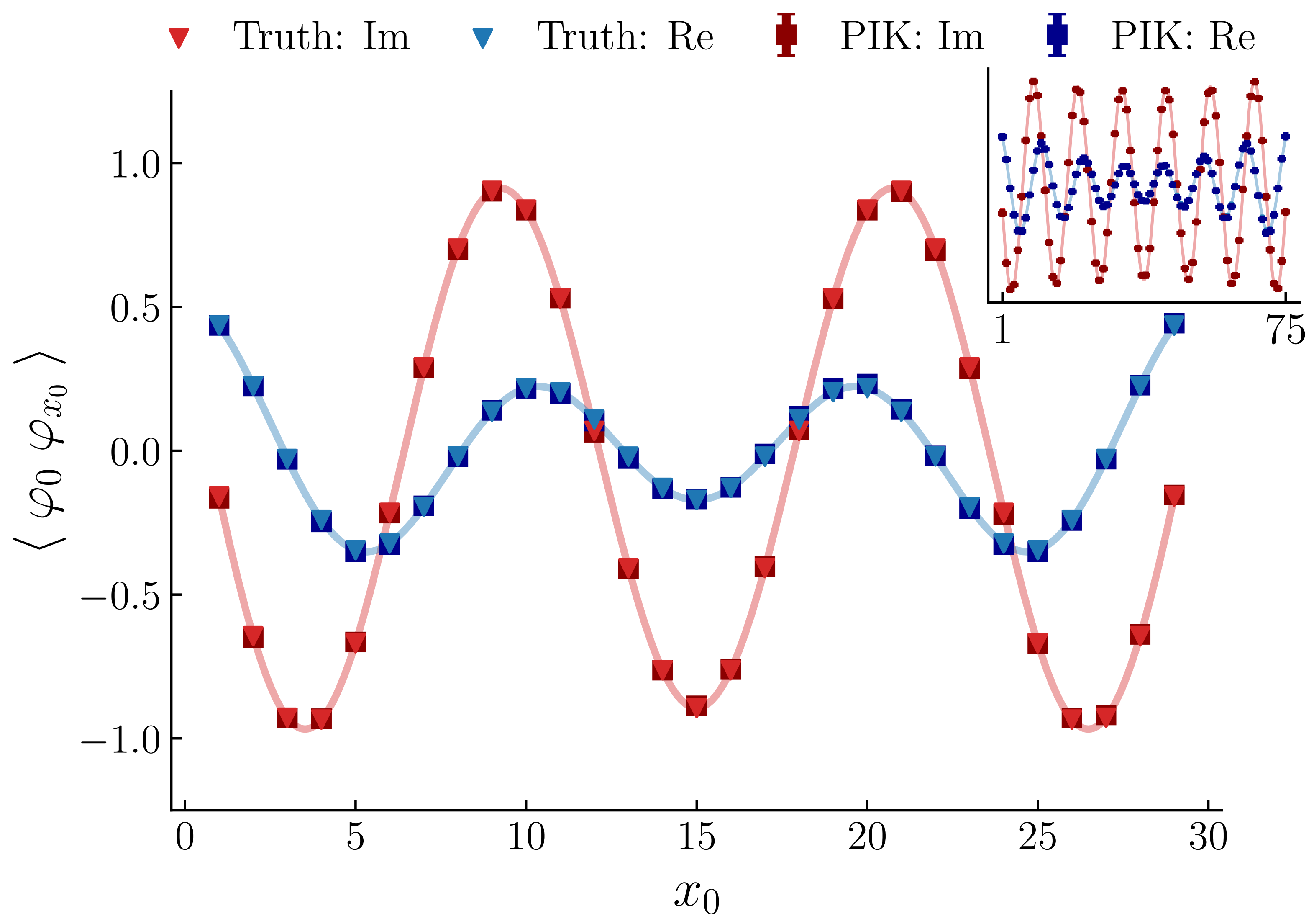
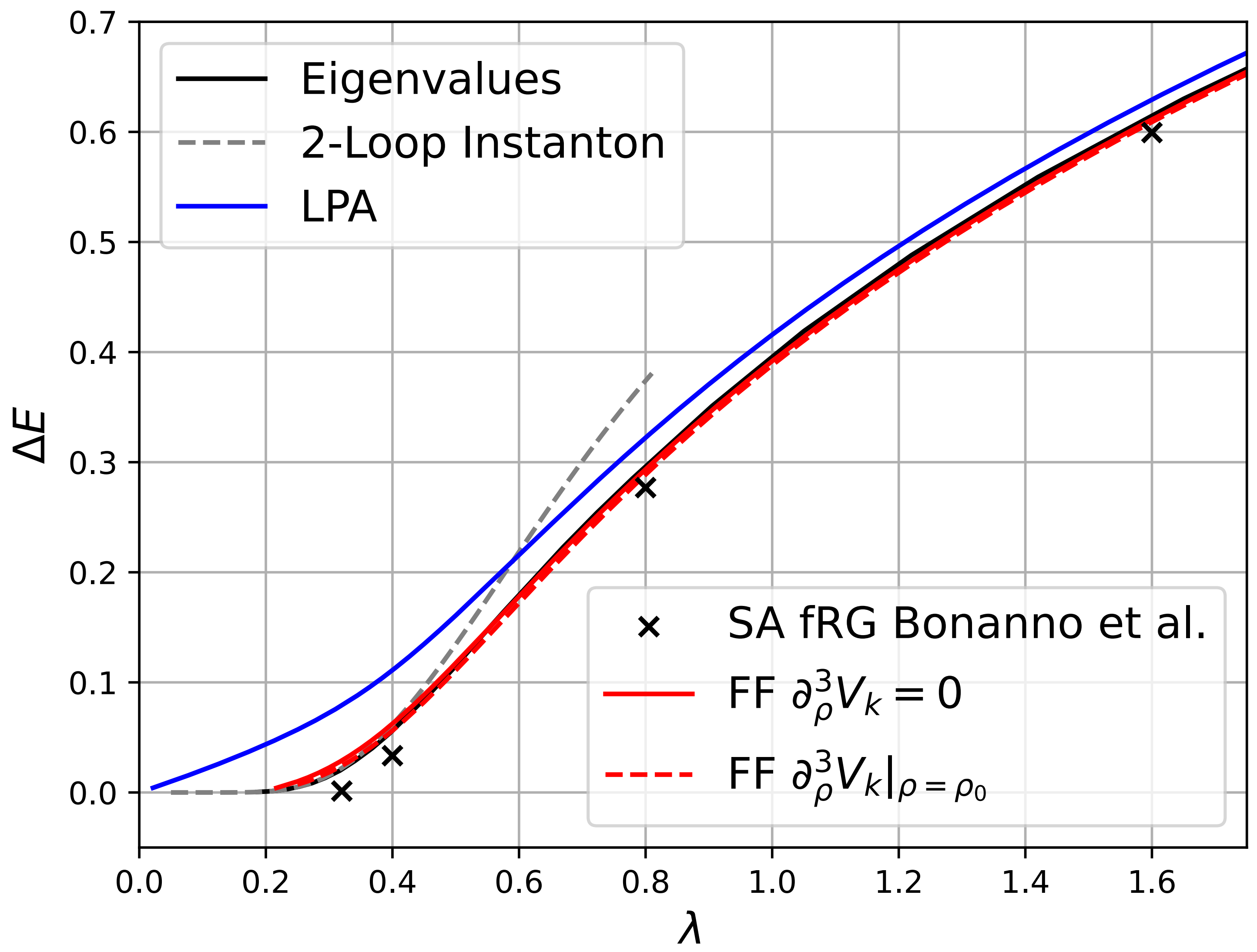
Applications of the Physics-Informed Renormalisation Group

From sampling to optimisation





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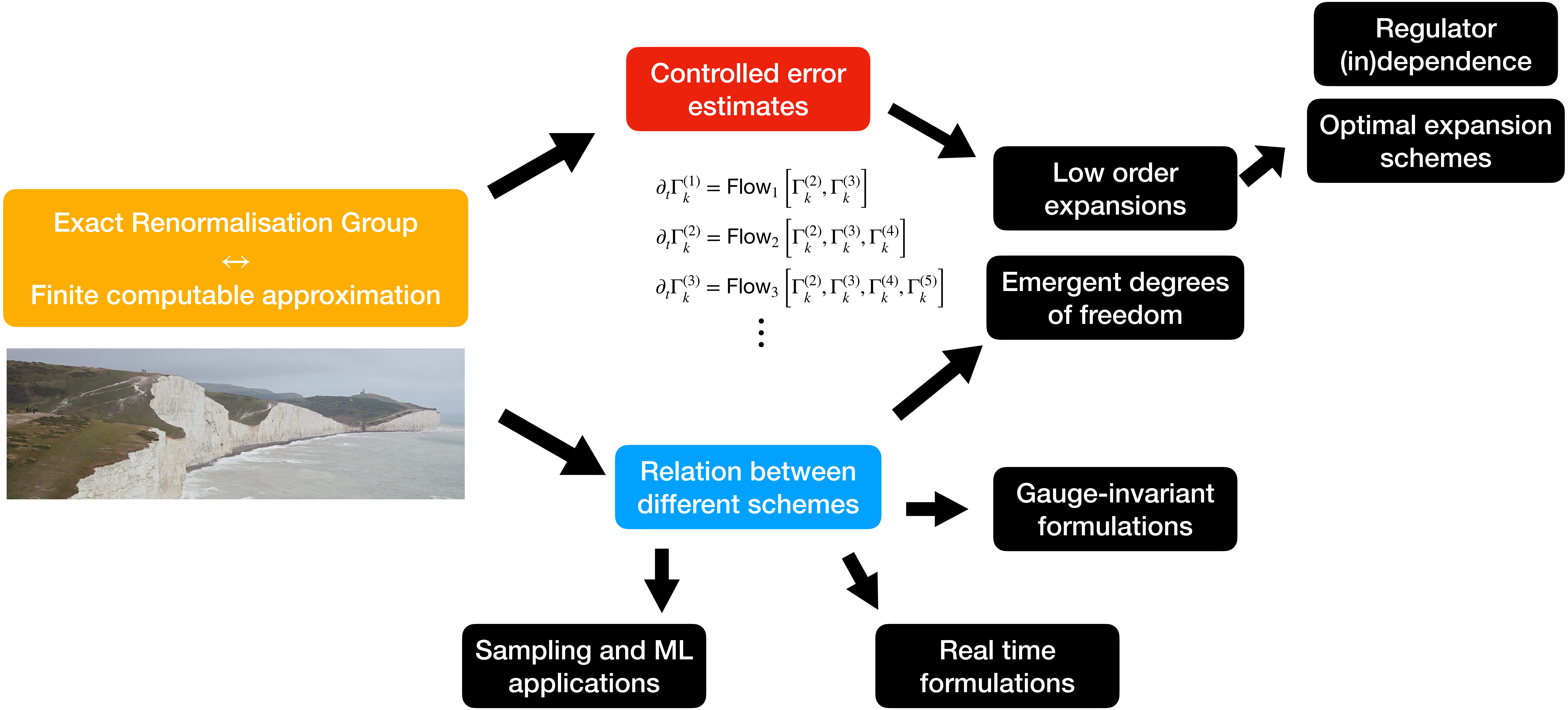
05.09 - ERG 2026



Exact Renormalisation Group
$\leftrightarrow$
Finite computable approximation
Controlled error estimates
$\partial_t \Gamma_k^{(1)} = \mathrm{Flow}_1\left[\Gamma_k^{(2)}, \Gamma_k^{(3)}\right]$
$\partial_t \Gamma_k^{(2)} = \mathrm{Flow}_2\left[\Gamma_k^{(2)}, \Gamma_k^{(3)}, \Gamma_k^{(4)}\right]$
$\partial_t \Gamma_k^{(3)} = \mathrm{Flow}_3\left[\Gamma_k^{(2)}, \Gamma_k^{(3)}, \Gamma_k^{(4)}, \Gamma_k^{(5)}\right]$
$\vdots$
Relation between different schemes
Low order expansions
Emergent degrees of freedom
Regulator (in)dependence
Optimal expansion schemes
Gauge-invariant formulations
Sampling and ML applications
Real time formulations

The RG as a transport task

- <u>Original idea:</u> Compression of information under repeated block-spinning transformations $\dot{\phi} : \phi_i(x) \mapsto \phi_{i+1}(x)$

→ Keep the number of sites intact/infinite lattice

  - Consider infinitesimal field reparametrisations $\phi \to \phi + \delta t \dot{\phi}(\phi)$

  → **Change of variables** in the path integral and changing distribution $p_t \to p_t + \delta t \dfrac{\partial}{\partial t} p[\phi]$

Wegner flow

Wegner, J. Phys. C 7 (1974) 2098

$$\frac{d\, p_t[\phi]}{dt} = - \int_x \frac{\partial}{\partial \phi(x)} \left\{ \dot{\phi}_t[\phi]\, p_t[\phi] \right\} \qquad p_t[\phi] \propto e^{-S_t[\phi]}$$

Diffusion

→ **Continuity equation** for the overall distribution

- Most modern momentum space RG does not use block-spinning anymore:

  → (Non-linear) **Fokker-Planck** dynamics:

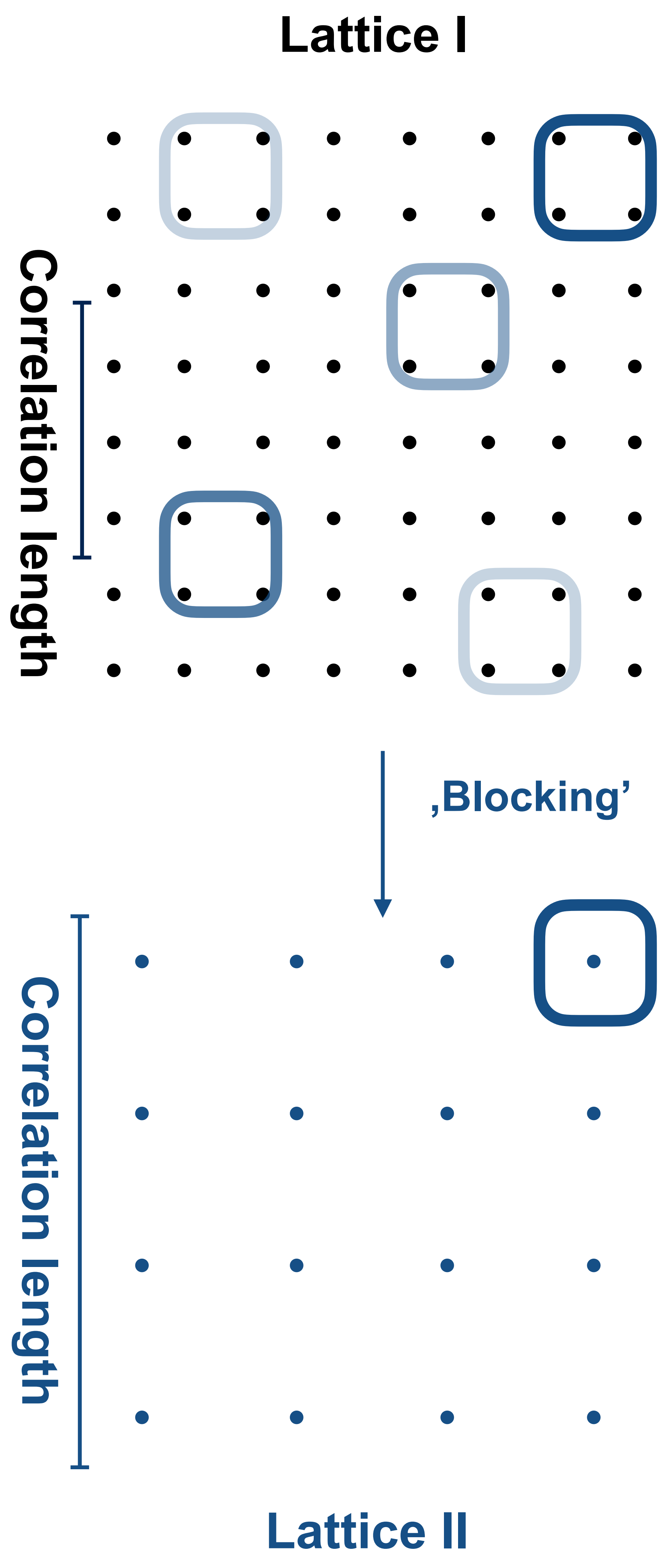
Scale transformations

Additional Reparametrisations

Polchinski-type kernel: $$\dot{\phi}(\phi) = \frac{1}{2} \mathscr{C}(\phi) \frac{\partial}{\partial \phi} S_t(\phi) \quad \boxed{+\dot{\phi}_{\mathrm{r}}(\phi)}$$

Selection/Filtering of propagating modes, e.g. $\mathscr{C} \propto -G \partial_t R_k G$

Gradient flow



Scale transformations in the path integral

Consider the generating functional for a theory: $Z_t[J] = \int D\phi\, p_t[\phi]\, e^{J\,\hat{O}_{\phi,t}[\phi]}$

Current coupled to some generic operator

Some effective degree of freedom

$\boxed{\textbf{Scale dependent distribution:}\quad p_t[\phi] = \frac{1}{\mathcal{N}} e^{-S_t[\phi]}}$ e.g. $S_t = S + \Delta S_t$

ϕ is in the **co-moving coordinate frame** of the block spinning, i.e. $\partial_t \phi = 0$ (passive transformation)

→ Use the scale dependent (composite) operator: $\boxed{\hat{O}_{\phi,t}[\phi] \quad \text{as a } \textbf{book-keeping device}}$ $\hat{O}_{\phi,t_0}[\phi] = \phi$

Physics-informed setup

$$\partial_t Z_t[J] = \int D\phi \left\{ \underbrace{\partial_t p_t[\phi]}_{\text{Wegner flow}} + \underbrace{J\,\partial_t \hat{O}_{\phi,t}\, p_t[\phi]}_{\text{Change of the composite operator}} \right\} e^{J\,\hat{O}_{\phi,t}[\phi]}$$

Two strategies

1) $\partial_t Z_t \equiv 0 \quad \Rightarrow \quad \hat{O}_\phi$ completely absorbs the flow
Sampling

2) $\partial_t Z_t \neq 0 \quad \Rightarrow \quad \hat{O}_\phi$ corrects the existing flow
Modifications in fRG

Sampling with the RG-flow

Inverse map: $\hat{O}_{\phi,t_0}[\phi]$

t_0 RG-Flow $\partial_t p_t \neq 0$, with passive transformation $\partial_t \phi = 0$ t_1

Active transformation: $\phi_{t_1}[\hat{O}_{\phi,t_0}[\phi]] = \phi$

Takeaway:

Integrating $\dot{\phi}[\phi]$ for microscopic transformations gives us a map for the field transformation

$$\phi_t = \phi_{t_0} + \int_{t_0}^{t} ds\, \dot{\phi}_s[\phi_s]$$

- We use $\partial_t Z = 0$ to **reconstruct the coarse-graining map**
- For general holomorphic operators $O[\phi]$ one can show

Observable in a hard theory

Sample from easy distr.

$$\int D\phi\, p_t[\phi] O[\phi] = \int D\varphi\, p_0[\varphi] O[\phi_t[\varphi]]$$

Map to $t = t_1$

$\rightarrow$ This is valid for any pair $\left(p_t[\phi], \dot{\phi}[\phi]\right)$ that fulfills the Wegner equation

$\rightarrow$ **Physics-informed kernels**: pick a suitable constraint to fix the pair, e.g.

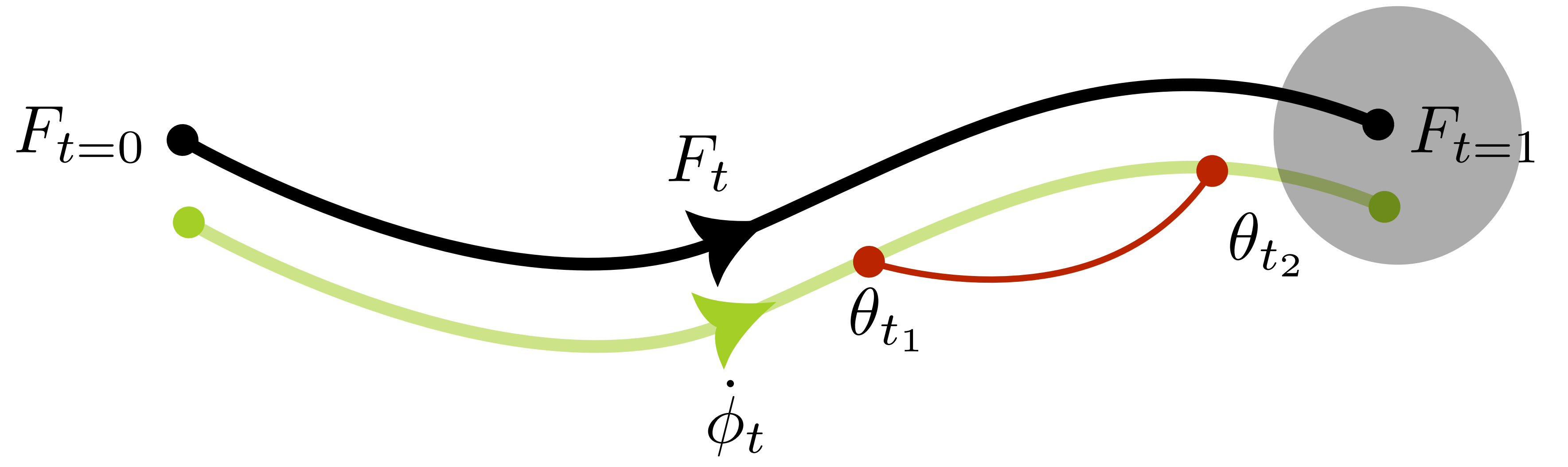
- Langevin evolution
- Diffusion models: Talk by L. Wang
- Polchinski kernel: Talk by Y. Ashida, M. Gubinelli
- …

Sample p_t with $\phi_t(\phi_0)$, where $\phi_0 \sim p_0$

FI, Kapust, Pawlowski, arxiv:2510.26678
FI, Kapust, Pawlowski, arxiv:2603.03159

The physics-informed mindset

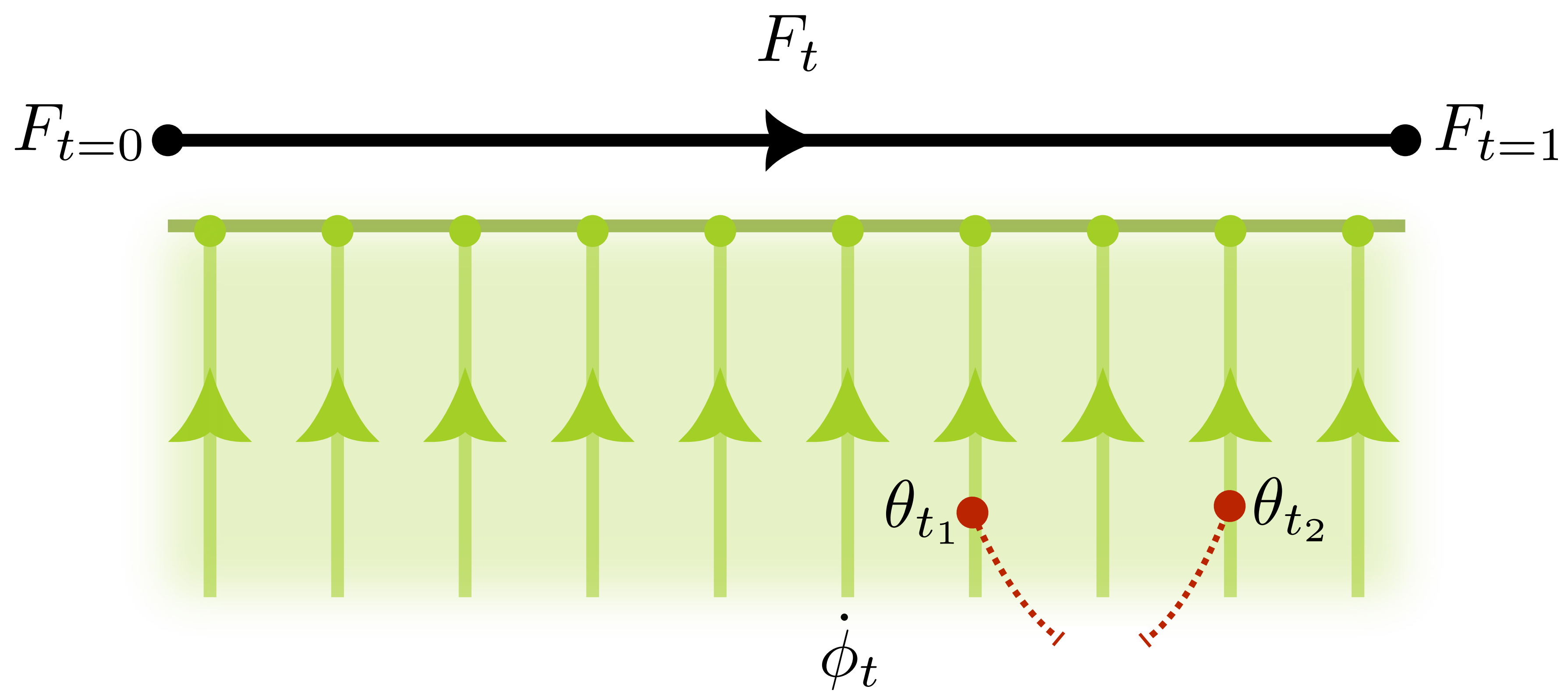
Allowing general time-dependent paths for distributions
$F_t = \{S_t, p_t, \ldots\}$ does not guarantee the desired physical distribution



…so we impose a trajectory analytically. Then the Wegner equation

$$\partial_t S_t + \dot{\phi}\frac{\delta}{\delta\phi}S_t = \frac{\delta}{\delta\phi}\dot{\phi} \longrightarrow RG_1 + \dot{\phi}RG_2 = \frac{\delta}{\delta\phi}\dot{\phi}$$

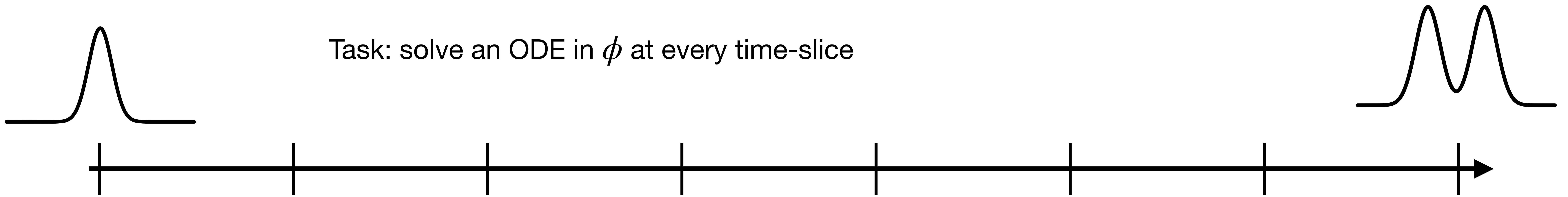
…is a differential equation at every time-slice!



Example: zero-dimension ϕ^4 - theory

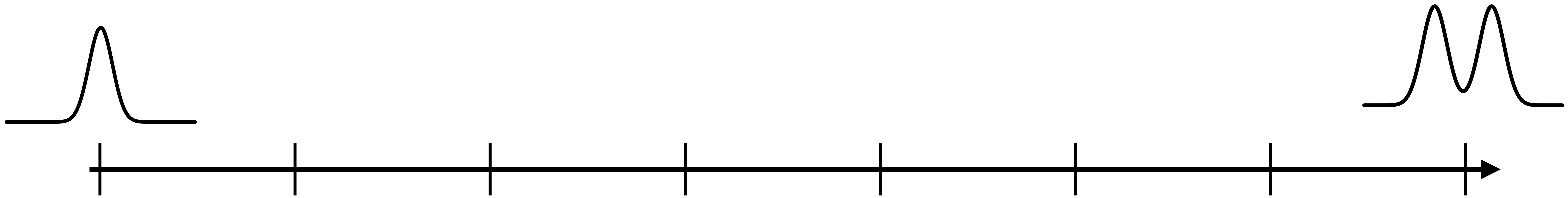
$$\hat{S}_t(\phi) = \frac{1}{2}m^2(t)\phi^2 + \frac{\lambda(t)}{4}\phi^4$$

Task: solve an ODE in ϕ at every time-slice

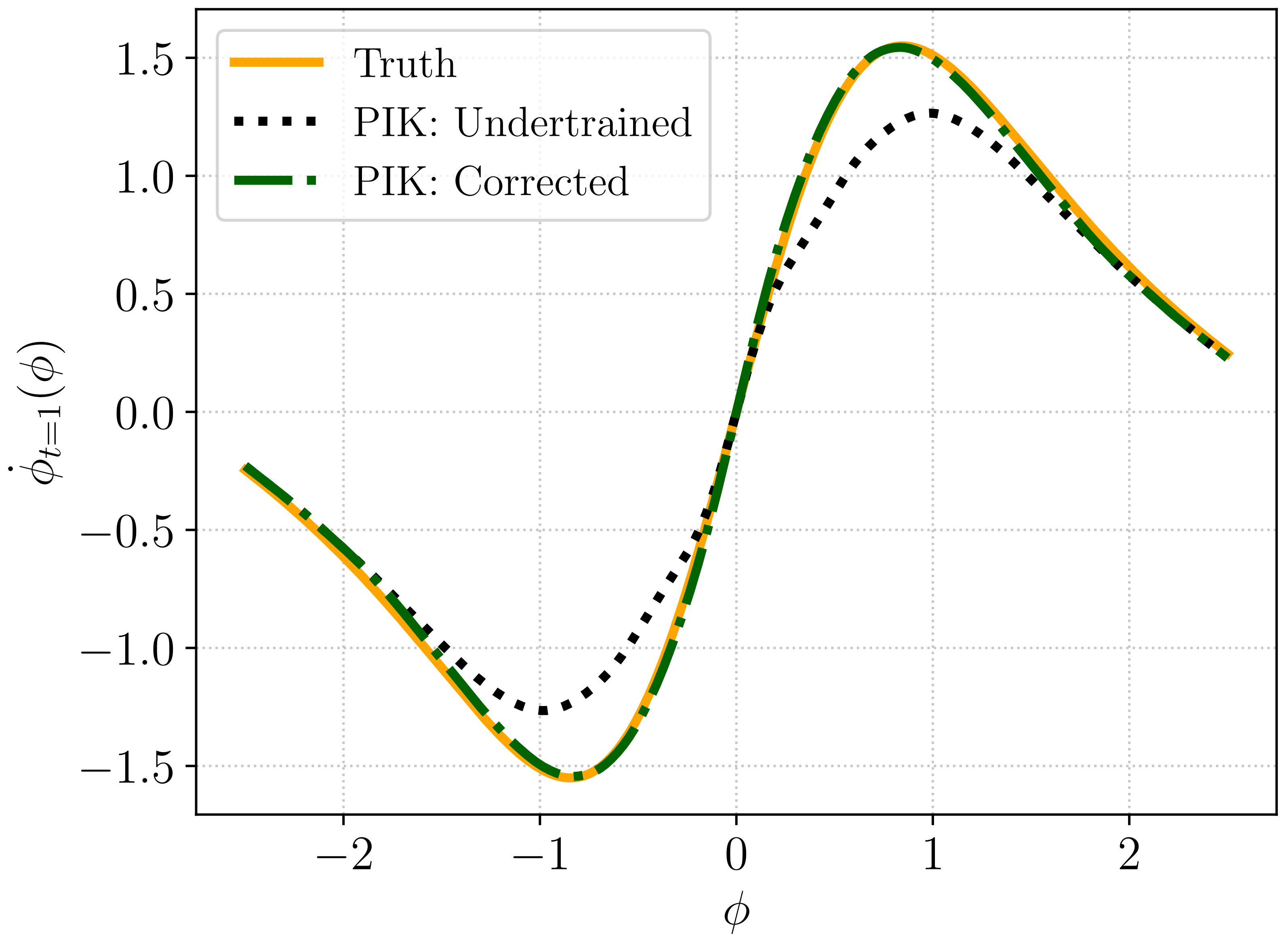


$(m_0^2\,,\,\lambda_0) = (1\,,\,0)$ $\longrightarrow$ Analytic interpolation between couplings $\longrightarrow$ $(m_1^2\,,\,\lambda_1) = (-2\,,\,1/2)$

FI, Kapust, Pawłowski, arxiv:2510.26678



$$(m_0^2\,,\,\lambda_0) = (1\,,\,0) \longrightarrow (m_t^2\,,\,\lambda_t) = \left(m_0^2 + t\,(m_1^2 - m_0^2)\,,\,\lambda_0 + t(\lambda_1 - \lambda_0)\right) \longrightarrow (m_1^2\,,\,\lambda_1) = (-2\,,\,1/2)$$

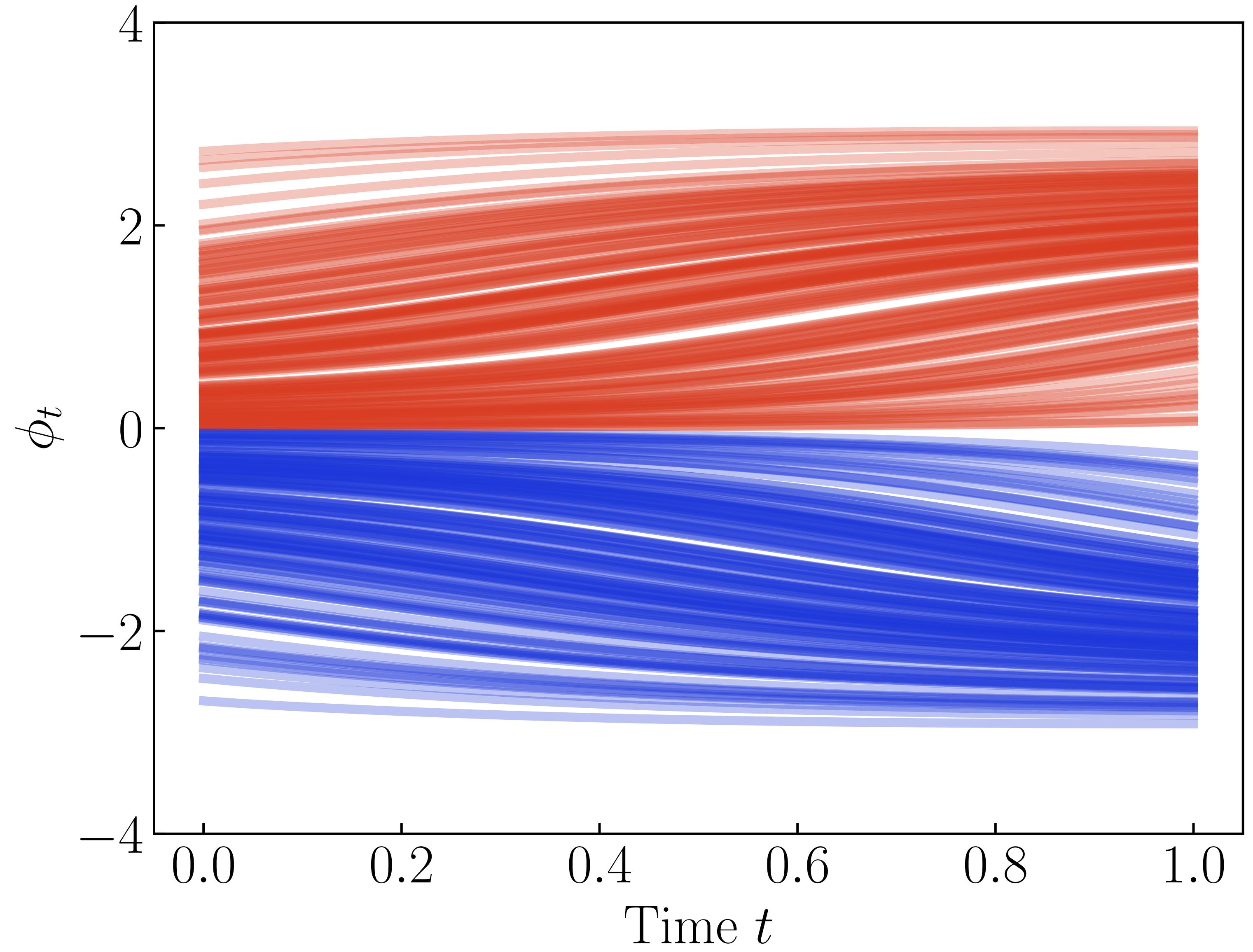


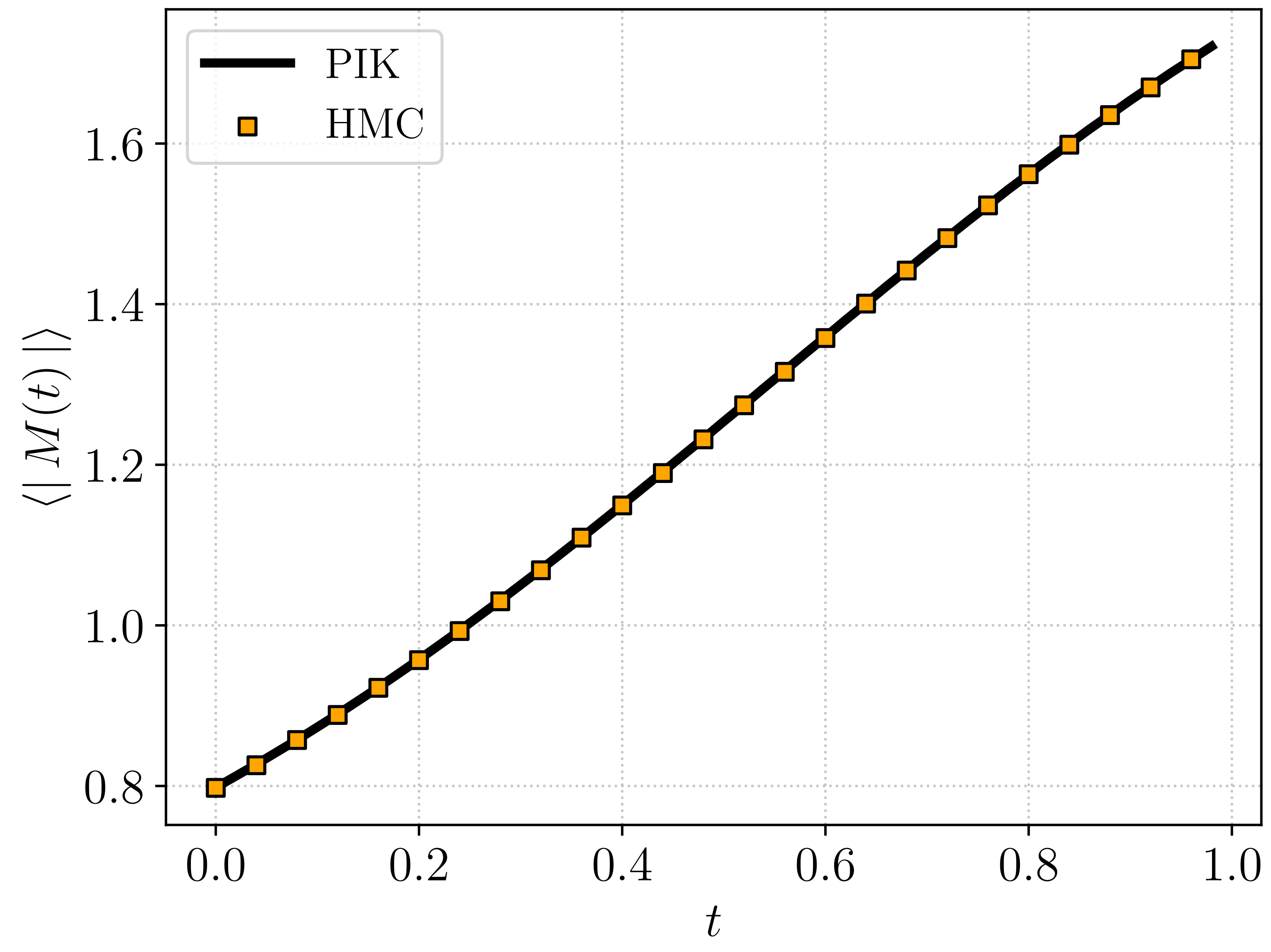
- Compute PIKs separately at every time-slice
- Linearity of the equation allows for systematic improvements

- Evolve samples separately with

$$\phi_t = \phi_{t_0} + \int_{t_0}^{t} ds\, \dot\phi_s[\phi_s]$$

- This is an **optimal transport flow**





- Sample operators along this trajectory

$$\langle O \rangle_t = \frac{1}{N} \sum_n O[\phi_{n,t}]$$

FI, Kapust, Pawlowski, arxiv:2510.26678

Part 1 (Strategy 1): Conclusion and outlook

PIK is an umbrella for sampling based strategies:

→ Fixing a target action S_t allows to sample this distribution directly <> Contrast to the Polchinski flow or complex Langevin

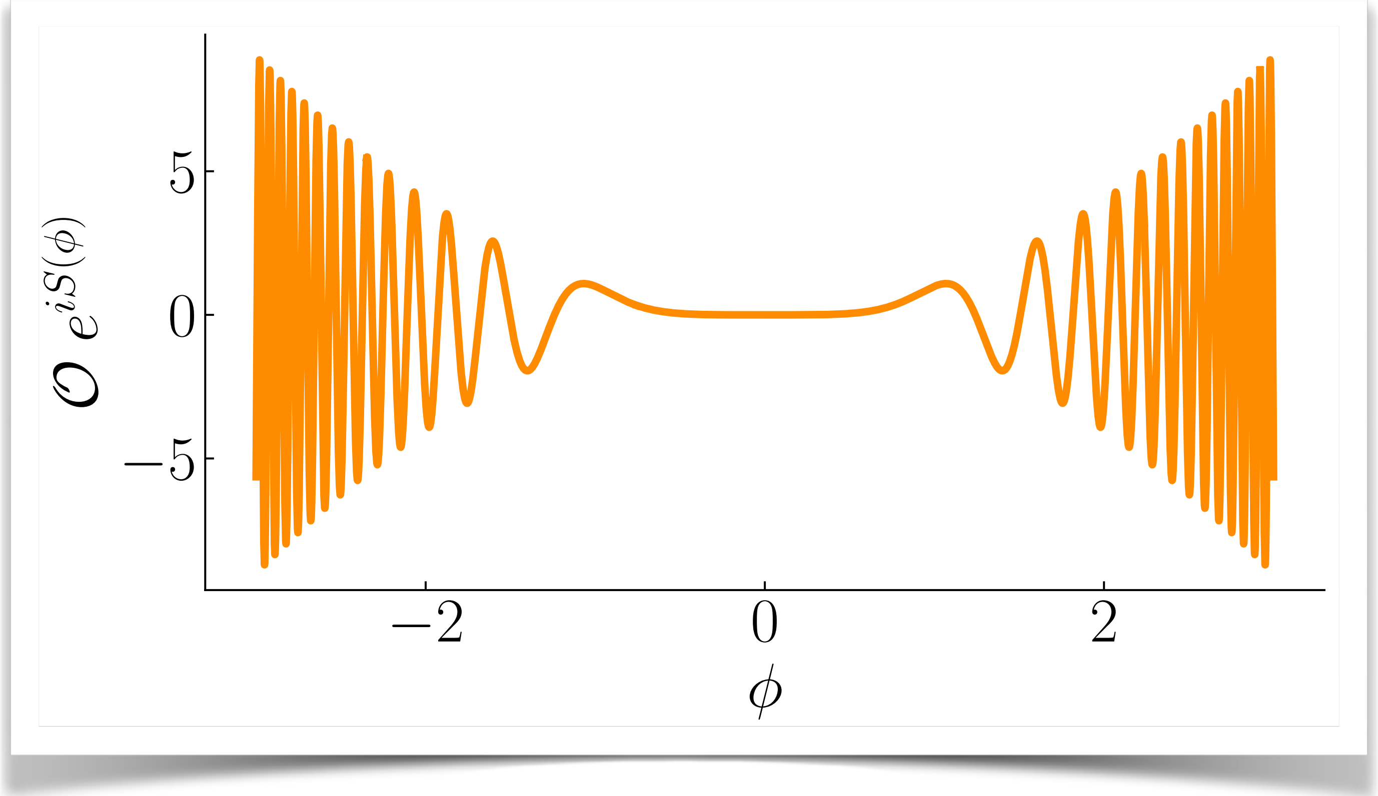
→ We have only used **general reparametrisation freedom**: 0d does not know about scale transformations

→ Particularly interesting for complex actions/systems with a sign problem

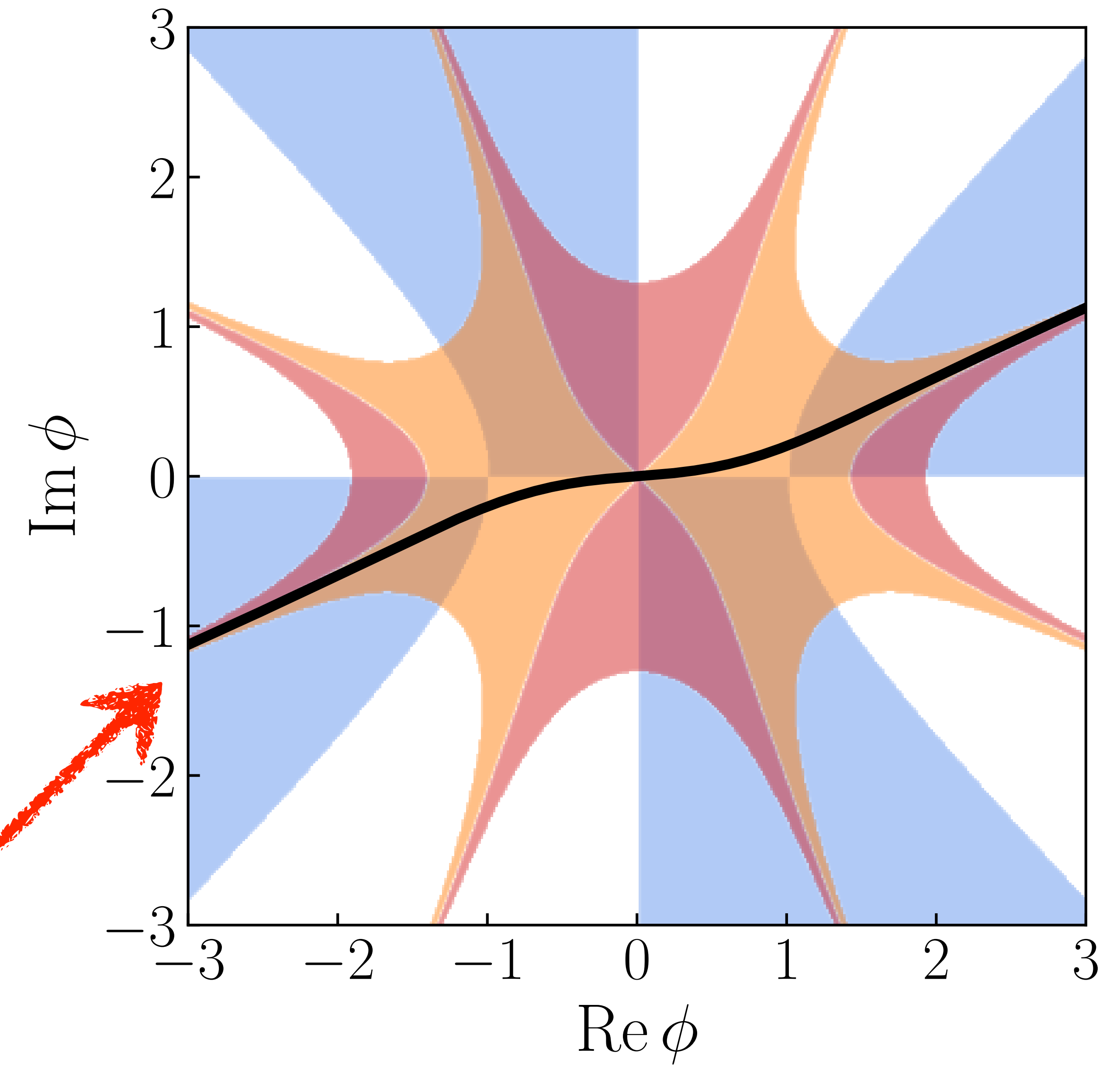
Weight preservation: $p_0\, d\mu_0 \in \mathbb{R} \longrightarrow p_t\, d\mu_t \in \mathbb{R}$

Complex contour deformation

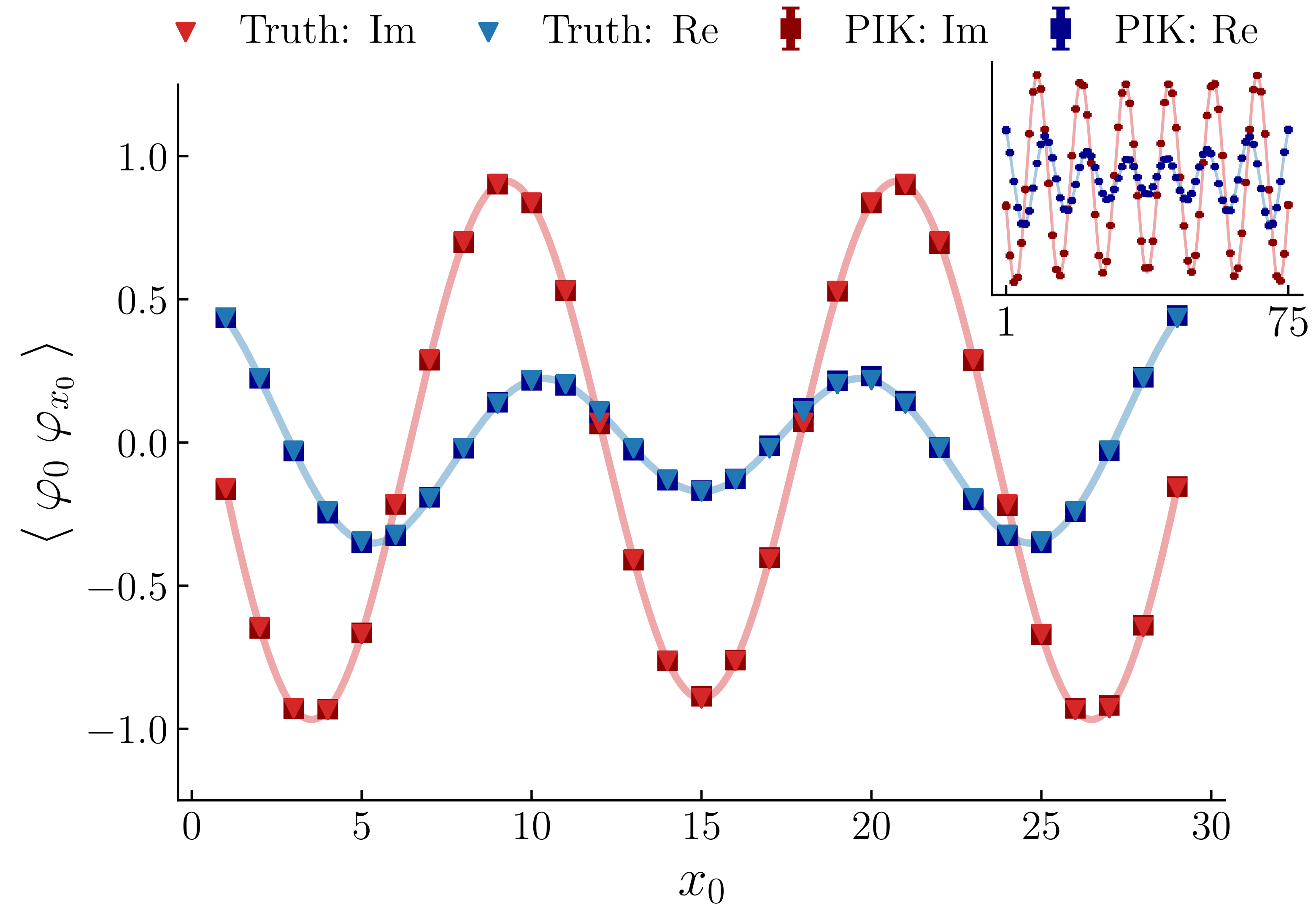
0-dim toy model of a real time theory



No guessing involved!



PIK implements a contour deformation to remain weight preserving



1+0 dim theory:
Quantum harmonic oscillator

FI, Kapust, Pawlowski, arxiv:2603.03159

Generalised RG flows: Strategy 2

$$\partial_t Z_t[J] = \int D\phi \left\{ \underbrace{\overset{\text{Scale transformations}}{\partial_t p_t[\phi]}}_{\text{Wegner flow}} + \underbrace{\overset{\text{Reparametrisations}}{J\, \partial_t \hat{O}_{\phi,t}\, p_t[\phi]}}_{\text{Change of the composite operator}} \right\} e^{J\, \hat{O}_{\phi,t}[\phi]}$$

Implement the **scale suppression/ Regulator** as usual

$$p_t[\phi] = \frac{1}{\mathcal{N}} e^{-S[\phi]-\Delta S_t[\phi]}$$

By allowing $\partial_t Z_t \neq 0$ we remain with a very general reparametrisation freedom

$$\partial_t \hat{O}_{\phi,t}$$

$$\dot{\phi} = \langle \partial_t \hat{O}_{\phi,t} \rangle\,, \quad \phi = \langle \hat{O}_{\phi,t} \rangle$$

Sampling

Direct computation & optimisation

$$\frac{d\,p_t(\phi)}{dt} = -\int_x \frac{\partial}{\partial\phi(x)} \left[\dot{\phi}_t(\phi)\, p_t(\phi)\right]$$

Legendre transform

$$\left(\partial_t + \int_x \dot{\phi}[\phi]\frac{\delta}{\delta\phi}\right)\Gamma_\phi[\phi] = \frac{1}{2}\mathrm{Tr}\left[G_\phi[\phi]\left(\partial_t + 2\frac{\delta\dot{\phi}}{\delta\phi}\right) R_k\right]$$

⋮

Wegner flow

Wegner, J. Phys. C 7 (1974) 2098

General field transformations

Generalized flow equation

Pawlowski, *Annals Phys.* 322 (2007) 2831

Wetterich equation $\dot{\phi} = 0$

Wetterich, PLB 301 (1993) 90

Scale transformation implemented with regulator → Remaining reparametrisation freedom

Lets go back to the generalised RG…

$$\left(\partial_t + \int_x \dot{\phi}[\phi]\frac{\delta}{\delta\phi}\right)\Gamma_\phi[\phi] = \frac{1}{2}\mathrm{Tr}\left[G_\phi[\phi]\left(\partial_t + 2\frac{\delta\dot{\phi}}{\delta\phi}\right)R_k\right]$$

$\dot{\phi} = 0$ Implements Wetterich RG (previous example)

$\dot{\phi} \neq 0$ Corrections of loop-order allow for optimization

Tree-level corrections change the theory

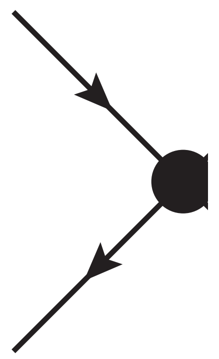
Key insights:

(1) $\dot{\phi}[\phi]$ is a functional and can be truncated the same as the effective action $\Gamma[\phi]$

(2) This additional freedom allows to implement constraints in the RG flow

- Example: Rebosonization — Talk by F. Rennecke, F. R. Sattler, …

Absorb the divergence into a Yukawa coupling $\propto h\phi_a\bar{\psi}M_a\psi$ $\quad \partial_t$

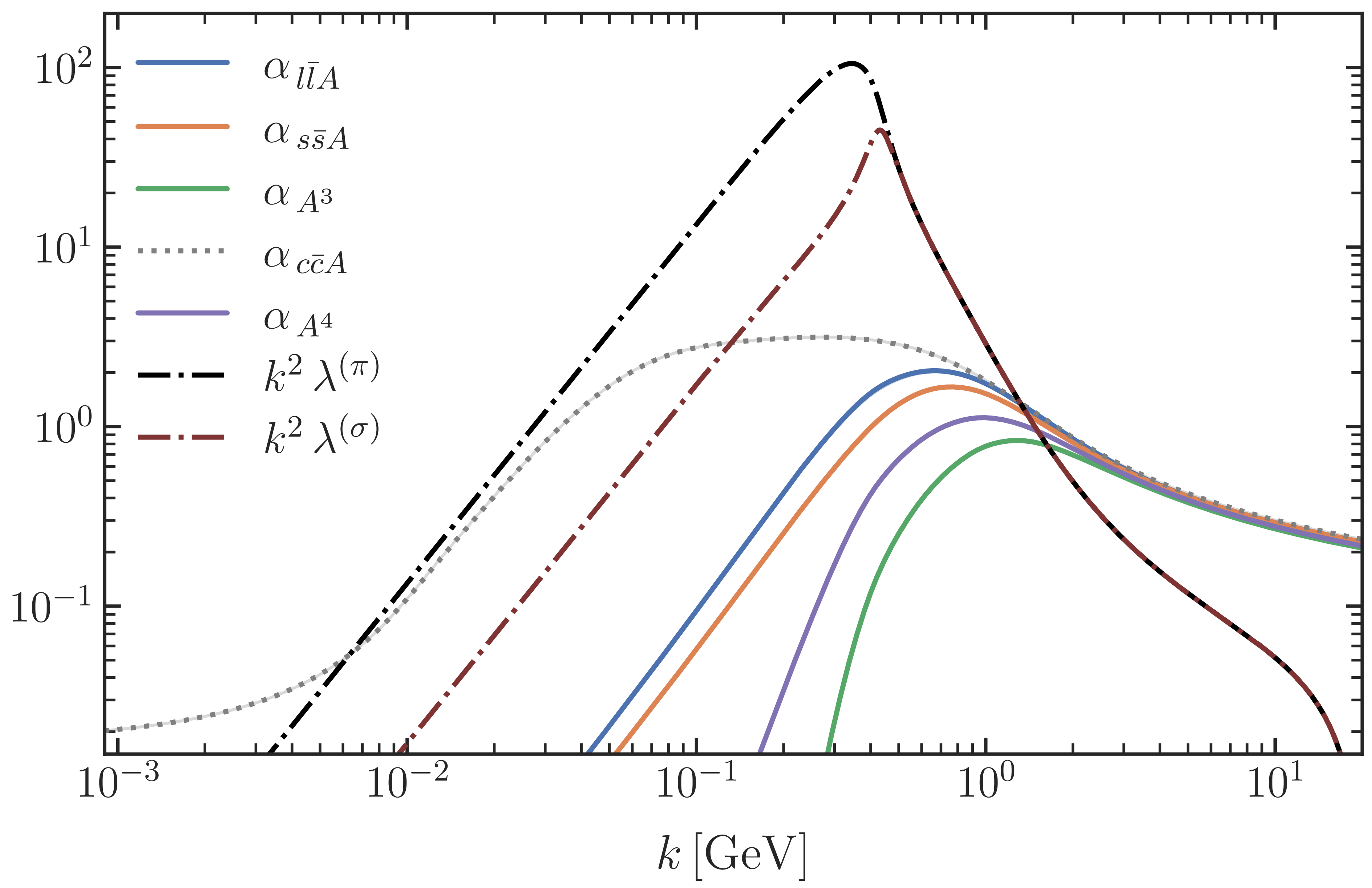


Ansatz: $\dot{\phi} = A_a(\phi)\,\bar{\psi}M_a\psi$ $\quad \boxed{A_a(\phi) \quad \to \quad \partial_t g_a = 0}$

Introduces corrections to the Yukawa coupling!

For linear transformations — Gies, Wetterich, PRD 65 (2002) 065001

Implementation in QCD — Fu, Pawlowski, Rennecke, PRD 101 (2020) 5, 054032; **FI,** Pawlowski, Sattler, Wink, PRD 113 (2026) 9, 094038 →



Effectively, a four-fermi vertex with a non-trivial momentum dependence is considered

→ No divergence

$$\lambda_i = \frac{h^2}{Z_\psi^2(m_i^2 + p^2 + R_k)}$$

…but there is more than that

Physics-informed RG $\boxed{\left(\Gamma_\phi[\phi], \dot\phi[\phi]\right)}$

Generally we can consider the pair and implement some physical constraint with it:

PIRG idea

$$\Gamma_{\phi,t_0}[\phi] \xrightarrow{\left(\Gamma_{\phi,t}[\phi], \dot\phi_t[\phi]\right)} \Gamma_{\phi,t_1}[\phi]$$

$$\left(\partial_t + \int_x \dot\phi \frac{\delta}{\delta\phi}\right)\Gamma_T[\phi] = \mathrm{RG}_1[\Gamma_T] + \mathrm{RG}_2[\Gamma_T]\frac{\delta\dot\phi}{\delta\phi}$$

Reconstruction & observables: Generalised operator flow

$$\left(\partial_t + \int_x \dot\phi \frac{\delta}{\delta\phi}\right)\mathcal{O}_t[\phi] = -\frac{1}{2}\mathrm{Tr}\left[G_T\left(\partial_t + 2\frac{\delta\dot\phi}{\delta\phi}\right)G_T\frac{\delta^2}{\delta\phi^2}\mathcal{O}_t[\phi]\right]\boxed{+F_\phi}$$

observable propagator insertion DoF for optimisation

Example: Direct Computation

Step 1: Solve approximate flow for $\Gamma_{t,\mathrm{app}}$

Step 2: Use $\Gamma_T = \Gamma_{t,\mathrm{app}}$ as target action

Solve generalized flow for $\dot\phi_t$

Advantages

(i) Trade intricate diffusion eq. for a simple linear diff. eq.

(ii) Optimised and more versatile expansion schemes

(a) Ground state expansion

(b) Feed-down flows

⋮

Disadvantage: Reconstruction!

FI, Pawlowski, Annals Phys. 481 (2025) 170177

FI, Pawlowski, PRD 113, 076003 (2026

The ground state-expansion

Topology in the anharmonic oscillator

- Quantum mechanics: i.e. 1+0 dimensional QFT

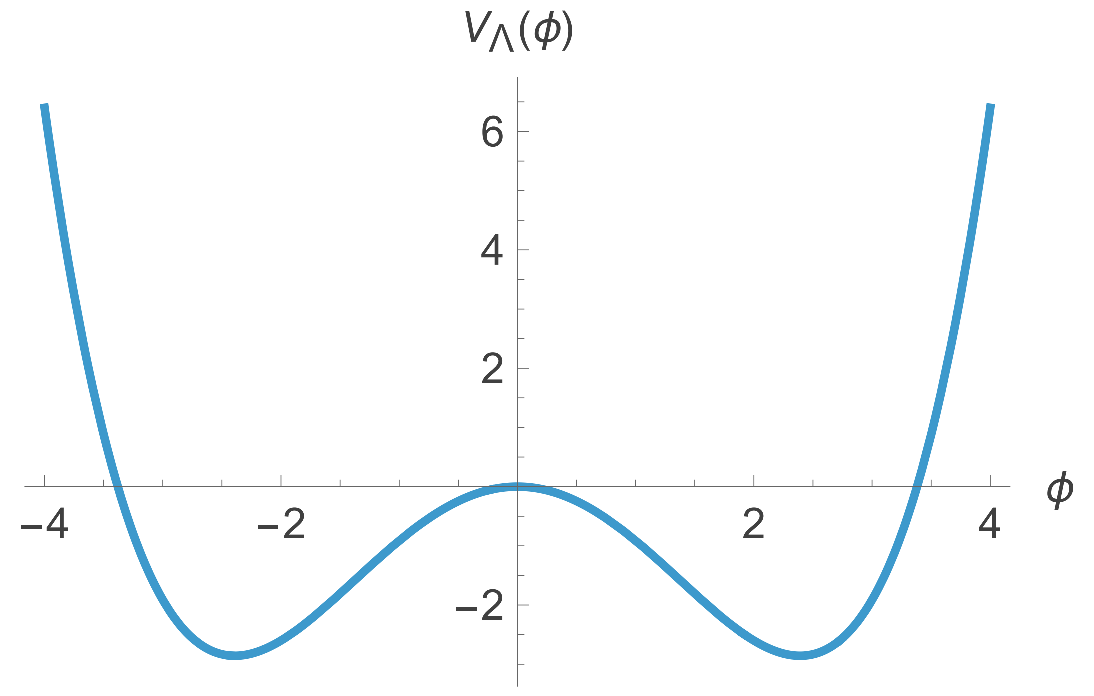
  → Setup with **tunneling**

  → Consider the **energy gap** between ground state and the first excited state Δ E

$$\langle \hat{q}(0)\hat{q}(\tau)\rangle = \sum_n |\langle 0|q|n\rangle|^2 \exp(-(E_0 - E_n)\tau)$$

  → Behavior at large imaginary times

$$\langle \hat{q}(0)\hat{q}(\tau)\rangle \propto \exp(-\Delta E\,\tau)$$



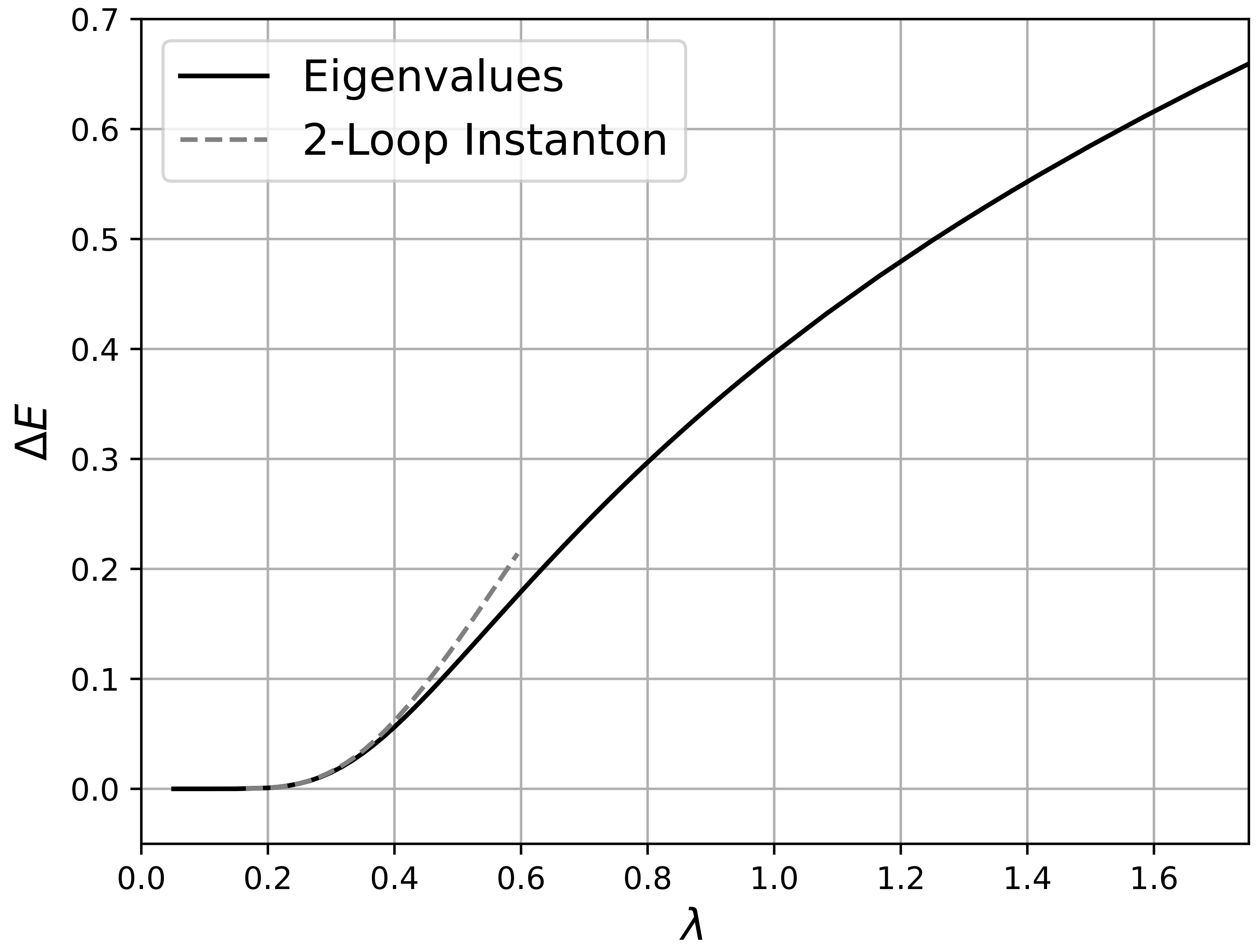
$$V_\Lambda(\hat{q}) = -\frac{1}{2}\hat{q}^2 + \frac{\lambda}{8}\hat{q}^4$$

This is all known from the **Schrödinger equation**

→ but also: Expansion about **Instanton solution**

BKT-type scaling

$$\Delta E(\lambda_\varphi \to 0) \propto \frac{1}{\sqrt{\lambda_\varphi}}\, e^{-a_{\text{inst}}/\lambda_\varphi}$$



Testing ground for novel methods with non-perturbative physics

→ In particular where fRG has struggled before

The anharmonic oscillator in the fRG

Chose a simple truncation with the derivative expansion (LPA)

$$\Gamma_k[\phi] = \int_\tau \left[\frac{1}{2}(\partial_\tau \phi)^2 + V_k(\phi)\right]$$

Solve the PDE for the Wetterich equation, and extract $\Delta E = \sqrt{V^{(2)}(\phi_{\text{eom}})}$

Solve the generalized equation in the same truncation for Γ_k

With an **optimized kernel**:
(Ground state expansion)

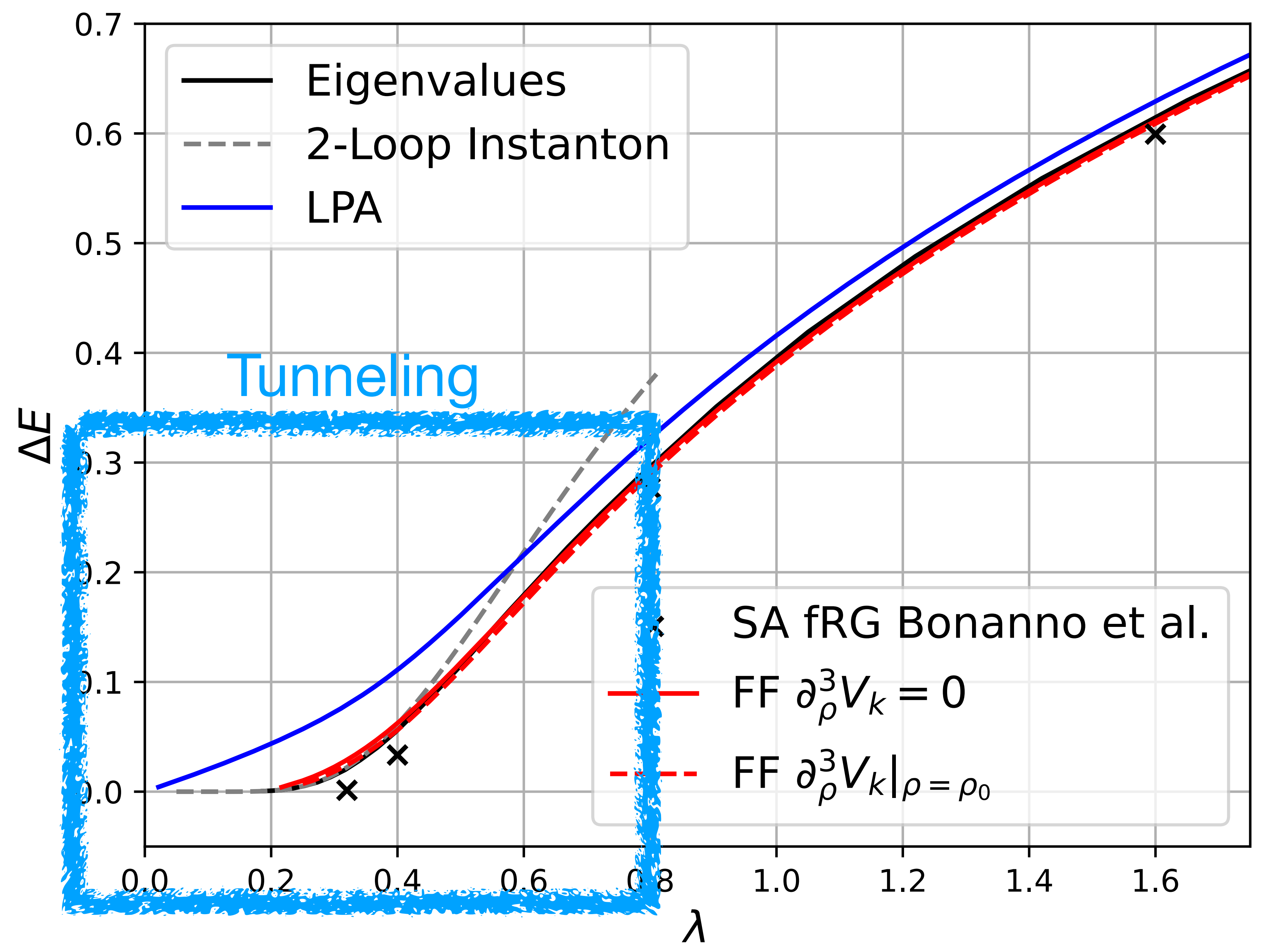
$$\dot{\phi}(\phi) \rightarrow \partial_t(\partial_{p^2}\Gamma_k^{(2)}[\phi]) = 0$$

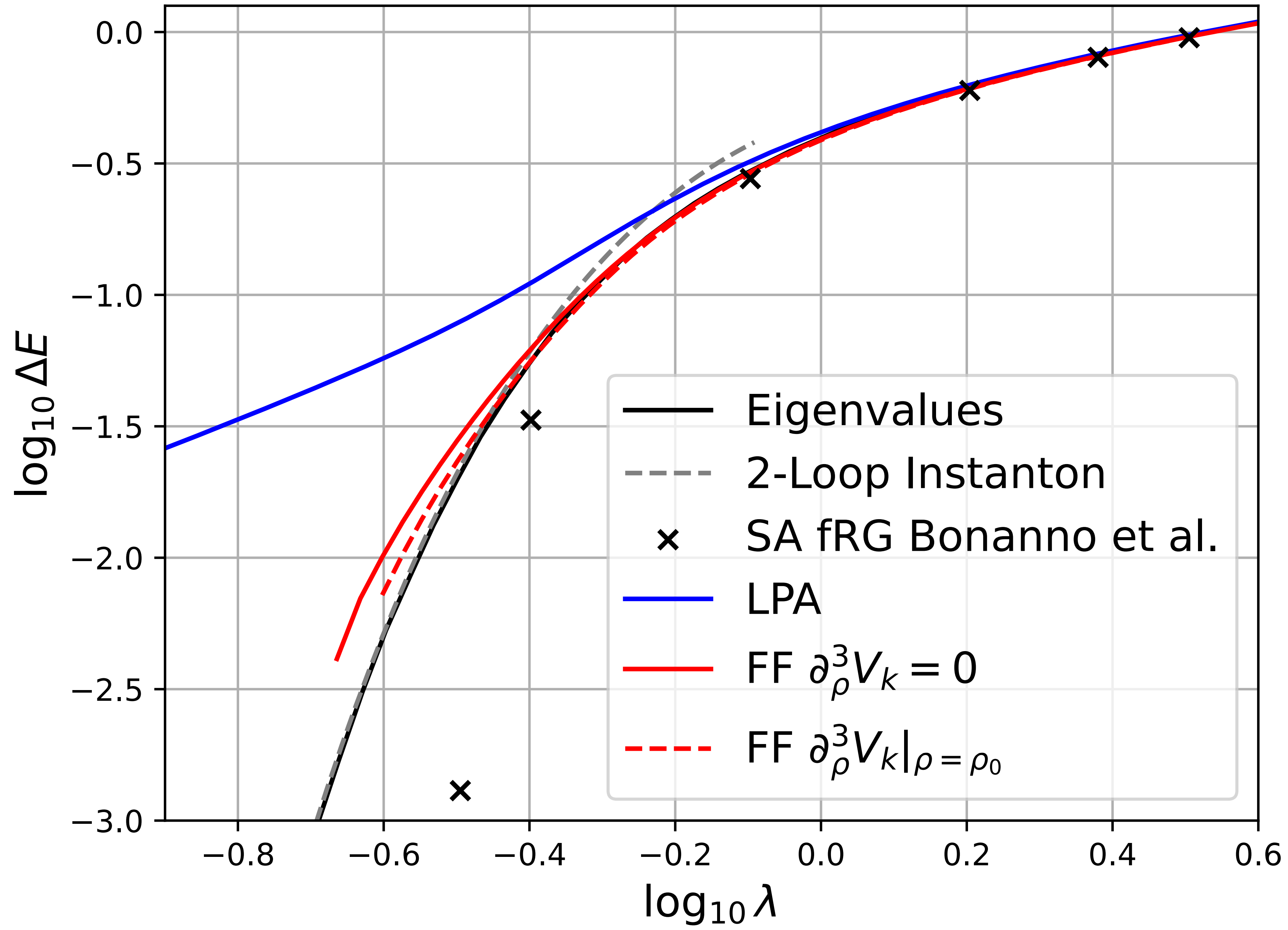
The optimized kernel **eliminates the next order of the expansion scheme.**

Results in a field dependent anomalous dimension:

This allows the extraction of the **instanton scaling** up to 1-2% exactness!

$a_{inst,ex} = 1.886....$ $\quad a_{inst,FF} = 1.910(2)$





Bonanno, **FI,** Pawlowski, SciPost Phys. Core 9, 005 (2026)

The ground-state expansion

Baldazzi, Ben Ali Zinati, Falls, *SciPost Phys.* 13 (2022) 4, 085
FI, Pawlowski, arXiv:2305.00816
Bonanno, **FI**, Pawlowski, *SciPost Phys.Core* 9 (2026) 005

Consider the standard derivative expansion:

$$\Gamma_k[\phi] = \int_\tau \left[\frac{1}{2}(\partial_\tau\phi)^2 + V_k(\phi)\right]$$

Zeroth-order expansion

$$\Gamma_k[\phi] = \int_\tau \left[\frac{1}{2}Z(\phi)(\partial_\tau\phi)^2 + V_k(\phi)\right]$$

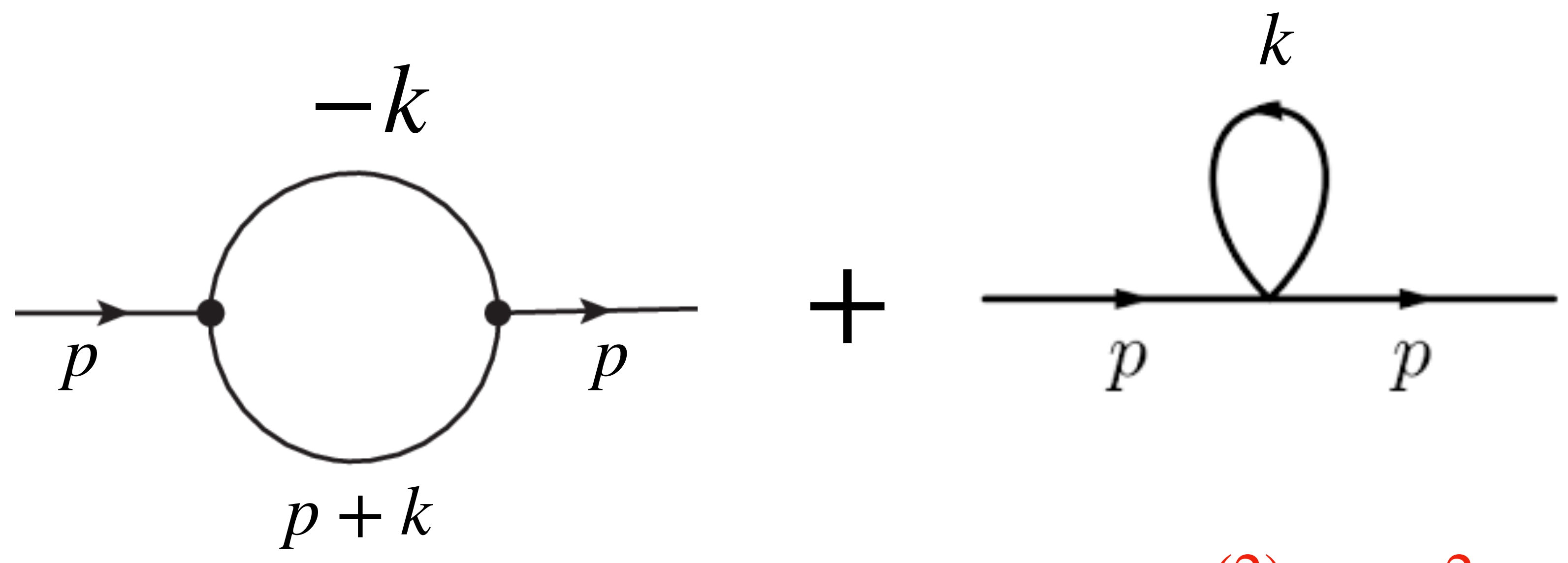
First-order expansion

RG-flow now accounts for fluctuation physics of order p^2

In this formulation the self-energy diagrams get momentum dependent corrections:

→It creates e.g. a momentum dependent Tadpole diagram for all n-point functions

$$\partial_t\Gamma^{(2)}_k[\phi] \propto$$



$\propto Z^{(2)}(\phi)p^2$

Consider an expansion about the classical dispersion:

$$\Gamma^{(2)}_k = p^2 + \tilde{m}^2(\phi)$$

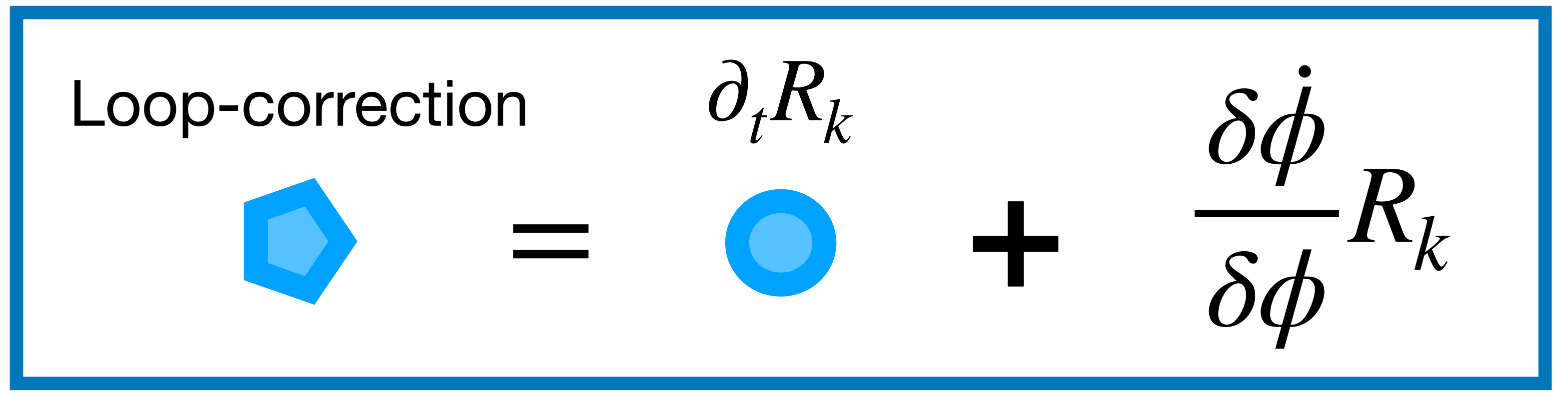
$$\dot{\phi}(\phi) \to \partial_t Z(\phi) \equiv 0$$

Effective action looks zeroth-order again, but **includes momentum dependent physics of all orders of tadpole diagrams** via $\dot{\phi}$

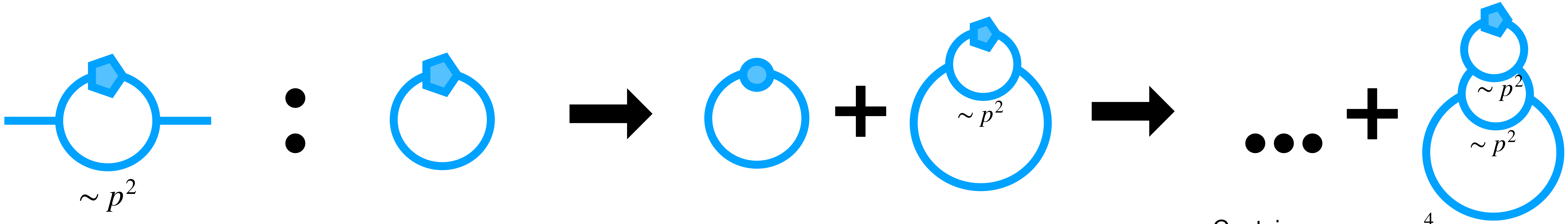
$$Z(\phi) \equiv 1$$

Vertex has no field dependence → No momentum dependent tadpole

The ‚absorption strategy‘



The ground-state expansion can be thought of as a **multi-loop scheme** (to infinite order):



p^2-correction to the flow

Appears in the loop-correction

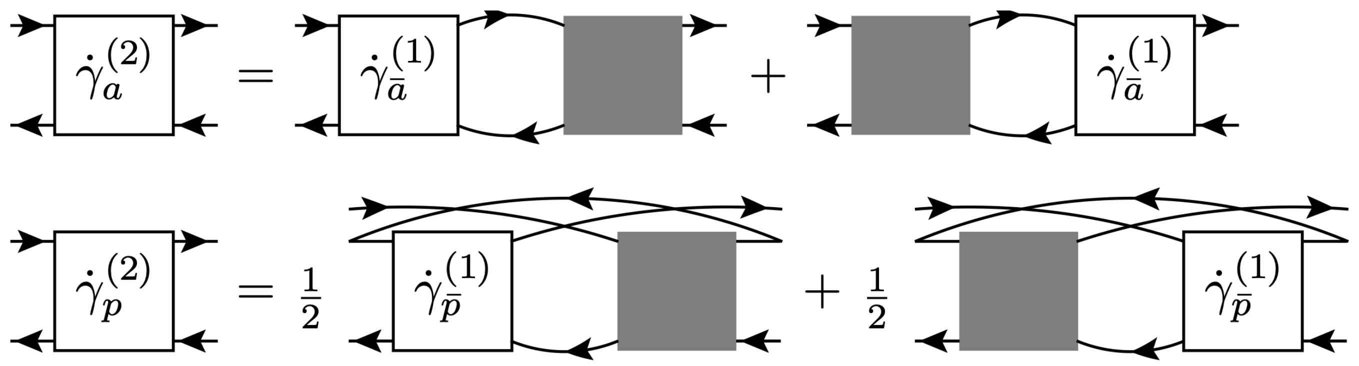
Appears in the loop-correction

Contains some $\sim p^4$ corrections

In this sense it is related to the very successful **multi-loop fRG** (Fermionic 6-pt vertex is absorbed)

Kugler, von Delft, PRL 120, 057403 (2018)

Kugler, von Delft, PRB 97, 035162 (2018)



$\rightarrow$ But automatically **includes all orders** (resummed scheme)

Application to Gauge theories (and other symmetry constraints): **Absorb terms that break the gauge invariance of the scheme!**

$\rightarrow$while feeding back their flow via implicit higher loop-corrections

E.g. Hard-wiring the modified Nielsen-Identities

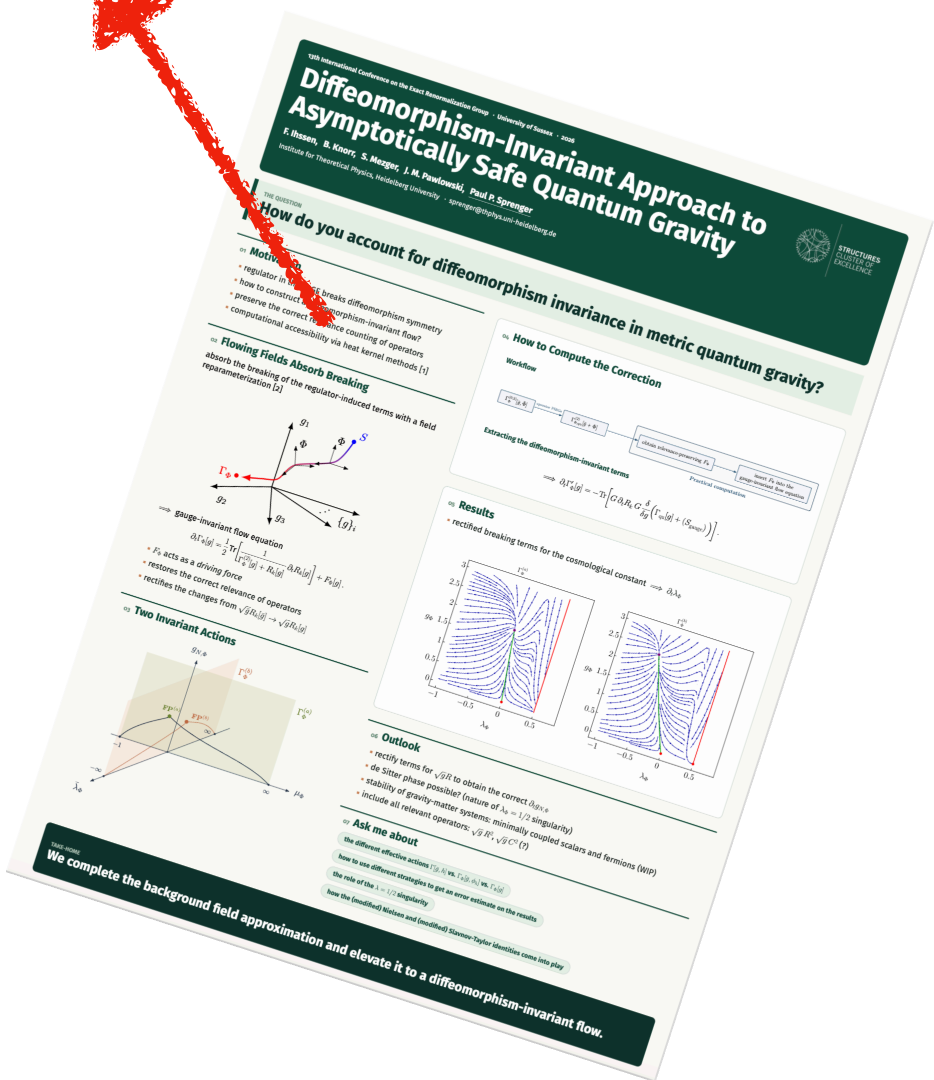
FI, Pawlowski, PRD 112 (2025) 10, 105005

Diffeomorphism-invariant Approach to Asymptotically Safe Quantum Gravity

Friederike Ihssen,[1,2] Benjamin Knorr,[2] Silas Mezger,[3,2] Jan M. Pawlowski,[2,4] and Paul P. Sprenger[2]

On the web tomorrow…

…since an entire week now

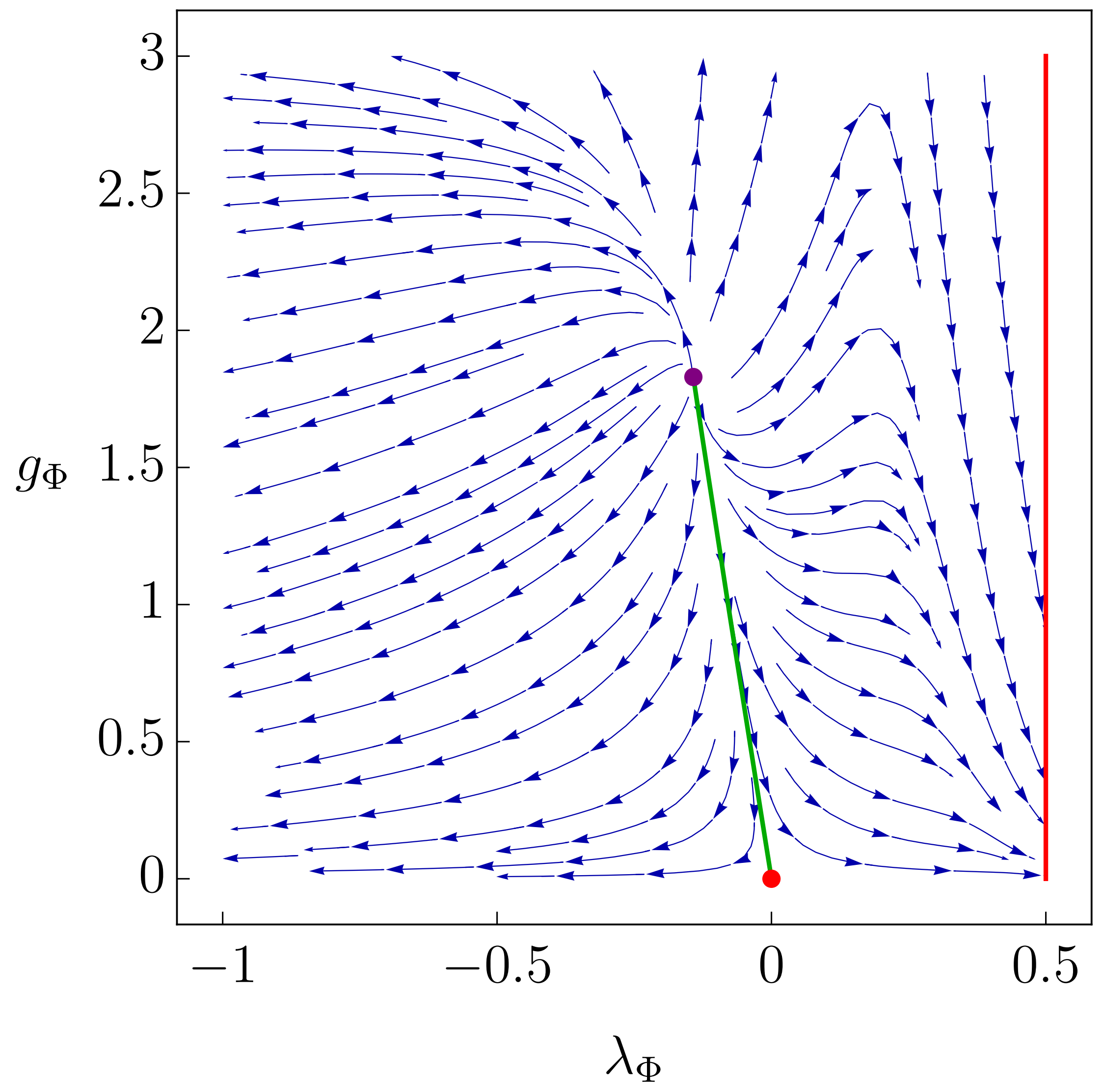


The regulator breaks gauge invariance.

Exact relations are around, e.g. modified Ward identities, Slavnov-Taylor, modified Nielsen Identities.

⇒ Embed a gauge invariant flow of your choice in the PIK — Constraint fixes Γ_k

⇒ Compute difference to the exact solution with $\dot{\Phi}$ — Contains relevant and symmetry breaking terms



猪$_k^{(a,b)}$	$(\lambda^*_\Phi, g^*_\Phi)$	$\{\theta_i\}$
$\Gamma^{(a)}_{\Phi,\mathrm{qu}}[g]$	$(-0.142,\ 1.830)$	$\{3.86,\ 1.95\}$
$\Gamma^{(b)}_{\Phi,\mathrm{qu}}[g]$	$(-0.473,\ 2.369)$	$\{4.66\,,\ 1.89\}$

Identify

Extract

Reinsert

$$\partial_t \Gamma_{\Phi,rp}[\bar{g}+\Phi] = \frac{1}{2}\mathrm{Tr}\, G_\Phi[\bar{g}+\Phi]\partial_t R_k[g] + F_\Phi[\bar{g}+\Phi]$$

Conclusion

- Generalised flows connect many of the topics of the conference!

  → Sampling strategies

  → Flows of the 1PI effective action

- **Analytical access** is very interesting to optimize the sampling task and understand AI algorithms

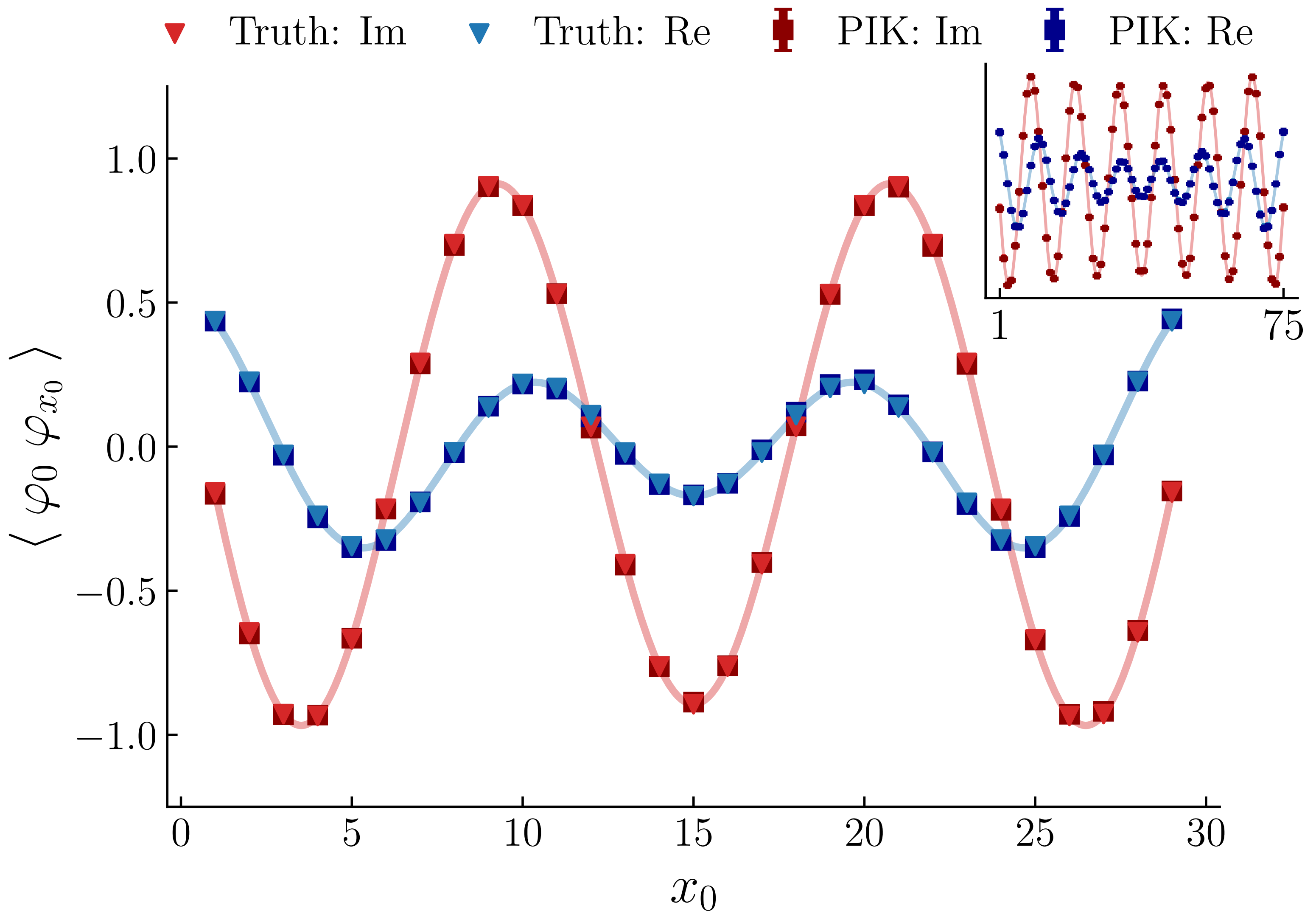
  → Complex actions + sign problem

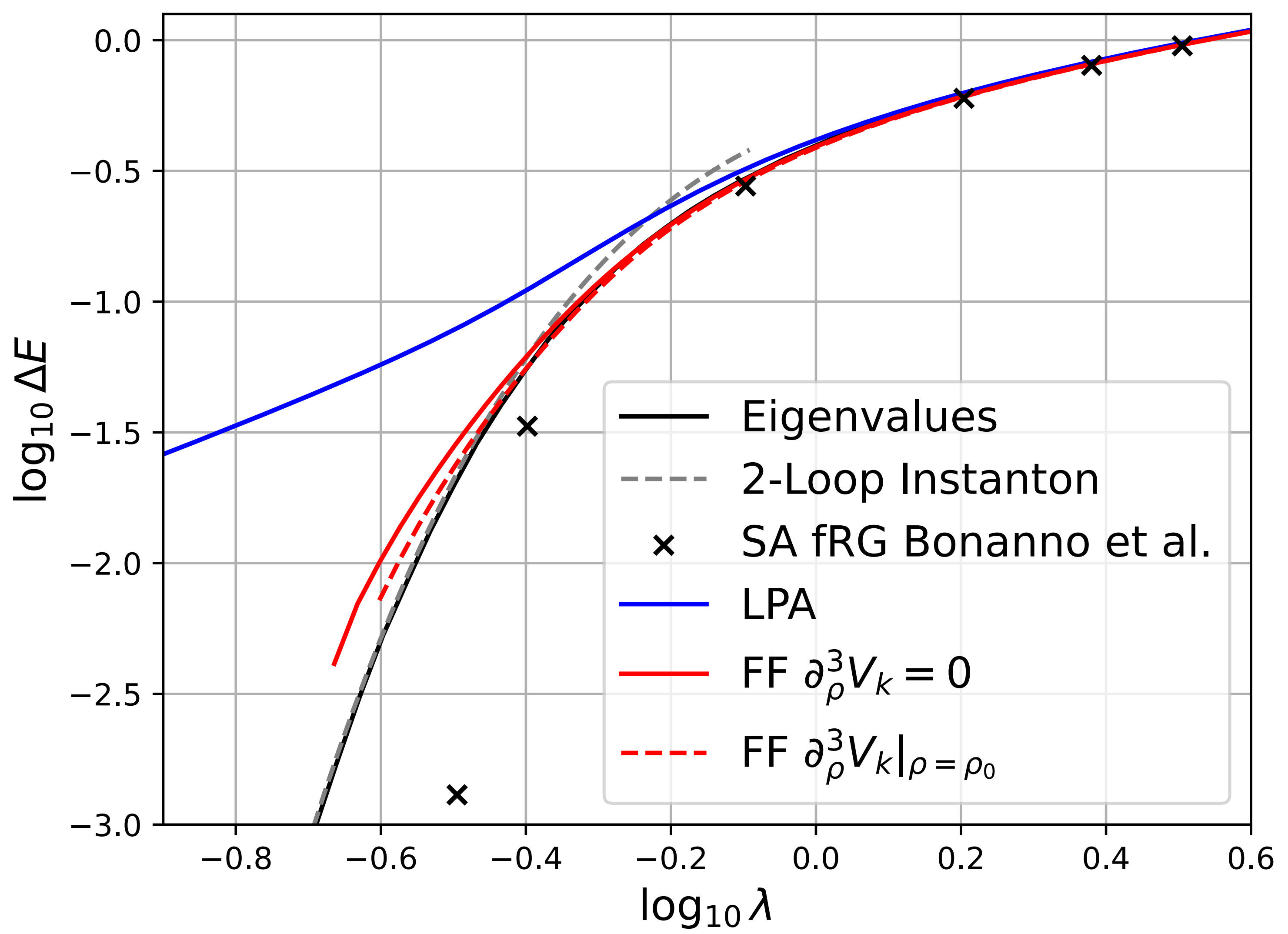
- **Optimisation** in the Wetterich equation

  → Test approximations

  → Absorption of couplings for faster convergence

  →Relevance counting preserved in gauge invariant formulations





Thank you for your attention!