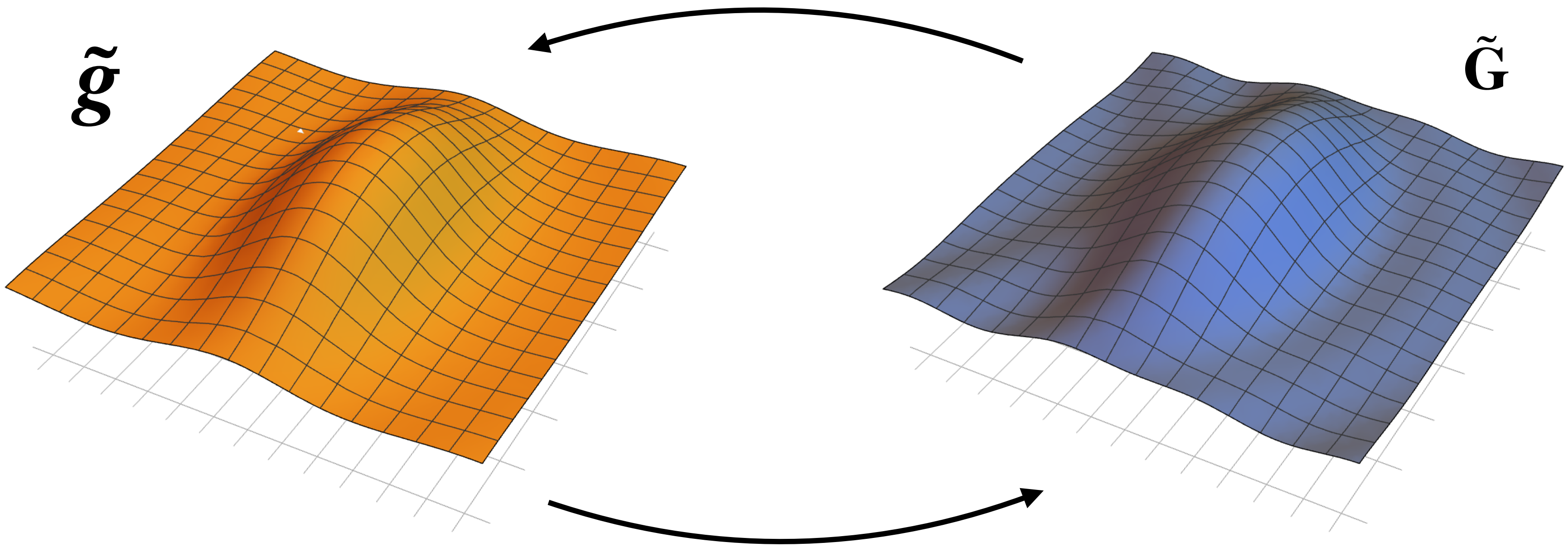
John Wheeler interpretations of Einstein equations

"*Matter tells spacetime how to curve, and curved spacetime tells matter how to move*"

$$\begin{tikzcd} R_{\mu\nu} \arrow[r, bend right] & T_{\mu\nu} \arrow[l, bend right] \end{tikzcd}$$

Can we elevate the role that in gravity has
the interplay between matter and geometry
as the key physical insight
to be reflected in the action?

GRAVITY FROM ENTROPY



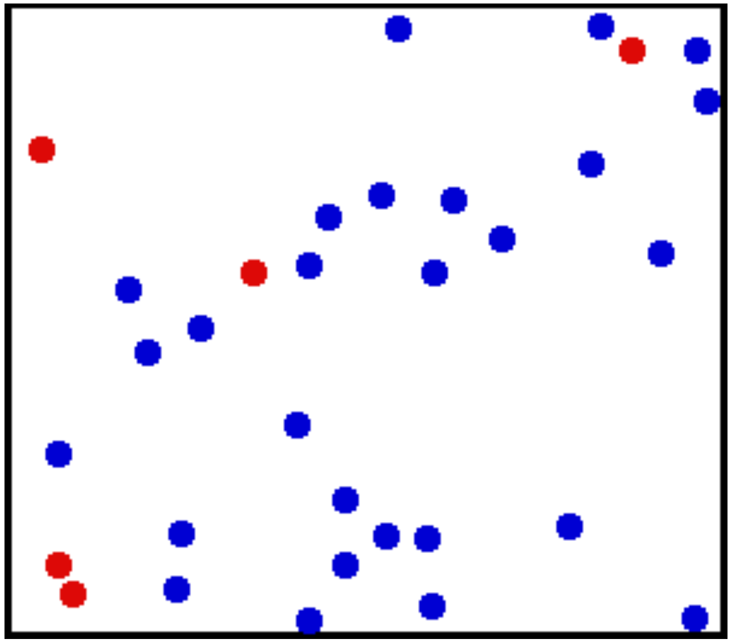
The Gravity from Entropy (GfE) Lagrangian is the Geometrical Quantum Relative Entropy (GQRE)

Gravity emerges

from the tension between the actual metric and the metric induced by matter and curvature

THE GEOMETRIC QUANTUM RELATIVE ENTROPY

ENTROPY



$$S = \ln \Omega$$

The entropy is the logarithm

of the number Ω of microstate configurations

compatible with a macrostate of the system

According to the

2nd principle of thermodynamics

the entropy of a system never decreases

QUANTUM -VON NEUMANN- ENTROPY

The Von Neumann entropy of a quantum system described by the density matrix ρ is given by

$$\boxed{H(\rho) = -Tr\rho \ln \rho}$$

which can also be expressed as

$$\boxed{H = -\sum_n \lambda_n \ln \lambda_n}$$

- The entropy quantifies the uncertainty in a quantum system
- Mixing increases the entropy (uncertainty)

QUANTUM RELATIVE ENTROPY

The quantum relative entropy

$$\boxed{S(\rho\|\sigma) = Tr\rho(\ln\rho - \ln\sigma)}$$

is a measure of distinguishability between two quantum states.

If ρ, σ commute

$$S(\rho\|\sigma) = \sum_n \lambda_n(\ln\lambda_n - \ln\tilde{\lambda}_n)$$

- **Quantum distinguishabilty never increases**
- (upon completely positive trace preserving maps)

STATISTICAL MECHANICS EMERGENCE AND INFORMATION IS PART OF NATURE

More is different

Phil W. Anderson

It from bit

John Wheeler

A theory

of quantum cosmology cannot be logically consistent

if it does not describe a complex universe

Lee Smolin, The Life of the Cosmos

STATISTICAL MECHANICS AS A FUNDAMENTAL THEORY

Can statistical mechanics and information theory

be actually part of our

fundamental understanding of nature?

GRAVITY

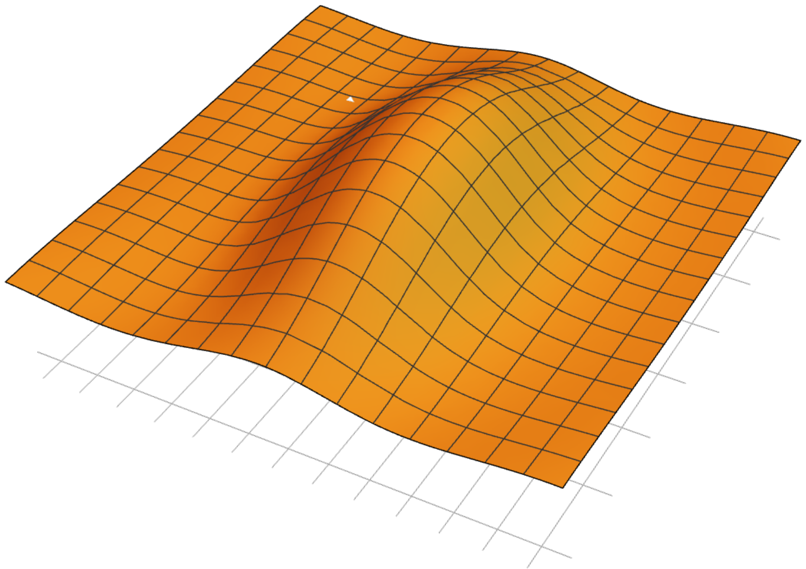
AS A THE FUNDAMENTAL STATISTICAL MECHANICS THEORY

OF GEOMETRY

which allows interactions to occur in spacetime

Gravity from Entropy

GRAVITY FROM ENTROPY



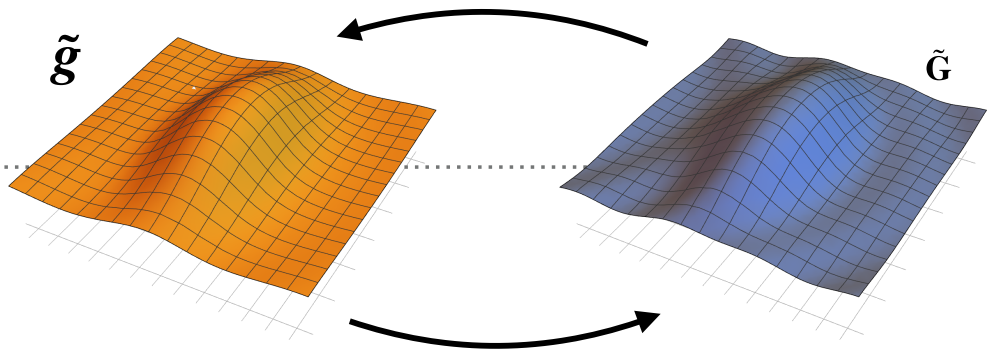
Gravity from Entropy

The metrics $g_{\mu\nu}, G_{\mu\nu}$

associated to the manifold

define quantum operators

GRAVITY FROM ENTROPY ACTION



Gravity from entropy

$$\boxed{\mathscr{L} = \mathrm{Tr}\tilde{g}\ln\tilde{\mathbf{G}}^{-1} = -\,\mathrm{Tr}_F\ln\tilde{\mathbf{G}}\tilde{g}^{-1}}$$

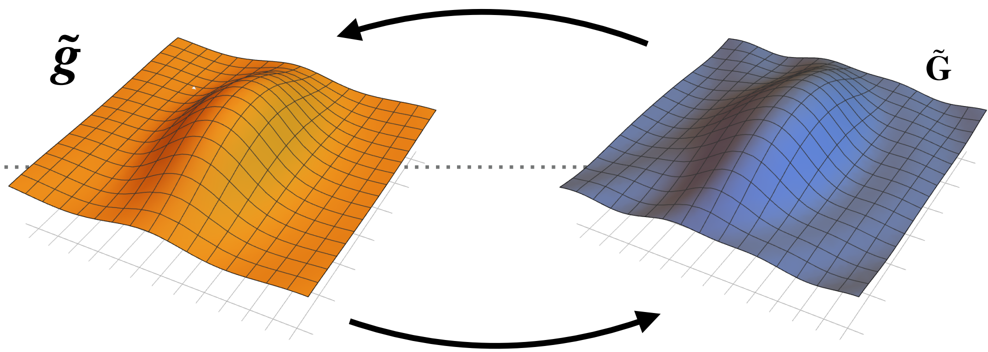
The Gravity from Entropy Lagrangian

is the geometrical quantum relative entropy between

the metric of the manifold and the metric induced by the matter fields and curvature

[Bianconi, Gravity from entropy. Phys. Rev. D 111, 066001 *(2025)*]

THE GFE ACTION



The GfE action for gravity

$$\mathcal{S} = \frac{1}{\ell_P^d} \int \sqrt{|-g|}\,\mathscr{L} d^d\mathbf{x}$$

where

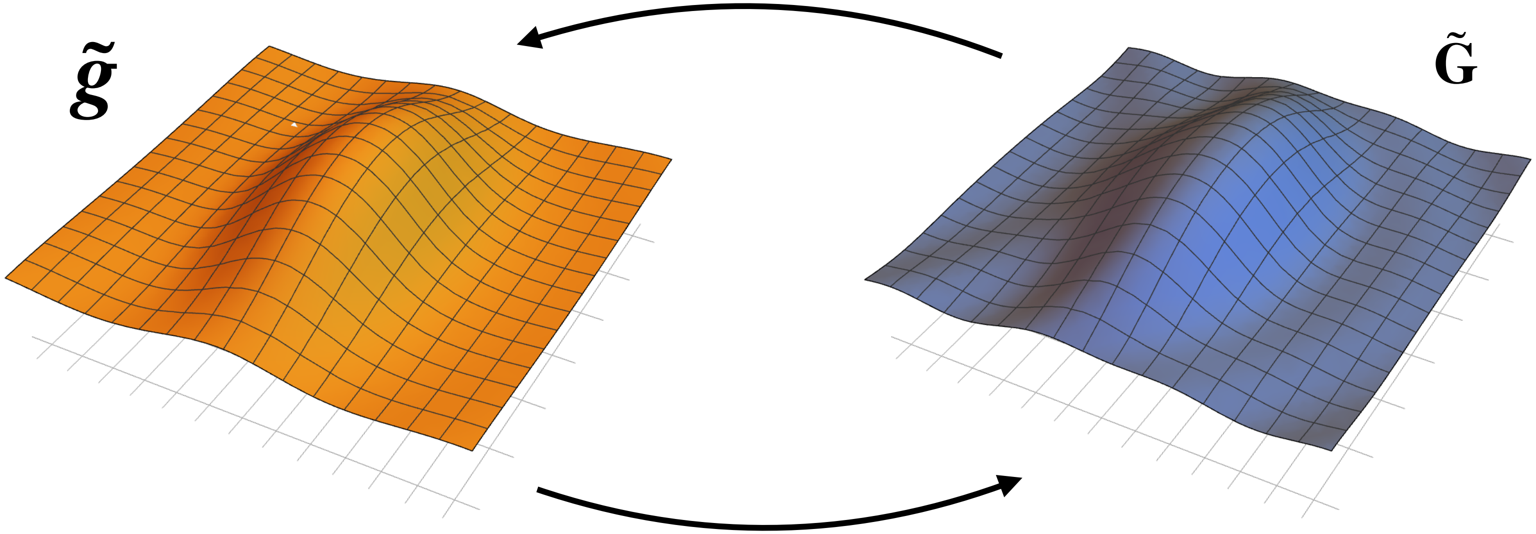
$$\mathscr{L} = \mathrm{Tr}\tilde{g} \ln \tilde{\mathbf{G}}^{-1} = -\,\mathrm{Tr}_F \ln \tilde{\mathbf{G}}\tilde{g}^{-1}$$

quantifies the information content

carried by the true metric that

that can be codified by the matter fields and the curvature

THE INTERPLAY BETWEEN MATTER AND GEOMETRY



$\tilde{g}$ encodes the actual metric

defines the scalar product at each point

$\tilde{G}$ encodes both the matter and the curvature

Depends on the connection and the curvature

Prescribes how the metric changes from point to point

WARM-UP SCENARIO

EIGENVALUES OF A METRIC TENSOR

Definition of eigenvalues

Given the metric tensor $G_{\mu\nu}$ at a given point p of the manifold $\mathscr{K}$,

we define its eigenvalues λ in a Lorentz invariant way,

$$\boxed{G_{\mu\nu}V^{\nu} = \lambda V_{\mu}}$$

This eigenvalue problem can be reduced to

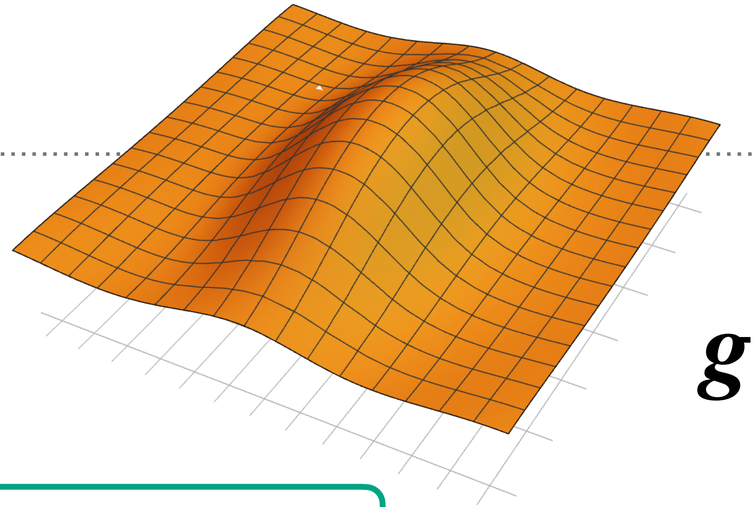
the eigenvalue problem of the the matrix $\mathbf{G}_{\mu\nu}\mathbf{g}^{\nu\alpha}$ as we have

$$\boxed{G_{\mu\nu}V^{\nu} = G_{\mu\nu}g^{\nu\alpha}V_{\alpha} = \lambda V_{\mu}}$$

The logarithm of G is given by

$$[\ln \mathbf{G}]_{\mu\nu} = \sum_{\lambda} (\ln \lambda) V_{\mu}^{(\lambda)} V_{\nu}^{(\lambda)} \text{ with } V_{\mu}^{(\lambda)} g^{\mu\nu} V_{\nu}^{(\lambda)} = 1$$

EIGENVALUES OF THE METRIC g



Eigenvalues of the metric g

- **Any invertible real metric $g_{\mu\nu}$ has only eigenvalues $\lambda = 1$.**

- Indeed we have that $g_{\mu\nu}V^{\nu} = V_{\mu}$ is an identity.

- Equivalently $g_{\mu\nu}g^{\nu\alpha} = \delta_{\mu}^{\ \alpha}$ is a matrix having only eigenvalues equal to one.

THE GEOMETRIC QUANTUM ENTROPY OF THE METRIC TENSORS

- The geometric quantum entropy of a general metric tensor $G_{\mu\nu}$ is given by

$$H(G) = \mathrm{Tr}\ G \ln G^{-1} = -\sum_n \lambda_n \ln \lambda_n$$

Since the metric $g_{\mu\nu}$ has all the eigenvalues λ_n equal to one, $\lambda_n = 1$

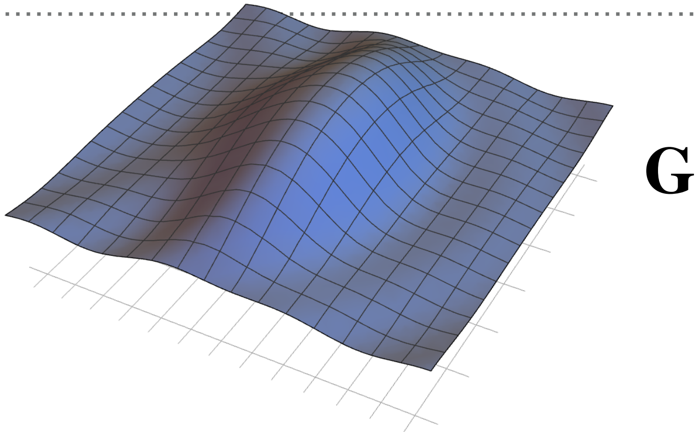
it follows that

the entropy associated to the metric $g_{\mu\nu}$ is null,

$$\mathbf{H(g) = Tr\ g \ln g^{-1} = 0}$$

GAUSS LEGACY

THE METRIC INDUCED BY THE MATTER FIELD

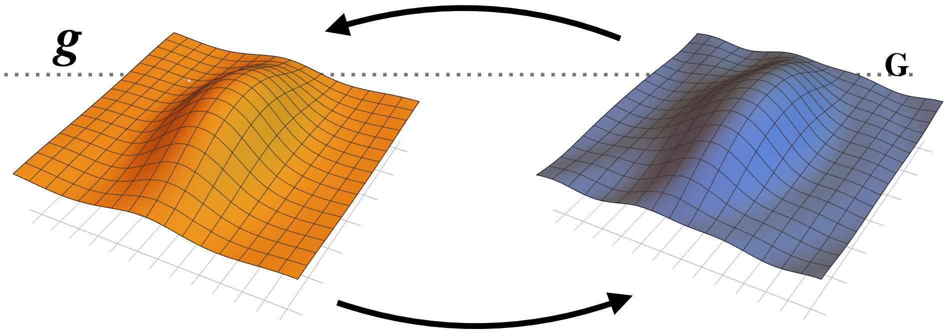


- Consider a boson matter field $\phi(\mathbf{x}) \in \mathbb{C}$, at any point p of $\mathscr{K}$ we consider the manifold $\mathscr{K} \otimes \mathbb{C}$ of coordinates $(\mathbf{x}, \phi(\mathbf{x}))$.

- The metric induced by this manifold on $\mathscr{K}$ is indicated by $\mathbf{G}$ of elements

$$\boxed{G_{\mu\nu} = g_{\mu\nu} + \alpha M_{\mu\nu}} \text{ with } \boxed{M_{\mu\nu} = (\nabla_\mu \bar{\phi})(\nabla_\nu \phi)} \text{ where } \alpha = \alpha' \ell_P^d$$

THE GFE LAGRANGIAN



- The Geometrical Quantum Relative Entropy between g and $\mathbf{G}$,

where $\mathbf{G}$ is the metric induced by the matter fields assumed to be positively defined is given by

$$\mathscr{L} = -\mathrm{Tr}g \ln g^{-1} + \mathrm{Tr}g \ln \mathbf{G}^{-1}$$

- Since the entropy of g is null, the GfE Lagrangian is given by

$$\boxed{\mathscr{L} = \mathrm{Tr}g \ln \mathbf{G}^{-1} = -\, \mathrm{Tr}_M \ln \mathbf{G}g^{-1}}$$

SOME TECHNICAL DETAILS

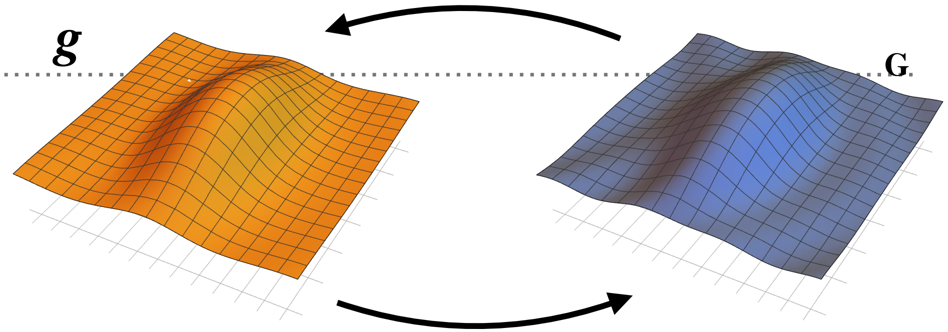
We want to discuss and prove that the adopted quantum relative entropy can be equivalently expressed as

$$\boxed{\mathscr{L} = \operatorname{Tr} g \ln \mathbf{G}^{-1} = -\operatorname{Tr}_M \ln \mathbf{G} g^{-1}}$$

- The eigenvalues λ_n of the metric $g_{\mu\nu}$ are equal to one.
- The eigenvalues $\tilde{\lambda}_n$ of the metric $G_{\mu\nu}$ induced by the matter field reduce to the eigenvalues of the matrix $G_{\mu\nu} g^{\nu\rho}$
- Thus we have

$$\mathscr{L} = \operatorname{Tr} g \ln \mathbf{G}^{-1} = -\sum_{n=1}^{d} \lambda_n \ln \tilde{\lambda}_n = -\sum_{n=1}^{d} \ln \tilde{\lambda}_n = -\operatorname{Tr}_M \ln \mathbf{G} g^{-1}$$

THE GFE THEORY ACTION



- The GfE Lagragian is given by

$$\mathscr{L} = \mathrm{Tr}g \ln \mathbf{G}^{-1} = -\, \mathrm{Tr}_M \ln \mathbf{G}g^{-1}$$

- The GFE action is

$$\boxed{\mathcal{S} = \frac{1}{\ell_P^d}\int \sqrt{|-g|}\,\mathscr{L} d^d\mathbf{x}}$$

THE MASSELESS KLEIN-GORDON EQUATION EMERGES FROM THE ENTROPIC ACTION

The action becomes

$$\mathscr{L} = -\ln(1 + \alpha|\nabla\phi|^2)$$

with

$$|\nabla\phi|^2 = \nabla_\mu\bar{\phi}\,\nabla^\mu\phi$$

In the low energy limit

the Lagrangian reduces to massless Klein-Gordon

$$\mathscr{L} = -\alpha\nabla_\mu\bar{\phi}\,\nabla^\mu\phi$$

but at the Planck scale it deviates from it already in flat geometry

THE ACHIEVEMENTS AND THE LIMITATIONS OF THIS WARM-UP SCENARIO

Achievements

A great achievement of this warm-up scenario is that we get

the Klein-Gordon equation out of the GfE action in the low energy limit

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Limitations

- A limitation is that we only get the **massless** Klein-Gordon equation
- The other limitation is that the Lagrangian $\mathscr{L} = -\ln(1 + |\nabla\phi|^2)$ is null in absence of matter field, so the metric g is undetermined in absence of matter fields

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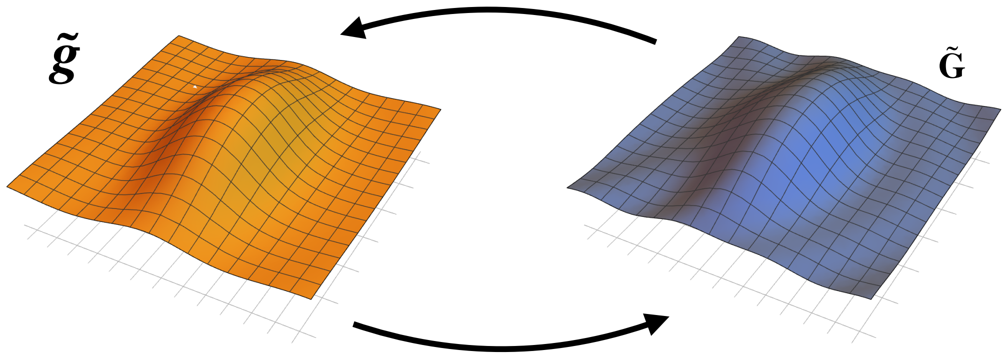
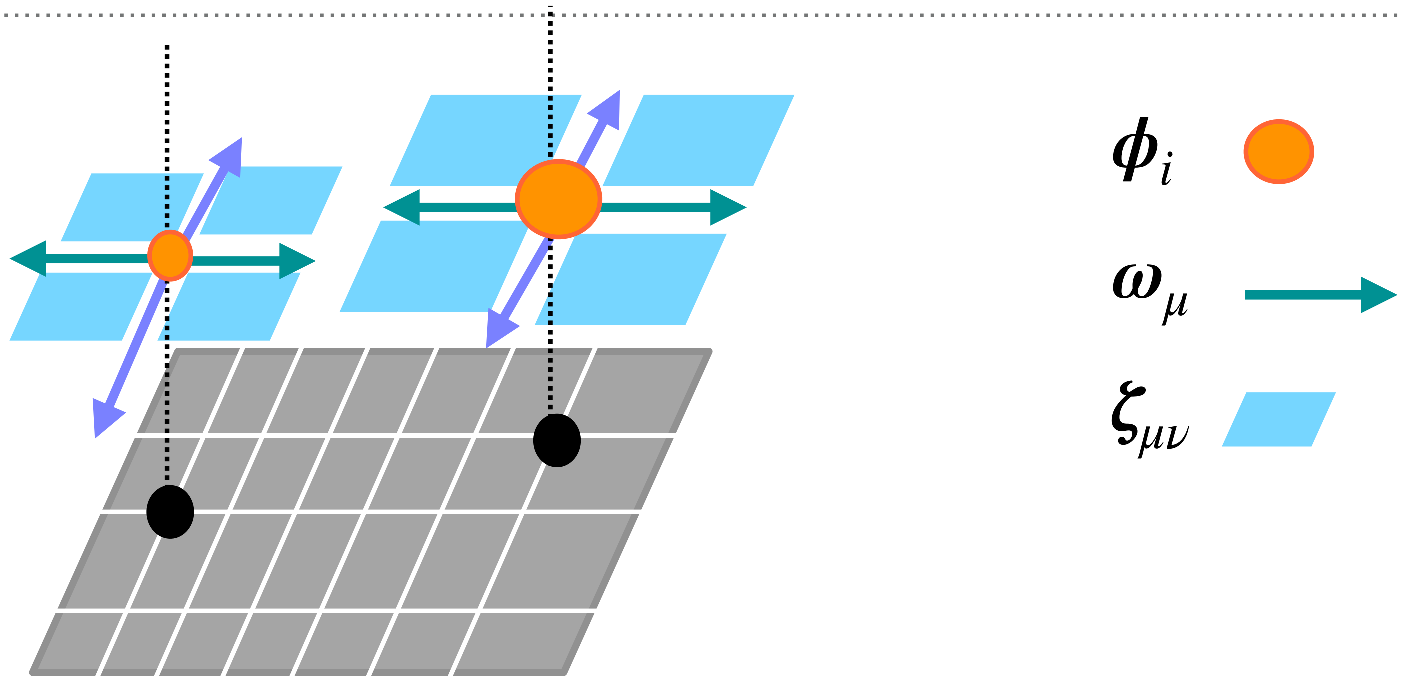
In the following we are going to address both limitations of this warm-up scenario

GENERAL THEORY

$$\mathscr{L} = -Tr\tilde{g}\ln\tilde{\mathbf{G}} = -\,Tr_M\ln\tilde{\mathbf{G}}\tilde{g}^{-1}$$



TOPOLOGICAL DESCRIPTION OF MATTER FIELDS



At each point of time we define matter fields

topologically as a direct sum of a zero form, a one-form and a two-form

TOPOLOGICAL MATTER FIELDS

The topological fields are vector of the Hilbert space

$$\boxed{|\Phi\rangle = \phi \oplus \omega_\mu dx^\mu \oplus \frac{1}{2}\zeta_{\mu\nu} dx^\mu \wedge dx^\nu}$$

and their adjoint is given by

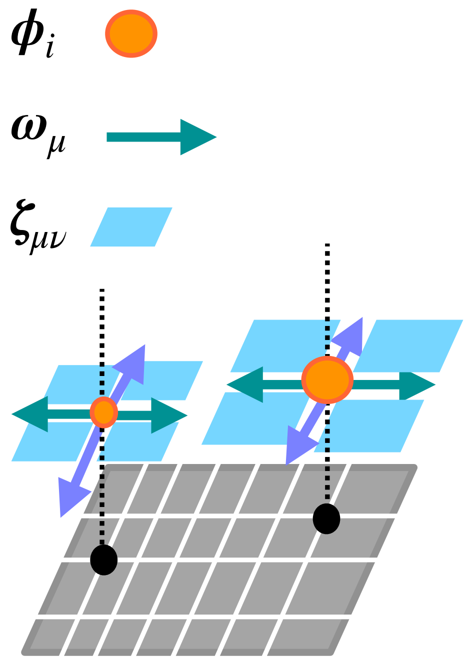
$$\langle\Phi| = \bar{\phi} \oplus \bar{\omega}_\mu dx^\mu \oplus \frac{1}{2}\bar{\zeta}_{\mu\nu} dx^\mu \wedge dx^\nu$$

The Hodge-Dirac operator $\boxed{\mathbf{D} = d + \delta}$ acts of the topological fields as

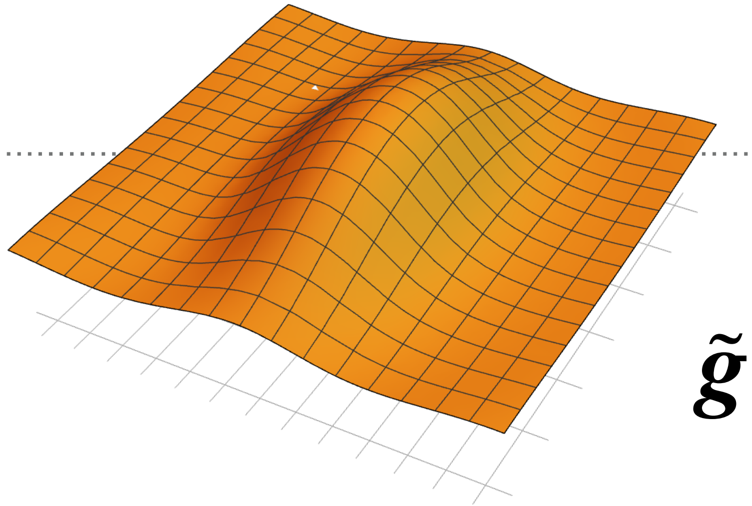
$$\mathbf{D}|\Phi\rangle = -\nabla^\mu \omega_\mu \oplus (\nabla_\mu \phi - \nabla^\rho \zeta_{\mu\rho}) dx^\mu \oplus \nabla_\mu \omega_\nu dx^\mu \wedge dx^\nu$$

its adjoint is

$$\langle\Phi|\mathbf{D}$$



TOPOLOGICAL METRIC OF THE MANIFOLD

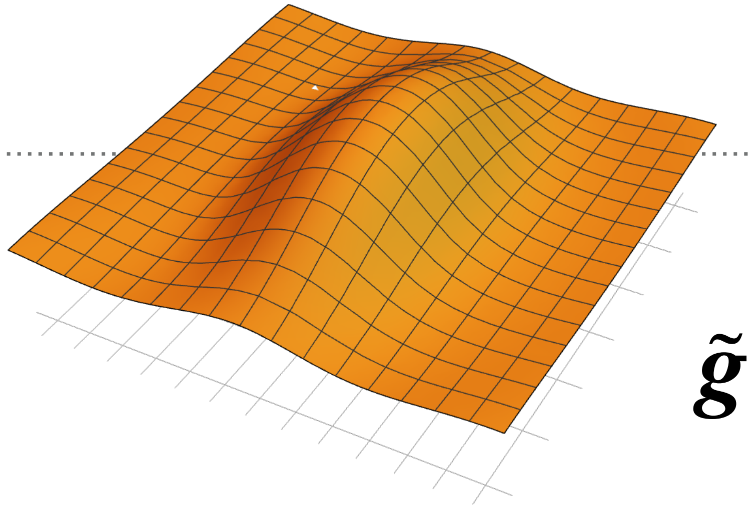


- We consider the topological metric $\tilde{g}$ given by

$$\boxed{\tilde{g} = 1 \oplus g_{\mu\nu} dx^\mu \otimes dx^\nu \oplus g^{(2)}{}_{\mu\nu\rho\sigma} (dx^\mu \wedge dx^\nu) \otimes (dx^\rho \wedge dx^\sigma)}$$

where $g^{(2)}_{\mu\nu\rho\sigma} = \dfrac{1}{2}(g_{\mu\rho} g_{\nu\sigma} - g_{\nu\rho} g_{\mu\sigma})$

EIGENVALUES OF THE METRIC $\tilde{g}$



Eigenvalues of the metric $\tilde{g}$

- Any invertible real metric $g_{\mu\nu}$ is associated to a metric $g^{(2)}$ which has only eigenvalues $\lambda = 1$.
- The entropy of $\tilde{g}$ is identically zero.

HIGHER_ORDER METRIC INDUCED BY MATTER FIELDS

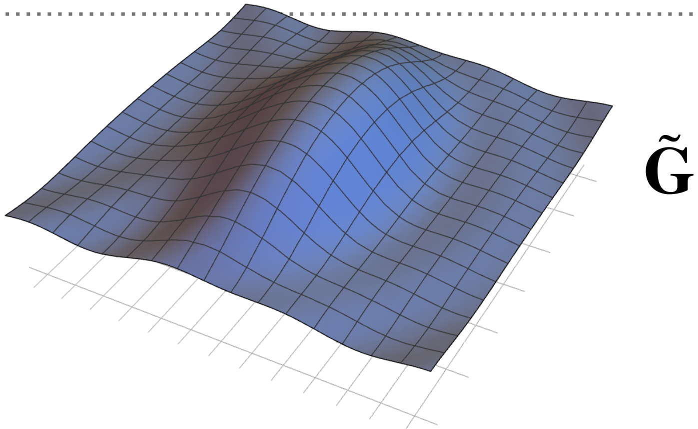
- The metrics induced by the matter fields and the geometry given in general by

$$\boxed{\tilde{G} = G^{(0)} \oplus G^{(1)}_{\mu\nu} dx^{\mu} \otimes dx^{\nu} \oplus G^{(2)}{}_{\mu\nu\rho\sigma}(dx^{\mu} \wedge dx^{\nu}) \otimes (dx^{\rho} \wedge dx^{\sigma})}$$

Observations

- We can easily extend the notion of eigenvalues, entropy and quantum relative entropy to metrics between n-forms [see G. Bianconi 2025 for details].

THE METRIC INDUCED BY THE MATTER FIELD AND THE GEOMETRY



- In the warm-up scenario the metric induced by the matter field was given by $\mathbf{G}$ of elements

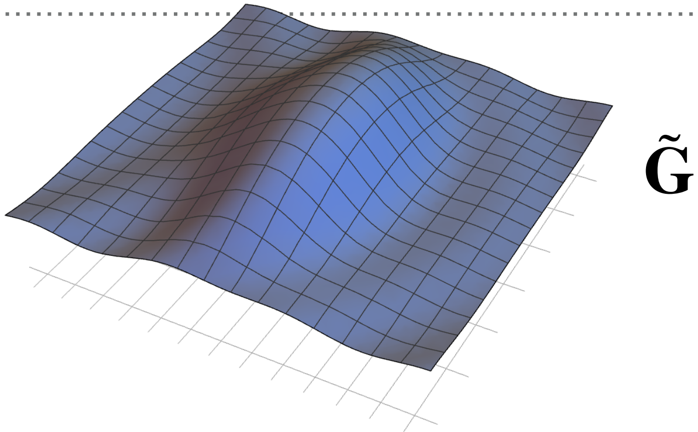
$$\boxed{G_{\mu\nu} = g_{\mu\nu} + \alpha M_{\mu\nu}} \text{ with } M_{\mu\nu} = (\nabla_\mu \bar{\phi})(\nabla_\nu \phi) \text{ where } \alpha = \alpha' \ell_P^d$$

- By considering topological matter fields it is natural to assume

$$\boxed{\tilde{G} = \tilde{g} + \alpha \mathbf{D} \,|\, \Phi \rangle \langle \Phi \,|\, \mathbf{D}}$$

where we substitute the covariant derivative with the Dirac operator $\mathbf{D}$

THE METRIC INDUCED BY THE MATTER FIELD AND THE GEOMETRY AS A DENSITY MATRICS



- If we interpret the metric $\tilde{G}$ as a density matrix $\mathbf{D}\,|\,\Phi\rangle\langle\Phi\,|\,\mathbf{D}$ can be seen as a projector, **in the spirit of Von Neumann algebras let us start adding relevant other terms…**

$$\boxed{\tilde{\mathbf{G}} = \tilde{g} + \alpha\tilde{\mathbf{M}}}$$

where

$$\boxed{\tilde{\mathbf{M}} = \mathbf{D}\,|\,\Phi\rangle\langle\Phi\,|\,\mathbf{D} + m^2\,|\,\Phi\rangle\langle\Phi\,|}$$

HIGHER-ORDER CURVATURE

- The curvature of the manifold $\mathscr{K}$ is described topologically as

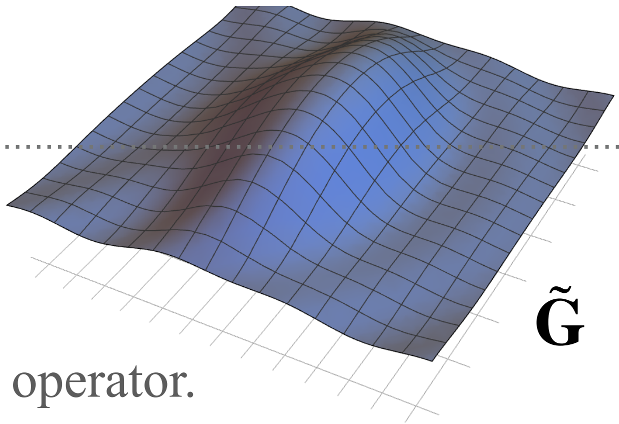
$$\boxed{\tilde{\mathscr{R}} = R \oplus R_{\mu\nu} dx^{\mu} \otimes dx^{\nu} \oplus R_{\mu\nu\rho\sigma}(dx^{\mu} \wedge dx^{\nu}) \otimes (dx^{\rho} \wedge dx^{\sigma})}$$

where R is the Ricci scalar, $R_{\mu\nu}$ is the Ricci tensor and $R_{\mu\nu\rho\sigma}$ is the Riemann tensor.

Observations

- We observe that $\tilde{\mathscr{R}}$ has the same structure as the general metric $\tilde{\mathbf{G}}$
- We also observe that $\tilde{\mathscr{R}}$ cannot be interpreted as a projector in general.

TOWARD A GEOMETRICAL QUANTUM MECHANICS



We interpret the metric $\tilde{\mathbf{G}}$ as a quantum operator.

Building on the first fundamental form of Gauss and

the theory of von Neumann algebra [Witten RMP 2018],

$\tilde{\mathbf{G}}$ for boson matter field is given by

$$\boxed{\tilde{\mathbf{G}} = \tilde{g} + \alpha\tilde{\mathbf{M}} - \beta\tilde{\mathscr{R}}}$$

where

$$\boxed{\tilde{\mathbf{M}} = \mathbf{D}\,|\,\Phi\rangle\langle\Phi\,|\,\mathbf{D} + m^2\,|\,\Phi\rangle\langle\Phi\,|}$$

Here $\tilde{\mathscr{R}}$ includes Ricci scalar, Ricci tensor, and Riemann tensor as well

ABELIAN GAUGE FIELDS

The Abelian gauge fields A_μ can be included in this framework by postulating that the metric induced by the matter fields and the geometry is given by

$$\tilde{\mathbf{G}} = \tilde{g} + \alpha \left[\tilde{\mathbf{M}} - \frac{1}{4} \tilde{\mathbf{F}} \right] - \beta \mathscr{R}$$

where includes the electromagnetic field

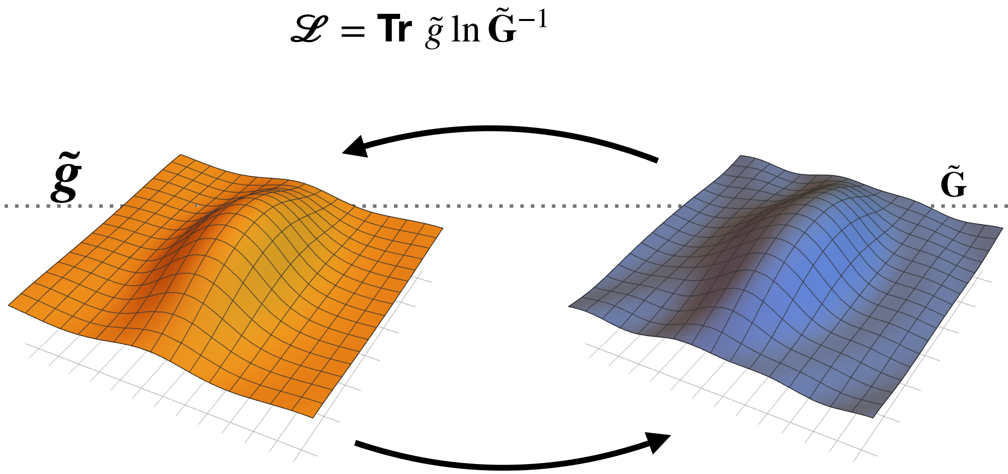
$$\tilde{\mathbf{F}} = 0 \oplus 0_{\mu\nu} dx^\mu \otimes dx^\nu \oplus F_{\mu\nu} F_{\rho\sigma} (dx^\mu \wedge dx^\nu \otimes dx^\rho \wedge dx^\sigma)$$

where here $F_{\mu\nu}$ is given by $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

and where we consider the minimal substitution to the covariant derivative

$$\nabla_\mu \rightarrow \nabla_\mu - \mathrm{i} e A_\mu$$

THE GFE ACTION



The GfE action reads

$$\mathscr{L} = -\mathrm{Tr}\tilde{g} \ln \tilde{g}^{-1} + \mathrm{Tr}\tilde{g} \ln \tilde{\mathbf{G}}^{-1}$$

Since the entropy of $\tilde{g}$ is null, the Lagrangian is given by

$$\boxed{\mathscr{L} = \mathrm{Tr}\tilde{g} \ln \tilde{\mathbf{G}}^{-1} = -\,\mathrm{Tr}_F \ln \tilde{\mathbf{G}}\tilde{g}^{-1}}$$

RESULTS

THE LOW ENERGY-SMALL CURVATURE LIMIT

In the low-energy, small curvature limit

we recover

the Einstein-Hilbert action

with zero-cosmological constant

$$\mathscr{L} = \hat{\mathscr{L}}_G + \hat{\mathscr{L}}_M$$

$$\hat{\mathscr{L}}_G = \frac{1}{2\kappa} R$$

LOW COUPLING REGIME: EINSTEIN-HILBERT ACTION WITH ZERO COSMOLOGICAL CONSTANT

In the low coupling regime the entropic action

$$\mathscr{L} - \mathrm{Tr}_M \ln \tilde{\mathbf{G}}\tilde{g}^{-1} = -\,\mathrm{Tr}_M \ln\left((\tilde{g} + \alpha\tilde{\mathbf{M}} - \beta\tilde{\boldsymbol{\mathscr{R}}})\tilde{g}^{-1}\right) = -\,\mathrm{Tr}_M \ln\left(\tilde{\mathbf{I}} + (\alpha\tilde{\mathbf{M}} - \beta\tilde{\boldsymbol{\mathscr{R}}})\tilde{g}^{-1}\right)$$

can be linearised and

reduces to the Einstein-Hilbert Lagrangian with zero cosmological constant

plus the matter field Lagrangian

$$\boxed{\mathscr{L} = \mathscr{L}_G + \mathscr{L}_M}$$

where we have

$$\boxed{\mathscr{L}_G = \beta\mathrm{Tr}\tilde{\boldsymbol{\mathscr{R}}}\tilde{g}^{-1} = 3\beta R, \quad \mathscr{L}_M = -\,\alpha\mathrm{Tr}\tilde{\mathbf{M}}\tilde{g}^{-1}}$$

THE G-FIELD

The geometrical quantum entropy action

leads to a modified theory of gravity

expressed in terms of

the G-field $\tilde{\mathcal{G}}$

THE INTRODUCTION OF THE G-FIELD

The considered Lagrangian

$$\mathscr{L} = \mathrm{Tr}\tilde{g} \ln \tilde{\mathbf{G}}^{-1} = -\, \mathrm{Tr}_F \ln \tilde{\mathbf{G}}\tilde{g}^{-1}$$

is not linear but only dependent on

$$\tilde{\mathbf{\Theta}} = \tilde{\mathbf{G}}\tilde{g}^{-1}$$

Imposing this set of constraints as Lagrangian multipliers using the auxiliary fields $\tilde{\mathscr{G}}, \tilde{\mathbf{\Theta}}$ we obtain the Lagrangian

$$\tilde{\mathscr{L}} = -\mathrm{Tr} \ln \tilde{\mathbf{\Theta}} - \mathrm{Tr}\ \tilde{\mathscr{G}}(\tilde{\mathbf{G}}\tilde{g}^{-1} - \tilde{\mathbf{\Theta}})$$

which is now a Lagrangian linear in the curvature.

G-FIELD

The G-field,

introduced as a set of Lagrangian multipliers,

is here interpreted as

a physical and measurable field

Other examples of physical Lagrangian multipliers

are

the temperature and the chemical potential

THE ROLE OF THE G-FIELD

The effective metric is dressed by the G-field

$$\boxed{\tilde{g}_{\mathscr{G}} = \tilde{g}\,\tilde{\boldsymbol{\mathscr{G}}}^{-1}}$$

The dressed Ricci scalar is given by the Ricci tensor

contracted with the dressed metric

$$\boxed{R_{\mathscr{G}} = \mathbf{Tr}_F\,\tilde{g}_{\mathscr{G}}^{-1}\,\tilde{\mathscr{R}}}$$

THE ACTION AS A FUNCTION OF THE G-FIELD

The Gravity from Entropy Lagrangian in terms of the G-field is given by

$$\mathscr{L} = \mathscr{L}_G + \mathscr{L}_M$$

where

$$\mathscr{L}_G = \frac{1}{2\kappa}(\mathscr{R}_{\mathscr{G}} - 2\Lambda_{\mathscr{G}}) \qquad \mathscr{L}_M = \mathbf{Tr}_F \tilde{g}_{\mathscr{G}}^{-1}\tilde{\mathbf{M}}$$

It has the same structure of the Einstein-Hilbert action + matter action with the metric $\tilde{g}_{\mathscr{G}} = \tilde{g}\mathscr{G}^{-1}$ dressed by the G-field and the emergent cosmological "constant" $\Lambda_{\mathscr{G}}$

EQUATIONS OF MOTION

The action depends now on

the metric, on the matter-fields and on the G-field.

By variational methods we get therefore three main equations of motion

1. Equation of motion for the matter fields, and the gauge fields
2. Modified gravity equations for the metric
3. Equations for the G-field (the Θ field is the inverse of the G-field)

GRAVITY FROM ENTROPY EQUATIONS

- **The Gravity from Entropy equations are**

$$\boxed{R^{\mathscr{G}}_{(\mu\nu)} - \frac{1}{2} g_{\mu\nu}\left(\mathscr{R}_{\mathscr{G}} - 2\Lambda_{\mathscr{G}}\right) + \mathscr{D}_{(\mu\nu)} = \kappa \mathscr{T}_{(\mu\nu)},}$$

Where

- $\mathscr{T}_{\mu\nu} = -\frac{1}{\sqrt{|-g|}} \frac{\delta \mathscr{S}_M}{dg_{\mu\nu}}$ is the dressed energy-momentum tensor,
- $R^{\mathscr{G}}_{\mu\nu}$ is the dressed Ricci tensor
- $\mathscr{D}_{\mu\nu}$ includes exclusively second order derivatives of the G-field

In Gravity from Entropy

$\Lambda_{\mathscr{G}}$, $\mathscr{D}_{\mu\nu}$ and $\mathscr{T}_{\mu\nu}$ might be interpreted as the dark energy and dark matter contributions

EMERGENT COSMOLOGICAL CONSTANT

Gravity is information

The cosmological “constant” is emergent

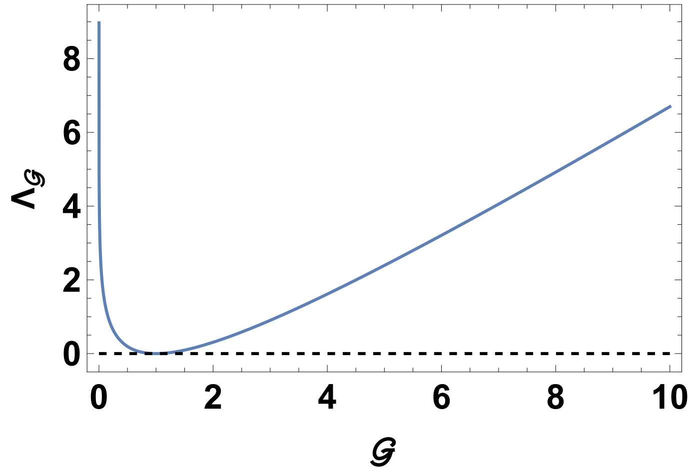
THE EMERGENT COSMOLOGICAL “COSTANT”

The emergent cosmological “constant” given by

$$\Lambda_{\mathscr{G}} = \frac{1}{\beta' \ell_P^2} \mathrm{Tr}_F[\tilde{\boldsymbol{\mathscr{G}}} - \tilde{\mathbf{I}} - \ln(\tilde{\boldsymbol{\mathscr{G}}})]$$

is positive

and only zero in the Minkowski vacuum



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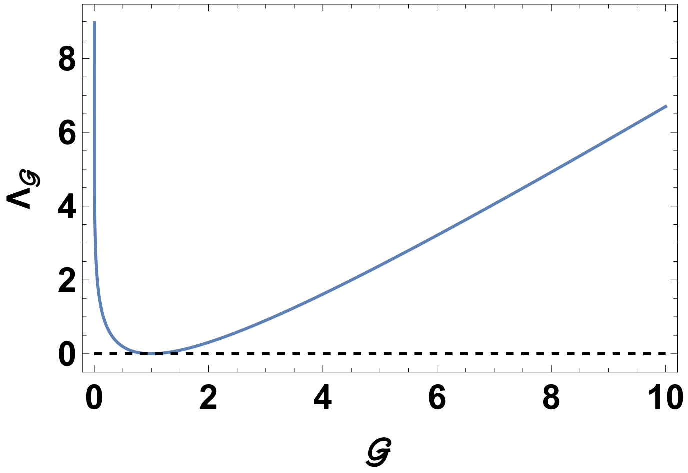


$\Lambda_{\mathscr{G}}$ is small in the low-energy limit but

it is dynamical and diverges

for high curvature and high energies

DERIVATION OF THE AREA LAW

HOLOGRAPHIC ARGUMENT

We consider the Gibbs-entropy

$$\mathcal{S} = \ln \Omega$$

Assuming that each elementary volume in space-time is associated to the same number of degrees of freedom

(example 2 for spin $s = 1$ and spin $s = -1$)

$$\Omega = 2^V$$

The entropy is therefore extensive on the volume of spacetime

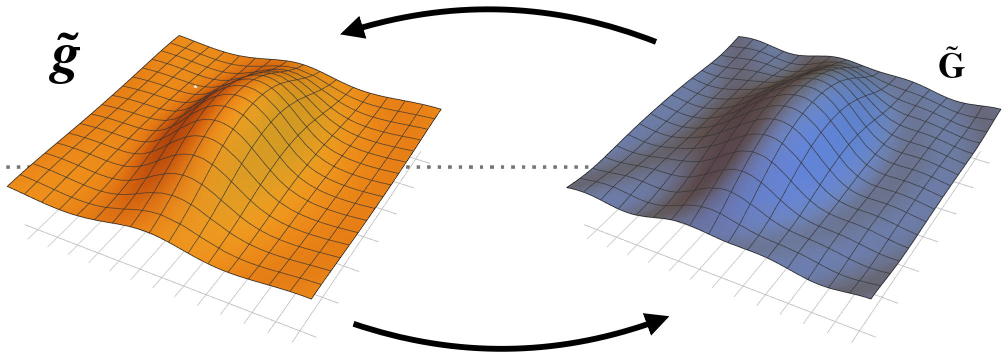
$$S = V \ln 2$$

Holographic argument:

In order to obey the area-law $S = \dfrac{A}{4G}$

the degrees of freedom must be only associated to the horizon.

GEOMETRIC QUANTUM RELATIVE ENTROPY



The Gravity from Entropy Lagrangian given by

$$\boxed{\mathscr{L} = \mathrm{Tr}\tilde{g}\ln \tilde{\mathbf{G}}^{-1} = -\,\mathrm{Tr}_F \ln \tilde{\mathbf{G}}\tilde{g}^{-1}}$$

can be expressed also as

$$\boxed{\mathscr{L} = \ln \Omega}$$

where

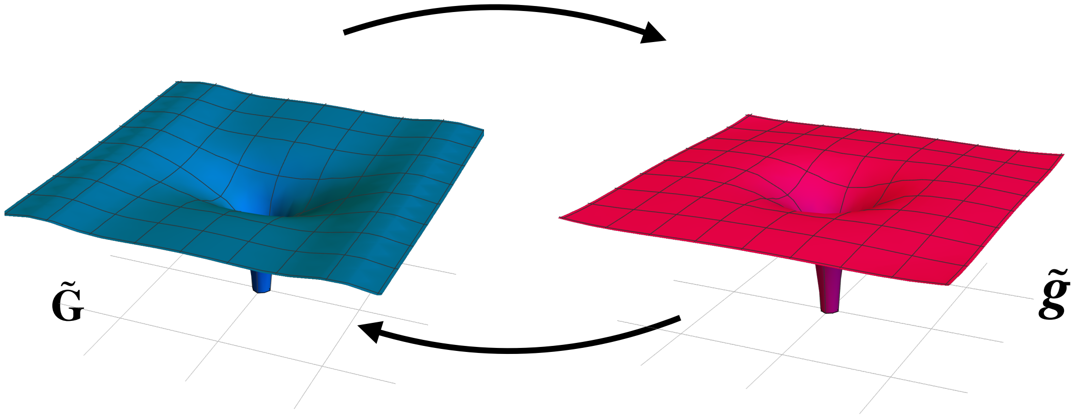
$$\boxed{\Omega = G_{(0)}^{-1}(\det(\mathbf{G}_{(1)}^{-1}g))(\det(\mathbf{G}_{(2)}^{-1}g_{(2)}))}$$

Ω evaluates the number of degrees of freedom of the metric,

and is not uniform for a black-hole

QUANTUM RELATIVE ENTROPY OF THE SCHWARZSCHILD METRIC

$$\mathscr{L} = \mathbf{Tr}\ \tilde{g} \ln \tilde{\mathbf{G}}^{-1} = -\,\mathbf{Tr}_F\ \ln \tilde{\mathbf{G}}\tilde{g}^{-1}$$



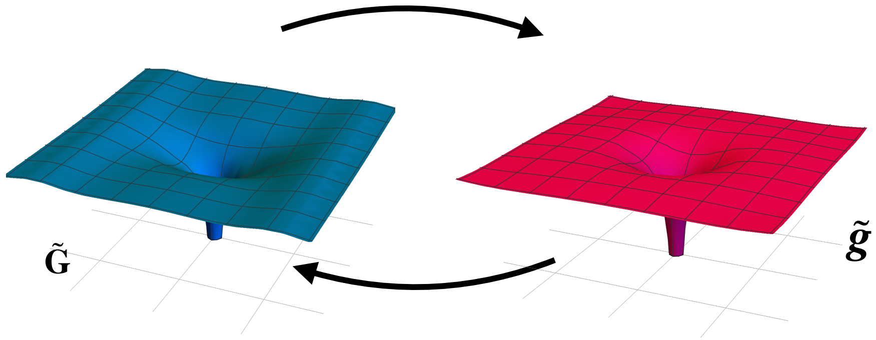
We can calculate the geometric quantum relative entropy of the Schwarzschild metric also if this metric is only the approximate solution of the equation of modified gravity

QUANTUM RELATIVE ENTROPY OF SCHWARZSCHILD METRIC

In absence of matter,

the metric induced by the curvature is

$$\boxed{\tilde{\mathbf{G}} = \tilde{g} - \beta\tilde{\mathcal{R}}}$$



The Schwarzschild metric

is an approximate solution

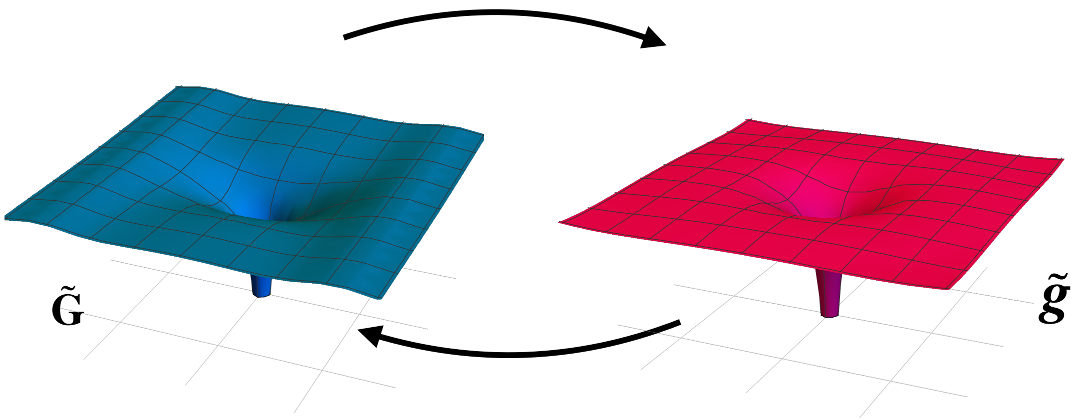
of the modified gravity equations for a black hole

good for large Schwarzschild radius

[Bianconi, G., Entropy 27, 266 (2025)]

QUANTUM RELATIVE ENTROPY OF SCHWARZSCHILD METRIC

$$\mathscr{L} = \textbf{Tr}\ \tilde{g} \ln \tilde{\mathbf{G}}^{-1}$$



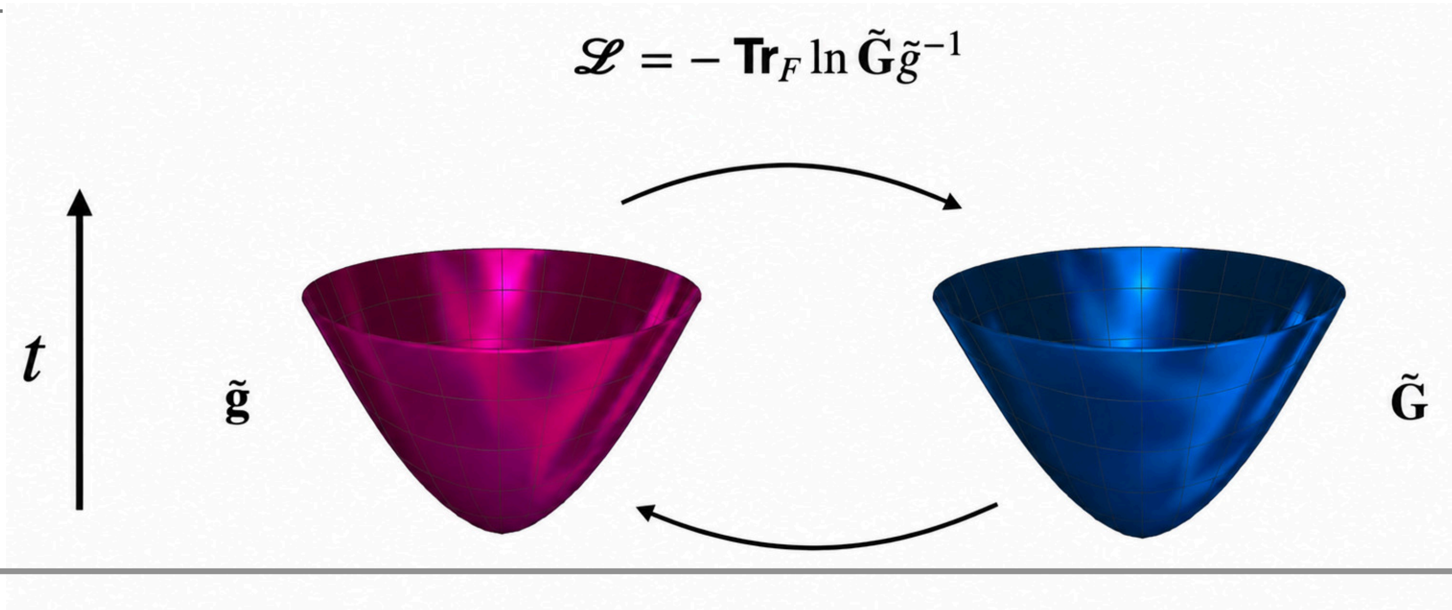
The GfE entropy associated

to the Schwarzschild metric o

beys the area law for large Schwarzschild radius

[Bianconi, G., Entropy 27, 266 (2025)]

THERMODYNAMICS OF GFE



By identifying $\Lambda_{\mathscr{G}}$ as energy density , GQRE $\mathscr{L}$ as the entropy density of GfE and $\sqrt{-|g|}$ as the volume density, and the GfE action $\mathcal{S}$ as total entropy we can formulate

the thermodynamics of GfE cosmologies in the FRW metric

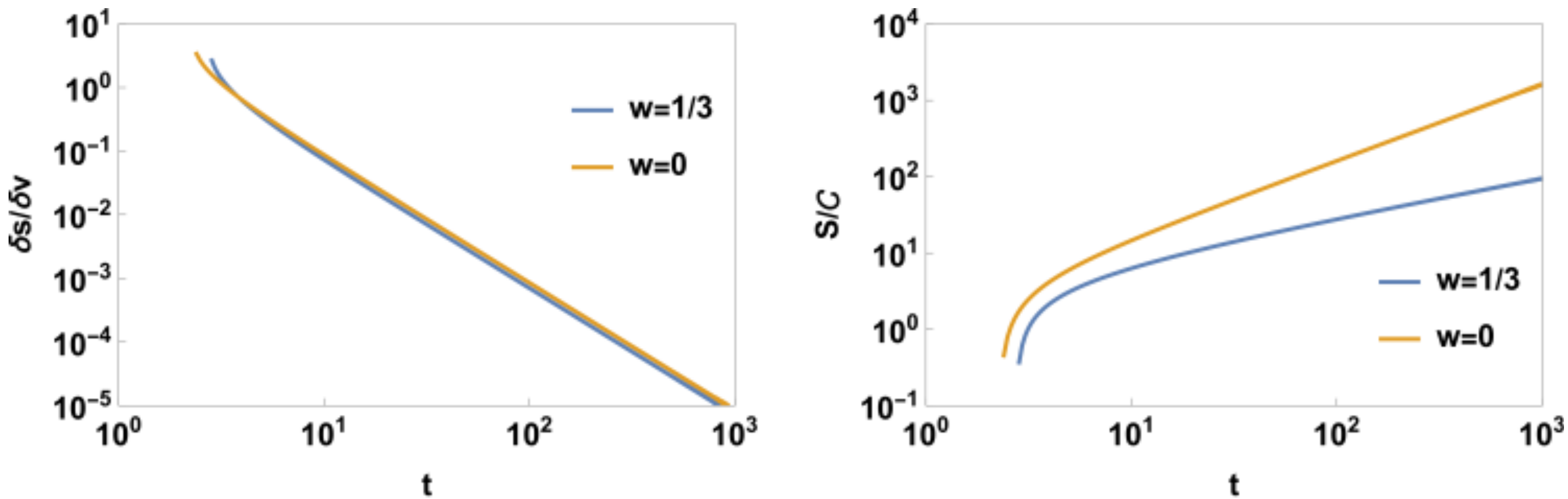
[G. Bianconi, The thermodynamics of the gravity from entropy theory PRD (in press)]

THERMODYNAMICS OF GFE

On Friedman universes the GQRE decreases in time,

while the entropy of the cosmology increases in time

for matter and radiation dominated universes



This thermodynamics opens new scenarios on how to reconcile

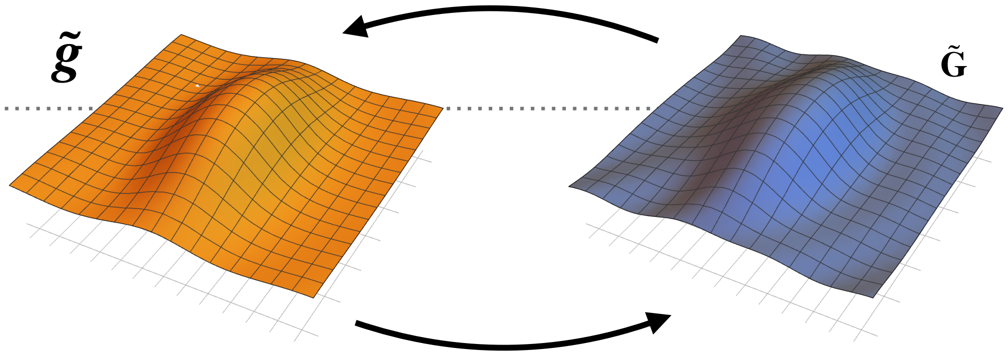
the second principle of thermodynamics with local structure of the Universe

BENEFITS OF THE GFE THEORY

- Field theory formulation
  - The GfE theory is the unique entropic gravity approach that stem from an action (reducing to Einstein-Hilbert action in the low-energy small-curvature limit)
- GfE leads to modified gravity
  - The GfE theory leads to modified equation of motion that can lead to testable predictions of the theory in cosmology and astrophysics.
- The theory is local, within its Dirac-Kalher treatment of quantum dynamics
  - Thus the theory can be potentially extended to all matter fields
- GfE Lagrangian is given by GQRE related to the Araki relative entropy
  - This connection provides a possible direction with quantum entanglement

SUMMARY

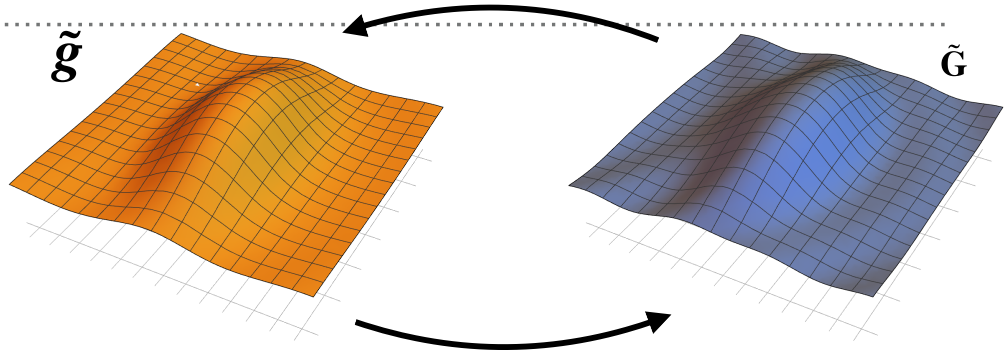
$$\mathscr{L} = -\operatorname{Tr}_F \ln \tilde{\mathbf{G}}\tilde{g}^{-1}$$



- The Gravity from Entropy action is proposed
- In the low-energy, small curvature limit we recover the Einstein-Hilbert action with zero-cosmological constant
- The Gravity from Entropy action whose Lagrangian is given by the Geometrical Quantum Relative Entropy leads to a modified theory of gravity expressed in terms of the G-field, predicting a dynamical cosmological constant
- The area law for the Schwarzschild black-holes is derived from the Gravity from Entropy approach

$$\mathscr{L} = -\operatorname{Tr}_F \ln \tilde{\mathbf{G}}\tilde{g}^{-1}$$

THE GFE THEORY



Gravity

results

from the GfE action

given by

Geometric Quantum Relative Entropy

between the metric and the metric associated to the matter fields and curvature.

The matter fields are described by a Dirac-Kahler type of description.

CONFERENCE AND OPENING

Gravity, Entanglement, and Entropy, 23 September 2026

Conference at the London Mathematical Society

Fantastic lineup of speakers! Possibility for online registration!

- https://sites.google.com/view/gravityentanglemententropy

JOHN TEMPLETON FOUNDATION

Inspiring Awe & Wonder

"Gravity from Entropy" is founded by JTF

There will be a PDRA Opening on this research field starting on Jan 2027.

Contact me if interested!

Location:
De Morgan House,
London Mathematical Society
57-58 Russell Square, London WC1B 4HS

Gravity Entanglement and Entropy

London Mathematical Society Conference
23 September 2026

Registration Link:
Deadline 8 September 2026

Invited Speakers and Panellists

Alejandra Castro (Cambridge University
Sougato Bose (University College London)
Damian Galante (King's College London)
Orlando Luongo (Università di Camerino)
Chiara Marletto (Oxford University)
Marika Taylor (Birmingham University)
Vlatko Vedral (Oxford University)

Chairs

Tarek Anous (QMUL)
Ginestra Bianconi (QMUL)

Website: https://sites.google.com/view/gravityentanglemententropy:

EXTRA SLIDES

METRICS BETWEEN ONE FORMS

- We consider a torsion-free, d-dimensional manifold $\mathscr{K}$ with metric $g_{\mu\nu}$ having Lorentz signature $(-1,1\ldots,1)$
- The metric g is assumed invertible and such that $g_{\mu\nu}V^\nu = V_\mu$, $g^{\mu\nu}V_\nu = V^\mu$ where at each point p of $\mathscr{K}$ the metric $g_{\mu\nu}$ is real and symmetric
- On the manifold $\mathscr{K}$ we consider a metric compatible Levi-Civita connection determining the covariant derivative ∇_μ
- In addition to the metric $g_{\mu\nu}$ we will consider alternative invertible metrics between vectors $G_{\mu\nu}$ which we might allow to be Hermitian

RELATION TO ARAKI ENTROPY

The adopted quantum relative entropy can be equivalently expressed as

$$\boxed{\mathscr{L} = \mathrm{Tr} g \ln \mathbf{G}^{-1} = -\mathrm{Tr}_M \ln \mathbf{G} g^{-1} = -\mathrm{Tr}_M \ln \Delta_{\mathbf{G},g}^{1/2}}$$

where $\Delta_{\mathbf{G},g}^{1/2} = \mathbf{G} g^{-1}$

- Here $\Delta_{\mathbf{G},g}$ plays the role of the modular operator and it square root is given by

$$\Delta_{\mathbf{G},g}^{1/2} = \mathbf{G} g^{-1} = \sqrt{\mathbf{G}\mathbf{G}^*}$$

- where $\mathbf{G}^* = g^{-1}\mathbf{G} g^{-1}$ is the adjoint metric operator of $\mathbf{G}$.
- Thus the adopted quantum relative entropy generalises the Araki entropy to metric tensors.

RELATION WITH THE FIRST FUNDAMENTAL FORM OF GAUSS

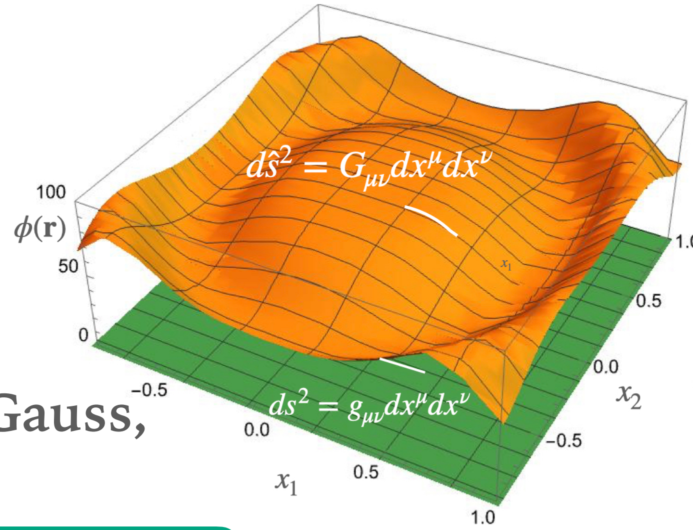
$$\boxed{\text{In general } G_{\mu\nu} = g_{\mu\nu} + \alpha M_{\mu\nu} \text{ with } M_{\mu\nu} = (\nabla_\mu \bar{\phi})(\nabla_\nu \phi)}$$

If the metric is Euclidean and flat, i.e.$g_{\mu\nu} = \delta_{\mu\nu}$ and ϕ is real, we have that the manifold $\mathscr{K} \otimes \mathbb{R}$ of coordinates $(\mathbf{x}, \phi(\mathbf{x}))$ has, in 2d,

elementary vectors $\mathbf{e}_x = (1,0,\phi_x)$, $\mathbf{e}_y = (0,1,\phi_y)$

defining elementary vectors $\mathbf{e}_x dx + \mathbf{e}_y dy$

the metric $\mathbf{G}$ reduces to the **first fundamental form of Gauss,**



$$\boxed{\mathbf{G} = \begin{pmatrix} \mathbf{e}_x^\top \mathbf{e}_x & \mathbf{e}_x^\top \mathbf{e}_y \\ \mathbf{e}_y^\top \mathbf{e}_x & \mathbf{e}_y^\top \mathbf{e}_y \end{pmatrix} = \begin{pmatrix} 1 + \phi_x^2 & \phi_x \phi_y \\ \phi_x \phi_y & 1 + \phi_y^2 \end{pmatrix}}$$