From Functional to Ricci Flows: Asymptotic Safety in Perturbation Theory

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Yannick Kluth, September 1st 2026

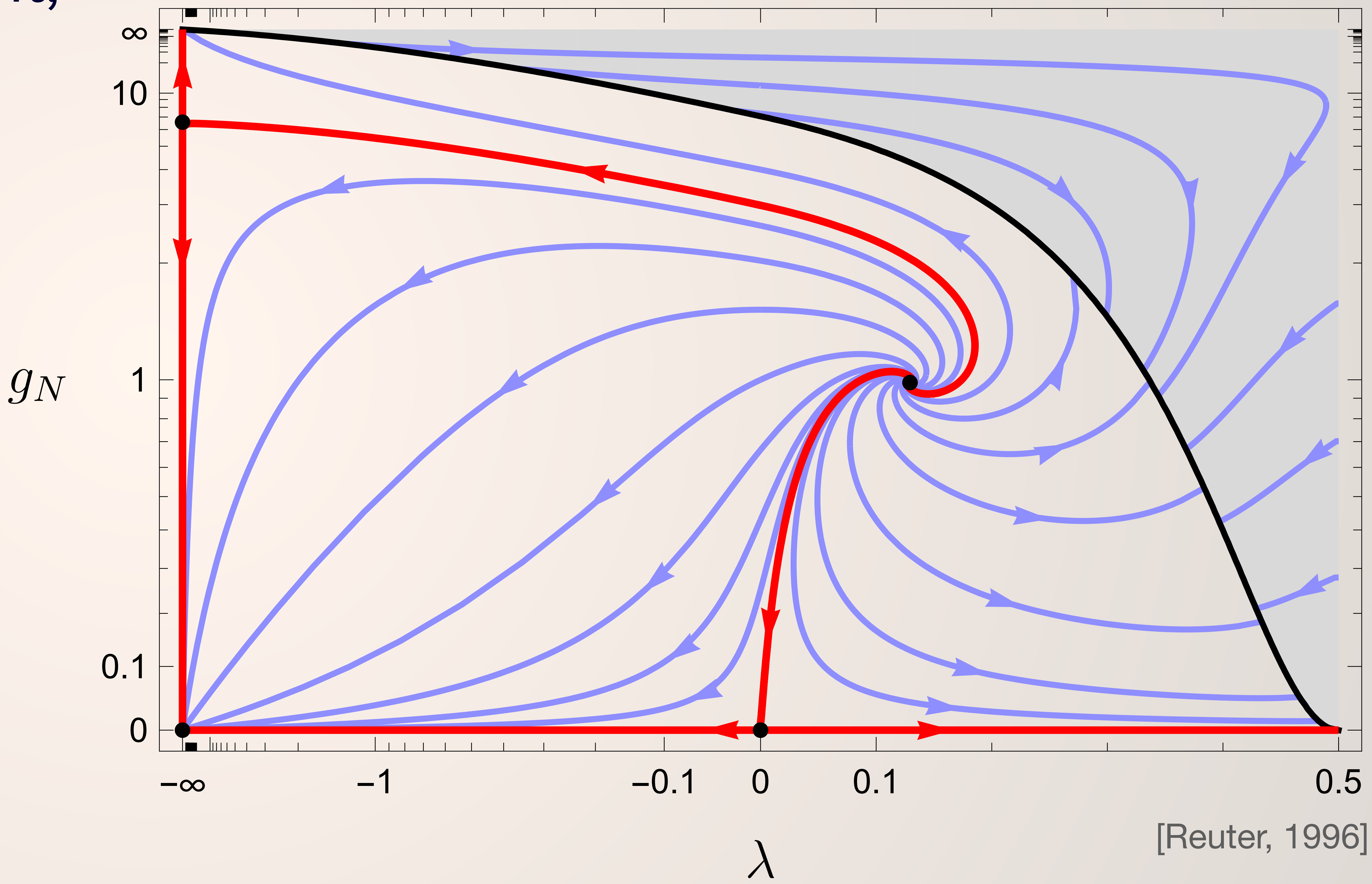
UNIVERSITY OF TORONTO

Walter Benjamin-Programm

DFG Deutsche Forschungsgemeinschaft

Asymptotically Safe Quantum Gravity

- Large Evidence for UV Fixed Point, almost entirely coming from FRG
  - 3-4 relevant directions
  - Near canonical scaling of eigenvalues
  - Effective Universality



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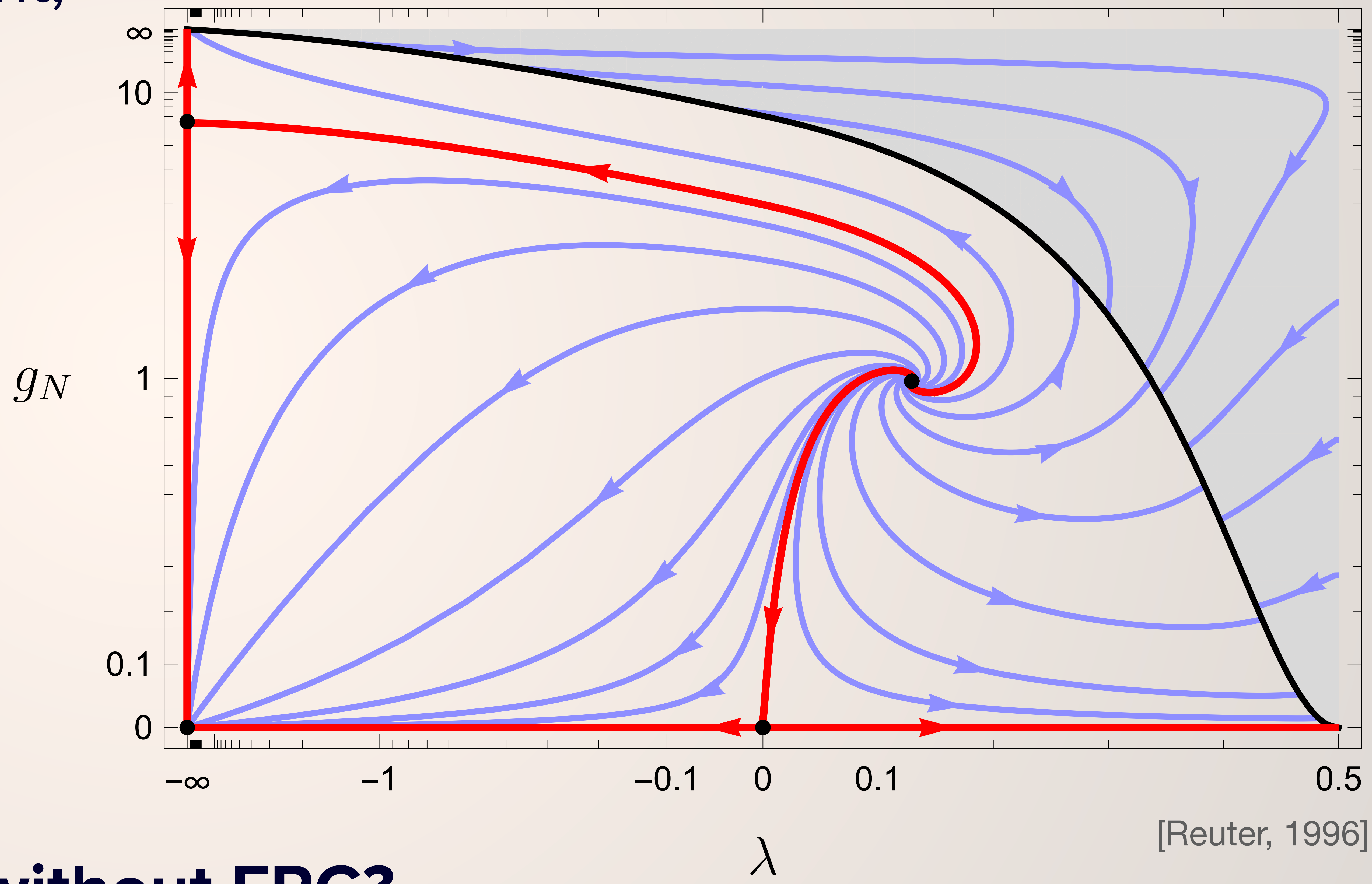

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can we see Asymptotic Safety without FRG?

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2. Ricci Flow and Asymptotic Safety

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$$\mu\frac{\mathrm{d}g}{\mathrm{d}\mu} = g\left(2 - B_2 g\lambda\right) + \mathcal{O}\left(g^3\right)$$

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  - $A_3 \neq -B_2$
  - **No one-loop fixed point**

Running Couplings in Quantum Gravity

- β-functions in 4d one-loop quantum gravity

quart div

$$\mu\frac{\mathrm{d}\lambda}{\mathrm{d}\mu} = -2\lambda - g\left(A_1 + A_2\lambda + A_3\lambda^2\right) + \mathcal{O}\left(g^2\right)$$

quad div

log div

$$\mu\frac{\mathrm{d}g}{\mathrm{d}\mu} = 2g - g^2\left(B_1 + B_2\lambda\right) + \mathcal{O}\left(g^3\right)$$

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Power-law divergences are necessary to obtain Fixed Point!

Power-Law Sensitive Dimensional Regularization

- How to identify power-law divergences in dimensional regularization?

$$\int \frac{\mathrm{d}^d p}{\pi^{d/2}} \frac{1}{p^2 + m^2} = \int \mathrm{d}p\, p^{d-1} \int \mathrm{d}\Omega_d \frac{1}{p^2 + m^2} = m^{d-2}\Gamma\left(1 - \frac{d}{2}\right)$$

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- The right-hand side is divergent in $d = 2,4,6,\ldots$
  - **Power-law divergences lead to divergences in $d < 4$** [Veltman, 1981]

Power Divergence Subtraction:

Renormalize bare couplings to remove divergences in $d \leq 4$

[Weinberg, 1979]
[Jack and Jones, 1990]
[Al-Sarhi and Jack and Jones, 1990]
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[Kaplan et al, 1998]
[**YK**, 2024]

Renormalization

- One-loop renormalization leads to β-functions [**YK**, 2024]

quart div $(d = 0)$

$$\beta_\lambda = \mu \frac{\mathrm{d}\lambda}{\mathrm{d}\mu} = -2\lambda + 0g - \frac{28}{3} g\lambda + \frac{4}{3\pi} g\lambda^2$$

quad div $(d = 2)$

log div $(d = 4)$

$$\beta_g = \mu \frac{\mathrm{d}g}{\mathrm{d}\mu} = 2g - \frac{2 \cdot 26}{3} g^2 - \frac{26}{3\pi} g^2 \lambda$$

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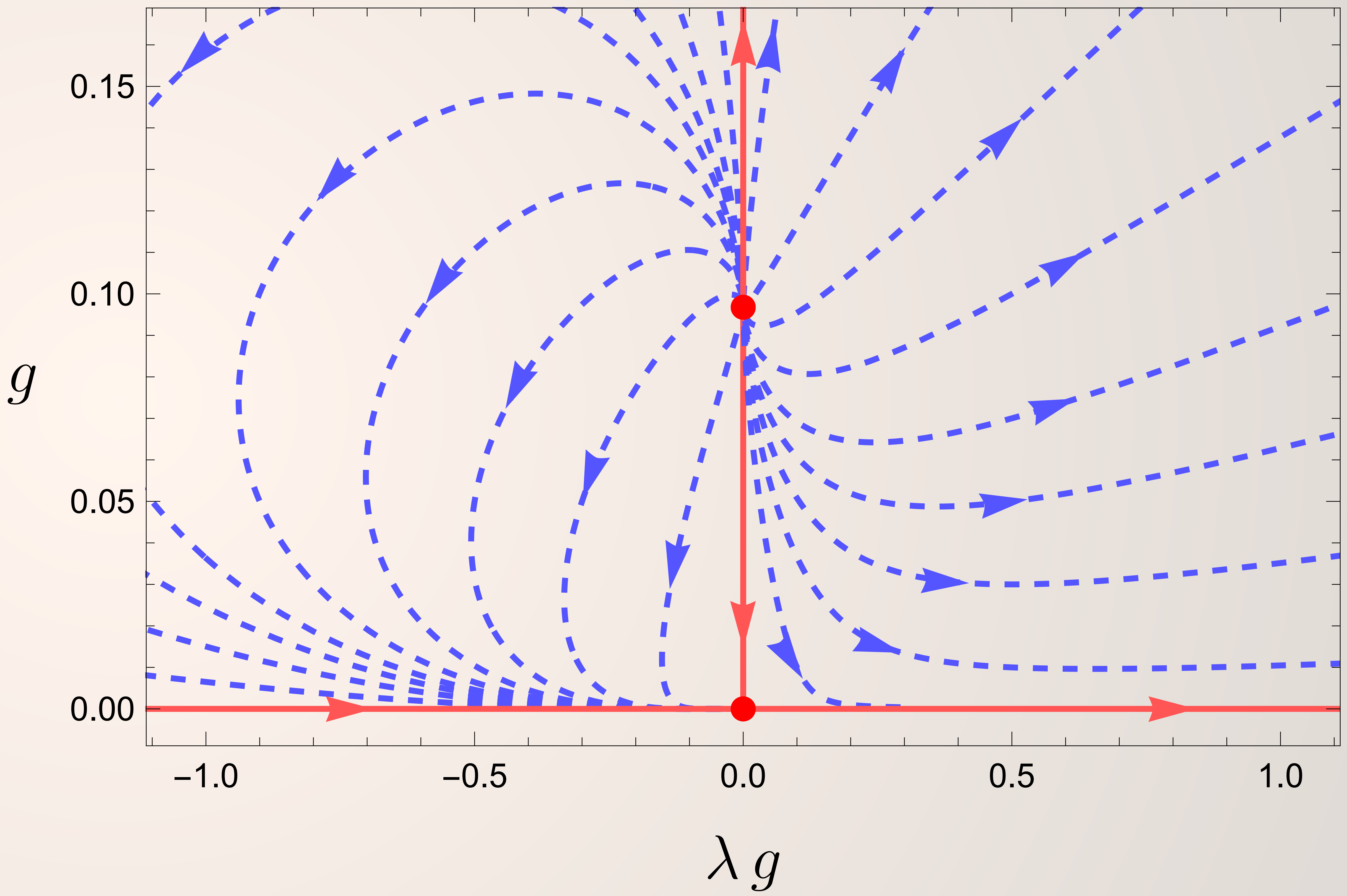
- No quartic divergences [Akhmedov, 2002] [Ossola and Sirlin, 2003]

Phase Diagram of One-Loop Gravity

	λ	g	θ_0	θ_1
$\mathbf{FP}_{\text{Gauss}}$	0	0	2	-2
$\mathbf{FP}_{\text{UV}}$	0	$\frac{3}{26}$	3.077	2
$\mathbf{FP}_{\text{unphys}}$	$-\frac{40\pi}{11}$	$-\frac{11}{78}$	$0.658 + 2.662i$	$0.658 - 2.662i$

- One-loop fixed point exists [YK, 2024]
  - Critical exponents similar to FRG
  - $\lambda = 0$ is a fixed point of the RG flow



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- Consider QCD with massive quark fields

$$\mathscr{L} = -\frac{1}{4g_0^2} F^{a\mu\nu} F^a_{\mu\nu} + \sum_{r=1}^{n_F} \left[\bar{\Psi}^r i \not{D} \Psi^r - M_{0,r} \bar{\Psi}^r \Psi^r \right]$$

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- **Renormalize coupling by subtracting poles in** $d \geq 4$ [**YK**, 2026]

Threshold Effects from Dimensional Regularization

- Subtracting all poles in $d \geq 4$ that are proportional to the coupling results in

$$\delta Z_{\mathrm{g}}^{\mathrm{f}} = \frac{2}{3(4\pi)^2} \sum_{r=1}^{n_F} \sum_{n=0}^{\infty} \frac{\left(-m_r^2\right)^n}{n!(2n+4-d)}$$

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- This gives the β-function

$$\beta_\alpha = (d-4)\,\alpha - \frac{22}{3} C_A \frac{\alpha^2}{4\pi} + \sum_{r=1}^{n_F} \frac{4}{3} e^{-m_r^2} \frac{\alpha^2}{4\pi}$$

- **Mass-Dependent Damping Terms!**

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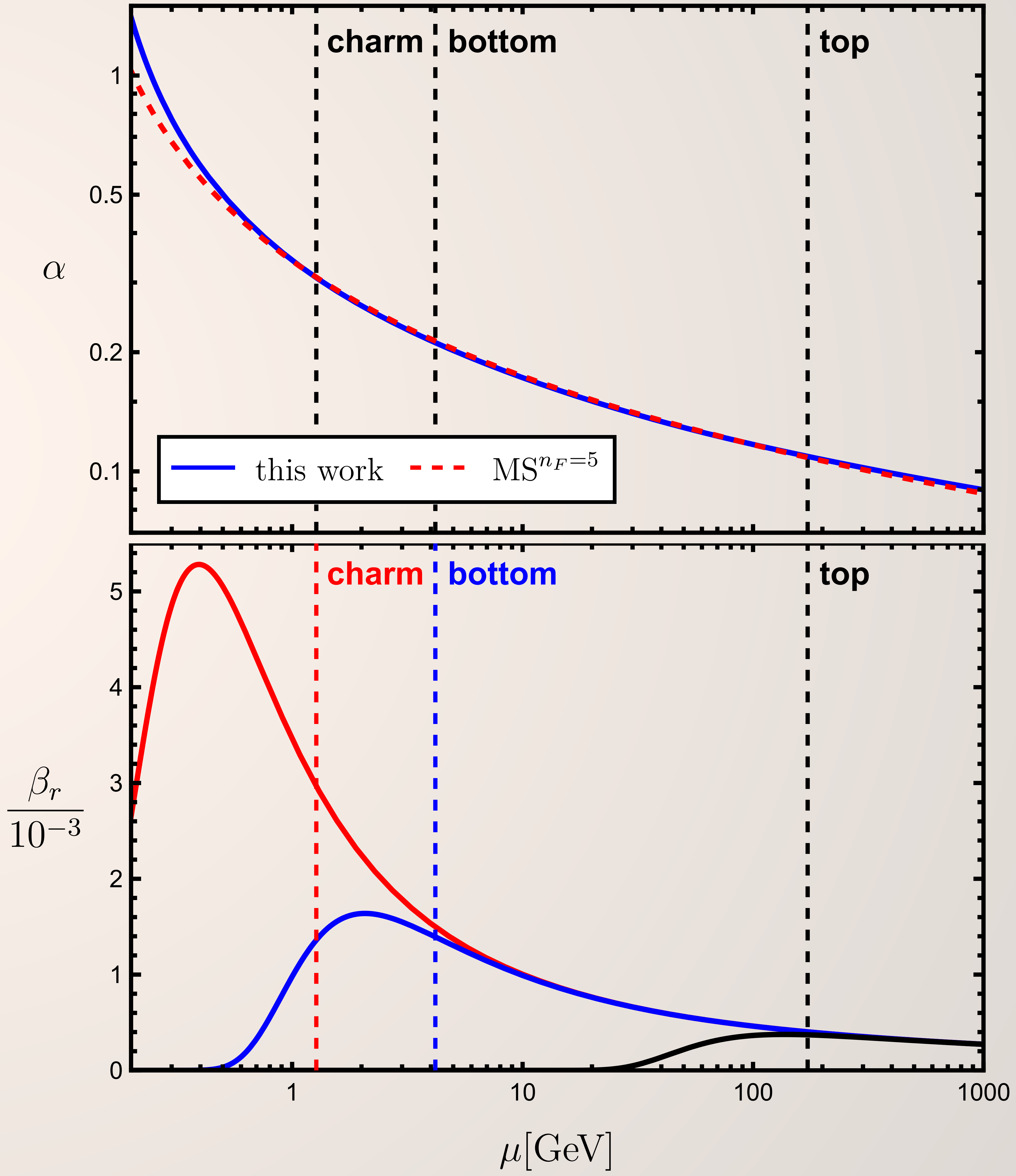

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  - **relies only on gravity in $d = 4$**

Ricci Flow

- Diffusion equation for the metric [Hamilton, 1982] [Perelman, 2003] [Perelman, 2006]

$$\partial_t \hat{g}_{\mu\nu}(t,x) = -2\hat{R}_{\mu\nu}(t,x) \qquad \hat{g}_{\mu\nu}(0,x) = g_{\mu\nu}(x)$$

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- We assume vanishing cosmological constant
  - Expand around flat space $\hat{g}_{\mu\nu} = \delta_{\mu\nu} + \hat{h}_{\mu\nu}$

$$\partial_t \hat{h}_{\mu\nu}(t,x) = \partial^2 \hat{h}_{\mu\nu}(t,x) + \dots$$

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- Can we define a Ricci flow version of Newton's coupling g_{RF}?
  - We will consider these correlators

$$\left\langle \int \mathrm{d}^d x \sqrt{\hat{g}} \right\rangle \qquad \left\langle \int \mathrm{d}^d x \sqrt{\hat{g}}\hat{R} \right\rangle$$

Feynman Diagram Description

- Introduce Lagrange multiplier $\hat{L}^{\mu\nu}$ to enforce Ricci flow

$$S_{\text{flow},h} = \int_0^\infty \mathrm{d}t \int \mathrm{d}^d x\, \hat{L}^{\mu\nu} \left[\partial_t \hat{h}_{\mu\nu} + 2\hat{R}_{\mu\nu} \right]$$

Lagrange multiplier (tensor density)

Ricci flow

[Harlander and **YK** and Kohnen and Werthenbach, 2026]

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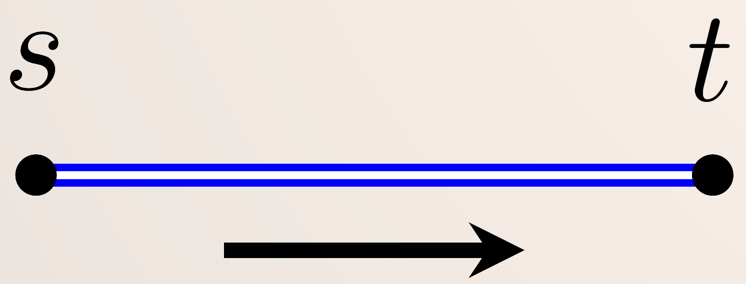
- New flow line and modified propagator



$$\langle \hat{L}\hat{h} \rangle \sim \theta(t-s) e^{-(t-s)p^2}$$

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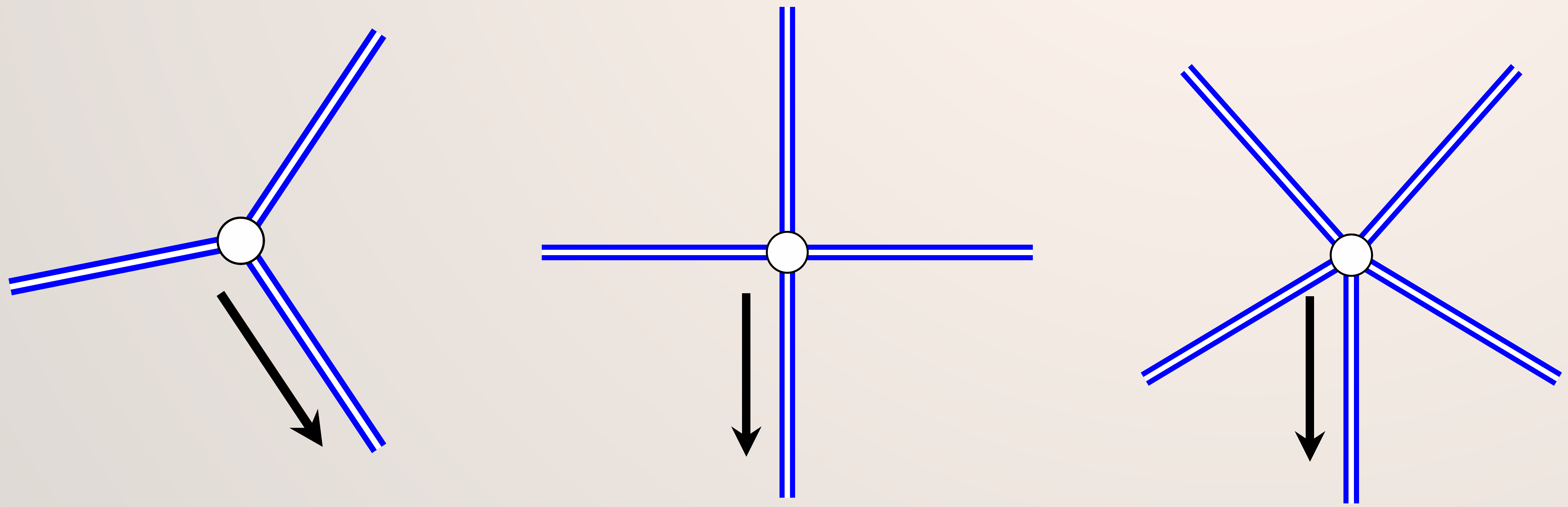
- Many new vertices



$+ \dots$

Renormalized Initial Condition

- All UV divergences can be removed by a renormalized initial condition

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$$\hat{c}_1^{\overline{MS}} = -\frac{107}{30}\frac{1}{\epsilon} \qquad \hat{c}_2^{\overline{MS}} = \frac{407}{30}\frac{1}{\epsilon}$$

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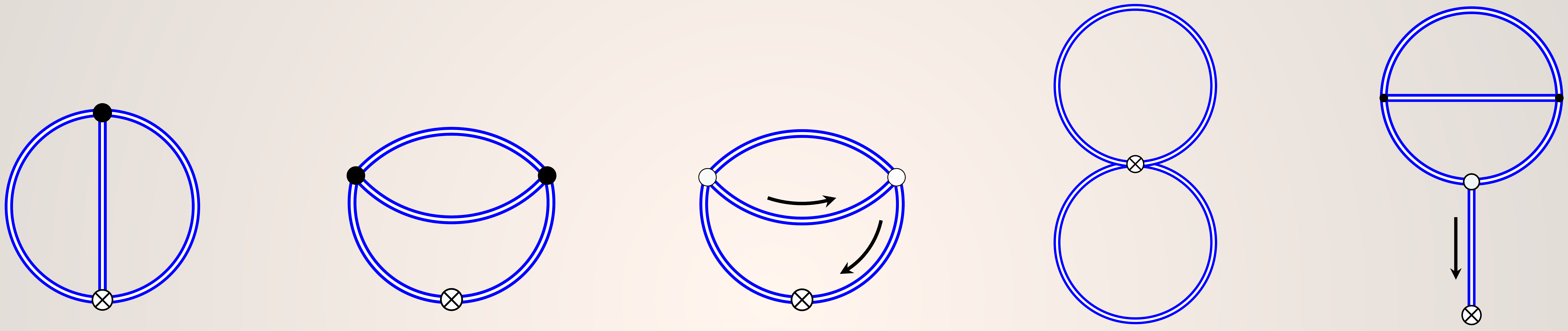
- This is sufficient to renormalize gauge invariant operators at NLO*

* (we ignore BRST exact operators here)

Computational Setup



- Toolchain developed for QCD gradient flow [Artz et al, 2019]

QGRAF [Noguiera, 1993] → tapir/exp [Harlander et al, 1997] [Gerlach et al, 2022] → FORM [Vermaseren, 2000] → Kira/Firefly [Usovitch et al, 2017] [Klappert et al, 2019]

$\overline{\text{MS}}$ Results

- After Renormalization, we obtain finite results for operators in $\overline{\text{MS}}$

$$\left\langle \sqrt{\hat{g}} \right\rangle^{\overline{\text{MS}}} = 1 - \frac{3G_N}{4\pi t}\left[1 + \frac{G_N}{4\pi t}\left(5\, l_{\mu t} + 9 \ln \frac{4}{3} + \frac{1231}{120}\right)\right]$$

$$\left\langle \sqrt{\hat{g}}\, \hat{R} \right\rangle^{\overline{\text{MS}}} = -\frac{3G_N}{4\pi t^2}\left[1 + \frac{G_N}{2\pi t}\left(5\, l_{\mu t} + 9 \ln \frac{4}{3} + \frac{931}{120}\right)\right]$$

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- Result is μ-**dependent**
  - Originates from field redefinitions used to remove UV divergences
  - Problematic to define Ricci flow coupling

Fixed Volume Scheme

- μ-dependence can be fixed by performing non-minimal subtraction

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- Use $\mathcal{O}(\varepsilon^0)$ terms to fix $\langle\sqrt{\hat{g}}\rangle$ to LO value:
  - In spirit similar to the ringed scheme of flowed quarks in gradient flow QCD

[Makino and Suzuki, 2014]

$$\langle\sqrt{\hat{g}}\rangle^{\text{FVS}} = 1 - \frac{3G_N}{4\pi t}$$

$$\langle\sqrt{\hat{g}}\hat{R}\rangle^{\text{FVS}} = -\frac{3G_N}{4\pi t^2}\left[1 - \frac{5G_N}{4\pi t}\right] + \ldots$$

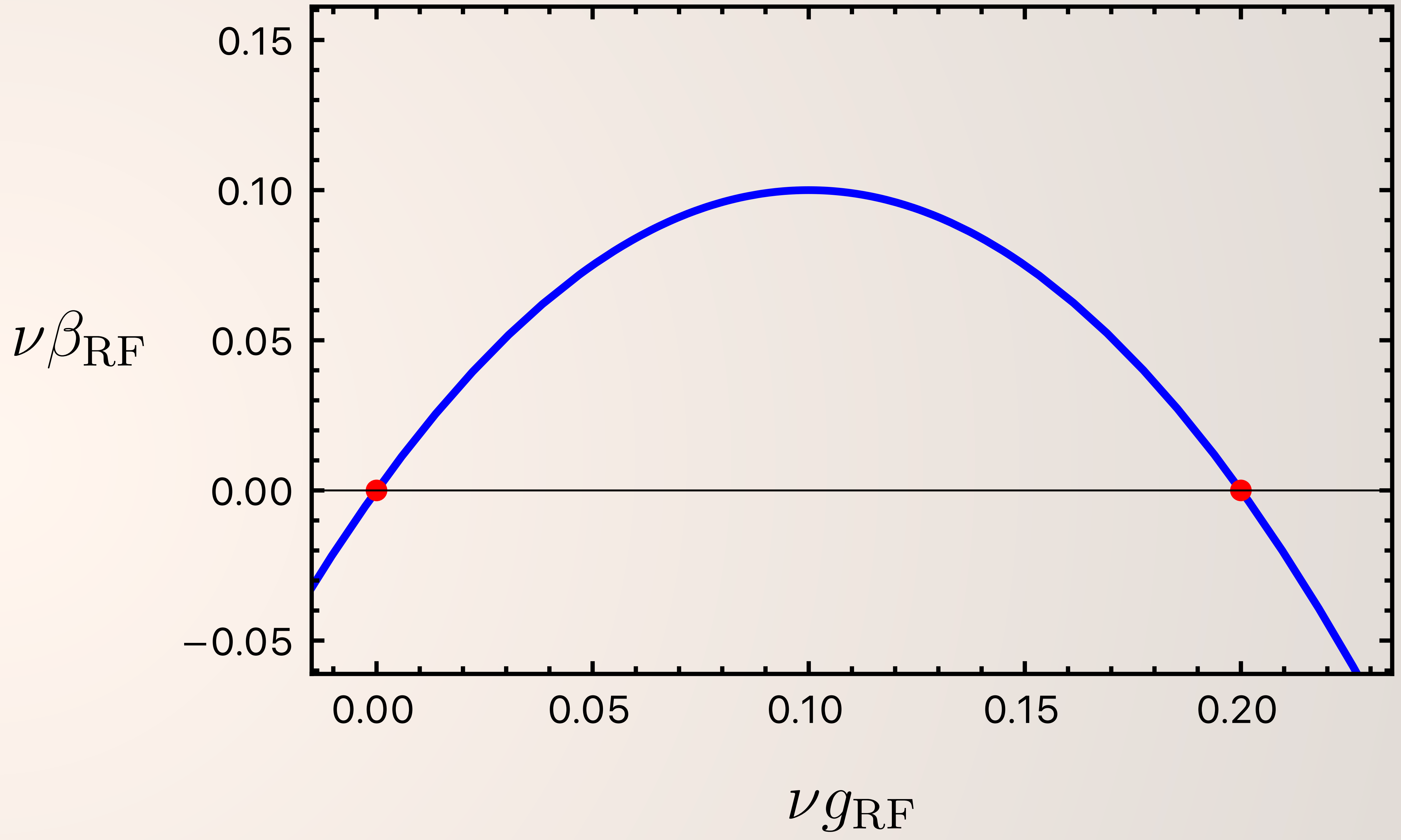
Ricci Flow Coupling

- We define Ricci flow coupling as

$$g_{RF} = -\frac{t}{3\nu}\langle\sqrt{\hat{g}}\hat{R}\rangle\Big|_{t=\rho/\mu^2}$$

$$\Rightarrow \beta_{RF} = 2g_{RF}(1-5\nu g_{RF})$$

- Right sign to get a fixed point!



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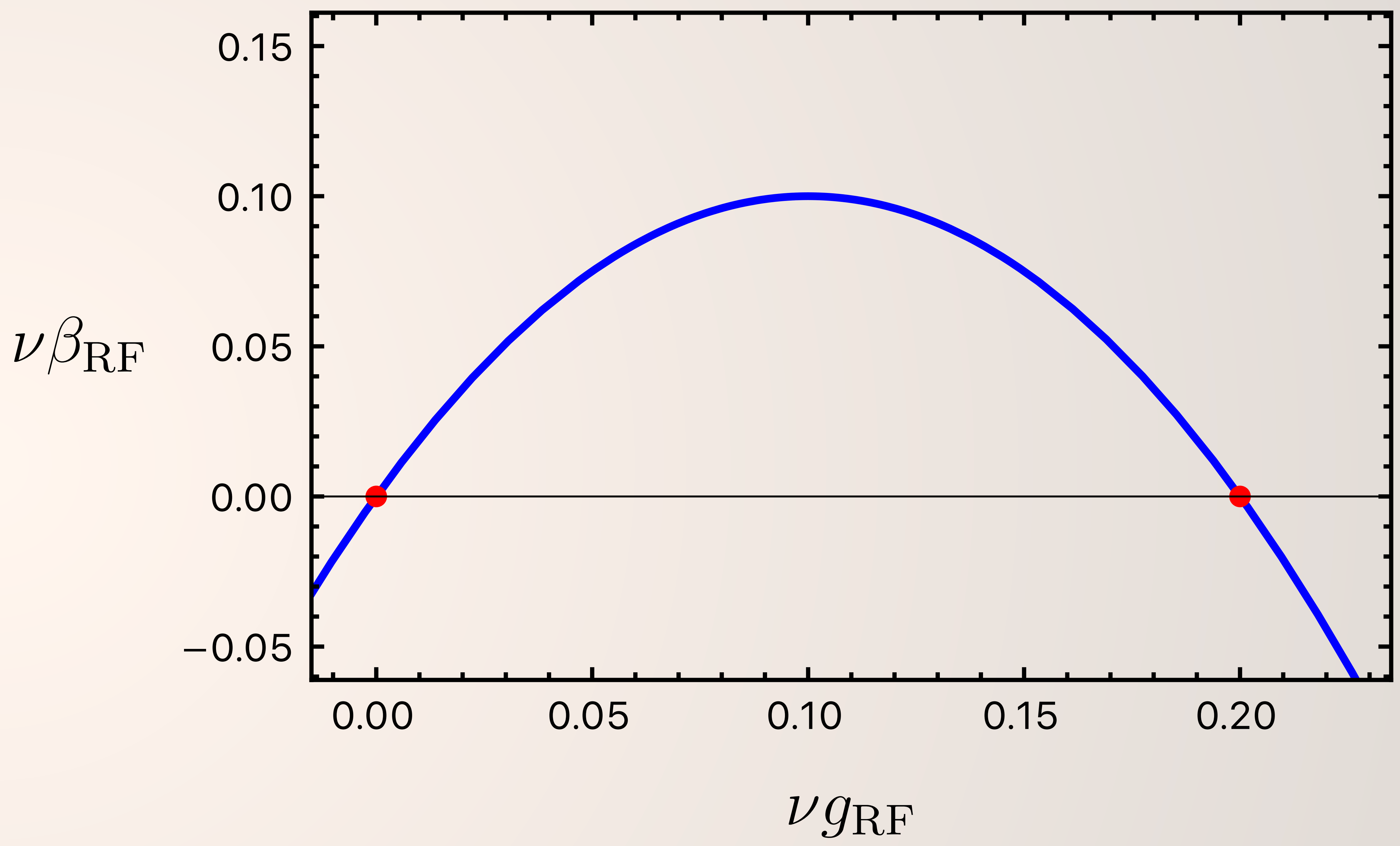
$$g_{RF} = -\frac{t}{3\nu}\langle\sqrt{\hat{g}}\hat{R}\rangle\Big|_{t=\rho/\mu^2}$$

$$\Rightarrow \beta_{RF} = 2g_{RF}(1-5\nu g_{RF})$$

- Right sign to get a fixed point!
    - Is $g^*_{RF} = 1/(5\nu)$ perturbative?
        - $g^*_{RF} < 1$ if $\nu > 1/5$



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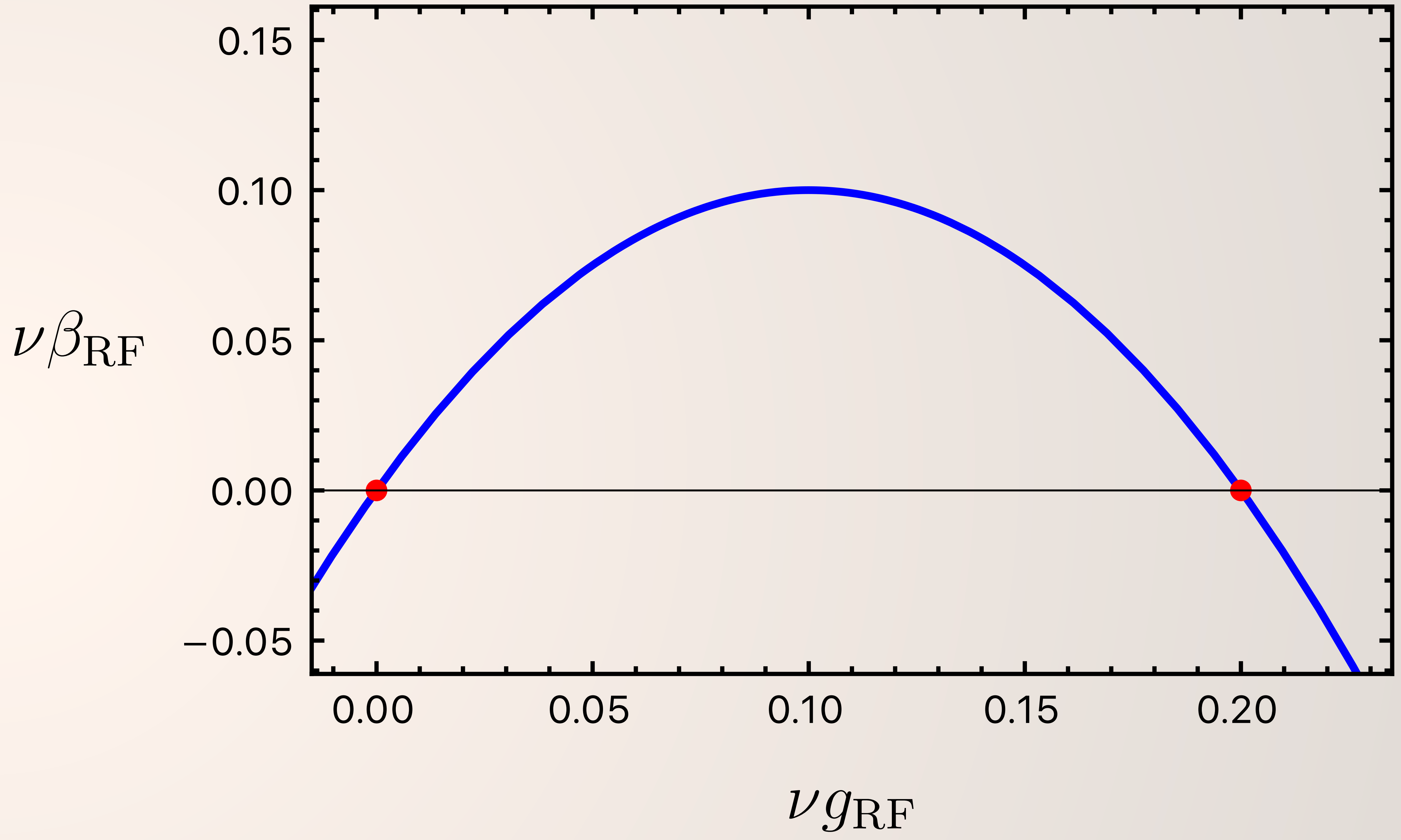

$$\frac{G_N}{4\pi t} = 3\nu g_{\mathsf{RF}} + 15\nu^2 g^2_{\mathsf{RF}} + \cdots$$

Summary and Outlook

- Mass-Dependent Renormalization Schemes are highly beneficial
    - UV Fixed Points, Threshold effects, etc

- Two covariant ways to find UV Fixed Point of Quantum Gravity
    - Subtracting poles in $d \leq 4$
    - Covariant Ricci Flow $\partial_t \hat{g}_{\mu\nu} = -2\hat{R}_{\mu\nu}$

- Can Perturbativity be confirmed at NNLO?
- Ricci Flow in EDT/CDT

Thanks for your attention!

Backup

Gauge fixed Ricci Flow

- In practice, it is highly beneficial to introduce a gauge-fixed Ricci flow
  - Done by using a flow-time dependent gauge transformation [Lüscher, 2010] [Lüscher and Weisz, 2011]

$$\hat{g}_{\mu\nu}(t,x) \to \hat{g}_{\mu\nu}(t,x) + \hat{\nabla}_{(\mu}\hat{\xi}_{\nu)}(t,x)$$

with $\partial_t \hat{\xi}^\mu = \frac{\alpha_0}{2}\hat{F}^\mu$

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$$S_{\mathrm{flow},h} = \int_0^\infty \mathrm{d}t \int \mathrm{d}^d x\, \underbrace{\hat{L}^{\mu\nu}}_{\text{Lagrange multiplier (tensor density)}} \Big[\partial_t \hat{h}_{\mu\nu} + \underbrace{2\hat{R}_{\mu\nu}}_{\text{Ricci flow}} \underbrace{-\alpha_0 \hat{\nabla}_{(\mu}\hat{F}_{\nu)}}_{\text{gauge-fixing}}\Big]$$

BRST Invariance

- BRST transformations

$$\mathfrak{s}\left(h_{\mu\nu}\right) = \nabla_\mu C_\nu + \nabla_\nu C_\mu \qquad \longrightarrow \qquad \mathfrak{s}(\hat{h}_{\mu\nu}) = \hat{\nabla}_\mu \hat{C}_\nu + \hat{\nabla}_\nu \hat{C}_\mu$$

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$$\mathfrak{s}\left(\overline{C}^\mu\right) = -\frac{2}{\alpha}\overline{g}^{\mu\nu} F_\nu$$

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$$\hat{C}^\mu\Big|_{t=0} = C^\mu \qquad \partial_t \hat{C}^\mu = \hat{e}^\mu$$

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- Flowed anti-ghosts not necessary!

BRST Invariance

- Ricci flow action is supplemented by flowed ghost fields

$$S_{\text{flow},h} = \int_0^\infty \mathrm{d}t \int \mathrm{d}^d x\, \hat{L}^{\mu\nu} \left[\partial_t \hat{h}_{\mu\nu} + 2\hat{R}_{\mu\nu} - \alpha_0 \hat{\nabla}_{(\mu}\hat{F}_{\nu)}\right]$$

$$S_{\text{flow},C} = \int_0^\infty \mathrm{d}t \int \mathrm{d}^d x\, \hat{D}_\mu \left[\partial_t \hat{C}^\mu - \frac{\alpha_0}{2}\left(\hat{F}^\rho \hat{\nabla}_\rho \hat{C}^\mu - \hat{C}^\rho \hat{\nabla}_\rho \hat{F}^\mu + 2\frac{\delta \hat{F}^\mu}{\delta \hat{h}_{\rho\sigma}} \hat{\nabla}_\rho \hat{C}_\sigma\right)\right].$$

- BRST transformations of Lagrange multipliers

$$\mathfrak{s}(\hat{L}^{\mu\nu}) = (\partial_\rho \hat{C}^\rho)\hat{L}^{\mu\nu} + \hat{C}^\rho \hat{\nabla}_\rho \hat{L}^{\mu\nu} - 2(\hat{\nabla}^\rho \hat{C}^{(\mu})\hat{L}^{\nu)}{}_\rho\,,$$

$$\mathfrak{s}(\hat{D}_\mu) = -2\hat{\Gamma}^\rho_{\rho\sigma}\hat{L}_\mu{}^\sigma - \hat{D}_\mu(\partial_\rho \hat{C}^\rho) - \hat{D}_\rho \hat{\nabla}_\mu \hat{C}^\rho + \hat{C}^\rho \hat{\nabla}_\rho \hat{D}_\mu + 2\hat{\nabla}_\rho \hat{L}^\rho{}_\mu\,.$$

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$$\mathfrak{s}(S_{\text{flow},h} + S_{\text{flow},C}) = 0$$

Total flow action is BRST invariant!

Flowed Ghosts are Irrelevant

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  - Flowed ghosts can only arise from flowed vertices…
    …and can only connect to other flowed vertices

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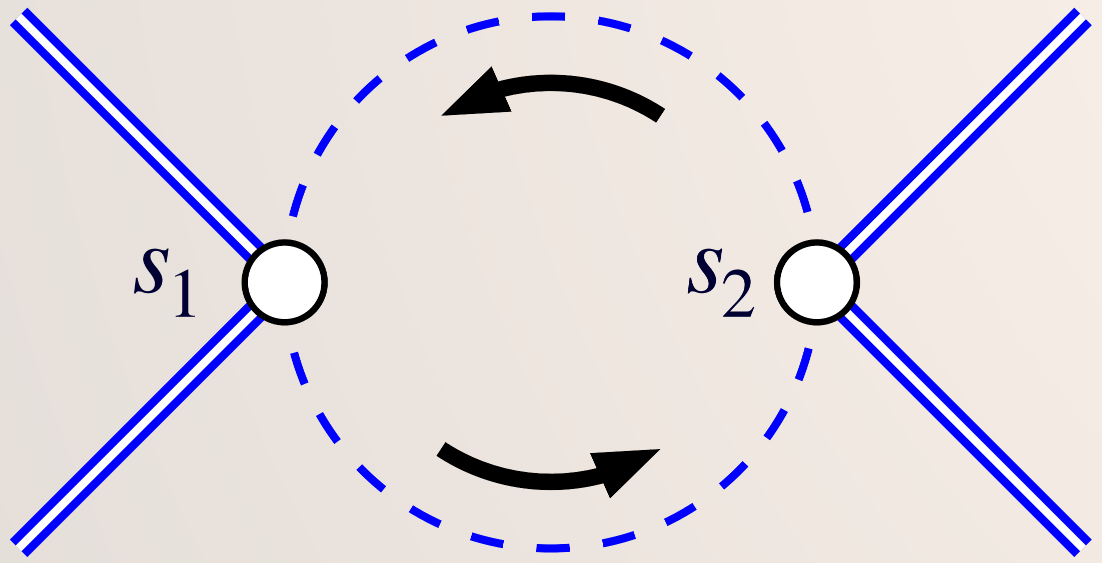

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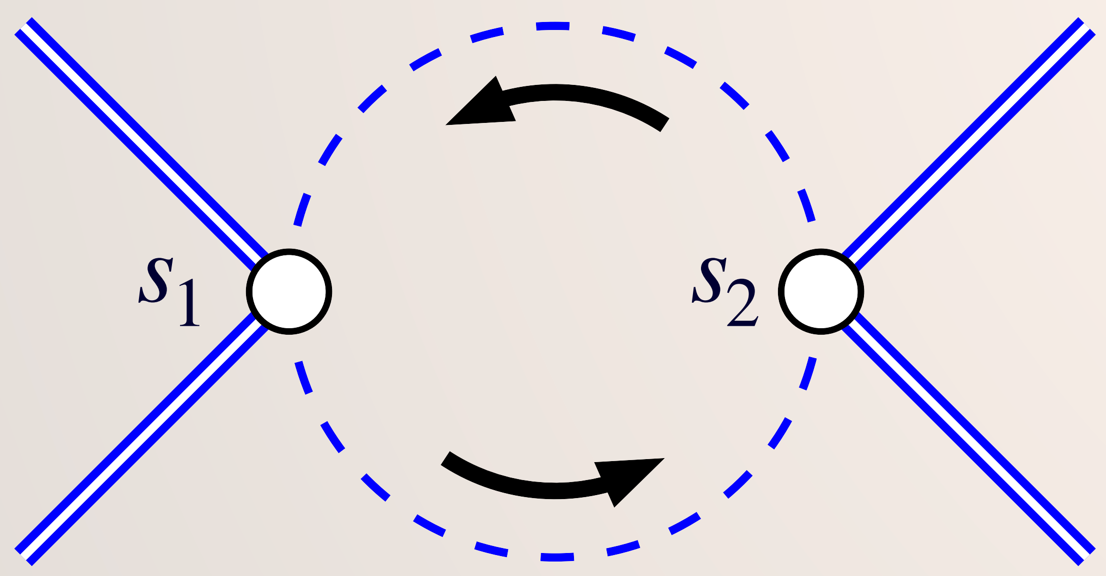

$$\sim \int_0^\infty \mathrm{d}s_1 \mathrm{d}s_2\, \theta(s_2 - s_1)\theta(s_1 - s_2) = 0$$

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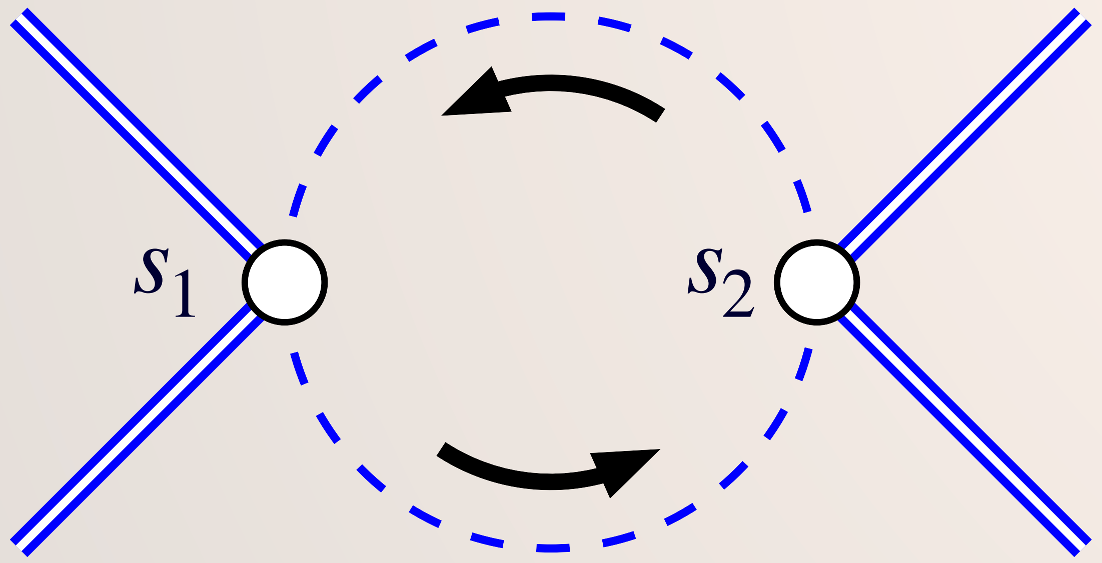

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- Resulting flow line loops vanish