Unitary Scattering in Asymptotic Safety

UNIVERSITY OF SUSSEX

Angelo P. Chiesa

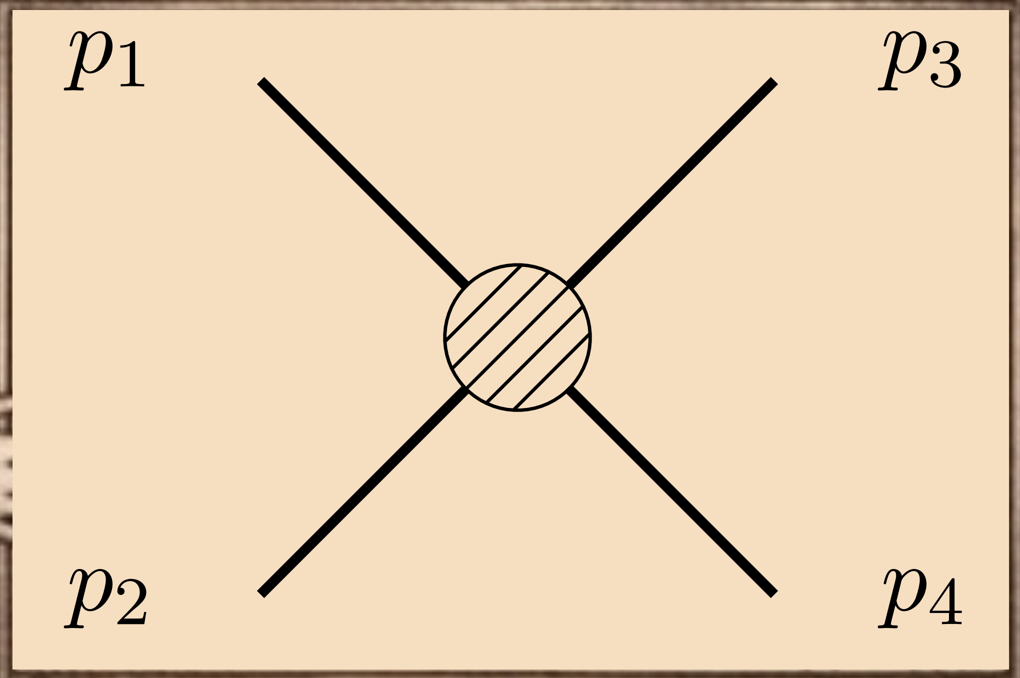
based on

[APC, Pawlowski, Reichert 2603.10168]

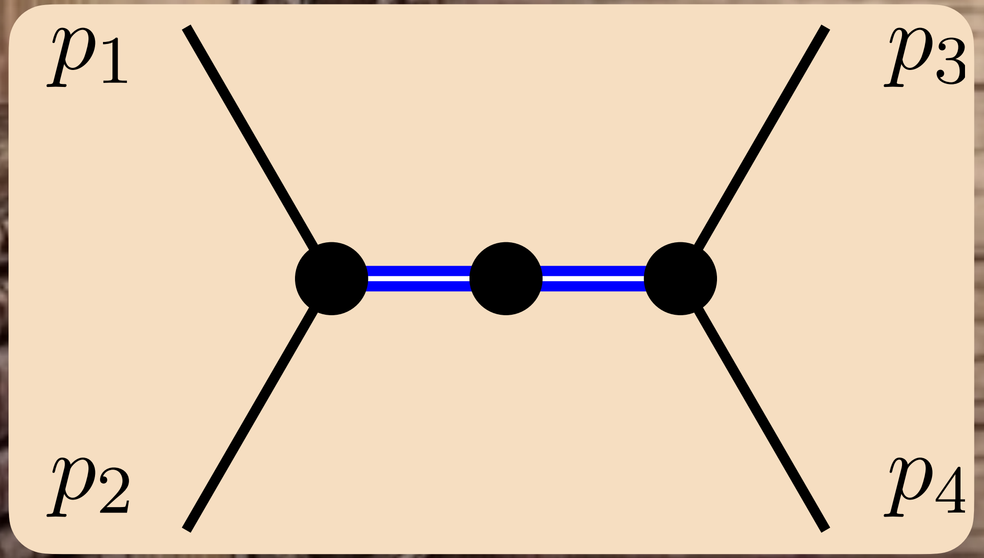
[APC, Reichert 261X.XXXXX]

“13th ERG Conference” @Sussex - 03/09/2026

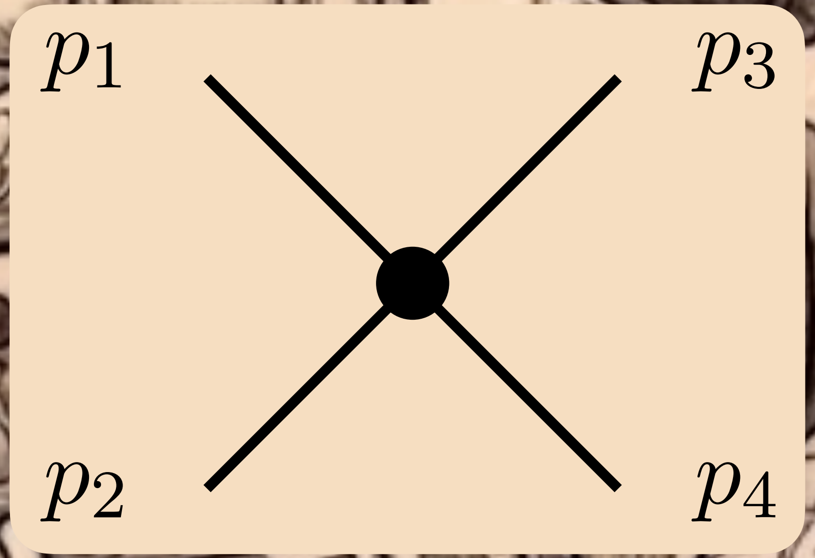
Overview



Euclidean:
[APC, Pawlowski, Reichert 2603.10168]



Lorentzian:
[APC, Reichert 261XX.XXXXX]

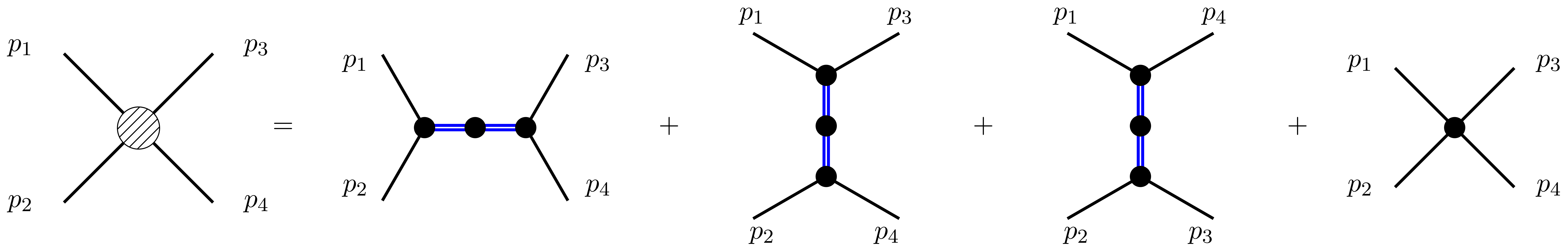


THE STAGE

Scattering in QG

$\phi\phi \to \phi\phi$ tree level diagrams

$$\Gamma_k = S_{EH} + S_{gf} + S_{gh} + \frac{1}{2}\int d^4x\sqrt{g}\,\partial_\mu\phi\partial^\mu\phi$$



Scattering in QG

$\phi\phi \to \chi\chi$ tree level diagrams

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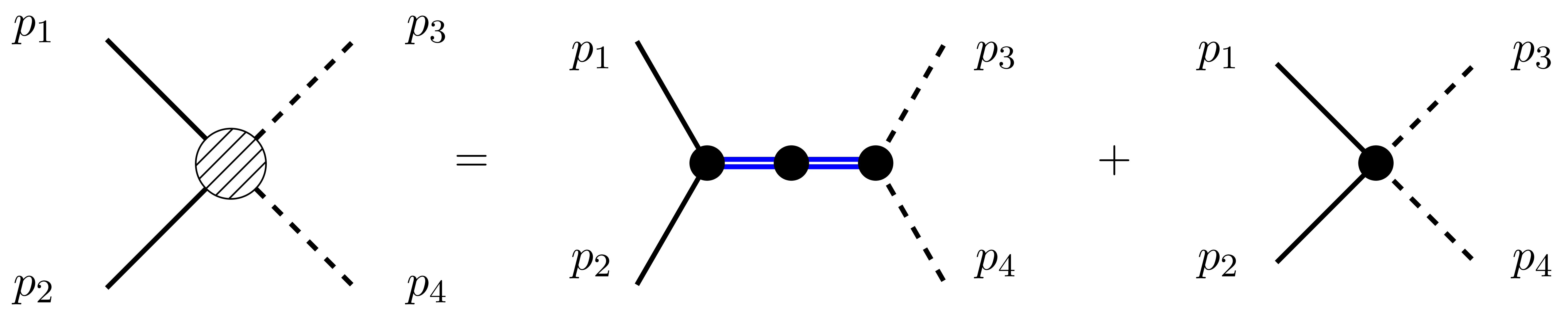

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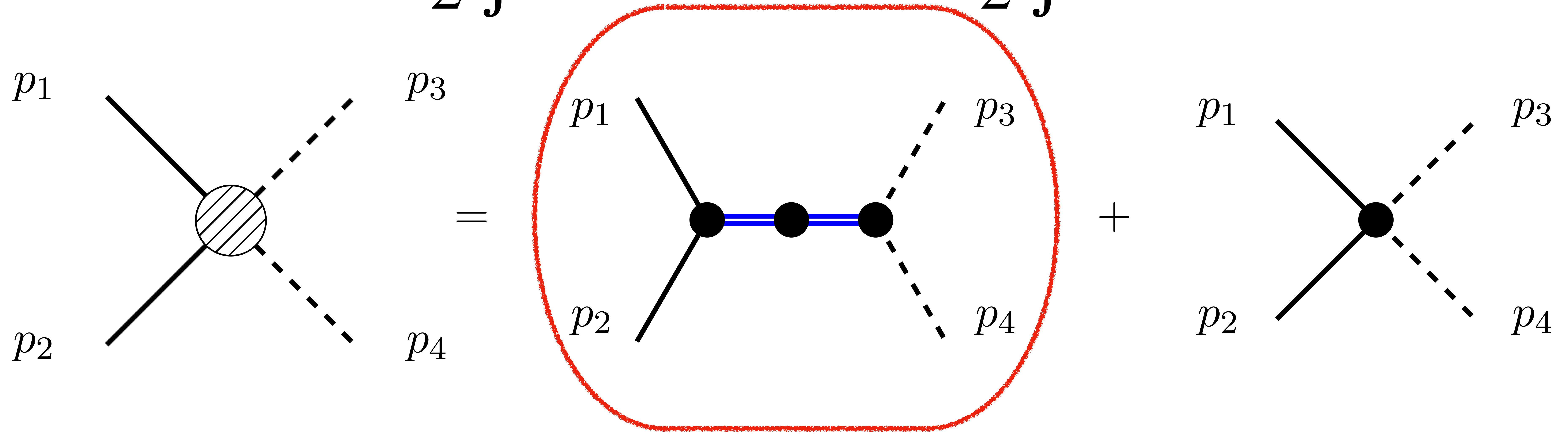

Gravity mediated

Scattering in QG

$\phi\phi \to \chi\chi$ tree level diagrams

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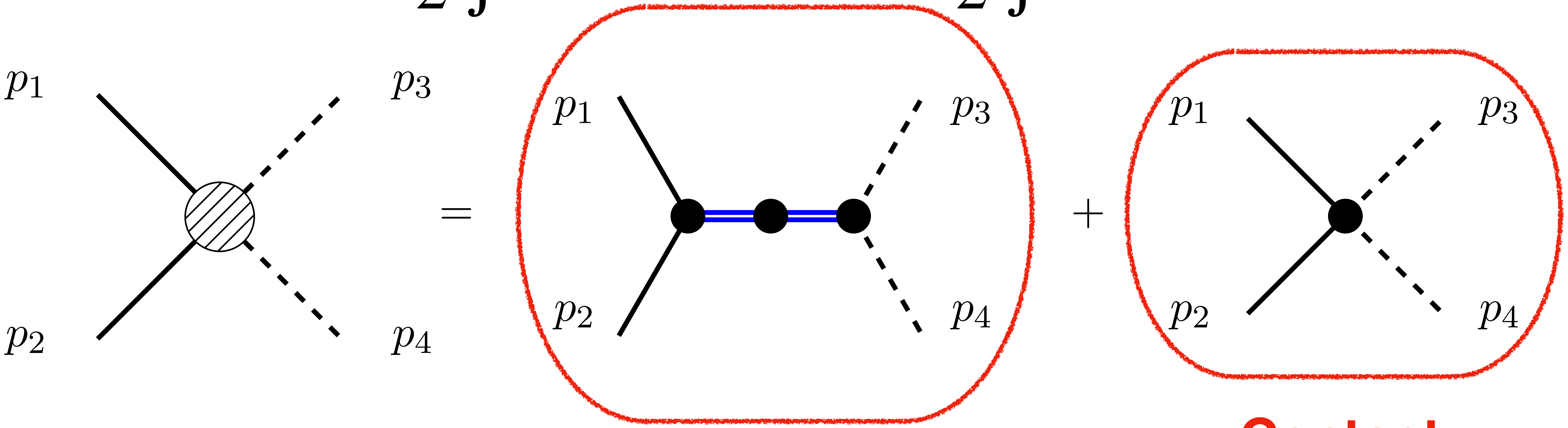


Gravity mediated

Contact (gravity induced)

Scattering in QG

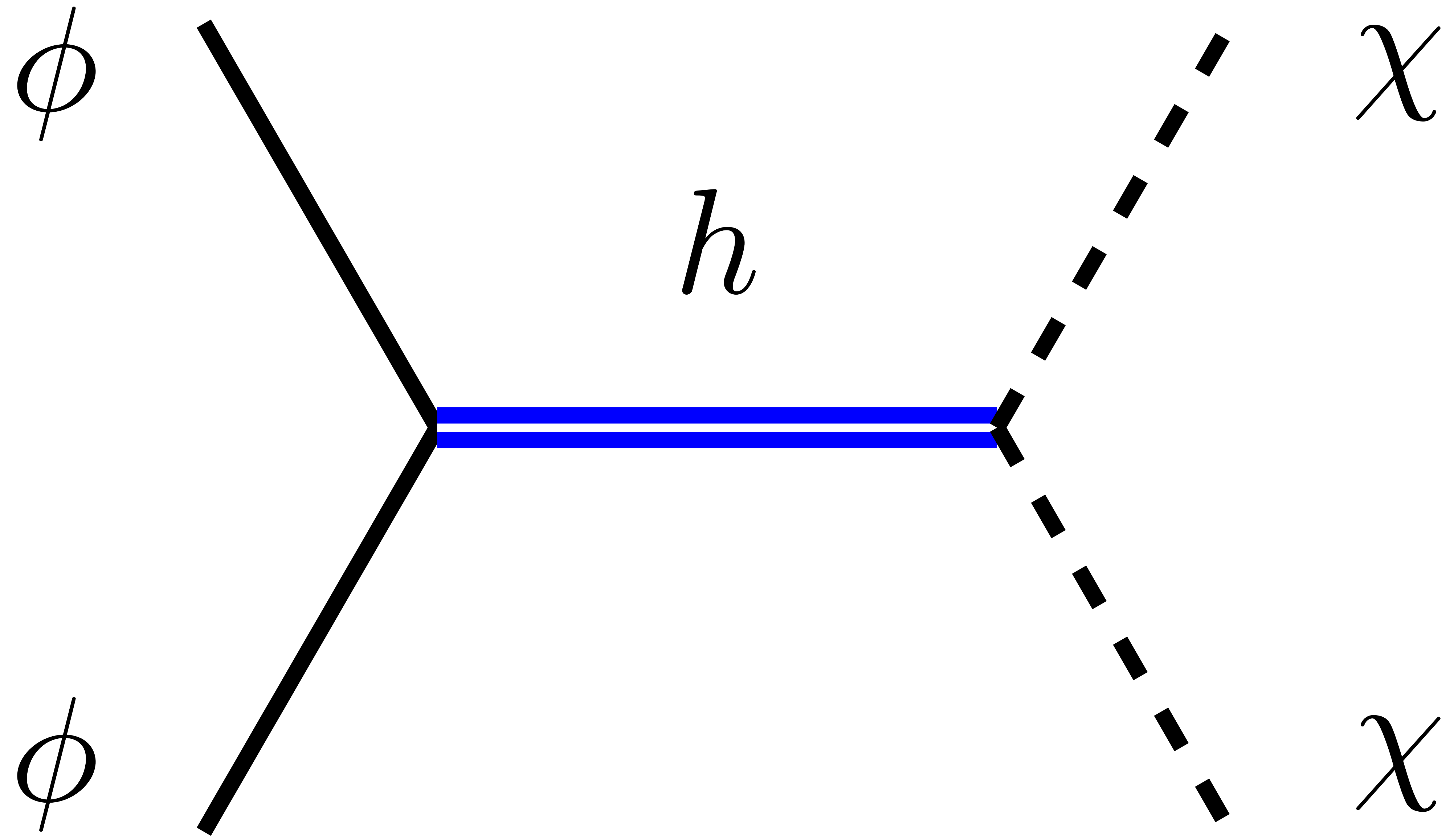
The cartoon

Scattering in QG

The cartoon

Graviton mediated process

$$\phi\phi \to \chi\chi$$



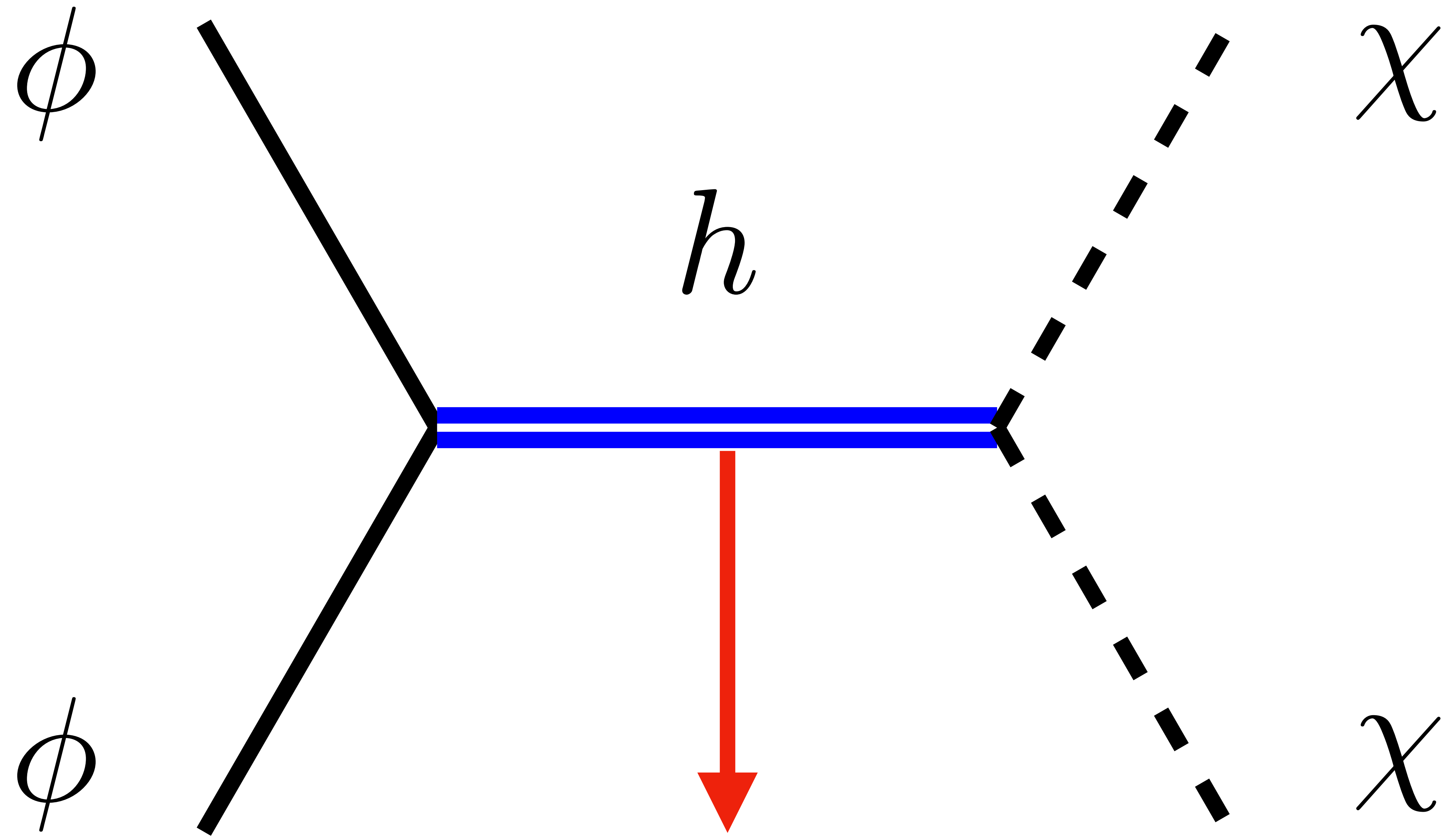
Classical $\mathscr{A} \sim$

Scattering in QG

The cartoon

Graviton mediated process

$\phi\phi \to \chi\chi$



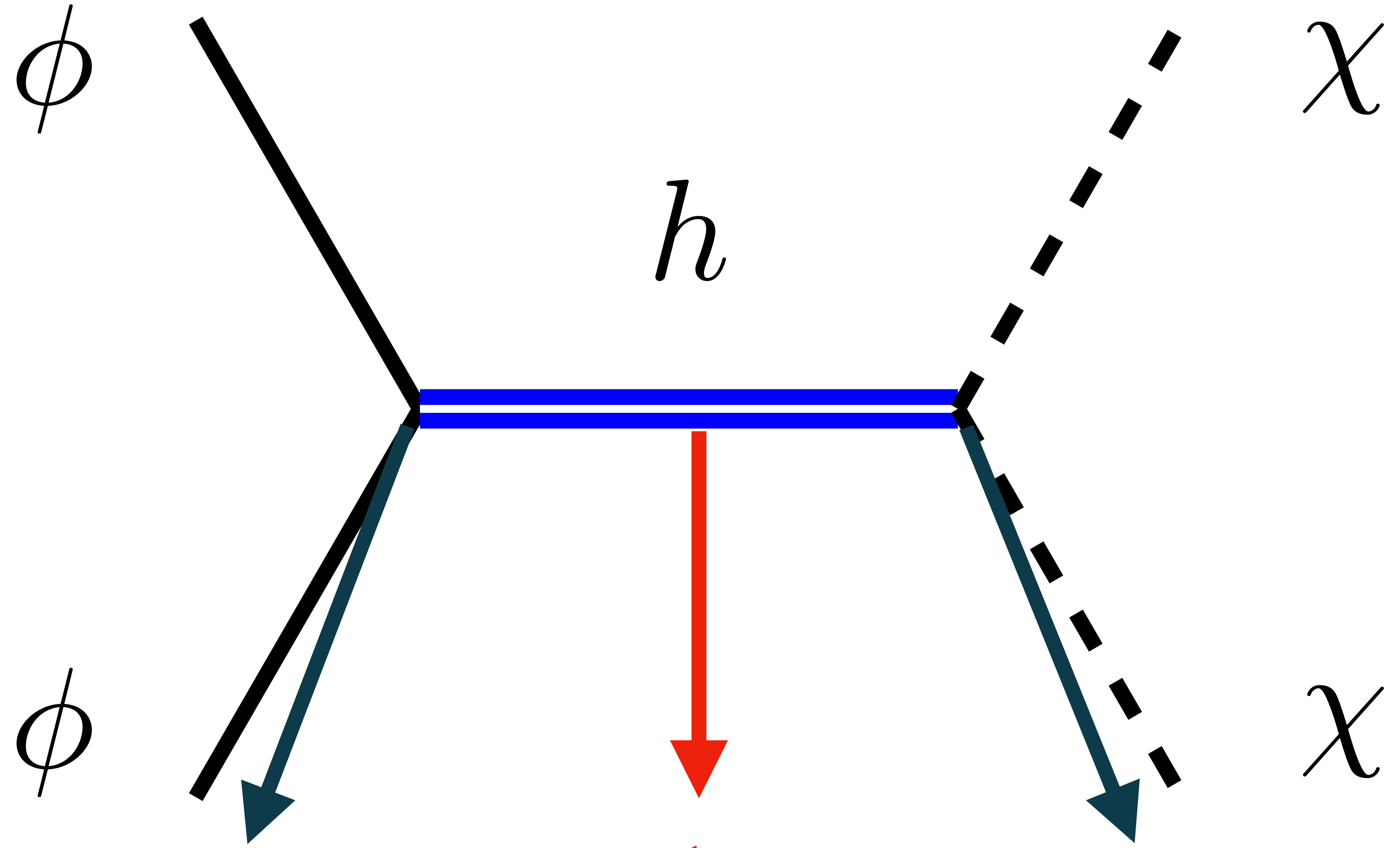
Classical $\mathscr{A} \sim \quad \times \; \frac{1}{s} \; \times$

Scattering in QG

The cartoon

Graviton mediated process

$$\phi\phi \to \chi\chi$$



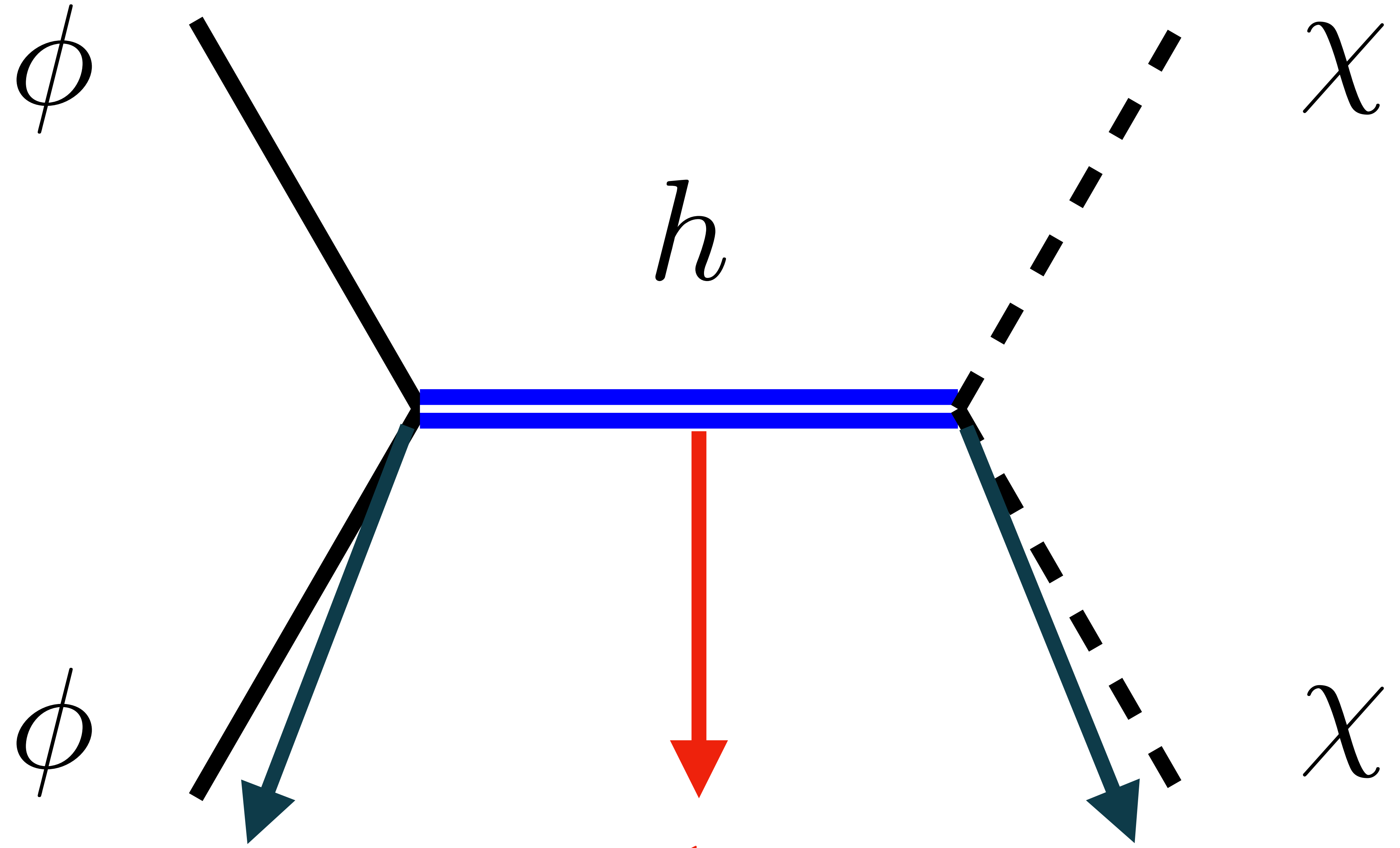
Classical $\qquad \mathscr{A} \sim \qquad \sqrt{G_N}\, s \quad \times \quad \frac{1}{s} \quad \times \quad \sqrt{G_N}\, s$

Scattering in QG

The cartoon

Graviton mediated process

$$\phi\phi \to \chi\chi$$



Classical $\quad \mathscr{A} \sim \sqrt{G_N}\, s \times \frac{1}{s} \times \sqrt{G_N}\, s \sim G_N\, s$

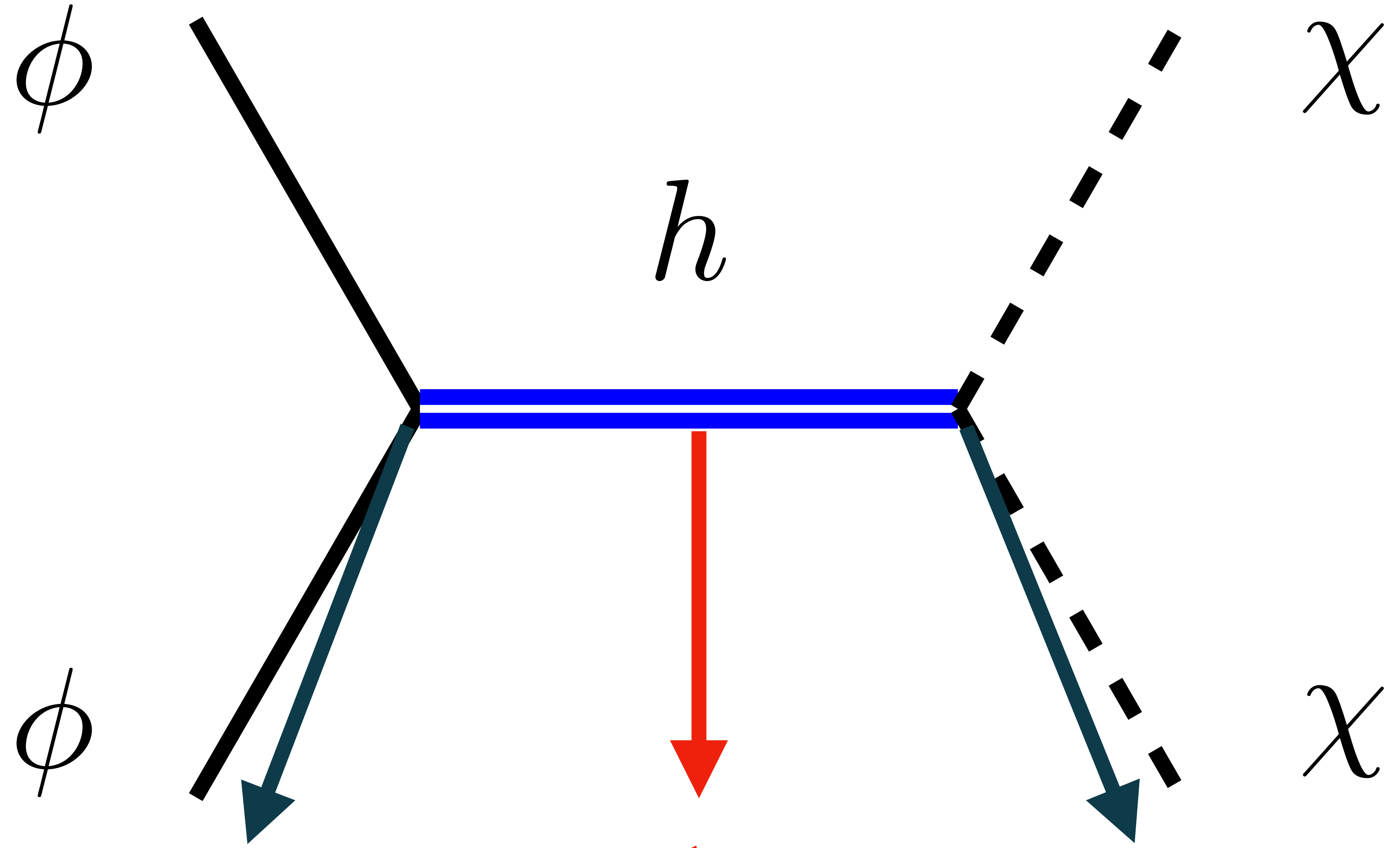
Unitarity at stake!!!

Scattering in QG

The cartoon

Graviton mediated process

$$\phi\phi \to \chi\chi$$



Classical $\quad \mathscr{A} \sim \sqrt{G_N\, s} \times \frac{1}{s} \times \sqrt{G_N\, s} \sim G_N\, s$ **Unitarity at stake!!!**

"The cure" $G_N = G_N(k) \sim \frac{1}{k^2}$ for $k > M_{pl}$ ➡ $\mathscr{A}$ **has a chance to be finite!**

Scattering in QG

The cartoon upgraded

1PI diagrams

$\phi\phi \to \chi\chi$

1PI version $\mathscr{A} \sim$

Scattering in QG

The cartoon upgraded

1PI diagrams

$\phi\phi \to \chi\chi$

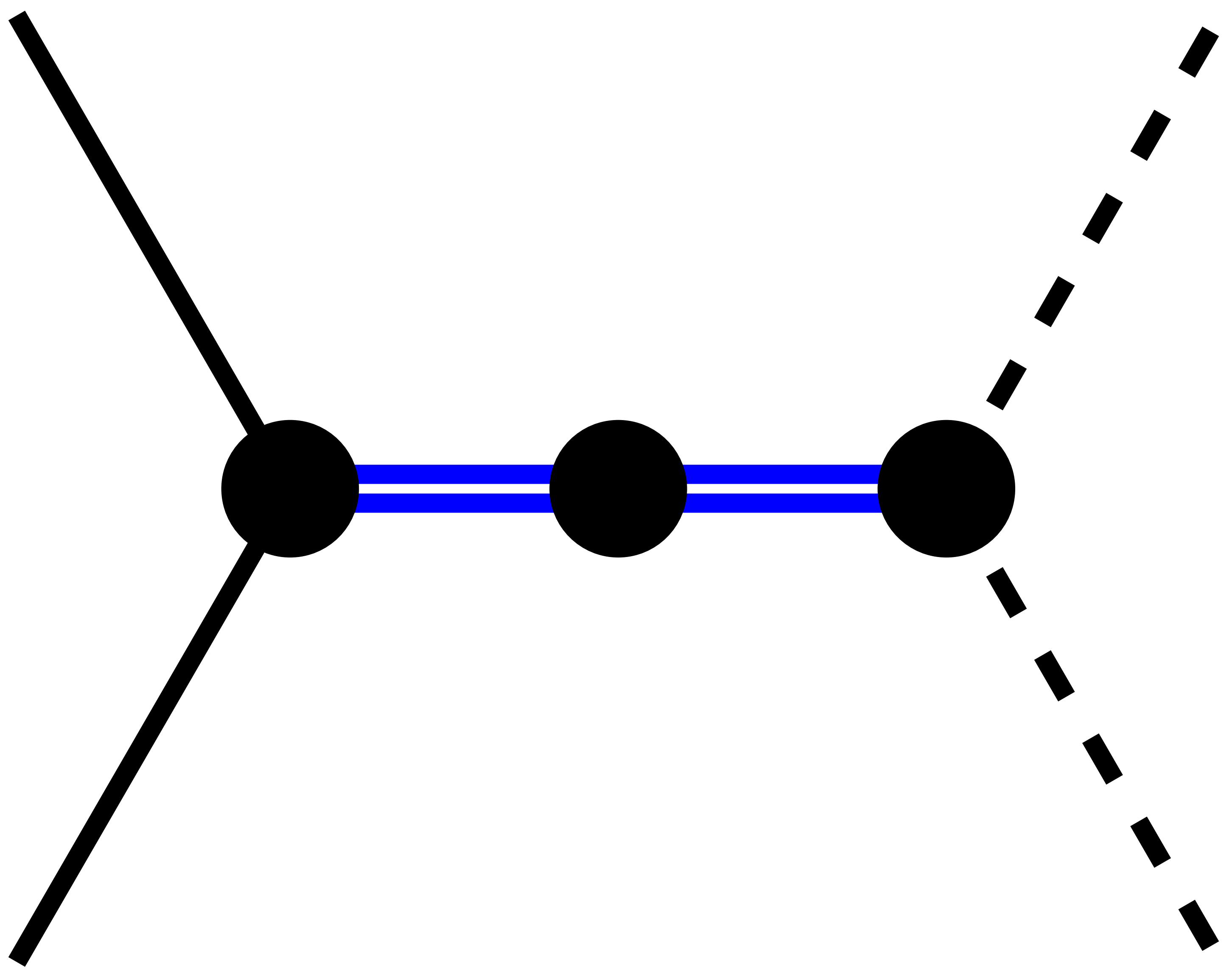
Fluctuation Approach

[Pawlowski, Reichert '21] [Pawlowski, Reichert '23]

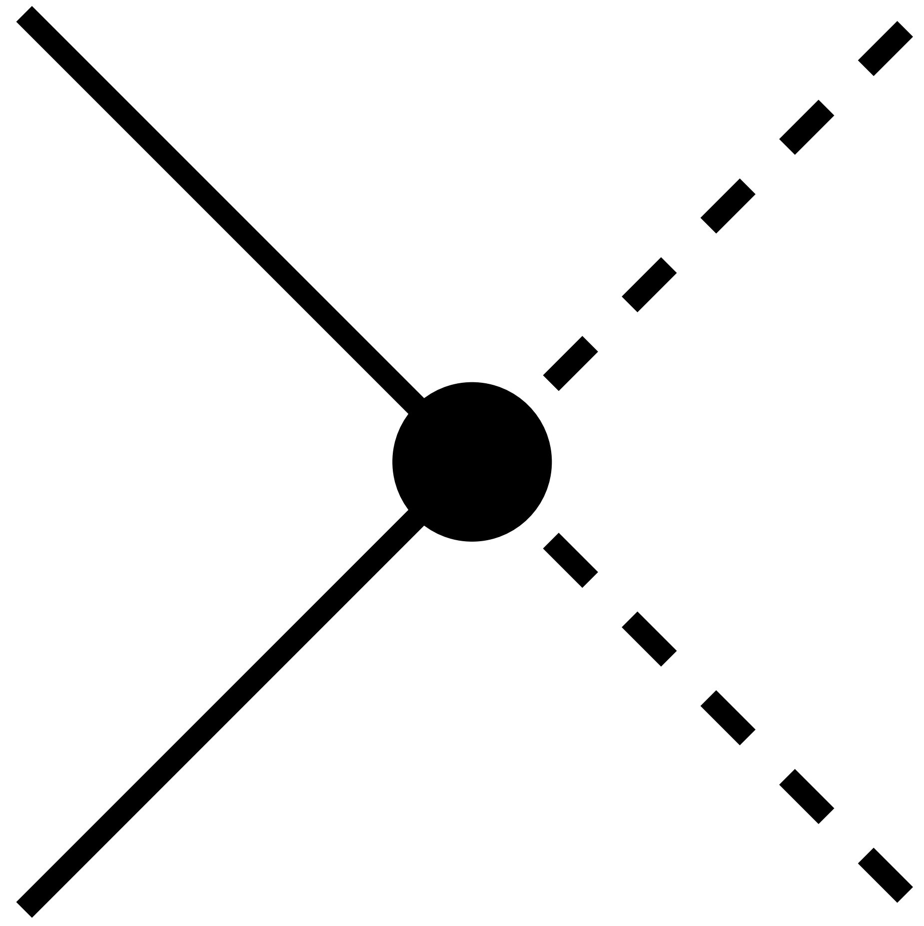
Form Factor Approach

[Knorr, Saueressig '18] [Draper, Knorr, Ripken, Saueressig '20]

1PI version $\mathscr{A} \sim$



$+$



Scattering in QG

The cartoon upgraded

1PI diagrams

$\phi\phi \to \chi\chi$

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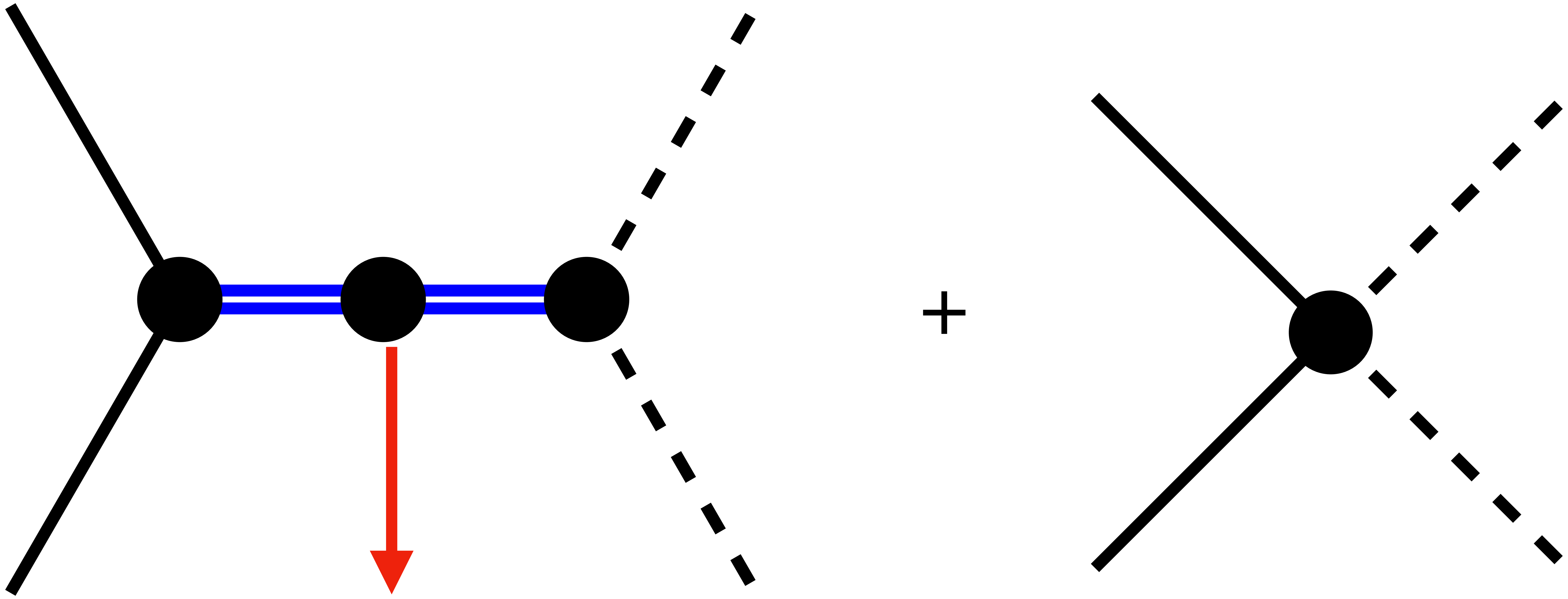

1PI version $\mathscr{A} \sim$ $\times \ \bar{\mathcal{G}}_{hh} \ \times$ $+$

Scattering in QG

The cartoon upgraded

1PI diagrams

$\phi\phi \to \chi\chi$

Fluctuation Approach

[Pawlowski, Reichert ’21] [Pawlowski, Reichert ’23]

Form Factor Approach

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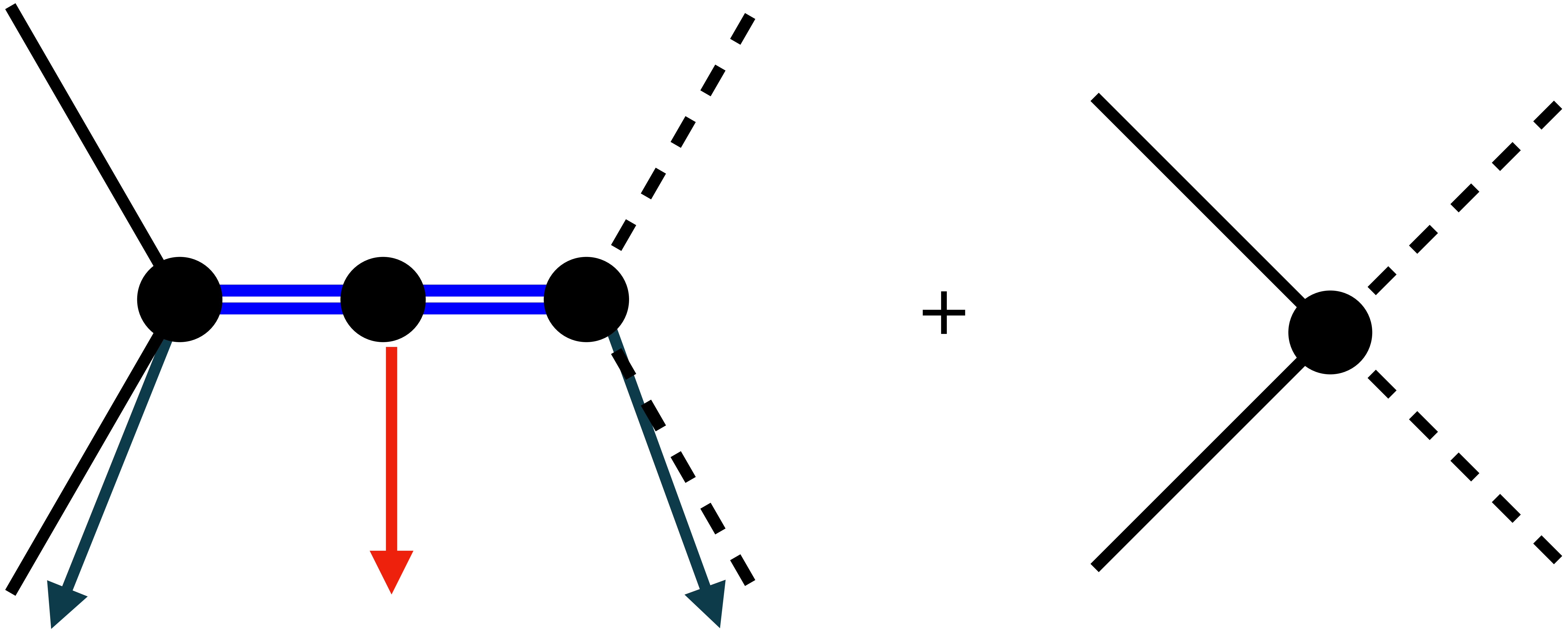

1PI version $\mathscr{A} \sim \bar{\Gamma}^{\phi\phi h} \times \bar{\mathcal{G}}_{hh} \times \bar{\Gamma}^{\phi\phi h}$

Scattering in QG

The cartoon upgraded

1PI diagrams

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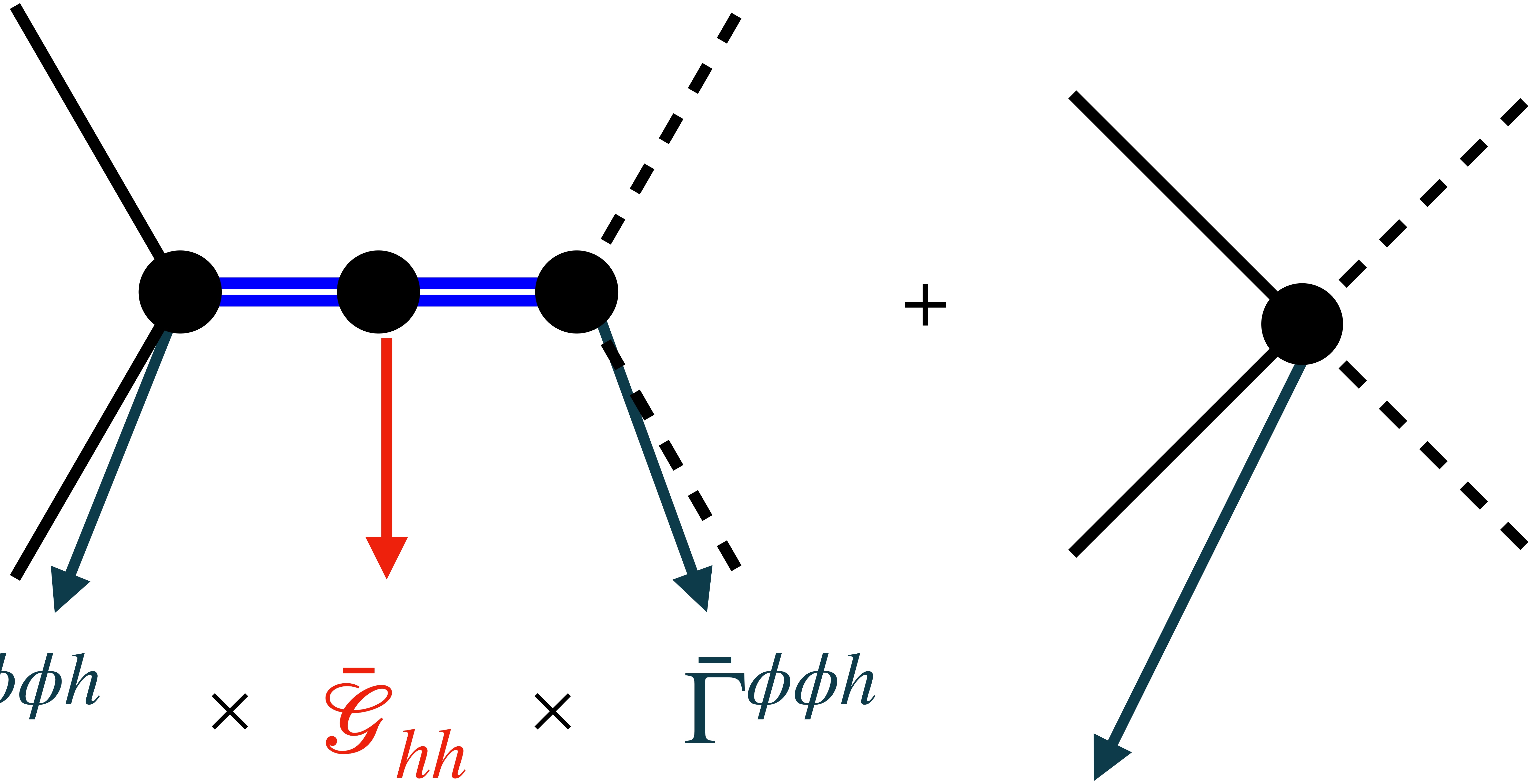
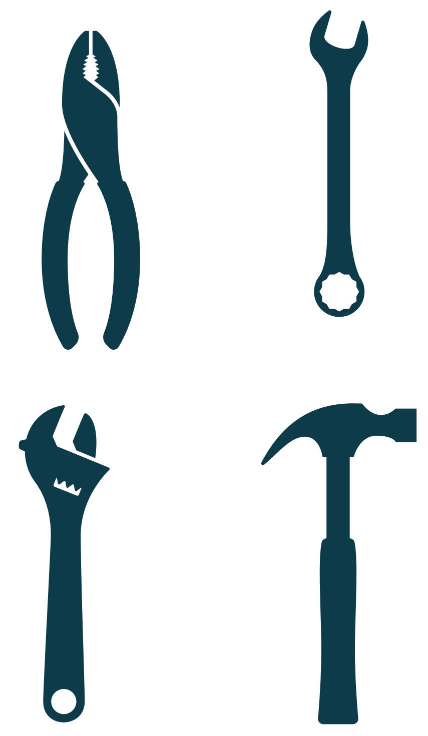


1PI version $\mathscr{A} \sim \quad \bar{\Gamma}^{\phi\phi h} \times \bar{\mathcal{G}}_{hh} \times \bar{\Gamma}^{\phi\phi h}$

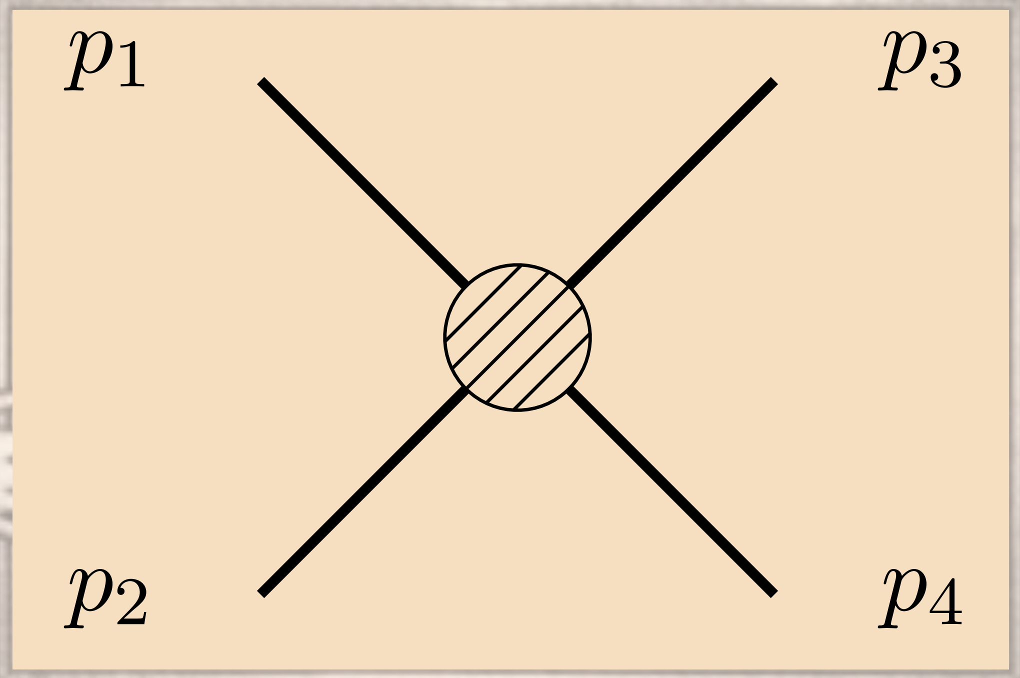
$$= V(s)\frac{1}{s}\left(t^2 - 4tu + u^2 - s^2\right) + \bar{\Gamma}^{(4)}(s,t)$$

HOW TO



Euclidean

✓	✗
Regulators Zoo	Numerical
30+ years	Wick Rotation



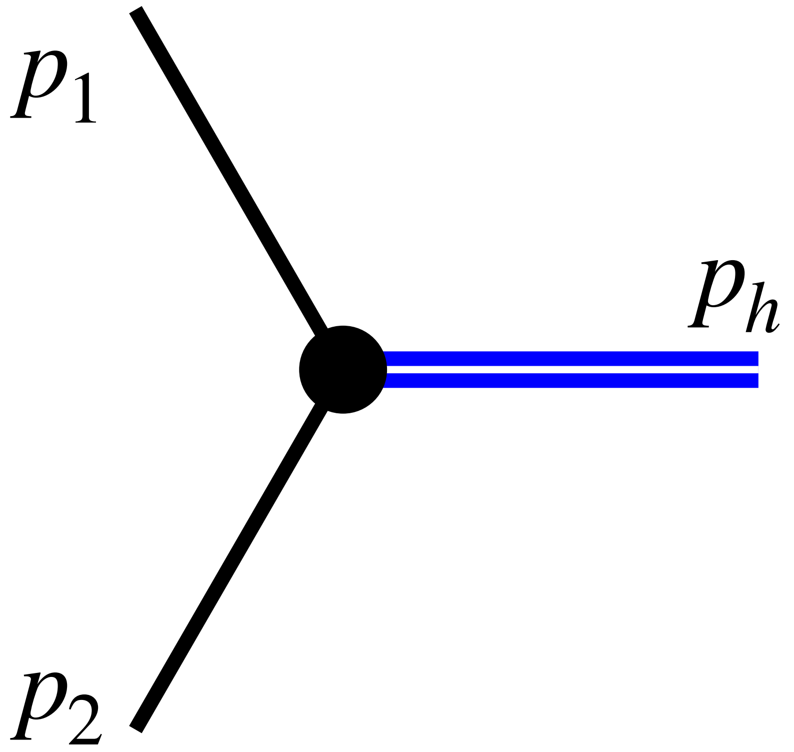
Lorentzian

✓	✗
Analytical	Only $R_k = k^2$
On-shell!	Unexplored

Euclidean $\Gamma^{\phi\phi h}$

"Through the looking glass"

[APC, Pawlowski, Reichert 2603.10168]



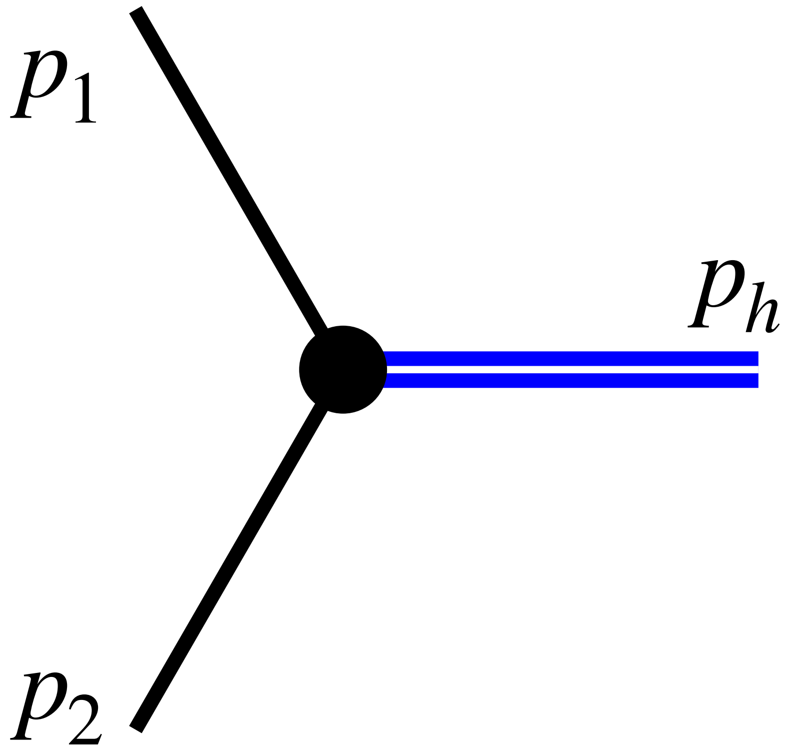
$$\sim \Gamma_k^{\phi\phi h}(p_1, p_2)$$

Euclidean $\Gamma^{\phi\phi h}$

“Through the looking glass“

[APC, Pawlowski, Reichert 2603.10168]

- Momenta $(\|p_1\|, \|p_2\|, z)$, $p_1 \cdot p_2 = \|p_1\| \, \|p_2\| \, z$



$\sim \Gamma_k^{\phi\phi h}(p_1, p_2)$

Euclidean $\Gamma^{\phi\phi h}$

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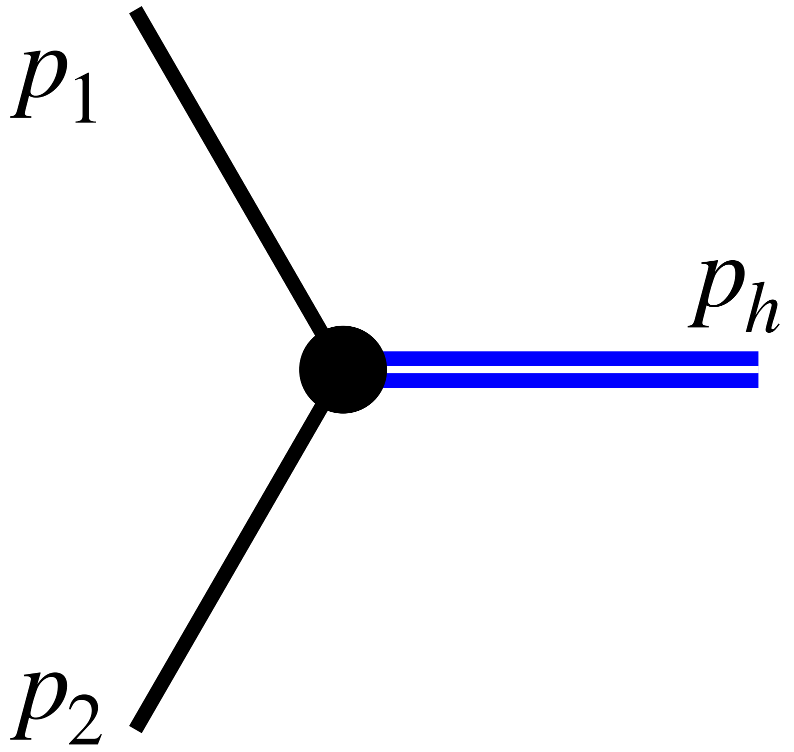
- Momenta $(\|p_1\|, \|p_2\|, z)$, $p_1 \cdot p_2 = \|p_1\| \, \|p_2\| \, z$
- Need Wick Rotation



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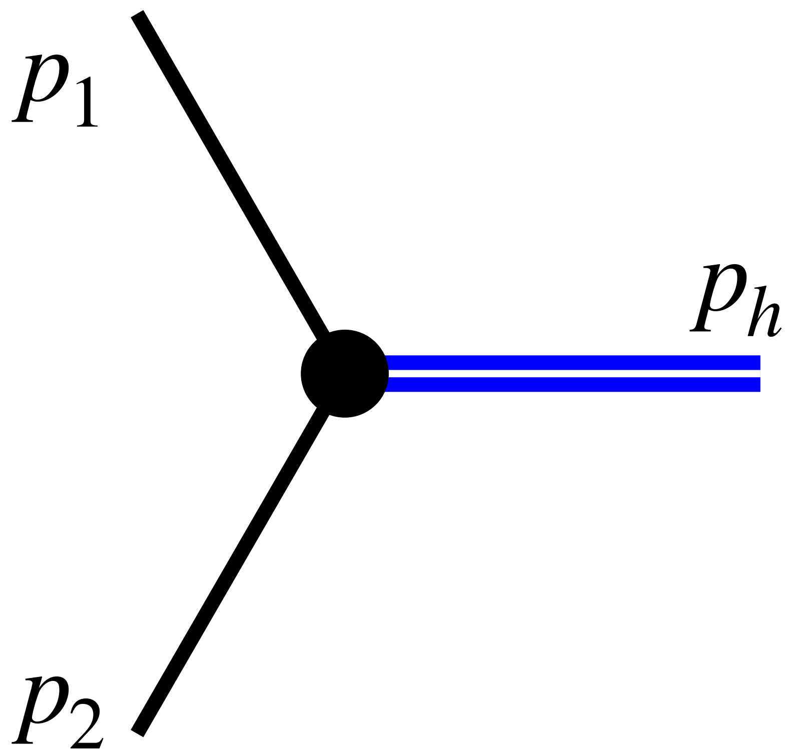
Euclidean $\Gamma^{\phi\phi h}$

[APC, Pawlowski, Reichert 2603.10168]

“Through the looking glass“

- Momenta $(\|p_1\|, \|p_2\|, z)$, $p_1 \cdot p_2 = \|p_1\| \, \|p_2\| \, z$
- Need Wick Rotation
- Reconstrution (see later)



$\sim \Gamma_k^{\phi\phi h}(p_1, p_2)$

Euclidean $\Gamma^{\phi\phi h}$

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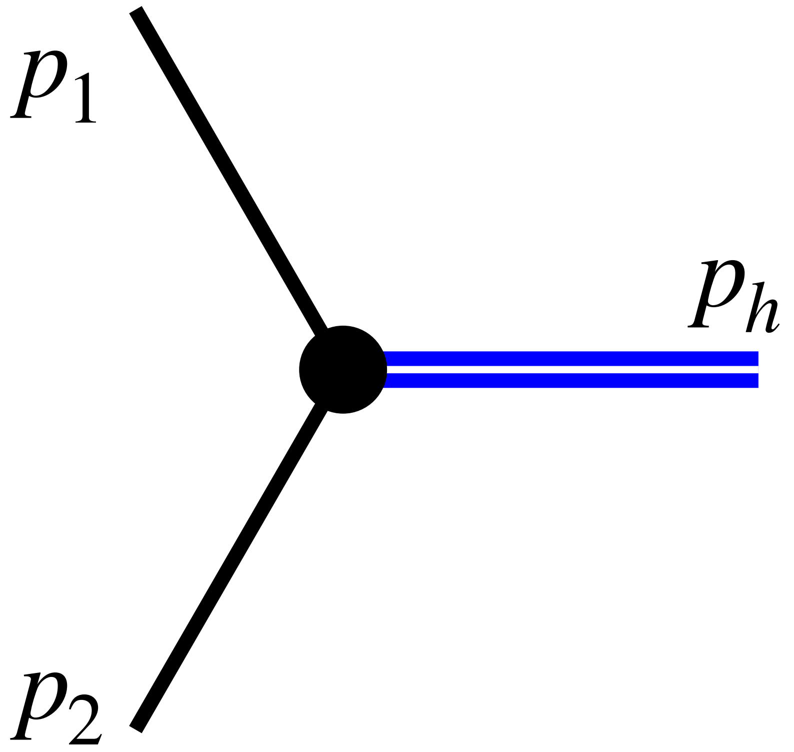

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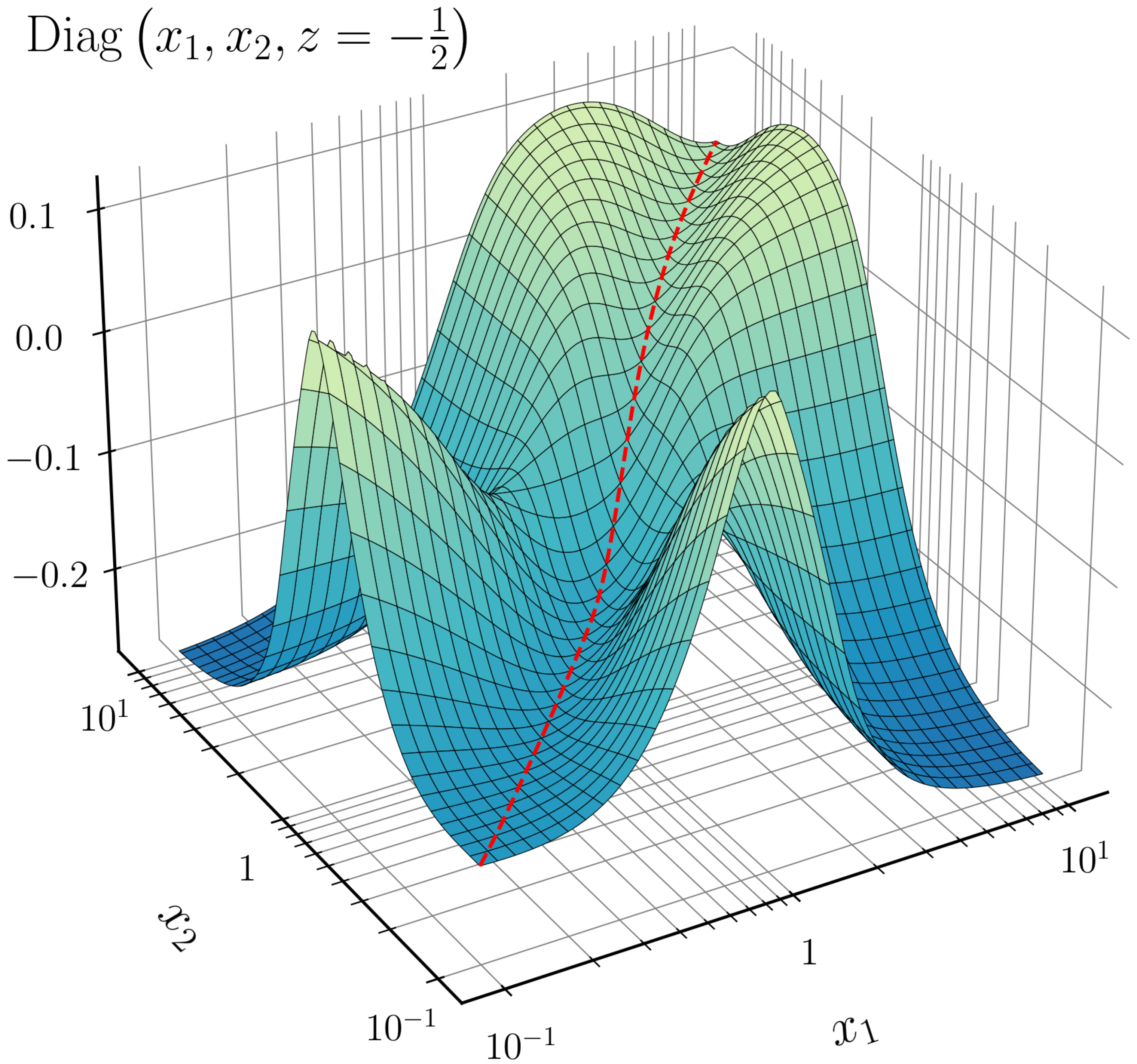
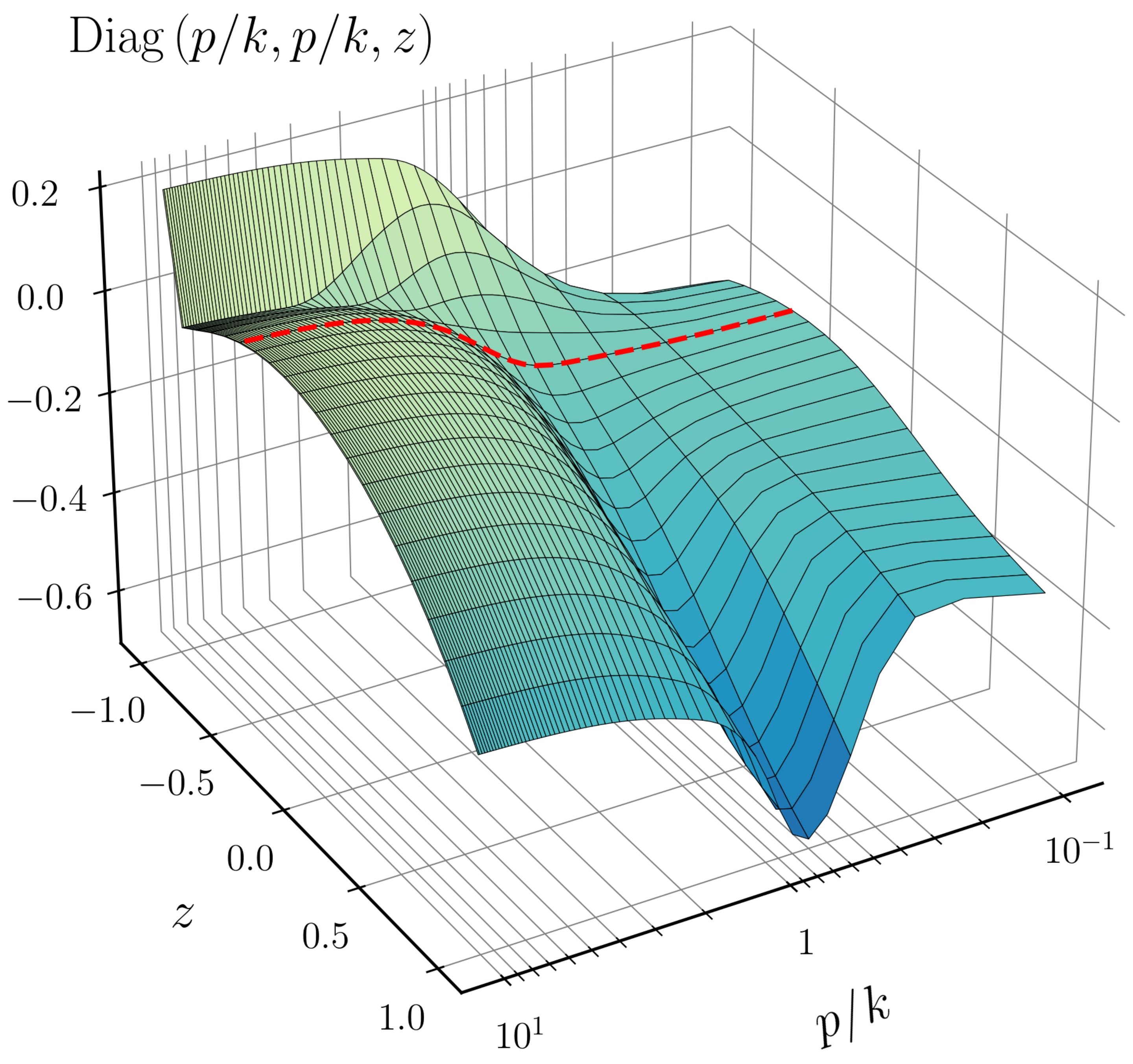
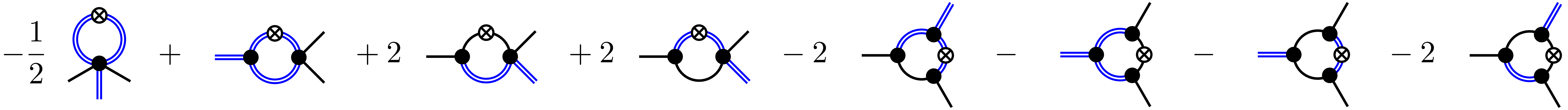




$\partial_t \boldsymbol{\Gamma}^{\phi\phi h} =$



Euclidean $\Gamma^{\phi\phi h}$

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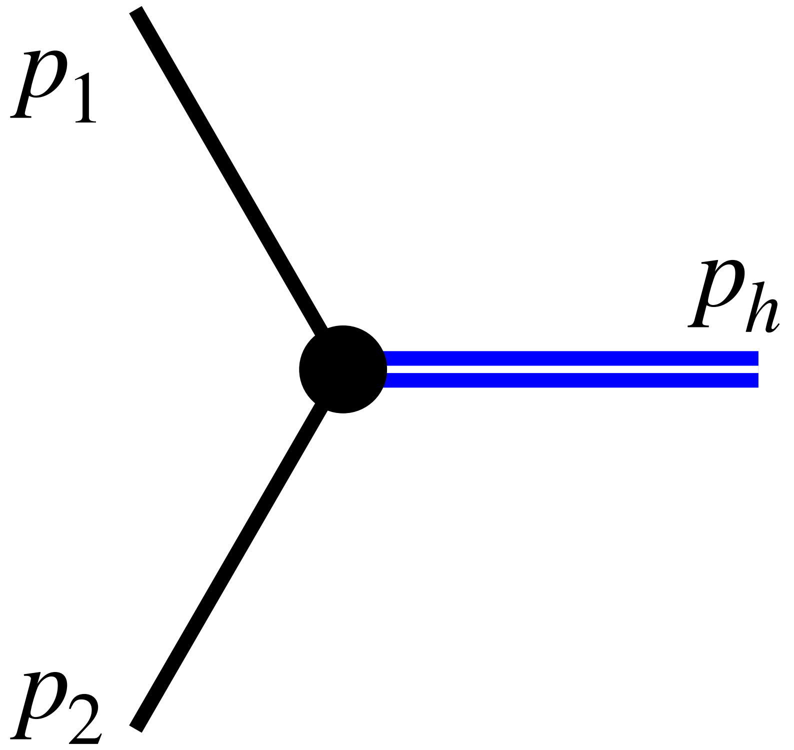
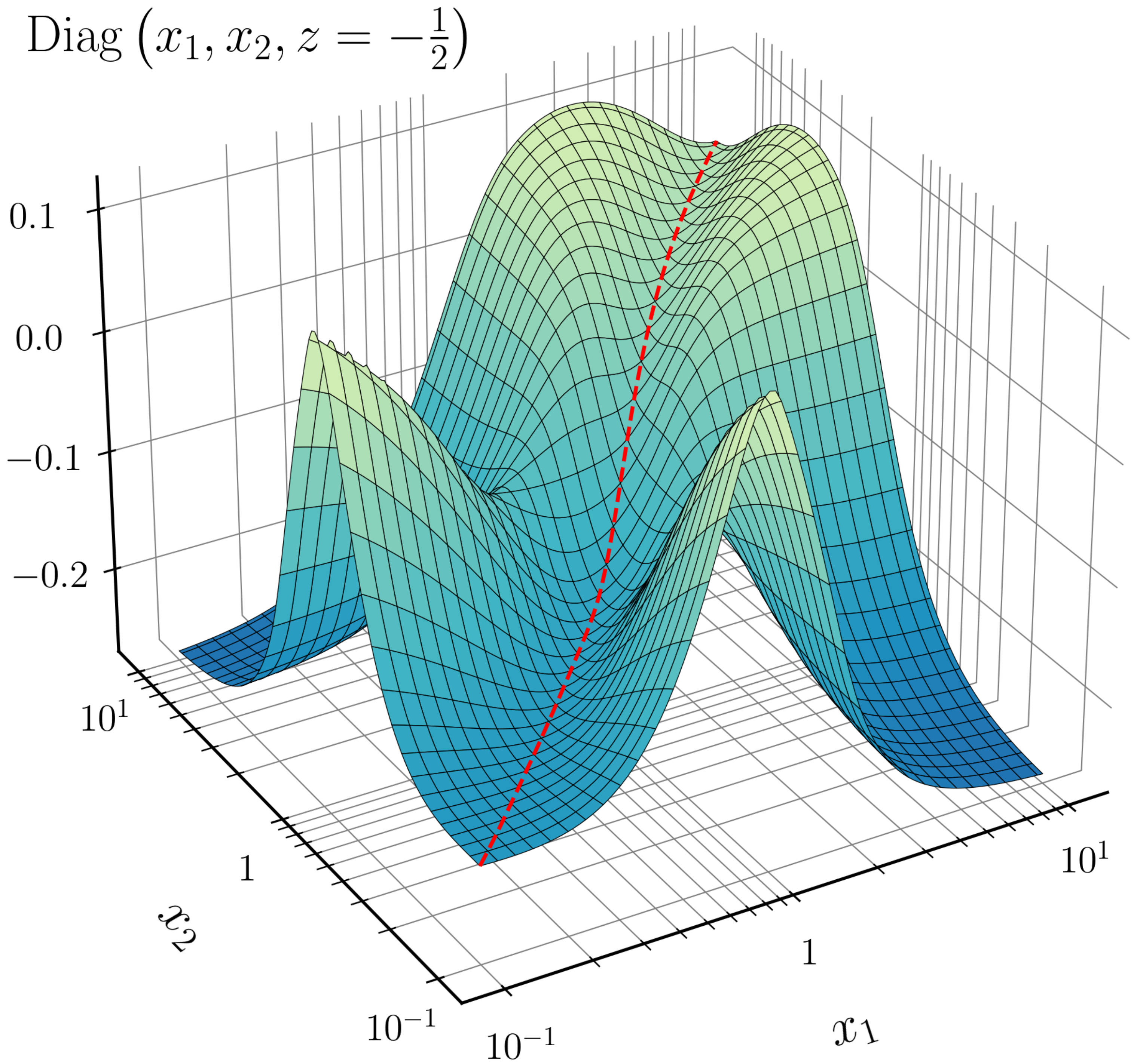
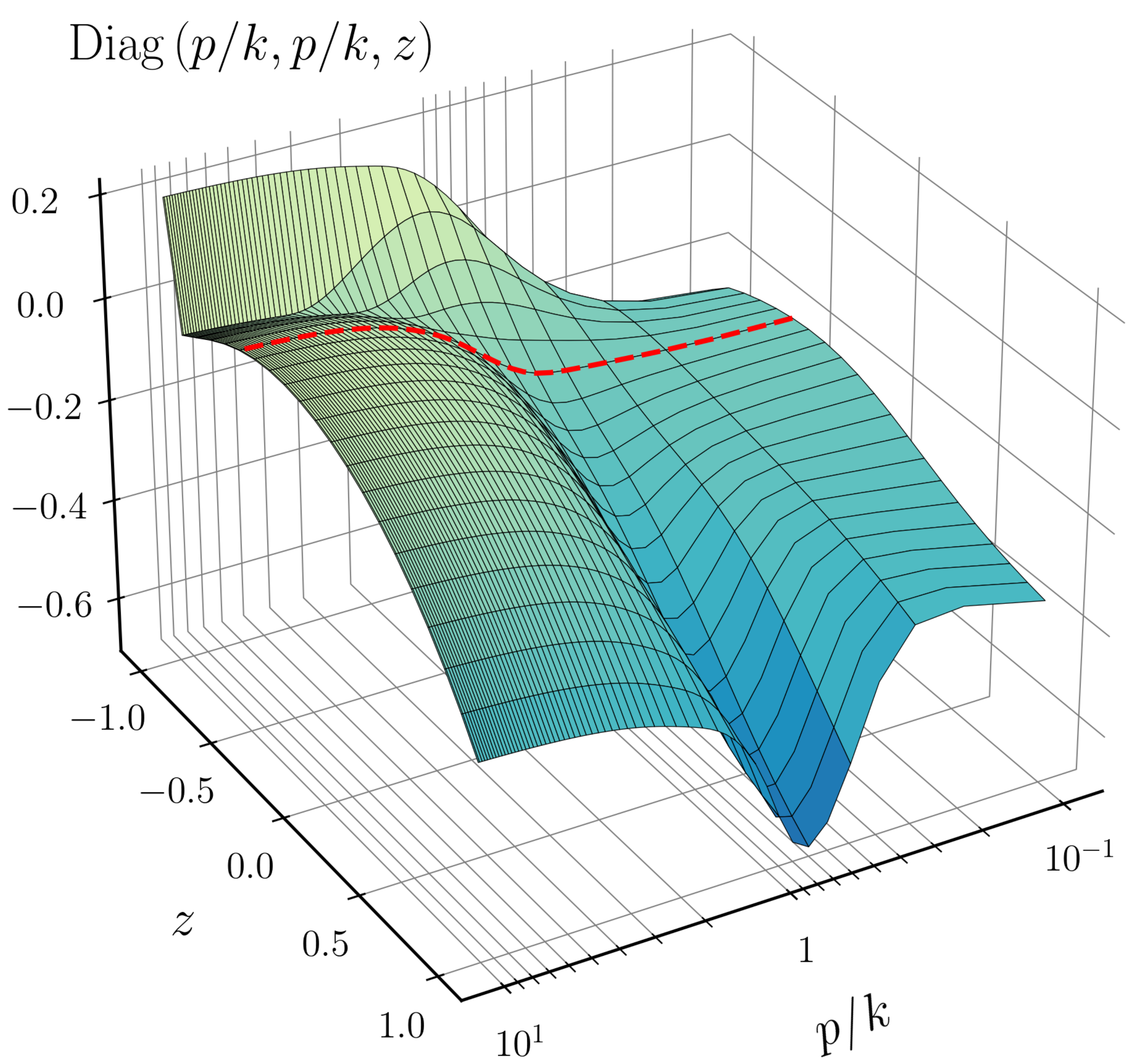
[APC, Pawlowski, Reichert 2603.10168]

See also talk by Benjamin Knorr

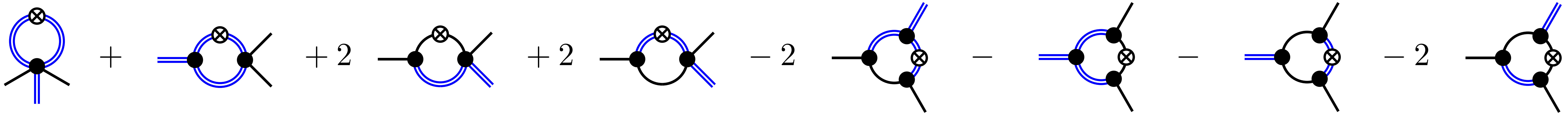


$\sim \Gamma_k^{\phi\phi h}(p_1, p_2)$





$\partial_t \mathbf{\Gamma}^{\phi\phi h} = -\frac{1}{2}$



Lorentzian $\Gamma^{(4)}$

[APC, Reichert 261X.XXXXX]

“Through the looking glass“

- Lorentzian fRG
- On-shell, Lorentzian flow $\partial_t \mathrm{Im}\, \Gamma_k^{(4)}(s,t)$
- $\Gamma_k \supset \frac{1}{2}\int d^4x\sqrt{g}\,\partial_\mu\phi\partial^\mu\phi + \frac{1}{2}\int d^4x\sqrt{g}\,\partial_\mu\chi\partial^\mu\chi$

Lorentzian $\Gamma^{(4)}$

[APC, Reichert 261X.XXXXX]

“Through the looking glass“

- Lorentzian fRG [Braun et al. ’22] [Fehre, Litim, Pawlowski, Reichert. ’21] [Kher, King, Litim, Reichert. ’25] [Pawlowski, Reichert, Wessely ‘25] [Assant, Litim, Reichert ’26]
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Lorentzian $\Gamma^{(4)}$

[APC, Reichert 261X.XXXXX]

“Through the looking glass“

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See talk by Gabriel Assant
See poster by Rowan Packer

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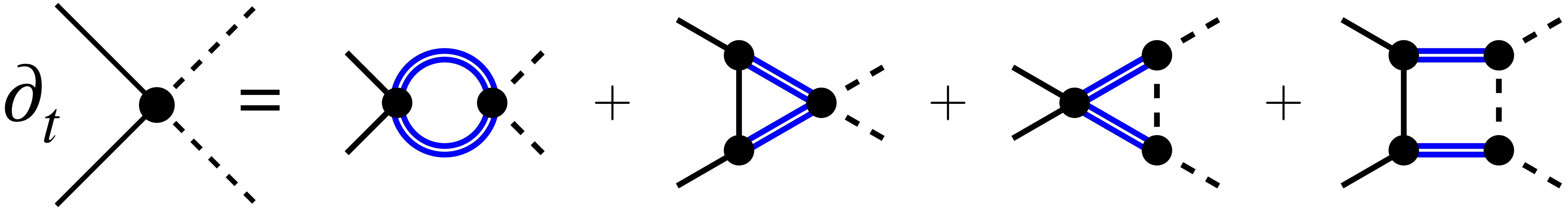

Lorentzian $\Gamma^{(4)}$

[APC, Reichert 261X.XXXXX]

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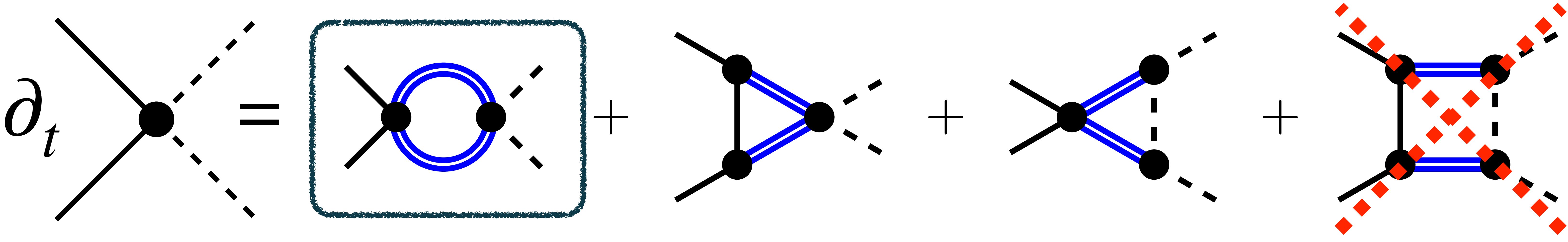


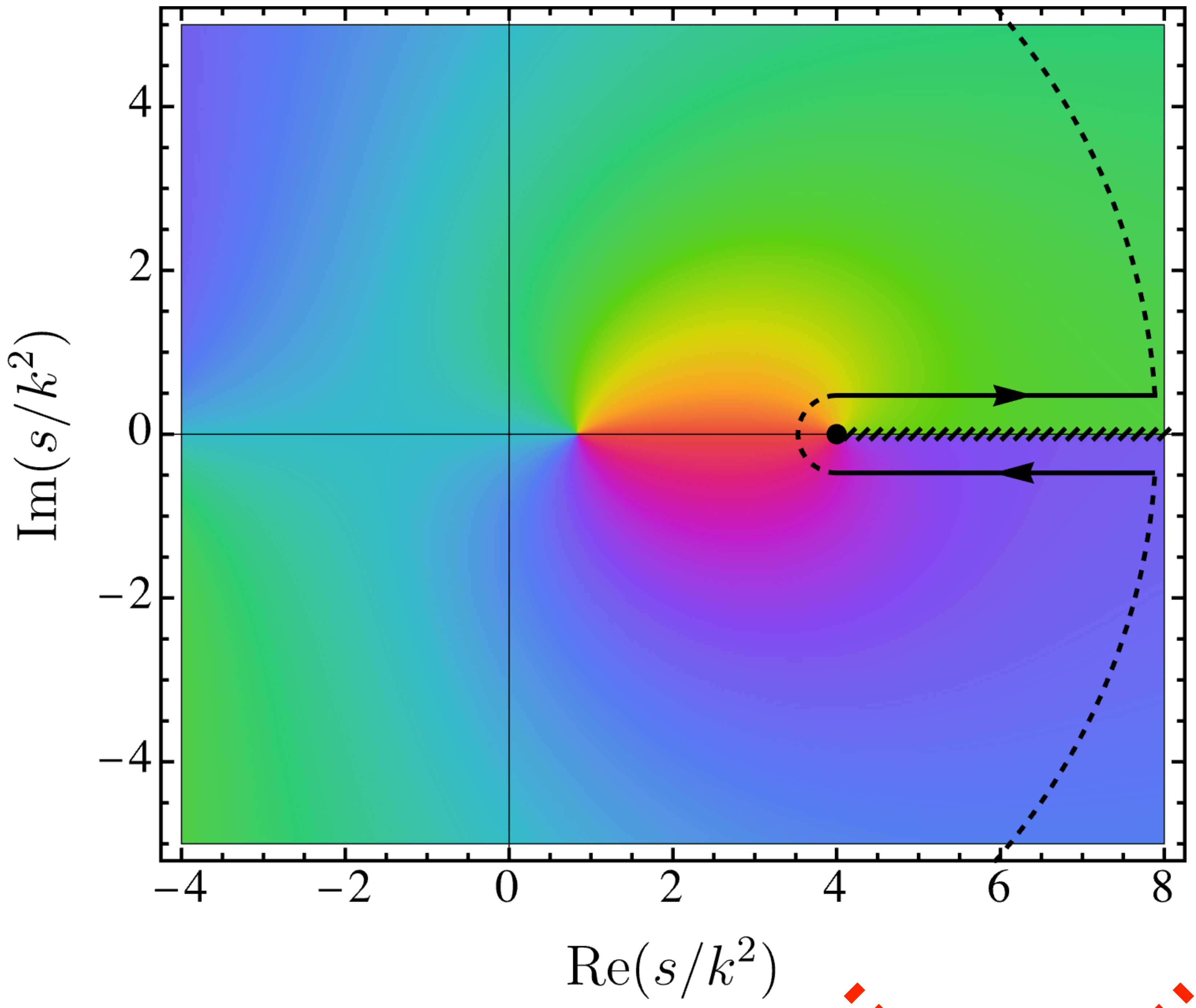
See also [Knorr ’26]

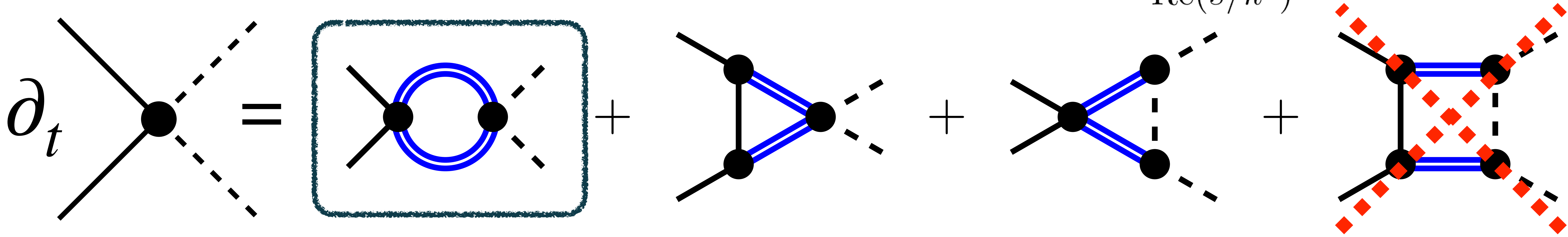
Lorentzian $\Gamma^{(4)}$

“Through the looking glass“

[APC, Reichert 261X.XXXXX]

- On-shell, Lorentzian flow



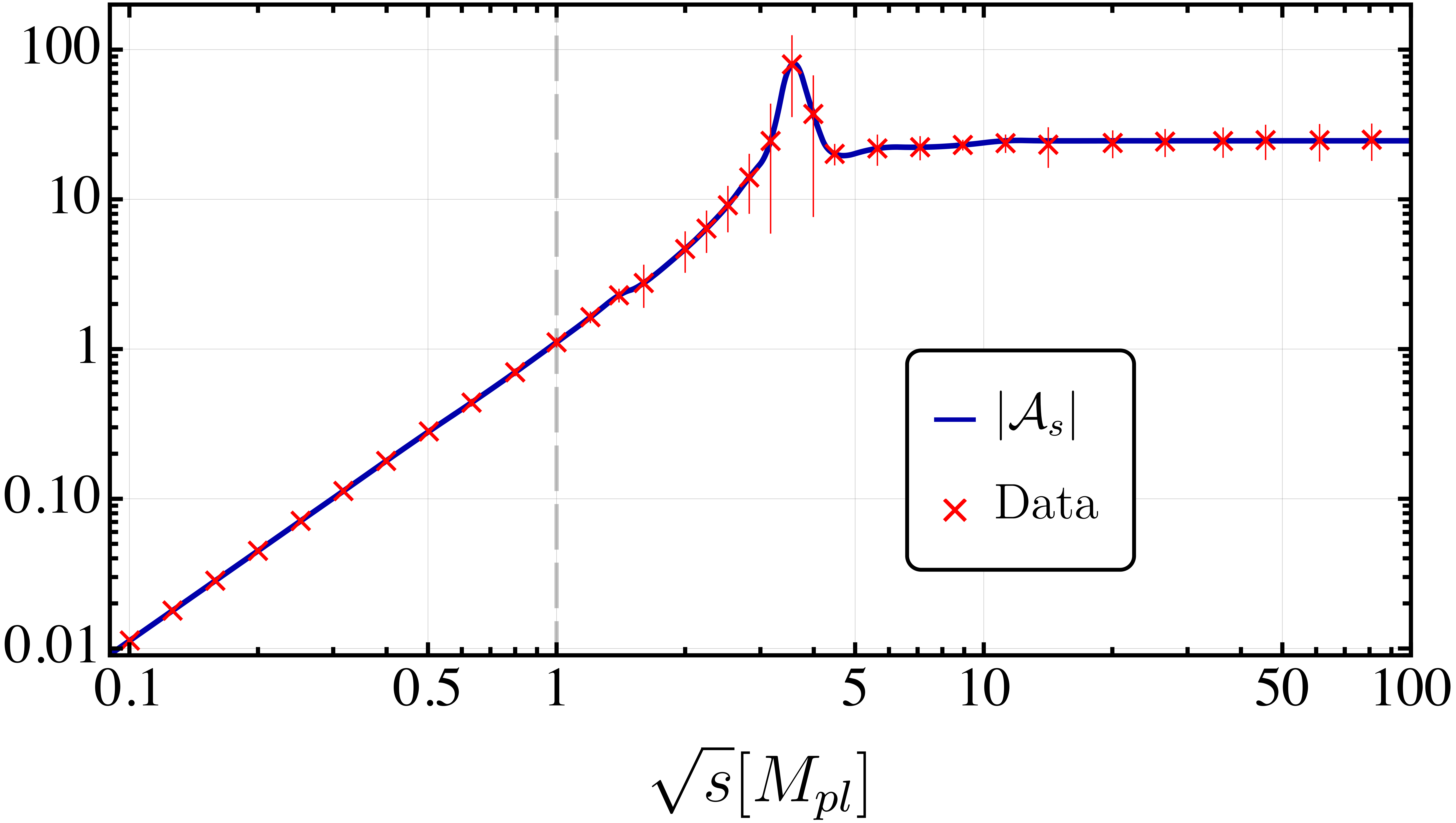


See also [Knorr ’26]

RESULTS

RESULT

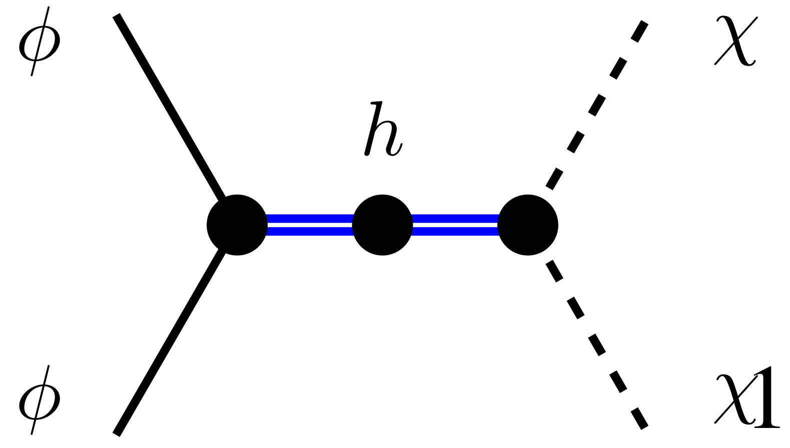
Finite Amplitude

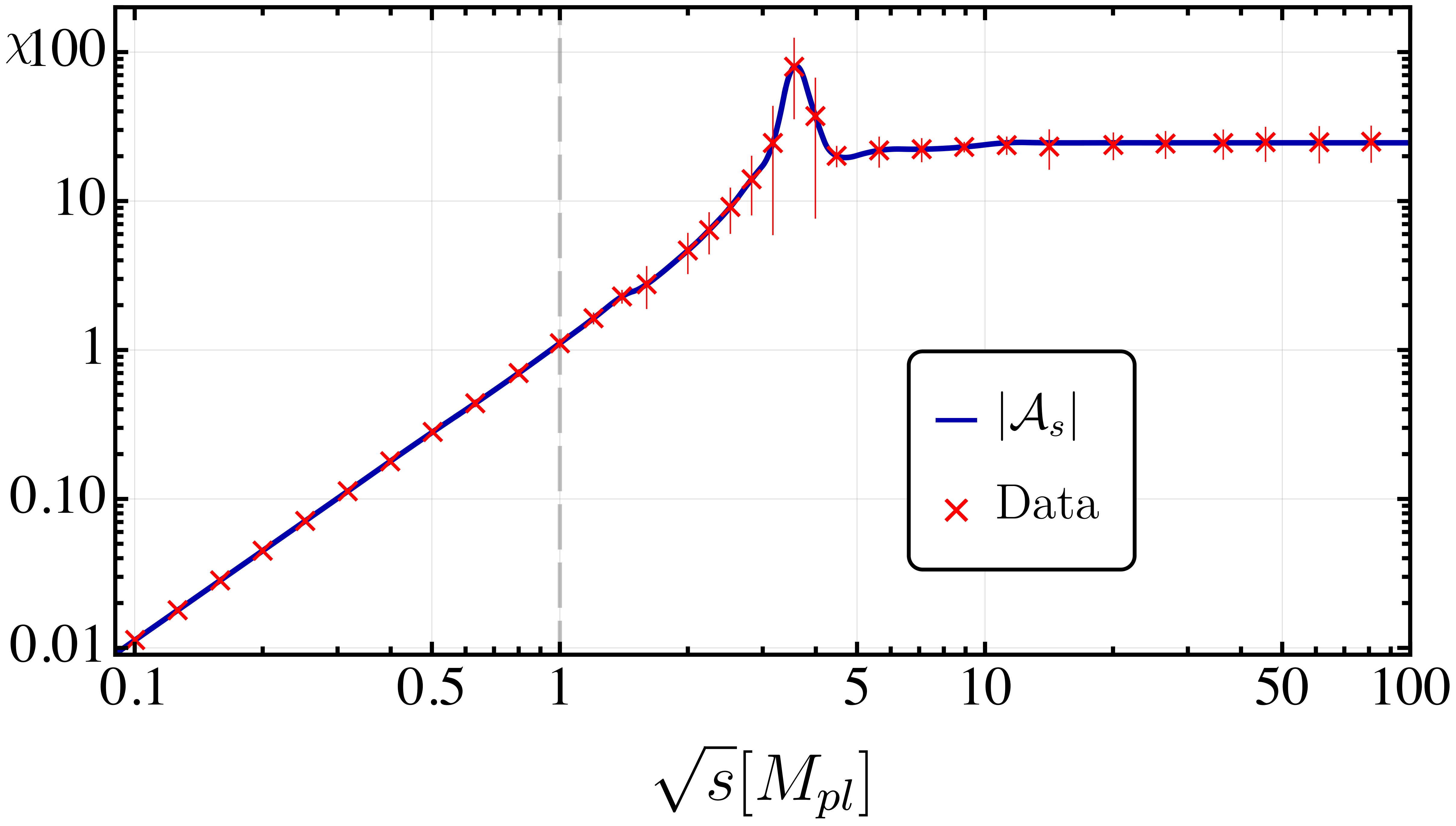


RESULT

Finite Amplitude

- $s-$channel

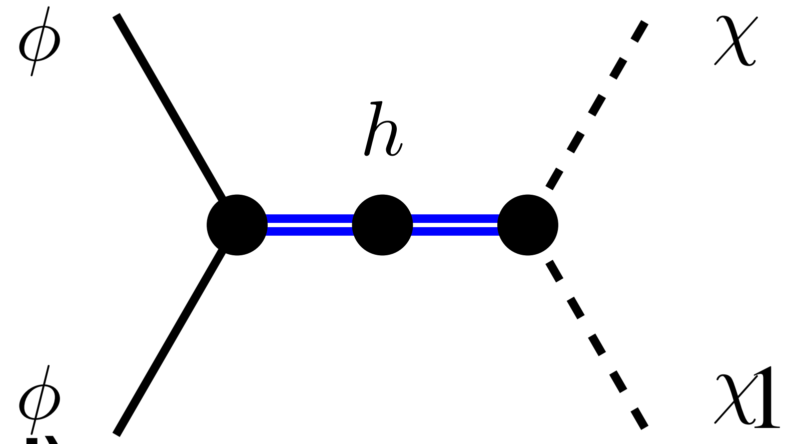


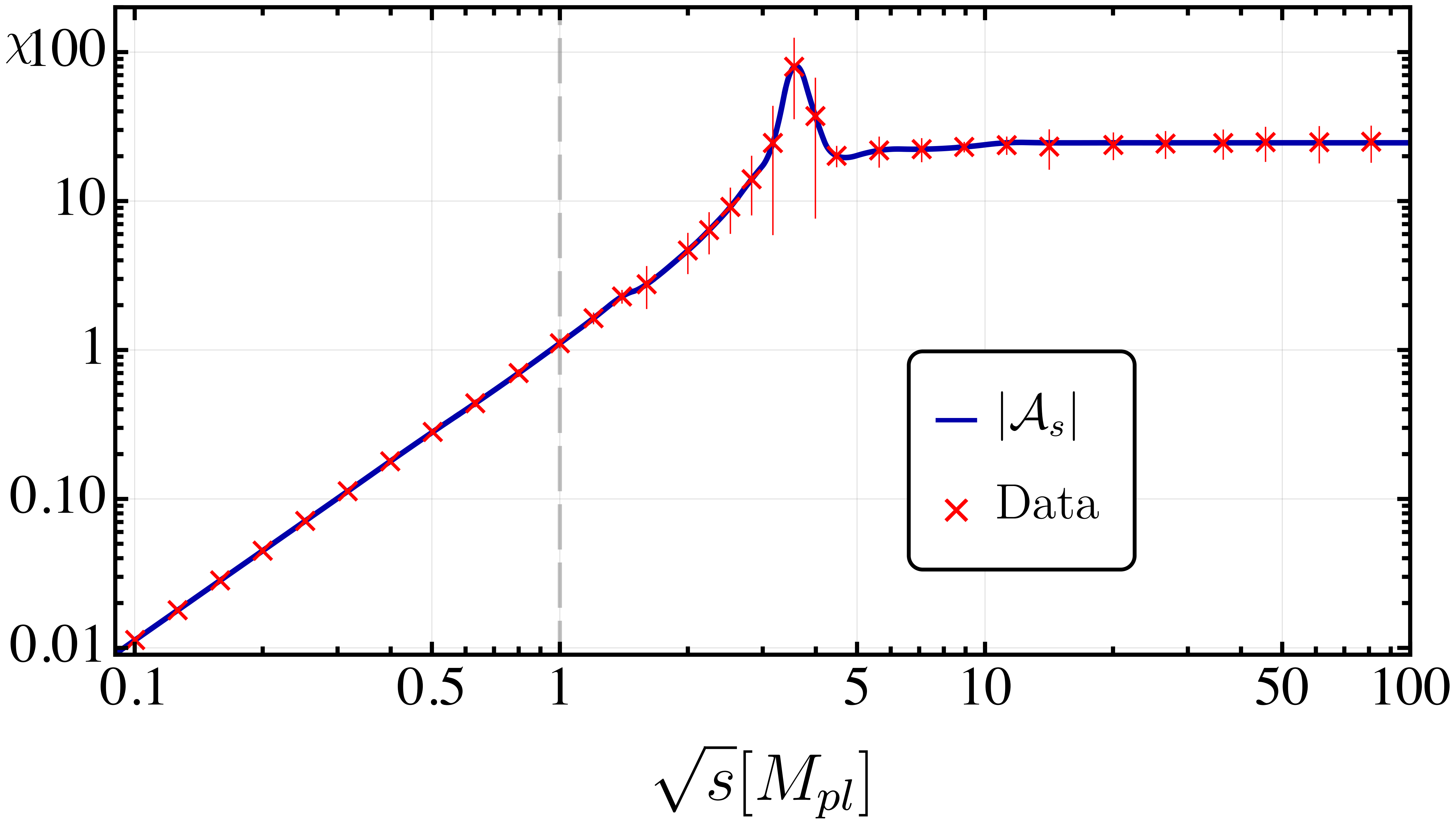


RESULT

Finite Amplitude

- $s-$channel
- Lorentzian (reconstructed)

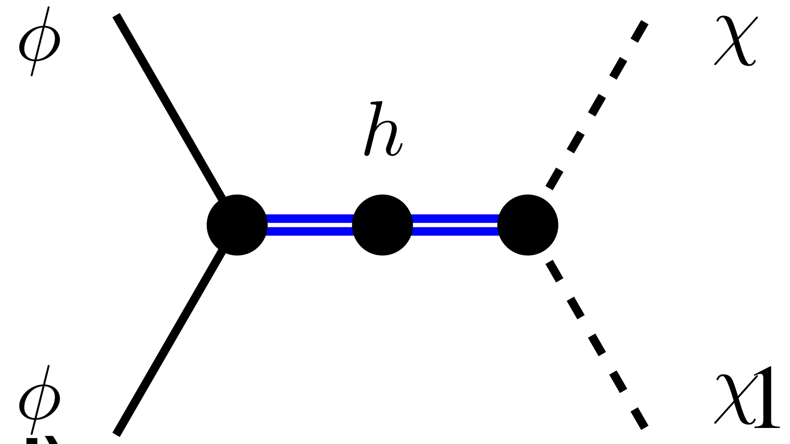


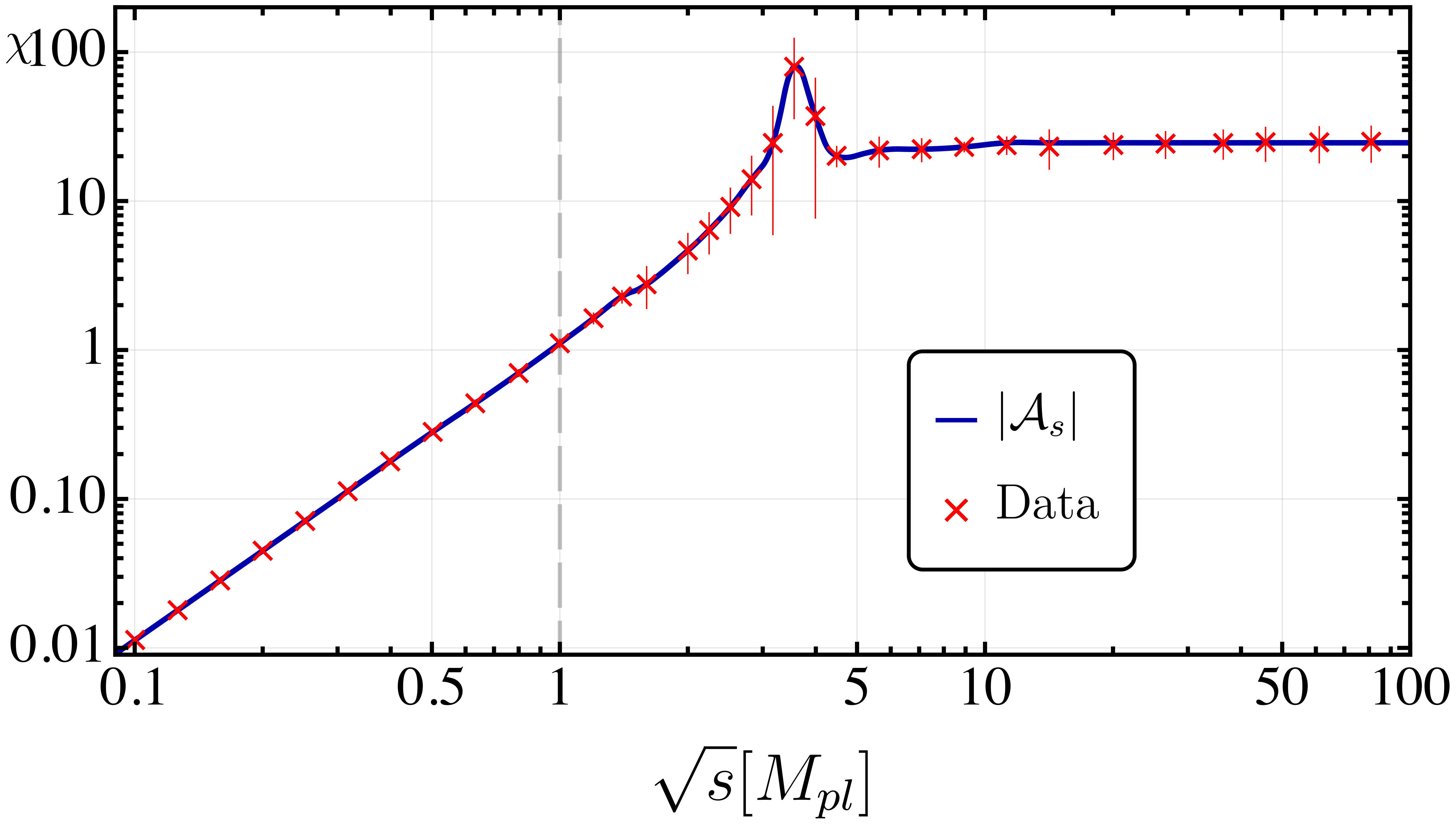


RESULT

Finite Amplitude

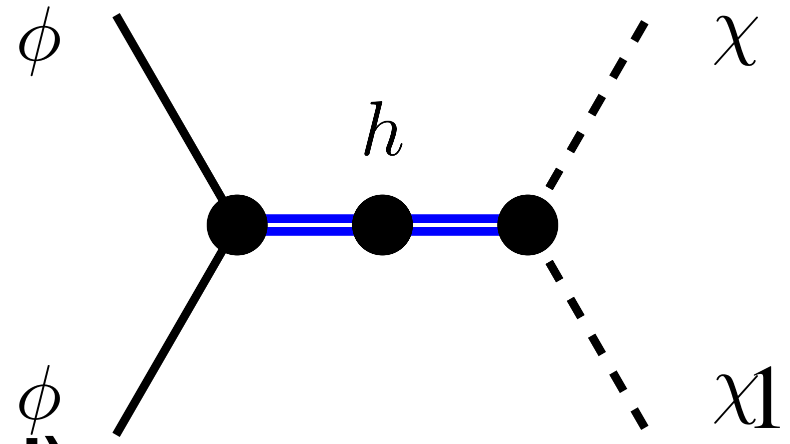
- $s-$channel
- Lorentzian (reconstructed)
- On-Shell $m_\phi^2 = 0$



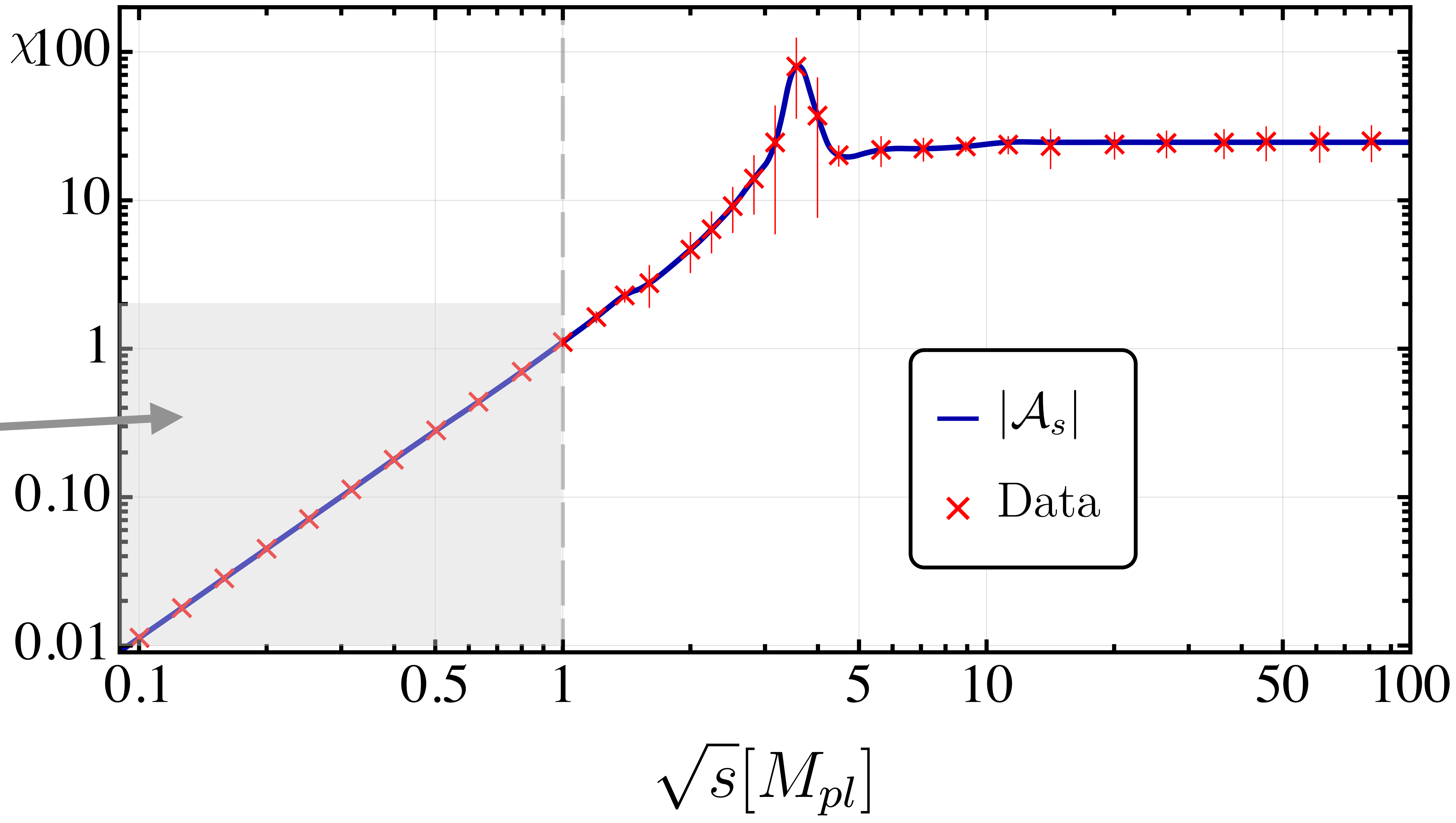


RESULT

Finite Amplitude



- $s-$channel
- Lorentzian (reconstructed)
- On-Shell $m_\phi^2 = 0$
- GR recovered



RESULT

Finite Amplitude

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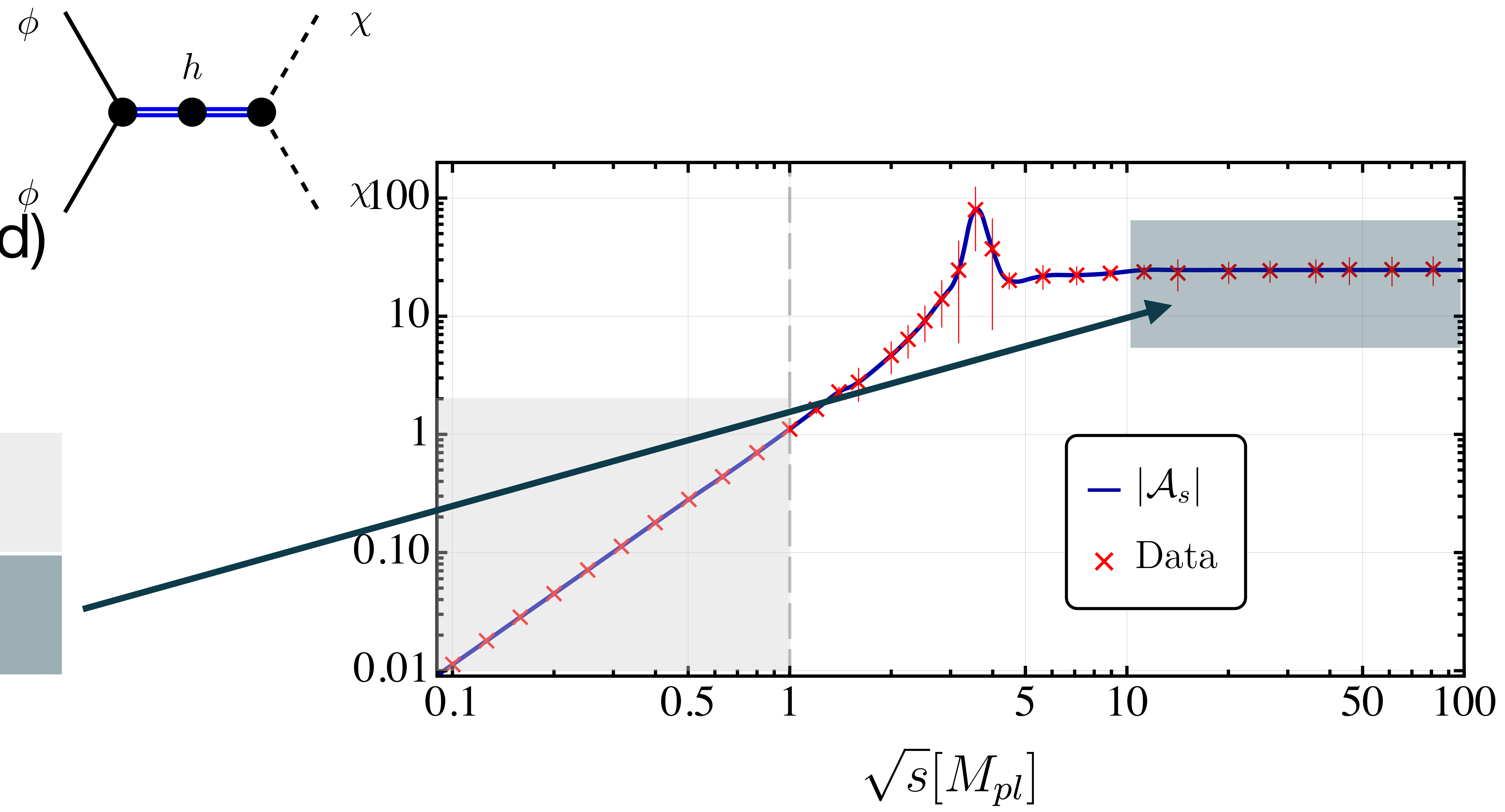
- On-Shell $m_\phi^2 = 0$
- GR recovered
- Fixed point $\mathscr{A}$ **FINITE**



RESULT

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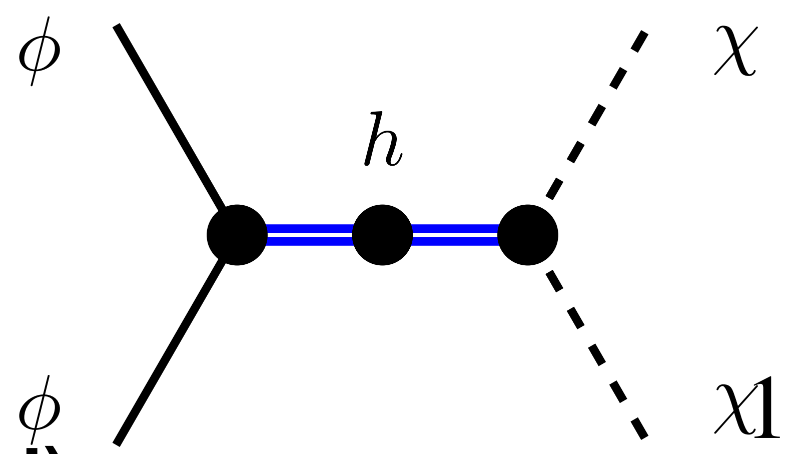
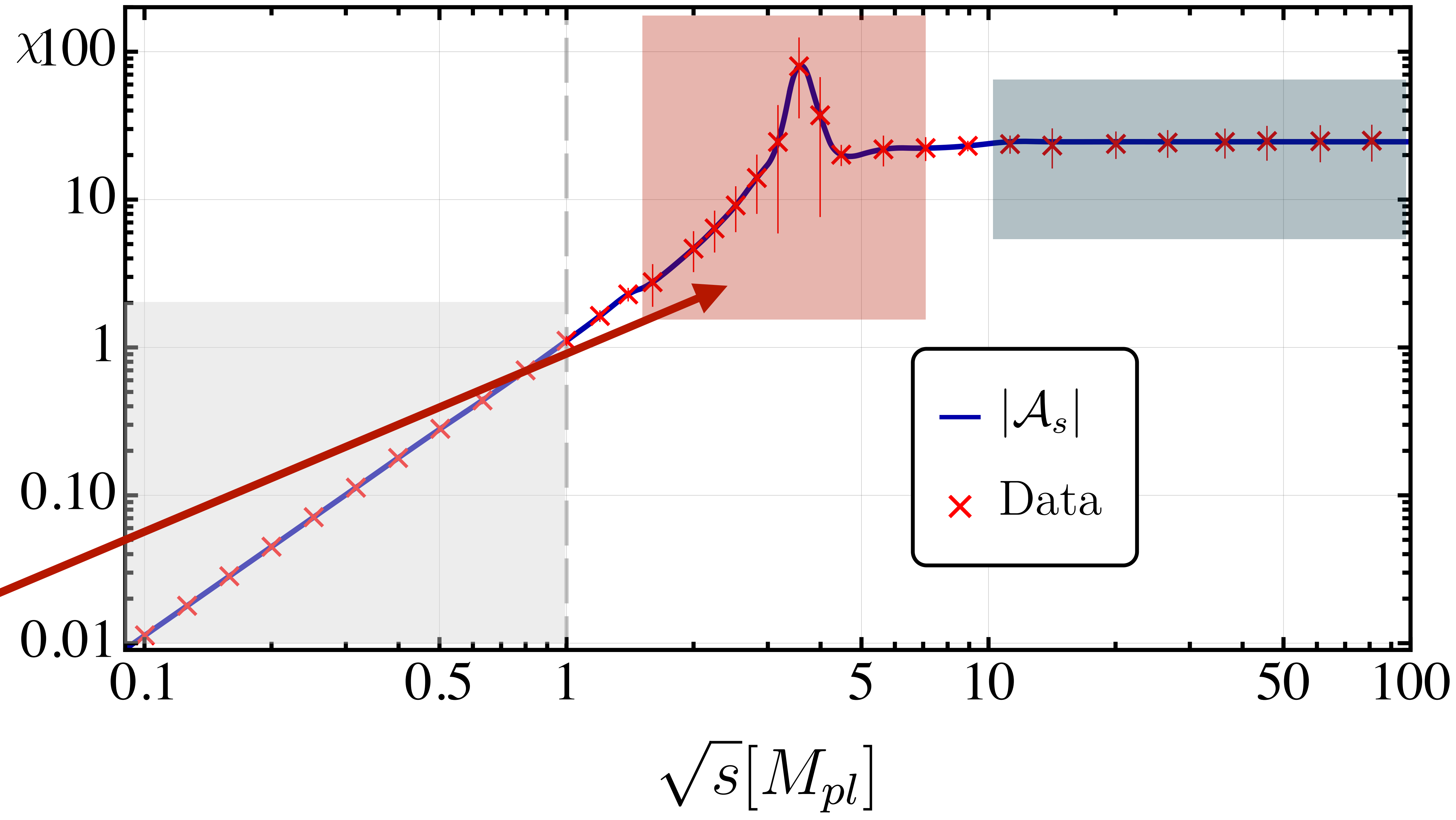
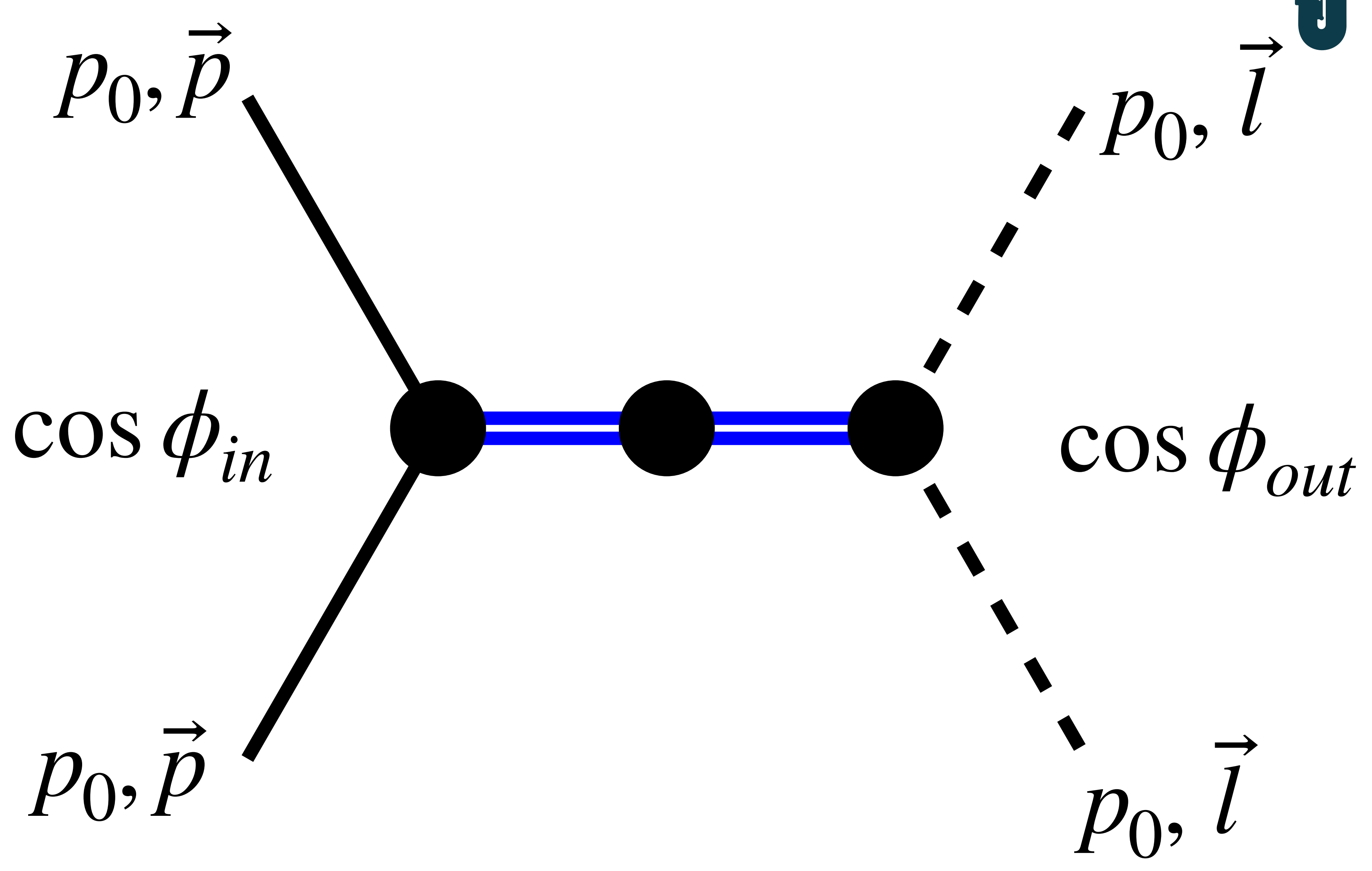
- On-Shell $m_\phi^2 = 0$
- GR recovered
- Fixed point $\mathscr{A}$ **FINITE**
- Graviball resonance?





Interlude

Momentum Variables



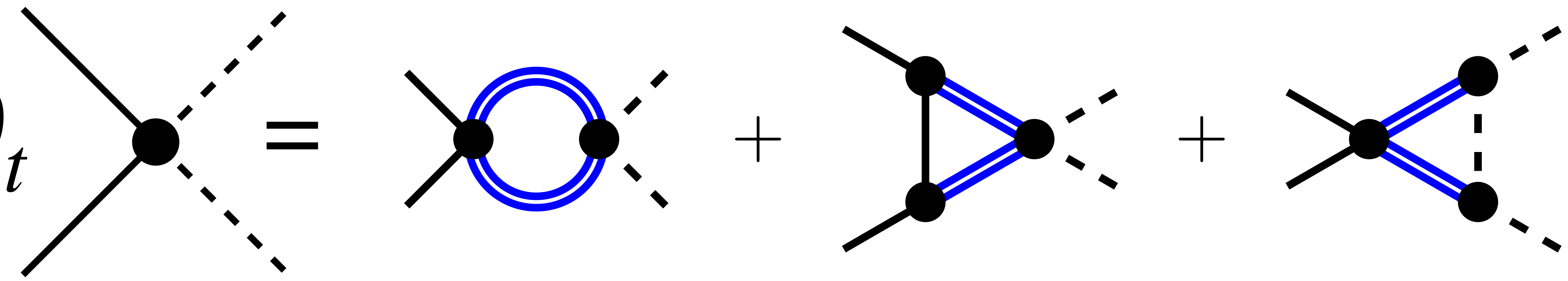
- (p, z) **Euclidean** $\Rightarrow$ **Wick rotation?**
- (p, z) to $p_0, \vec{p}, \vec{l}$ + angles ϕ_{in}, ϕ_{out} in **CoM frame**

$$\mathscr{A} = \sqrt{G_{\phi\phi h}(p_0, ||\vec{p}||, \phi_{in})}\sqrt{G_{\phi\phi h}(p_0, ||\vec{l}||, \phi_{out})} \times T.C. = V_E(p_0) \times T.C.$$

- Reconstruct **only** p_0 **Multivariable reconstruction**
- On-shell: $p_0^2 = ||\vec{p}||^2$

Lorentzian $\Gamma^{4\phi}$

“Through the looking glass“

∂_t 

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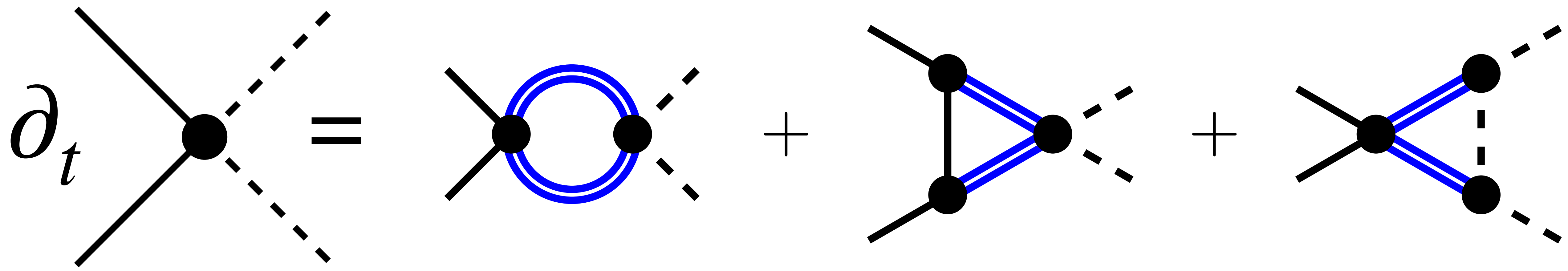

Structure:

$$\partial_t \,\mathrm{Im}\,\Gamma^{4\phi}(s,t) = G_N(\mathbf{x})^2 \;\mathrm{Im\ Diag}_k(s,t)$$

Approximations:

1. $G_N(\mathbf{x}) \equiv G_N$
2. $G_N(\mathbf{x}) = G_N(k) = \dfrac{g^*}{k^2 + g^* M_{\mathrm{pl}}^2}$
3. $G_N(\mathbf{x}) = G_N(k,s) = \dfrac{g^*}{k^2 + s + g^* M_{\mathrm{pl}}^2}$

Educated by Euclidean results

Lorentzian $\Gamma^{4\phi}$

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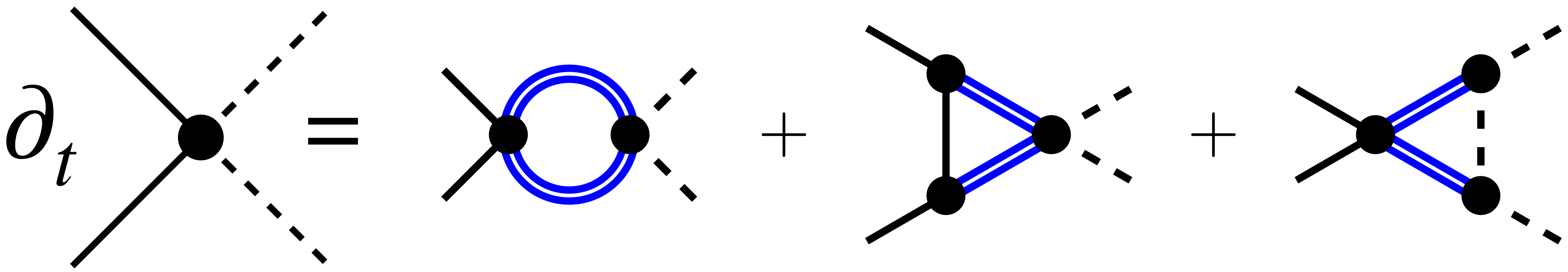

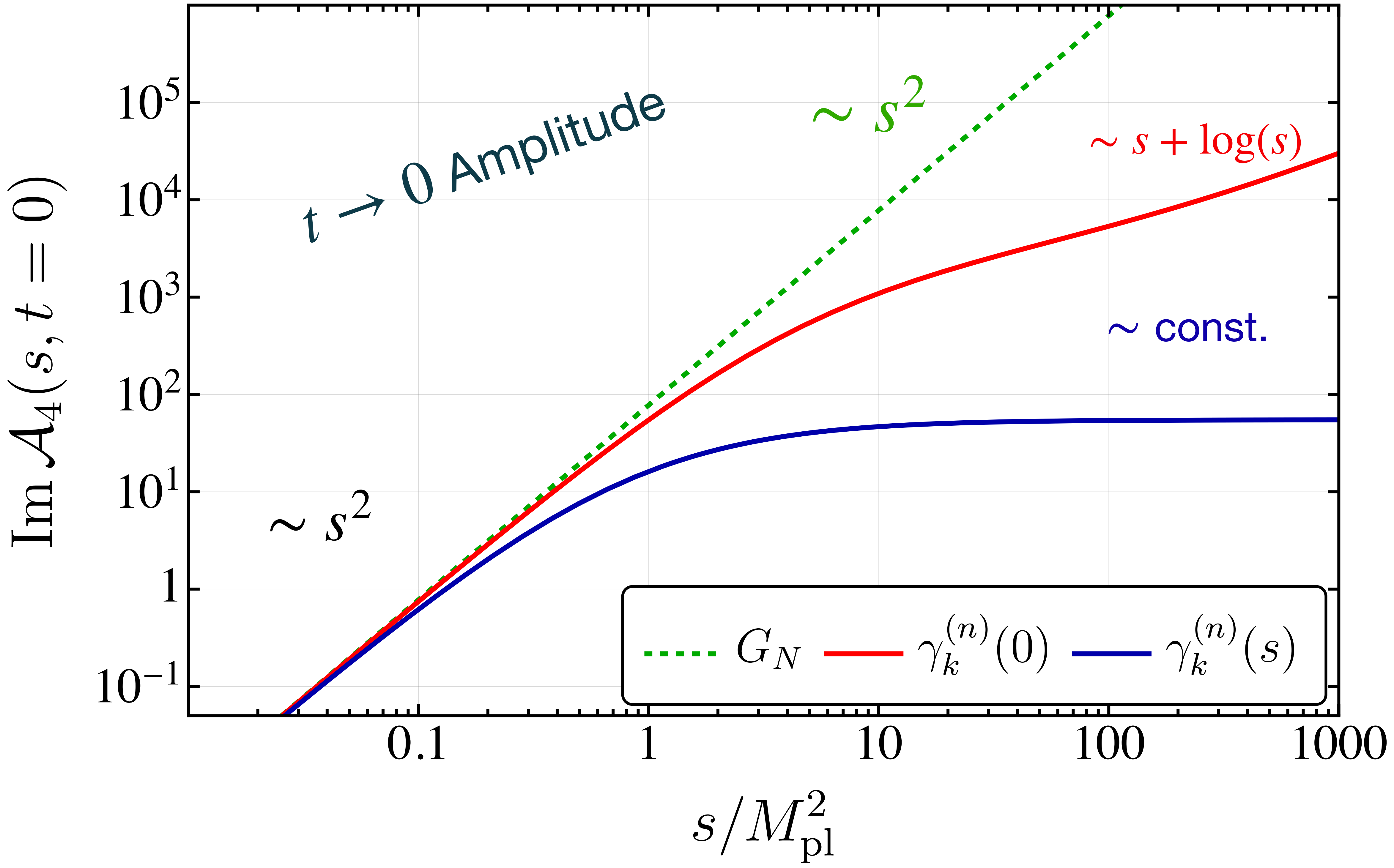

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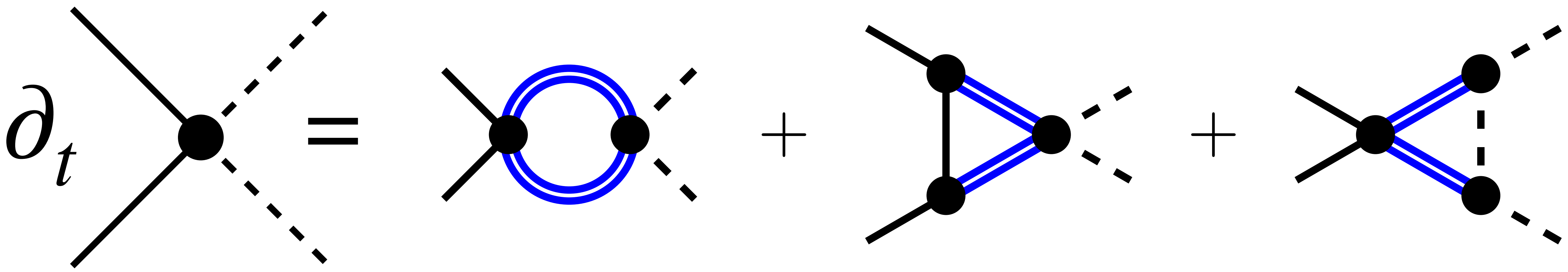
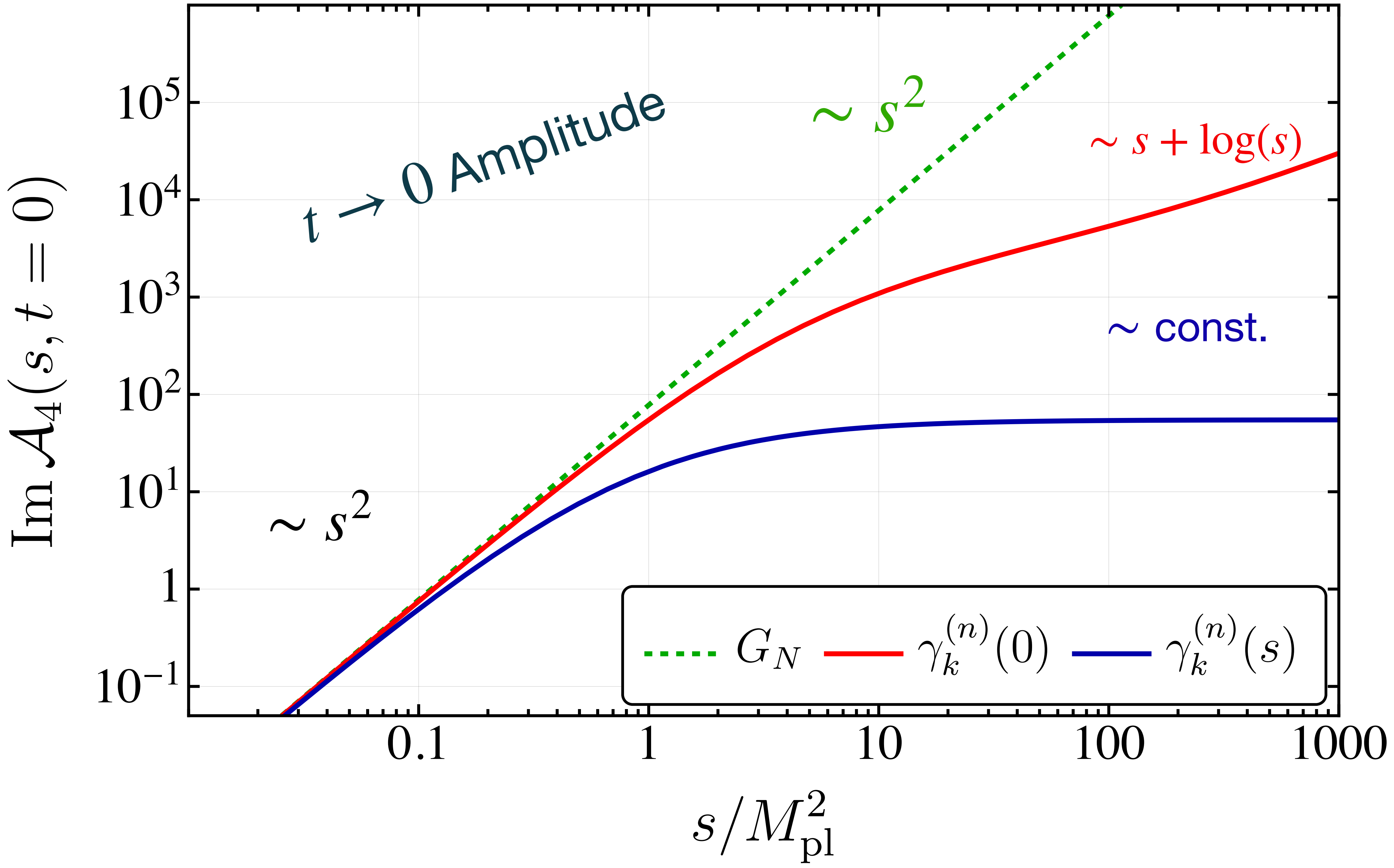
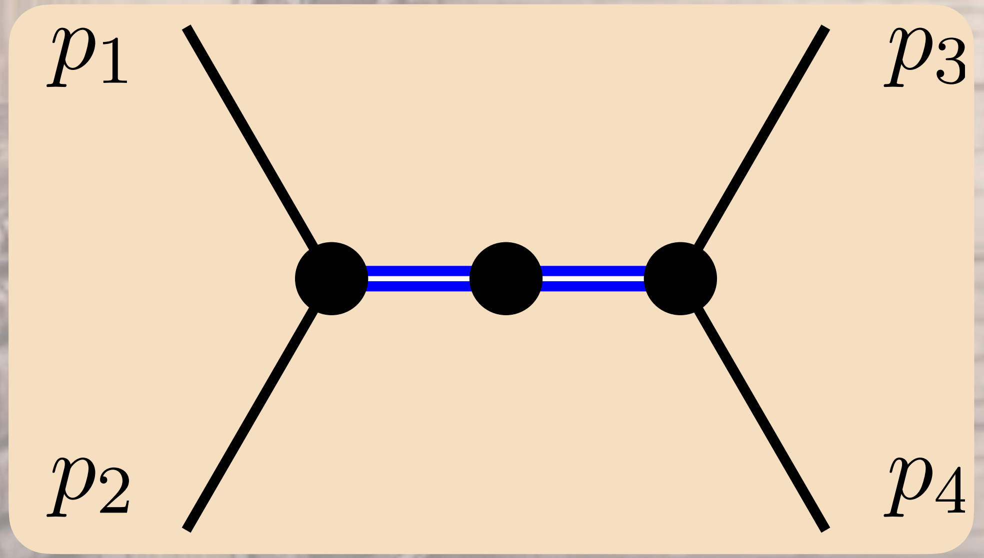




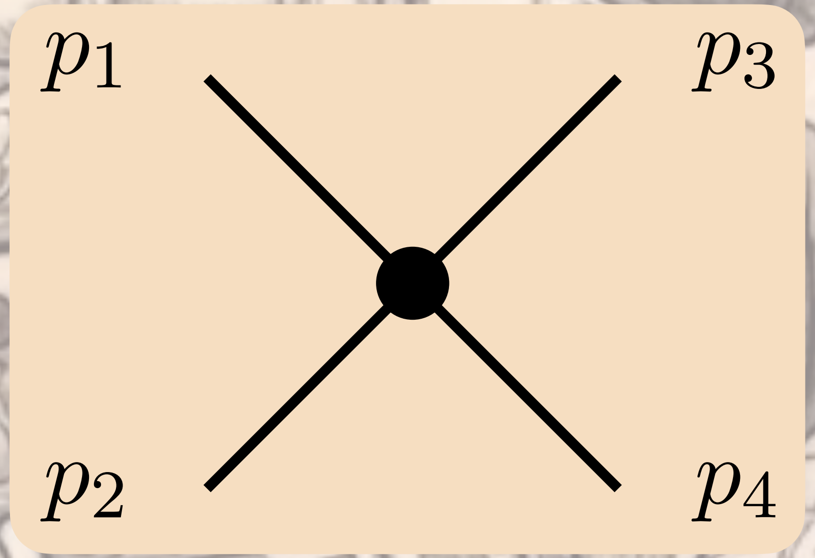
$$\boxed{G_N(\mathbf{x}) = G_N(k, s) = \frac{g^*}{k^2 + s + g^* M_{\text{pl}}^2}}$$

Different Schemes and Gauges

Euclidean:
[APC, Pawlowski, Reichert 2603.101



Lorentzian:
[APC, Reichert 261XX.XXXXX]



Interlude

Unitarity & Bounds

- S-matrix Unitarity: $SS^\dagger = \mathbb{1}$

- Partial Waves (PW) Unitarity:

$$2\mathrm{Im}\ a_j(s) \gtrsim |a_j(s)|^2 \text{(non linear)} \qquad 0 \leq \mathrm{Im}\ a_j(s) \leq 1 \text{ (linear)}$$

- Partial Waves: $\mathscr{A}(s,\theta) = 32\pi \sum_j a_j(s) P_j(\cos\theta)$

Weak-Gravity Bound

[Eichhorn, Held '17]

From unitarity

Weak-Gravity Bound

[Eichhorn, Held '17]

From unitarity

- $\text{Im}\, a_j(s) = \text{Im}\, a_j^{med}(s) + \text{Im}\, a_j^{(4)}(s)$

Weak-Gravity Bound

[Eichhorn, Held ’17]

From unitarity

- $\text{Im}\, a_j(s) = \cancel{\text{Im}\, a_j^{med}(s)} + \text{Im}\, a_j^{(4)}(s)$
- $j = 1$

Weak-Gravity Bound

[Eichhorn, Held '17]

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- Existence of FP $0 \leq \mathrm{Im}\,(a_1^*) \leq 1$?

Weak-Gravity Bound

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[Eichhorn, Held ’17]

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Weak-Gravity Bound

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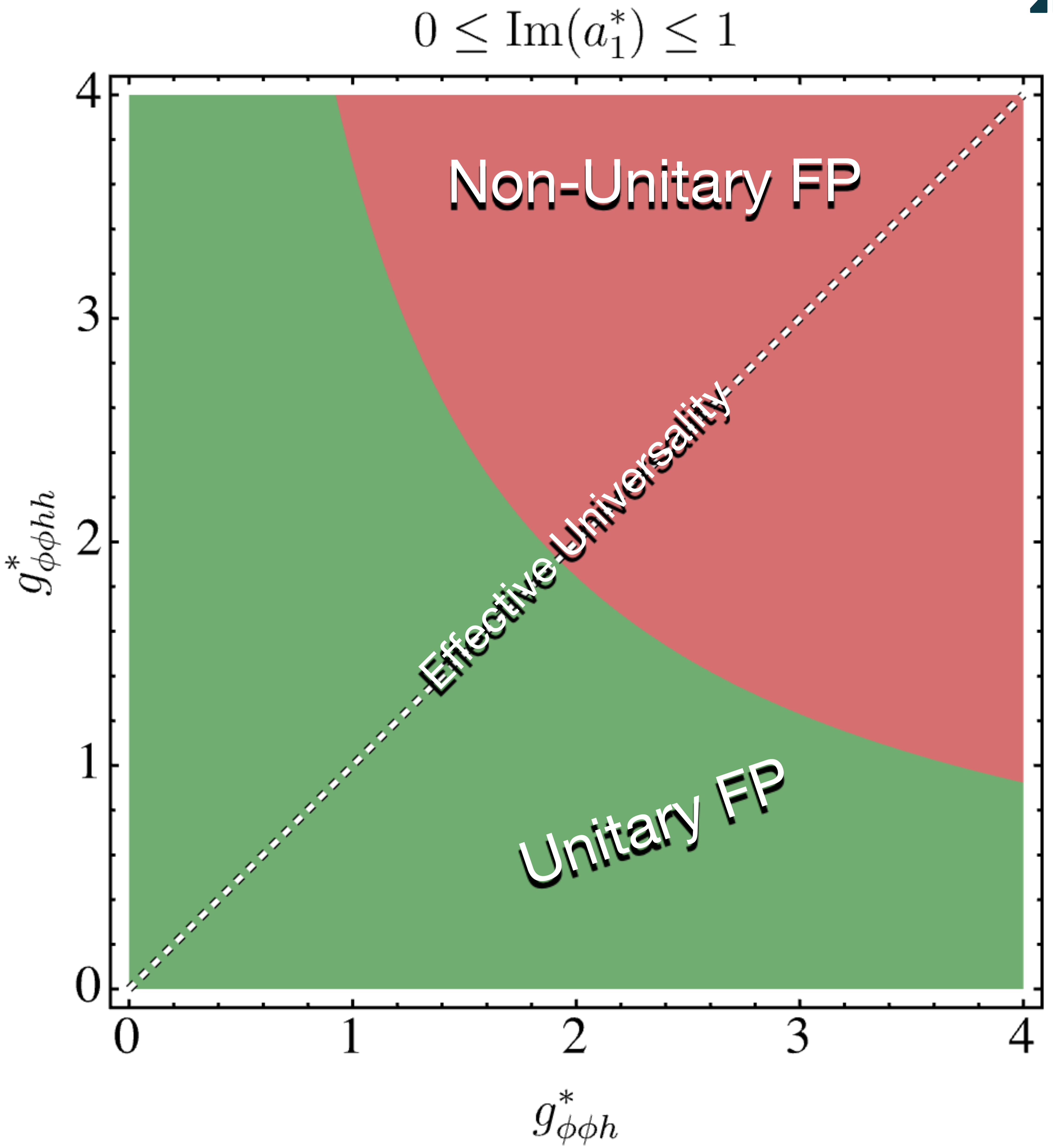

Weak-Gravity Bound

[Eichhorn, Held '17]

From unitarity

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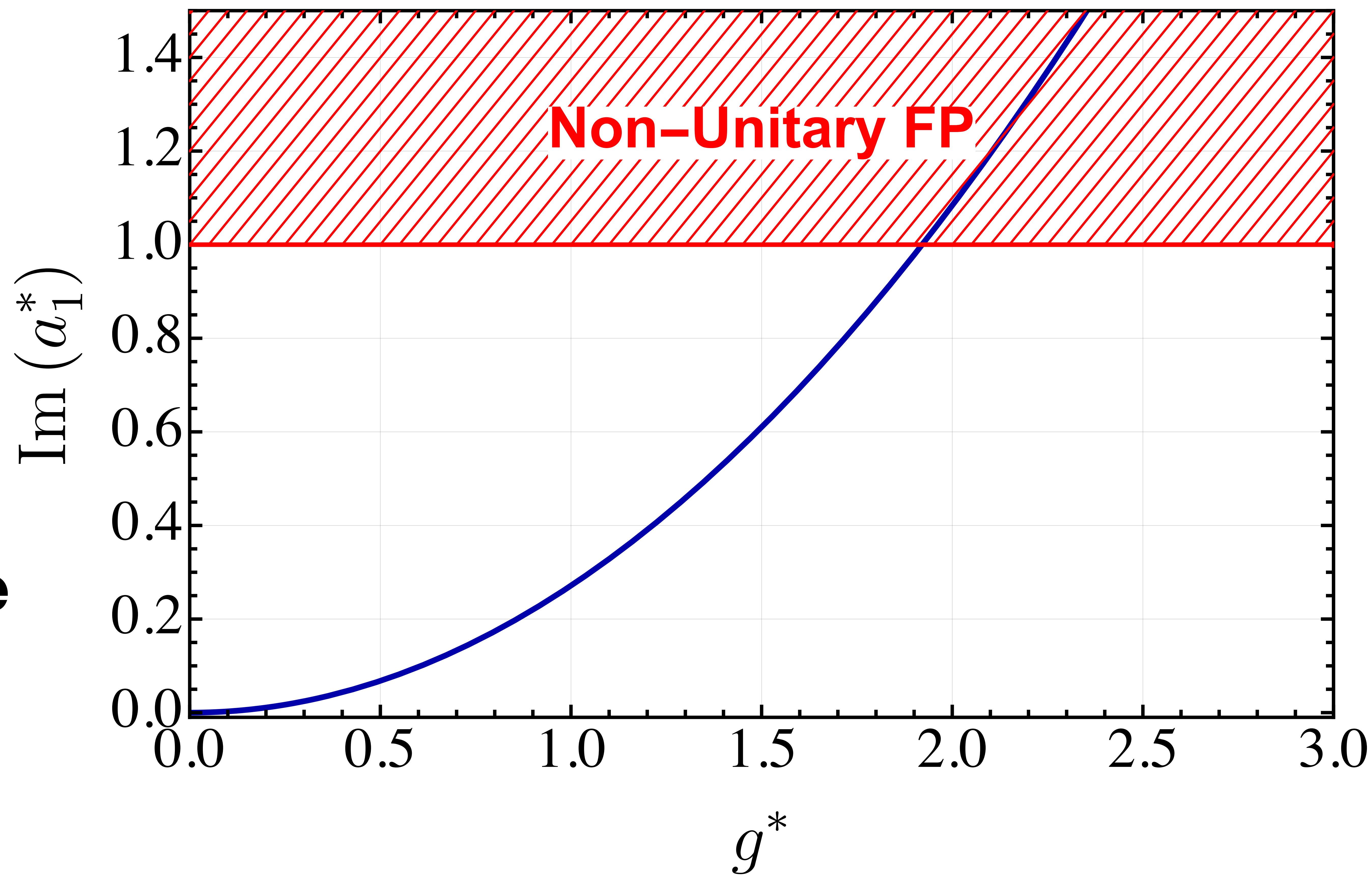
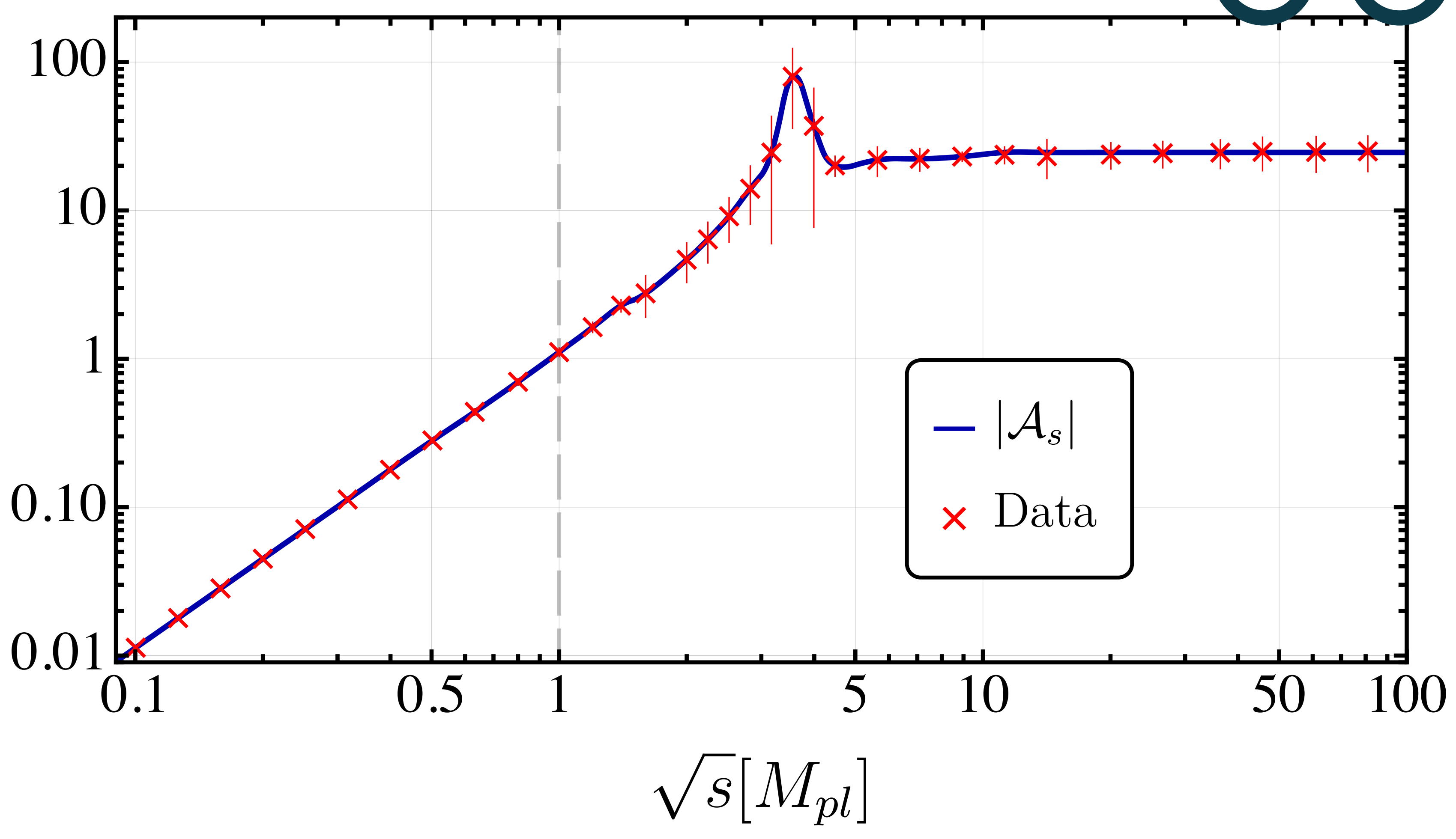
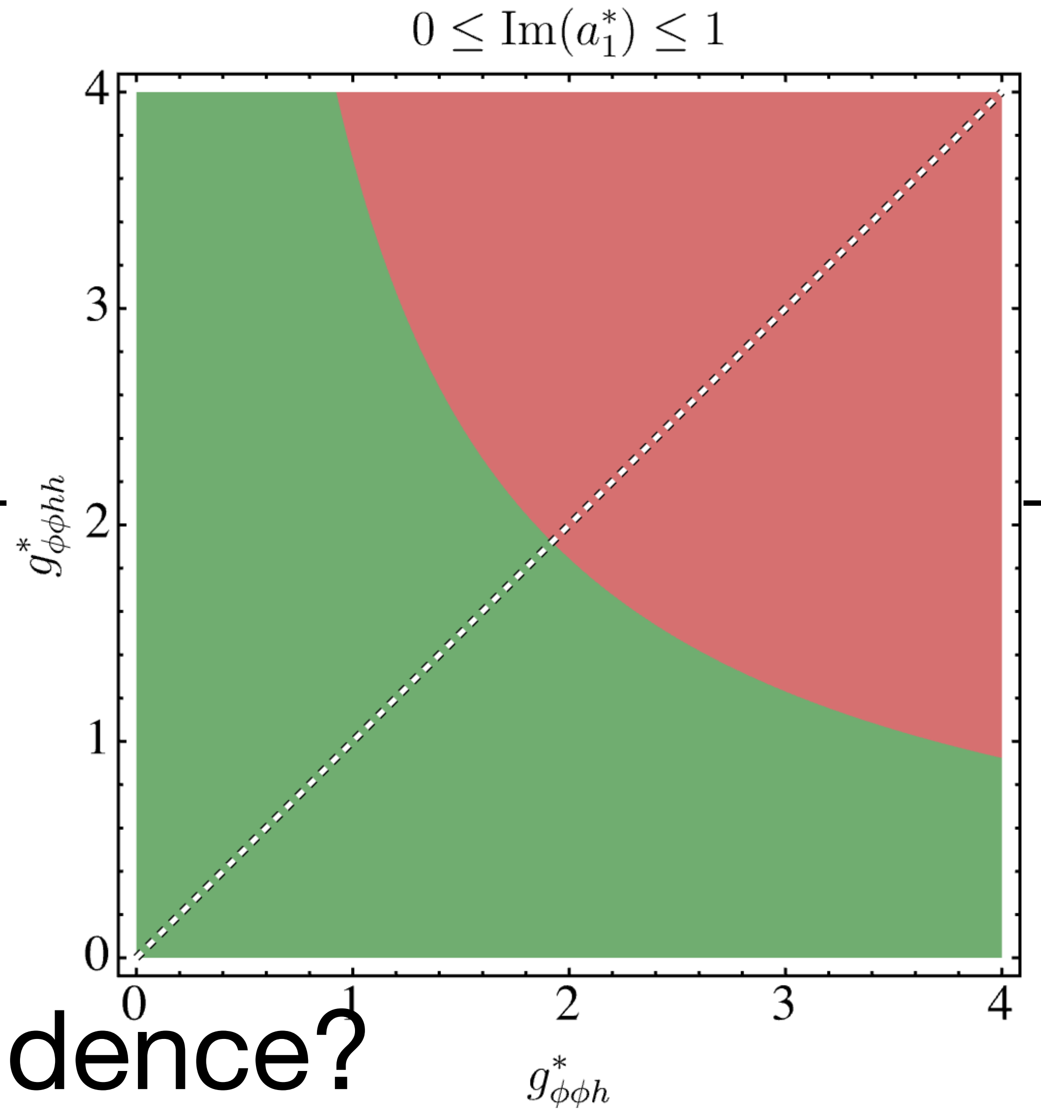
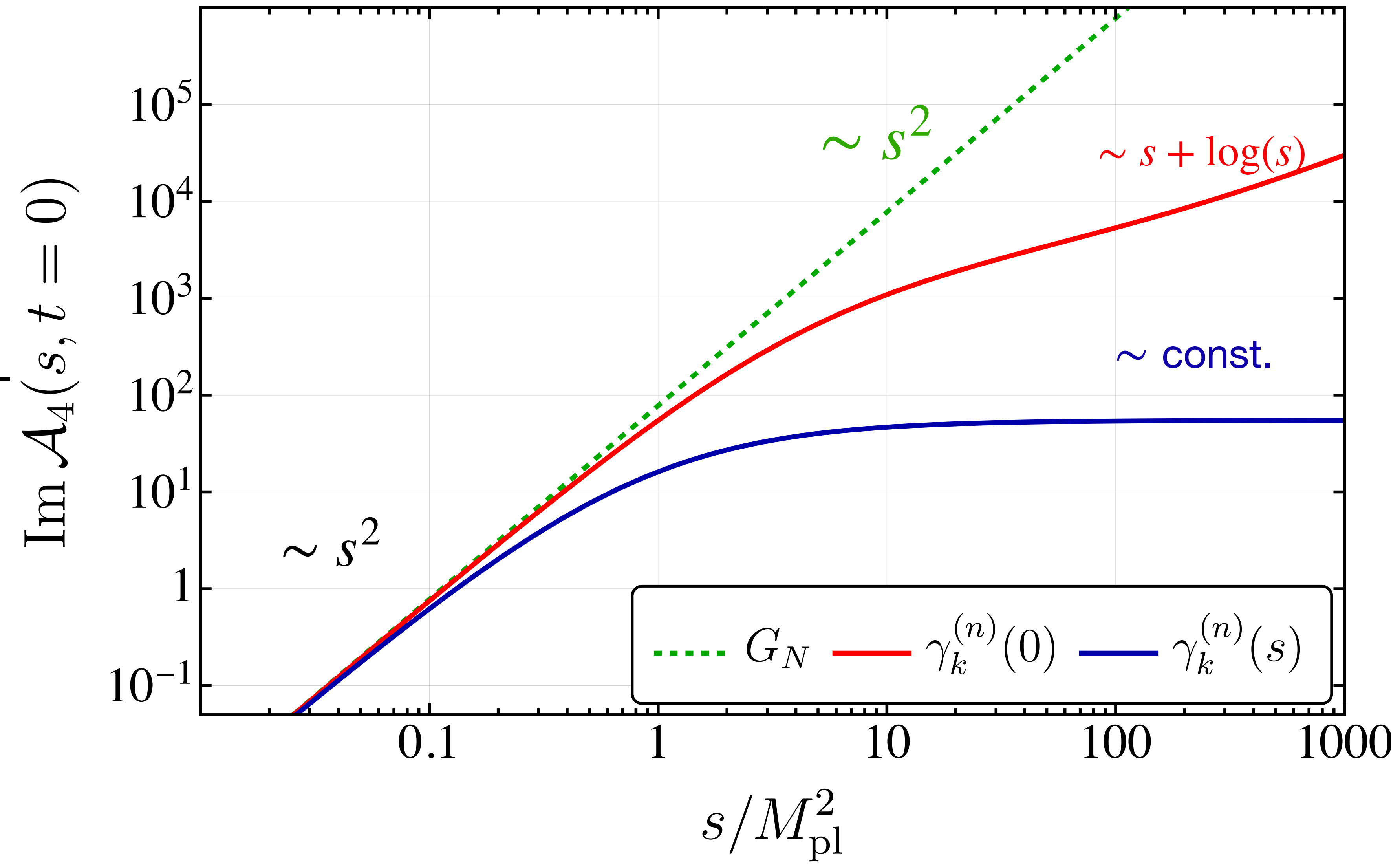


Conclusions & Outlook



- **Scattering Amplitudes in ASQG**
- **Finite mediated & finite contact amplitude**

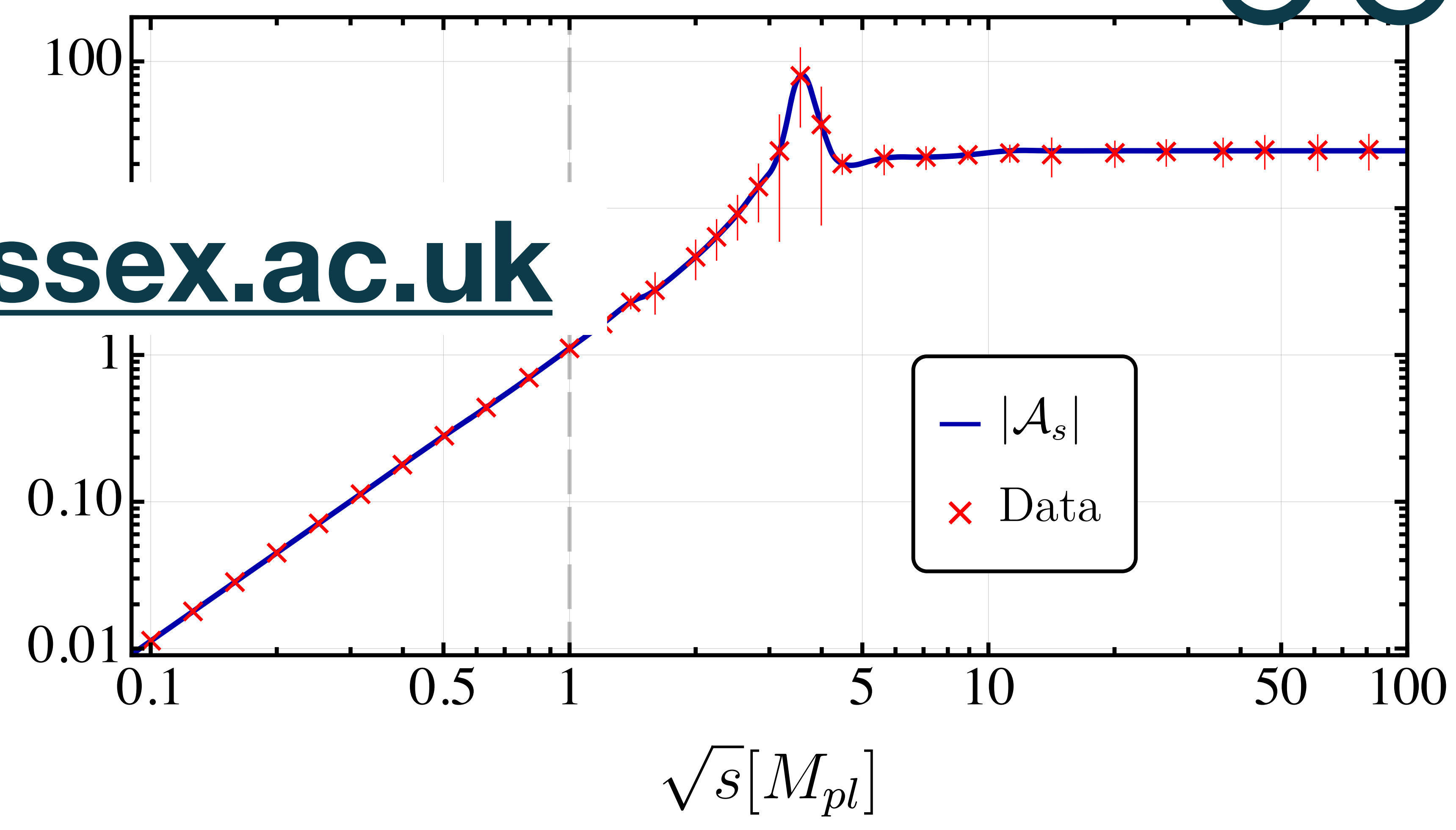




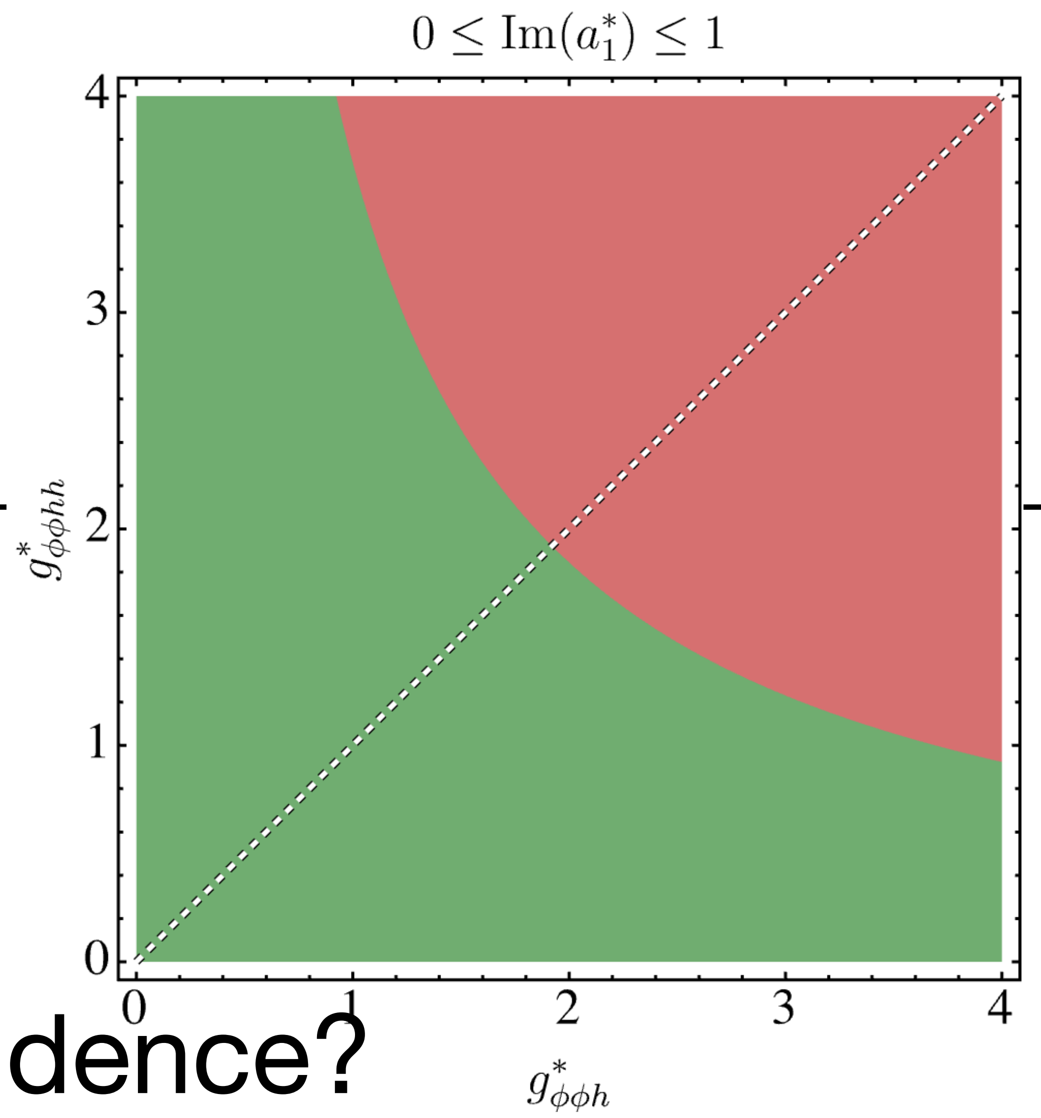
- **Unitarity Bounds on FP**
- $t-$**channel?**
  - a_0, a_2 **bounds:** fixed point independence?
  - **Lorentzian Mediated??**

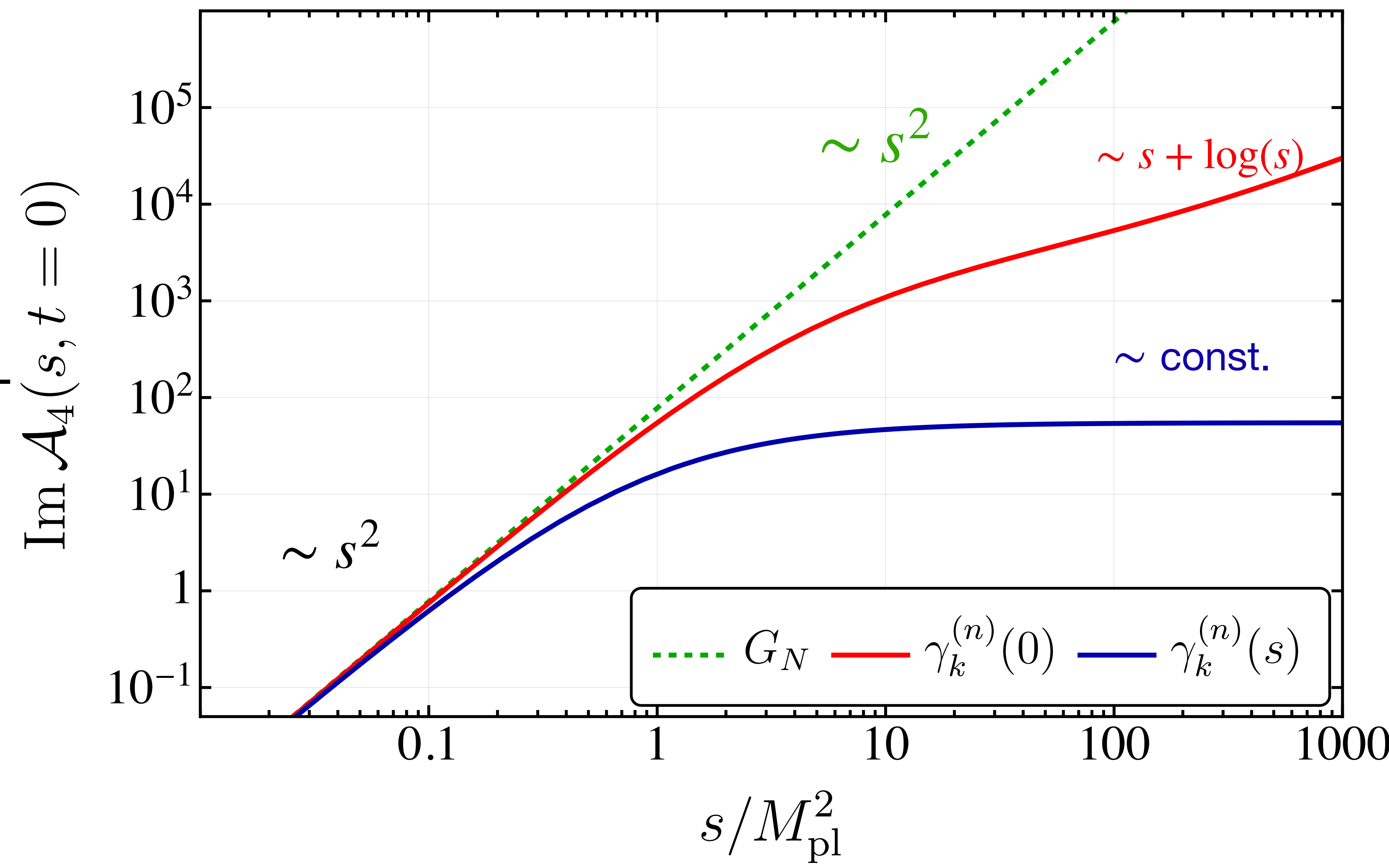
Thanks for your attention!

(More) DETAILS 2603.10168 or a.portas-chiesa@sussex.ac.uk



- **Scattering Amplitudes in ASQG**
- **Finite mediated & finite contact amplitude**
- **Unitarity Bounds on FP**



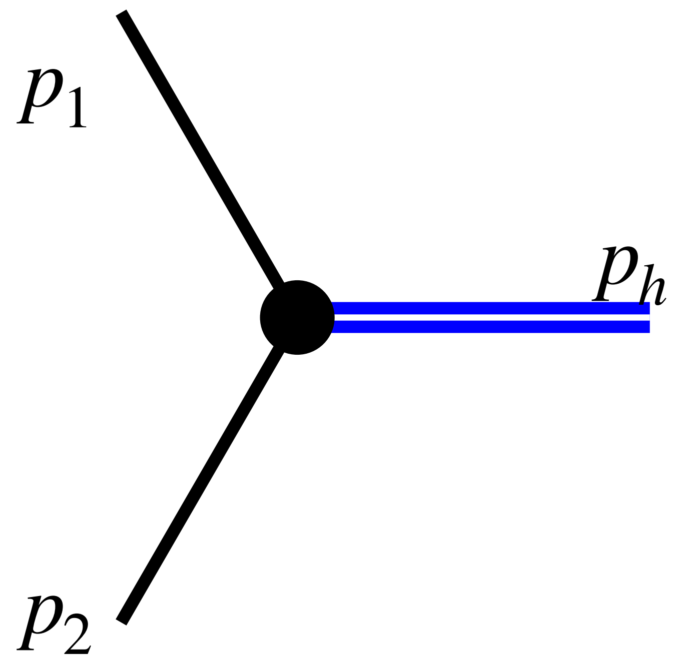


- **$t-$channel?**
  - a_0, a_2 **bounds:** fixed point independence?
  - **Lorentzian Mediated??**

BACKUP

Scalar-Graviton vertex in AS

Massage

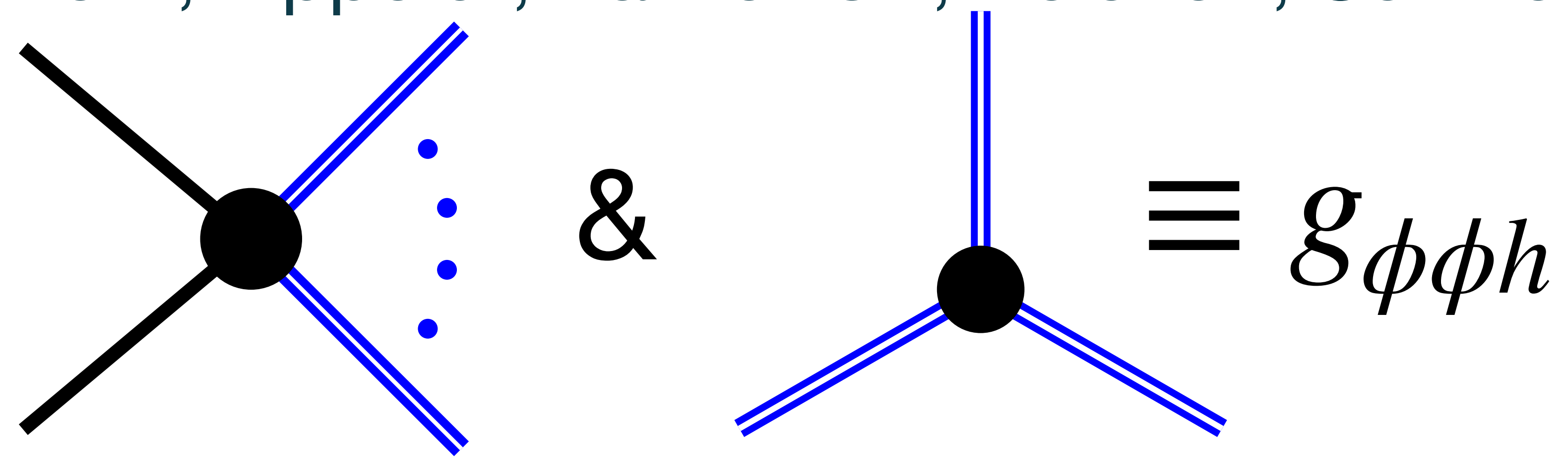


$$\partial_t g_{\phi\phi h,k}(p_1,p_2) = \left(2+\eta_{h,k}(p_h^2)\right) g_{\phi\phi h,k}(p_1,p_2) + g^{1/2}_{\phi\phi h,k}(p_1,p_2)\,\frac{\partial_t \bar{\Gamma}^{\phi\phi h}_{\mu\nu}\;\mathcal{T}^{\phi\phi h,\mu\nu}}{\mathcal{N} k^{-1}}$$

Effective Universality

[Eichhorn, Labus, Pawlowski, Reichert '18]

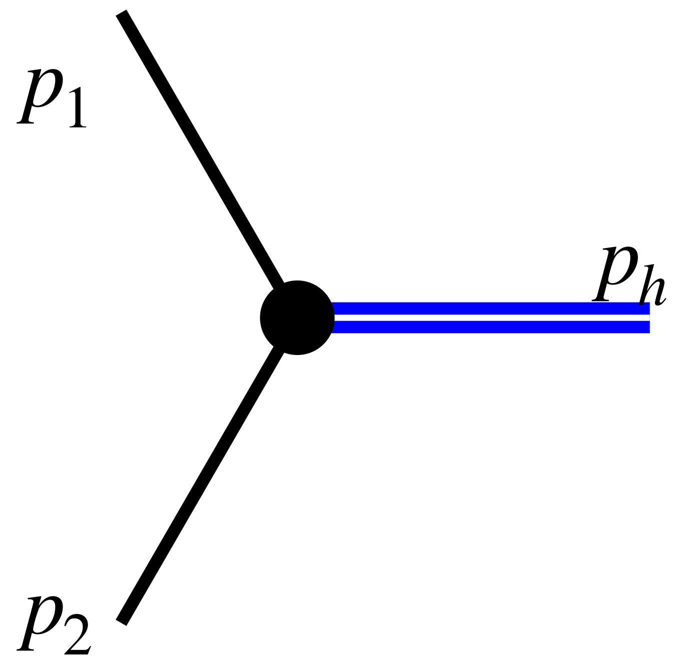
[Eichhorn, Lippoldt, Pawlowski, Reichert, Schiffer '18]



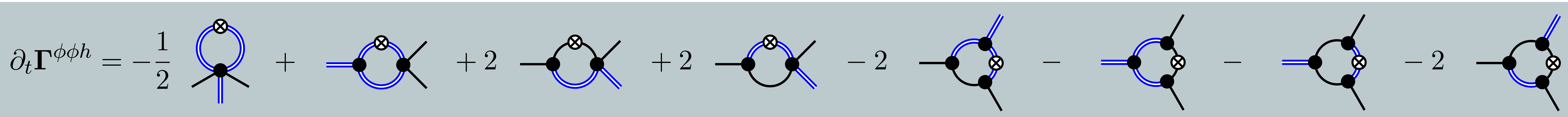
- $g_{n,m}$ loop independent
- $\mu = \lambda_i = 0$
- Loops: $\eta_h(q) \equiv 0$
- $\mathcal{T}^{\phi\phi h,\mu\nu}$ classical

Scalar-Graviton vertex in AS

Flow Equation



Diagrams generated in $\partial_t \Gamma_k^{\phi\phi h}$



Flow Equation

$$\partial_t g_{\phi\phi h,k}(p,z) = (2 + \eta_{h,k}(p,z))\, g_{\phi\phi h,k}(p,z) + g^2_{\phi\phi h,k}(p,z)\, \mathrm{Diag}_k(p,z)$$

$(p/k, z)$ **dependence** $\rightarrow k = 0$ **(physics)**

Momentum Variables

Full glory

$$z = \frac{p_1 \cdot p_2}{|p_1||p_2|}$$

$$p_{\text{in}}^2 = p_0^2 + |\vec{p}\,|^2 \qquad z_{\text{in}} = 1 + \frac{|\vec{p}\,|^2}{|\vec{p}\,|^2 + p_0^2}\Big(\cos\left(\phi_{\text{in}}\right) - 1\Big)$$

$$p_{\text{out}}^2 = l_0^2 + |\,\vec{l}\,|^2 \qquad z_{\text{out}} = 1 + \frac{|\,\vec{l}\,|^2}{|\,\vec{l}\,|^2 + l_0^2}\Big(\cos\left(\phi_{\text{out}}\right) - 1\Big)$$

$$l_0^2 = \frac{1}{2}\Big(2p_0^2 + |\vec{p}\,|^2\left(1 + \cos\phi_{\text{in}}\right) - |\,\vec{l}\,|^2(1 + \cos\phi_{\text{out}})\Big)$$

$$\vec{p}\cdot\vec{l} = |\vec{p}\,||\,\vec{l}\,|\cos\theta_{\text{sc}}$$

$$s = 2p_{\text{in}}^2(1 + z_{\text{in}}) = 2p_{\text{out}}^2(1 + z_{\text{out}})$$

$$t = p_{\text{in}}^2 + p_{\text{out}}^2 + 2(p_0 l_0 + |\vec{p}\,||\,\vec{l}\,|\cos\theta_{\text{sc}})$$

$$u = 2p_{\text{in}}^2 + 2p_{\text{out}}^2 - s - t$$

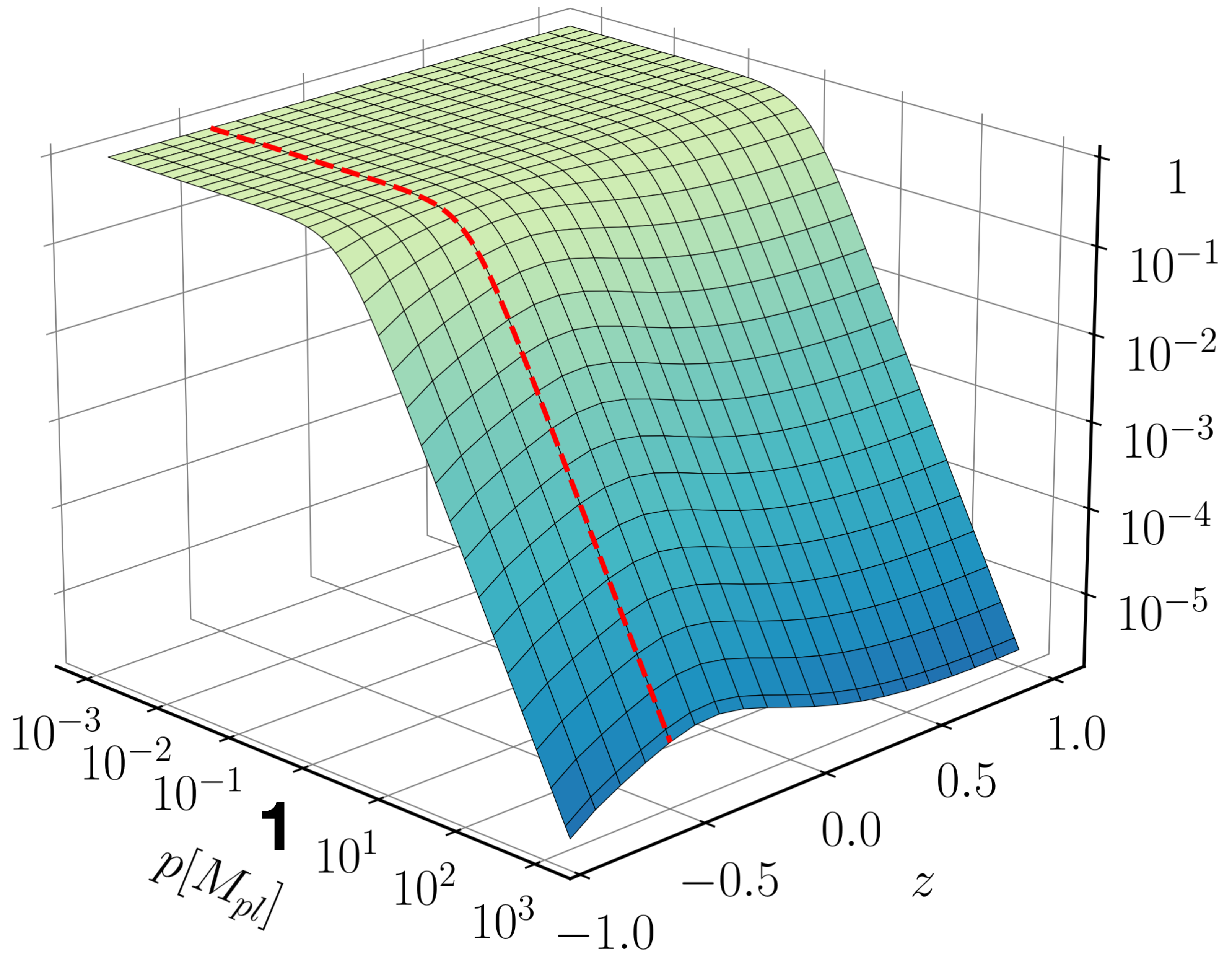
RESULTS

Newton Coupling

- $G_{\phi\phi h}$ const. in the IR $p \ll 1$
- Non-perturbative corrections: $G_{\phi\phi h} \sim p^{-2}$

Asymptotic Safety hallmark

- z influences the transition



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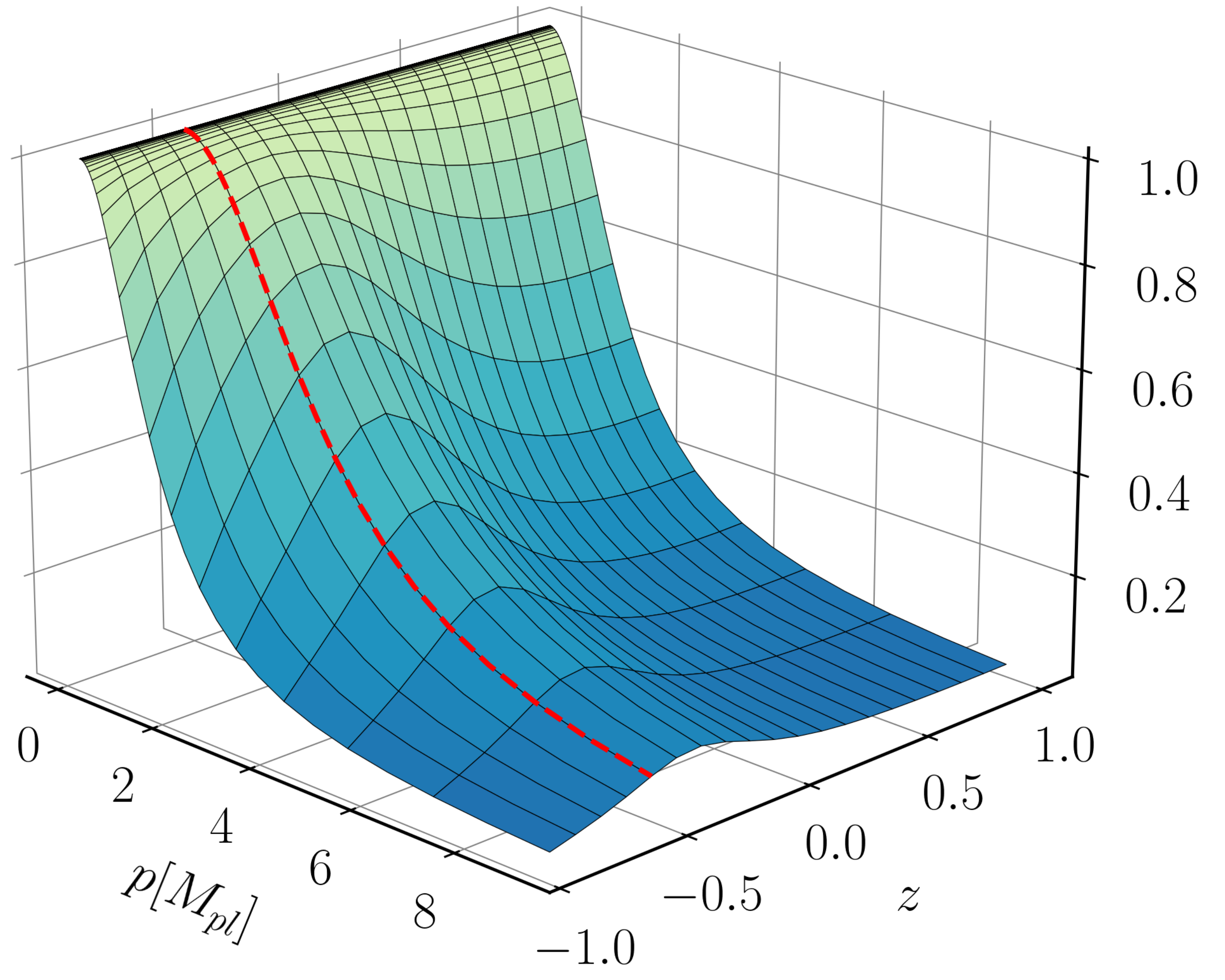
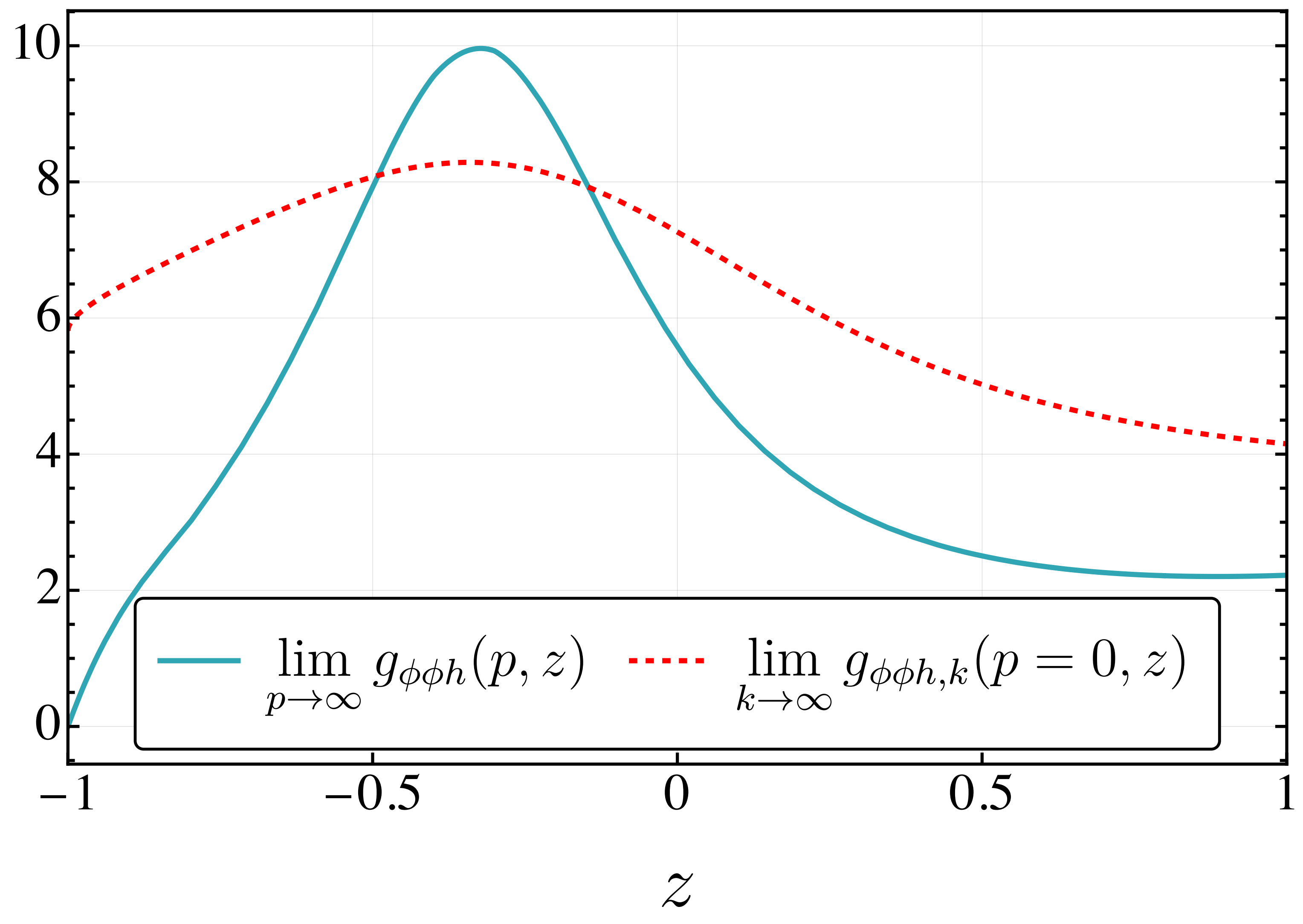




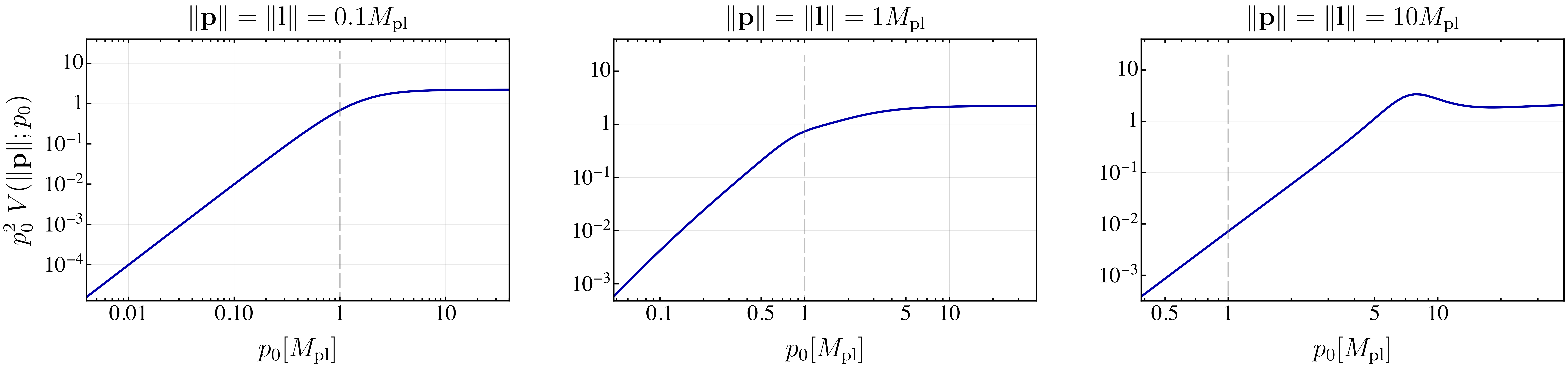
RESULTS

Euclidean Vertex

$$\mathscr{A} = V_E(p_0) \times T.C.$$

Quantum corrections, **Physical AS**

$$= \frac{1}{s}(t^2 - 4tu + u^2 - s^2)$$



Reconstruction

Technical details - Pipeline

$$C_N(x) = \cfrac{y_1}{1 + \cfrac{a_1(x - x_1)}{1 + \cfrac{a_2(x - x_2)}{\vdots \; \dots a_{N-1}(x - x_{N-1})}}}$$

- Assume a continued fraction for $V_E(\|\vec{p}\|; p_0)$
- Get the coefficient → **Hard Part**
- Derive the spectral function $\rho(\|\vec{p}\|; \lambda) = 2\ \mathrm{Im} V(\|\vec{p}\|; -\mathrm{i}(\lambda + \mathrm{i}0^+))$
- Compute $V_{rec}(\|\vec{p}\|; p_0) = \int_0^\infty \frac{d\lambda\lambda}{\pi} \frac{\rho(\|\vec{p}\|; \lambda)}{\lambda^2 + p_0^2}$.
- Compute $V_L(\|\vec{p}\|) = \int_0^\infty \frac{d\lambda\lambda}{\pi} \frac{\rho(\|\vec{p}\|; \lambda)}{\lambda^2 - \|\vec{p}\|^2 + i0^+}$ **On-shell Lorentzian Vertex**

Technical details - Algorithm

Fixed $||\vec{p}||$ $\longrightarrow$ Repeat $||\vec{p}|| \in [0, 10^3]$

See also [Binosi, Tripolt '19]

M Euclidean Data

$(p_0^{(i)}, V^i)$

Sample N<M

Get

- $C_N(p_0)$
- $\rho_M(\lambda)$

$$V_{rec}(||\vec{p}||; p_0) = \int_0^\infty \frac{d\lambda\lambda}{\pi} \frac{\rho(||\vec{p}||;\lambda)}{\lambda^2 + p_0^2}\,.$$

$$\chi^2 = \frac{1}{M}\sum_{i=1}^{M} \frac{\left(V^i - V_{rec}\left(p_0^{(i)}\right)\right)^2}{V^i}$$

$\chi > \chi_T$

$\chi < \chi_T$

$$V_L(||\vec{p}||) = \int_0^\infty \frac{d\lambda\lambda}{\pi} \frac{\rho(||\vec{p}||;\lambda)}{\lambda^2 - ||\vec{p}||^2 + i0^+}$$

Iterate 80 times

RESULTS

Amplitude & Cross Section

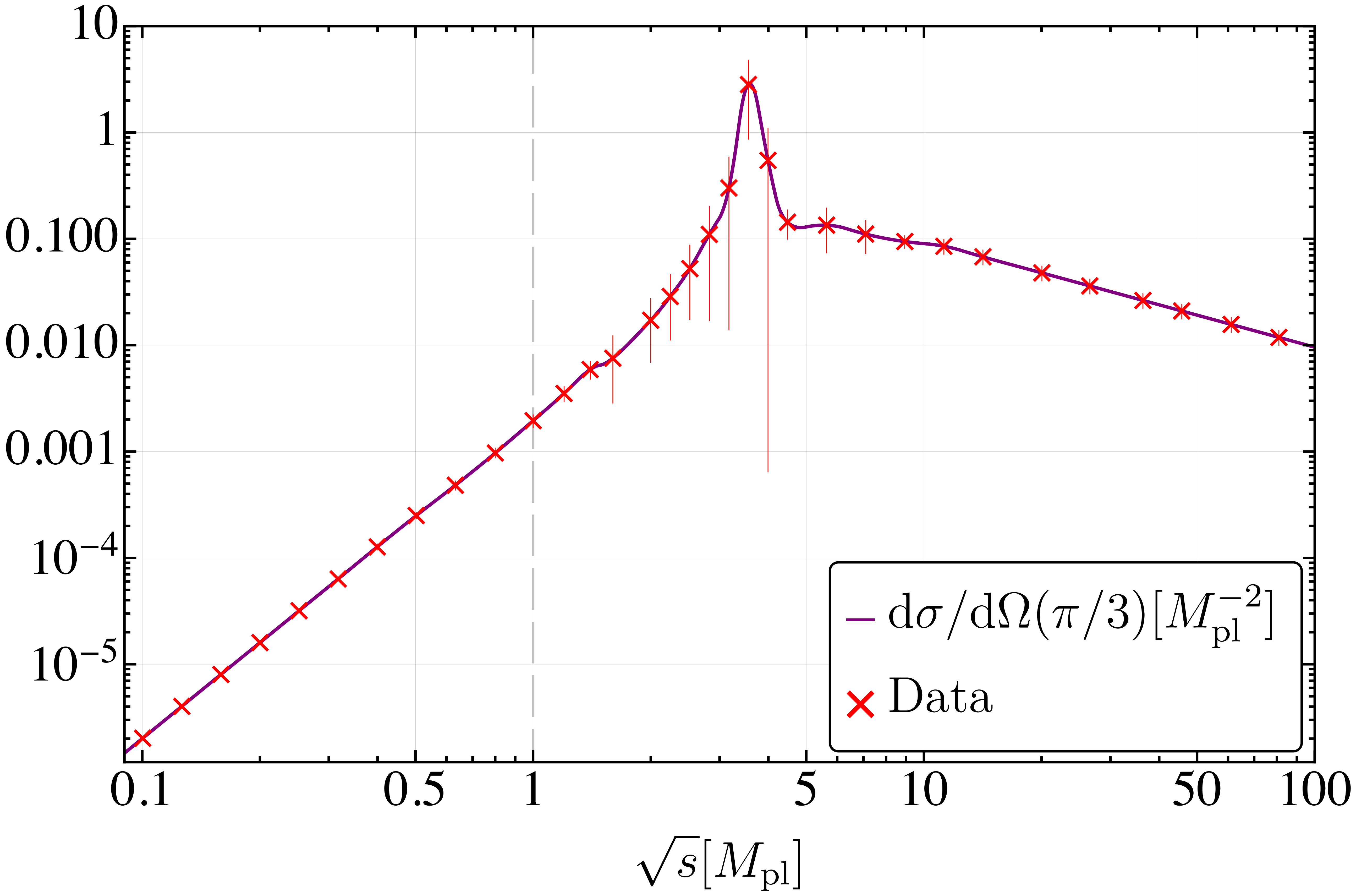
Numerical Nightmare

- Reconstruct and $p_0 \to -ip_0$
- On-shell $\to$ s, t, u dependence

$$\mathscr{A}_s = V_L(s)\frac{1}{s}\left(t^2 - 4tu + u^2 - s^2\right)$$

- Cross Section

$$\frac{d\sigma_s}{d\Omega} = \frac{1}{64\pi^2 s}\,|\mathscr{A}_s|^2$$



RESULTS

$$\mathcal{A}_s = \boxed{V_L(s)} \frac{1}{s}(t^2 - 4tu + u^2 - s^2)$$

RGi comparison

See also [Pastor-Gutiérrez, Pawlowski, Reichert, Ruisi '24]

- Standard RGi $p = 0, k = \sqrt{s}$

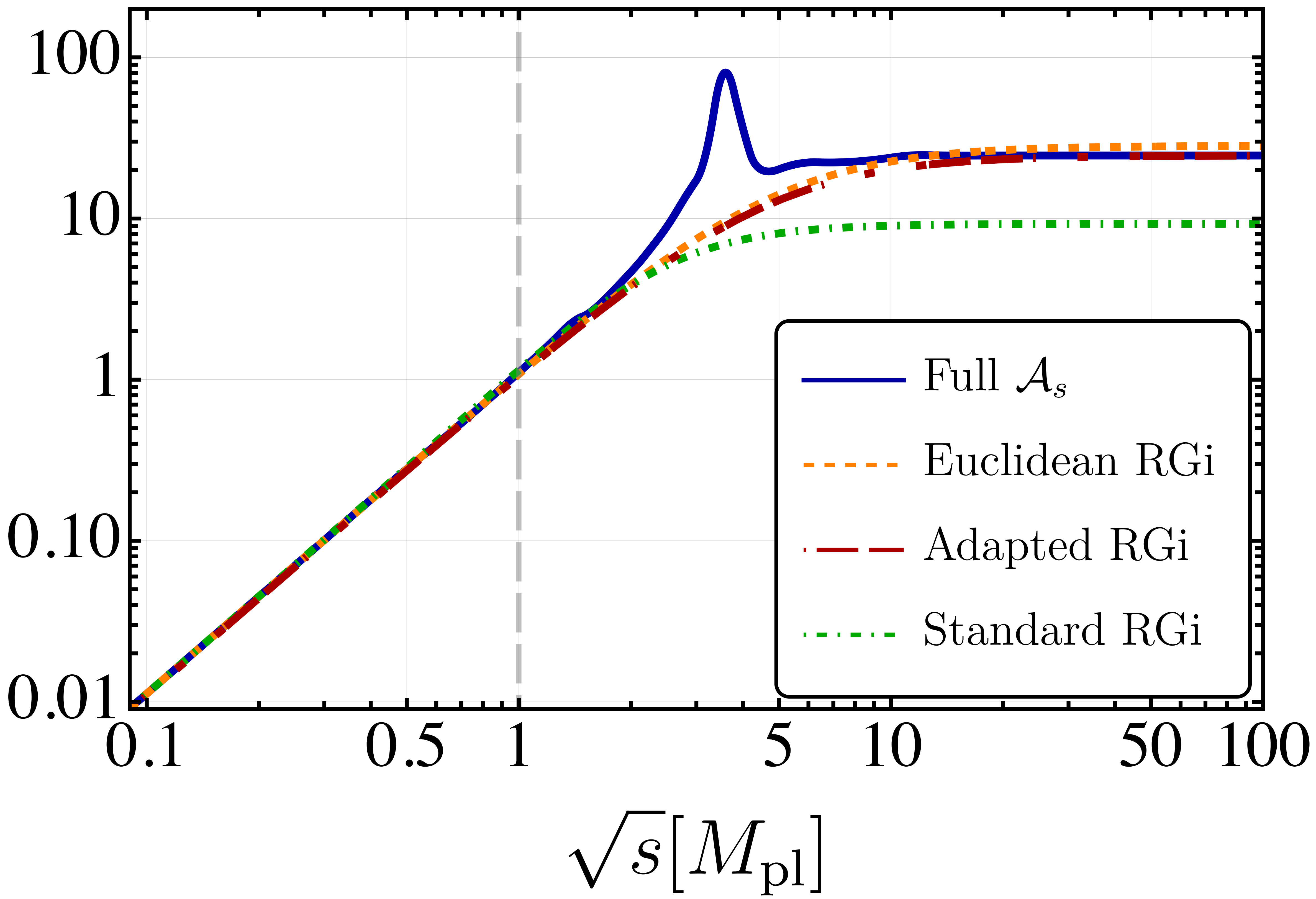
$$\boxed{V_L(s)} \longrightarrow \boxed{G(\sqrt{s},0)}$$

- Euclidean RGi $k \to 0, p^2 = s$

$$\boxed{V_L(s)} \longrightarrow \boxed{G(0,\sqrt{s})}$$

- Adapted RGi: use fixed point of

$$\boxed{V_L(s)} \longrightarrow \boxed{\frac{V^*}{s + V^* \mathrm{M}_{\mathrm{pl}}^2}}$$



Qualitative agreement

Miss peak structure!

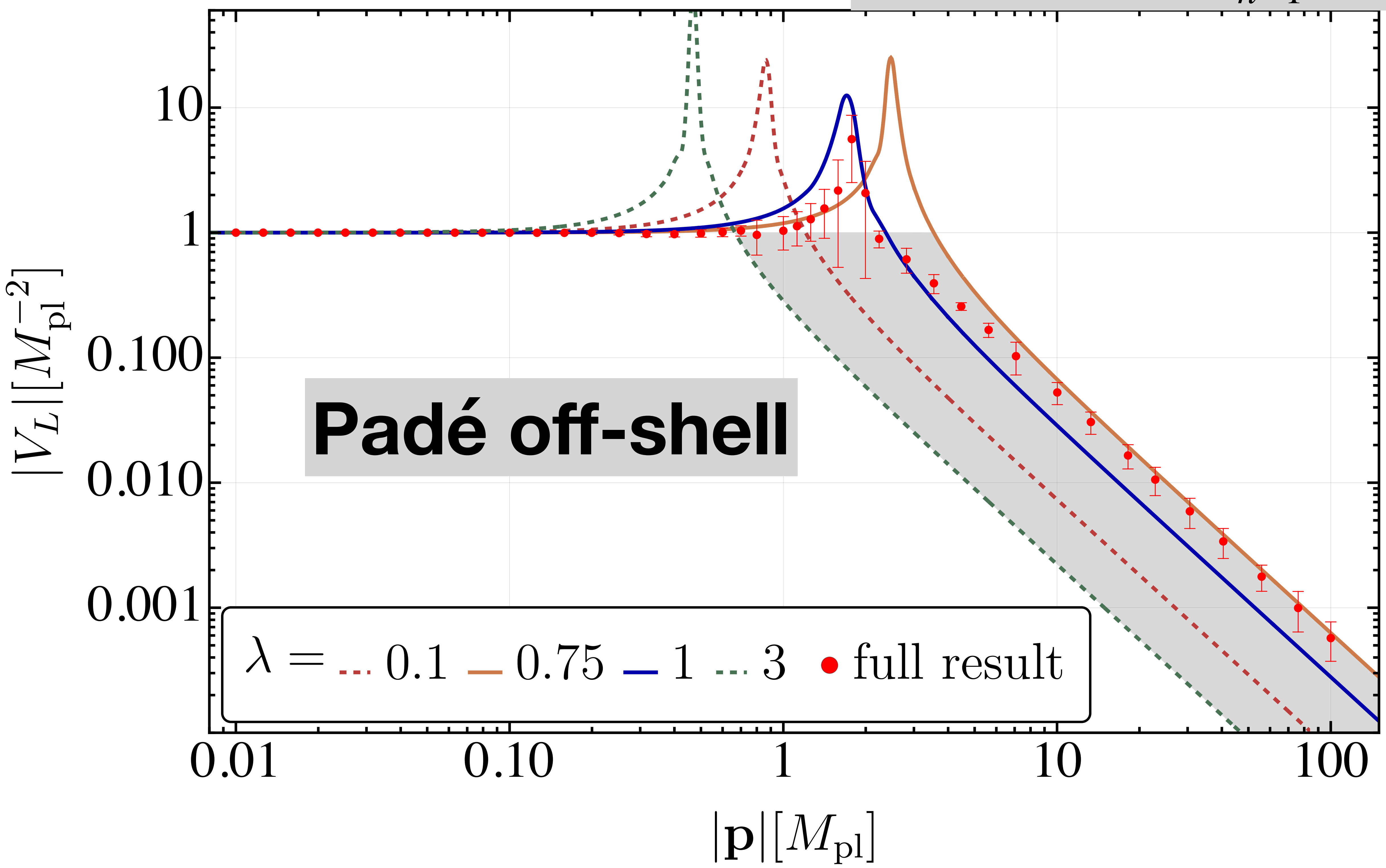
Comparison

Other reconstructions

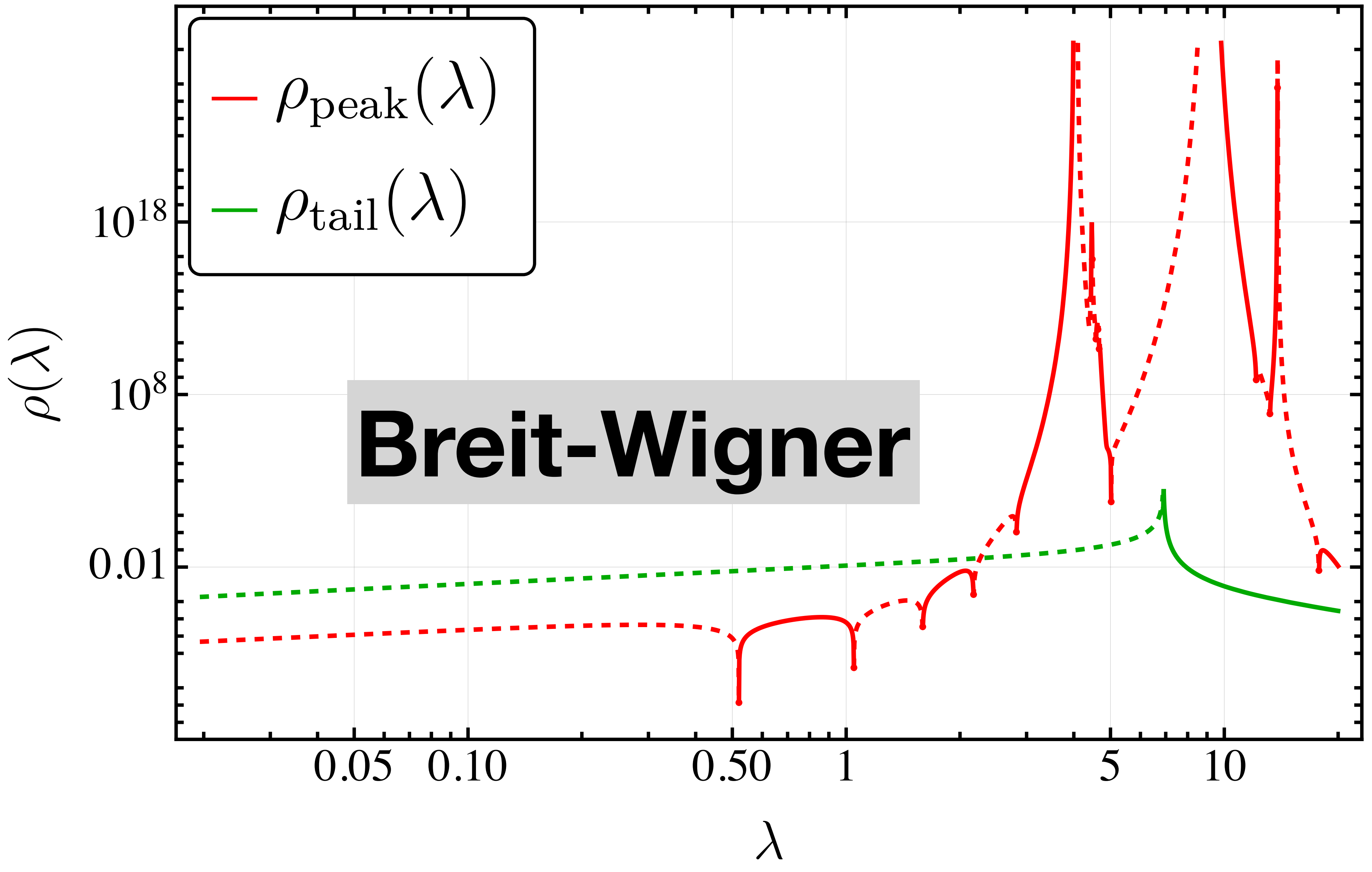
$$V_{BW}(p_0) = \sum_{i=1}^{3} N_i \prod_{k=1}^{5} \left(\frac{1}{(p_0 + \Gamma_{i,k})^2 + M_{i,k}^2} \right)^{\delta_{i,k}}$$

$$p_E^2 = (1 + \lambda^2)\,|\vec{p}\,|^2, \;\; z = \frac{-1 + \lambda}{1 + \lambda}$$

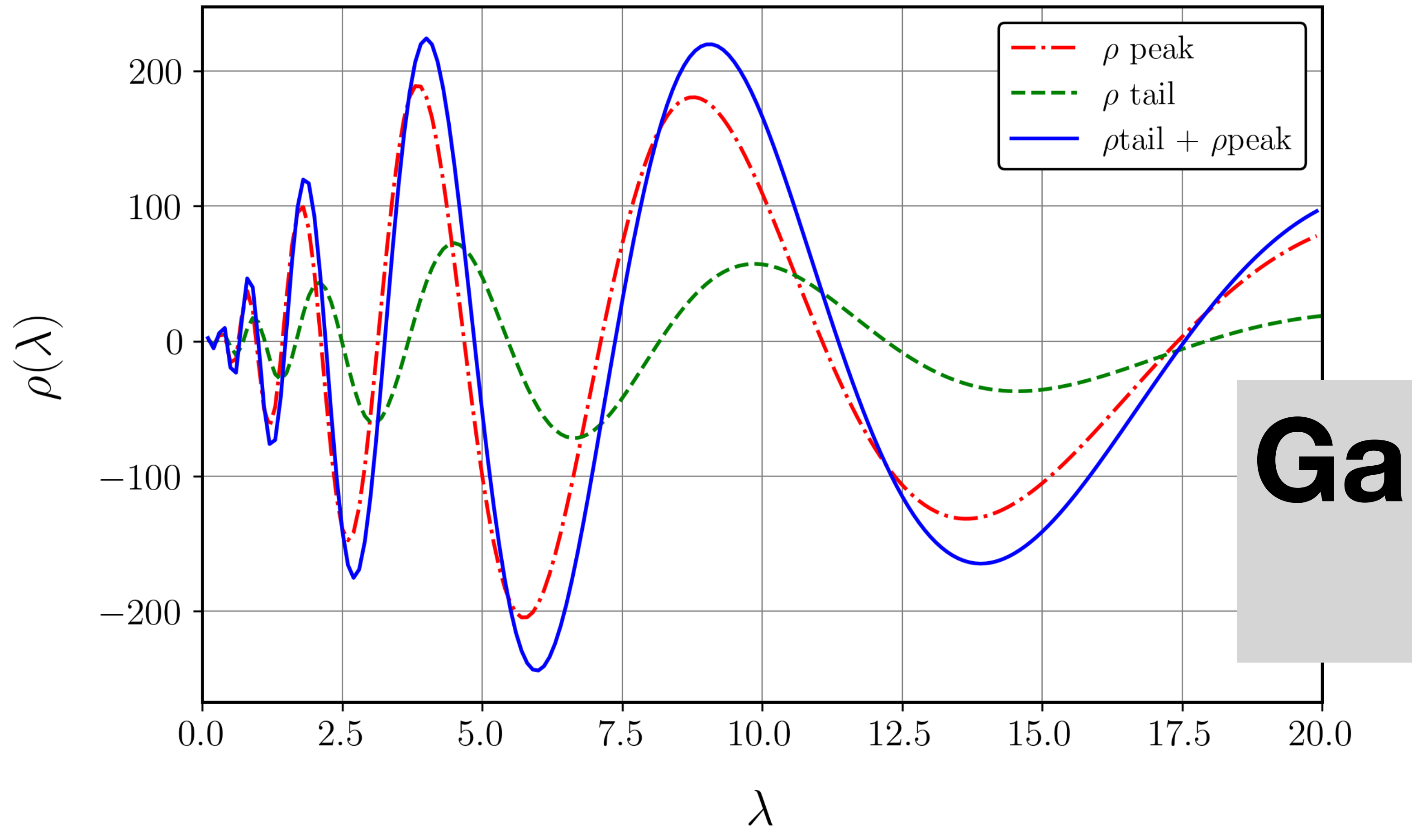
$$V_{L,\lambda}(p_0) = \frac{1 + \sum_{n=1}^{N} a_n p_0^n}{1 + \sum_{n=1}^{N} b_{n+2} p_0^{n+2}}$$



Padé off-shell



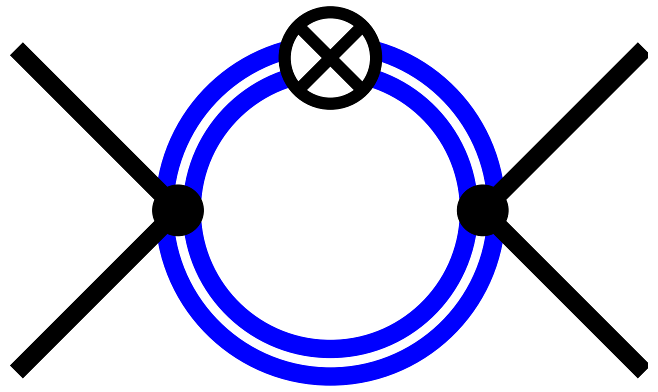
Breit-Wigner



Gaussian Process Regression

Interlude

Avatars

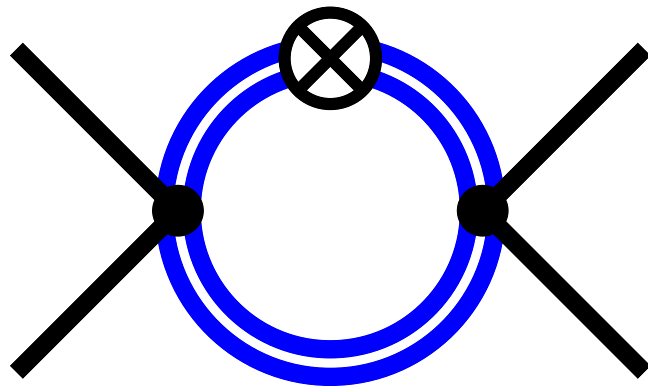


$$= \int_q \Gamma_k^{\phi\phi hh}(p_1, p_2, q)\Gamma_k^{\phi\phi hh}(p_3, p_4, q) f(q, p_1, p_2, p_3, p_4) \times$$
$$\mathcal{G}_k(q + p_1 + p_2)(\mathcal{G}_k \partial_t R_k \mathcal{G}_k)(q)$$

IR

$$G^{\phi\phi hh}(0) = G_N$$

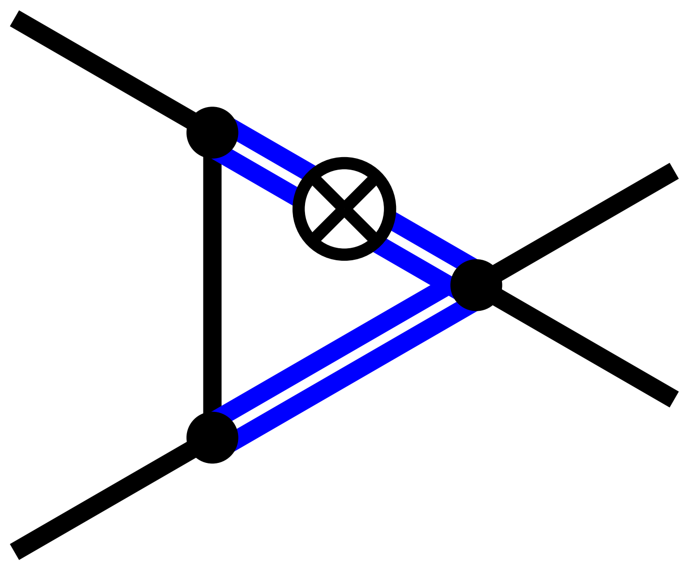
Main approximation: $\Gamma_k^{\phi\phi hh}(\mathbf{p}, q) \approx G_k^{\phi\phi hh}(\mathbf{p},0)\, T^{\mu\nu\rho\sigma}(\mathbf{p}, q)$



$$\approx G_k^{\phi\phi hh}(s, t)^2\, \text{Diag}_k(s, t)$$

Interlude

Avatars



$$= \int_q \Gamma_k^{\phi\phi h}(p_1, q)\Gamma_k^{\phi\phi h}(p_2, q)\Gamma_k^{\phi\phi hh}(p_3, p_4, q) f(q, p_1, p_2, p_3, p_4) \times \mathcal{G}_k(q + p_1)\mathcal{G}_k(q + p_1 + p_2)(\mathcal{G}_k \partial_t R_k \mathcal{G}_k)(q)$$

IR

$$G^{\phi\phi hh}(0) = G_N$$

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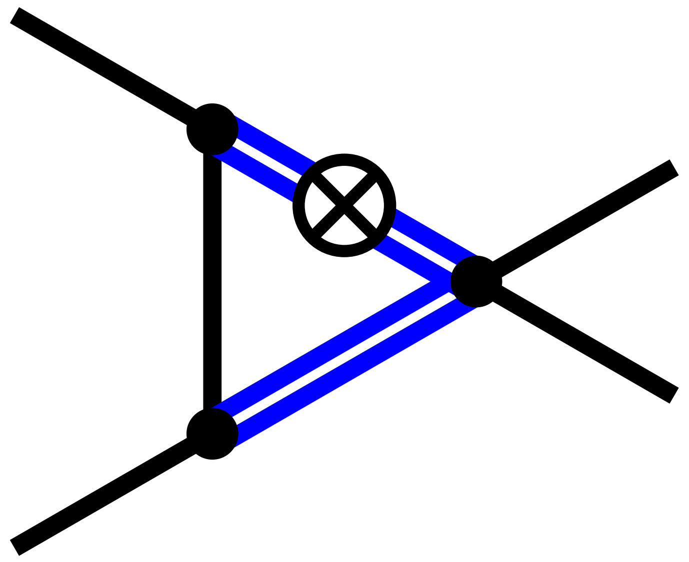
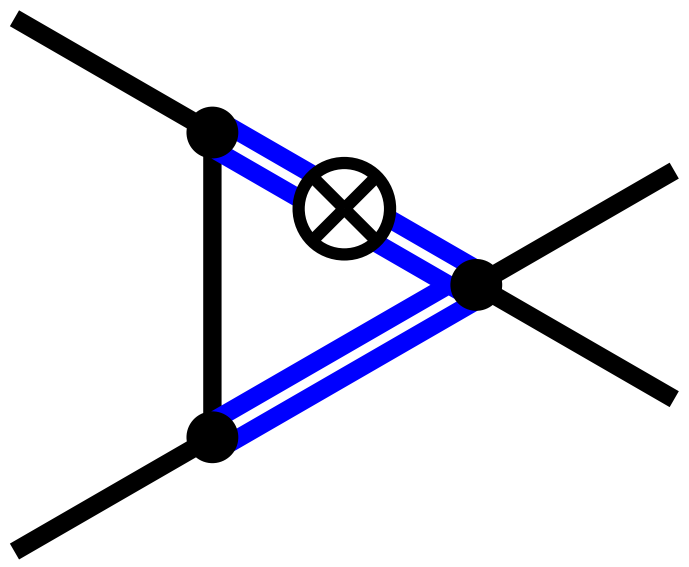


$$\approx G_k^{\phi\phi h}(s, t) G_k^{\phi\phi hh}(s, t)\, \text{Diag}_k(s, t)$$

Interlude

Avatars



$$= \int_q \mathrm{I} \quad \cdots \quad) f(q, p_1, p_2, p_3, p_4) \times$$

$$\cdots {}_k \partial_t R_k \mathcal{G}_k)(q)$$

UV

$$k^2 G^{\phi\phi hh}(\infty) \to g^*_{\phi\phi hh}$$

$$\neq$$

$$k^2 G^{\phi\phi h}(\infty) \to g^*_{\phi\phi h}$$

IR

$$G^{\phi\phi hh}(0) = G_N$$

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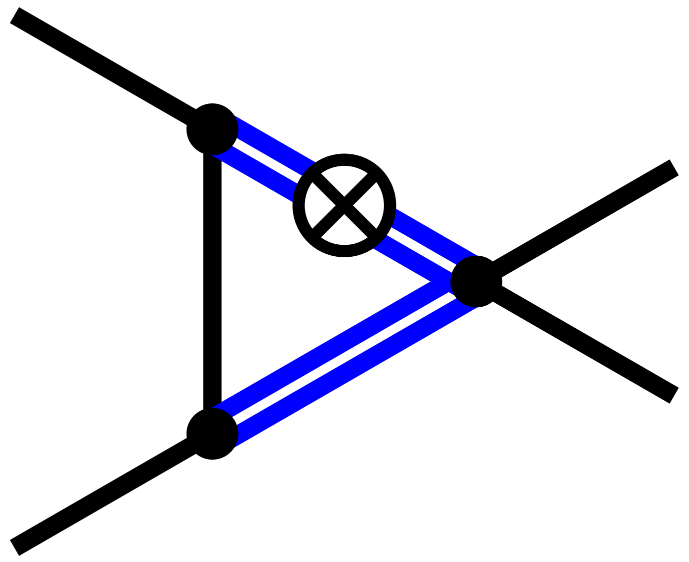
Main approximation:



$$\approx G_k^{\phi\phi} \cdots$$