ERG 2026, University of Sussex

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Wilsonian Cosmology

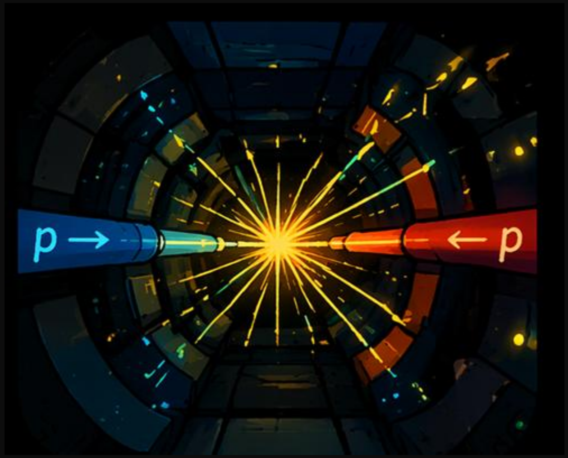
Inflation & Quantum Corrections?

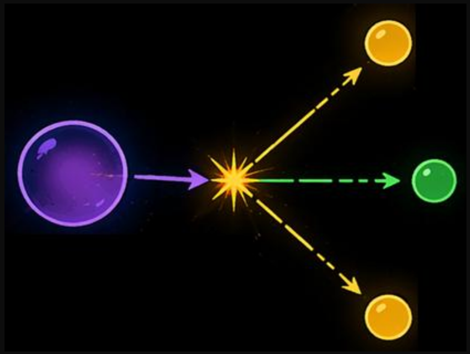
Based on ArXiv:2511.05296 and 2608.23676– J. Alexandre, LH, S. Pla

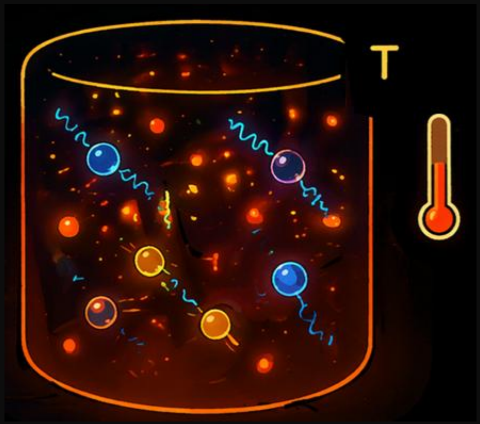
In QFT, renormalisation is crucial

Observables independent on renormalisation scale μ, but dependent on the energy scale of the system

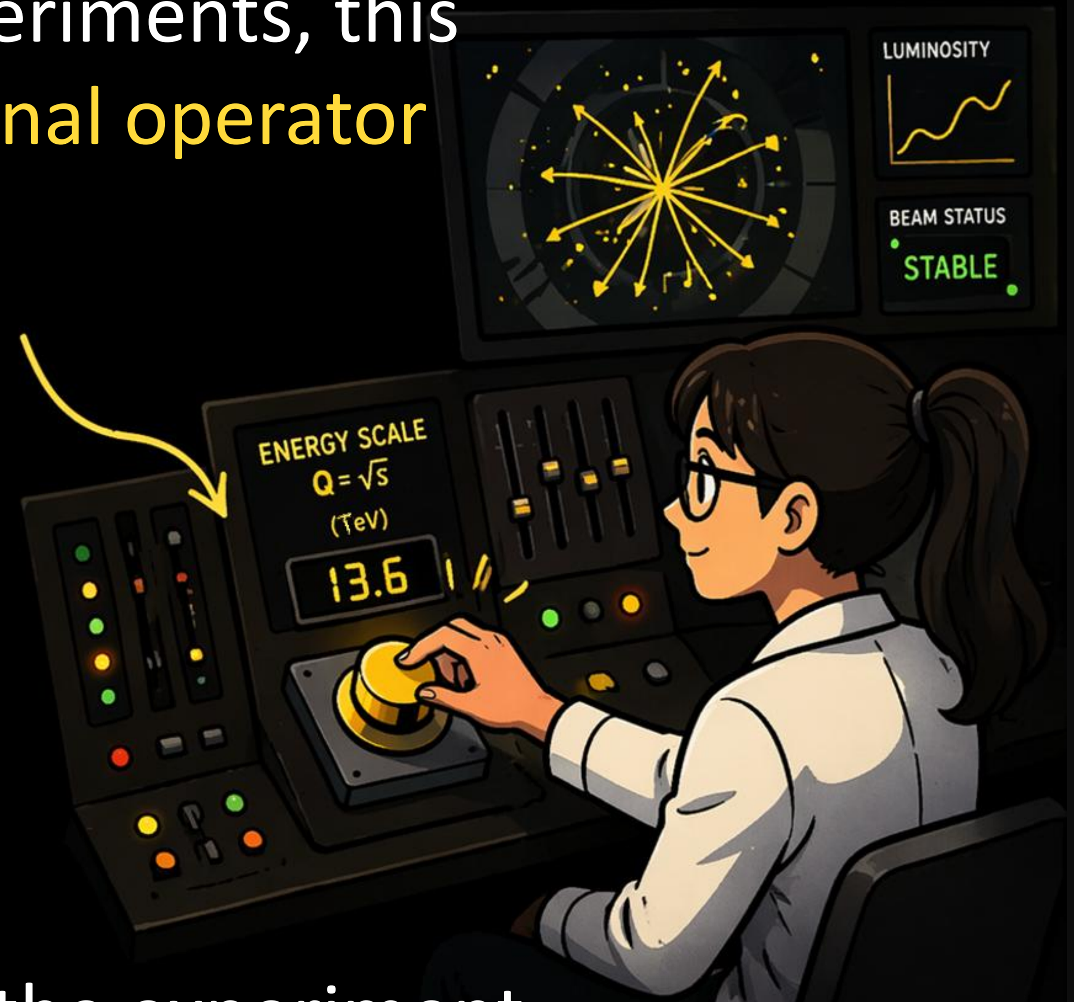
- Collision $\longrightarrow$ $\sqrt{s}$ (CoM energy)
- Particle decay $\longrightarrow$ m (vacuum/thermal mass)
- Thermal bath $\longrightarrow$ T (temperature)







In particle physics experiments, this scale is set by an external operator

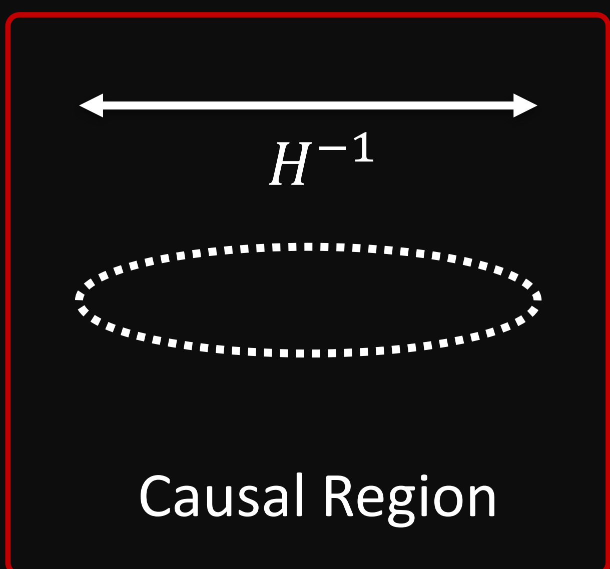


What happens inside the experiment does not affect this choice.

… unlike in cosmology

In cosmology

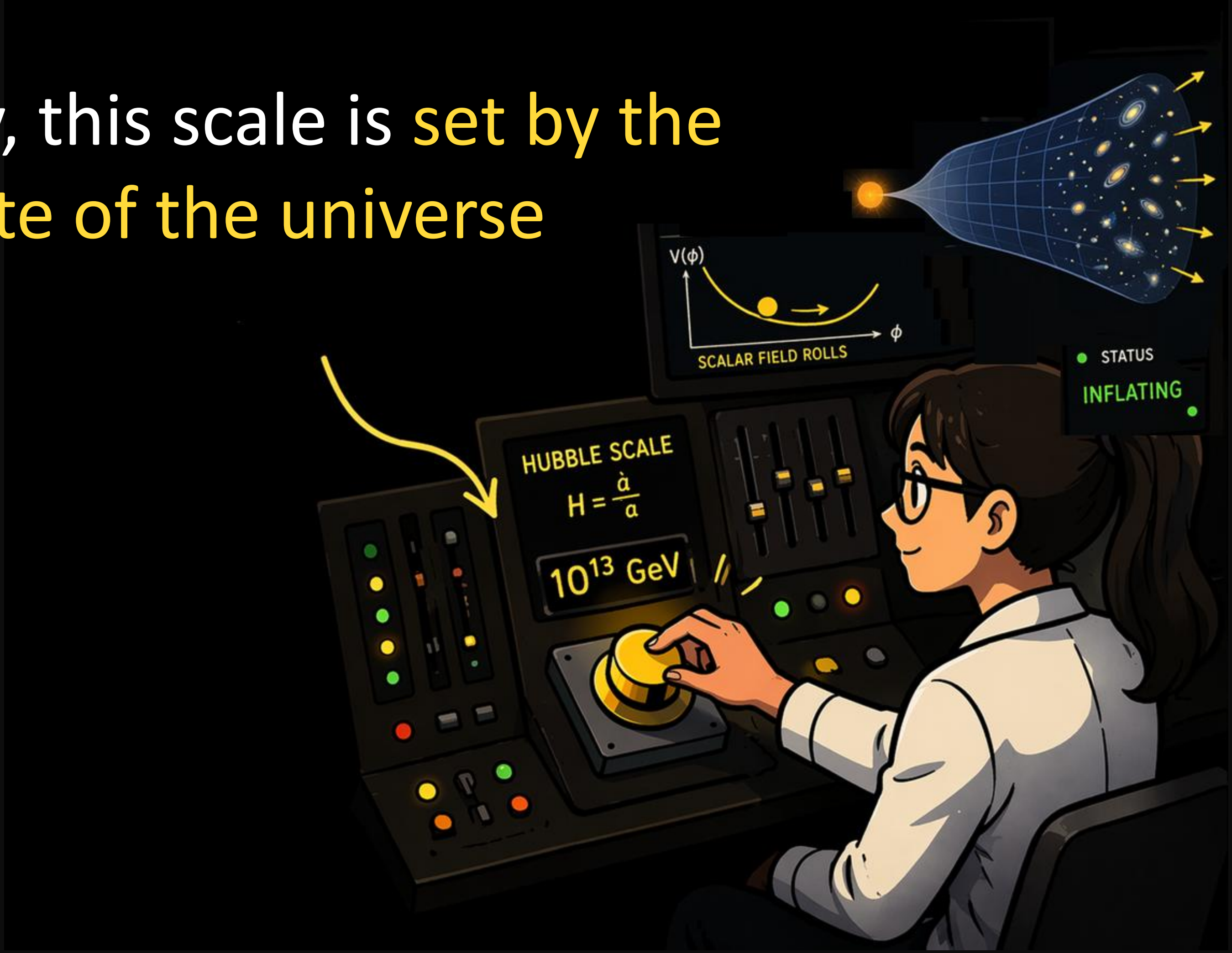
$$\frac{a'(t)}{a(t)} \equiv H(t)$$ Hubble scale



Small-scale fluctuations $\omega \gg H$ Ultraviolet theory

Large-scale fluctuations $\omega \ll H$ Infrared Theory

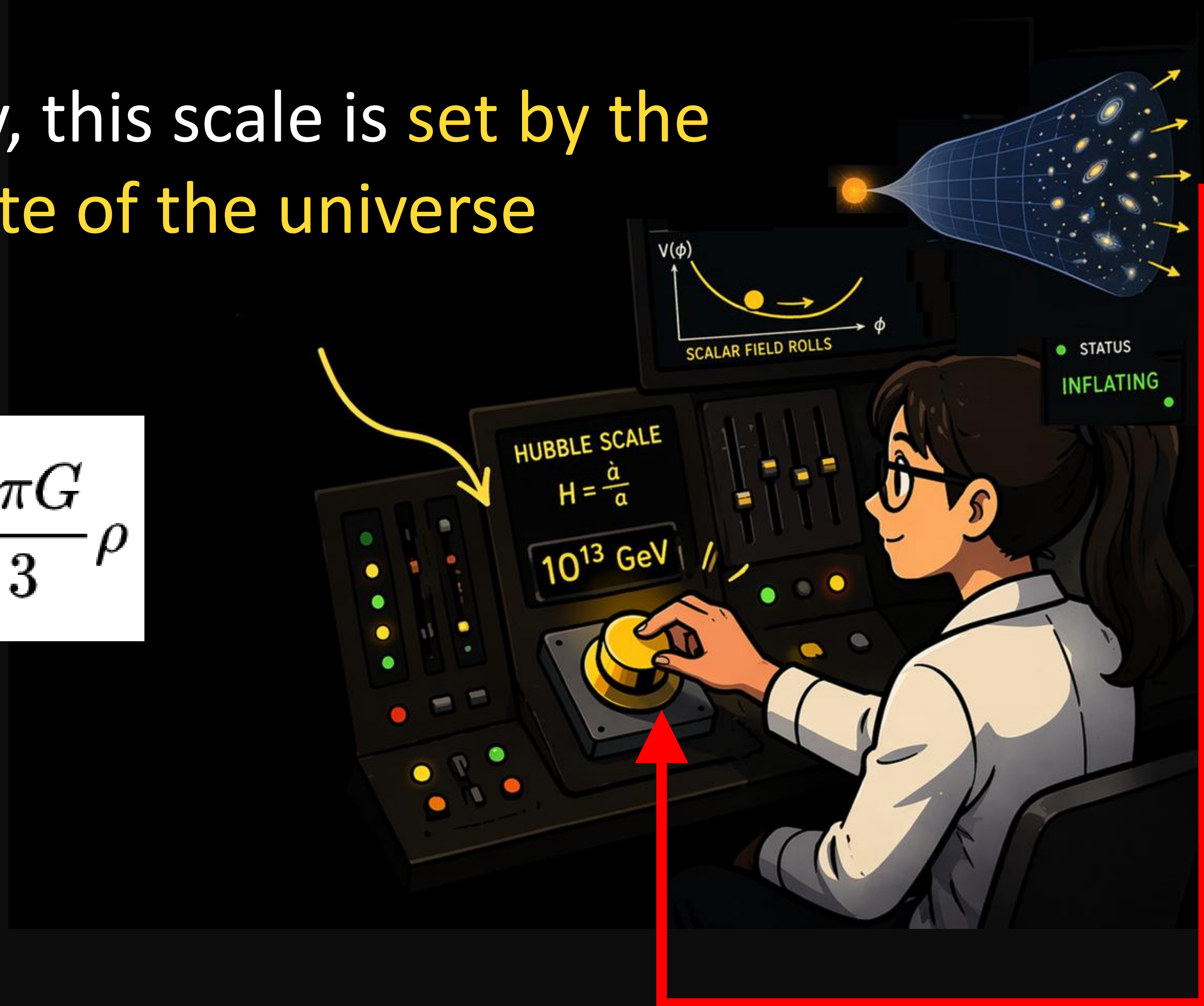
In cosmology, this scale is set by the expansion rate of the universe



In cosmology, this scale is set by the expansion rate of the universe

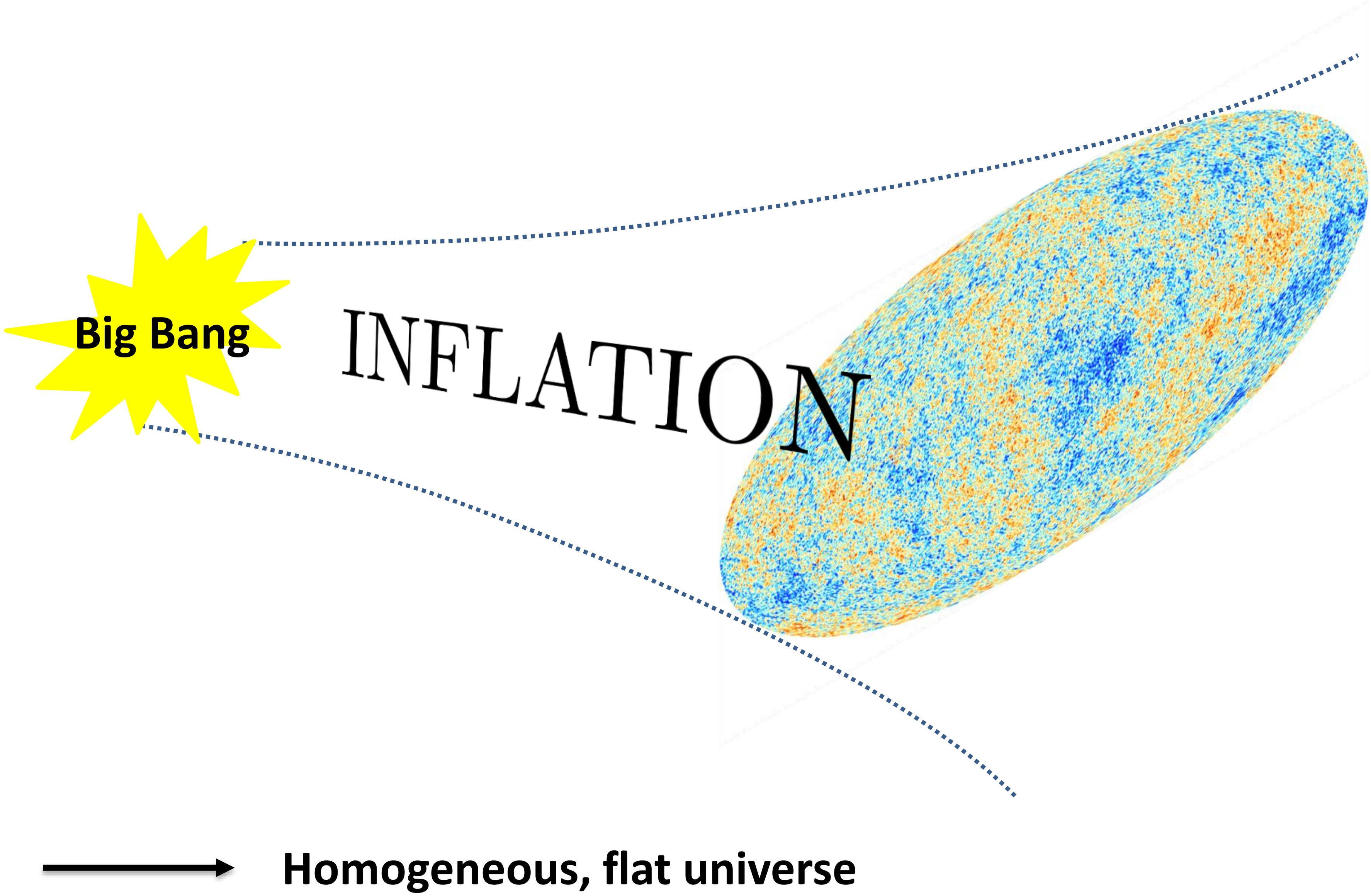
$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho$$

Friedmann equations



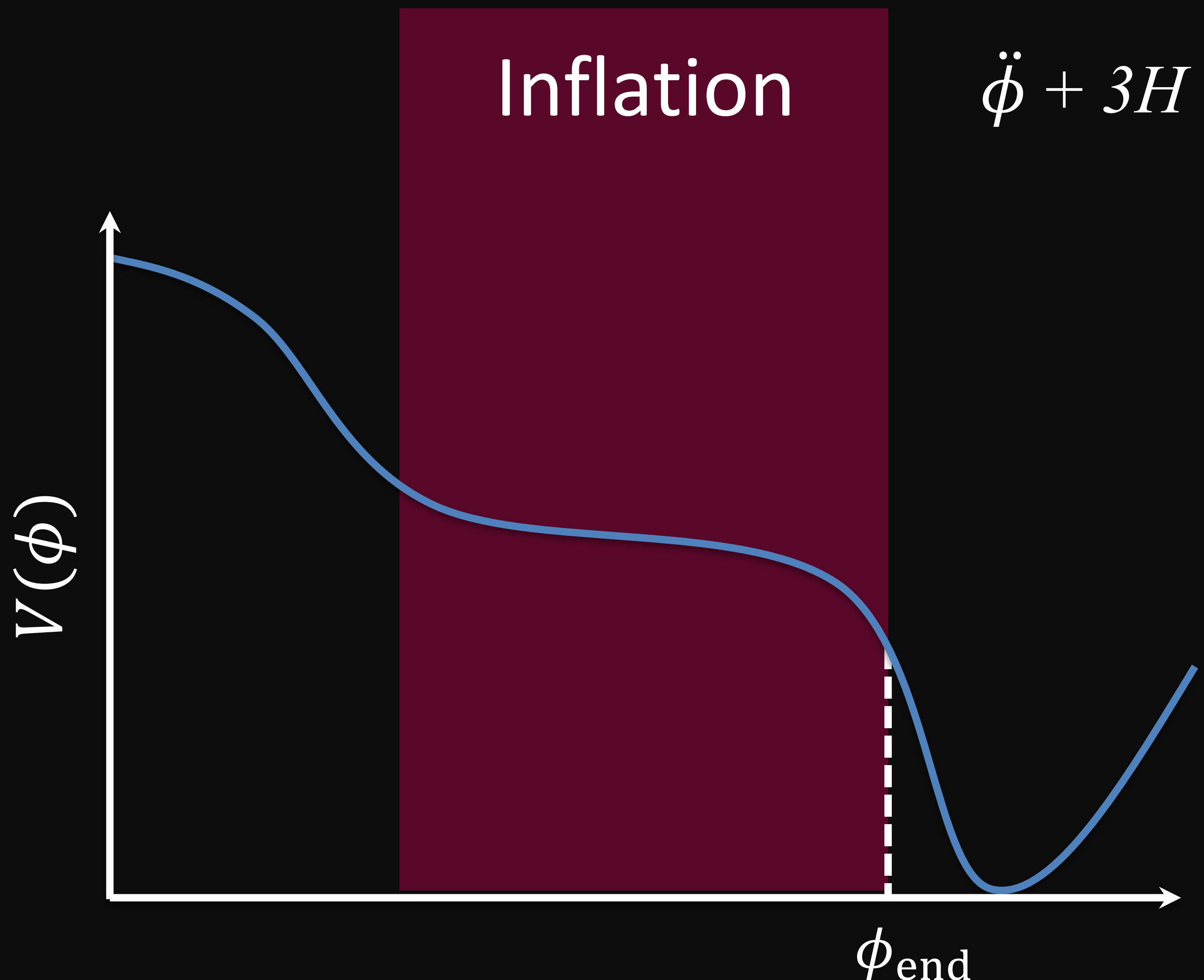
What happens inside the experiment perturbs the operator!

Cosmic Inflation

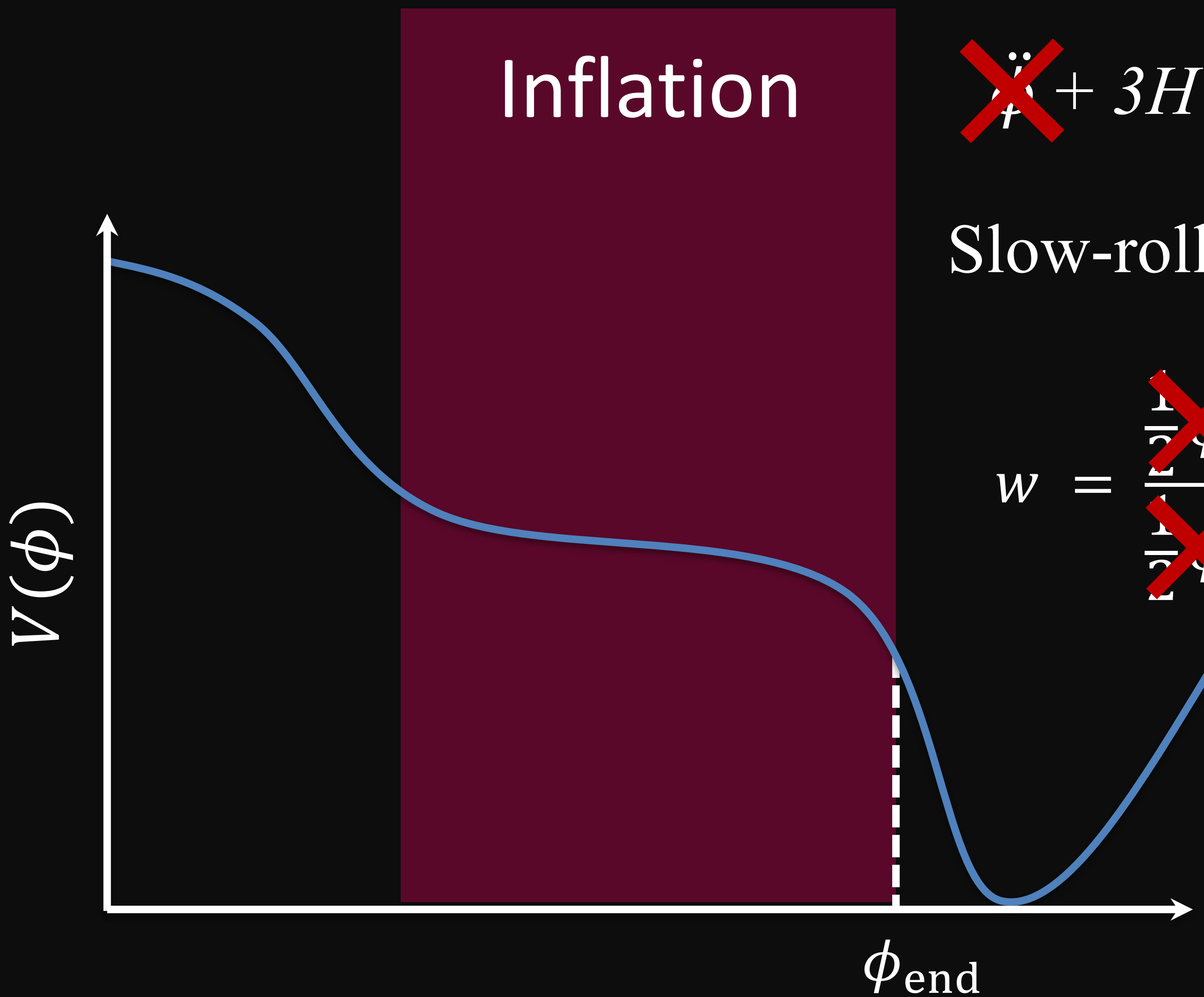


Inflation Potential

$$\ddot{\phi} + 3H\,\dot{\phi} + V'(\phi) = 0$$



Inflation Potential



$$\cancel{\ddot{\phi}} + 3H\,\dot{\phi} + V'(\phi) = 0$$

Slow-roll: $3H\,\dot{\phi} \sim V'(\phi)$

$$w = \frac{\cancel{\frac{1}{2}\dot{\phi}^2} - V(\phi)}{\cancel{\frac{1}{2}\dot{\phi}^2} + V(\phi)} \approx -1$$

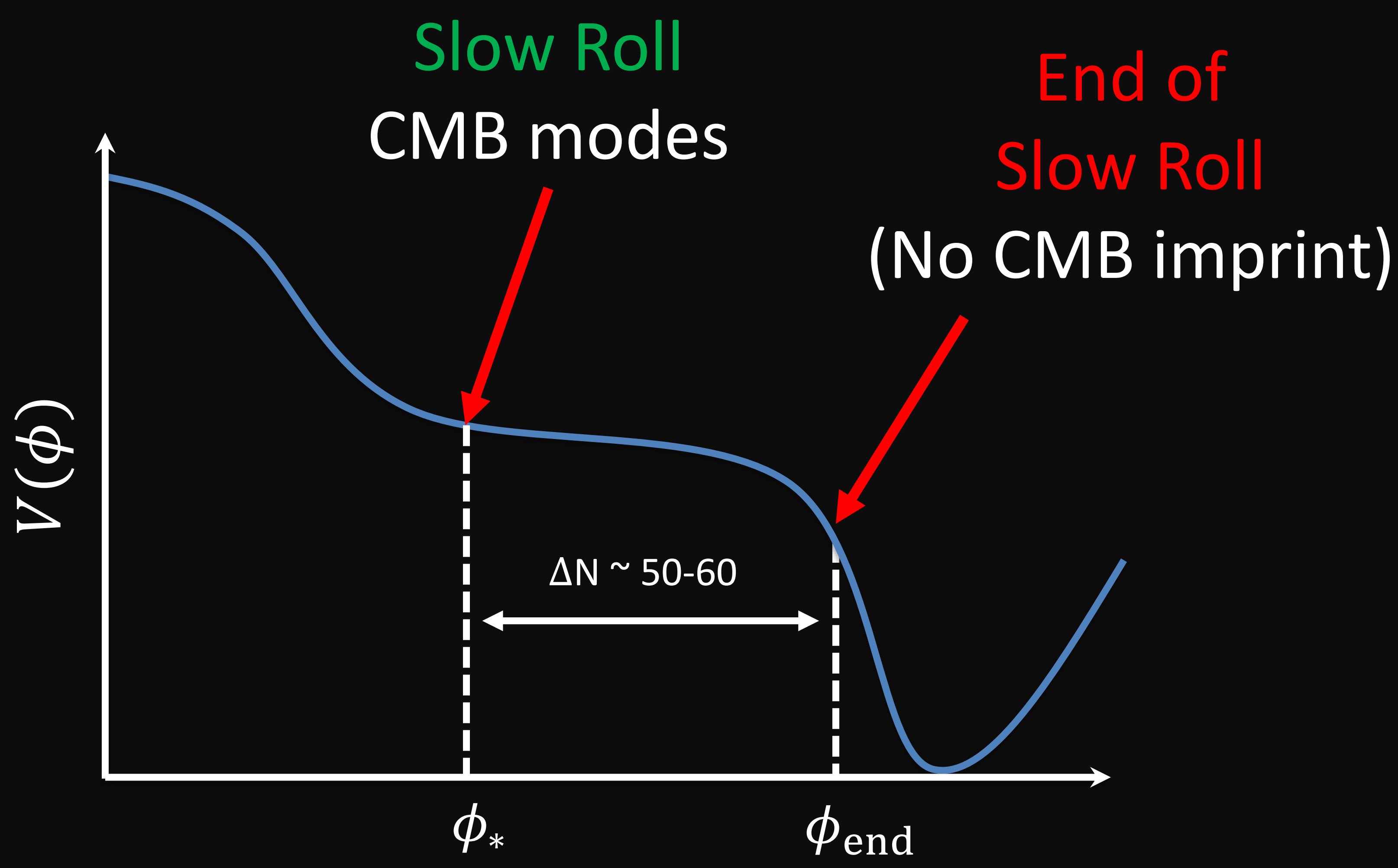
Inflation Potential

$$V(\phi) = V_0 + g_1\,\phi + g_2\phi^2 + g_3\phi^3 + \cdots$$

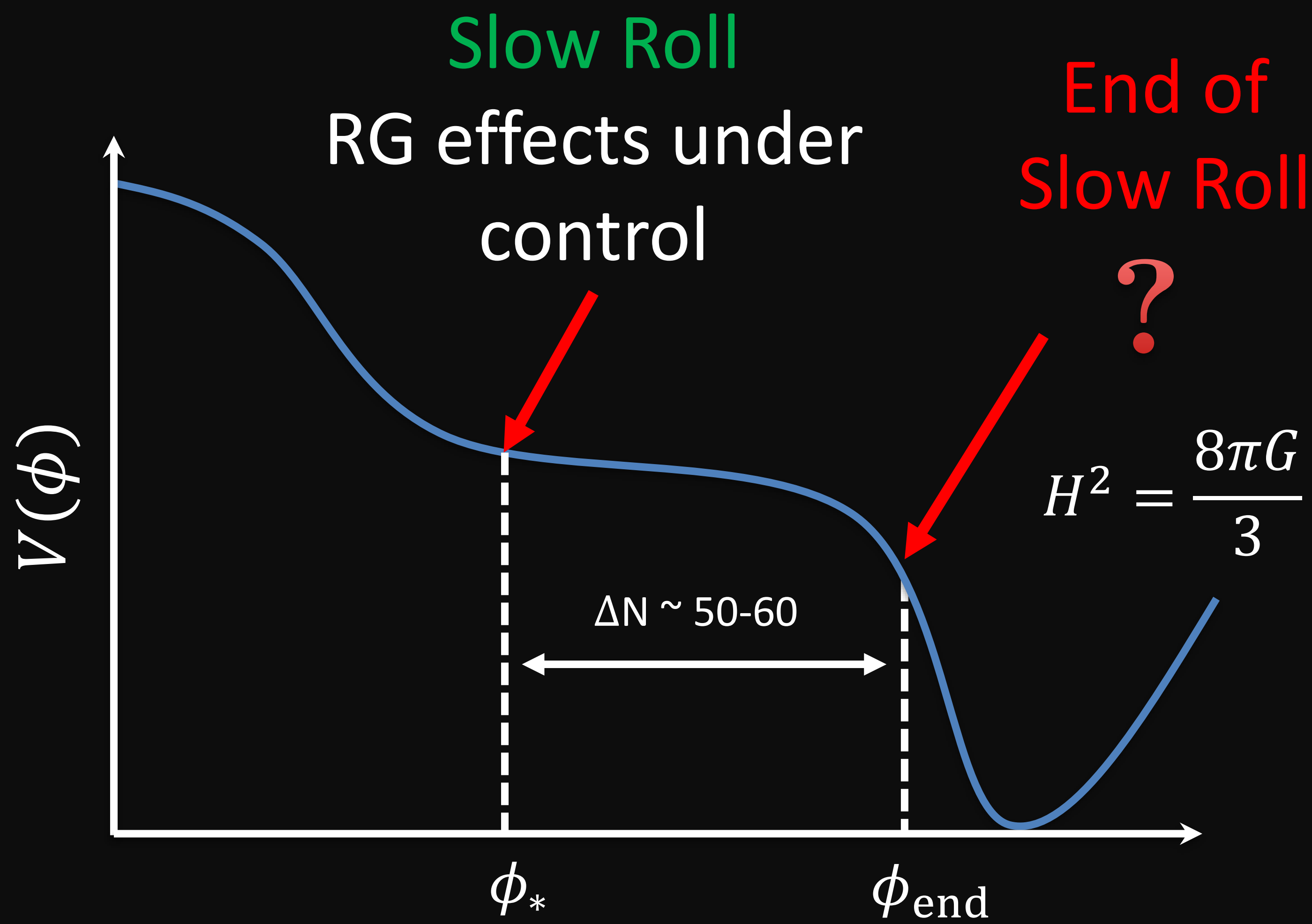
All couplings measured in the CMB

$(id\ est\ @\ H = H_{\mathrm{CMB}})$

Inflation Observables



Inflation Observables



$$H^2 = \frac{8\pi G}{3}\left(\frac{\dot{\phi}^2}{2} + V(\phi)\right)$$

Inflation Potential

$$V(\phi) = V_0 + g_1\,\phi + g_2\phi^2 + g_3\phi^3 + \cdots$$

All couplings measured in the CMB

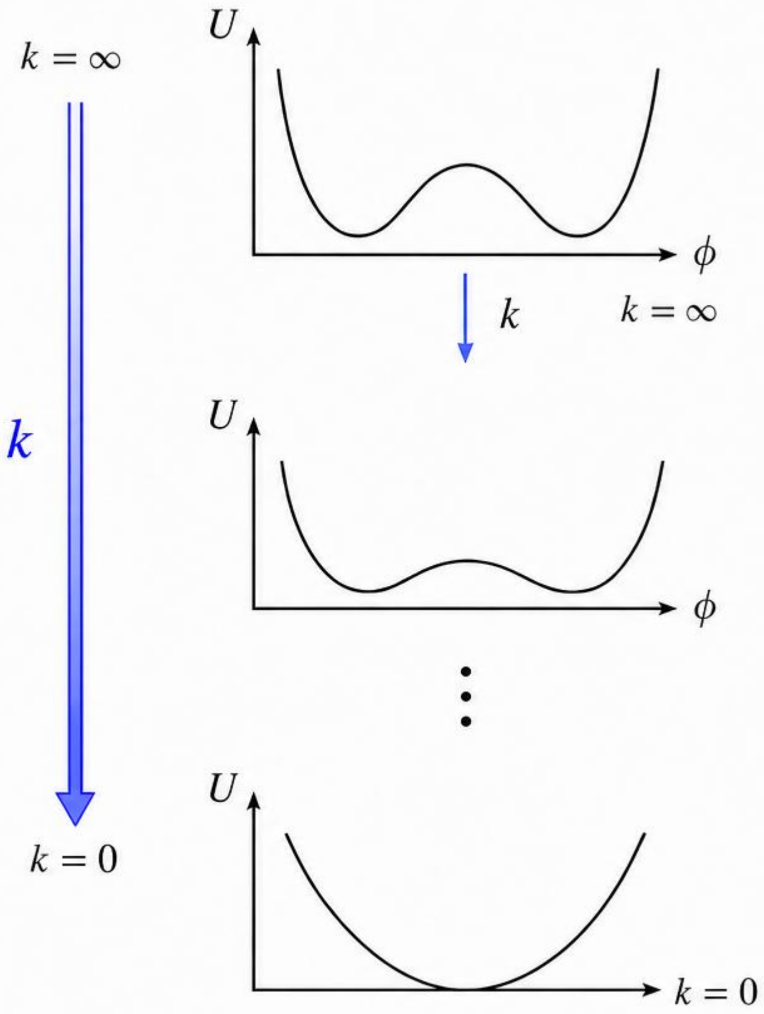
$(id\ est\ @\ H = H_{\mathrm{CMB}})$

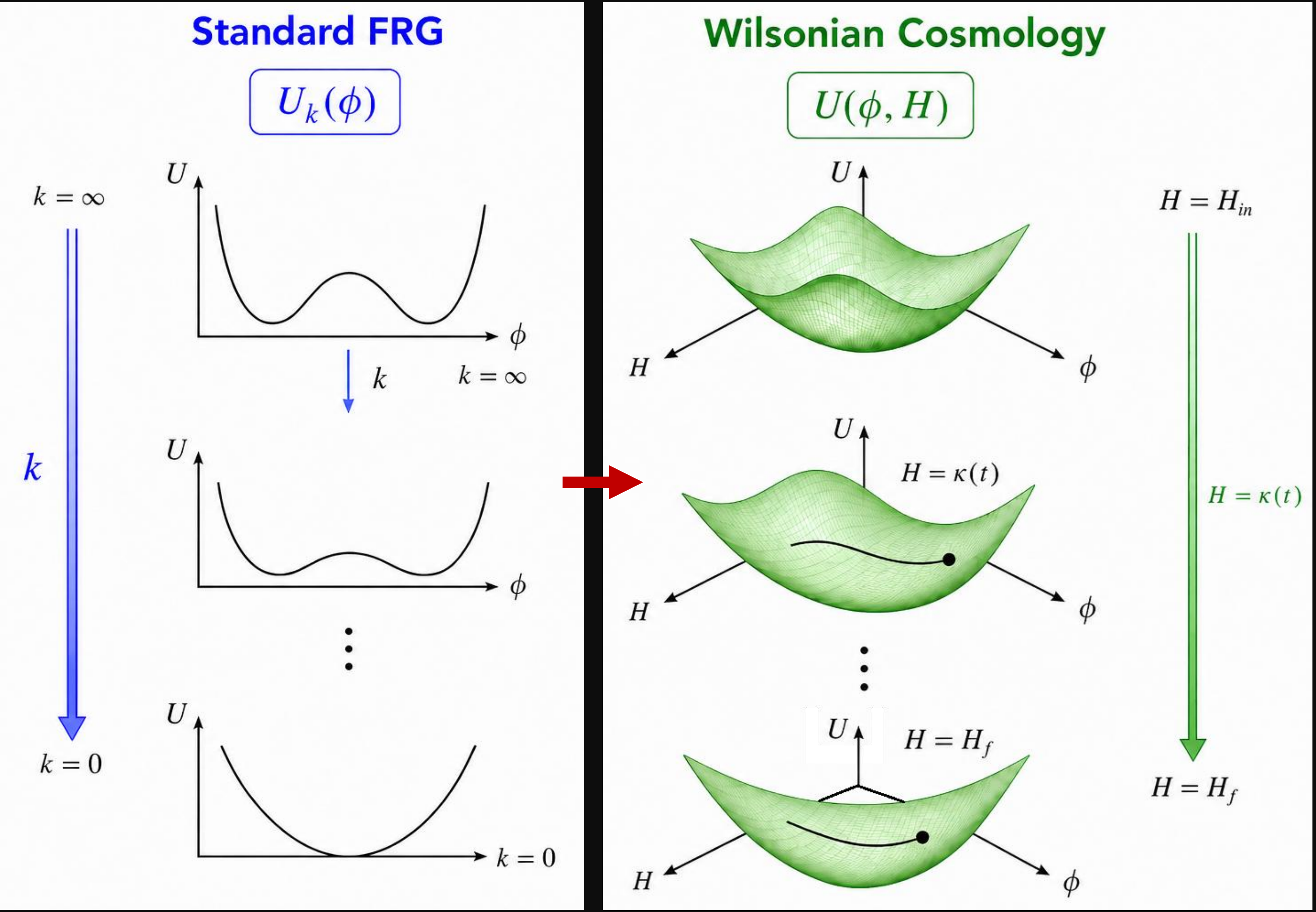
When Hubble (and the field) evolves, the theory must be described by

$$U(\phi, H) =$$
$$U_0 + g_1(H)\,\phi + g_2(H)\phi^2 + g_3(H)\phi^3 + \cdots$$

Standard FRG

$$\boxed{U_k(\phi)}$$




Standard FRG
$U_k(\phi)$
$k=\infty$
k
$k=0$
U
ϕ
k
$k=\infty$
U
ϕ
U
$k=0$
Wilsonian Cosmology
$U(\phi, H)$
U
H
ϕ
$H=H_{in}$
U
$H=\kappa(t)$
H
ϕ
$H=\kappa(t)$
U
$H=H_f$
H
ϕ
$H=H_f$

$$ds^2 = -dt^2 + a^2(t)(d\vec{x})^2$$

FLRW metric

$$\Phi = \phi(t) + \varphi(t, \vec{x})$$

Background + Fluctuation

Time-dep. cutoff

Coarse-grain over spatial slices only

$$S_k[\Phi] = \int dt \sqrt{-g} \int d^3x \left(-\frac{g^{\mu\nu}}{2} \partial_\mu \Phi \partial_\nu \Phi - \frac{\xi}{2} R \Phi^2 - U_i(\Phi) \right) \boxed{-\frac{1}{2} \int dt \sqrt{-g} \int_p \varphi(t, \vec{p}) \varphi(t, -\vec{p}) \left(C_k(p^2) - i\varepsilon \right)},$$

Partition function

$$Z_k[j] \equiv \exp\left(iW_k[j]/\hbar\right) = \int \mathcal{D}[\varphi] \exp\left((i/\hbar) S_k[\Phi] + (i/\hbar) \int dt \sqrt{-g} \int d^3x \; j\varphi \right)$$

Effective action

$$\Gamma_k[\phi + \varphi_b] = W_k[j] - \int dt' \sqrt{-g} \int d^3x \; j\varphi_b \boxed{+\frac{1}{2} \int dt' \sqrt{-g} \int_p \varphi_b(t', \vec{p}) \varphi_b(t', -\vec{p})(C_k - i\varepsilon)},$$

1) Functional derivative wrt the cutoff function

$$\frac{\delta\Gamma_k}{\delta k(t)} = i\hbar\frac{\sqrt{-g}}{2}\int_p\int_q \delta(\vec{p}+\vec{q})\int dt'\delta(t-t') \times \left(\sqrt{-g(t)}(C_k - i\varepsilon)\delta(\vec{p}+\vec{q})\delta(t-t') - \frac{\delta^2\Gamma_k}{\delta\varphi_b(t,\vec{p})\delta\varphi_b(t',\vec{q})}\right)^{-1}\partial_k C_k$$

2) Local Potential Approximation (LPA)

$$\Gamma_k[\Phi] = \int dt\sqrt{-g}\int d^3x\left(-\frac{g^{\mu\nu}}{2}\partial_\mu\Phi\partial_\nu\Phi - \frac{\xi}{2}R\Phi^2 - U_k(\Phi)\right)$$

3) Flow Equation

$$\boxed{\partial_k U_k(\phi) = \hbar\frac{T^{-1}}{2}\int_p \mathcal{D}_E^{-1}\left(a^{-3}\partial_k C_k\right)}$$

where

$$\mathcal{D}_E = -\frac{d^2}{dt^2} - 3H\frac{d}{dt} + \kappa^2 - \xi R + \partial_\phi^2 U_\kappa(\phi)$$

+ Litim Regulator

$$C_k(p^2) = a^{-2}(k^2 - p^2)\Theta(k^2 - p^2)$$

$$\kappa \equiv k/a$$

$$\boxed{\partial_\kappa U_\kappa(\phi) = \hbar \frac{aT^{-1}}{6\pi^2} \mathcal{D}_E^{-1} \left(a^{-1}\kappa^4\right)}$$

$$\mathcal{D}_E = -\frac{d^2}{dt^2} - 3H\frac{d}{dt} + \kappa^2 - \xi R + \partial_\phi^2 U_\kappa(\phi)$$

+ Fix frequency cutoff in the adiabatic limit

$$H = 0 \text{ and } \dot{\kappa} = \dot{\phi} = \dot{T} = 0$$

$$\partial_\kappa U_\kappa(\phi) = \frac{T^{-1}}{6\pi^2} \frac{\hbar\kappa^4}{\kappa^2 + \partial_\phi^2 U_\kappa(\phi)}$$

Identify T^{-1} with $3\kappa/16$

+ Wick rotate

+ Inverse operator

$$\boxed{\mathcal{D}\left(T\partial_\kappa U_\kappa(\phi)\right) = \frac{\hbar\kappa^4}{6\pi^2}}$$

$$\begin{aligned} \mathcal{D} &\equiv \frac{d^2}{dt^2} + H\frac{d}{dt} + \kappa^2 + \mathcal{M}^2 \\ \mathcal{M}^2 &\equiv \partial_\phi^2 U_\kappa(\phi) + (\xi - 1/6)R \ , \end{aligned}$$

+ Hubble size Coarse-Graining

$$\kappa = H$$

$$\ddot{X} + H\dot{X} + \left(H^2 + \partial_\phi^2 U - \frac{R}{6}\right) X = \frac{\hbar H^4}{32\pi^2} \ ,$$

with

$$X \equiv H^{-1}\partial_H U \ ,$$

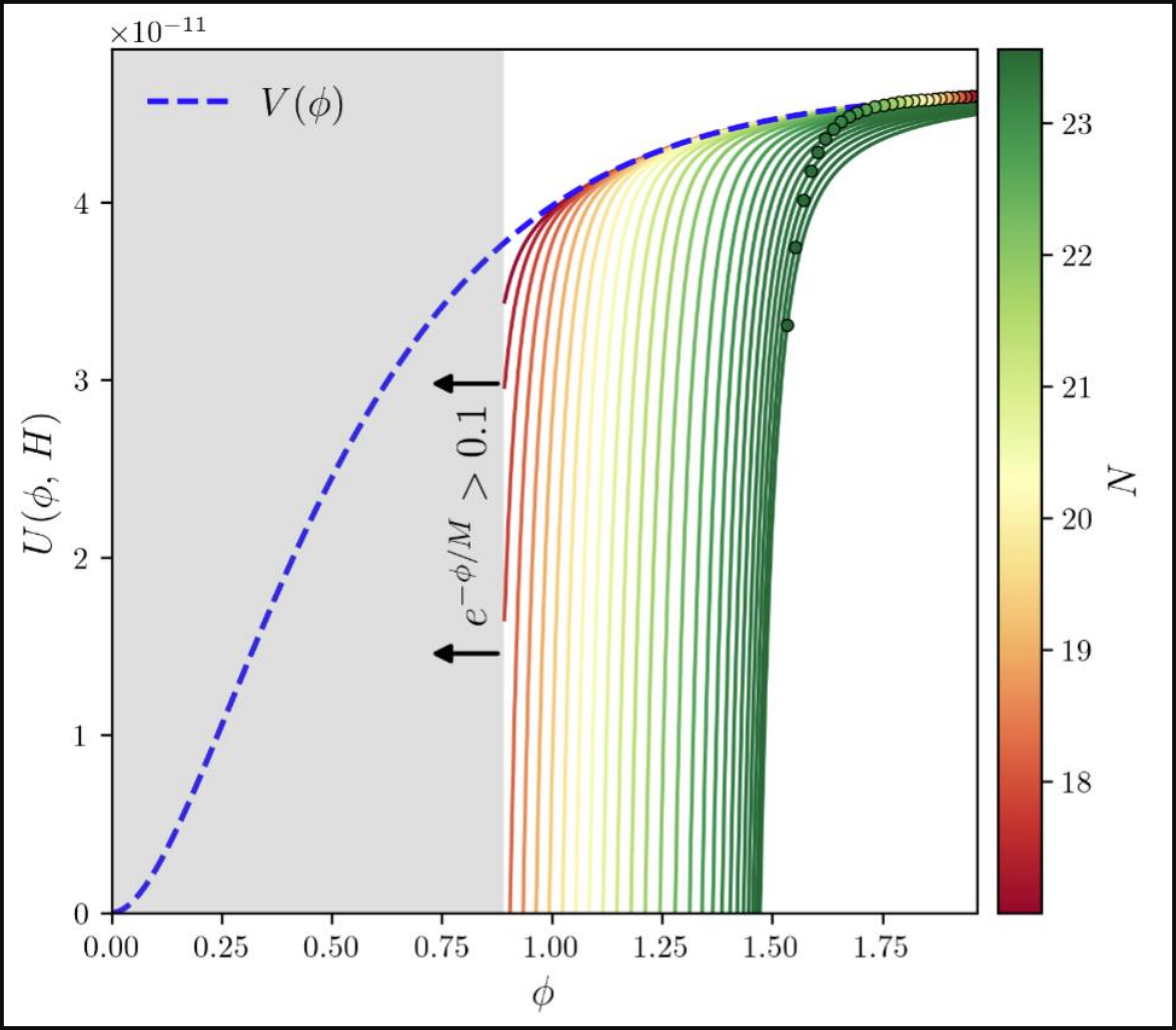
+ Einstein equations

With U = U(phi,H)

$$X'' = \frac{\hbar H^2}{32\pi^2} - \left(1 + \frac{H'}{H}\right) X' - \left(\frac{\partial_\phi^2 U}{H^2} - \frac{H'}{H} - 1\right) X \,,$$

$$\phi'' = -\left(3 + \frac{H'}{H}\right)\phi' - \frac{\partial_\phi U}{H^2} \,,$$

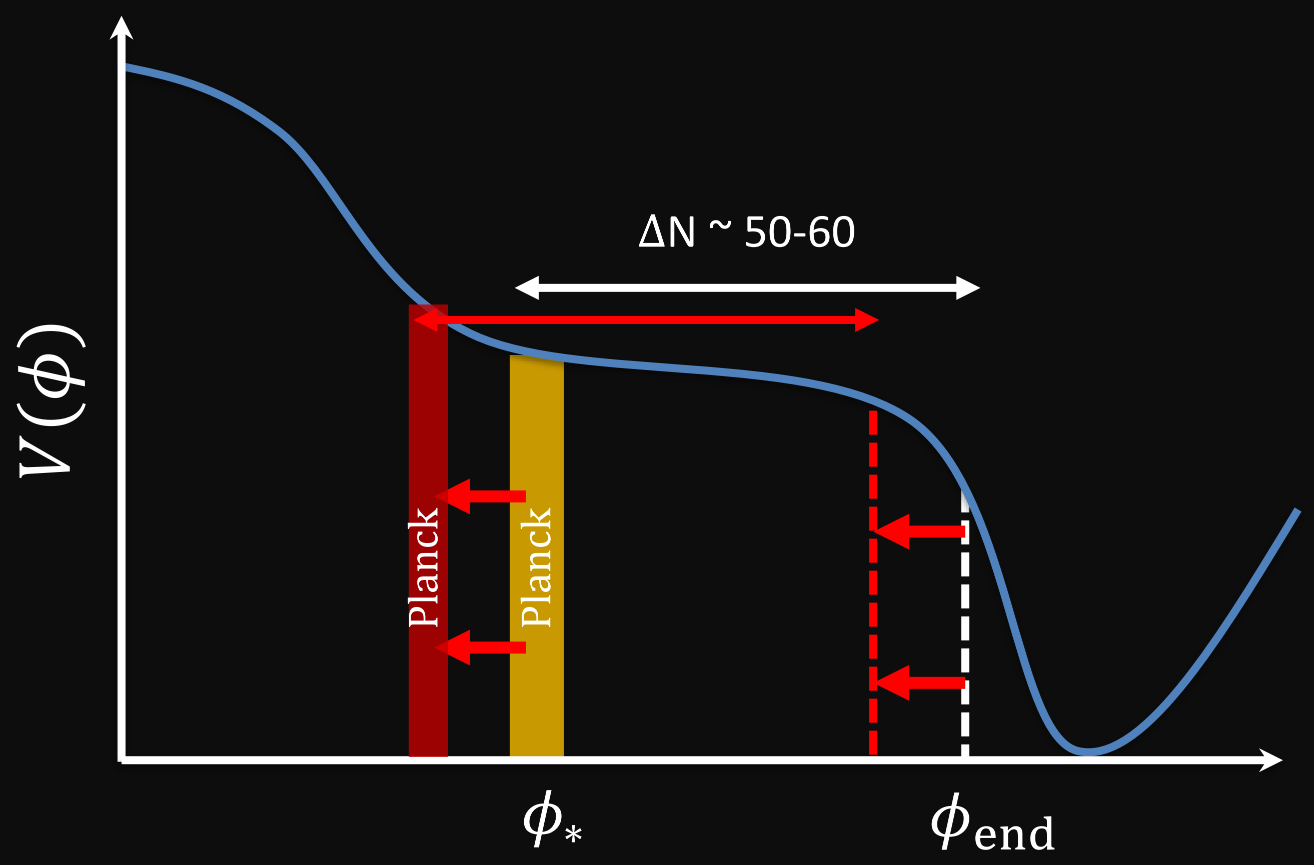
$$H' = -\frac{1}{2M_p^2}\left[H(\phi')^2 + \frac{1}{3}(HX)'\right] \,.$$



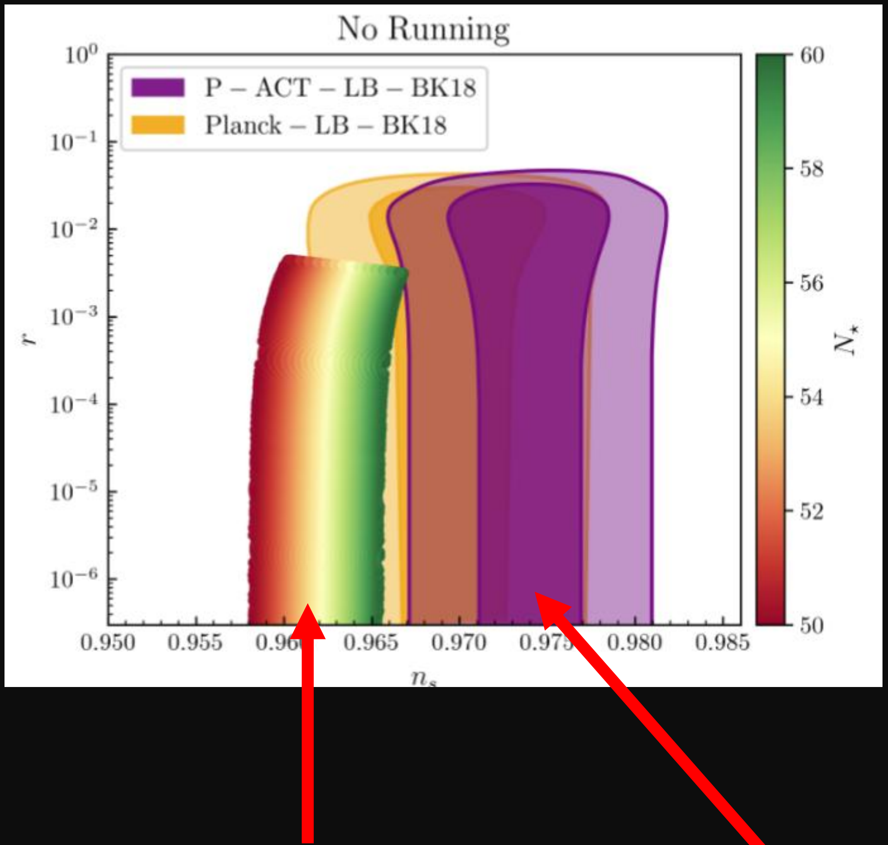
ArXiv:2511.05296 – J. Alexandre, LH, S. Pla

CMB measurements

Planck:	$k \sim 0.05\ \mathrm{Mpc}^{-1}$



Effect on CMB observables

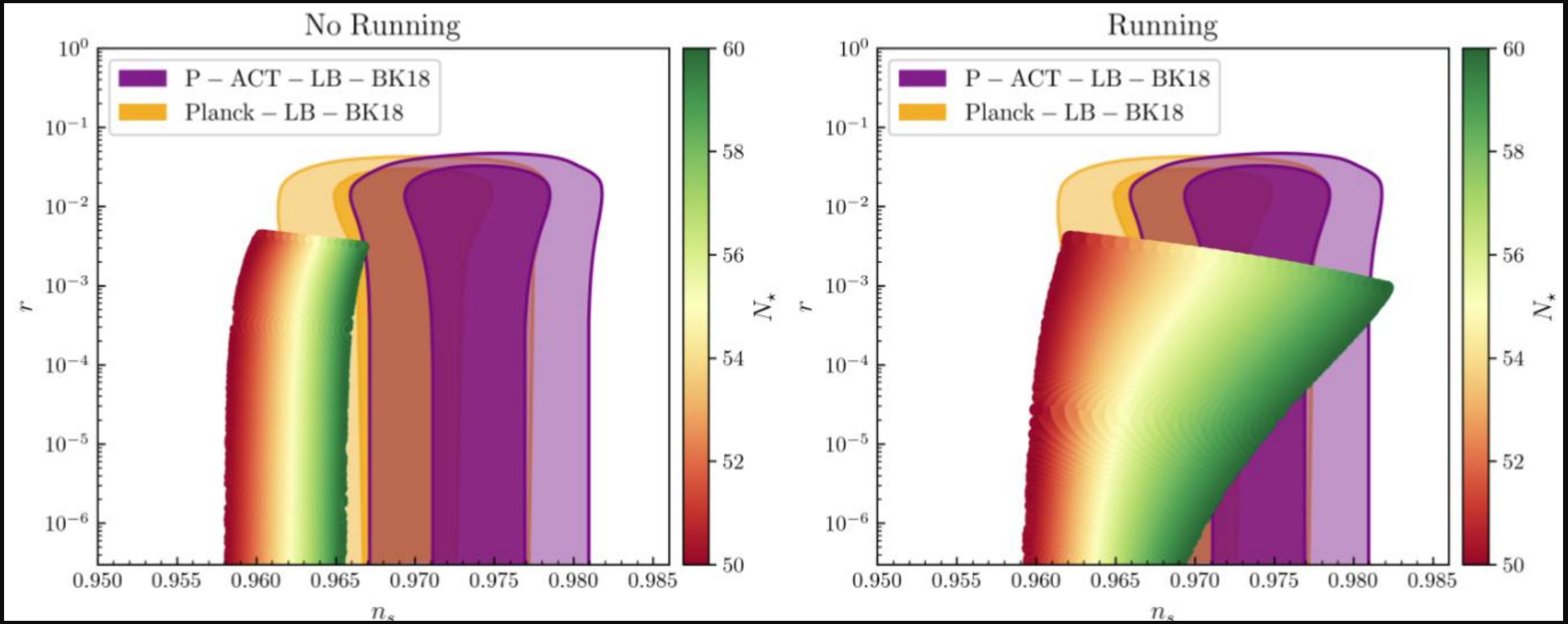


A popular model prediction

The Data

ArXiv:2511.05296 – J. Alexandre, LH, S. Pla

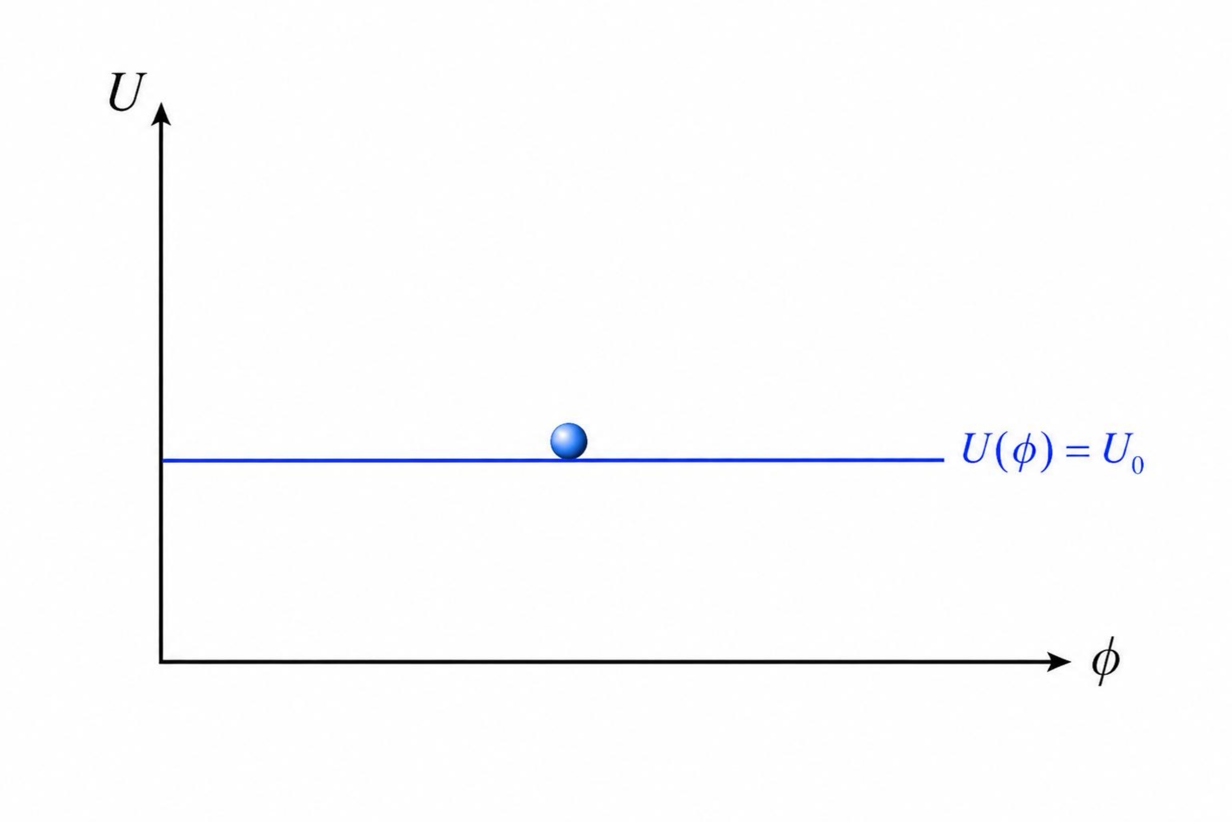
Effect on CMB observables



Reduced number of e-folds → Shift to larger n_s

ArXiv:2511.05296 – J. Alexandre, LH, S. Pla

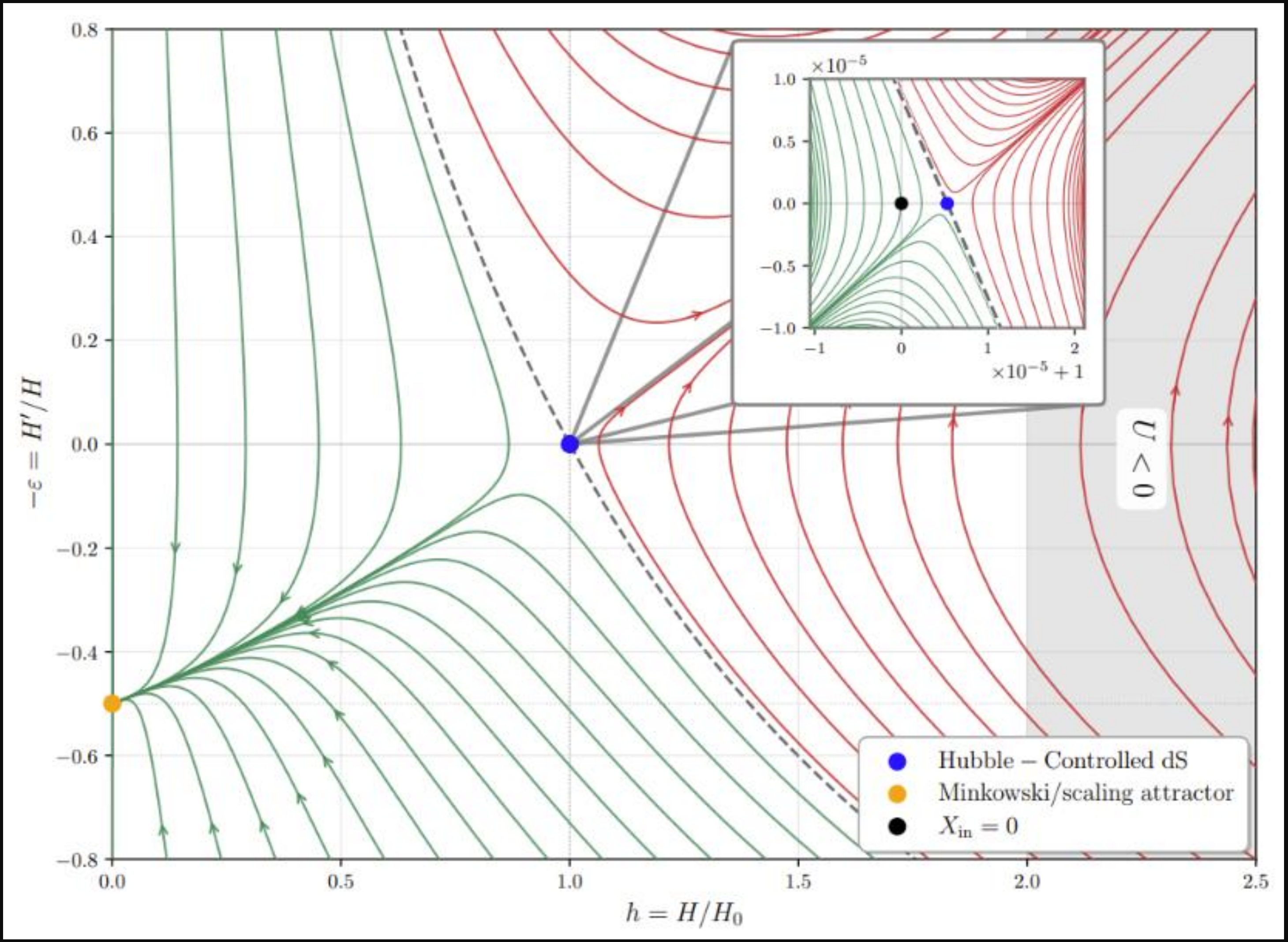
Effect on de Sitter solutions



Flat potential (CC):

ArXiv:2608.23676 – J. Alexandre, LH, S. Pla

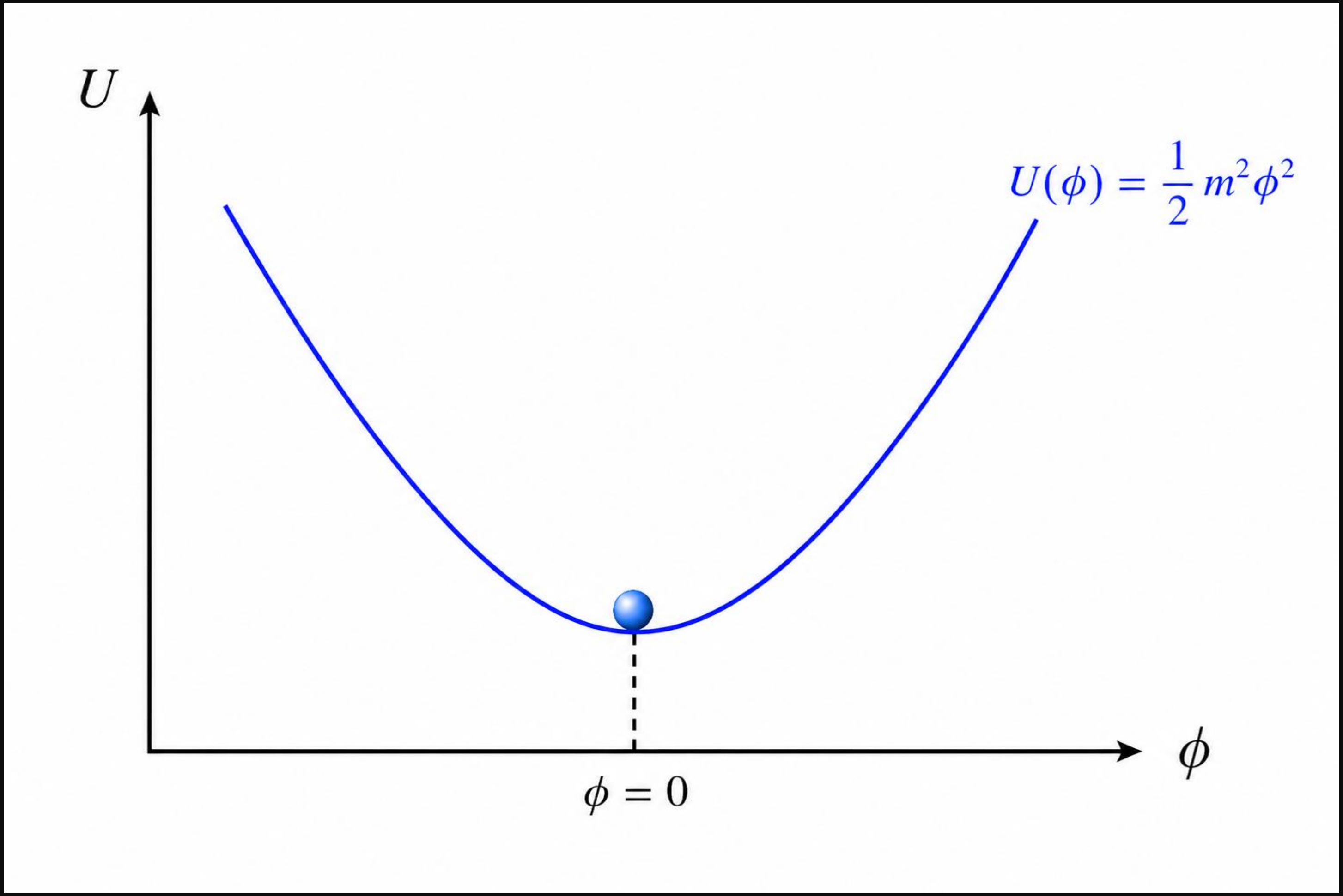
Effect on de Sitter solutions



Flat potential (CC): de Sitter is a saddle

ArXiv:2608.23676 – J. Alexandre, LH, S. Pla

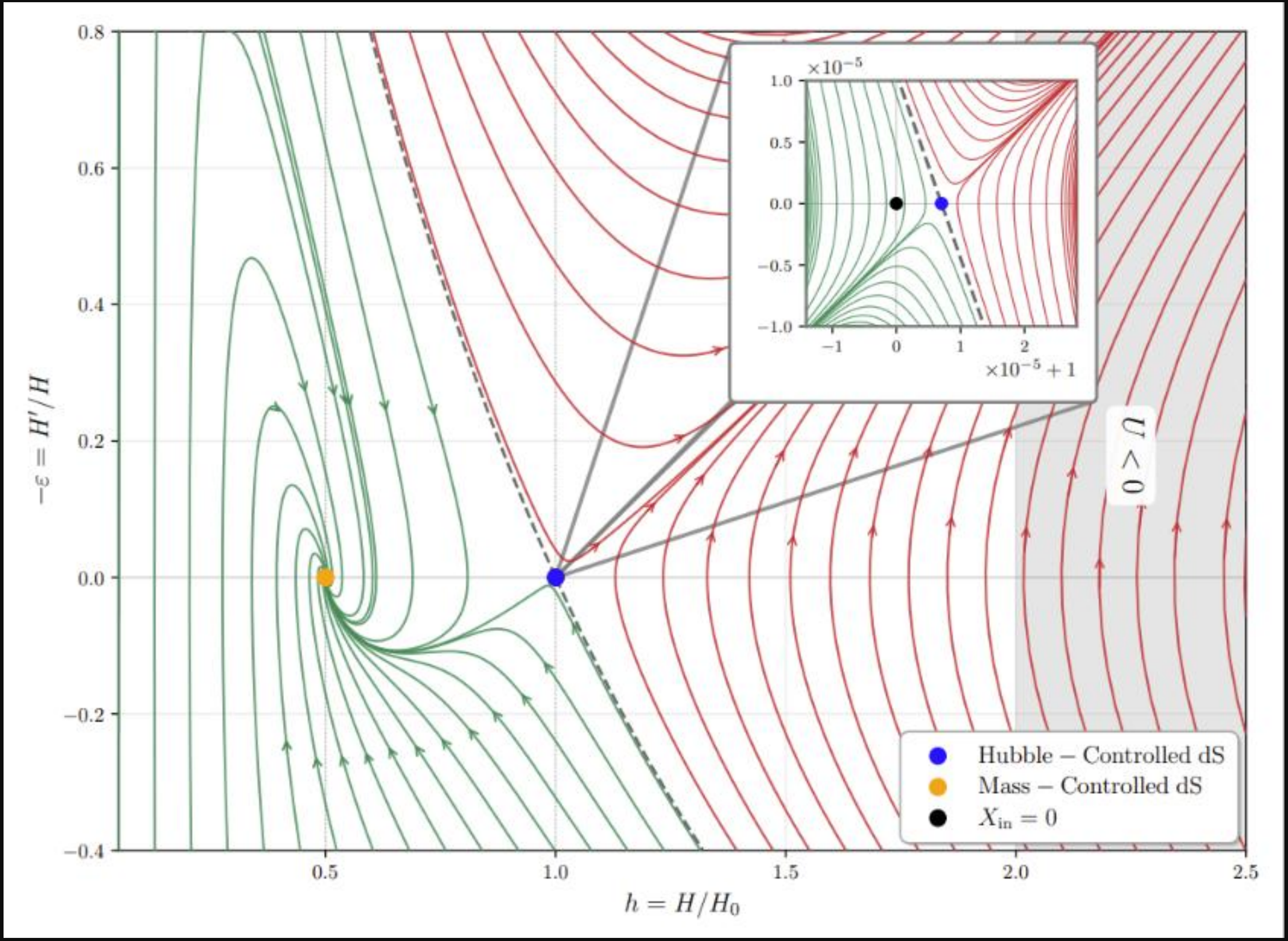
Effect on de Sitter solutions



Quadratic potential

ArXiv:2608.23676 – J. Alexandre, LH, S. Pla

Effect on de Sitter solutions

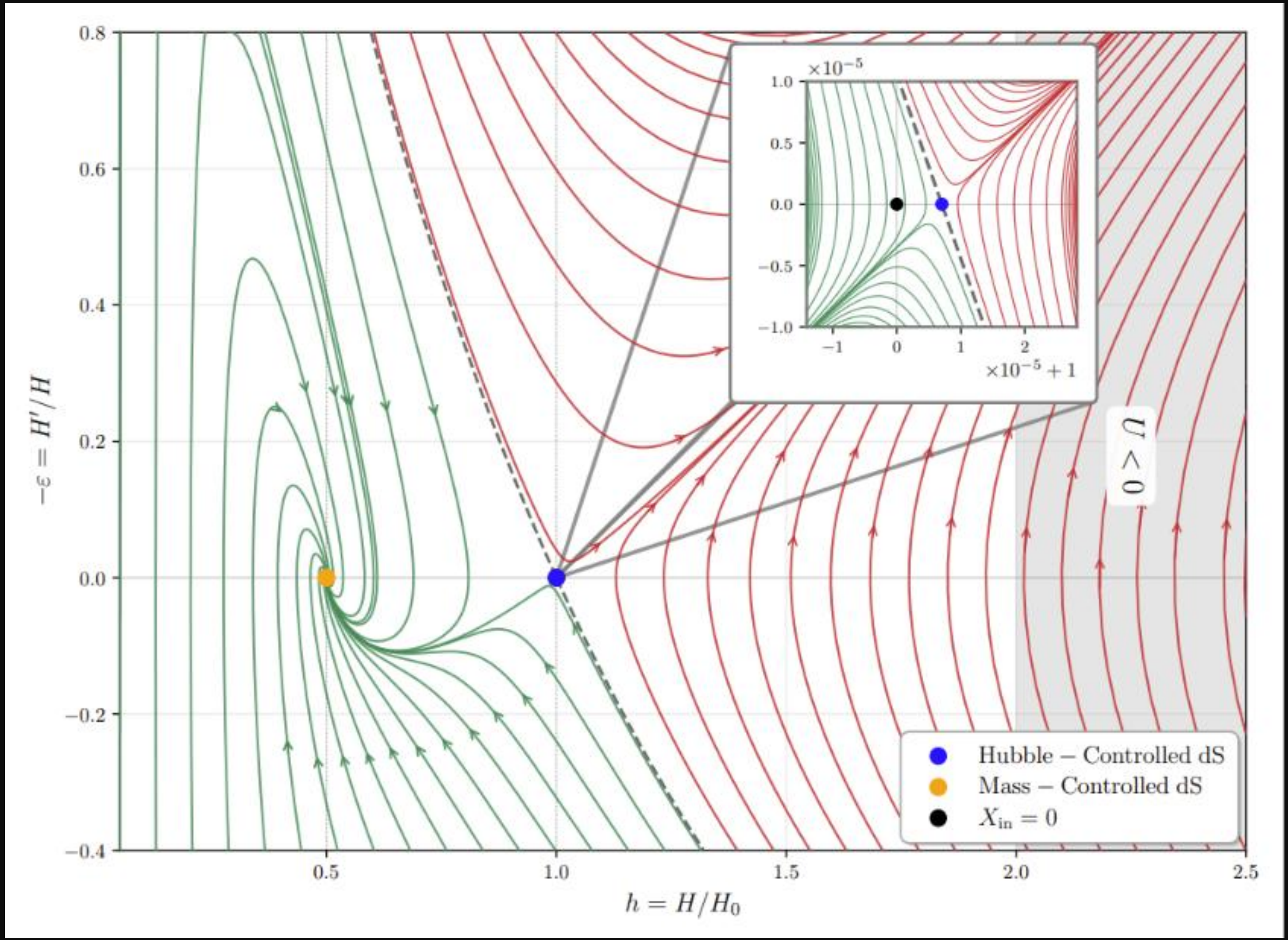


Quadratic potential (Hin > m): de Sitter is a saddle

+ new attractor (H ~ m)

ArXiv:2606.23676 – J. Alexandre, LH, S. Pla

Effect on de Sitter solutions



Quadratic potential (Hin < m): de Sitter is an attractor

+ new saddle (H ~ m)

ArXiv:2608.23676 – J. Alexandre, LH, S. Pla

Many more avenues to be explored:

Analytical: Metric fluctuations, spinodal instability, initial conditions vs boundary conditions

Phenomenological: Axion DM, dark energy dynamics, cosmological constant problem, bouncing cosmologies, etc.

Thank You!

1PI Effective Action

Classical Theory

$V(\phi)$

RG Improved

$V(\phi) + \Delta V(\phi)$

Markannen *et al*, '18

Ellis *et al*, '25

$$\Delta V = \frac{1}{64\pi^2}\left[\mathcal{M}_\phi^4\left(\log\frac{|\mathcal{M}_\phi^2|}{\mu^2} - \frac{3}{2}\right) + 2b_s \log\frac{|\mathcal{M}_\phi^2|}{\mu^2}\right]$$

$$R = 6(2H^2 + \dot{H})$$

$$b_s = -\frac{1}{180}R_{\mu\nu}R^{\mu\nu} + \frac{1}{180}R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} \simeq -\frac{H^4}{15}$$

$$\mathcal{M}_\phi^2 = V''(\phi) + \frac{1}{6}R$$ [redacted]

1PI Effective Action

Classical Theory

$V(\phi)$ $\longrightarrow$ RG Improved $V(\phi) + \Delta V(\phi)$

Markannen *et al*, '18
Ellis *et al*, '25

Slow-roll

$$R = 6(2H^2 + \cancel{\dot{H}})$$

$$H \approx H(\phi)$$

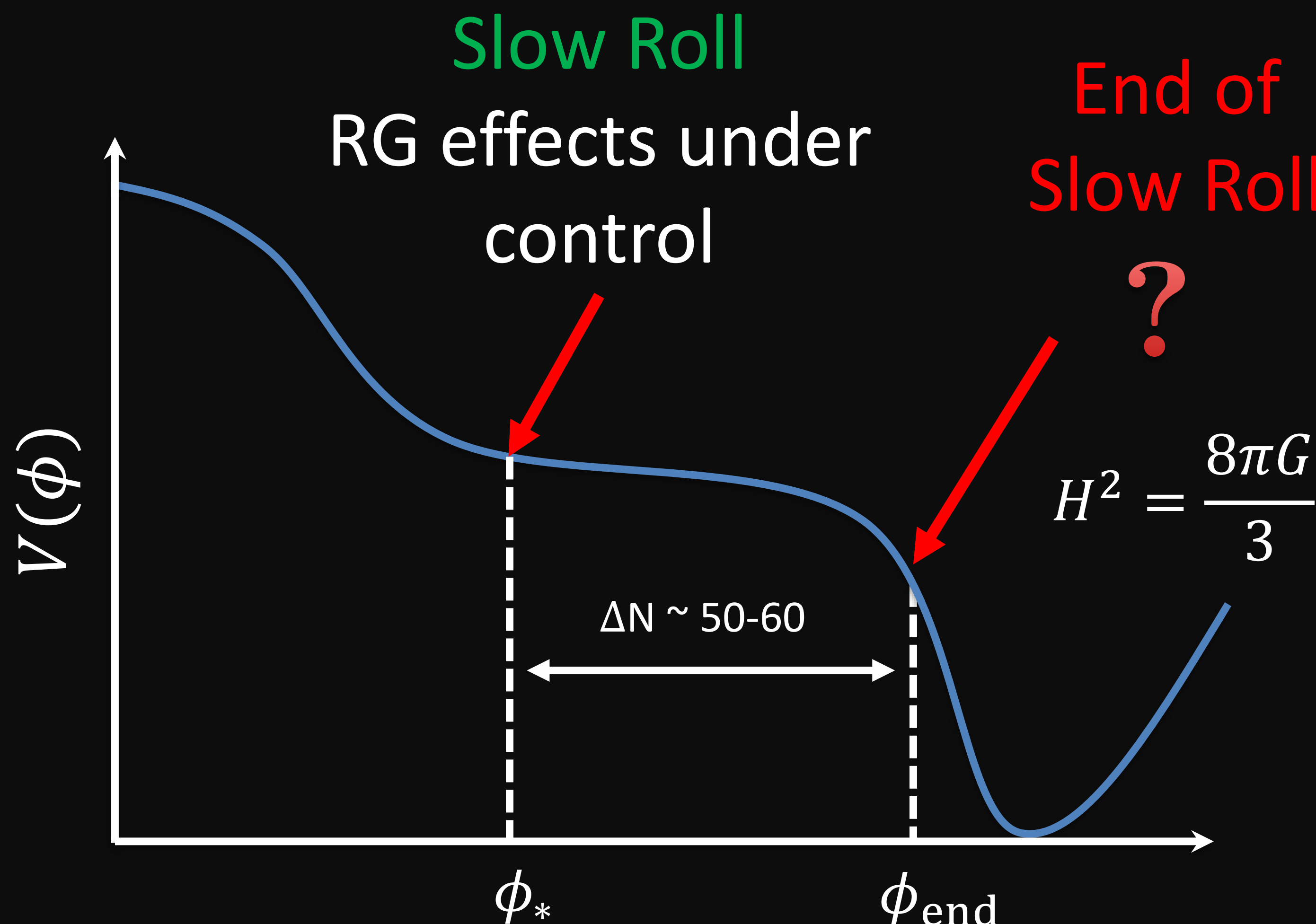
$$\Delta V = \frac{1}{64\pi^2}\left[\mathcal{M}_\phi^4\left(\log\frac{|\mathcal{M}_\phi^2|}{\mu^2} - \frac{3}{2}\right) + 2b_s \log\frac{|\mathcal{M}_\phi^2|}{\mu^2}\right]$$

$$b_s = -\frac{1}{180}R_{\mu\nu}R^{\mu\nu} + \frac{1}{180}R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} \simeq -\frac{H^4}{15}$$

$$\mathcal{M}_\phi^2 = V''(\phi) + \frac{1}{6}R \simeq \cancel{V''(\phi)} - 2H^2$$

Negligible Self-Int. $\Delta V \sim H^4 \sim 10^{-10} V$

Inflation Observables



$$H^2 = \frac{8\pi G}{3}\left(\frac{\dot{\phi}^2}{2} + V(\phi)\right)$$

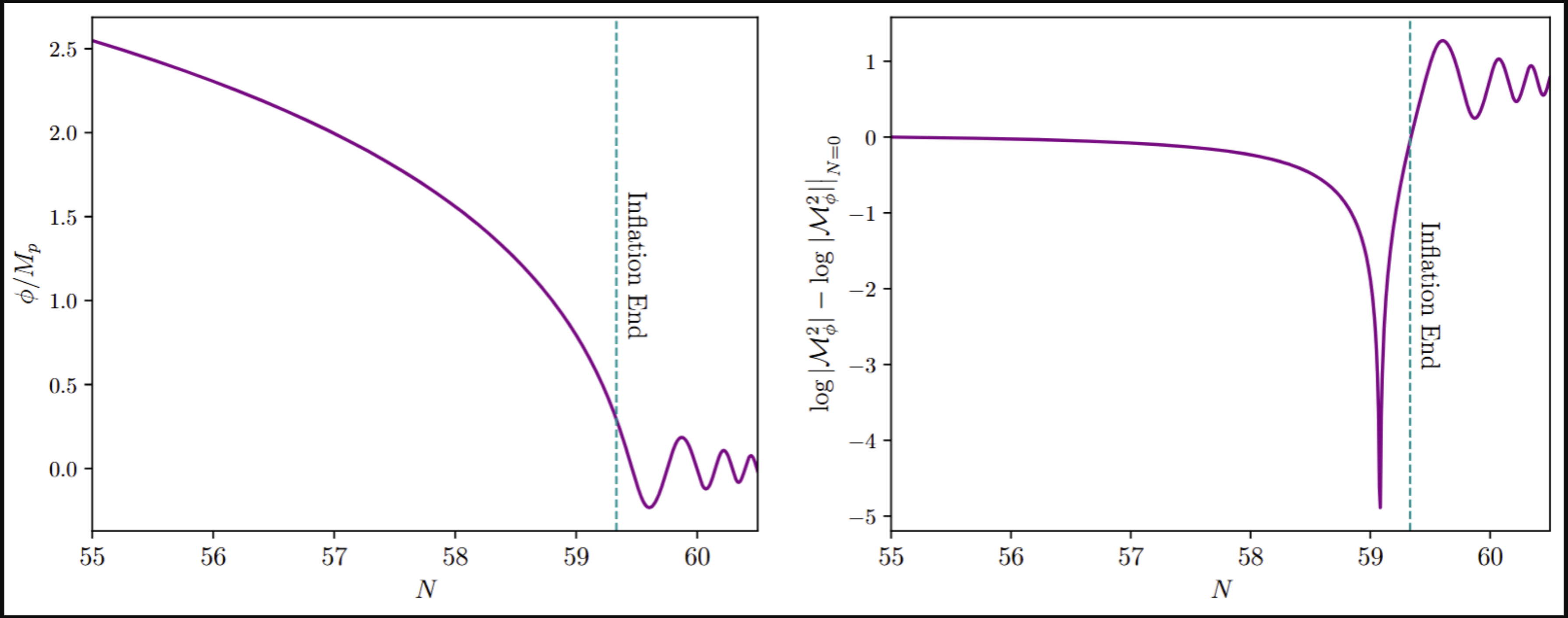
1PI Effective Action

Classical Theory

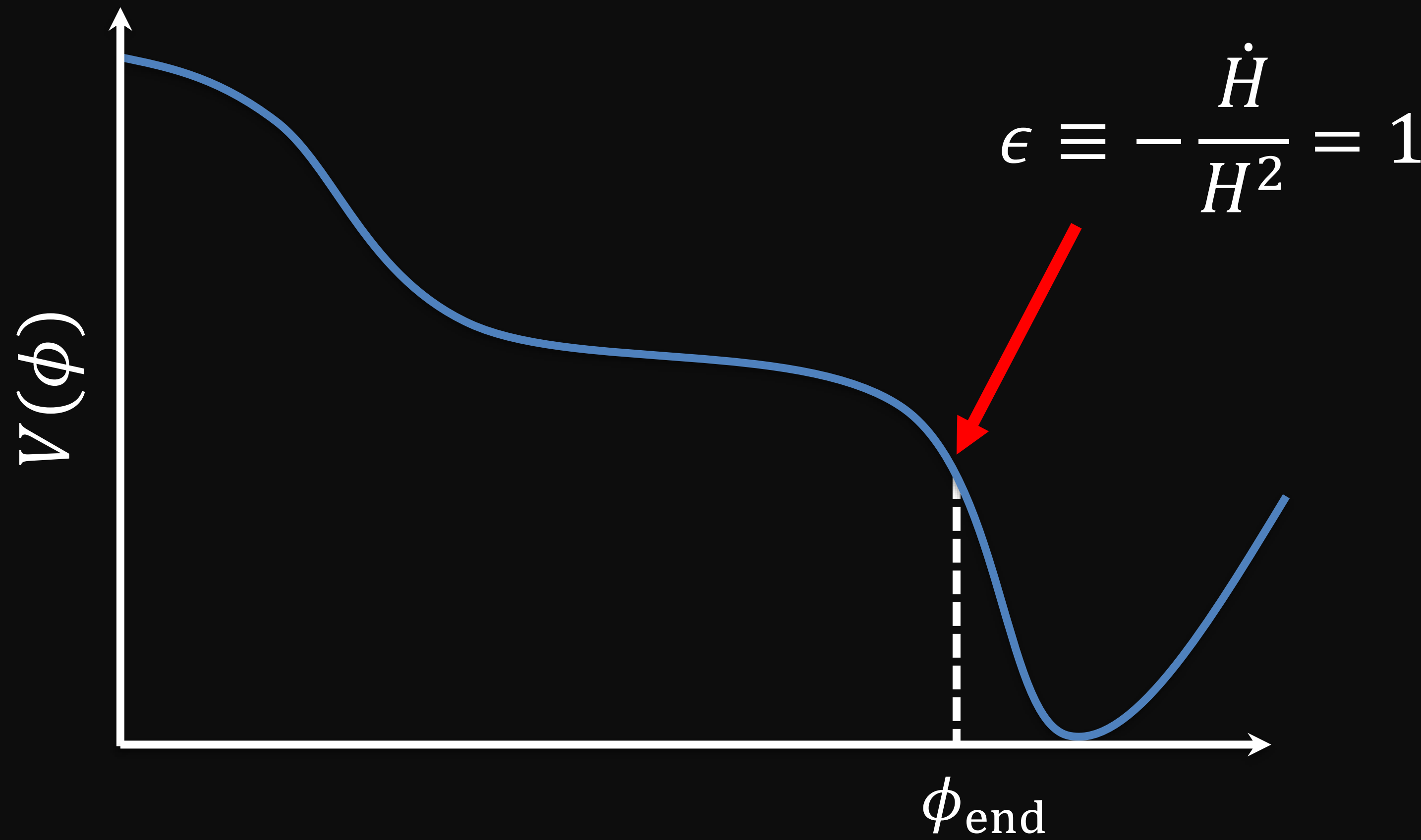
$$V(\phi)$$

RG Improved

$$V(\phi) + \Delta V(\phi)$$



Inflation Observables



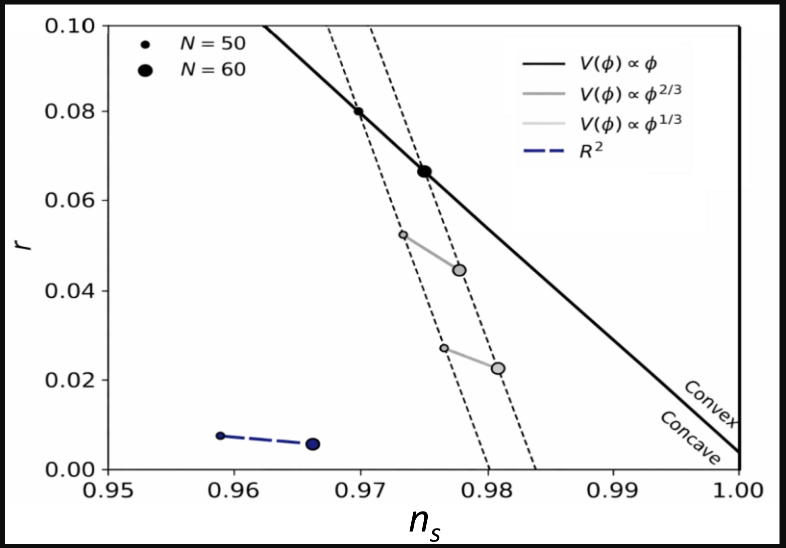
Inflation Observables

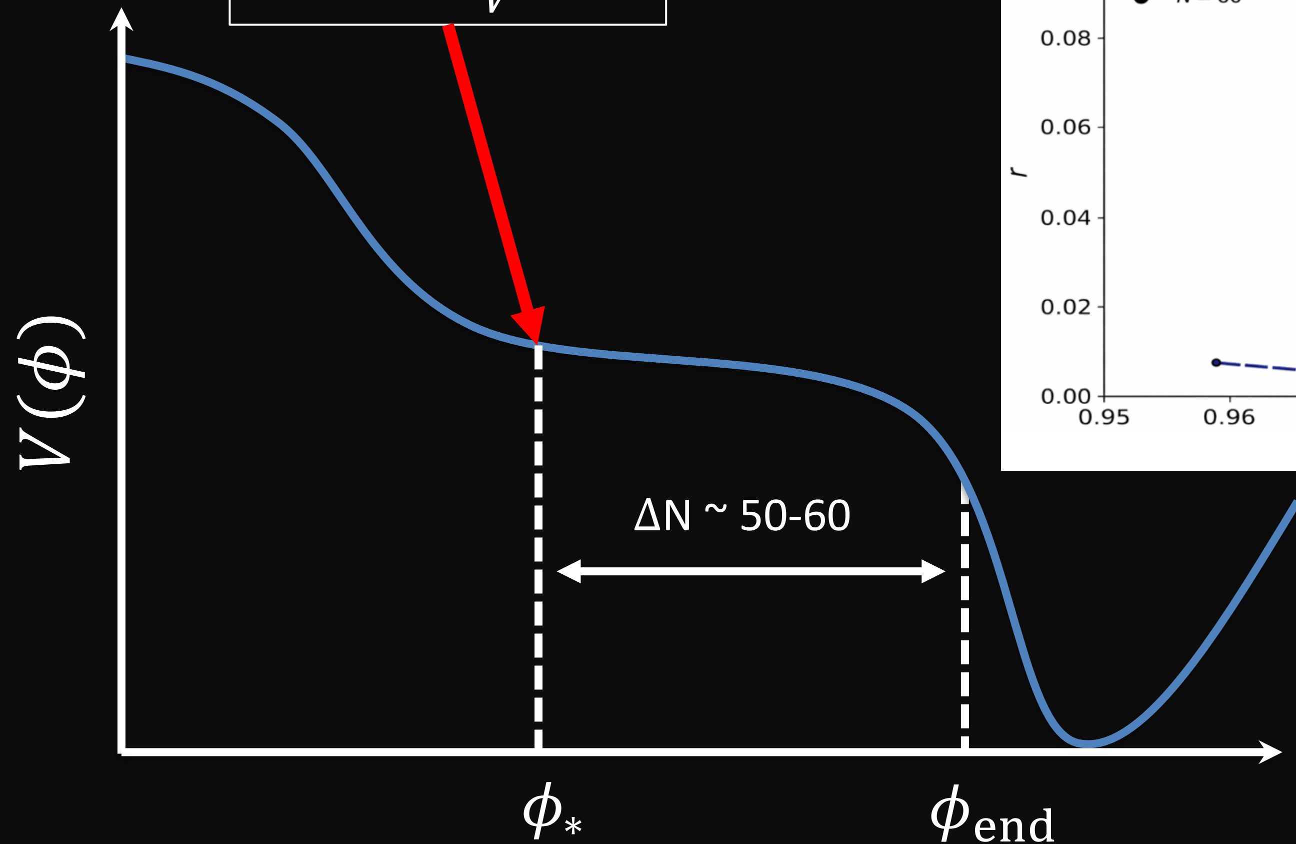
$$\epsilon_V \equiv \frac{M_p^2}{2}\frac{V'}{V}\,,$$

$$\eta_V \equiv \frac{M_p^2 V''}{V}$$

$$r = 16\,\epsilon_V,$$

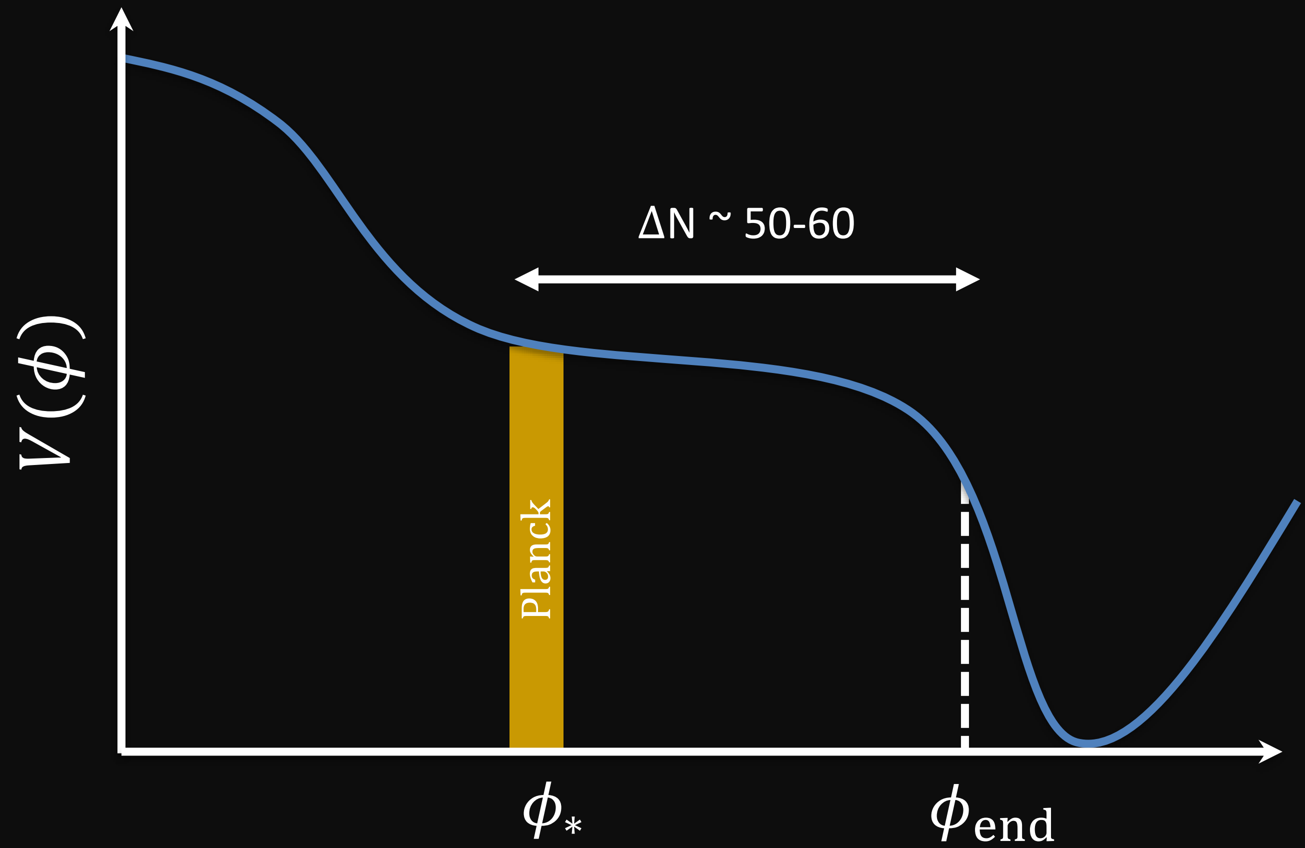
$$n_s = 1 - 6\epsilon_V + 2\eta_V$$





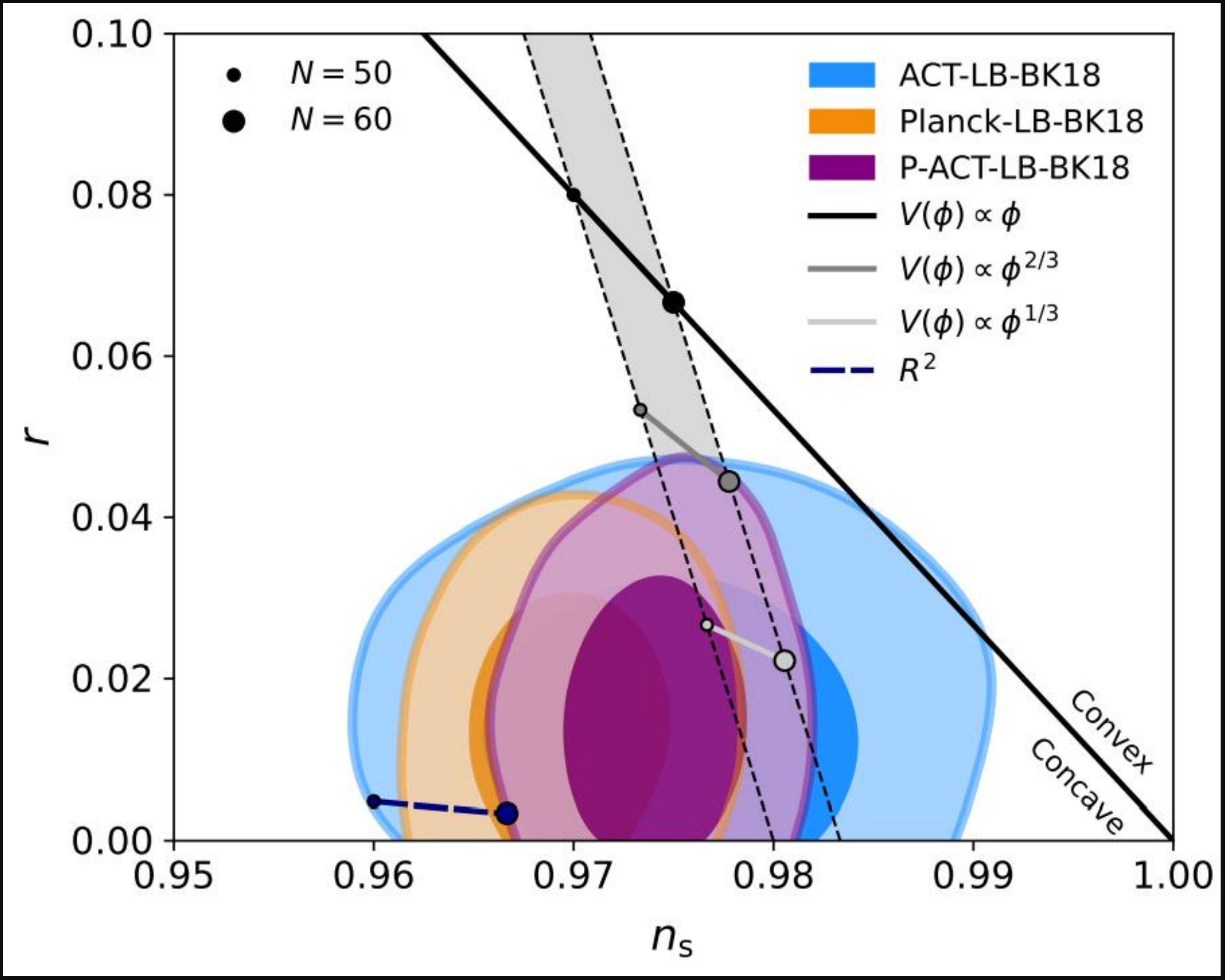
Inflation Model Building

Planck: $k \sim 0.05\ \mathrm{Mpc}^{-1}$



Spectral Index Running

ACT DR6, March 2025



What about

quantum corrections?

1PI Effective Action

Classical Theory

RG Improved

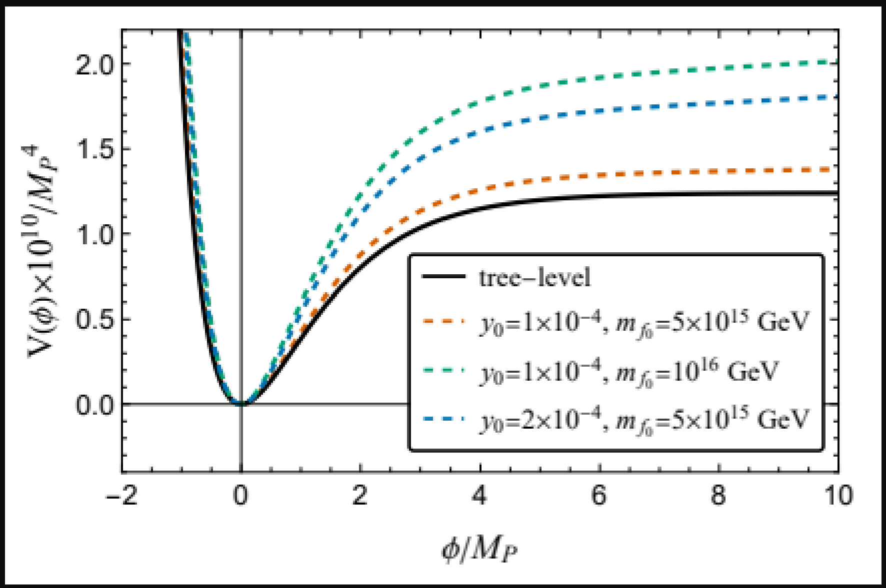
$$V(\phi) \longrightarrow V(\phi) + \Delta V(\phi)$$

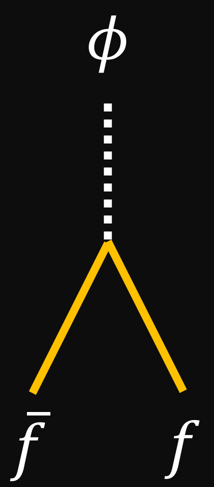
Markannen *et al*, '18
Ellis *et al*, '25

Slow-roll

$$\boxed{R = 6(2H^2 + \cancel{\dot{H}})}$$

$$H \approx H(\phi)$$





1PI Effective Action

Classical Theory

RG Improved

$$V(\phi) \longrightarrow V(\phi) + \Delta V(\phi)$$

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$$\Delta V = \frac{1}{64\pi^2}\left[\mathcal{M}_\phi^4\left(\log\frac{|\mathcal{M}_\phi^2|}{\mu^2} - \frac{3}{2}\right) + 2b_s \log\frac{|\mathcal{M}_\phi^2|}{\mu^2}\right]$$

$$R = 6(2H^2 + \dot{H})$$

$$b_s = -\frac{1}{180}R_{\mu\nu}R^{\mu\nu} + \frac{1}{180}R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$$ [redacted]

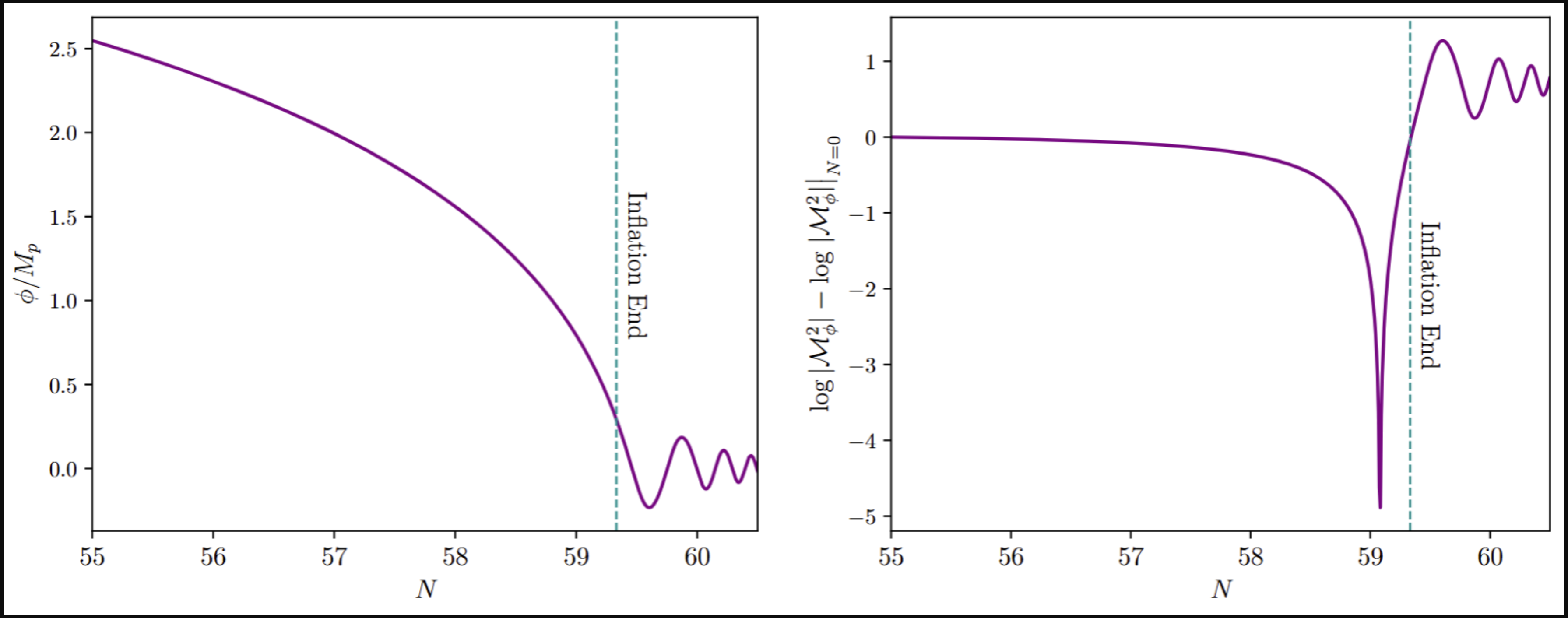
$$\mathcal{M}_\phi^2 = V''(\phi) + \frac{1}{6}R$$ [redacted]

1PI Effective Action

Classical Theory

RG Improved

$$V(\phi) \rightarrow V(\phi) + \Delta V(\phi)$$



RG running of inflation

Bare potential $\boxed{V(\phi)} \longrightarrow$ RG-Improved potential $\boxed{U(\phi)}$

Static field: $\boxed{\phi \sim \text{cst.}} \longrightarrow \boxed{H = H(\phi)} \longrightarrow \boxed{U = V_{RGI}(\phi)}$

Valid in the deep slow-roll era only

Rolling field: $\boxed{\phi(t), \dot{\phi}(t)} \longrightarrow \boxed{\phi(t), H(t)} \longrightarrow \boxed{U = U(\phi, H)}$

Allows for deviations from slow-roll