Perturbatively Exact Fixed Points in Three Dimensions

Tom Steudtner
TU Dortmund University

based on
Kvedaraitė, TS, Uetrecht [2502.07880]
Schröder, Stamou, TS, Uetrecht [2512.02123]

ERG 2026

03.09.2026 University of Sussex

technische universität
dortmund

Perturbative Fixed Point

$$\beta = -\epsilon\,\alpha^2 + C\,\alpha^3 + \dots$$

$$\alpha^* = \epsilon/C + \mathcal{O}(\epsilon^2)$$

$\longrightarrow$ both LO and NLO

$\longrightarrow$ sign $C\epsilon > 0$

$\longrightarrow$ perturbative $|\epsilon| \ll |C|$

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» $|\epsilon| \neq 0$ but small and tunable $\longrightarrow$ systematic expansion

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» NLO orders in perturbation theory !

» unicorns !

Perturbative Critical Phenomena

d = 4: Non-Abelian Gauge sector

[Coleman & Gross 1973]

- Asymptotic Freedom

  [Gross & Wilczek, Politzer 1973]

- IR gauge fixed-points

  [Caswell 1974, Banks & Zaks 1982]

- IR or UV gauge-Yukawa

  [Litim, Sannino 2014]
  [Bond, Litim 2016 & 2018]

d = 3

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d = 3

[1] » UV in pure scalar theories

[Heinz & Appelquist 1981]
[Pisarski 1982]
[Kvedaraitė & TS & Uetrecht 2025]

[2] » IR in scalar & fermionic

[Schröder, Stamou, TS, Uetrecht 2025]

Facts of Life in d=3

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{i}{2}\overline{\psi}\gamma^\mu\partial_\mu\psi + \mathcal{L}_{\text{gauge}}$$
$$- \frac{M}{2}\overline{\psi}\psi - \frac{y}{2}\phi\overline{\psi}\psi - \frac{Y}{4}\phi^2\overline{\psi}\psi$$
$$- \frac{m^2}{2}\phi^2 - \frac{h}{6}\phi^3 - \frac{\lambda}{4!}\phi^4 - \frac{\tau}{5!}\phi^5 - \frac{\eta}{6!}\phi^6$$

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» fermions in d=4: Dirac 4 complex $\longrightarrow$ Weyl 2 complex
d=3: Dirac 2 complex $\longrightarrow$ Majorana 2 real

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» fermions in d=4: Dirac 4 complex $\longrightarrow$ Weyl 2 complex
d=3: Dirac 2 complex $\longrightarrow$ Majorana 2 real

» RGEs only at even loops: $\underbrace{\text{2-loop, 4-loop}}_{\text{critical phenomena in d=3}}$, 6-loop, ...

Part 1 – Scalar Theories

Tricritical O(N) model

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& large N

perturbation theory

[Applequist, Heinz (1981-82)]
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saddle point methods

[Bardeen, Moshe, Bander (1984)]
[David et al. (1984-85)]

FRG [Litim et al. (2017-20)]
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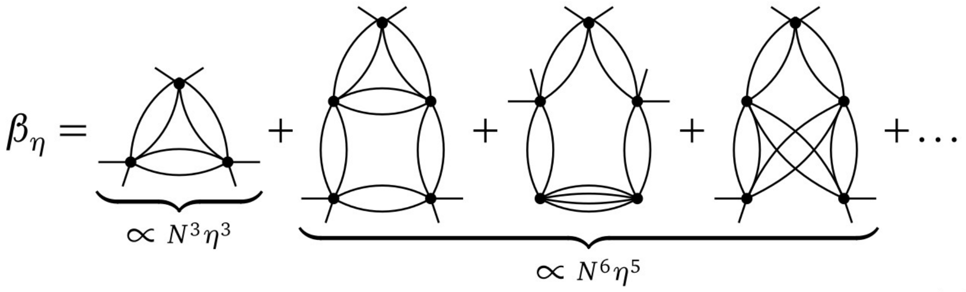
Only “N = ∞” (???)

line or single FP?
breaking of scale invariance?

Line of FPs?



$$\hat{\eta} \propto N^{3/2}\eta$$

$$\epsilon = N^{-1/2}$$

$$\beta_{\hat{\eta}} = \frac{\epsilon}{5}\hat{\eta}^2 - \frac{\pi^2}{7200}\hat{\eta}^3 + \ldots$$

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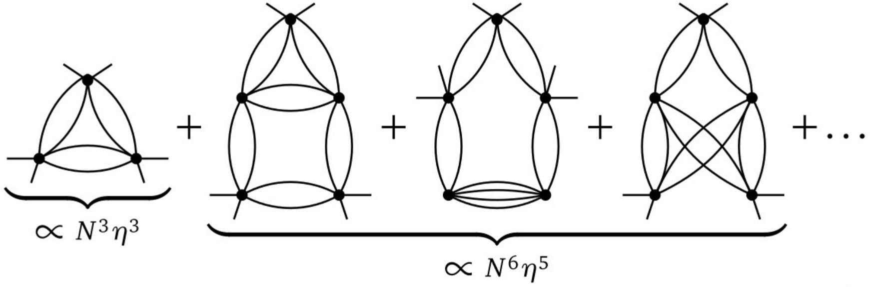
$\beta_\eta =$ 

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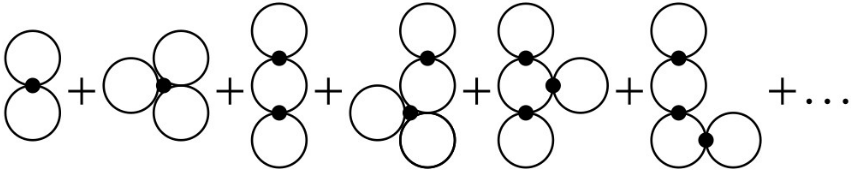
$V_{\text{eff}} =$ 

$$\bar{\eta} \propto N^2\eta$$

$$\beta_{\bar{\eta}} = \frac{24\bar{\eta}^2 - 2\pi^2\bar{\eta}^3}{N} + \mathcal{O}(N^{-2})$$

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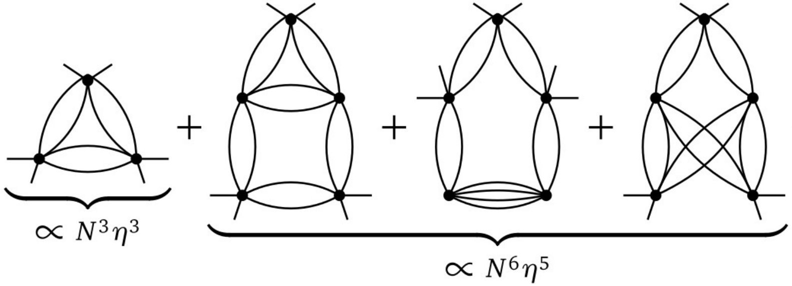

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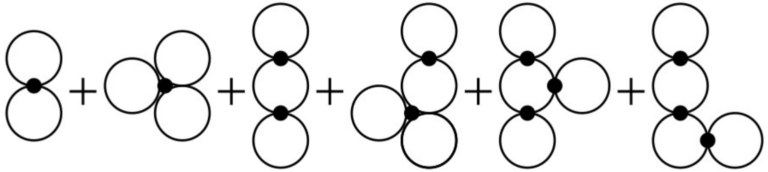

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premature $N \longrightarrow \infty$ limit: $\beta_{\bar{\eta}} = 0$ line of FPs

no smooth limit for finite N

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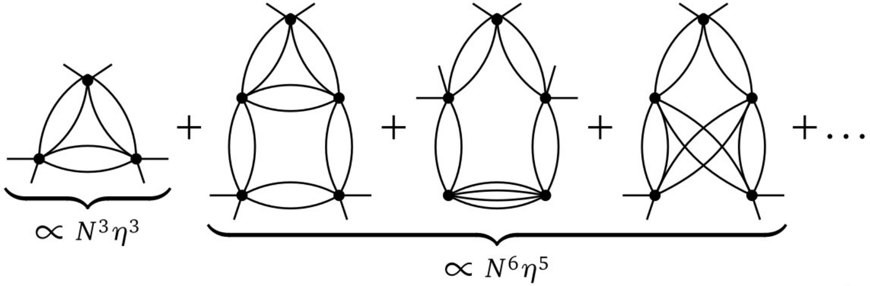
$\beta_\eta =$ 

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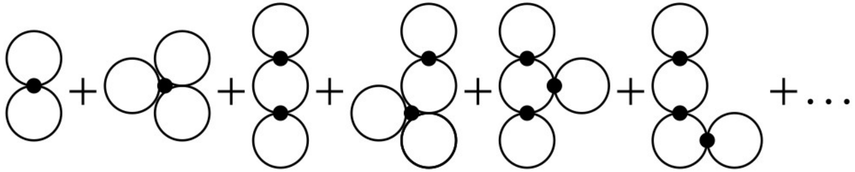
$V_{\text{eff}} =$ 

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FRG: large-N too early!

LPA truncation!

premature N $\longrightarrow \infty$ limit: $\beta_{\bar\eta} = 0$ line of FPs

no smooth limit for finite N

Stability

» (quantum) effective potential: computed to NLO in 1/N

$\longrightarrow$ no sign of instability

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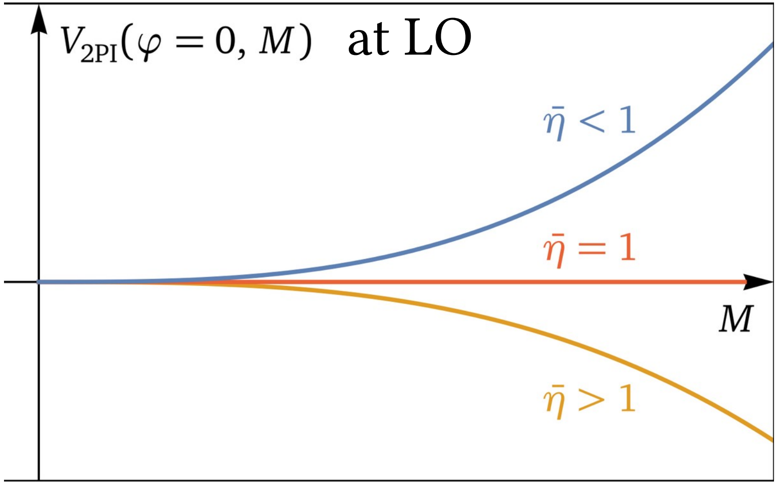
» BMB caputured by 2PI effective potential

[Cornwall, Jackiw 1974]

$$V_{2\mathrm{PI}}^{\mathrm{LO}} = \frac{NM^3}{24\pi^2} + \frac{(4\pi)^2}{6N^2}\bar{\eta}\left(\varphi^2 - \frac{NM}{4\pi}\right)^3$$



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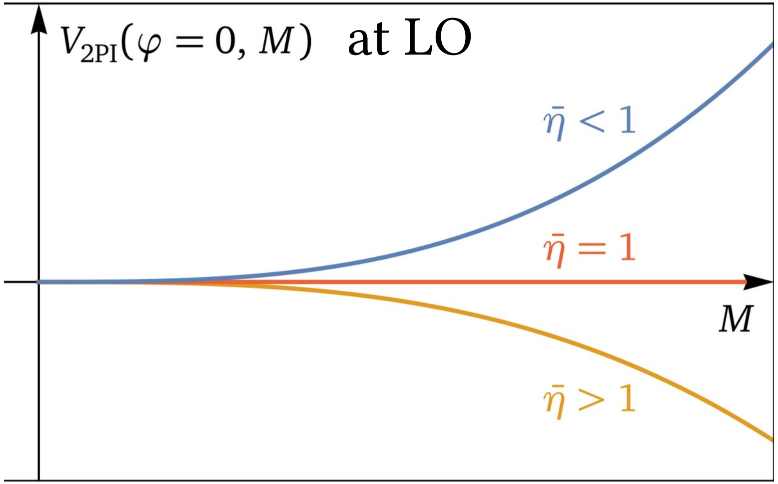
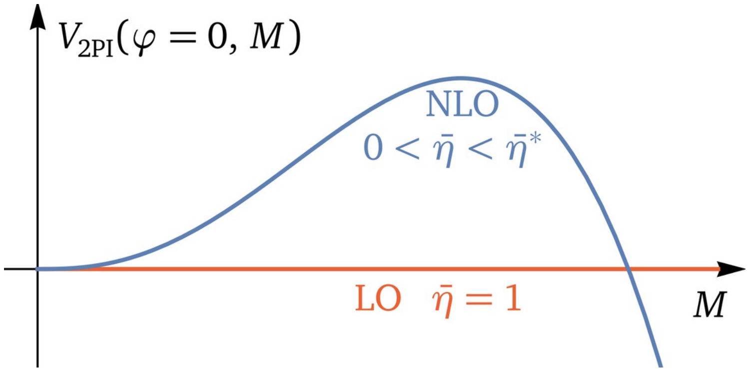


NLO:

$$V_{2\mathrm{PI}}^{\mathrm{NLO}}(0, M) = M^3\left[c(\bar{\eta}) - \beta_{\bar{\eta}}\log\frac{M}{\mu}\right]$$



$\longrightarrow$ quantum corrections break scale symmetry, not BMB

$\longrightarrow$ no sign of BMB, line of fixed-points

Part 2 – Scalar & Fermionic Theories

Template Formalism

$$\mathcal{L} = \frac{1}{2}\partial_\mu \phi_a \partial^\mu \phi_a + \frac{i}{2}\overline{\psi}_i \gamma^\mu \partial_\mu \psi_i - \frac{1}{4} Y^{abij} \phi_a \phi_b \overline{\psi}_i \psi_j - \frac{1}{6!}\eta^{abcdef} \phi_a \phi_b \phi_c \phi_d \phi_e \phi_f \,,$$

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field anomalous dimensions

$$\gamma_\phi^{ab} = \underset{\substack{\uparrow \\ \text{coefficient}}}{\gamma_{\phi,1}}\,\underbrace{Y^{acij}Y^{cbji}}_{\text{tensor structures}} + \ldots \quad = [1\text{ term}]_{\text{2-loop}} + [9\text{ terms}]_{\text{4-loop}}$$

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+ permutations

vertex corrections

$$\hat{\beta}_Y^{abij} = [5\text{ terms}]_{\text{2-loop}} + [132\text{ terms}]_{\text{4-loop}}$$

$$\hat{\beta}_\eta^{abcdef} = [5\text{ terms}]_{\text{2-loop}} + [150\text{ terms}]_{\text{4-loop}}$$

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IRA mass [Chetyrkin, Misiak, Munz, 1997]

MaRTIn [Brod, Hüdepohl, Stamou, TS 2401.04033]

+ FMFT [Pikelner, 1707.01710]

+ PSLQ & York Schröder

Crosscheck: SUSY

Generalised WZ theory $W(\Phi) = \frac{1}{4!}\lambda^{ABCD}\,\Phi_A\Phi_B\Phi_C\Phi_D$

$\mathcal{N} = 1:$ real superfields $\Phi \longrightarrow$ real scalar ϕ, fermion ψ

$$\mathcal{L}_{\mathcal{N}=1} = -\frac{1}{4}\lambda^{ABCD}\,\phi_A\phi_B(\psi_C\psi_D) - \frac{1}{36}\lambda^{ABCG}\lambda^{GDEF}\,\phi_A\phi_B\phi_C\phi_D\phi_E\phi_F + \ldots$$

$$\Rightarrow Y^{ABCD} = \lambda^{ABCD}\,, \qquad \eta^{ABCDEF} = \lambda^{ABCG}\lambda^{GDEF} + \text{perms}$$

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11 + 214 conditions ✅

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$\mathcal{N}=2:$ complex superfields $\Phi \longrightarrow$ complex scalar ϕ, fermion ψ

$$\mathcal{L}_{\mathcal{N}=2} = \left[-\tfrac{1}{4}\lambda^{ABCD}\,\phi_A\phi_B(\psi_C\psi_D) + \text{h.c.}\right] - \tfrac{1}{36}\lambda^{ABCG}(\lambda^{GDEF})^*\,\phi_A\phi_B\phi_C\phi_D^*\phi_E^*\phi_F^*$$

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$$\left.\begin{aligned} &\gamma_\phi^{AB} = \gamma_\psi^{AB}\,, \quad \hat\beta_Y^{ABCD} = 0\,, \\ &\beta_\eta^{ABCDEF} = \beta_Y^{ABCG}(\lambda^{GDEF})^* + \lambda^{ABCG}(\beta_Y^{GDEF})^* + \text{perms} \end{aligned}\right\}$$

6 + 46 conditions ✅

SUSY across Dimensions

$d = 3$ at 4-loop
[Schröder, Stamou, TS, Uetrecht, 2512.02123]

$d = 4$ at 4-loop [TS 2507.18710]

$\mathcal{N} = 2$ ✅ → $\mathcal{N} = 1$ ✅

$\mathcal{N} = 1$ ✅ → $\mathcal{N} = 1/2$ ❌

$\mathrm{Tr}\left[\gamma^{\mathrm{odd}}\right] \neq 0$

$\mathrm{Tr}\left[\gamma^{\mathrm{odd}}\right] = 0$

[Zerf, Mihaila, Marquard, Herbut, Scherer, 1709.05057]

Perturbative Fixed Points ?

$$\beta_\eta = \# \, \eta^2 + \# \, Y^2 \eta - \# \, Y^4$$

$$\beta_Y = \boxed{\# \, Y^3} + \# \, Y^5 + \# \, \eta^2 Y - \# \, \eta Y^3$$

small? negative?

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$$(4\pi)^2\beta_Y^{ab} = \frac{1}{12}\mathcal{S}_2\mathrm{tr}\left[Y^{ac}Y^{cd}\right]Y^{bd} + \frac{1}{12}\mathcal{S}_2 Y^{ab}Y^{cd}Y^{cd} + \frac{1}{2}\mathrm{tr}\left[Y^{ac}Y^{bd}\right]Y^{cd} + \frac{1}{2}Y^{cd}Y^{ab}Y^{cd} + \frac{1}{2}\mathcal{S}_4 Y^{ac}Y^{bd}Y^{cd}$$

» naive: all positive, no cancellations $\longrightarrow$ no fixed point?

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$$\beta_Y = \boxed{\#\,Y^3} + \#\,Y^5 + \#\,\eta^2 Y - \#\,\eta Y^3$$

small? negative?

$$(4\pi)^2\beta_Y^{ab} = \frac{1}{12}\mathcal{S}_2 \mathrm{tr}\left[Y^{ac}Y^{cd}\right]Y^{bd} + \frac{1}{12}\mathcal{S}_2 Y^{ab}Y^{cd}Y^{cd} + \frac{1}{2}\mathrm{tr}\left[Y^{ac}Y^{bd}\right]Y^{cd} + \frac{1}{2}Y^{cd}Y^{ab}Y^{cd} + \frac{1}{2}\mathcal{S}_4 Y^{ac}Y^{bd}Y^{cd}$$

» naive: all positive, no cancellations ⟶ no fixed point?

» remove tensor structure: $\mathrm{Tr}\left[Y^{ab}\,\beta_Y^{ab}\right]$

⟶ almost always positive

⟶ subtle opportunity when Y^{ab} furnish clifford algebra

Example

$$\mathcal{L} = \partial_\mu \phi_i^* \partial^\mu \underset{\uparrow}{\phi_i} + i\overline{\psi}_{ia} \not{\partial} \underset{\uparrow}{\psi_{ia}} - Y\,(\phi_i^* \sigma^A_{ij} \phi_j)(\overline{\psi}_{ak} \sigma^A_{kl} \psi_{al}) - \tfrac{1}{36}\eta(\phi_i^* \phi_i)^3$$

$SO(3) \times U(N):$ $\quad \mathbf{2} \times \mathbf{1} \quad \mathbf{2} \times \mathbf{N}$

Example

$$\mathcal{L} = \partial_\mu \phi_i^* \partial^\mu \phi_i + i\overline{\psi}_{ia} \partial\!\!\!/ \psi_{ia} - Y\,(\phi_i^* \sigma_{ij}^A \phi_j)(\overline{\psi}_{ak} \sigma_{kl}^A \psi_{al}) - \tfrac{1}{36}\eta(\phi_i^* \phi_i)^3$$

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» large-N: $\epsilon = \dfrac{1}{N}$ $\qquad \alpha_Y = \dfrac{NY^2}{(4\pi)^2}$ $\qquad \alpha_\eta = \dfrac{\eta}{(4\pi)^2}$

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spinor rep SO(3)

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spinor rep SO(3)

» perturbative control

$$\alpha_Y^* = \frac{\epsilon}{6+\pi^2} + \mathcal{O}(\epsilon^2)$$

$$\alpha_\eta^* = \frac{12(3+\pi^2)}{(6+\pi^2)^2}\epsilon^2 + \mathcal{O}(\epsilon^3)$$

Example

$$\mathcal{L} = \partial_\mu \phi_i^* \partial^\mu \phi_i + i\overline{\psi}_{ia} \not{\partial} \psi_{ia} - Y\,(\phi_i^* \sigma^A_{ij} \phi_j)(\overline{\psi}_{ak} \sigma^A_{kl} \psi_{al}) - \tfrac{1}{36}\eta(\phi_i^*\phi_i)^3$$

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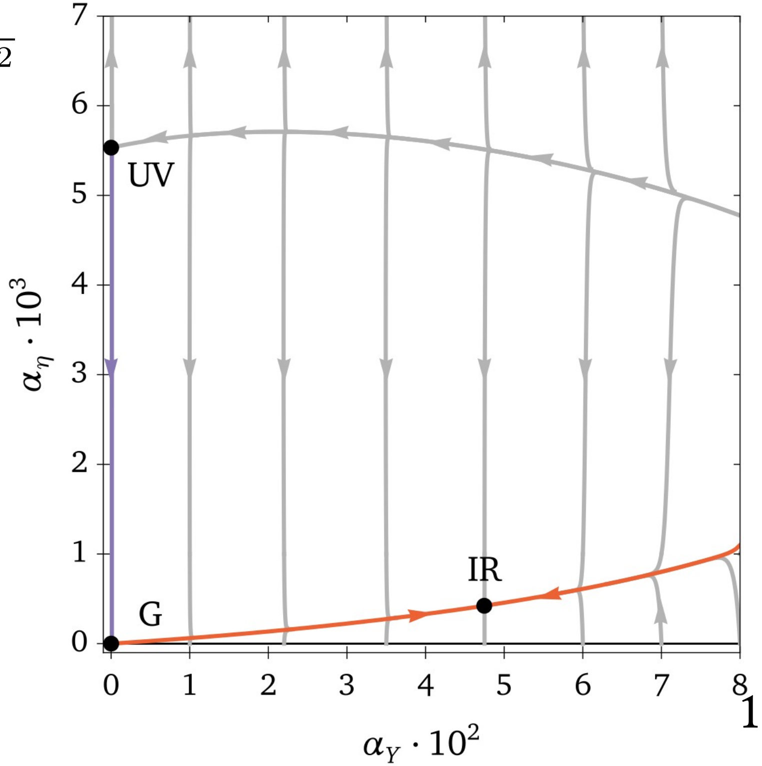

Conclusion

» towards overview of perturbative critical phenomena in d=3

» purely scalar theory: NLO effective potential, BMB argument weakened

» scalar-fermionic theory: 4L templates
new kind of IR fixed points

» up next: gauge sector