Modified Forward Scattering and Unitarity in Matter Chern—Simons Theory

with L. Di Pietro, E. Di Salvo & E. Stamou — arXiv:2609.XXXXX (in preparation)

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Motivation

- **S Matrix:** Central Object in QFT $S = \langle f | \mathcal{S} | i \rangle = \mathbb{I} + iT$

  $\mathbb{I}$: No scattering/interaction T: scattering from $|i\rangle$ to $|f\rangle$

- Fulfills **Unitarity** $SS^\dagger = \mathbb{I}$ & **Analyticity** (S analytic in kinematic vars)

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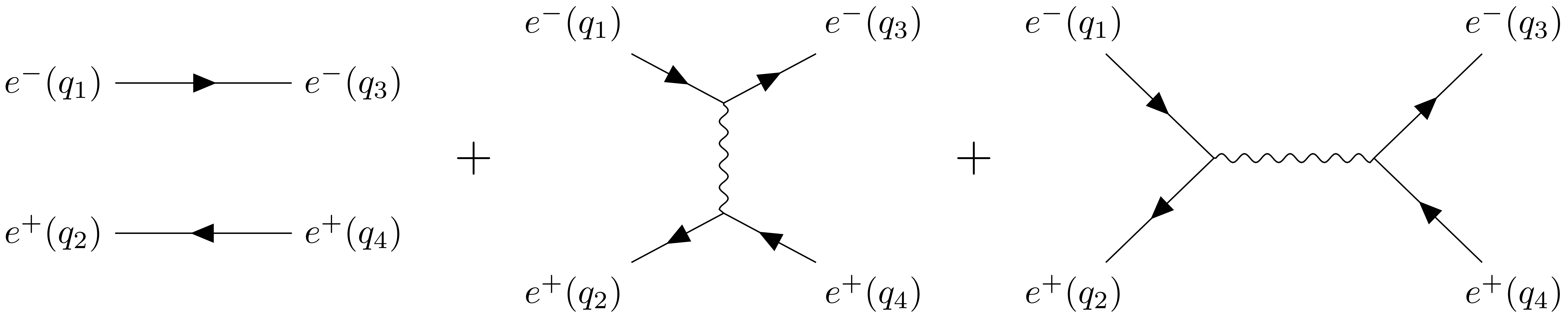
- Example: **Bhabha—Scattering**   Crossing: $\mathscr{M}_t \longleftrightarrow \mathscr{M}_s$ using $q_2 \leftrightarrow -q_3$



t-channel s-channel

$\mathbb{I}$ iT

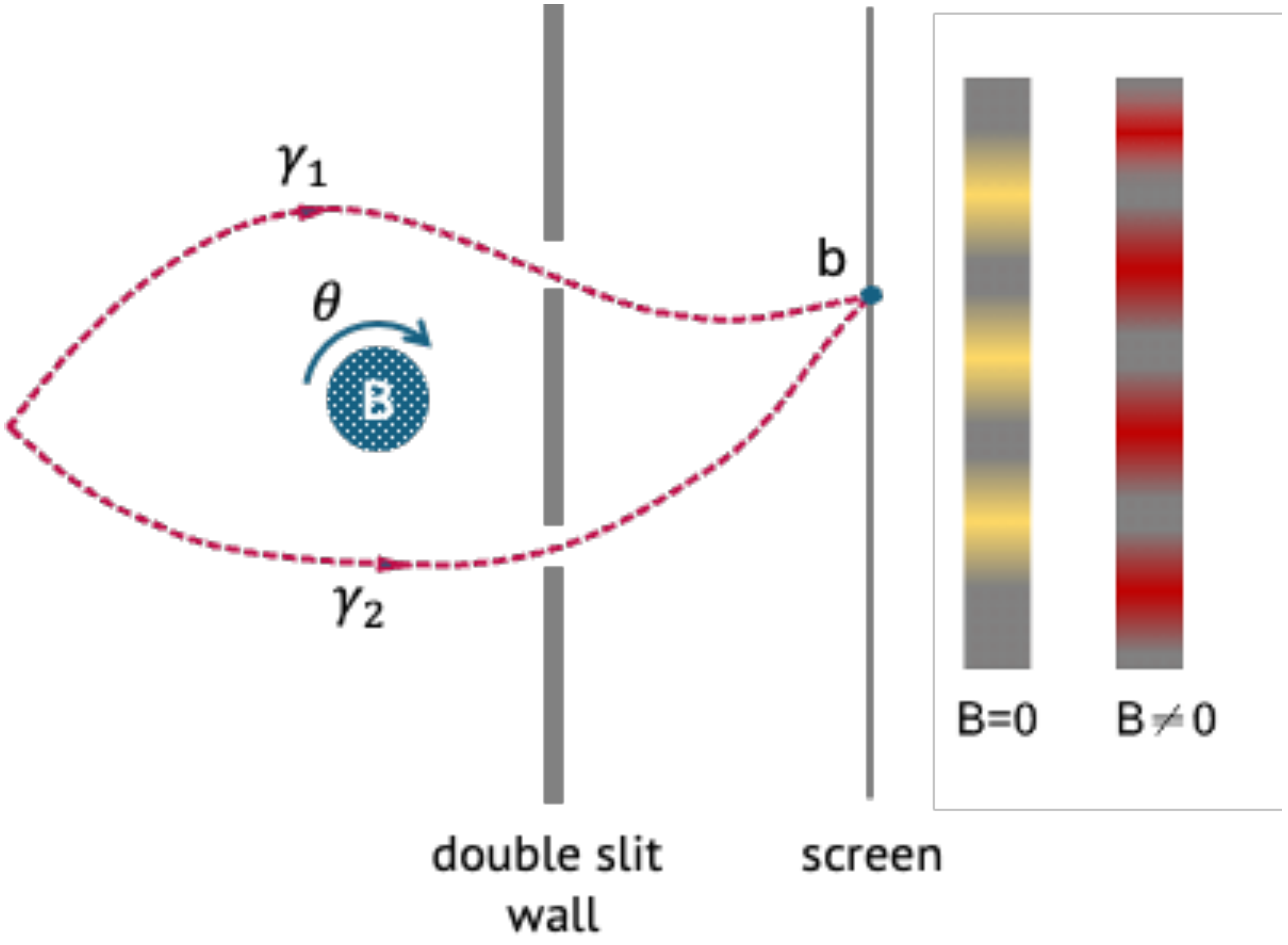
Motivation: Aharonov—Bohm Effect

But: Identity $\mathbb{I}$ not always trivial

QM (non relativ.) : Charged Particle and Solenoid

$$i\frac{\partial \Psi}{\partial t} = \left[\frac{1}{2m}\left(-i\vec{\nabla} - q\vec{A}\right)^2\right]\Psi$$

Static potential $\vec{A}$



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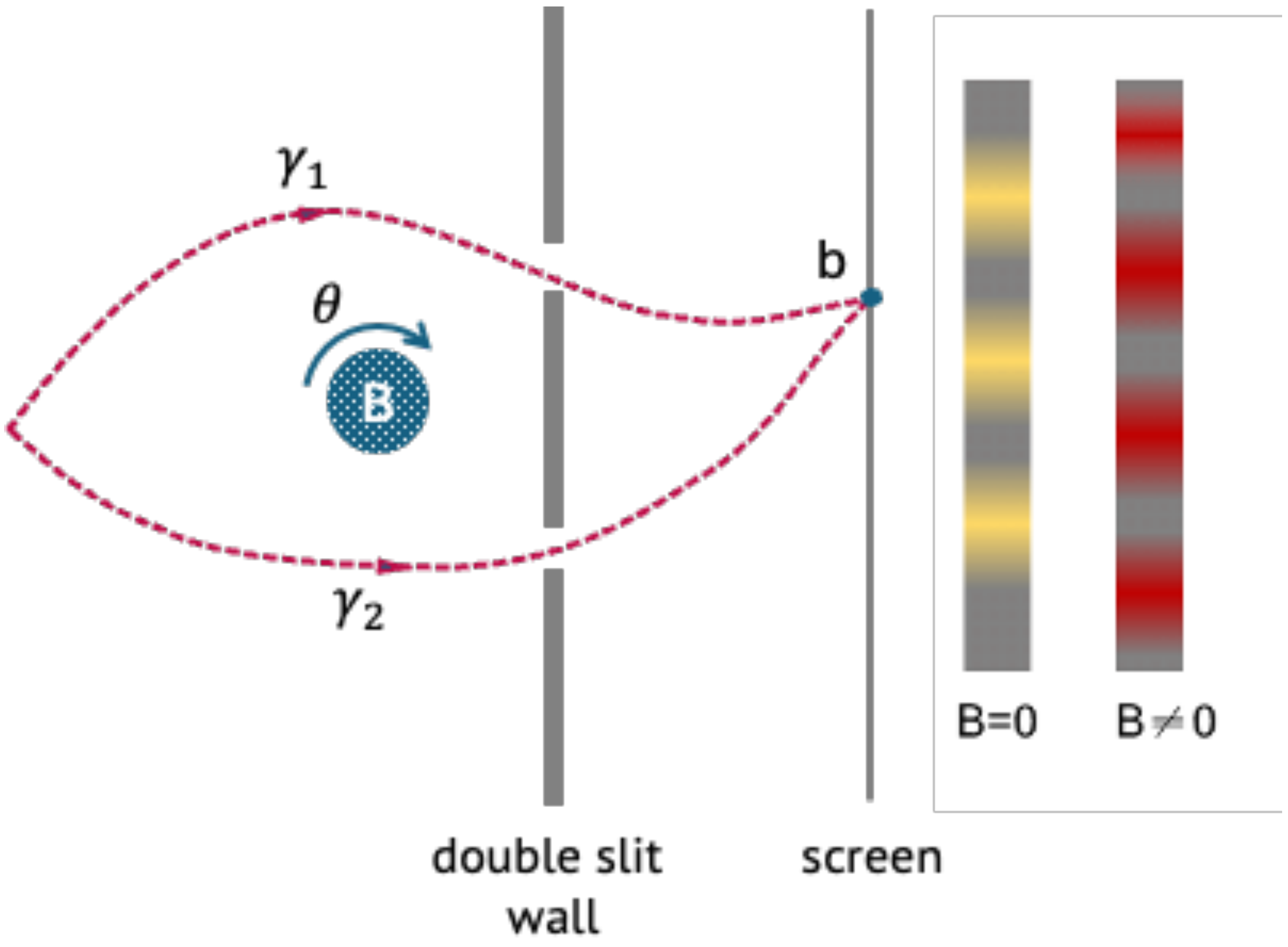

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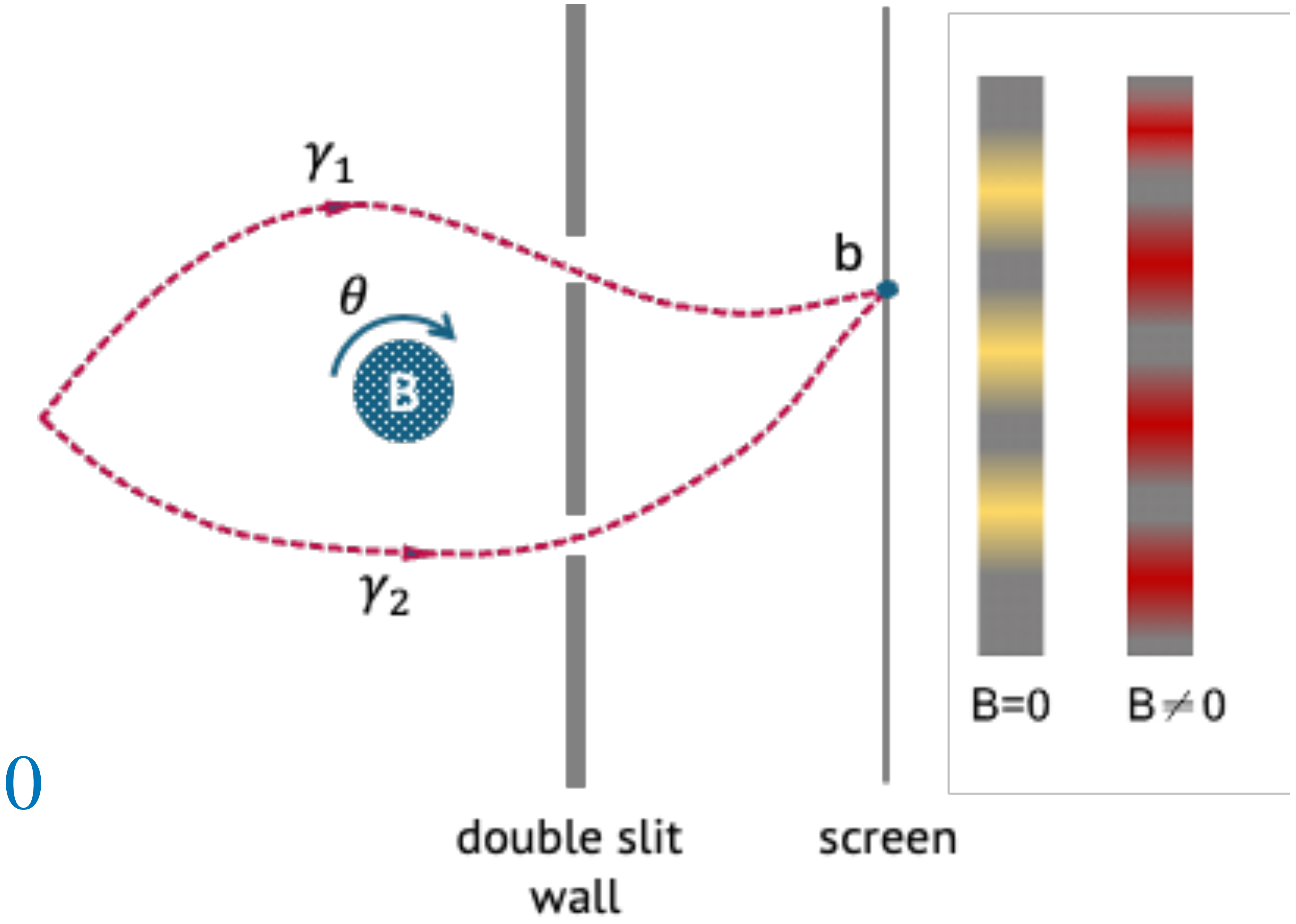

Interference depends on flux Φ even where $\vec{B} = 0$

$$\Psi_i = \exp\left(iq\int_{\gamma_i}\vec{A}\cdot d\vec{r}\right)\Psi_0 \qquad \frac{\Psi_1}{\Psi_2} = \exp\left(iq\oint_{\gamma_1-\gamma_2}\vec{A}\cdot d\vec{r}\right)e^{iq\Phi}$$

Wilson Loop → topol. phase from enclosed flux Φ

Matter Chern—Simons Theory

QFT description of **Aharonov—Bohm** by **Matter Chern—Simons in $d = 2 + 1$**

[Boz, Fainberg, Pak, Annals of Physics 246, 347-368 ’96]

$$\mathscr{L} = \frac{k}{4\pi}\epsilon^{\mu\nu\rho}\, Tr\left(A_\mu \partial_\nu A_\rho + \frac{2i}{3} A_\mu A_\nu A_\rho\right) + \mathscr{L}_{ghost} + \begin{cases} \bar{\psi}^i \left(i\gamma^\mu D_\mu - m \right) \psi_i \\ (D_\mu \phi)^2 - m^2 \phi^{\dagger i} \phi_i \end{cases}$$

$$\supset A_\mu J^\mu \sim A_\mu \epsilon_{\mu\nu\rho} F^{\nu\rho} \implies \rho = \frac{k}{2\pi} B$$

- **Charged Matter** carries magnetic flux → acts as **Solenoid**

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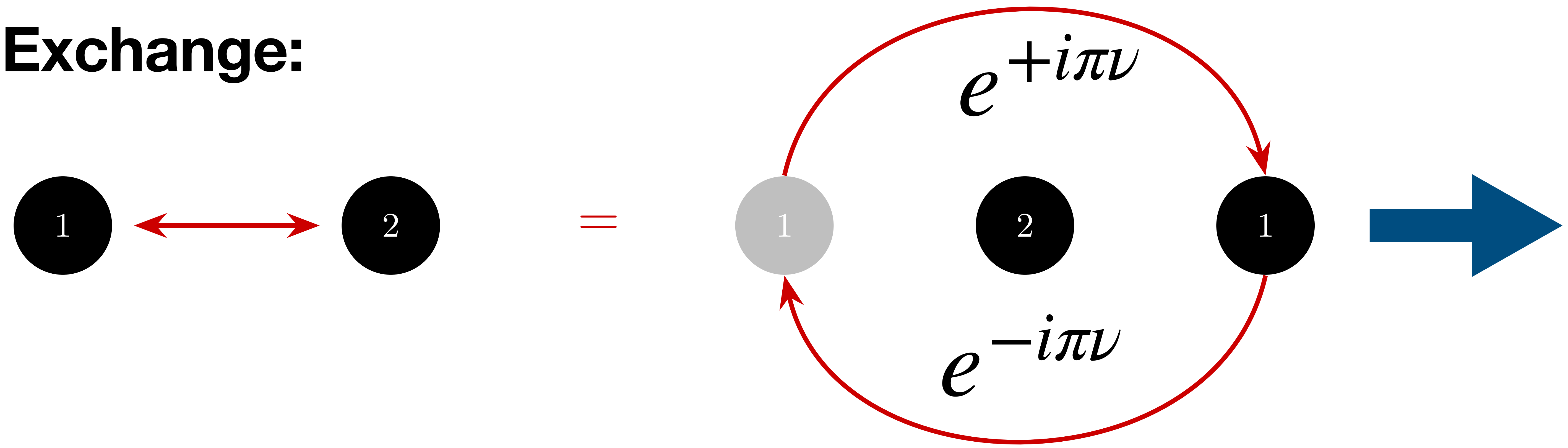
Particle Exchange:

Gauge inv. states dressed with Wilson lines



$\mathbb{I}$ **modified by topol. phase**

Matter Chern—Simons Theory: 2 → 2 Scattering

$$\mathscr{L} = \frac{k}{4\pi}\epsilon^{\mu\nu\rho}\, Tr\left(A_\mu \partial_\nu A_\rho + \frac{2i}{3} A_\mu A_\nu A_\rho\right) + \mathscr{L}_{ghost} + \mathscr{L}_{matter}$$

- 2015: $2 \to 2\ U(N)$ **fund. Matter Scattering** @ planar large-N , fixed $\lambda = N/k$

[Jain, Minwalla et. al., JHEP 04 ’15, 129]

$$P_i(q_1) + \bar{P}^j(q_2) \longrightarrow P_k(q_3) + \bar{P}^\ell(q_4)$$

$$[S_{P\bar{P}}]^{k;j}_{i;l} \simeq \mathbf{S}_{\text{sing}} \oplus \mathbf{S}_{\text{adj}}$$

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- Used **Crossing** to obtain S_{sing} from $S_{adj,sym,asym}$ **but** $S_{sing}S^\dagger_{sing} \neq \mathbb{I}$ **Violated Unitarity**

- **Fails** non-relativistic limit **Violated Aharonov—Bohm**

Matter Chern—Simons Theory: $2 \to 2$ Scattering

- 2015: $2 \to 2\ U(N)$ **fund. Matter Scattering** @ planar large-N , fixed $\lambda = N/k$

$$S^{naive}_{\rm sing} = \mathbb{I} + iT^{naive}_{\rm sing}\,, \qquad S_{\rm sing}S^{\dagger}_{\rm sing} \neq \mathbb{I}$$

[Jain, Minwalla et. al., JHEP 04 '15, 129]

Proposed Resolution: Modify physical S matrix → Restore **Non-Relativistic Limit** and **Unitarity**

2022: **Generalized conjecture for all channels** *[Mehta, Minwalla et. al., Adv.Theor.Math.Phys. 27, 193-310]*

Modified Crossing + Modified Identity $\to S^{phys}_{\rm sing}(S^{phys}_{\rm sing})^{\dagger} = \mathbb{I}$ ✓

Motivated by braiding of Wilson-lines

$2 \to 2$ Scattering: Testing Conjectured Modifications (We)

- **We (present work): Test modifications** @ large-k & finite N at one loop

$$S_M^{\text{phys}} = \mathbb{I} + i\widehat{T}_M \,,\; \widehat{T}_M = [1 + \mathcal{O}(1/k^2)]T_M^{naive} - i\left[\cos(\pi\nu_M) - 1\right]\mathbb{I}$$

Modified Crossing

Modified Identity

→ **One loop** $(1/k^2)$: Modification only in **identity piece** $T_M^{\text{naive}} = \mathcal{O}(1/k)$

$$S_M^{\text{phys}} = \mathbb{I} + i\widehat{T}_M = S_M^{\text{naive}} + \frac{i\pi^2}{2}(\nu_M)^2\mathbb{I} + \mathcal{O}(1/k^3)$$

$\nu_M \propto 1/k$: channel—dep. phase

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Unitarity has to hold for every channel M (center-of-mass frame) $\mathbb{I} = 8\pi\sqrt{s}\,\delta(\alpha)$

$$-i\Big(\widehat{T}_M(s,\alpha) - \widehat{T}^*_M(s,-\alpha)\Big) = \frac{1}{8\pi\sqrt{s}}\int_0^{2\pi} d\theta\,\widehat{T}_M(s,\theta)\widehat{T}^*_M(s,\theta-\alpha)$$

$\propto 1/k^2$ $\propto 1/k \times 1/k$

$2 \to 2$ Scattering: Unitarity

$\to$ **One loop** $(1/k^2)$: $S_M^{\text{phys}} = \mathbb{I} + i\widehat{T}_M = S_M^{\text{naive}} + \frac{i\pi^2}{2}(\nu_M)^2\mathbb{I} + \mathcal{O}(1/k^3)$

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$\mathbb{I} = 8\pi\sqrt{s}\,\delta(\alpha)$ $\to$ **How does $\delta(\alpha)$ result from integral?**

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$$T_{\text{sing}} = W(s)\ \text{PV}\cot\frac{\alpha}{2} \ + \ R(s,\alpha) \ + \mathcal{O}(1/k^2)$$

Singular at $\alpha = 0$ **Regular** at $\alpha = 0$

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$$\text{PV}\int_0^{2\pi} d\theta\,\cot\frac{\theta}{2}\cot\frac{\alpha-\theta}{2} = 2\pi - 4\pi^2\delta(\alpha)$$

[Mehta, Minwalla et. al., Adv.Theor.Math.Phys. 27, 193-310]

$2 \to 2$ Scattering: Singlet Channel

$\to$ **One loop** $(1/k^2)$ **:** $S_M^{\text{phys}} = \mathbb{I} + i\widehat{T}_M = S_M^{\text{naive}} + \frac{i\pi^2}{2}(\nu_M)^2\mathbb{I} + \mathcal{O}(1/k^3)$

$\mathscr{L}_{\text{matter}} = \mathscr{L}_\Psi$ **: Unitarity in Singlet Channel** *[Di Pietro, Di Salvo, Stamou, **MU**, in preparation — prelim. results]*

$$-i\left(\widehat{T}_{\text{sing}}(s,\alpha) - \widehat{T}^*_{\text{sing}}(s,-\alpha)\right) = \frac{N^2}{k^2}\left[\frac{\pi^2 e^{-i\alpha}}{2s\sqrt{s}}\Big(-8s(s-4m^2) + \left[16m^4 + 8(4N+3)m^2 s + (8N+1)s^2\right]\cos\alpha\right.$$
$$\left. + 8im\sqrt{s}\left[4m^2 + (4N+1)s\right]\sin\alpha\Big) + 8\pi^3\sqrt{s}\,\delta(\alpha)\right] + \mathcal{O}(1/k^3).$$

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$\Rightarrow$ $S_{\text{sing}}^{\text{phys}}(S_{\text{sing}}^{\text{phys}})^\dagger = \mathbb{I}$ ✓

δ-piece $\mathcal{O}(N^0)$ @ fixed $\lambda = N/k$

$\to$ visible in planar limit

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$\to$ visible in planar limit

Analogous checks for other channels & scalar matter ✓

Conclusion & Outlook

- Naïve $S = \mathbb{I} + iT$ **violates Unitarity & Non-Rel. Limit** in Matter Chern—Simons

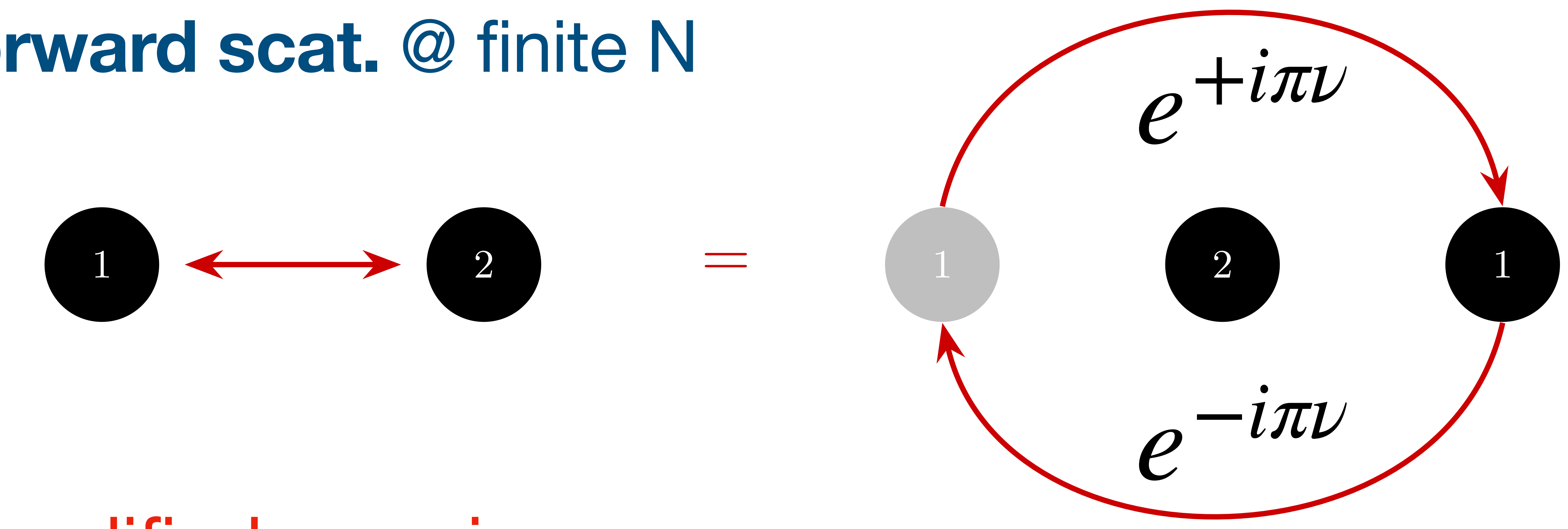
[Jain, Minwalla et. al., JHEP 04 ’15, 129]

- **Restored** by modified forward scat. & crossing

[Mehta, Minwalla et. al., Adv.Theor.Math.Phys. 27, 193-310]

- $U(N)$ fund. $2 \to 2$ matter @ one loop: Only mod. **forward scat.** $\propto \delta(\alpha)$

- **Our work: First perturbative verification of forward scat.** @ finite N



Future Work: **higher loops:** $1/k^3 \to$ check modified crossing

other theories: $\mathcal{N} = 6$ ABJM, … *[Aharony, Bergmann, Jafferis, Maldacena, JHEP 10 ’08, 091]*

Backup Slides

$2 \to 2$ Scattering: Adjoint Channel

$$\mathscr{L} = \frac{k}{4\pi}\epsilon^{\mu\nu\rho}\, Tr\Big(A_\mu \partial_\nu A_\rho + \frac{2i}{3} A_\mu A_\nu A_\rho\Big) + \mathscr{L}_{ghost} + \mathscr{L}_{matter}$$

$\to$ **One loop** $(1/k^2)$: $S_M^{\text{phys}} = \mathbb{I} + i\widehat{T}_M = S_M^{\text{naive}} + \frac{i\pi^2}{2}(\nu_M)^2\mathbb{I} + \mathcal{O}(1/k^3)$

$$\begin{aligned}
&-i\left(\widehat{T}^{G}_{\psi\bar{\psi},\text{adj}}(s,\alpha) - \widehat{T}^{G*}_{\psi\bar{\psi},\text{adj}}(s,-\alpha)\right)\\
&= \frac{\pi^2 e^{-i\alpha}}{2k^2N^2 s^{3/2}}\Big\{ -8(1-\eta_G)s(s-4m^2)\\
&\quad + \Big[N^2(s^2+24m^2 s+16m^4) - 8N(1-\eta_G)s(s+4m^2)\Big]\cos\alpha\\
&\quad + 8imN\sqrt{s}\Big[N(s+4m^2) - 4(1-\eta_G)s\Big]\sin\alpha\Big\} + \frac{8\pi^3\sqrt{s}(1-\eta_G)}{k^2N^2}\delta(\alpha) + \mathcal{O}(1/k^3).
\end{aligned}$$

$\eta_G = 1$ for $U(N)$, $\eta_G = 0$ for $SU(N)$

δ-piece $\mathcal{O}(1/N)$ @ fixed $\lambda = N/k$

$\to$ **not** visible in planar limit

[Di Pietro, Di Salvo, Stamou, **MU**, 2609.XXXXX]

$2 \to 2$ Scattering: Symmetric Channel

$$\mathscr{L} = \frac{k}{4\pi}\epsilon^{\mu\nu\rho}\, Tr\Big(A_\mu \partial_\nu A_\rho + \frac{2i}{3} A_\mu A_\nu A_\rho\Big) + \mathscr{L}_{ghost} + \mathscr{L}_{matter}$$

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*[Di Pietro, Di Salvo, Stamou, **MU**, 2609.XXXXX]*

$2 \to 2$ Scattering: Antisymmetric Channel

$$\mathscr{L} = \frac{k}{4\pi}\epsilon^{\mu\nu\rho}\,Tr\left(A_\mu\partial_\nu A_\rho + \frac{2i}{3}A_\mu A_\nu A_\rho\right) + \mathscr{L}_{ghost} + \mathscr{L}_{matter}$$

$\to$ **One loop** $(1/k^2)$: $S_M^{\text{phys}} = \mathbb{I} + i\widehat{T}_M = S_M^{\text{naive}} + \frac{i\pi^2}{2}(\nu_M)^2\mathbb{I} + \mathcal{O}(1/k^3)$

$\eta_G = 1$ for $U(N)$, $\eta_G = 0$ for $SU(N)$

$$-i\left(\widehat{T}^{G}_{\psi\psi,U_A}(s,\alpha) - \widehat{T}^{G*}_{\psi\psi,U_A}(s,-\alpha)\right) = \frac{8\pi^3\sqrt{s}}{k^2}\frac{(N+1-\eta_G)^2}{N^2}\left[\delta(\alpha) + \delta(\alpha-\pi)\right] + \mathcal{O}(1/k^3).$$

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Motivation: Analyticity and Crossing

- Example: **Bhabha—Scattering**