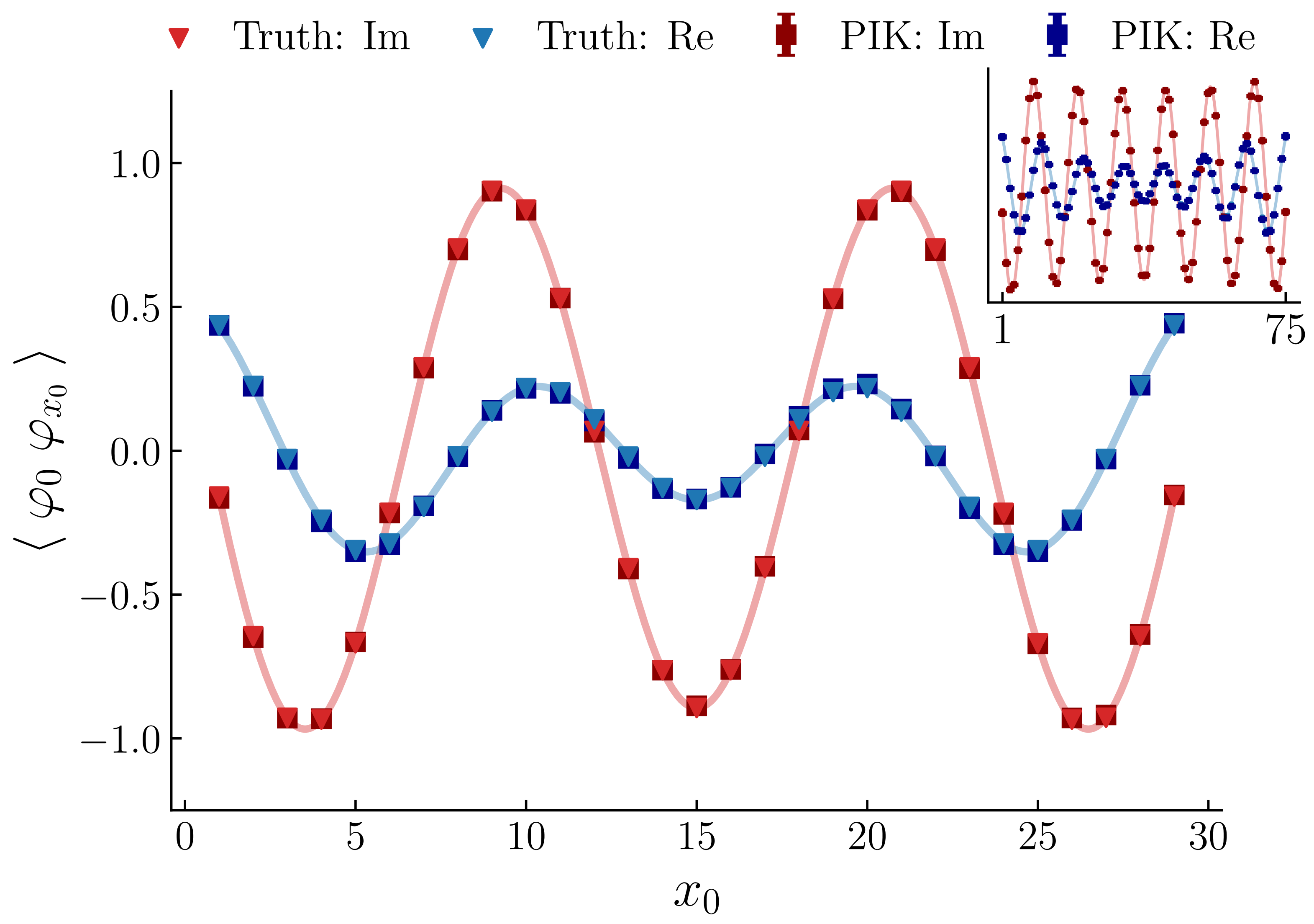
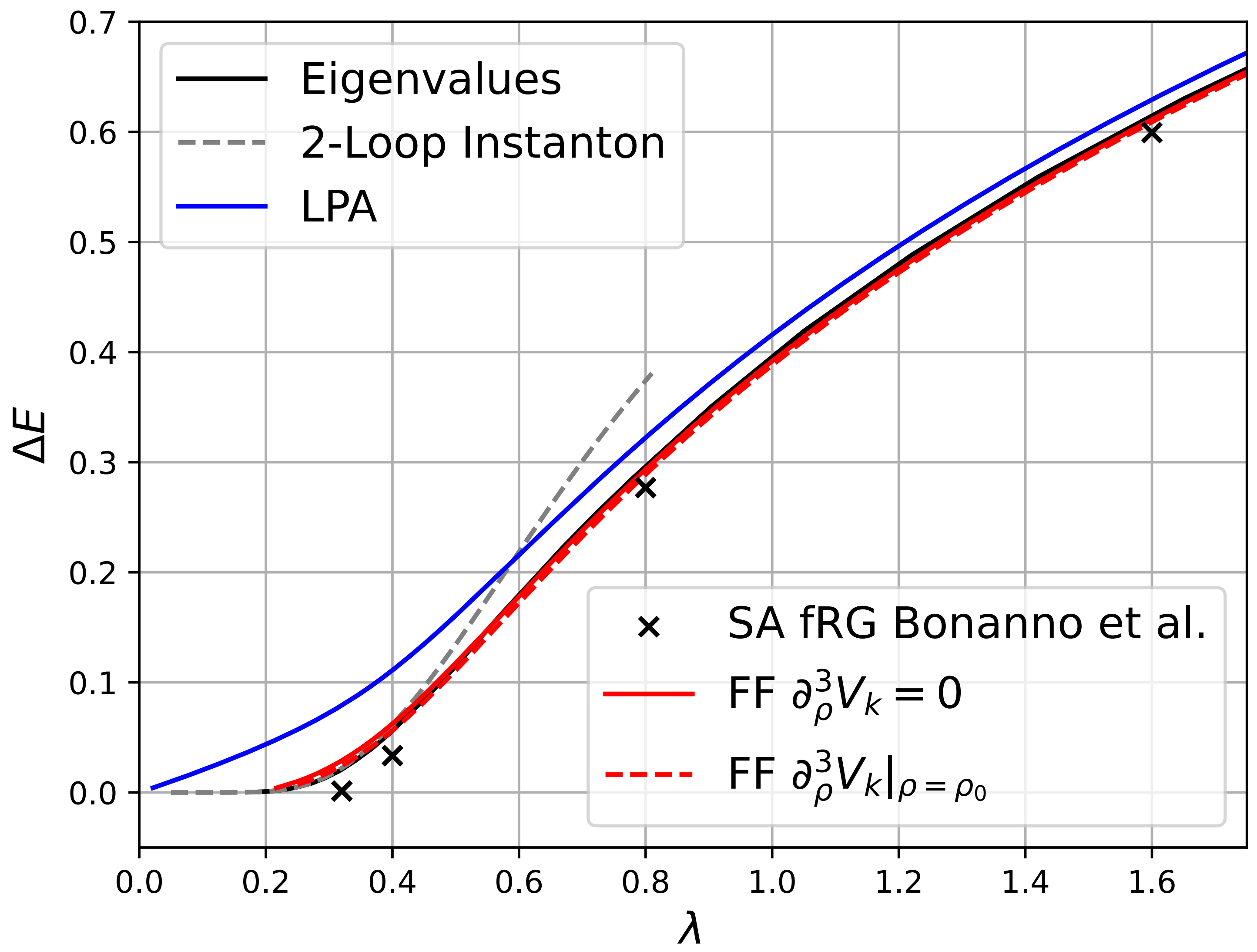
Applications of the Physics-Informed Renormalisation Group

From sampling to optimisation





Friederike Ihssen, Ruhr-Universität Bochum

05.09 - ERG 2026

Exact Renormalisation Group

$\leftrightarrow$

Finite computable approximation

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Finite computable approximation

Controlled error estimates

$$\partial_t \Gamma_k^{(1)} = \text{Flow}_1 \left[\Gamma_k^{(2)}, \Gamma_k^{(3)}\right]$$

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Relation between different schemes

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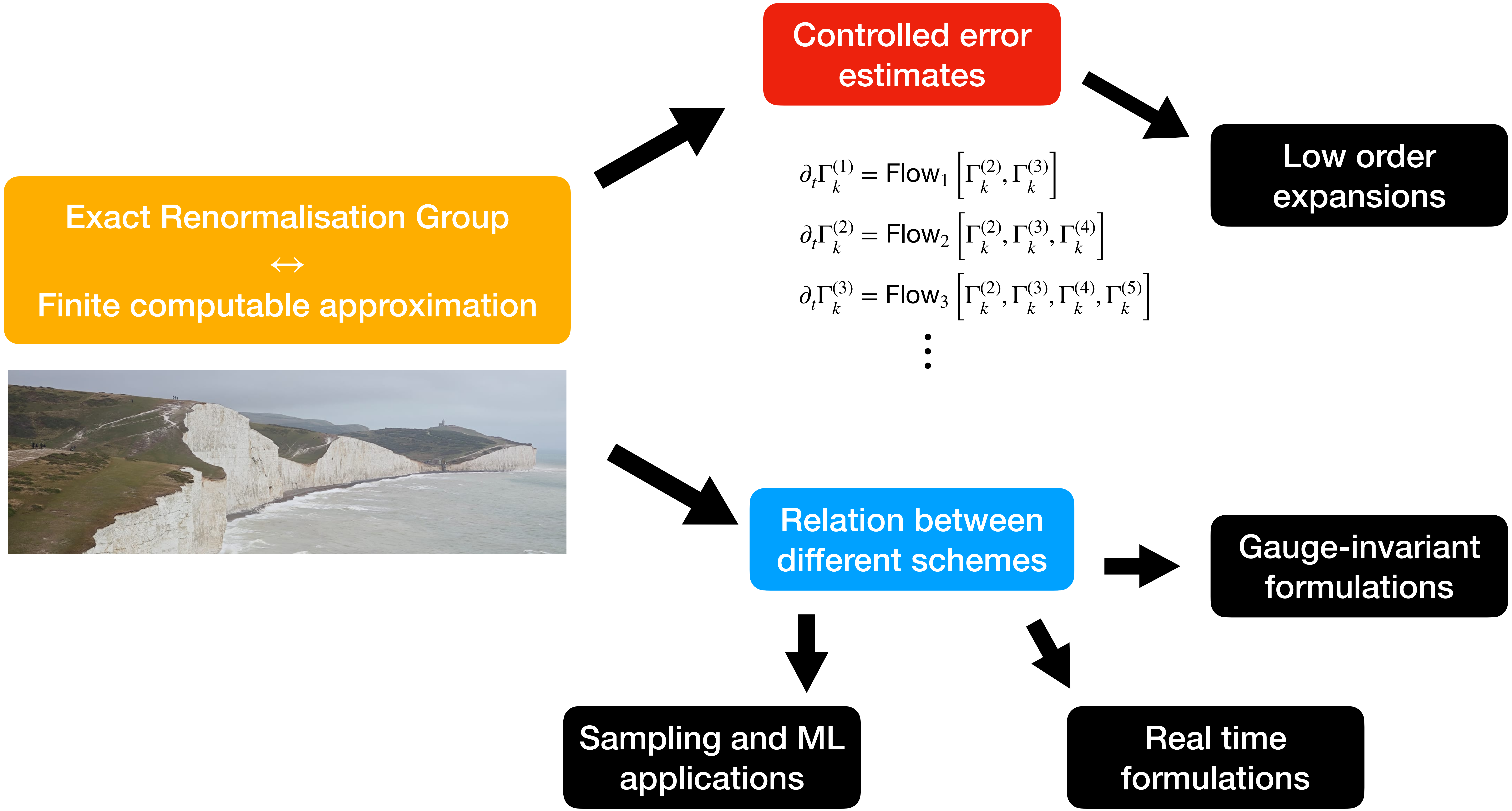

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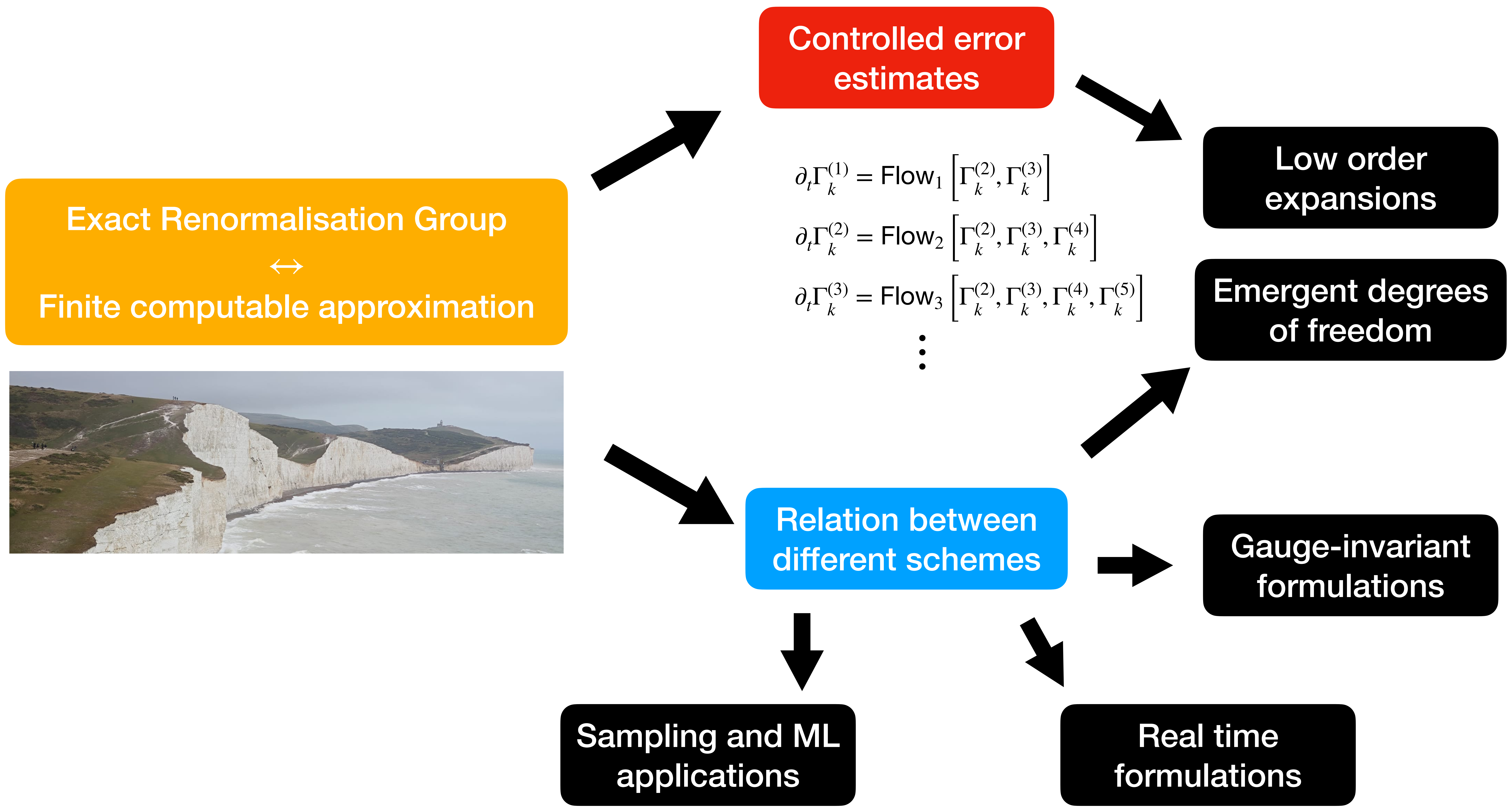
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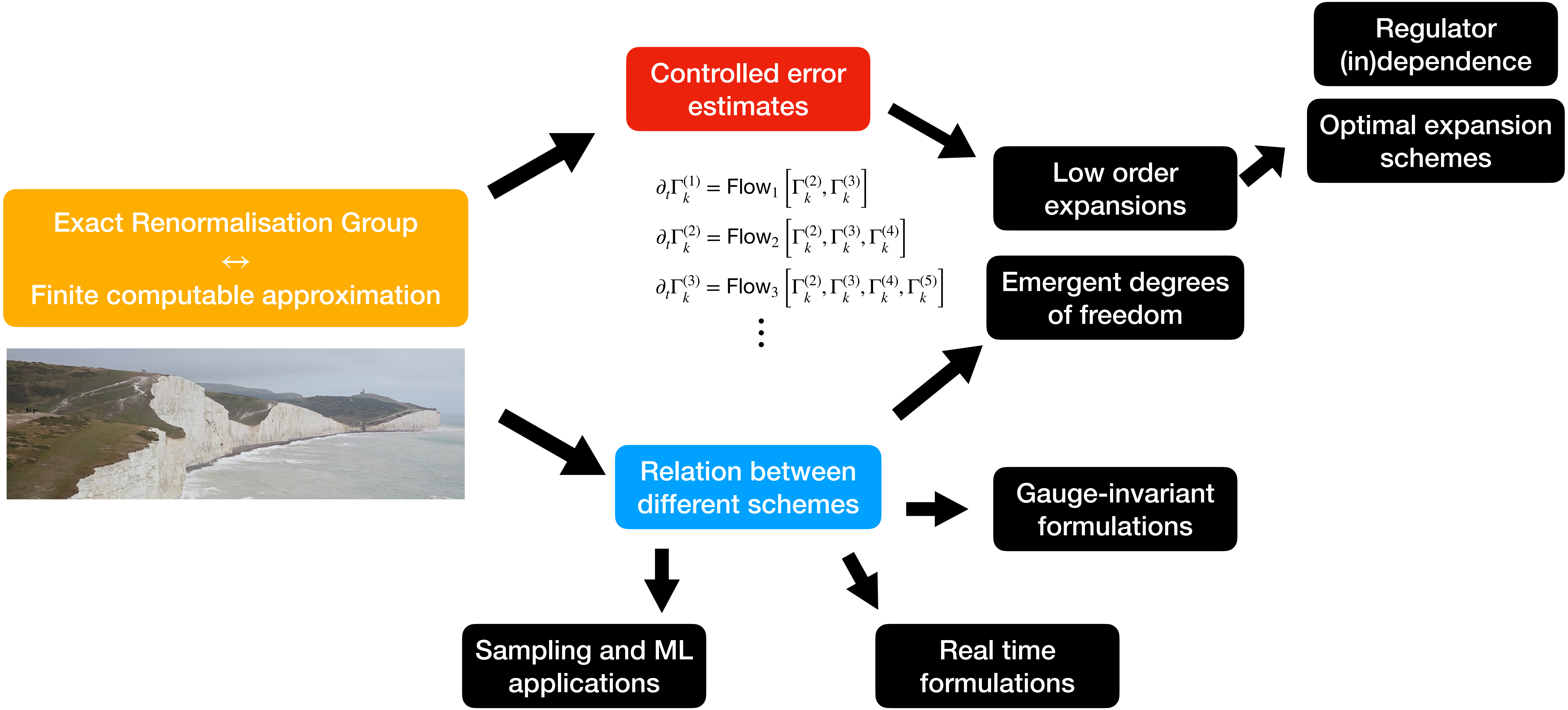

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Optimal expansion schemes
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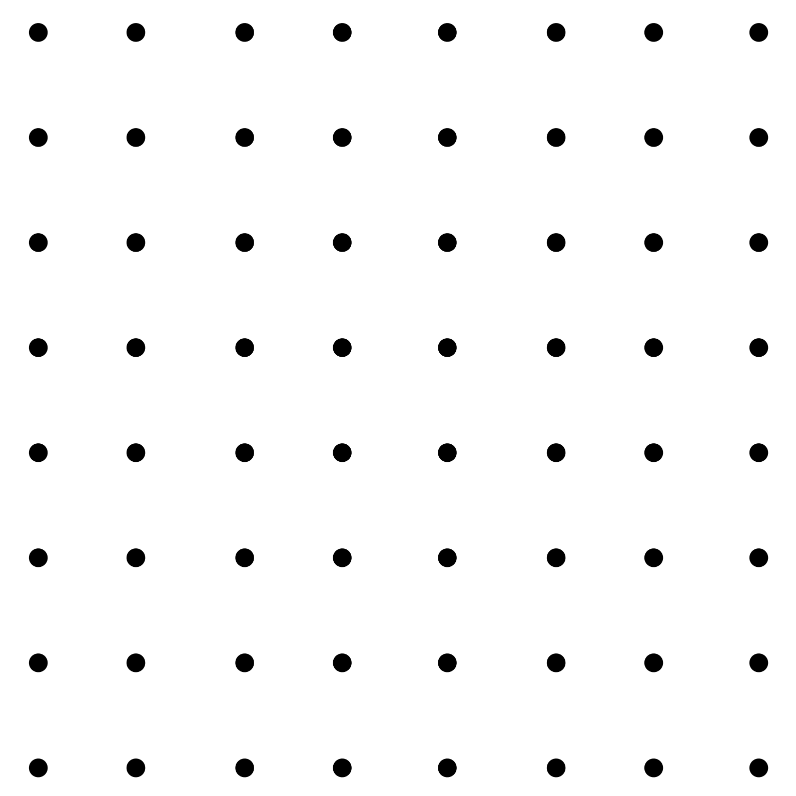
Real time formulations

The RG as a transport task

Lattice I



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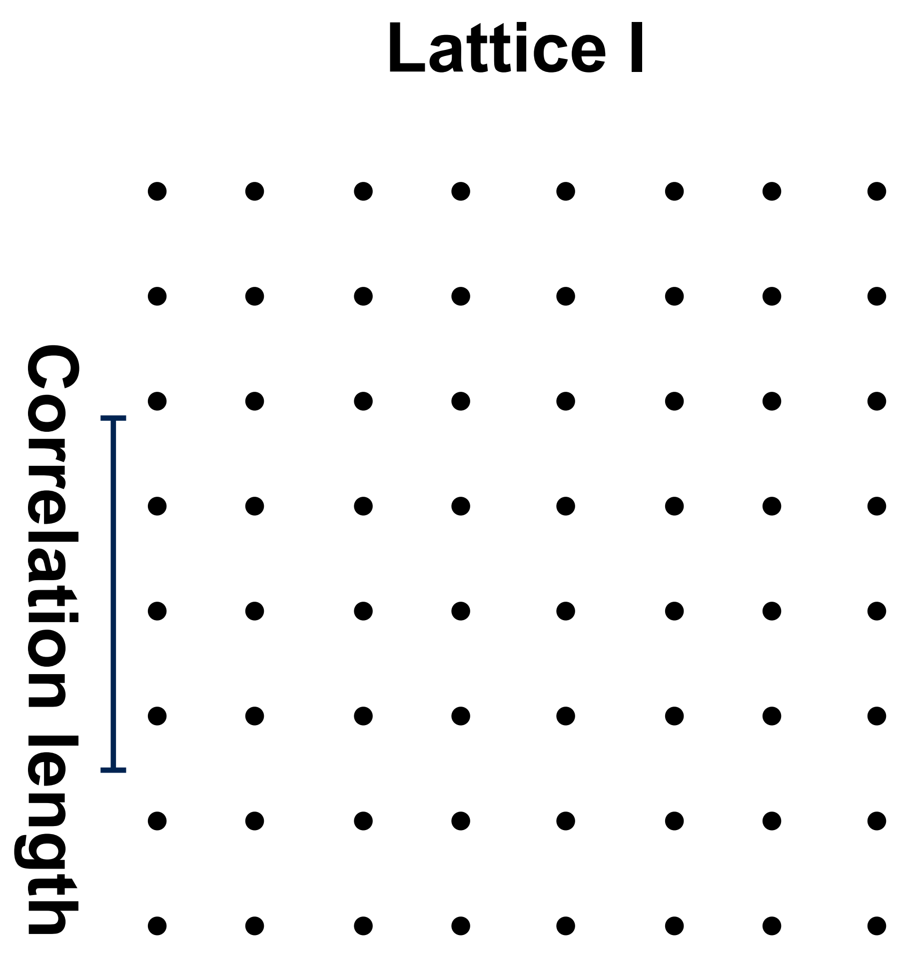
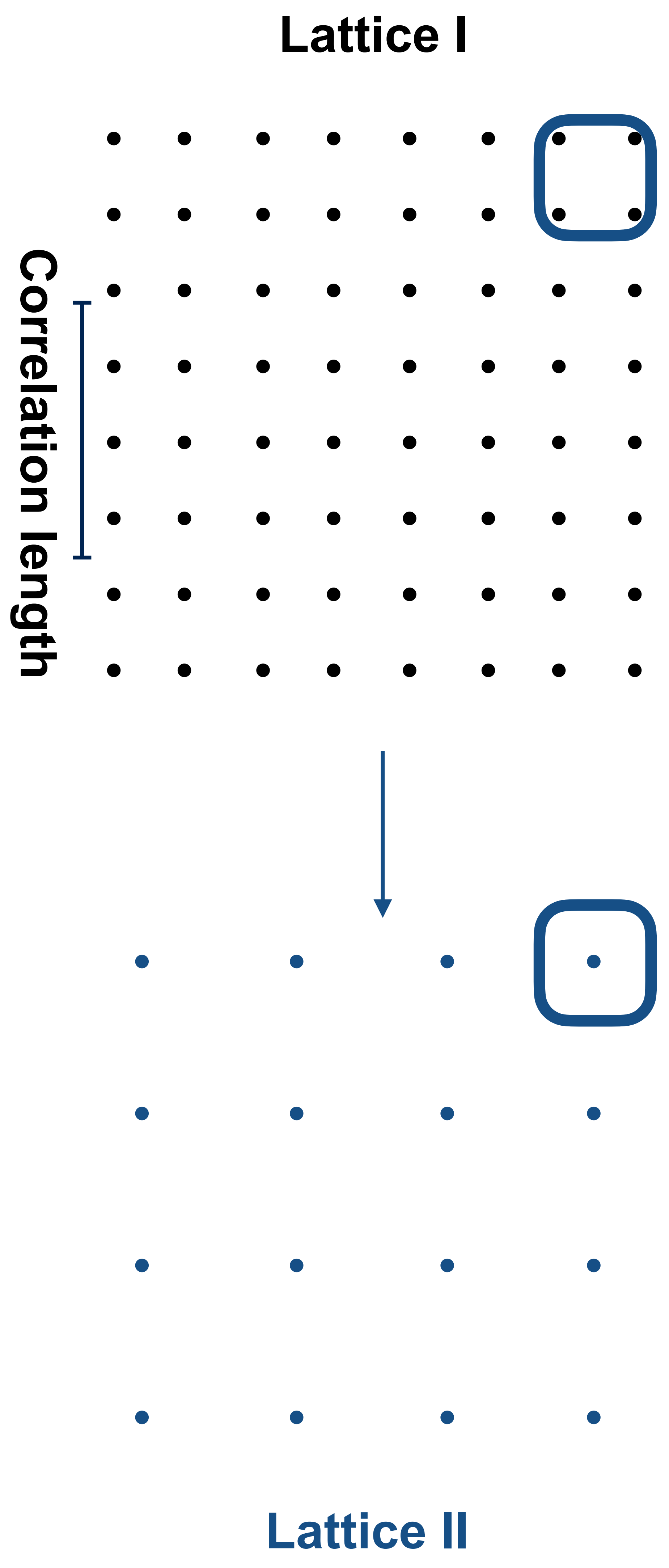


The RG as a transport task

- Original idea: Compression of information under repeated block-spinning transformations



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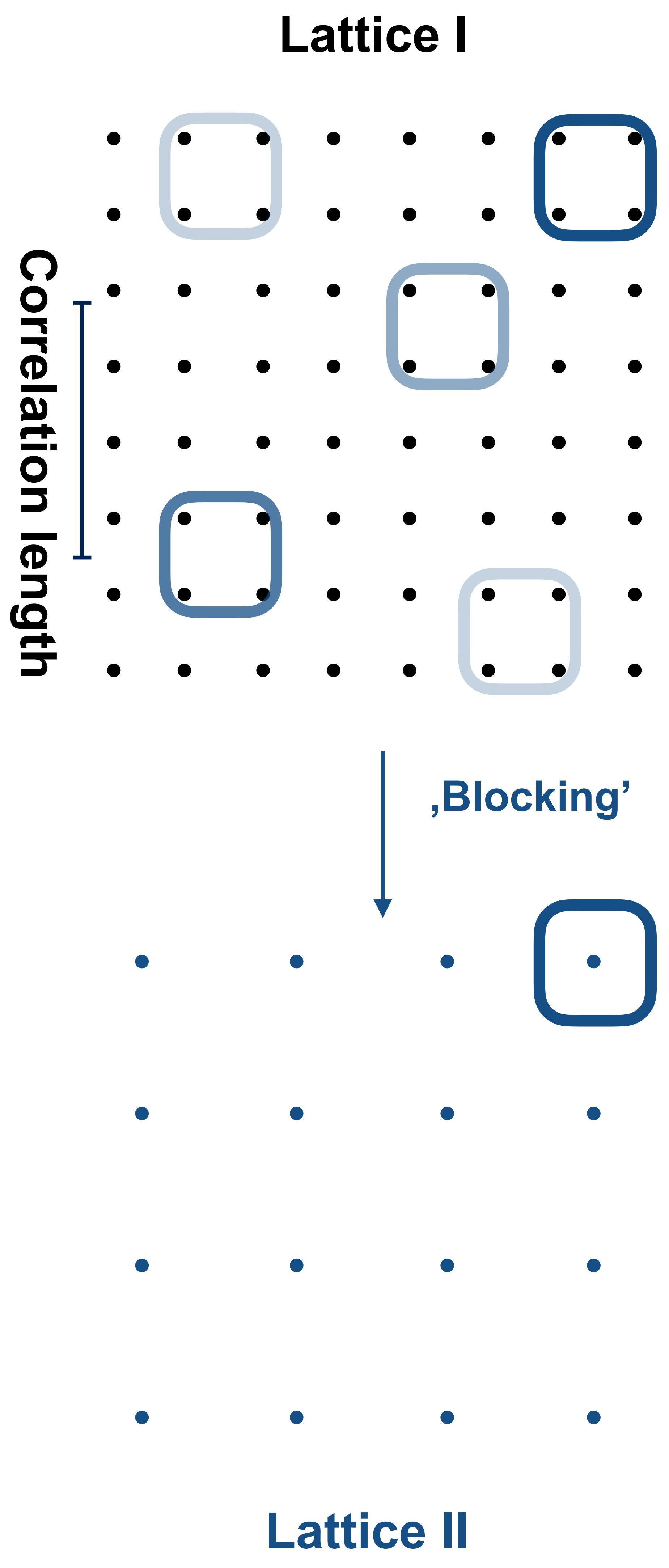
- Original idea: Compression of information under repeated block-spinning transformations $\dot{\phi} : \phi_i(x) \mapsto \phi_{i+1}(x)$



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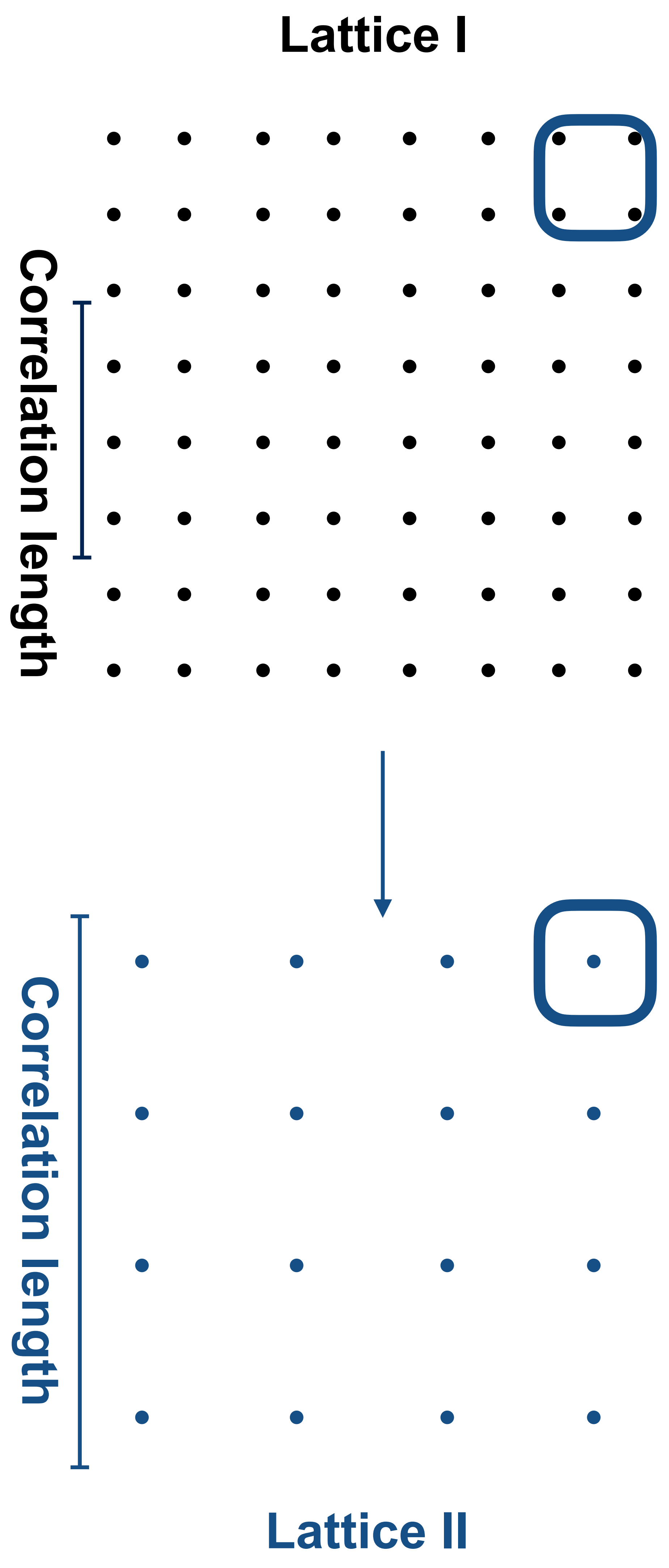

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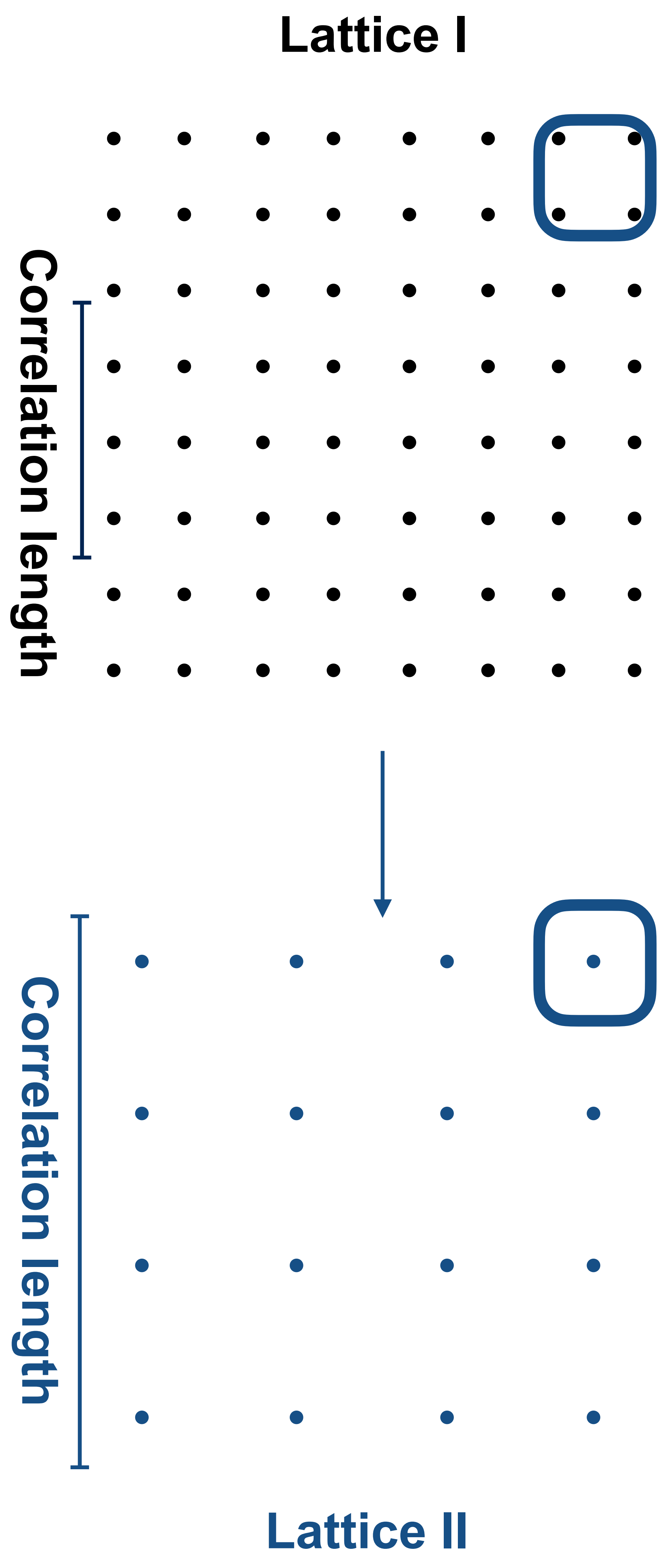

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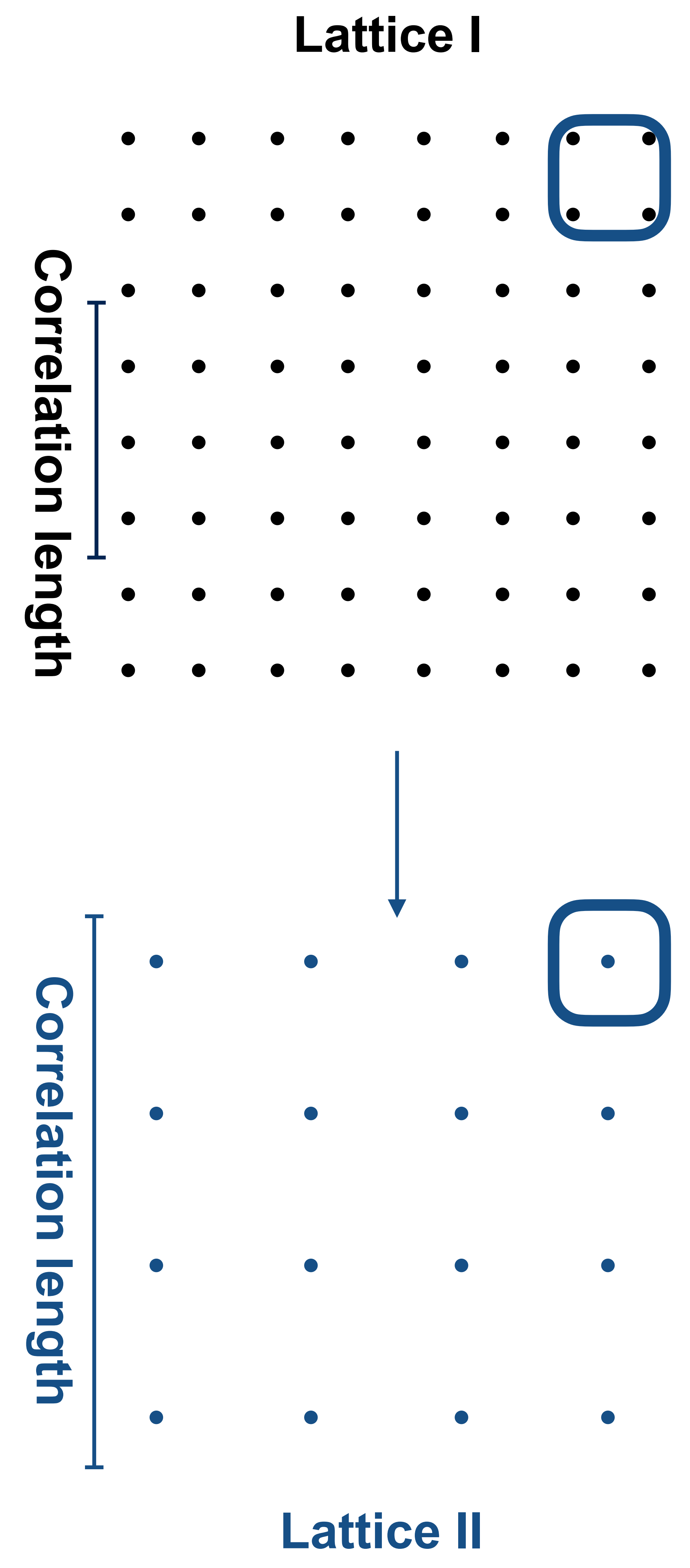

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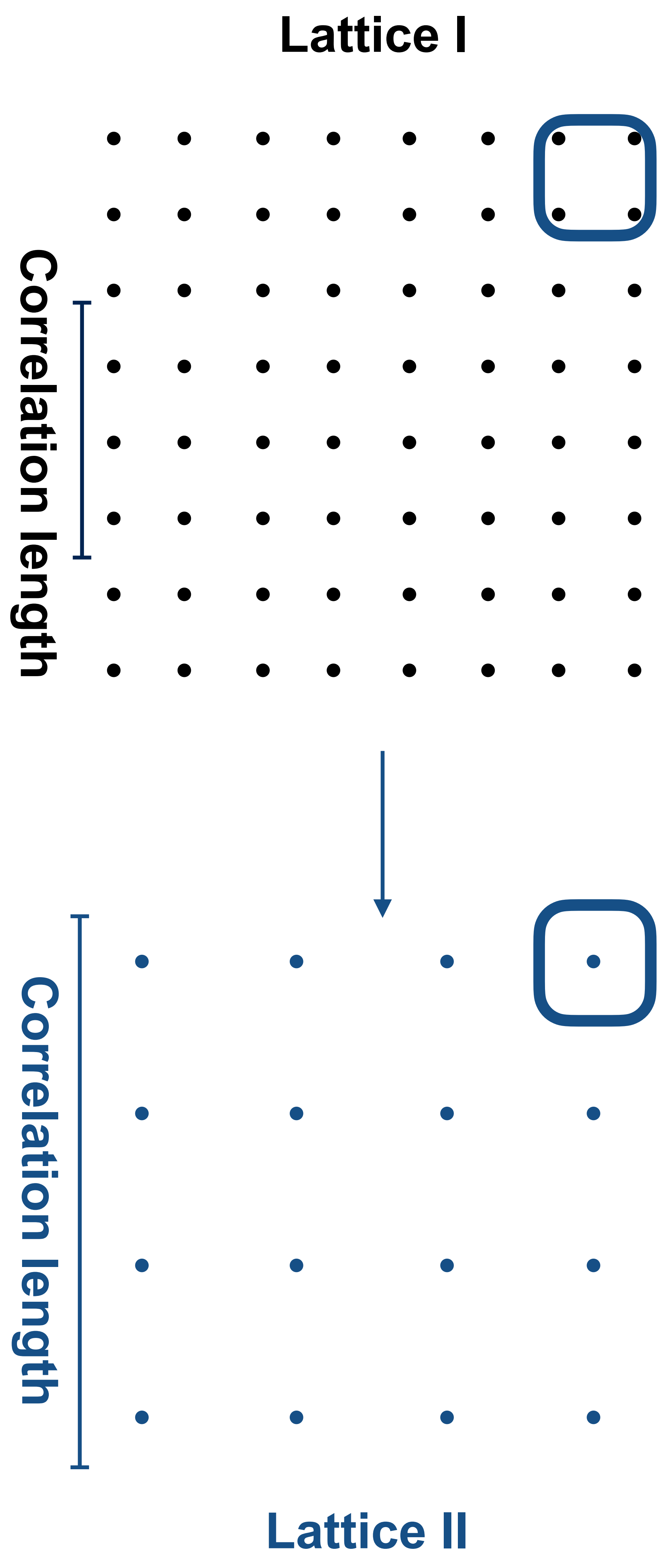

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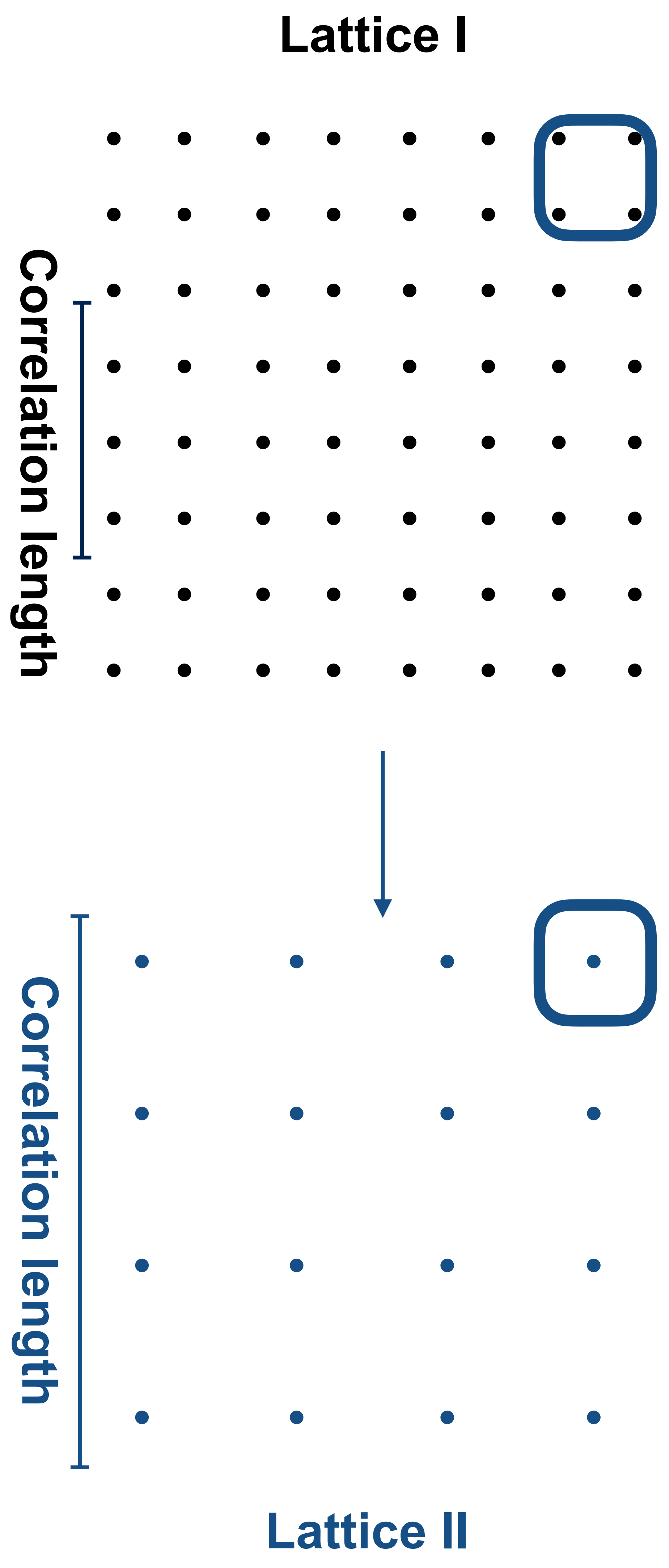

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Wegner flow

Wegner, J. Phys. C 7 (1974) 2098

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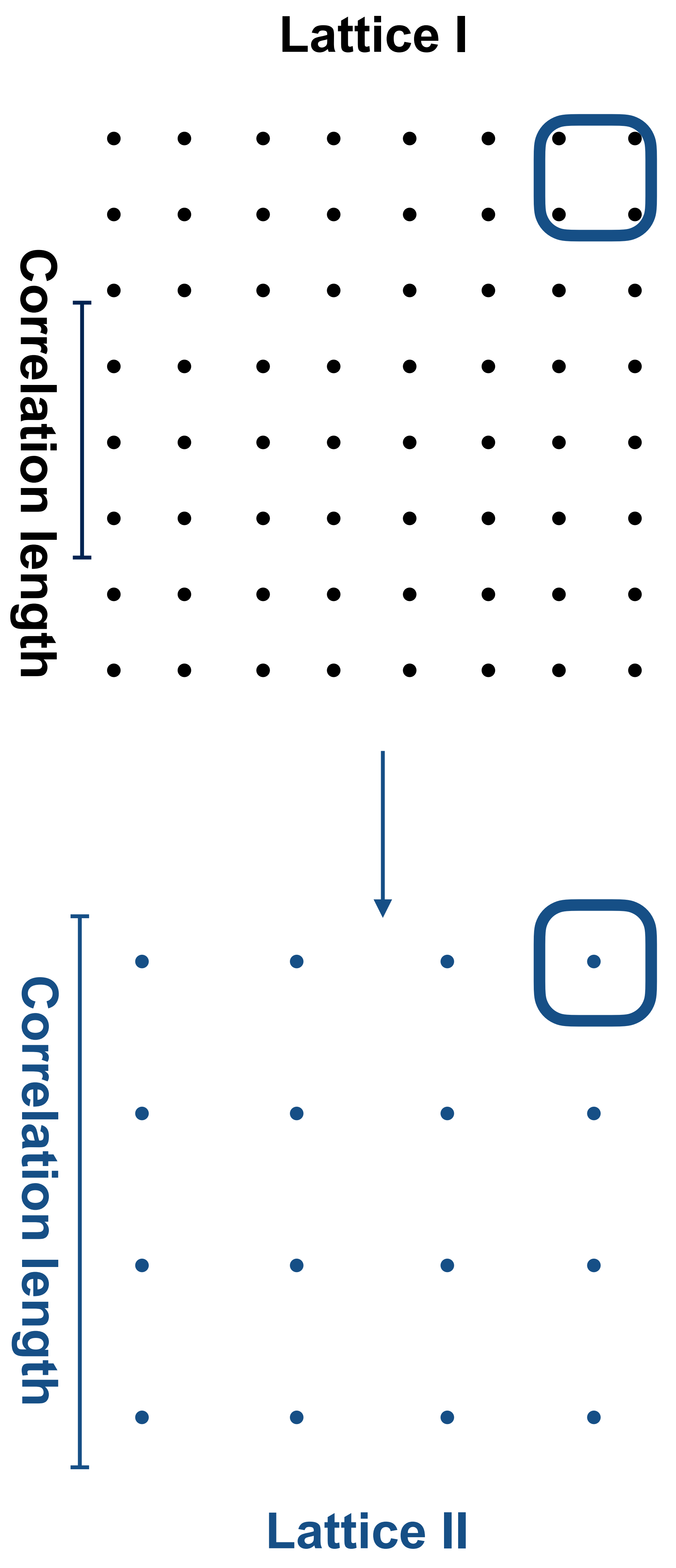

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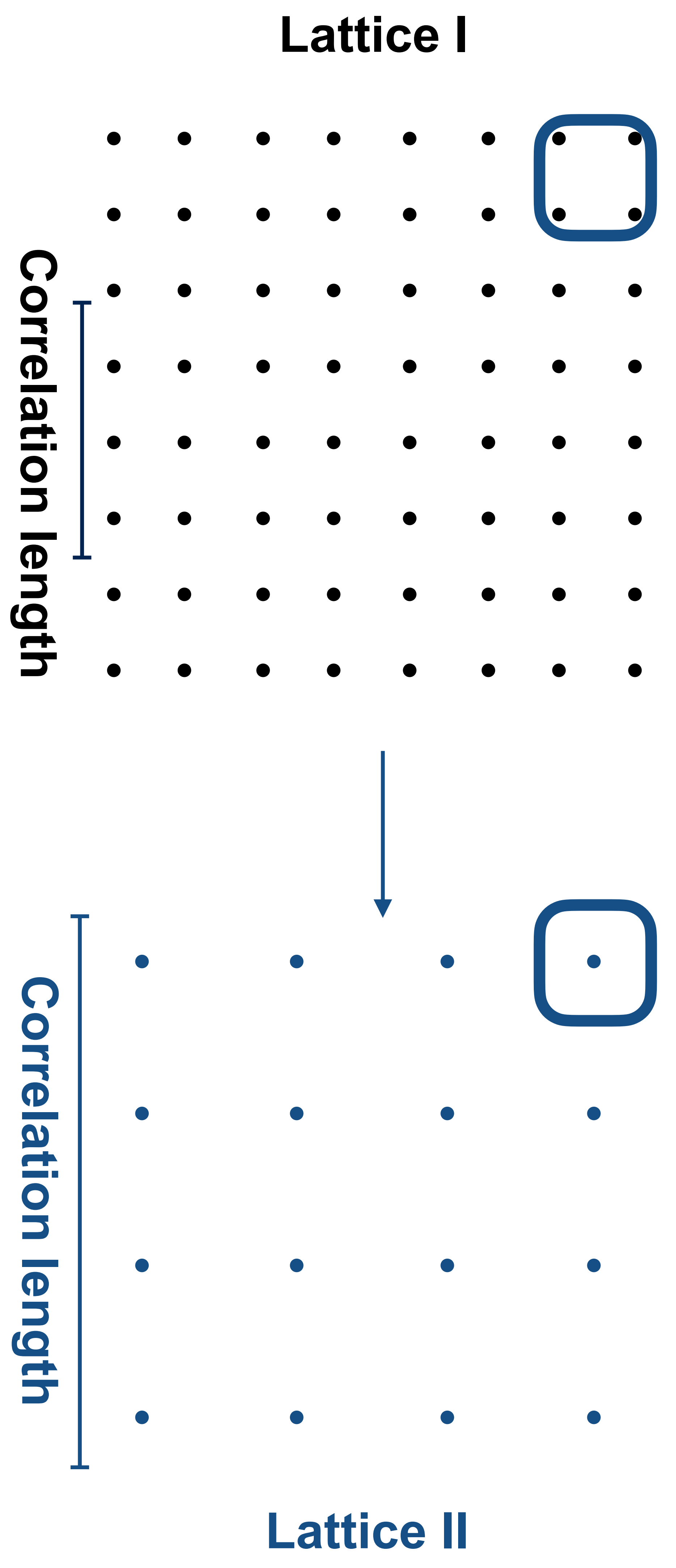

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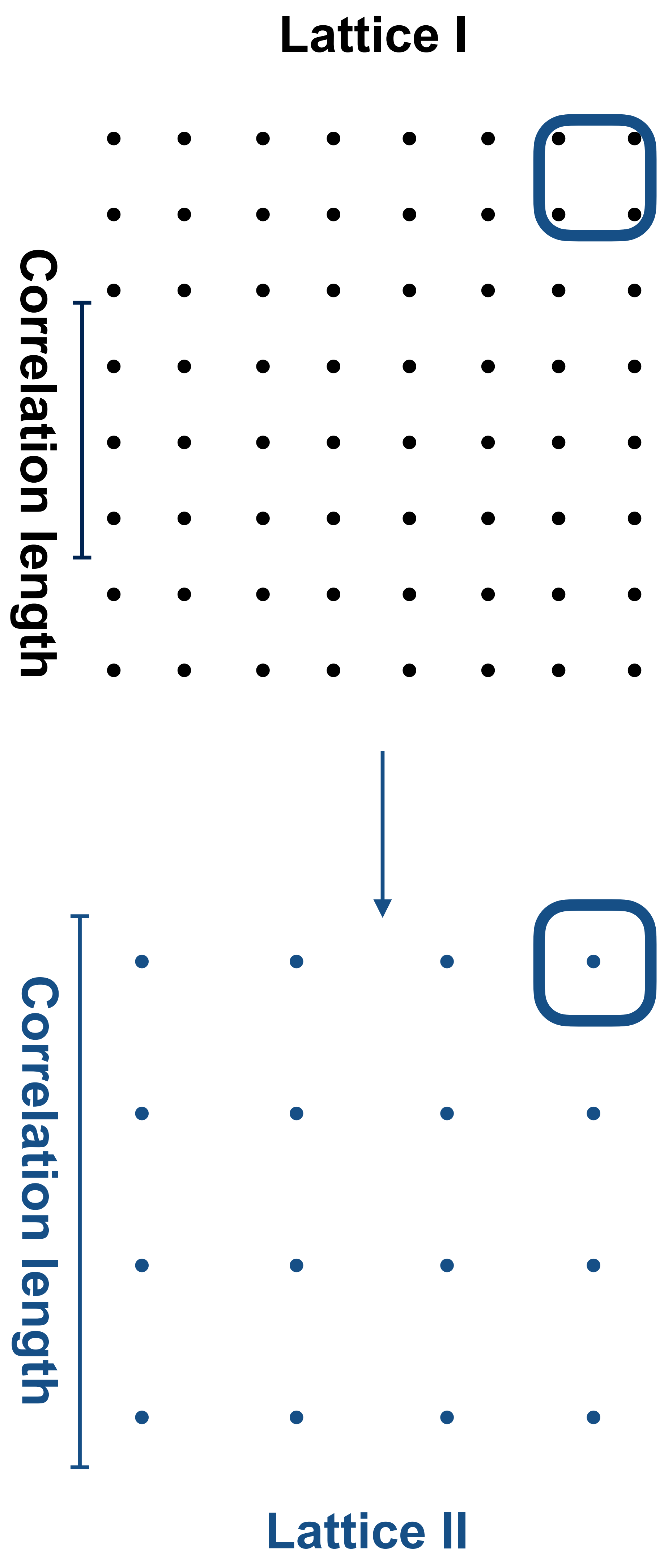

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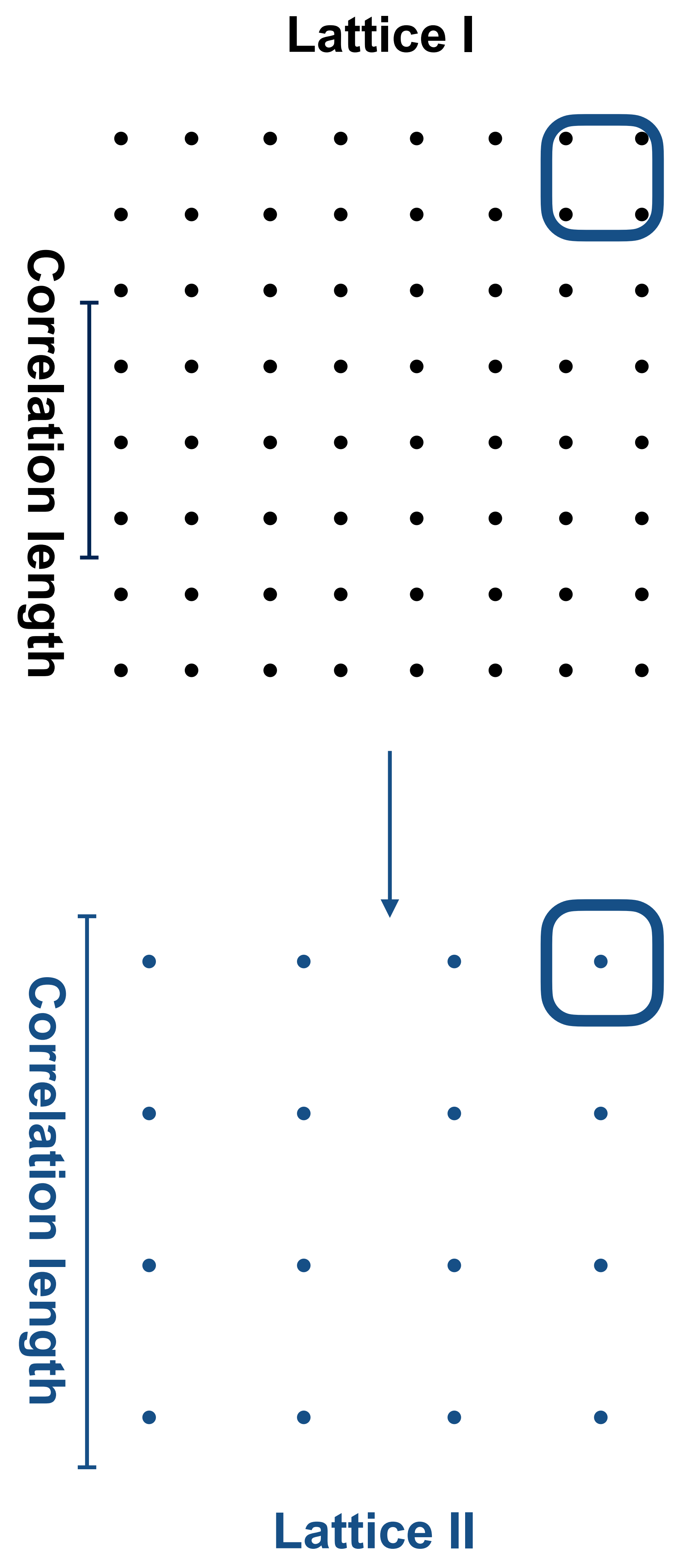

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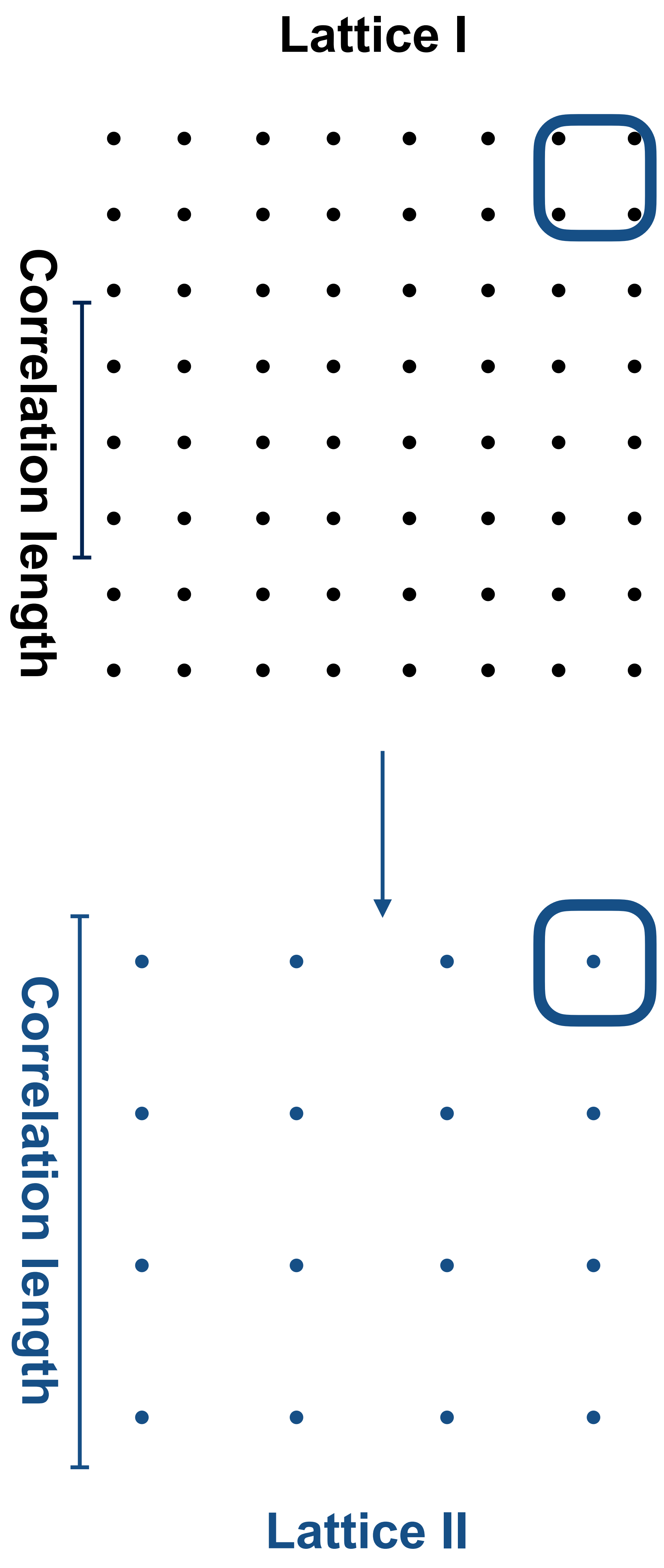

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Diffusion

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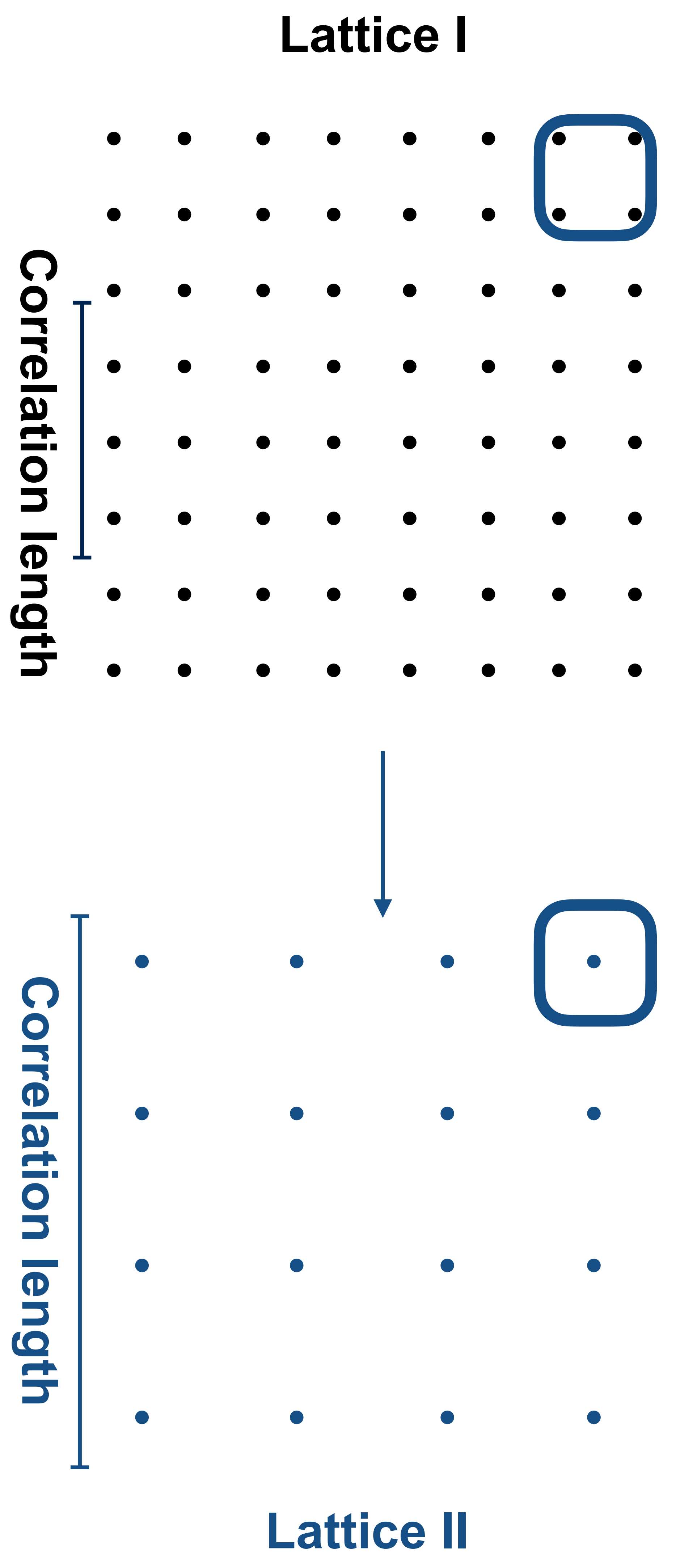

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Scale transformations

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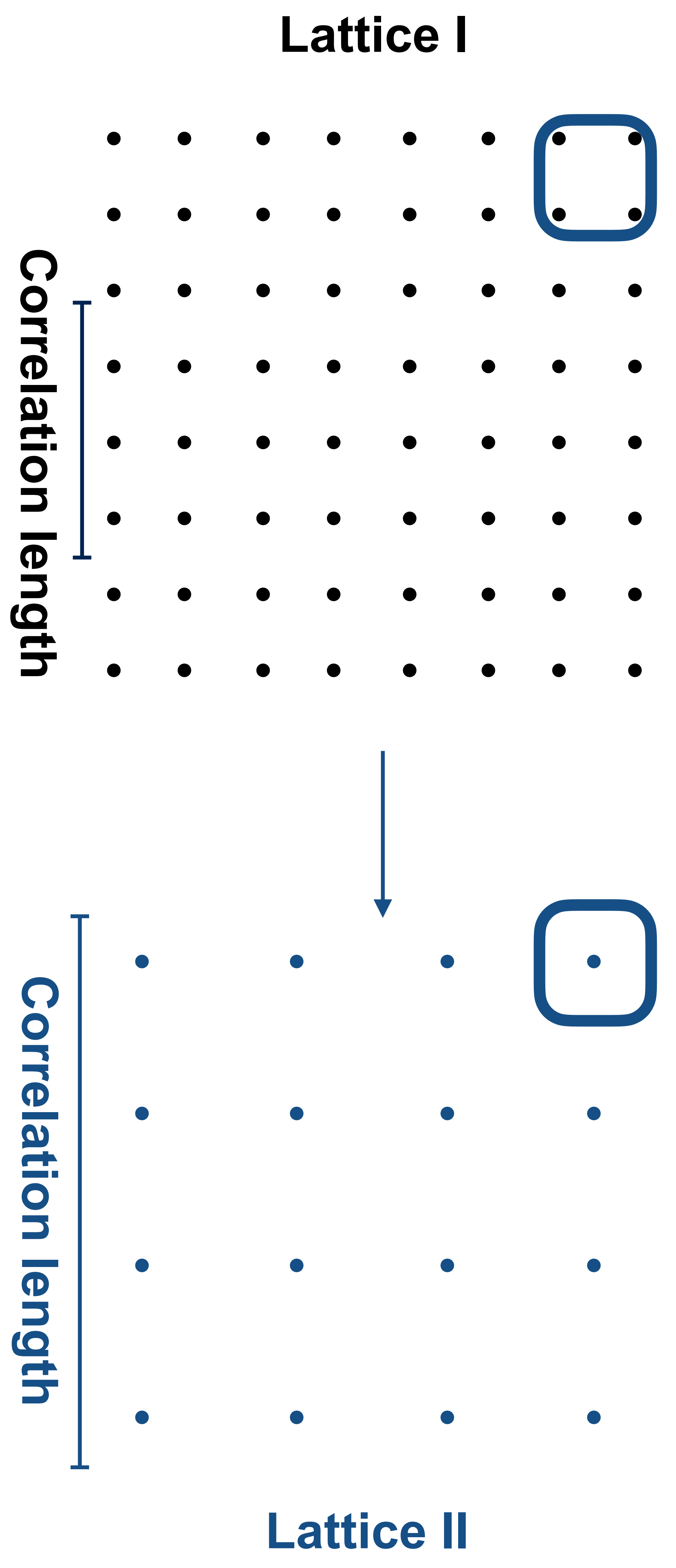

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Additional Reparametrisations

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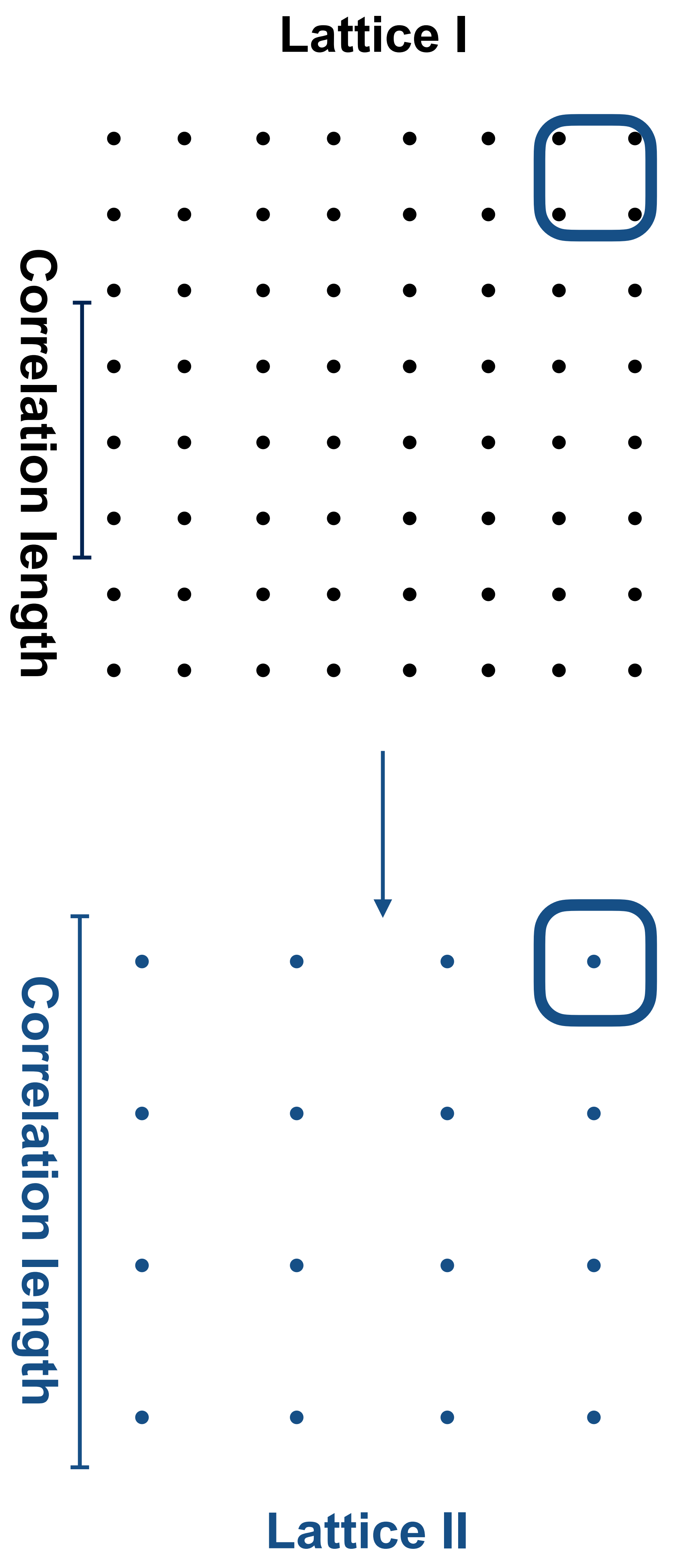

Scale transformations in the path integral

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Consider the generating functional for a theory: $$Z_t[J] = \int D\phi\, p_t[\phi]\, e^{J\,\hat{O}_{\phi,t}[\phi]}$$

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Physics-informed setup

Scale transformations in the path integral

Consider the generating functional for a theory: $$Z_t[J] = \int D\phi \, p_t[\phi] \, e^{J \hat{O}_{\phi,t}[\phi]}$$

Current coupled to some generic operator

Some effective degree of freedom

Scale dependent distribution: $p_t[\phi] = \frac{1}{\mathcal{N}} e^{-S_t[\phi]}$

e.g. $S_t = S + \Delta S_t$

ϕ is in the **co-moving coordinate frame** of the block spinning, i.e. $\partial_t \phi = 0$ (passive transformation)

→ Use the scale dependent (composite) operator: $\hat{O}_{\phi,t}[\phi]$ as a **book-keeping device** $\hat{O}_{\phi,t_0}[\phi] = \phi$

Physics-informed setup

$$\partial_t Z_t[J] = \int D\phi \left\{ \partial_t p_t[\phi] + J \, \partial_t \hat{O}_{\phi,t} \, p_t[\phi] \right\} e^{J \hat{O}_{\phi,t}[\phi]}$$

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Modifications in fRG

Sampling with the RG-flow

FI, Kapust, Pawlowski, arxiv:2510.26678
FI, Kapust, Pawlowski, arxiv:2603.03159

Sampling with the RG-flow

- We use $\partial_t Z = 0$ to **reconstruct the coarse-graining map**

📄 **FI,** Kapust, Pawlowski, arxiv:2510.26678
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t_0 RG-Flow $\partial_t p_t \neq 0$, with passive transformation $\partial_t \phi = 0$ t_1

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Integrating $\dot{\phi}[\phi]$ for microscopic transformations gives us a map for the field transformation

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Map to $t = t_1$

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- Langevin evolution
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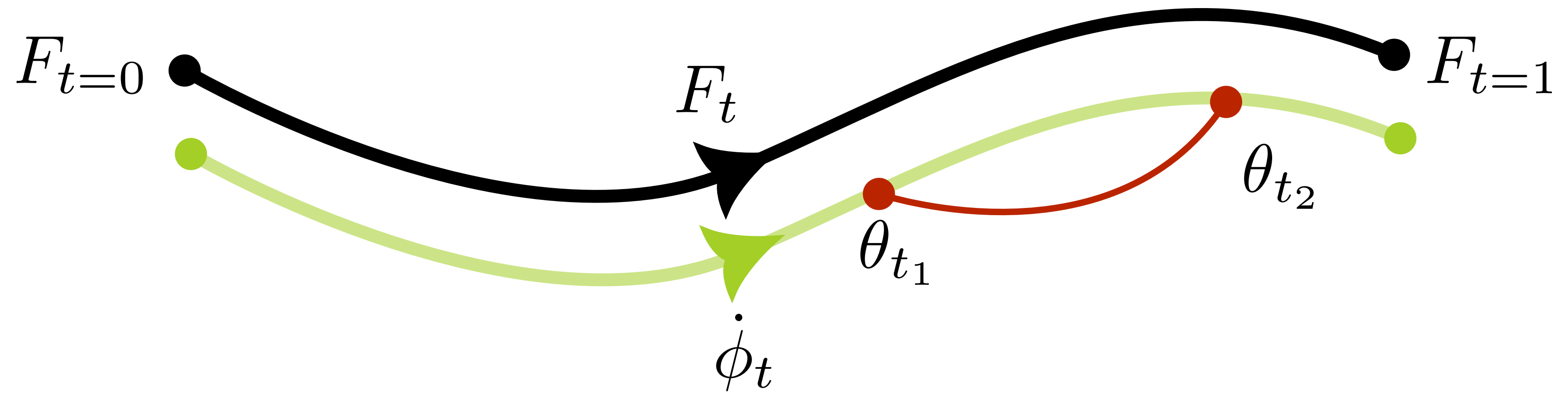
The physics-informed mindset

The physics-informed mindset

Allowing general time-dependent paths for distributions
$F_t = \{S_t, p_t, \ldots\}$ does not guarantee the desired physical distribution



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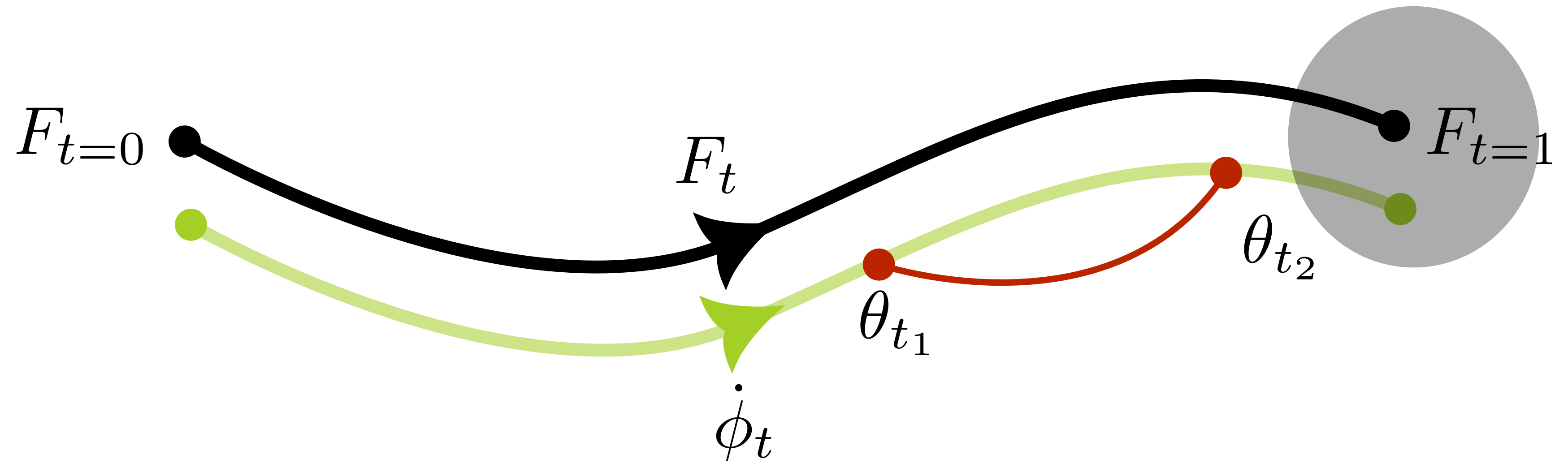

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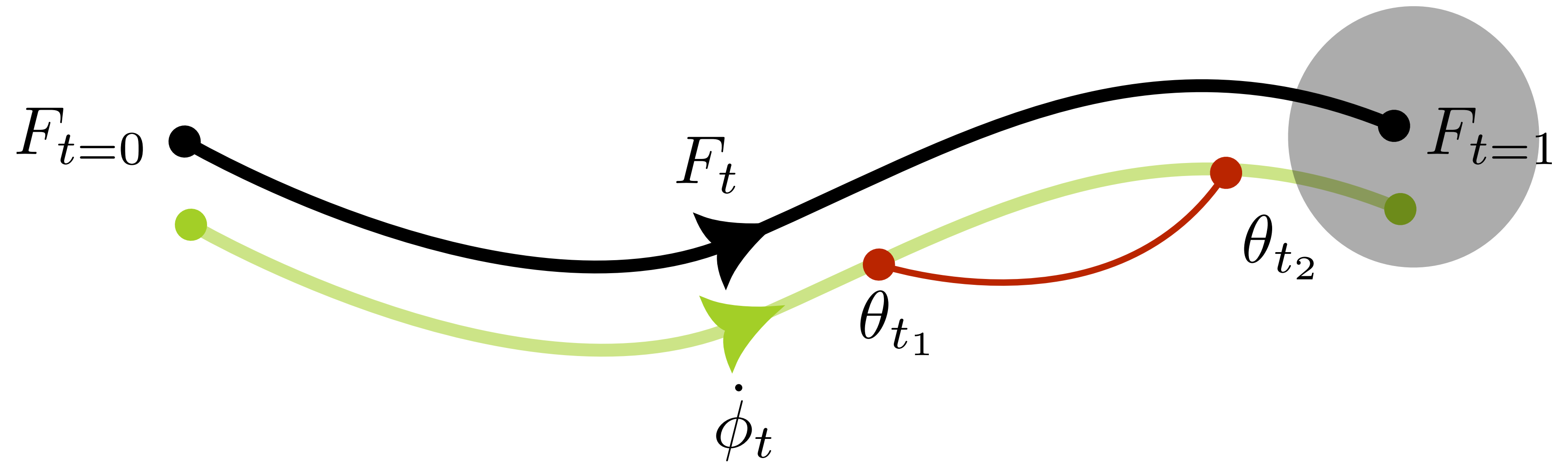

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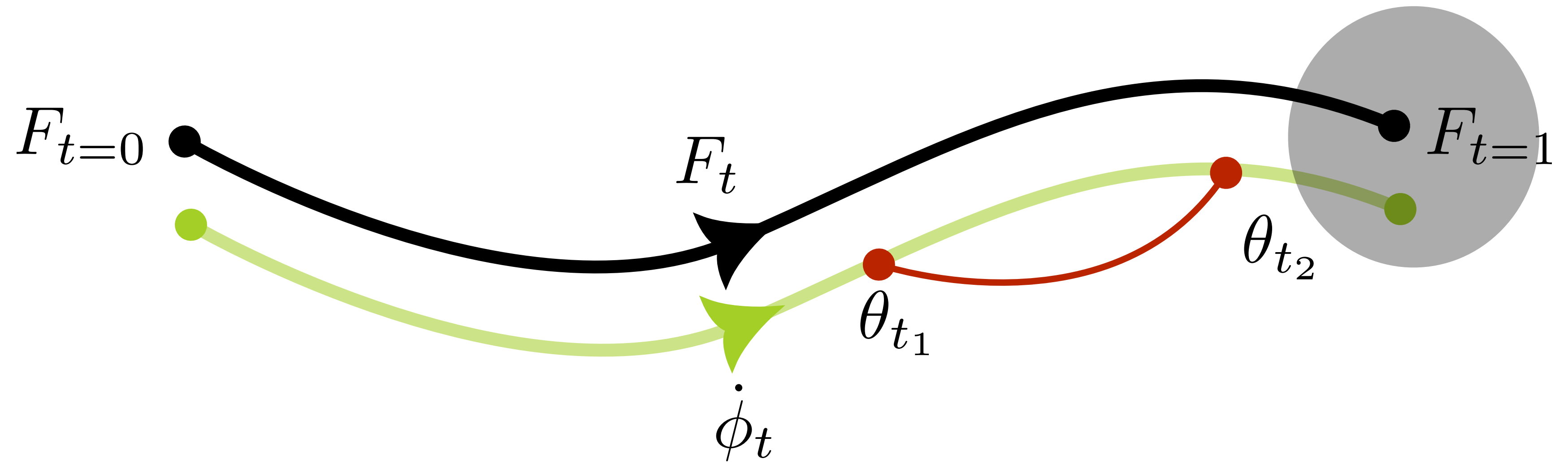

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…so we impose a trajectory analytically. Then the Wegner equation

$$\partial_t S_t + \dot{\phi}\frac{\delta}{\delta\phi} S_t = \frac{\delta}{\delta\phi}\dot{\phi}$$

…is a differential equation at every time-slice!

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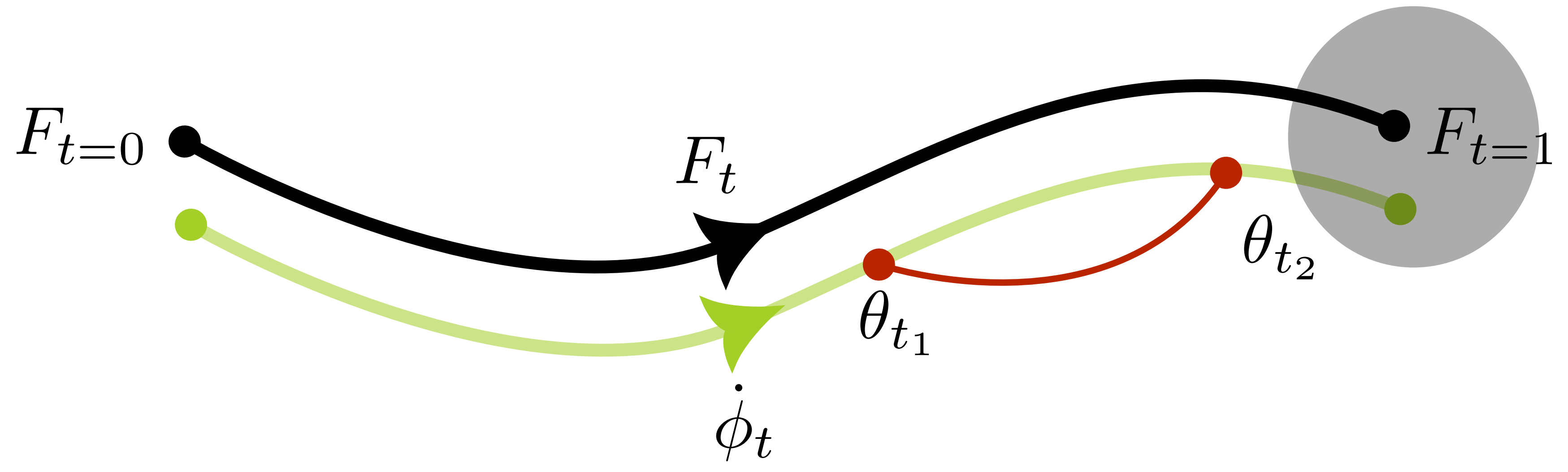

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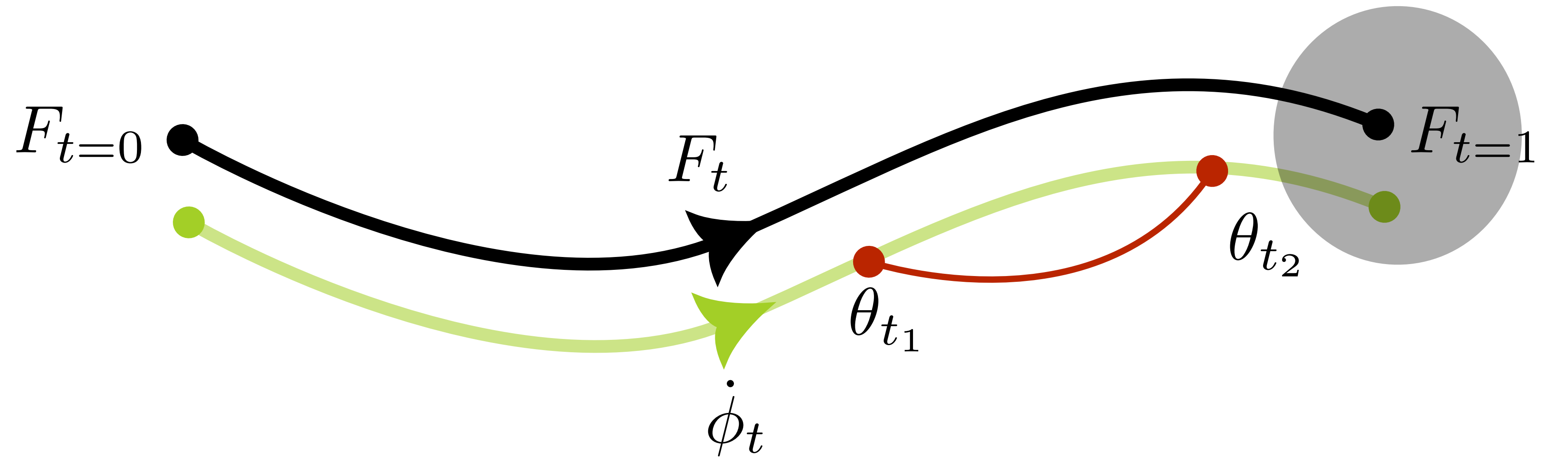

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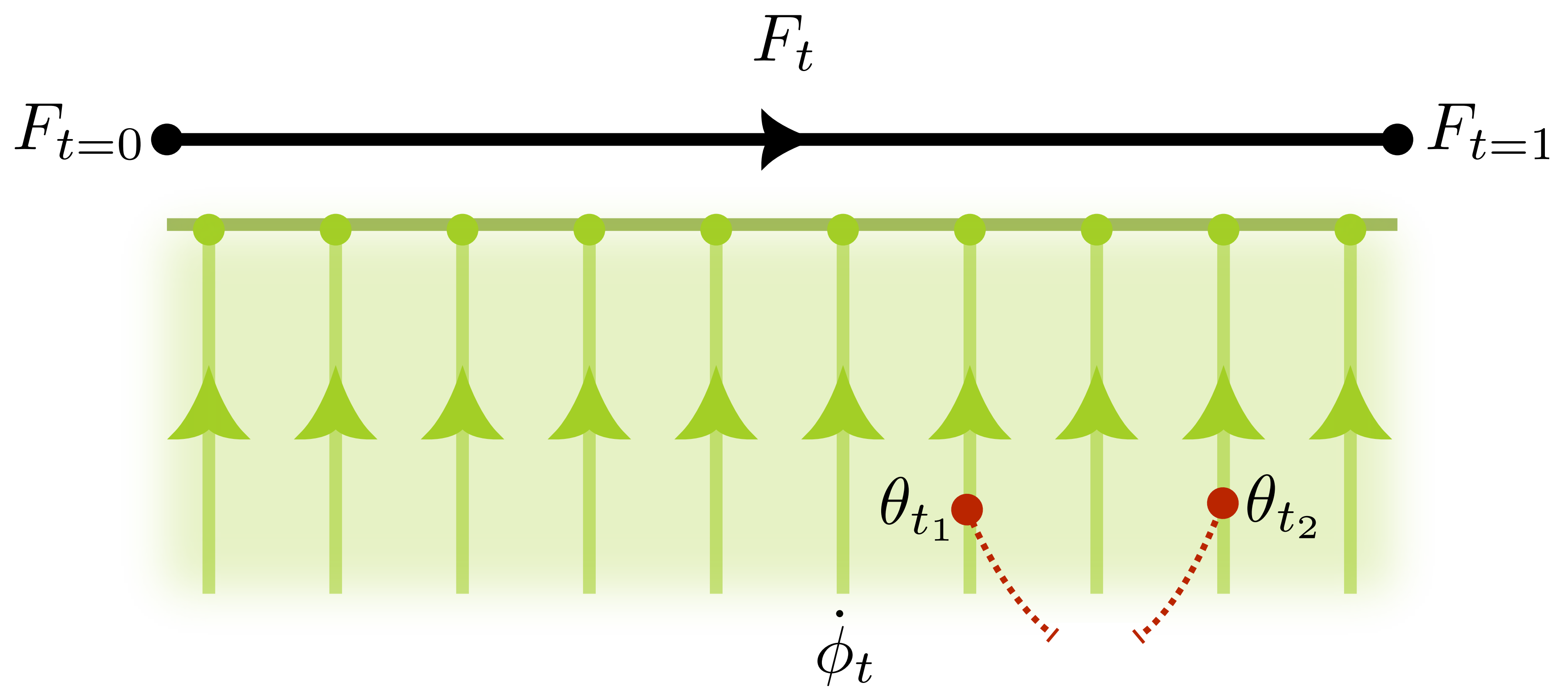

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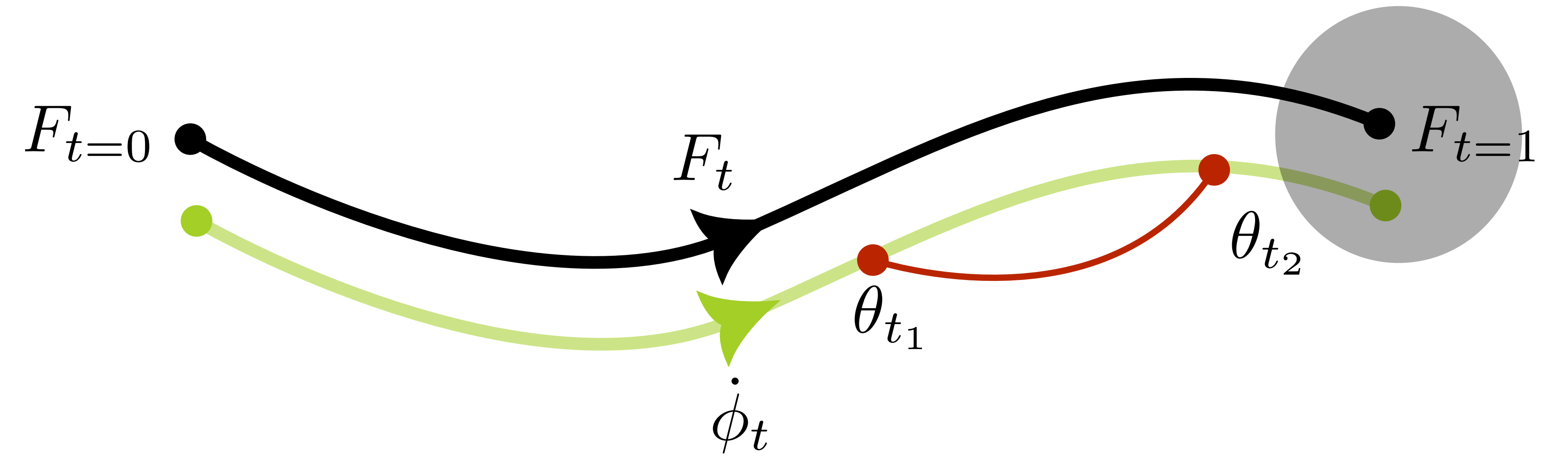

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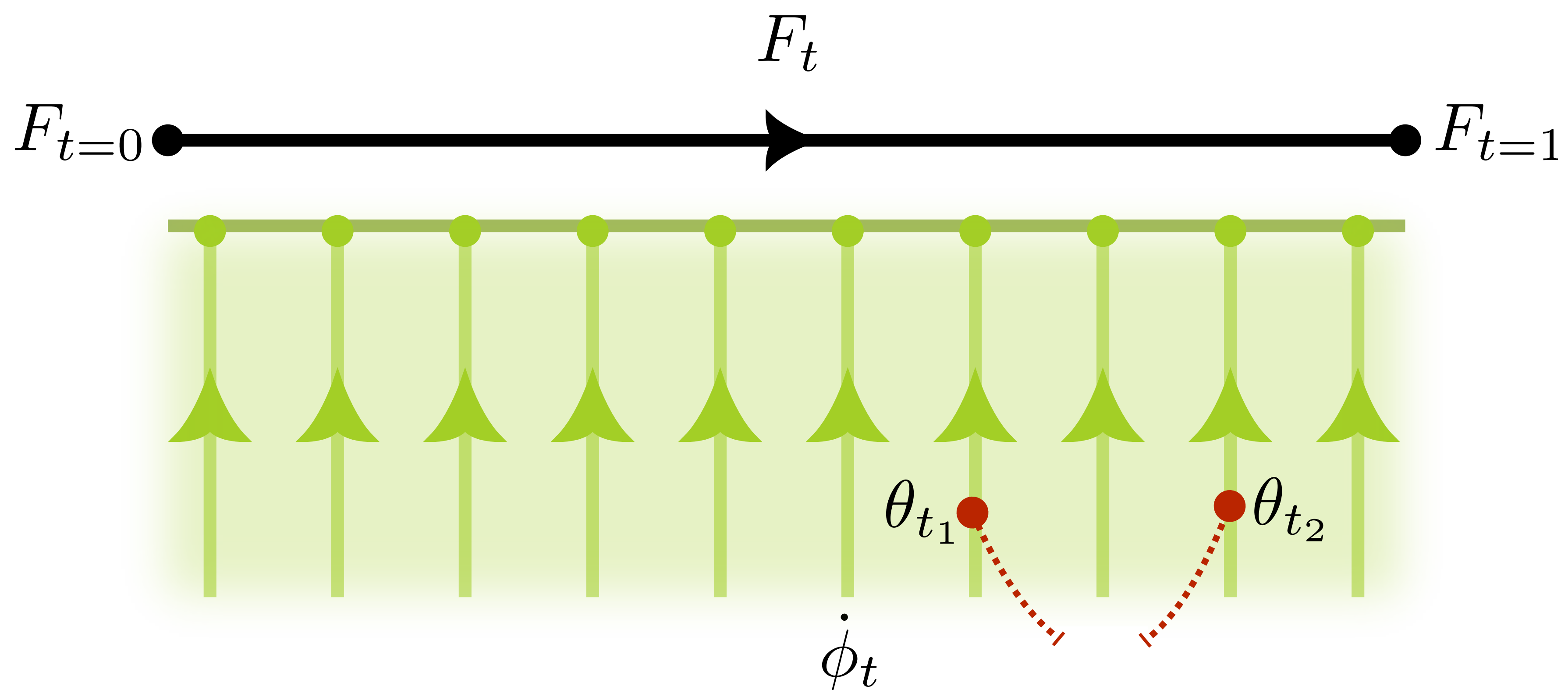

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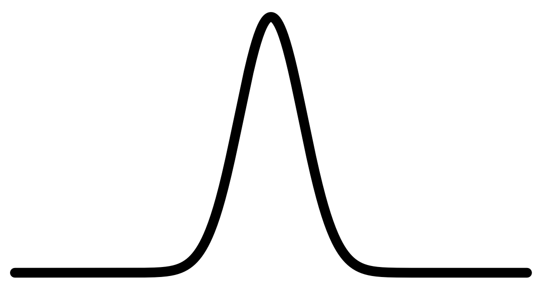
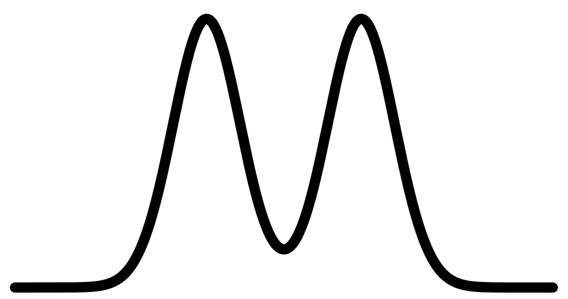
Example: zero-dimension ϕ^4 - theory

$$\hat{S}_t(\phi) = \frac{1}{2}m^2(t)\phi^2 + \frac{\lambda(t)}{4}\phi^4$$

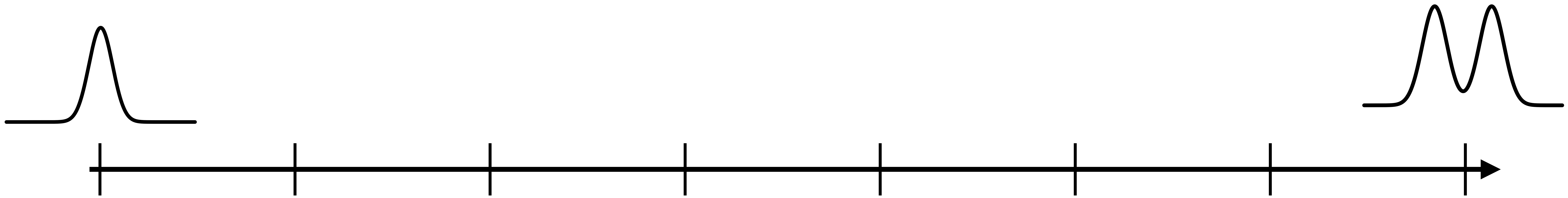


Task: solve an ODE in ϕ at every time-slice



$\left(m_0^2\,,\,\lambda_0\right) = (1\,,\,0)$ → Analytic interpolation between couplings → $\left(m_1^2\,,\,\lambda_1\right) = (-2\,,\,1/2)$

FI, Kapust, Pawłowski, arxiv:2510.26678



$$\left(m_0^2\,,\,\lambda_0\right) = (1\,,\,0) \longrightarrow \left(m_t^2\,,\,\lambda_t\right) = \left(m_0^2 + t\,(m_1^2 - m_0^2)\,,\,\lambda_0 + t(\lambda_1 - \lambda_0)\right) \longrightarrow \left(m_1^2\,,\,\lambda_1\right) = (-2\,,\,1/2)$$

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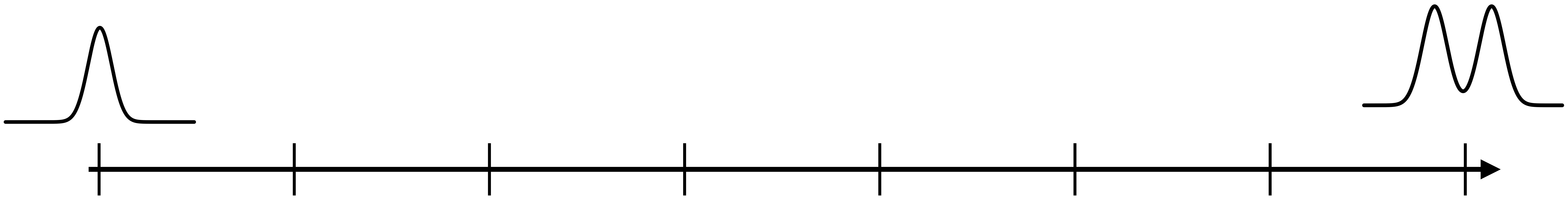

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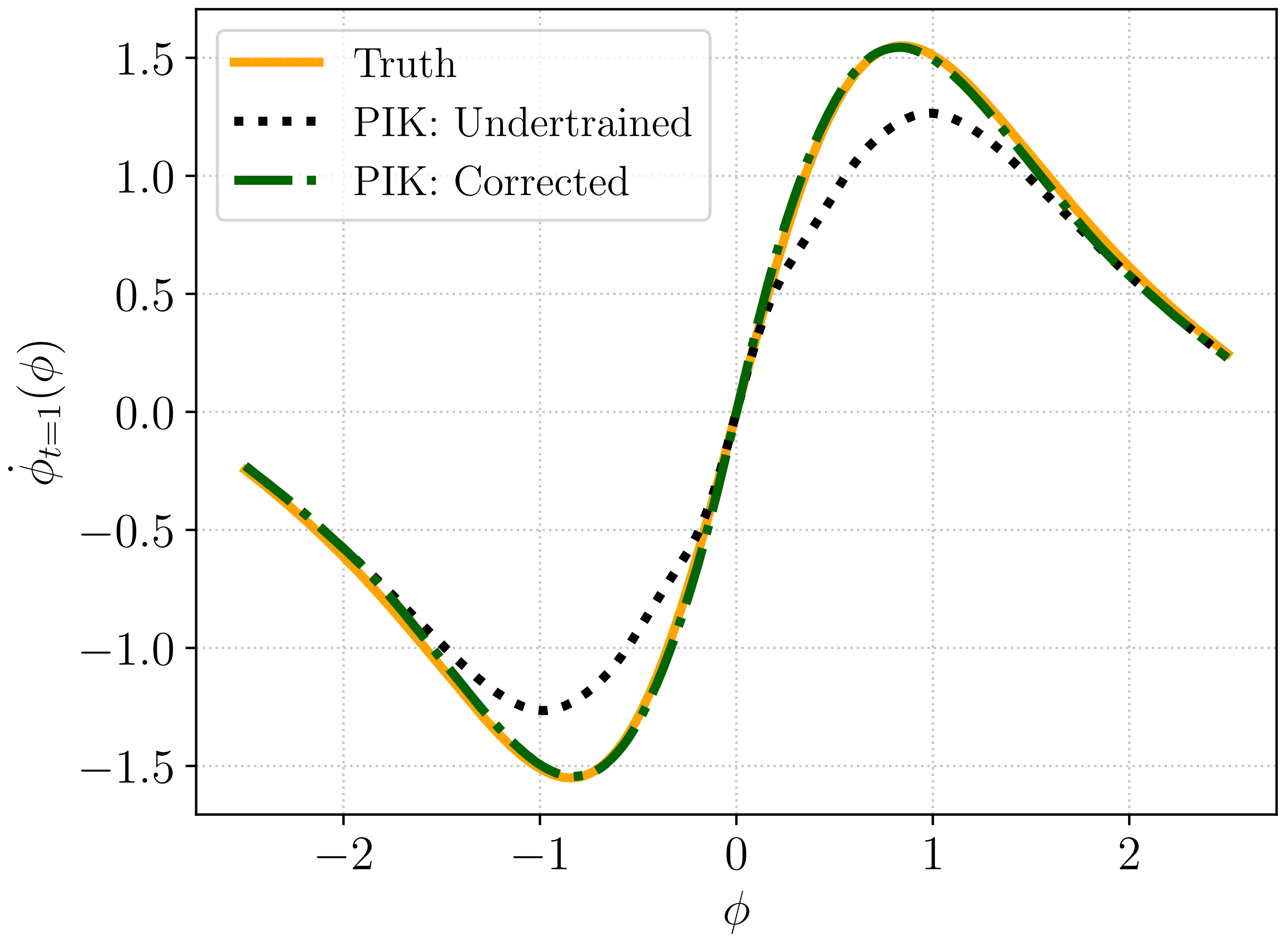
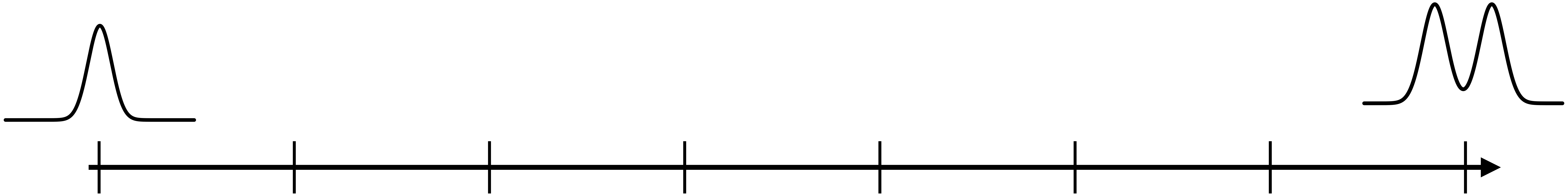


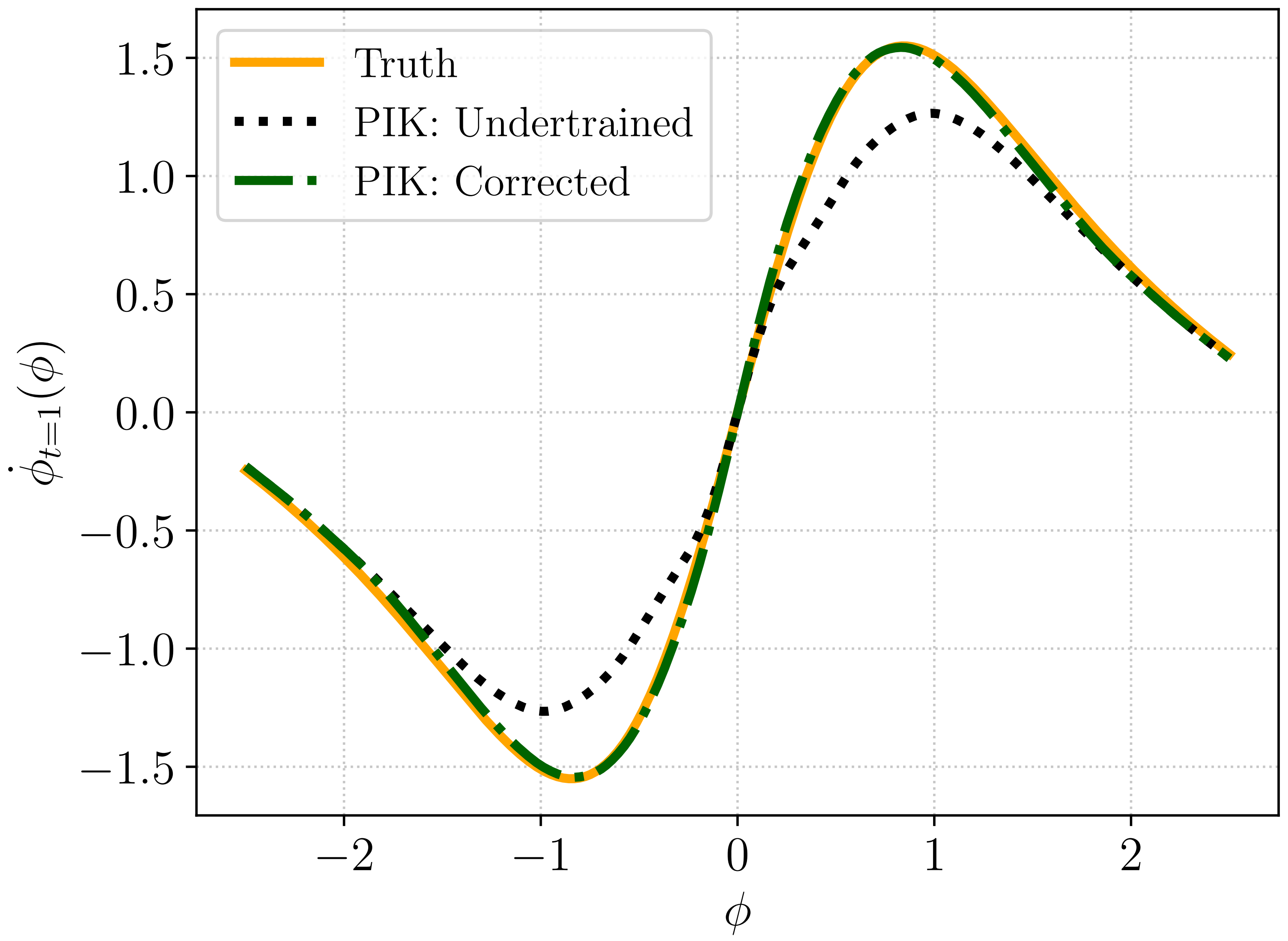
- Compute PIKs separately at every time-slice

- Linearity of the equation allows for systematic improvements

FI, Kapust, Pawlowski, arxiv:2510.26678



$$\left(m_0^2\,,\,\lambda_0\right) = (1\,,\,0) \longrightarrow \left(m_t^2\,,\,\lambda_t\right) = \left(m_0^2 + t\,(m_1^2 - m_0^2)\,,\,\lambda_0 + t(\lambda_1 - \lambda_0)\right) \longrightarrow \left(m_1^2\,,\,\lambda_1\right) = (-2\,,\,1/2)$$



- Compute PIKs separately at every time-slice
- Linearity of the equation allows for systematic improvements

- Evolve samples separately with

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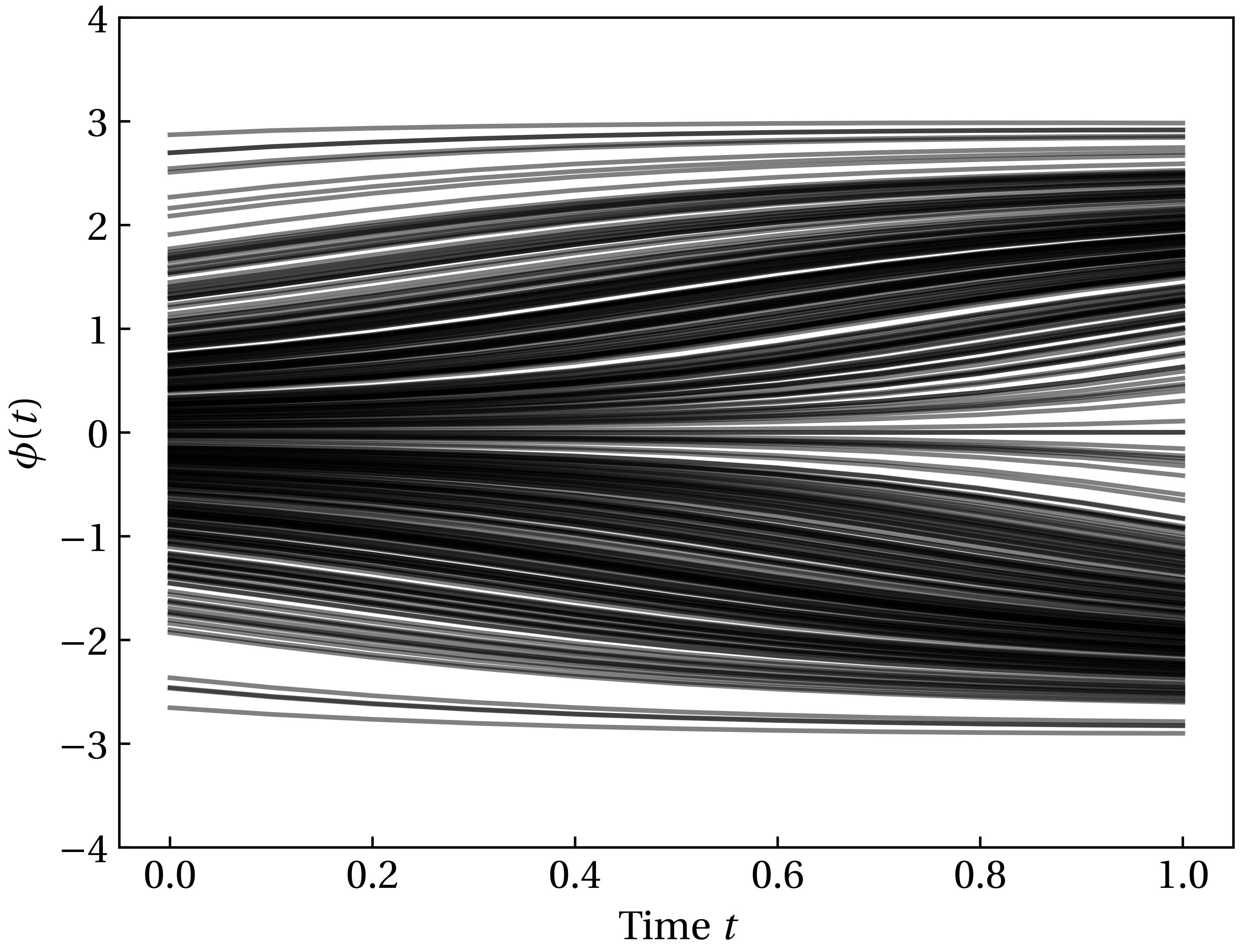

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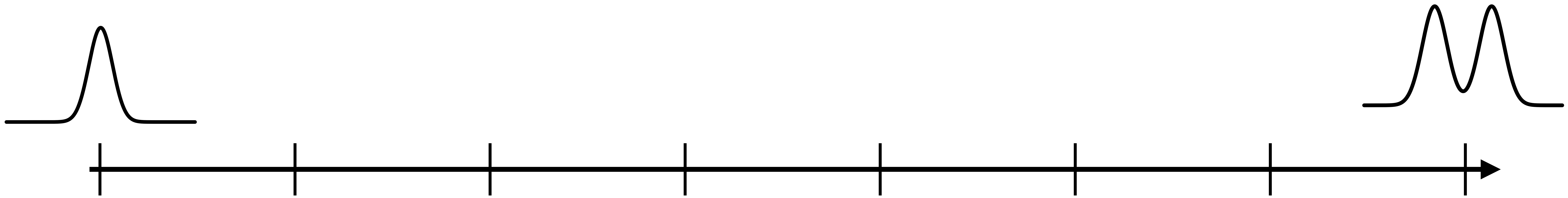

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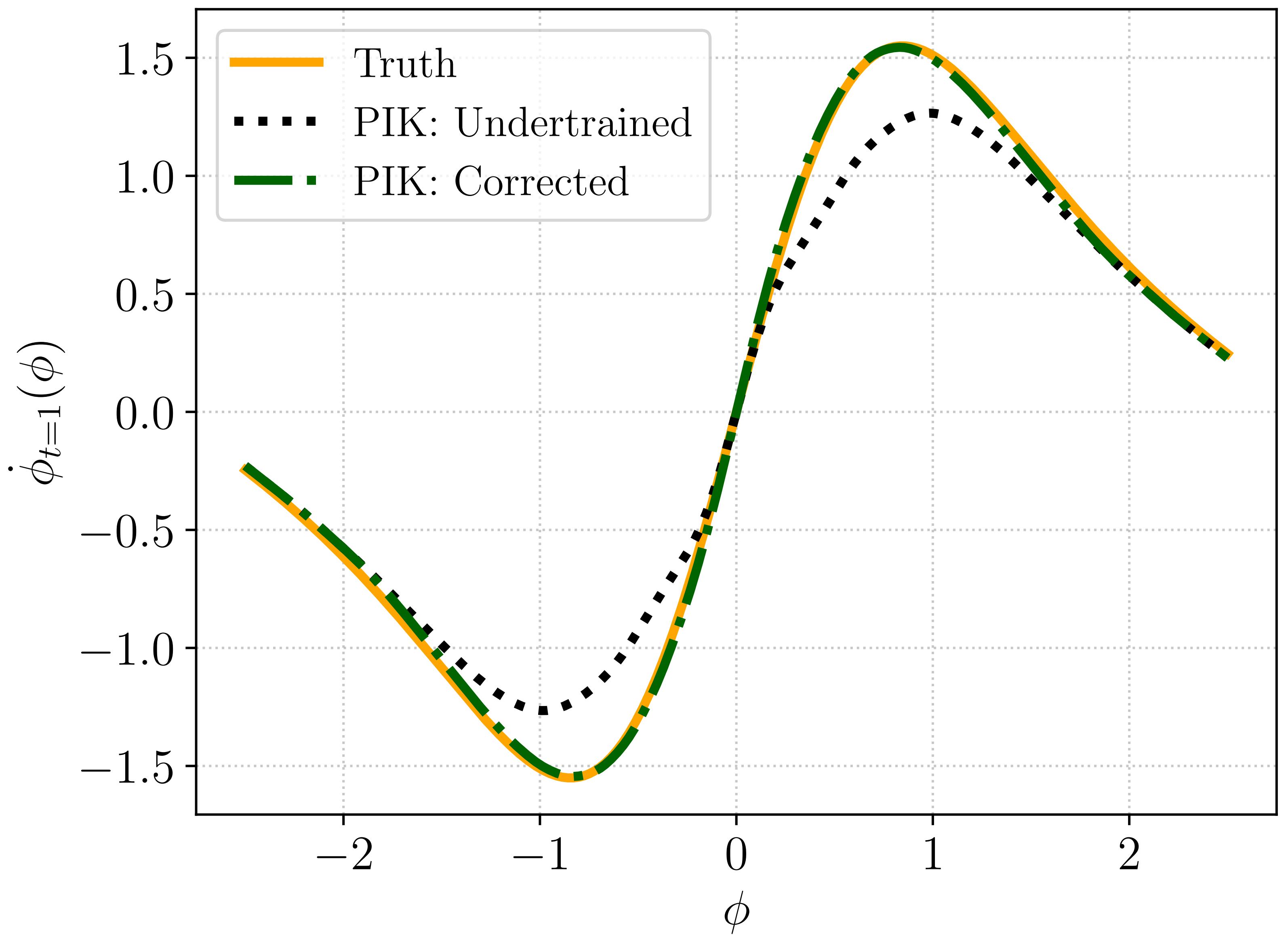

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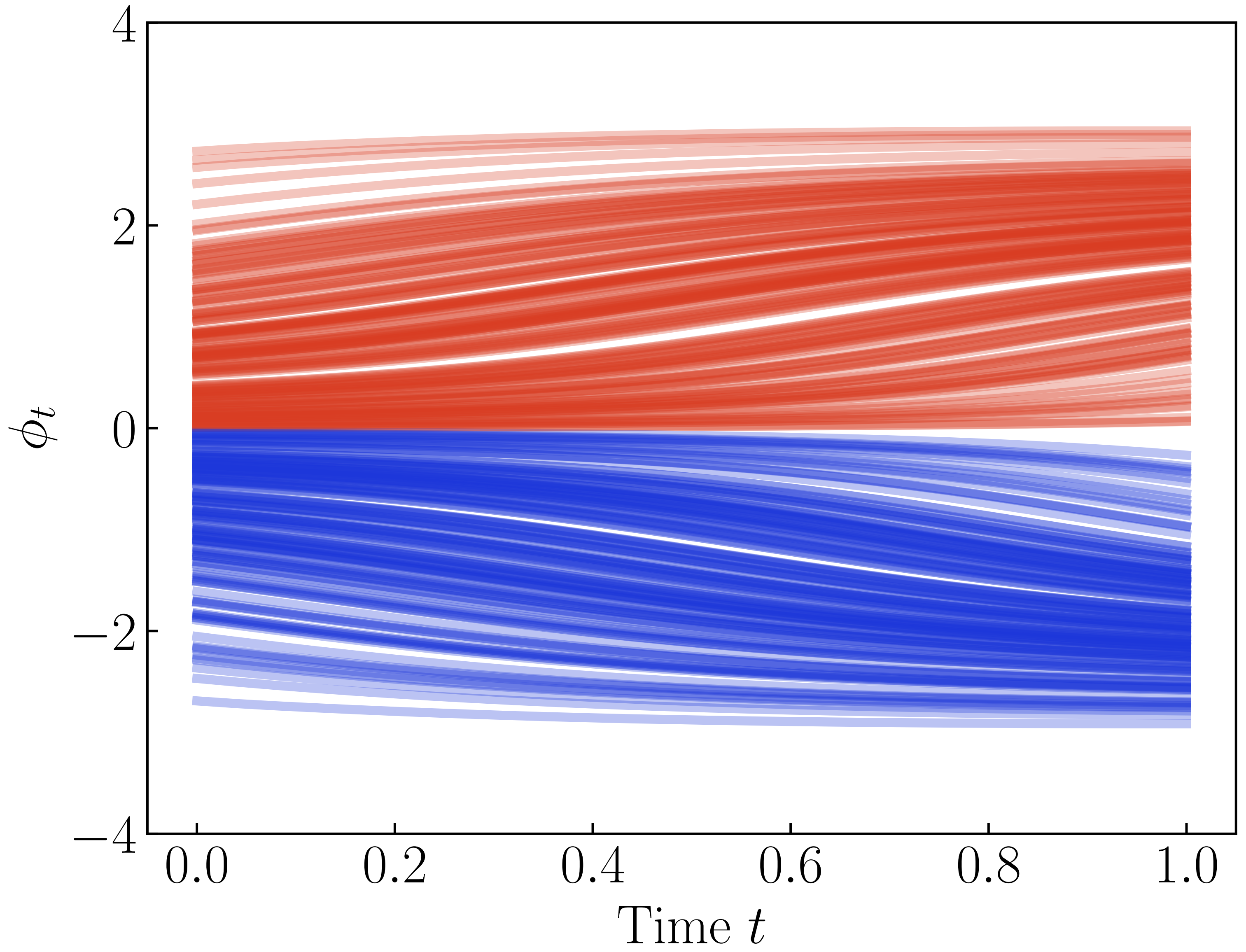

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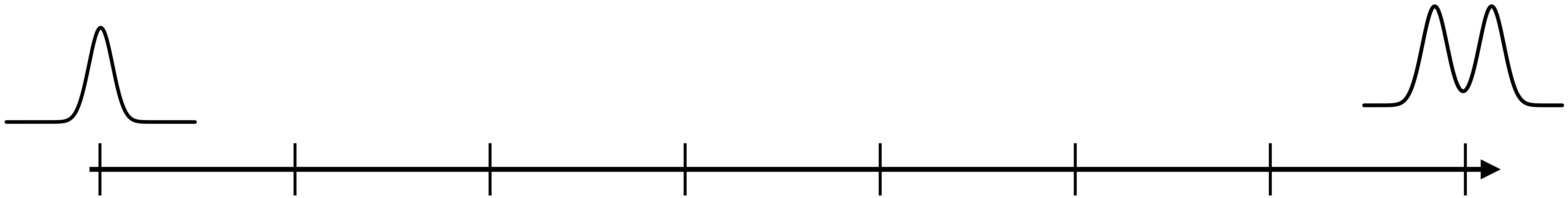

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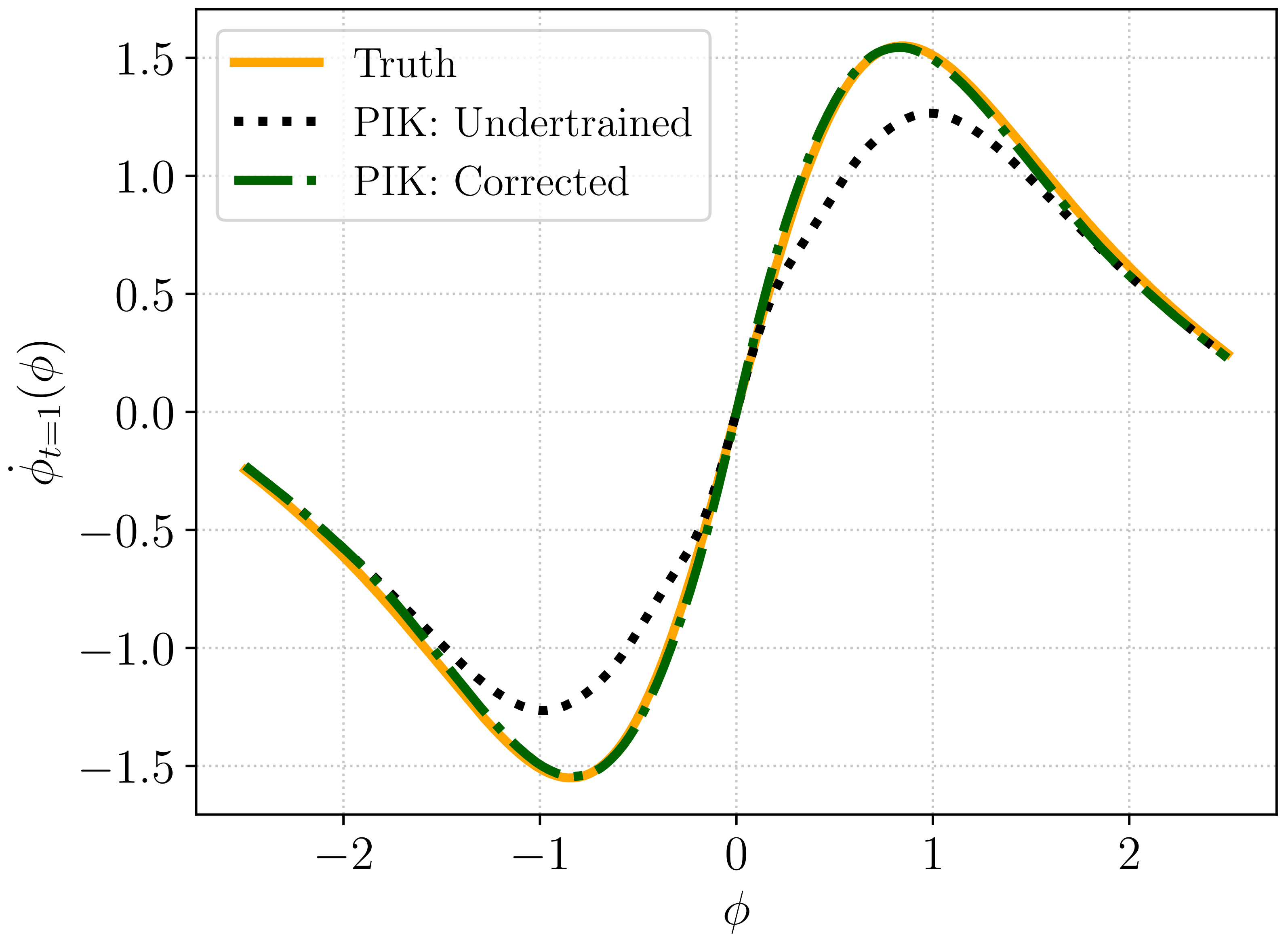

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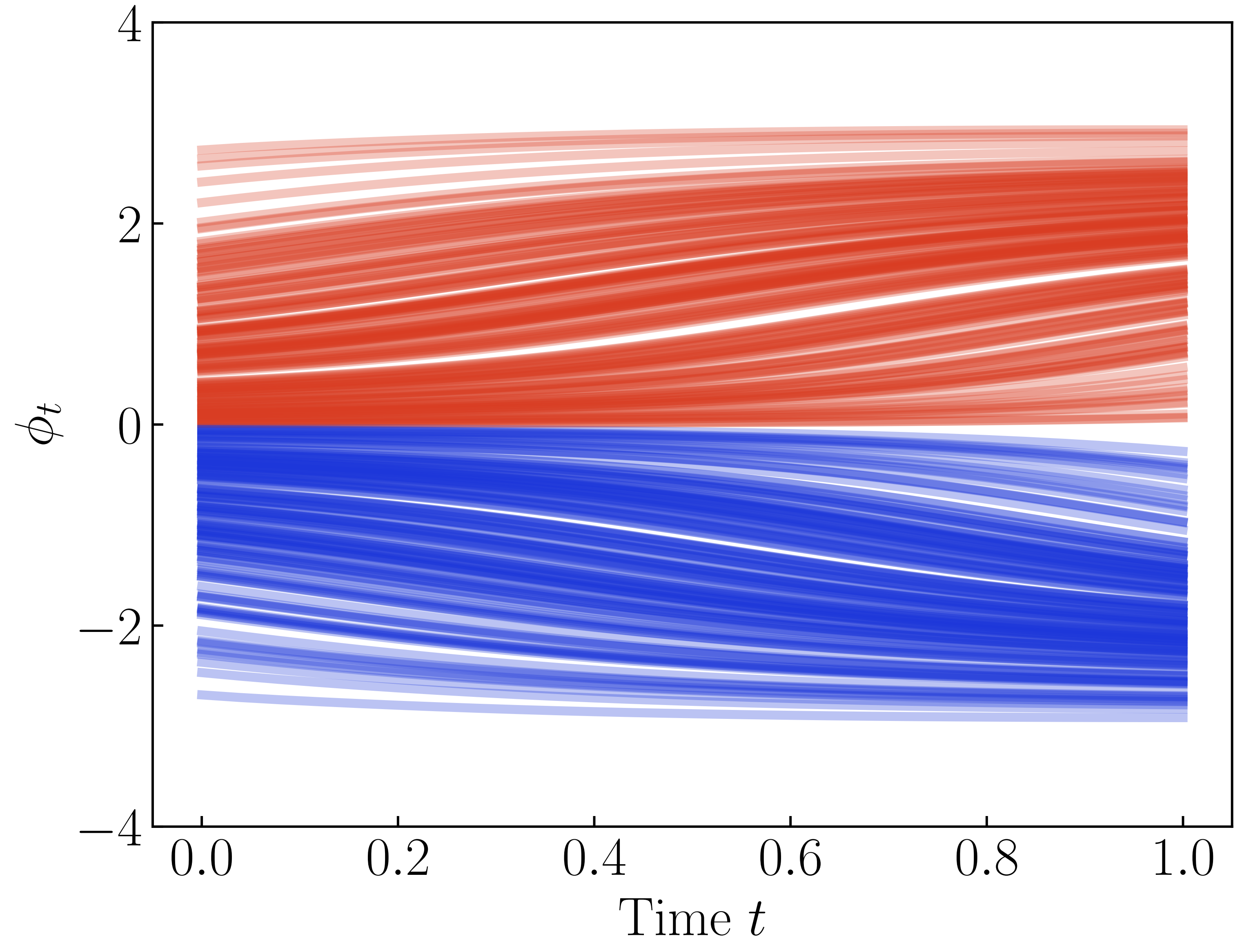

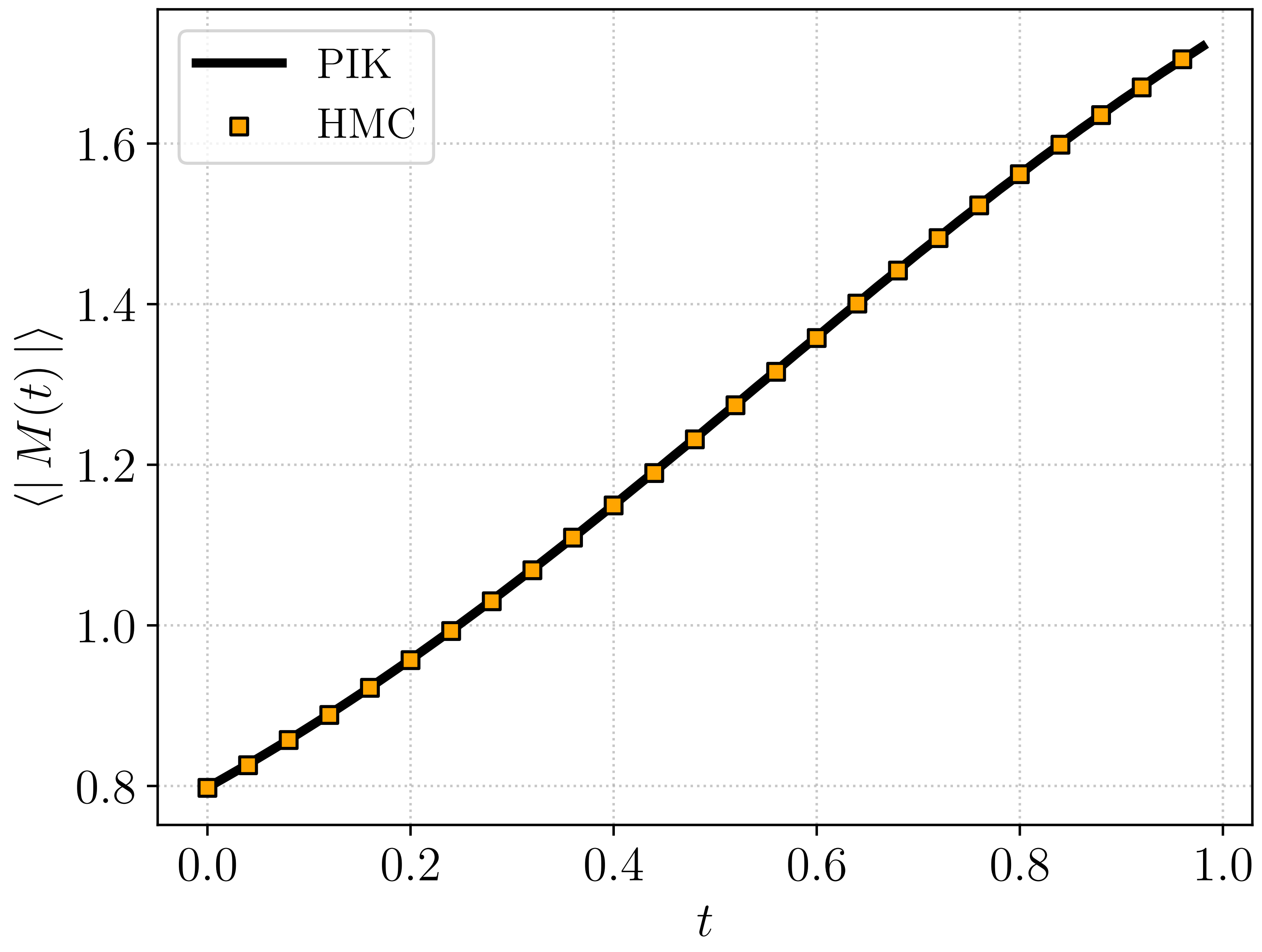

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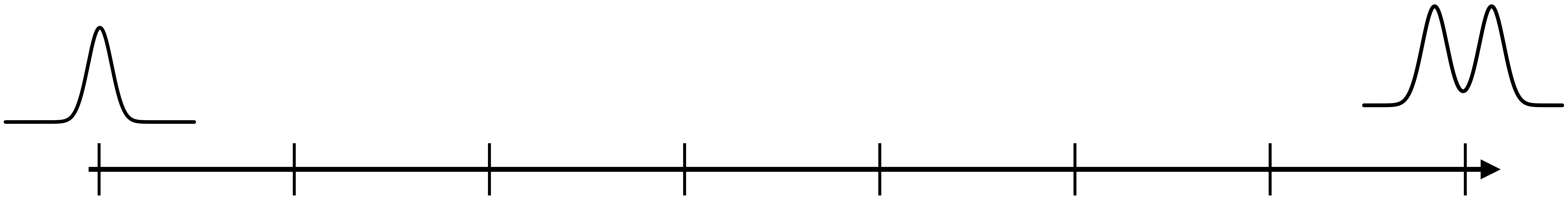

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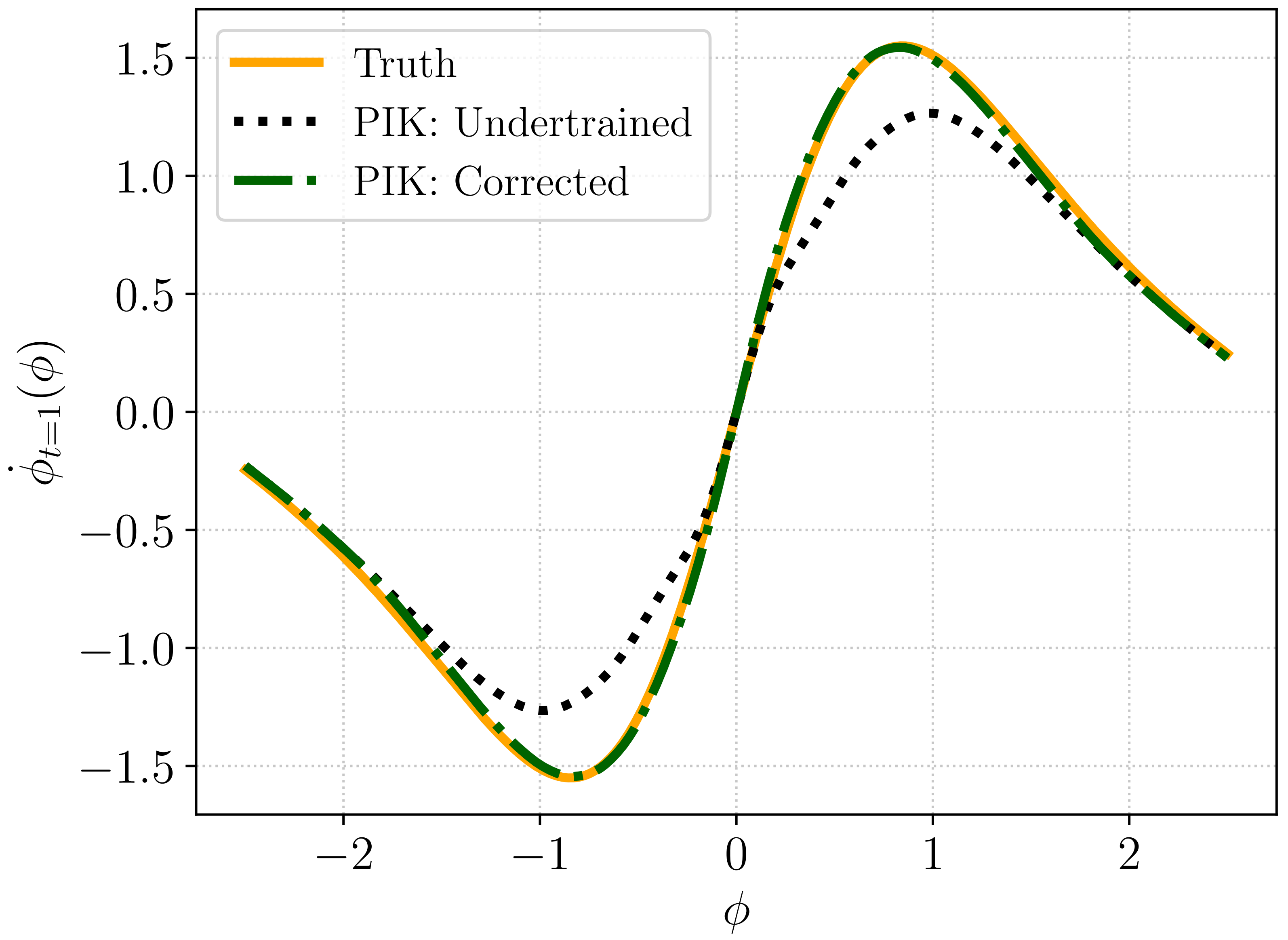

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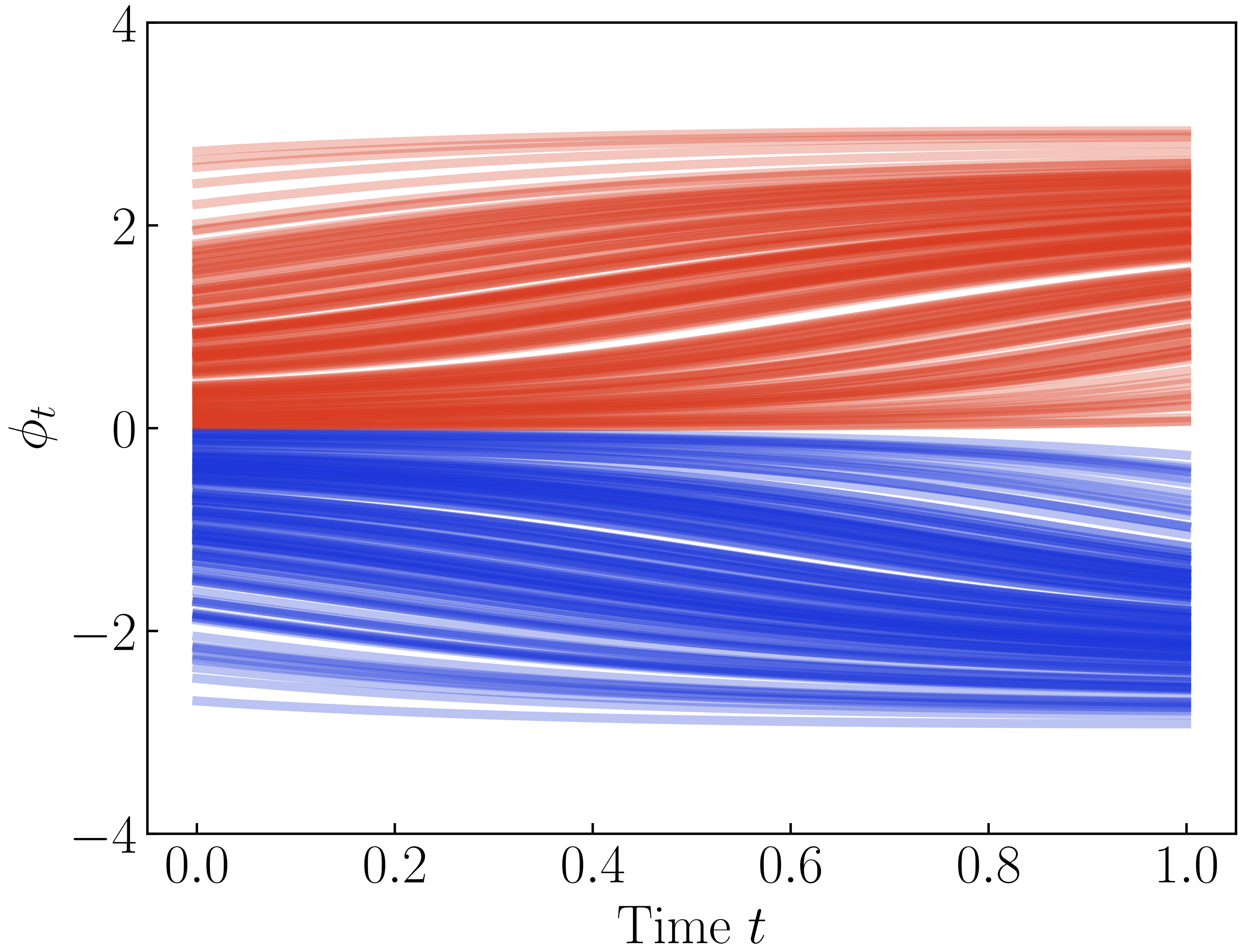

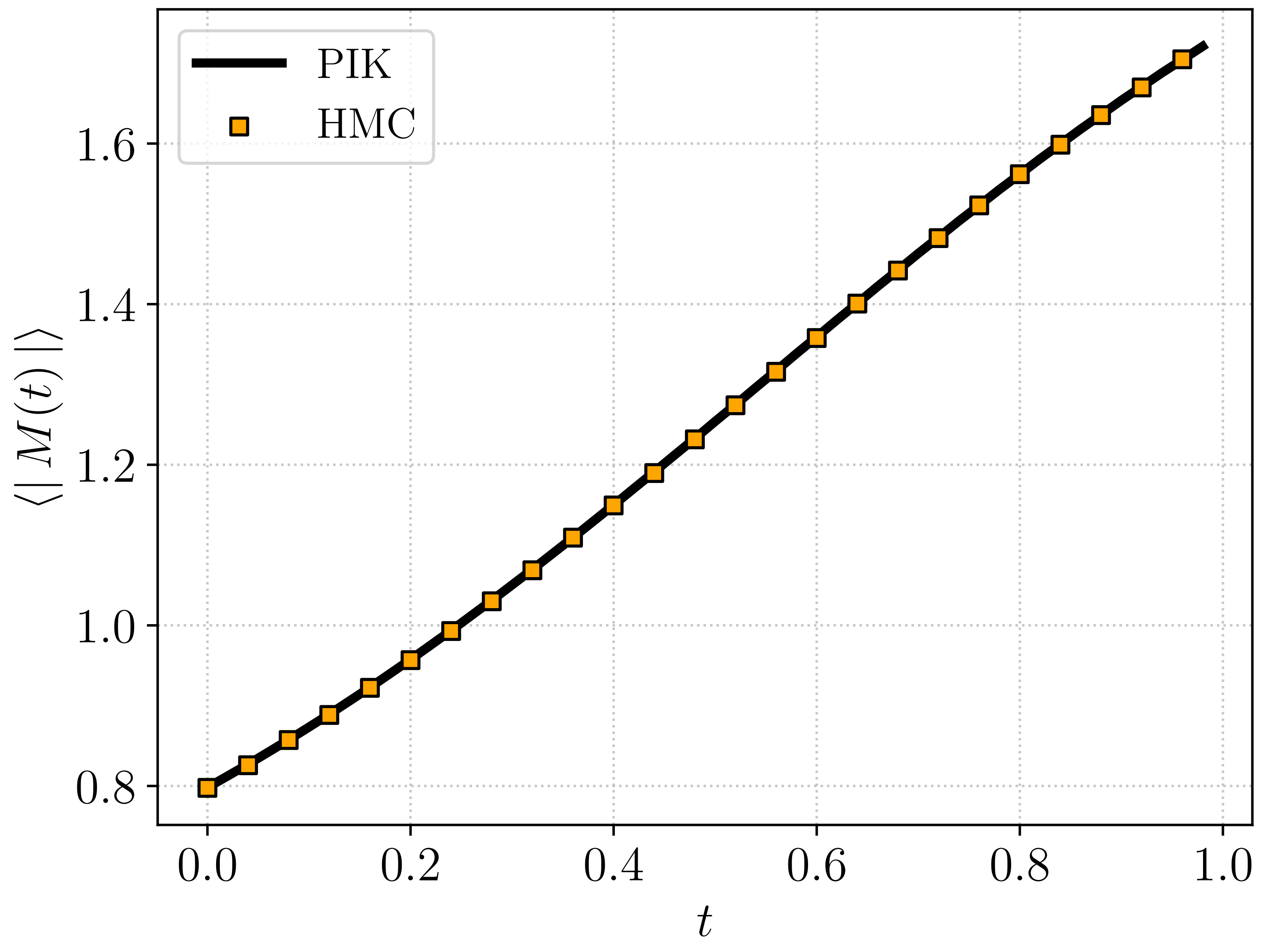

- Sample operators along this trajectory

$$\langle O \rangle_t = \frac{1}{N} \sum_n O[\phi_{n,t}]$$

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Part 1 (Strategy 1): Conclusion and outlook

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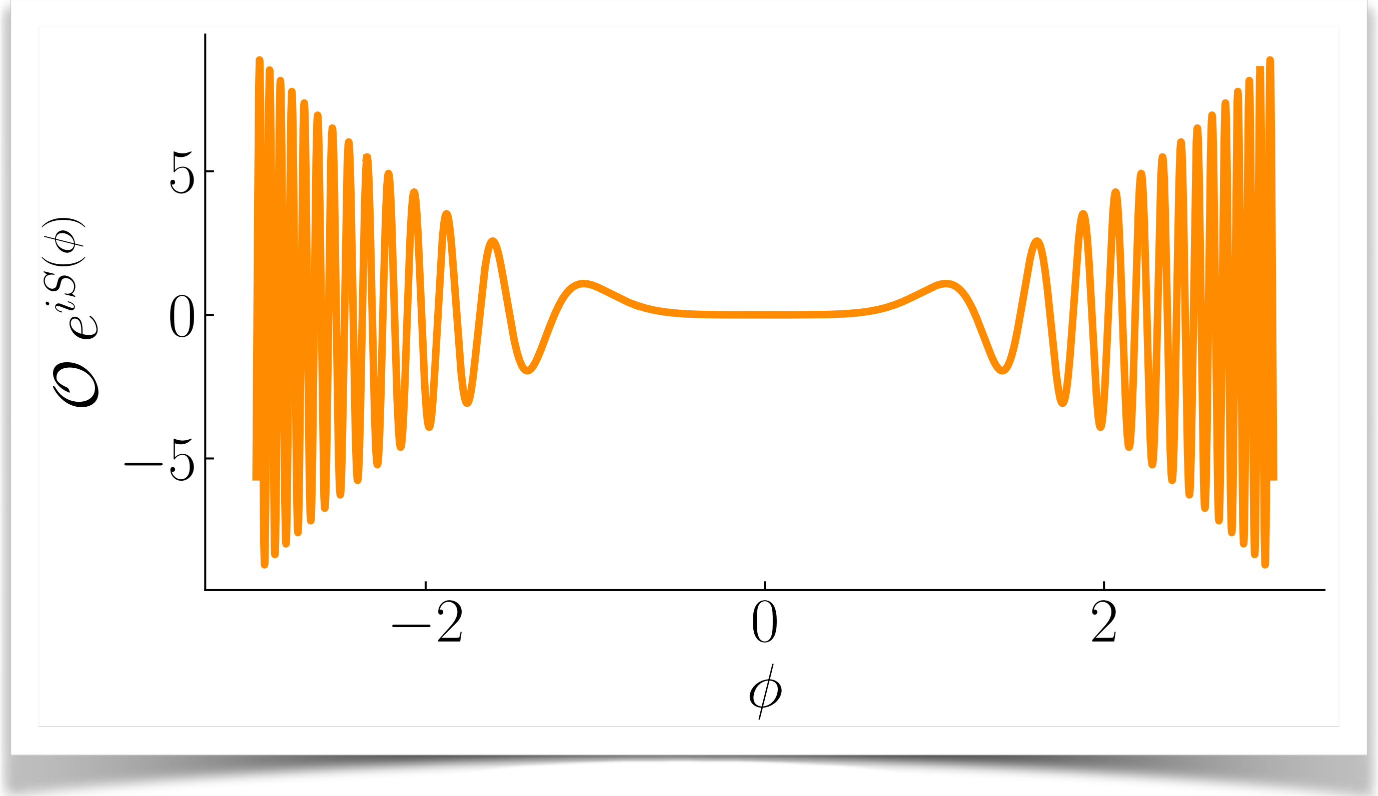
0-dim toy model of a real time theory



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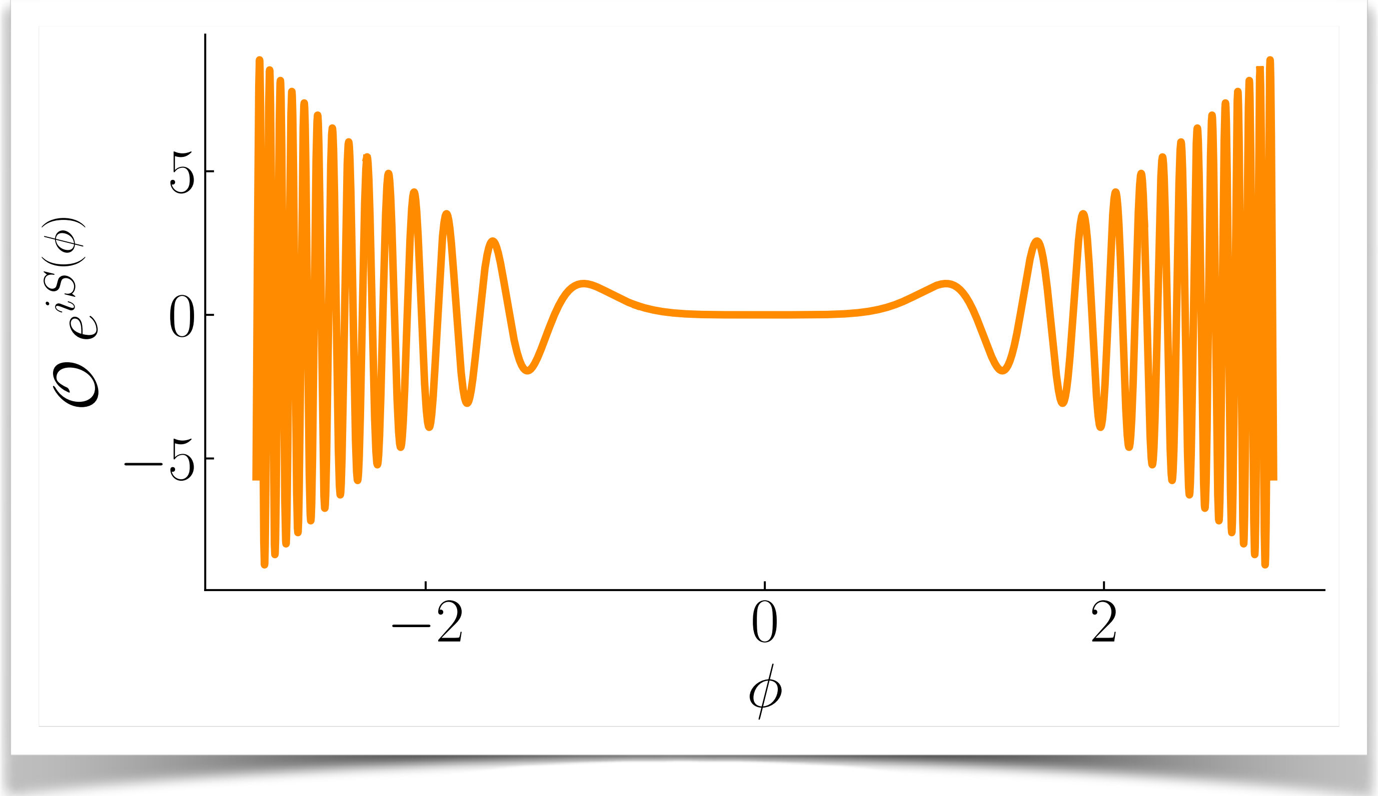

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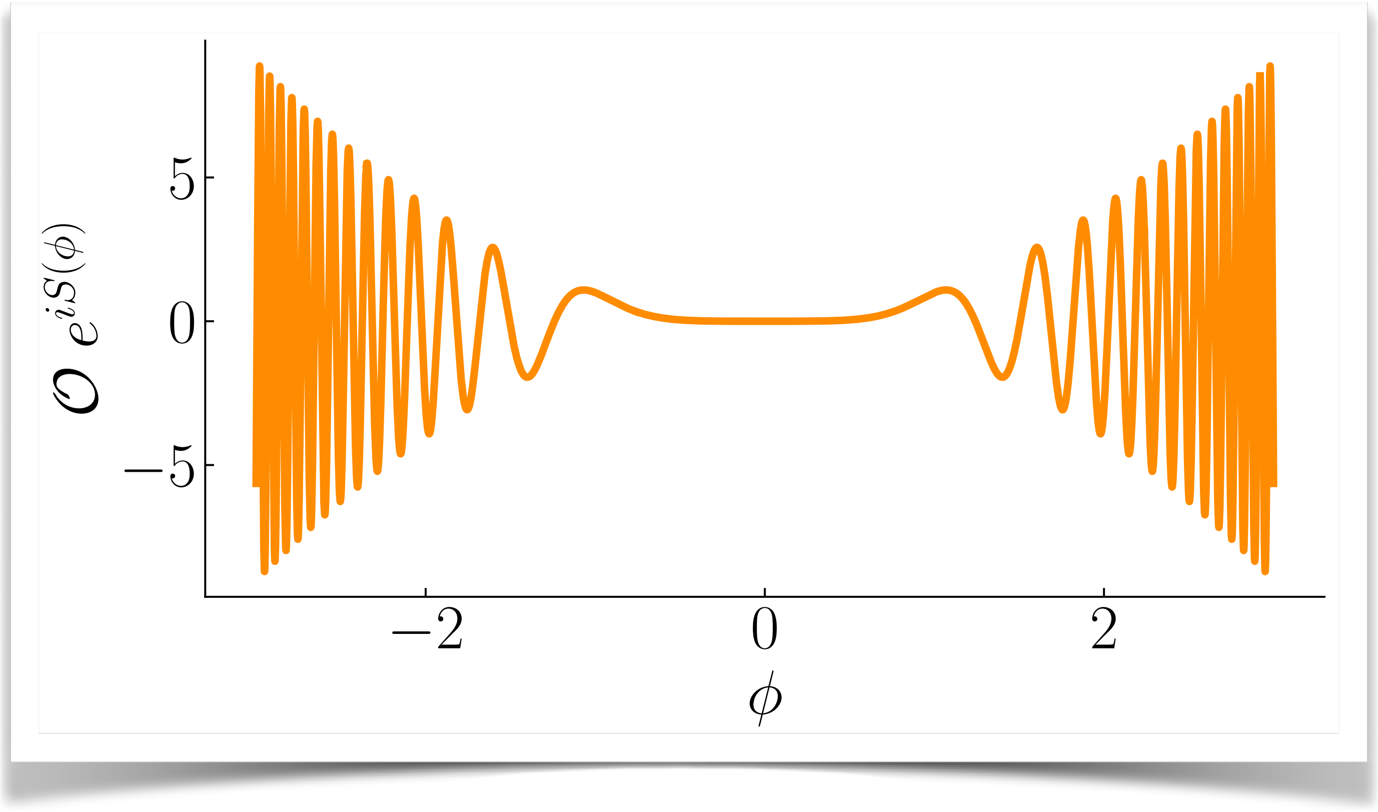

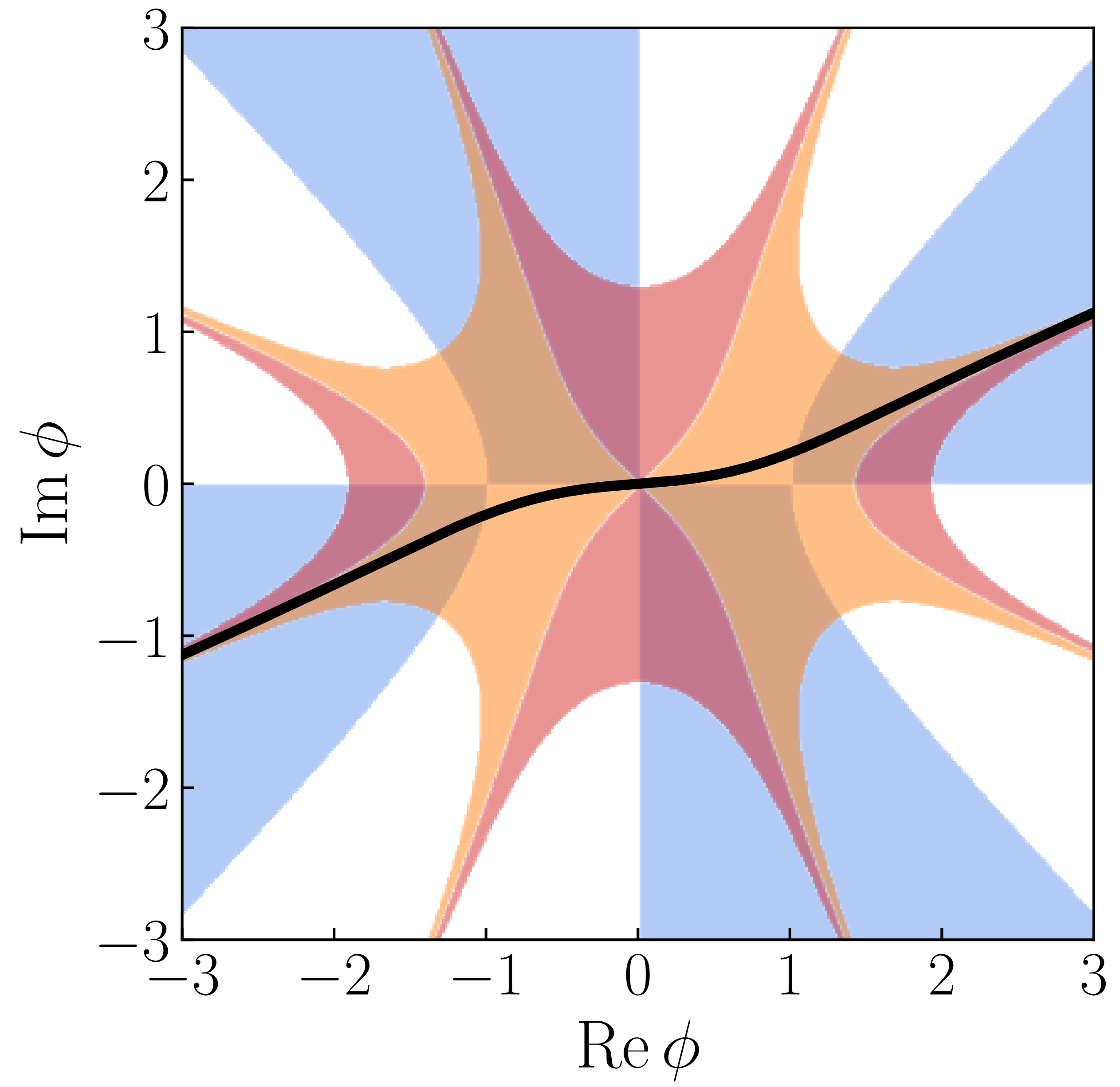

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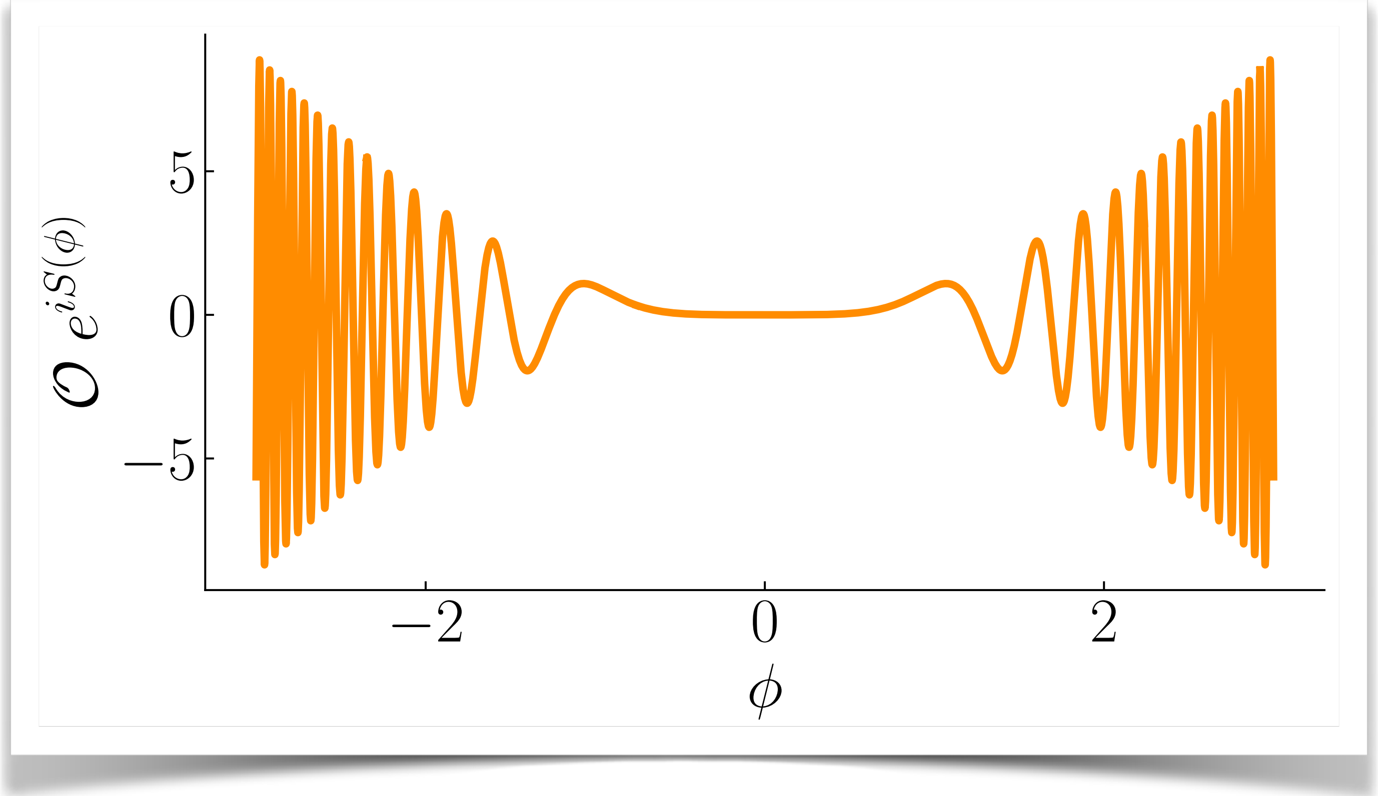

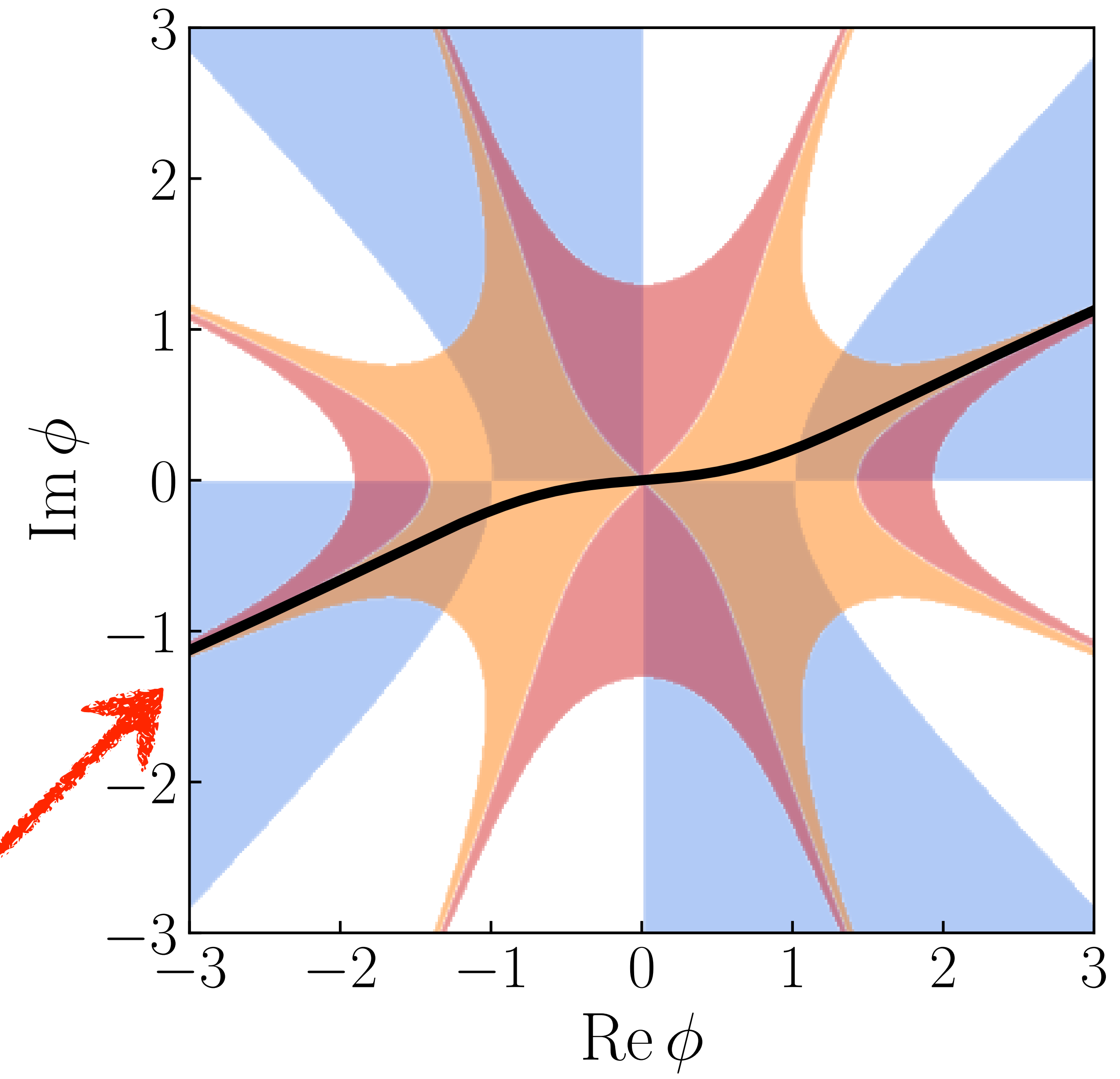

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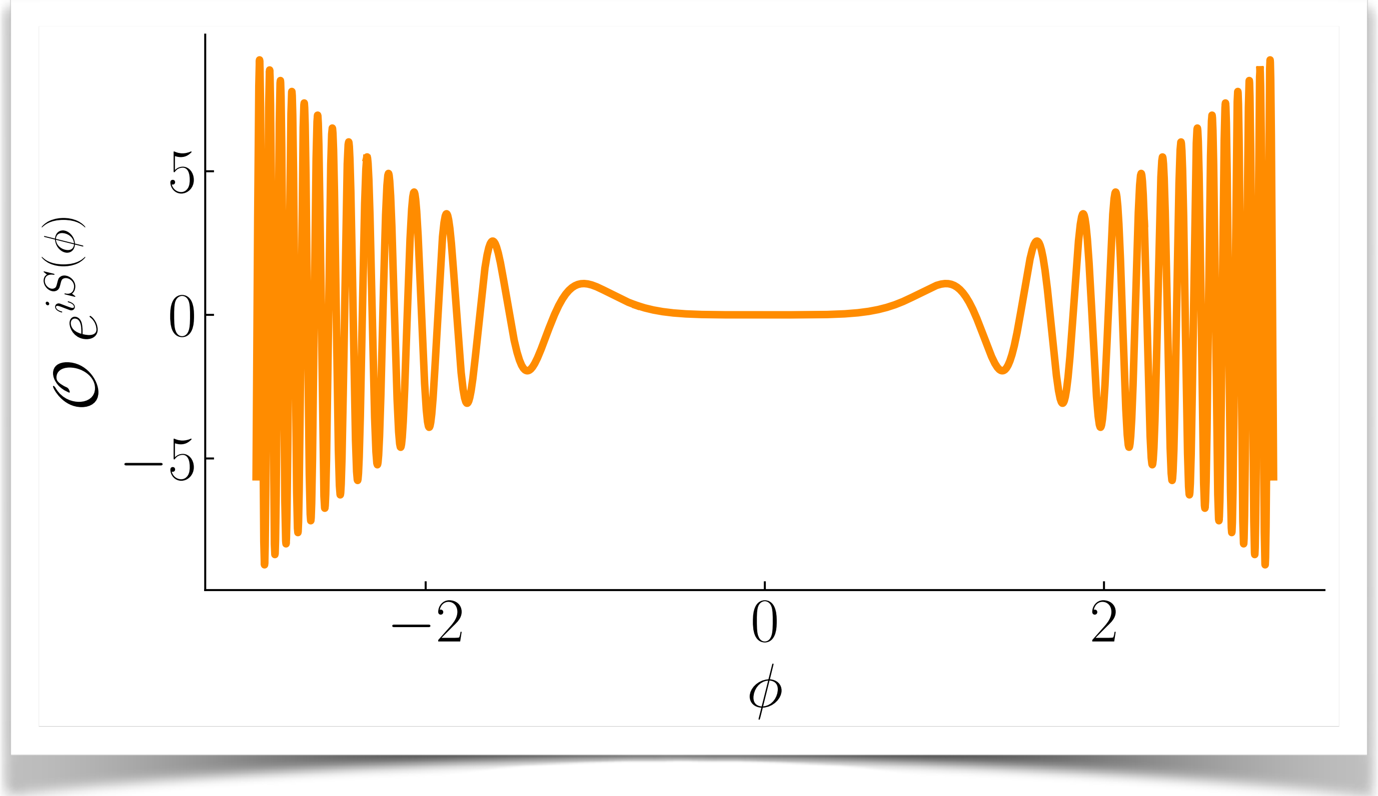

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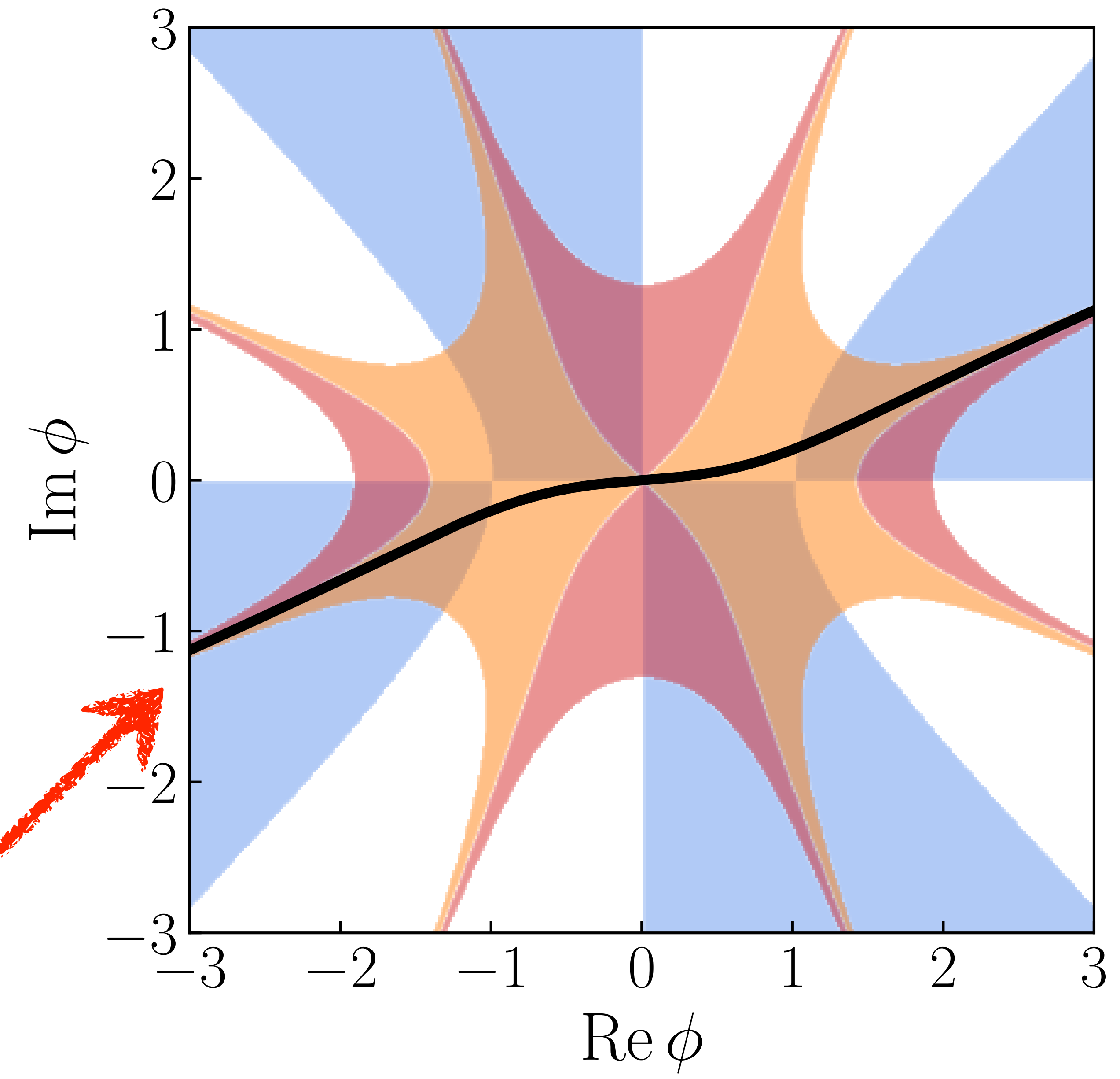

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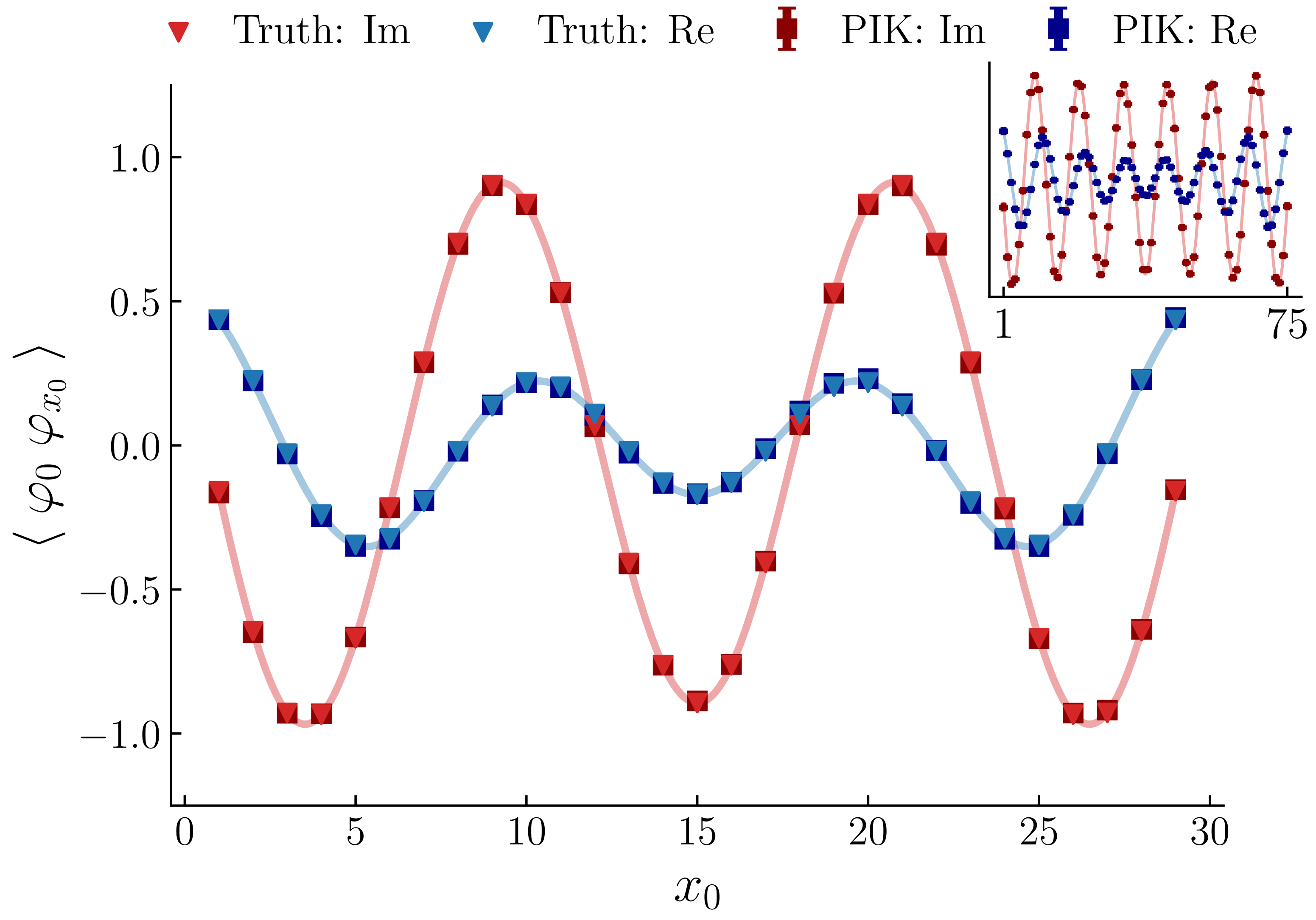

1+0 dim theory:
Quantum harmonic oscillator

FI, Kapust, Pawlowski, arxiv:2603.03159

Generalised RG flows: Strategy 2

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Wegner flow

Wegner, J. Phys. C 7 (1974) 2098

General field transformations

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Sampling

$$\frac{d\, p_t(\phi)}{dt} = -\int_x \frac{\partial}{\partial \phi(x)} \Big[\dot{\phi}_t(\phi)\, p_t(\phi) \Big]$$

$\longleftrightarrow$ **Legendre transform**

$\vdots$

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Wegner flow

Wegner, J. Phys. C 7 (1974) 2098

General field transformations

Generalized flow equation

Pawlowski, *Annals Phys.* 322 (2007) 2831

Generalised RG flows: Strategy 2

Scale transformations   Reparametrisations

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Wegner, J. Phys. C 7 (1974) 2098

General field transformations

<u>Generalized flow equation</u>

Pawlowski, *Annals Phys.* 322 (2007) 2831

<u>Wetterich equation</u> $\dot{\phi} = 0$

Wetterich, PLB 301 (1993) 90

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Sampling

Direct computation & optimisation

$$\frac{d\,p_t(\phi)}{dt} = -\int_x \frac{\partial}{\partial\phi(x)} \left[\dot{\phi}_t(\phi)\, p_t(\phi) \right]$$

Legendre transform

$$\left(\partial_t + \int_x \dot{\phi}[\phi] \frac{\delta}{\delta\phi} \right) \Gamma_\phi[\phi] = \frac{1}{2} \mathrm{Tr} \left[G_\phi[\phi] \left(\partial_t + 2 \frac{\delta \dot{\phi}}{\delta \phi} \right) R_k \right]$$

⋮

Wegner flow

Wegner, J. Phys. C 7 (1974) 2098

General field transformations

Generalized flow equation

Pawlowski, *Annals Phys.* 322 (2007) 2831

Wetterich equation $\dot{\phi} = 0$

Wetterich, PLB 301 (1993) 90

Scale transformation implemented with regulator → Remaining reparametrisation freedom

Lets go back to the generalised RG…

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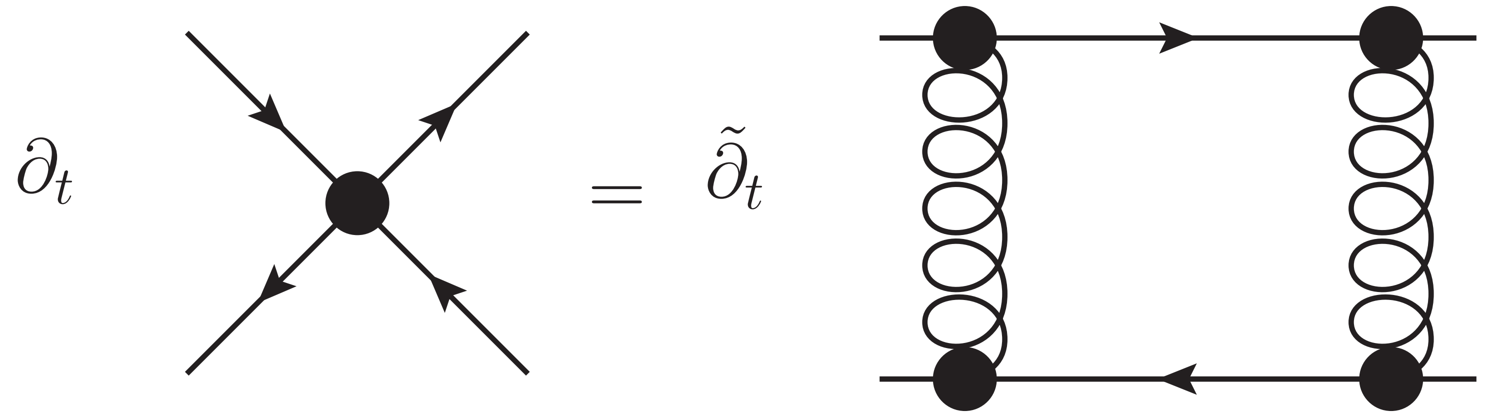

Fukushima, Pawlowski, Strodthoff, Annals Phys. 446 (2022) 169106

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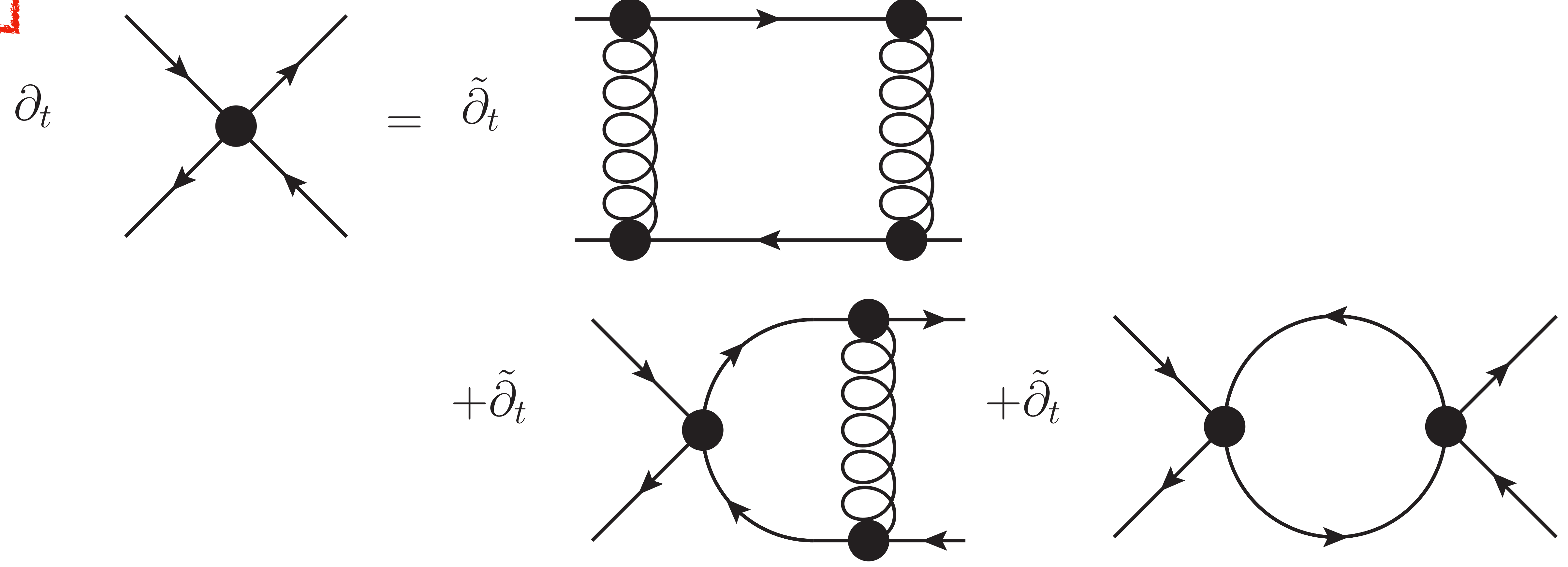

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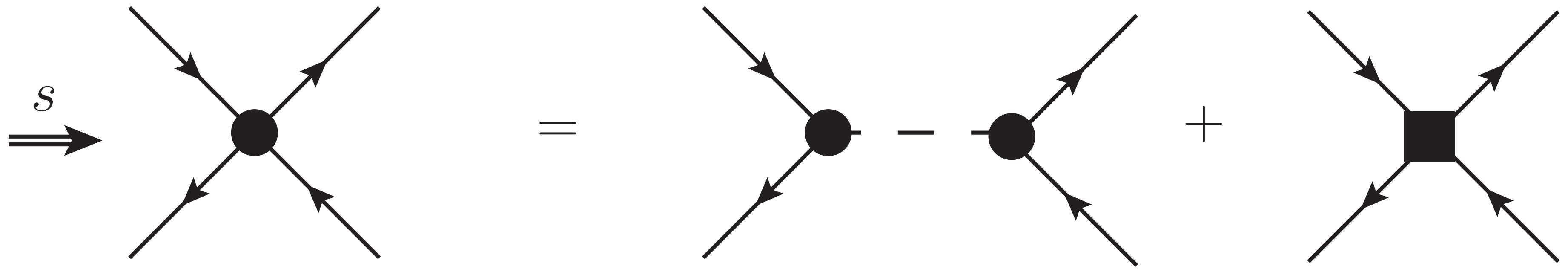

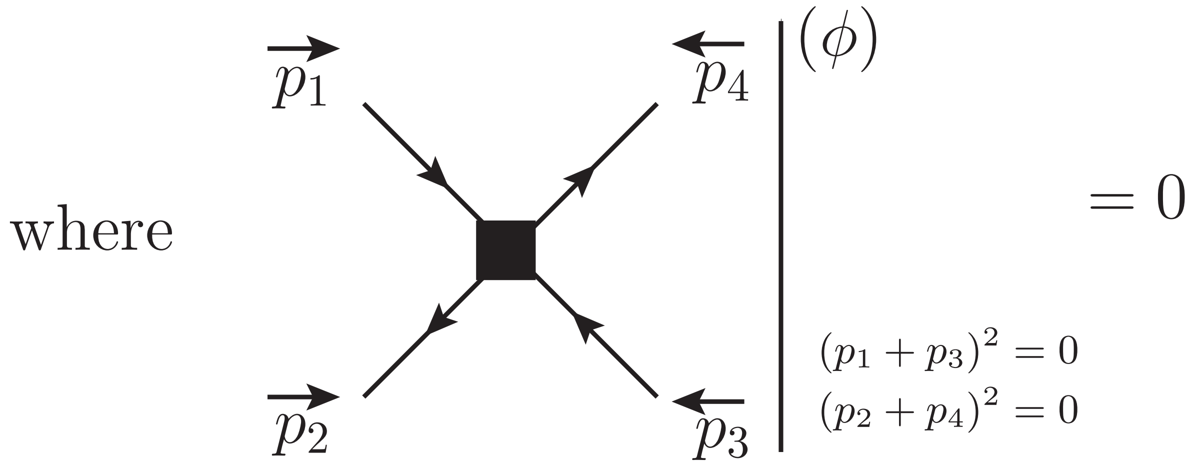

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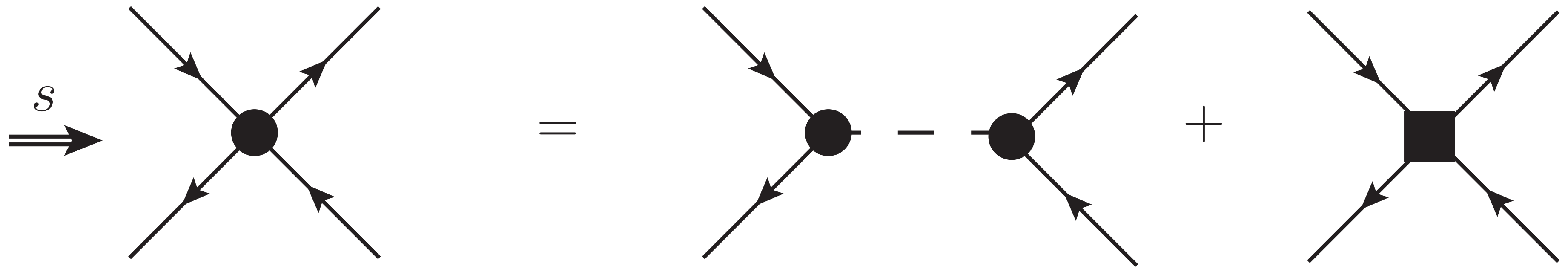

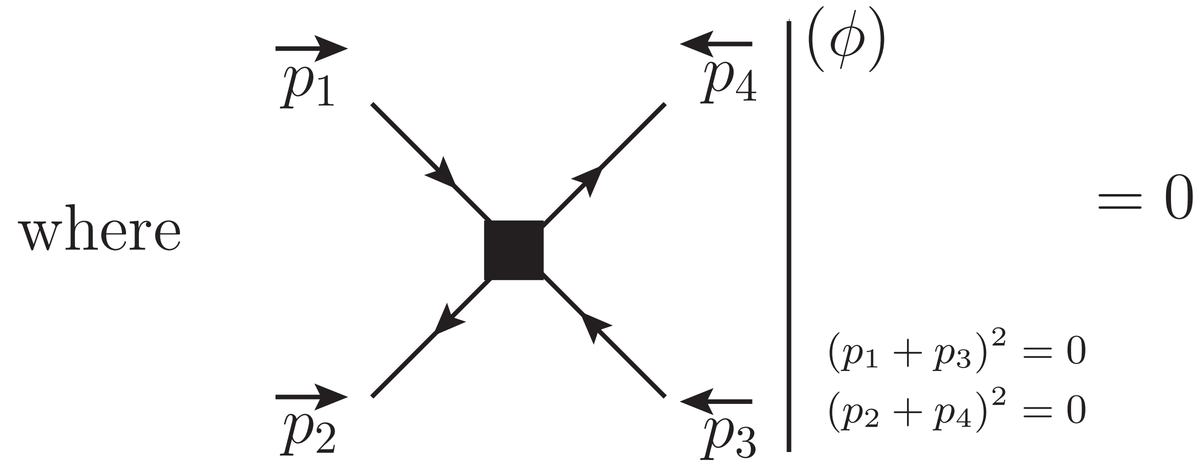

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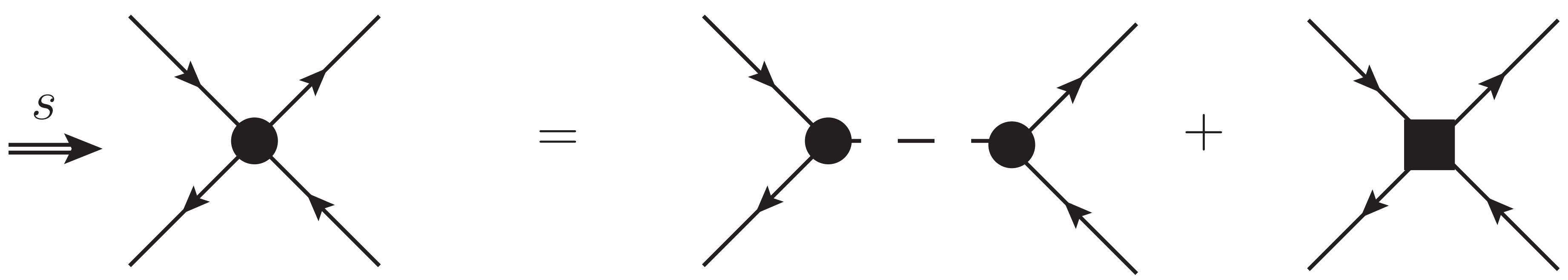

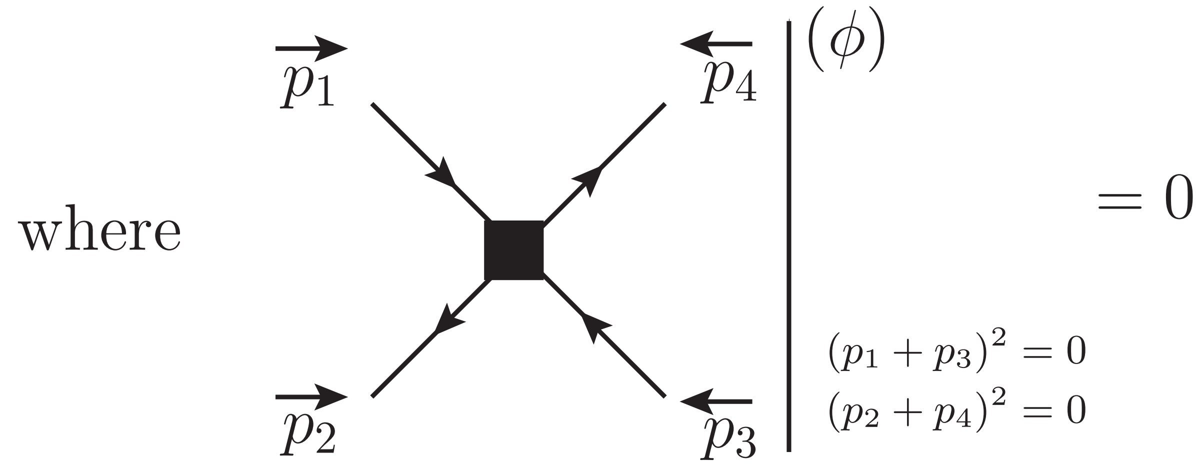

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Implementation in QCD Fu, Pawlowski, Rennecke, PRD 101 (2020) 5, 054032
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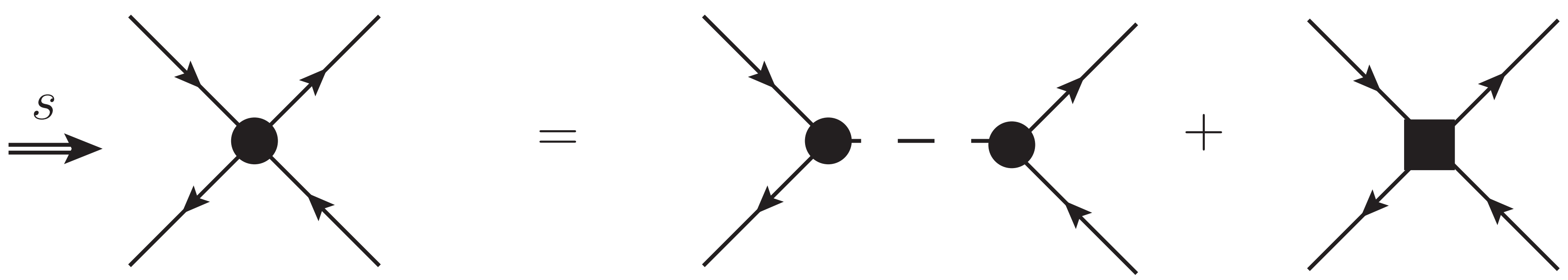

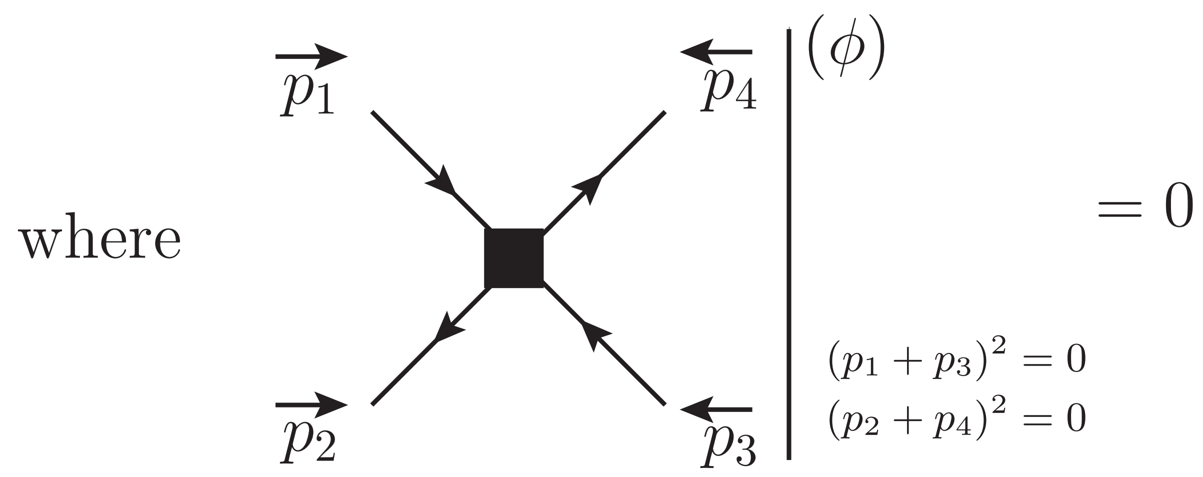

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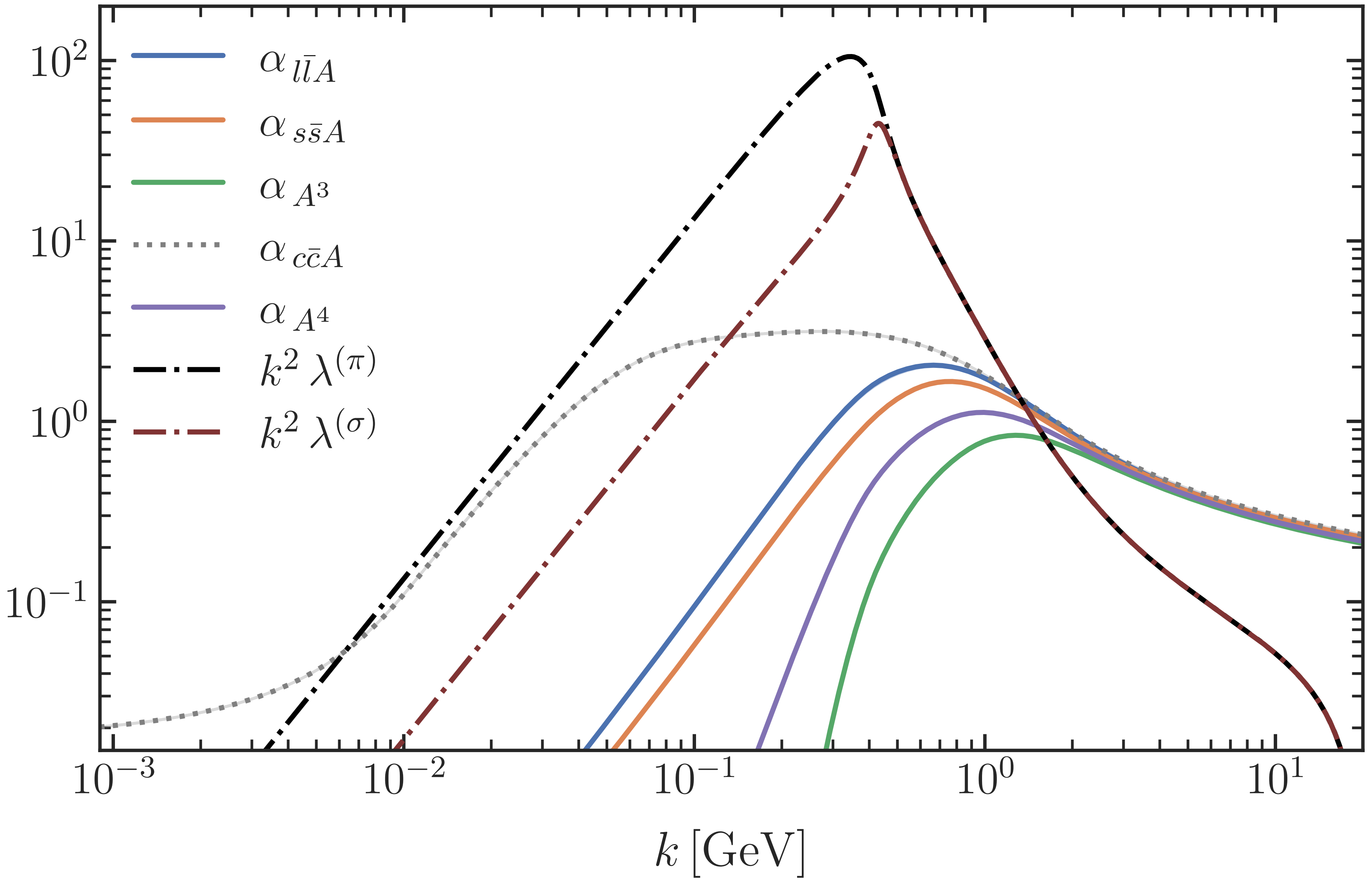

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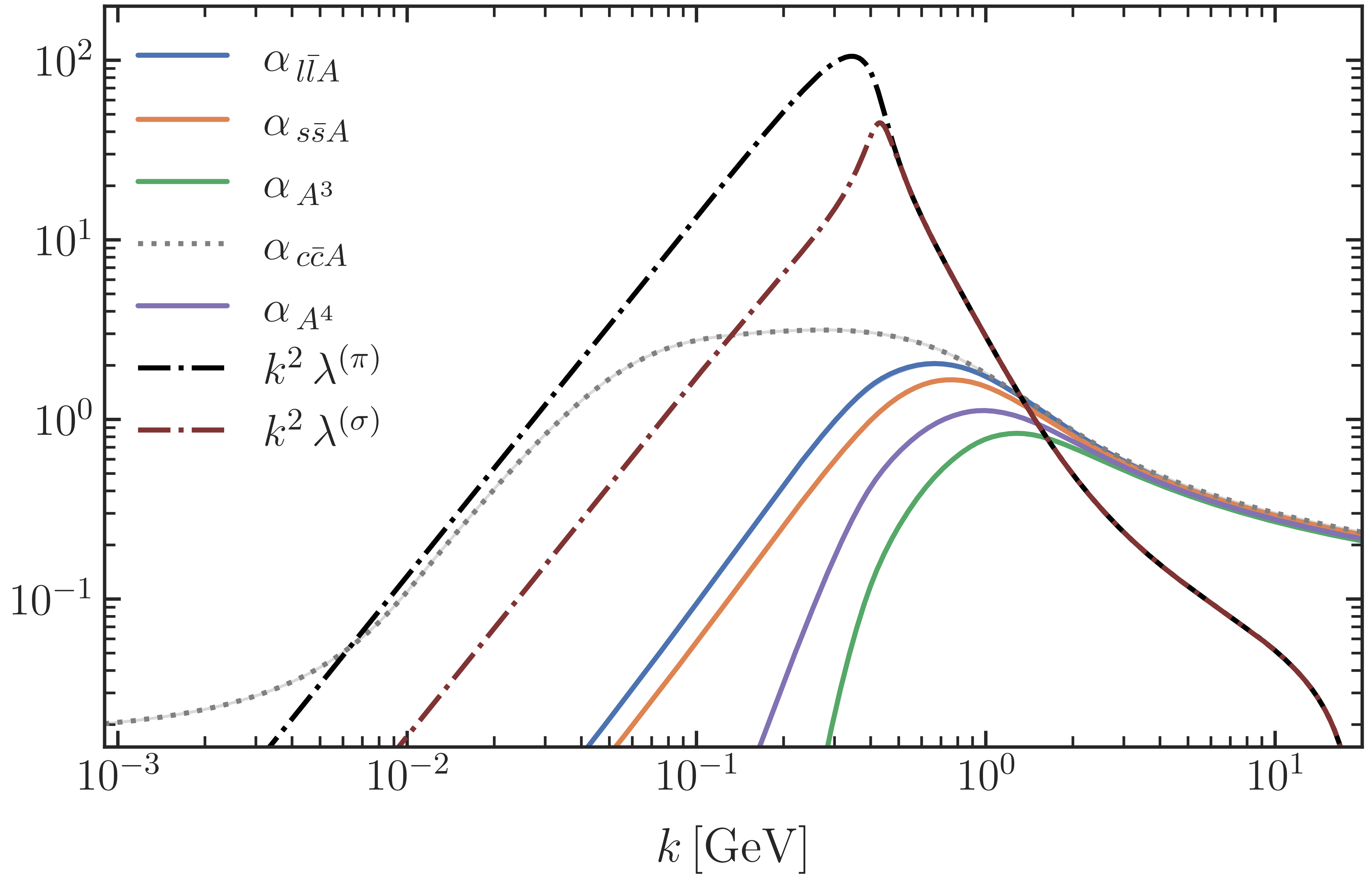

Effectively, a four-fermi vertex with a non-trivial momentum dependence is considered
→ No divergence

$$\lambda_i = \frac{h^2}{Z_\psi^2(m_i^2 + p^2 + R_k)}$$

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Step 1: Solve approximate flow for $\Gamma_{t,\mathrm{app}}$

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(i) Trade intricate diffusion eq. for a simple linear diff. eq.

(ii) Optimised and more versatile expansion schemes

(a) Ground state expansion

(b) Feed-down flows

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Disadvantage: Reconstruction!

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Reconstruction & observables: Generalised operator flow

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observable propagator insertion DoF for optimisation

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The ground state-expansion

Topology in the anharmonic oscillator

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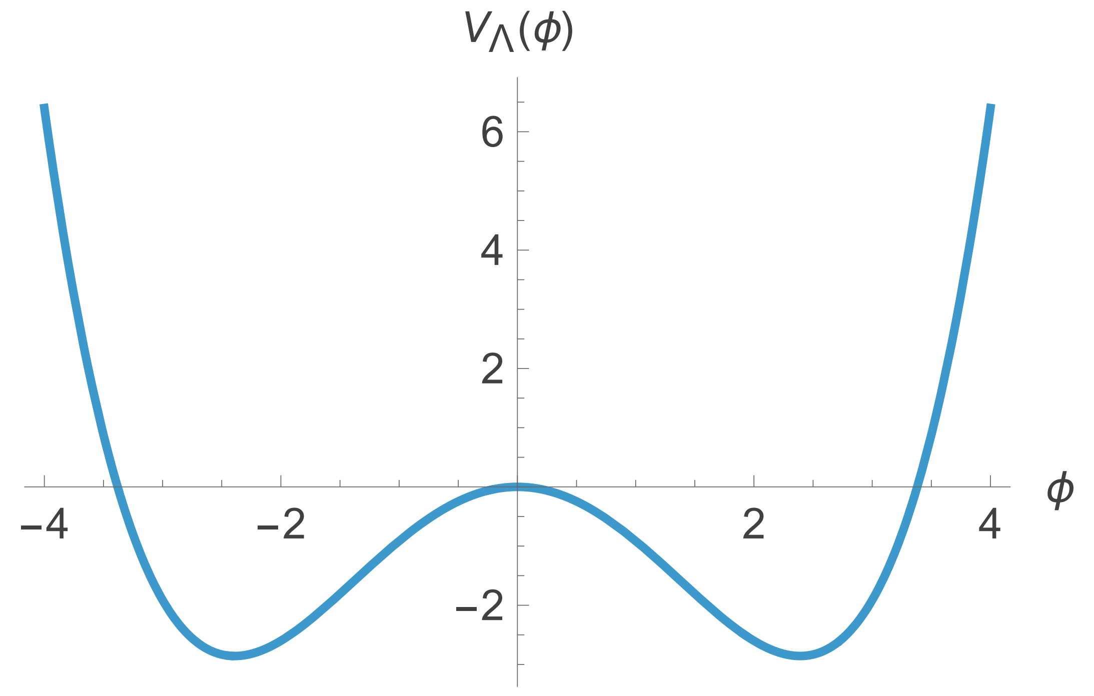
Topology in the anharmonic oscillator

- Quantum mechanics: i.e. 1+0 dimensional QFT

$\rightarrow$ Setup with **tunneling**



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The ground state-expansion

Topology in the anharmonic oscillator

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→ Consider the **energy gap** between ground state and the first excited state Δ E

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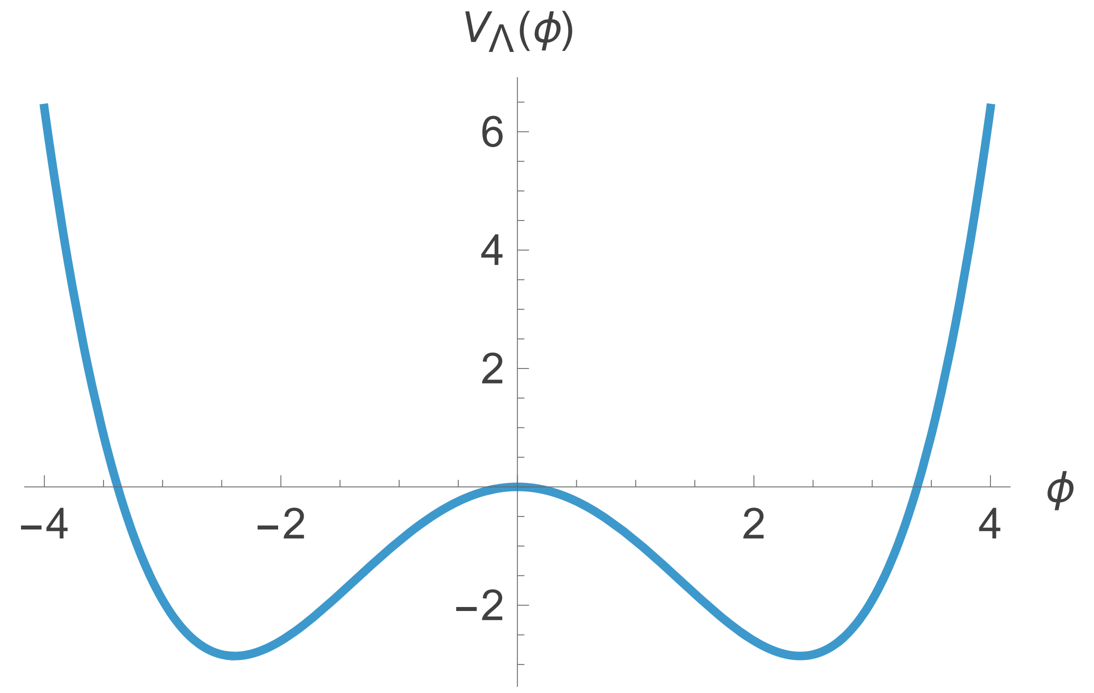

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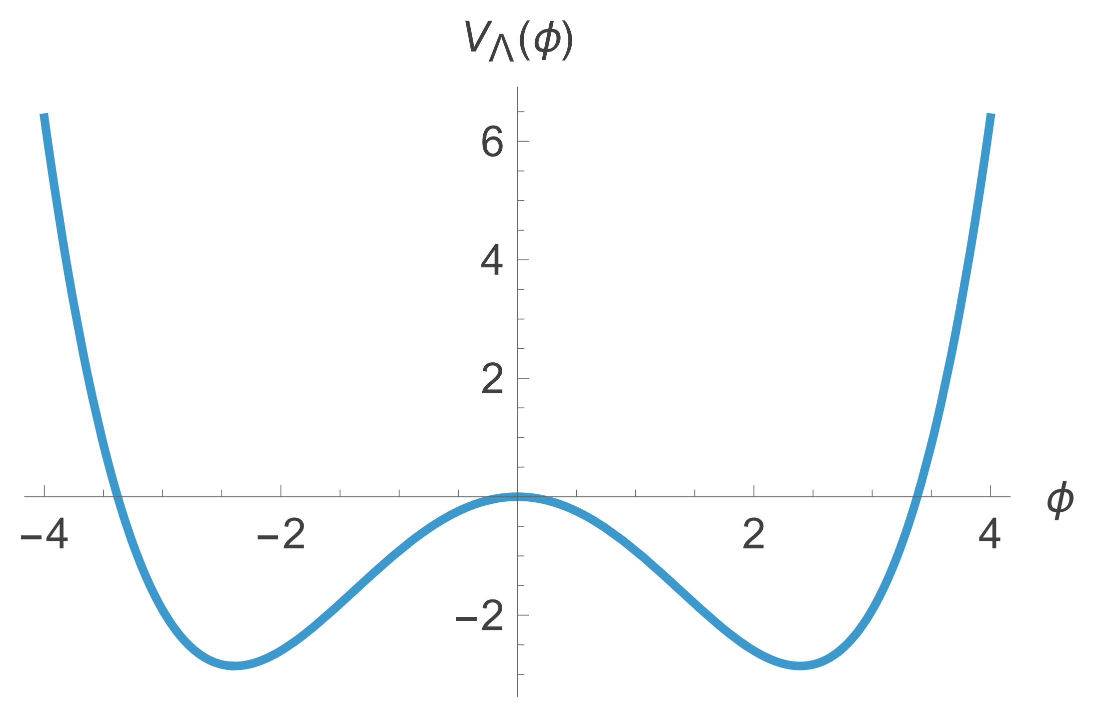

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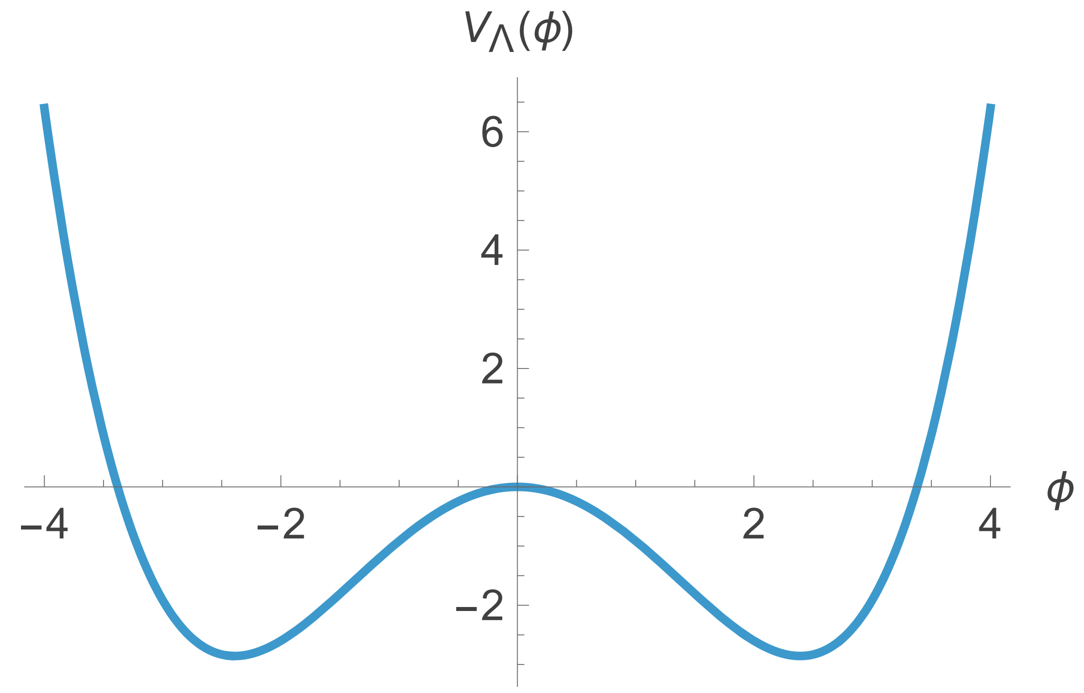
This is all known from the **Schrödinger equation**



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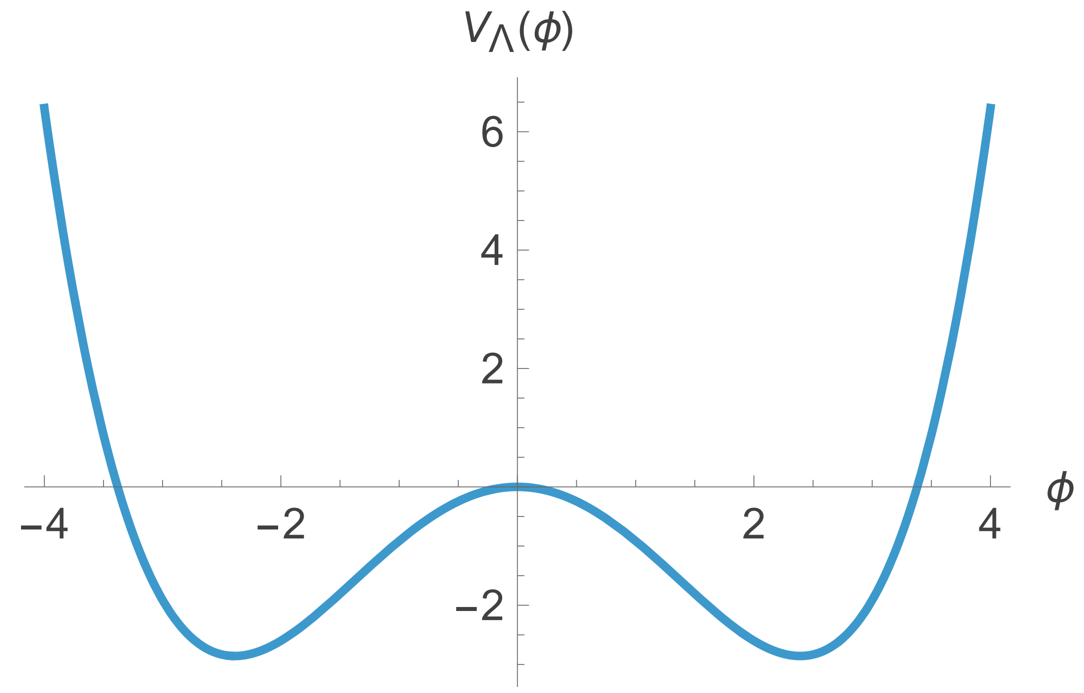

$$V_\Lambda(\hat{q}) = -\frac{1}{2}\hat{q}^2 + \frac{\lambda}{8}\hat{q}^4$$

This is all known from the **Schrödinger equation**

→ but also: Expansion about **Instanton solution**

$$\Delta E(\lambda_\varphi \to 0) \propto \frac{1}{\sqrt{\lambda_\varphi}}\, e^{-a_{\mathrm{inst}}/\lambda_\varphi}$$

The ground state-expansion

Topology in the anharmonic oscillator

- Quantum mechanics: i.e. 1+0 dimensional QFT

→ Setup with **tunneling**

→ Consider the **energy gap** between ground state and the first excited state Δ E

$$\langle \hat{q}(0)\hat{q}(\tau)\rangle = \sum_n |\langle 0|q|n\rangle|^2 \exp(-(E_0 - E_n)\tau)$$

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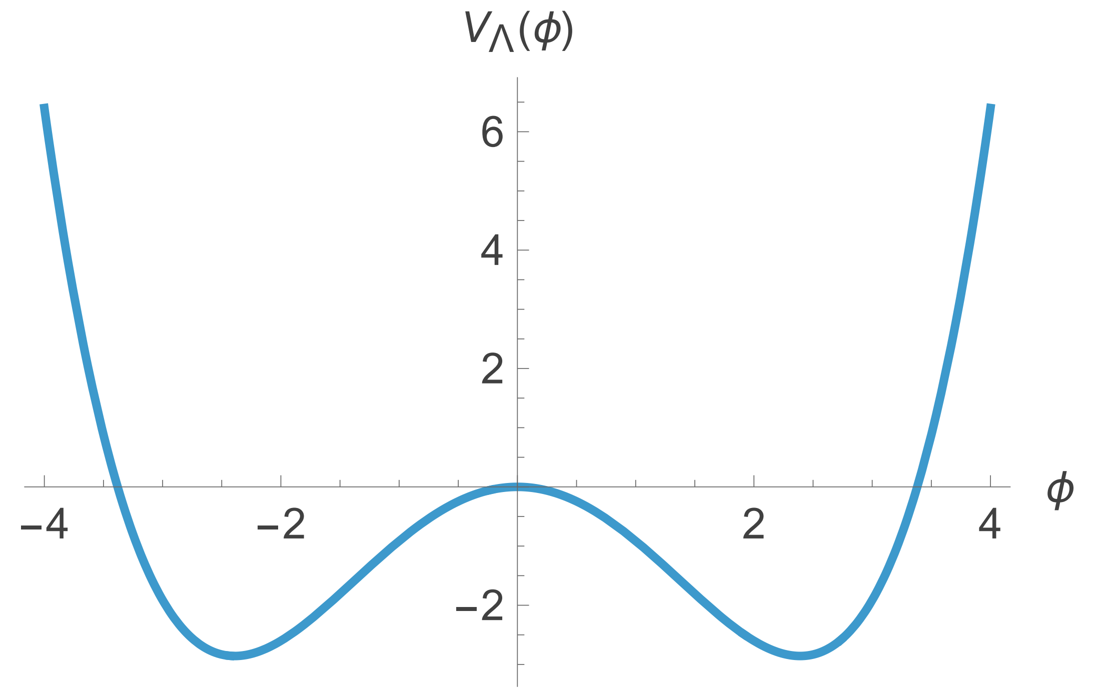

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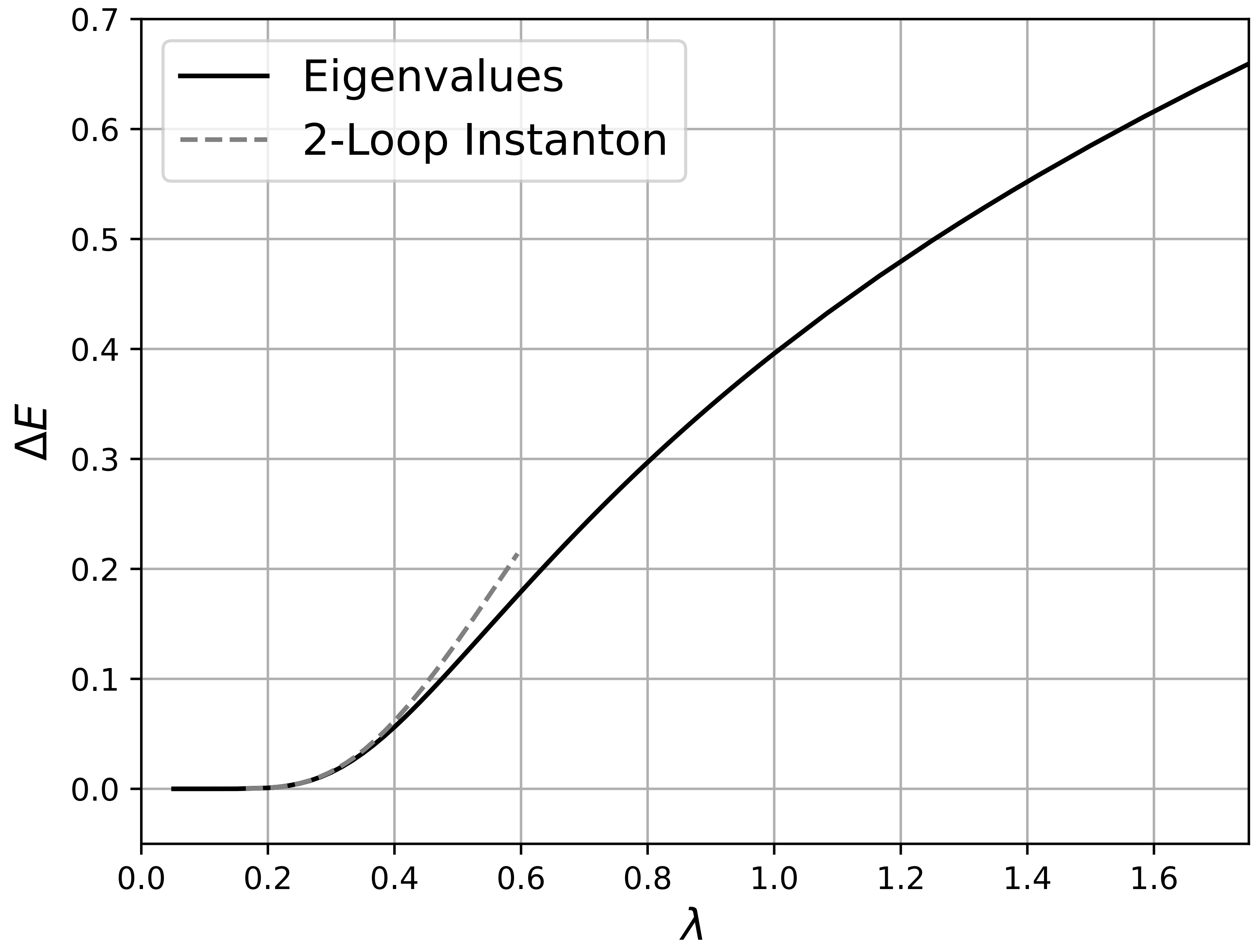

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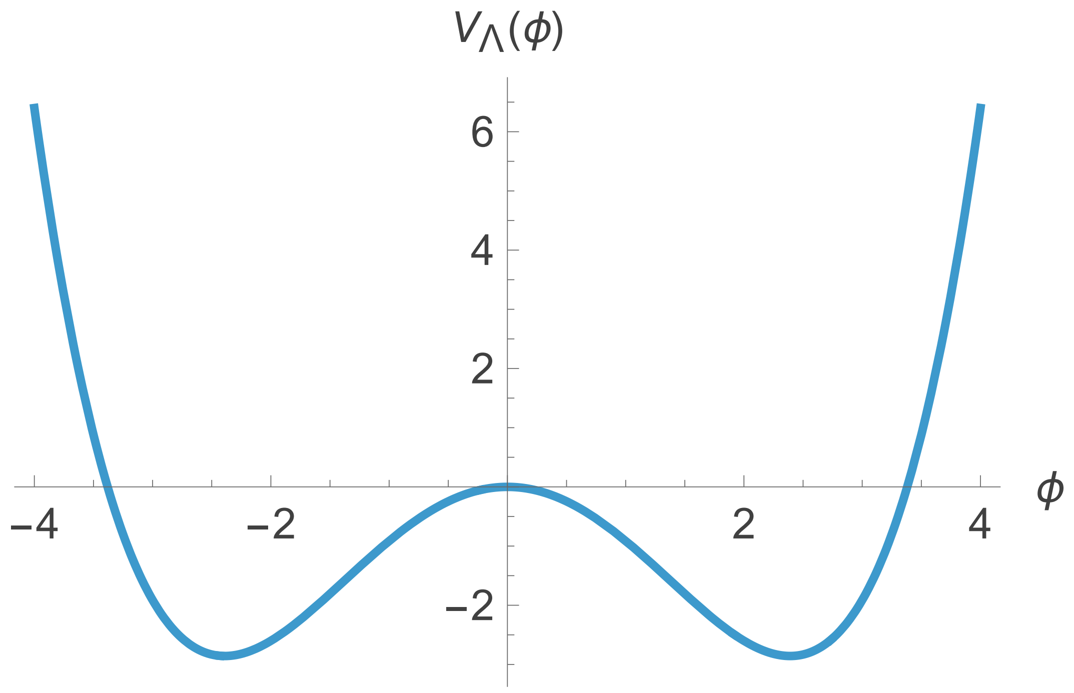

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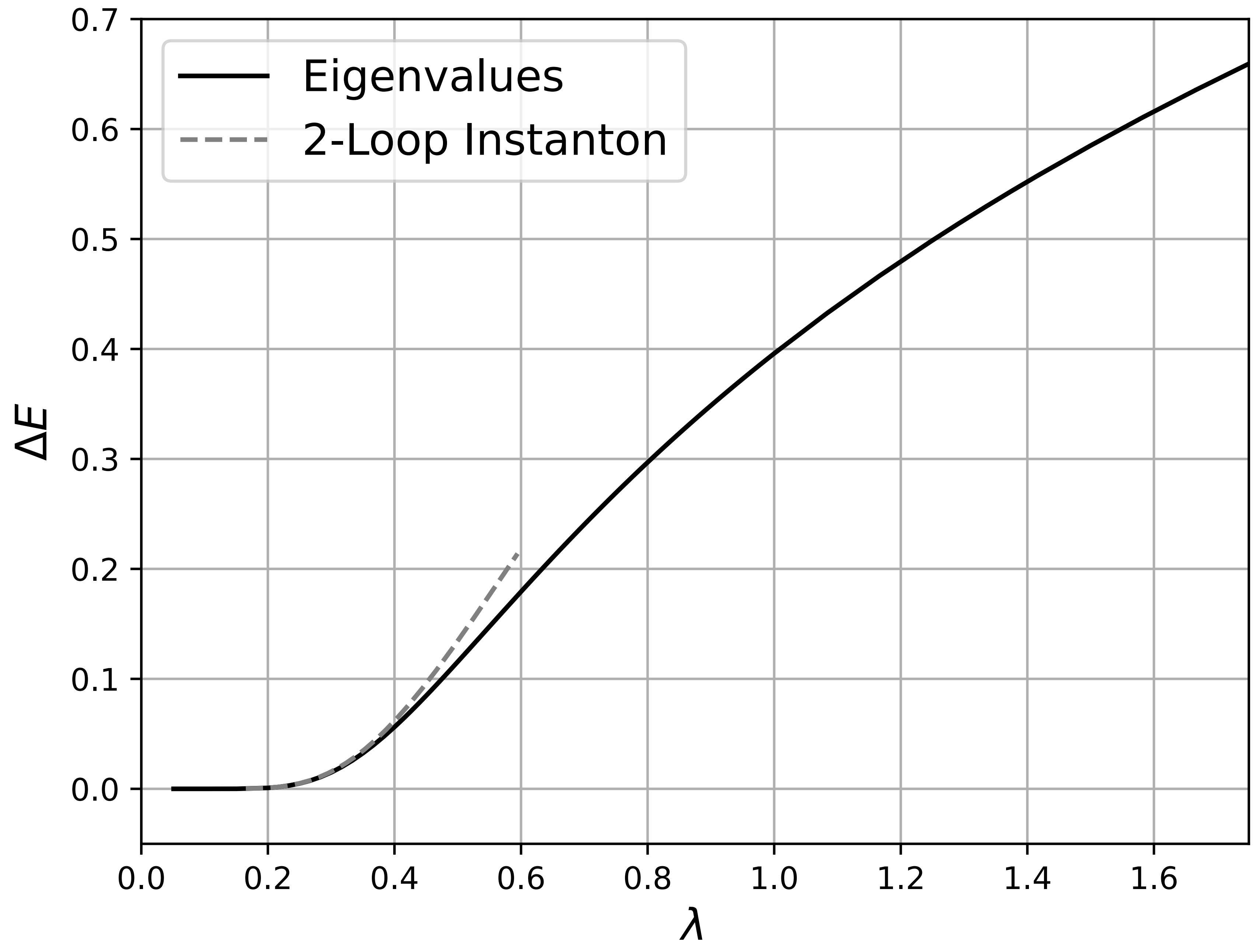

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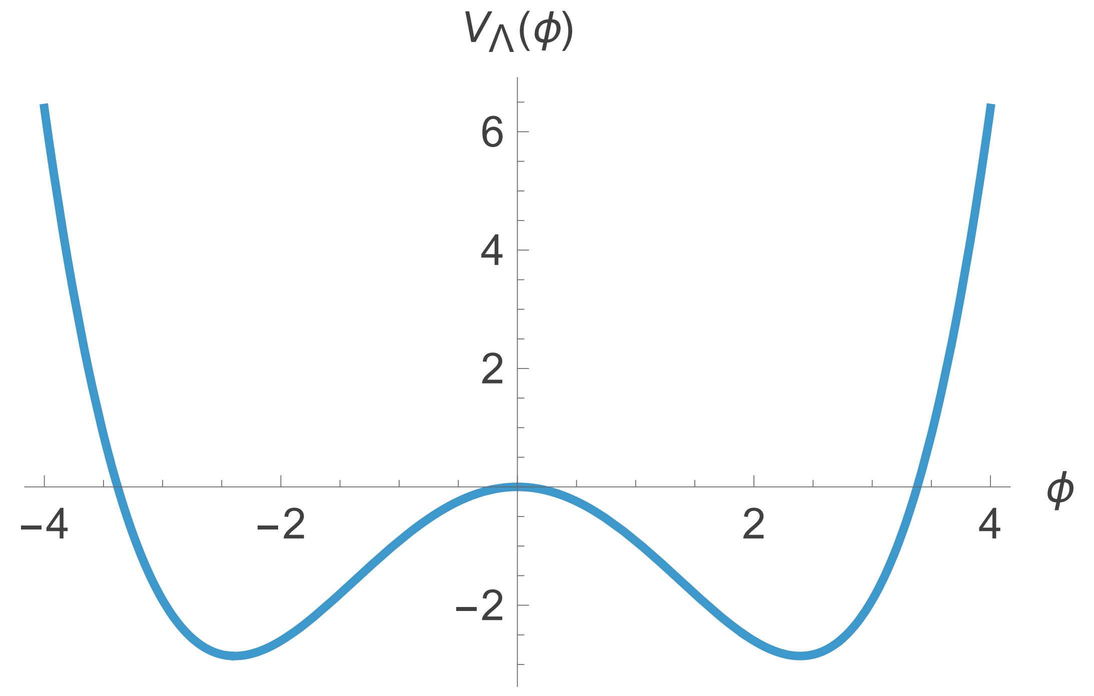

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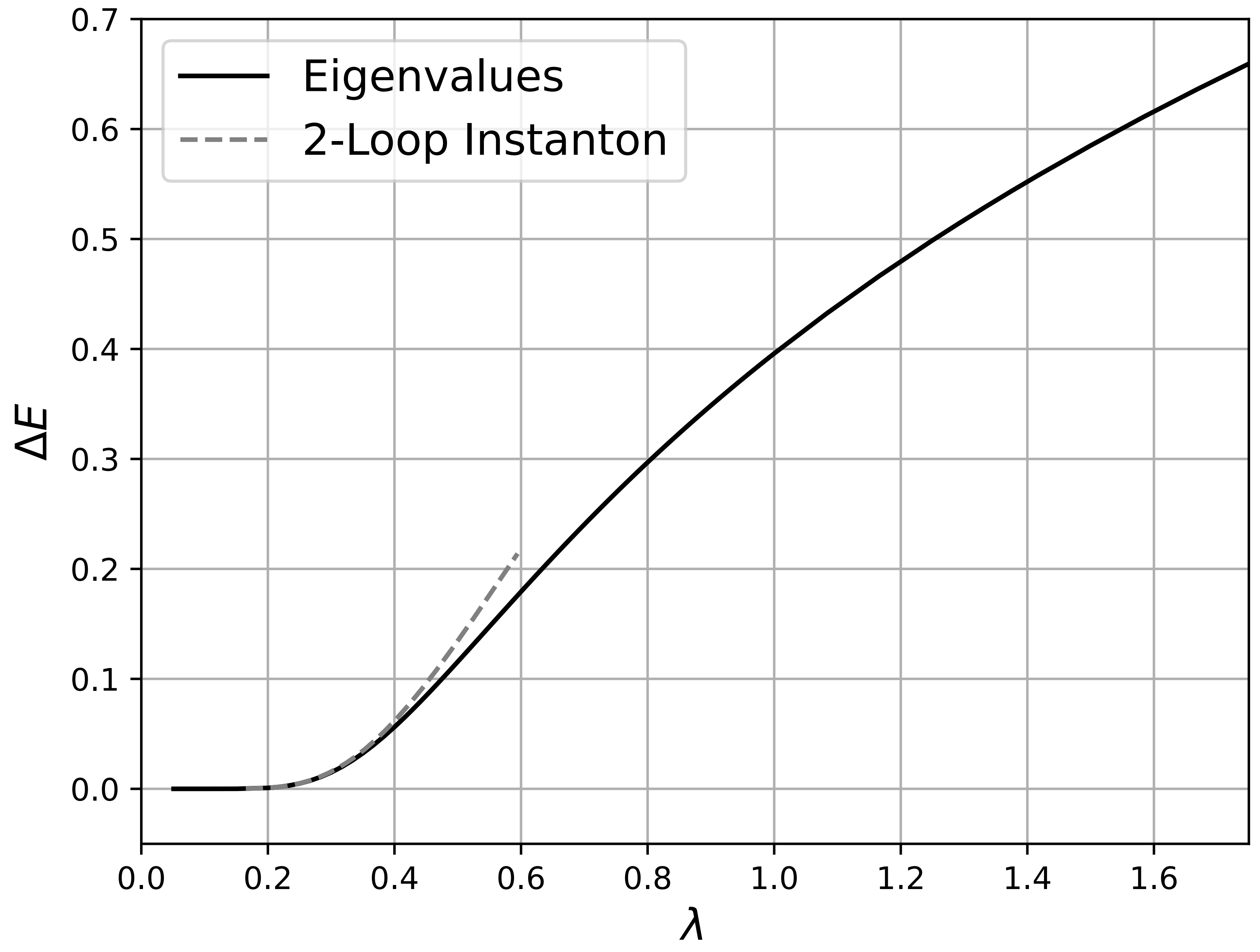

Testing ground for novel methods with non-perturbative physics

→ In particular where fRG has struggled before

The anharmonic oscillator in the fRG

Bonanno, **FI,** Pawlowski, SciPost Phys. Core 9, 005 (2026)

The anharmonic oscillator in the fRG

Chose a simple truncation with the derivative expansion (LPA)

$$\Gamma_k[\phi] = \int_\tau \left[\frac{1}{2}(\partial_\tau \phi)^2 + V_k(\phi)\right]$$

Solve the PDE for the Wetterich equation, and extract $\Delta E = \sqrt{V^{(2)}(\phi_{\mathrm{eom}})}$

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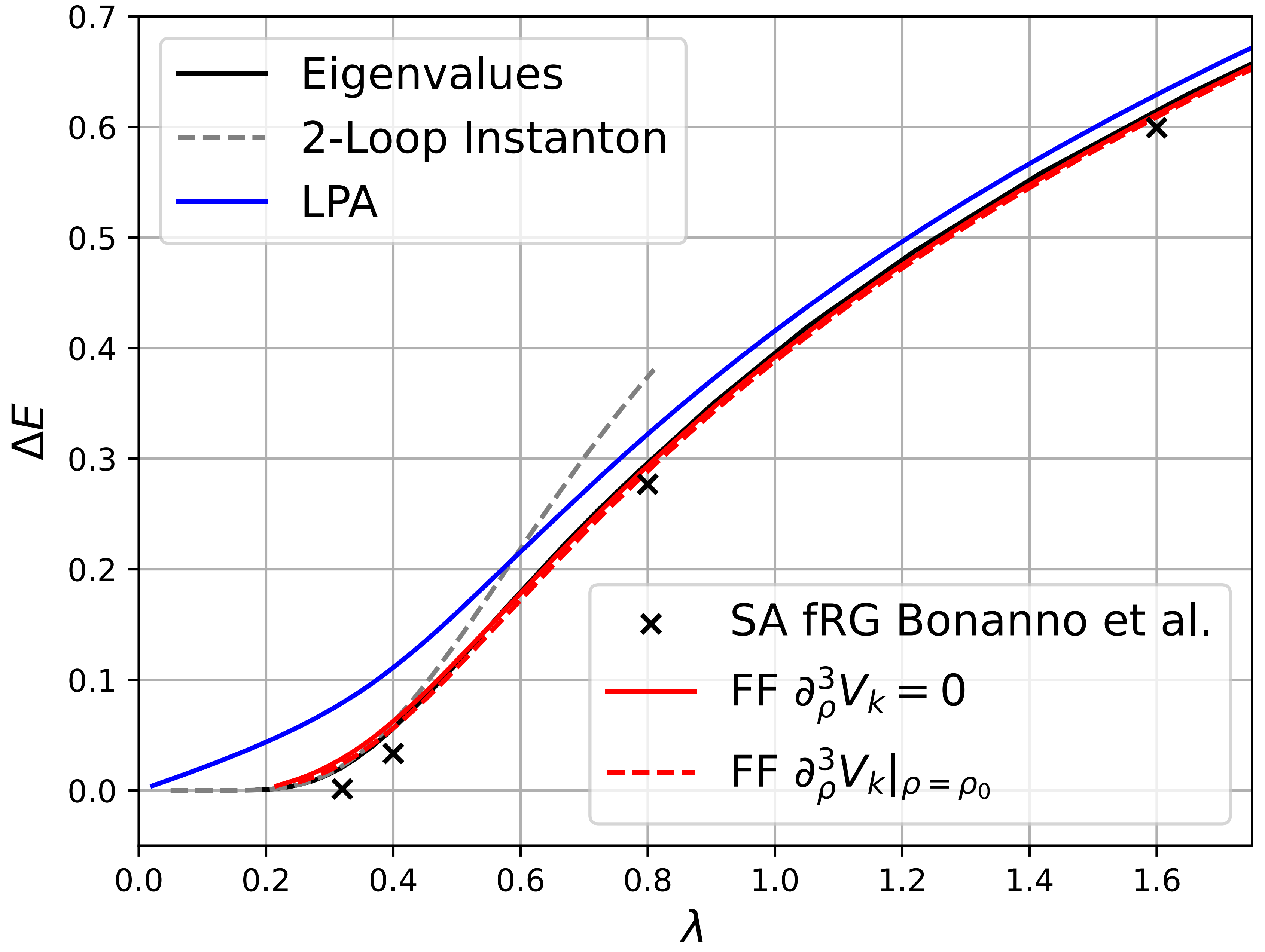

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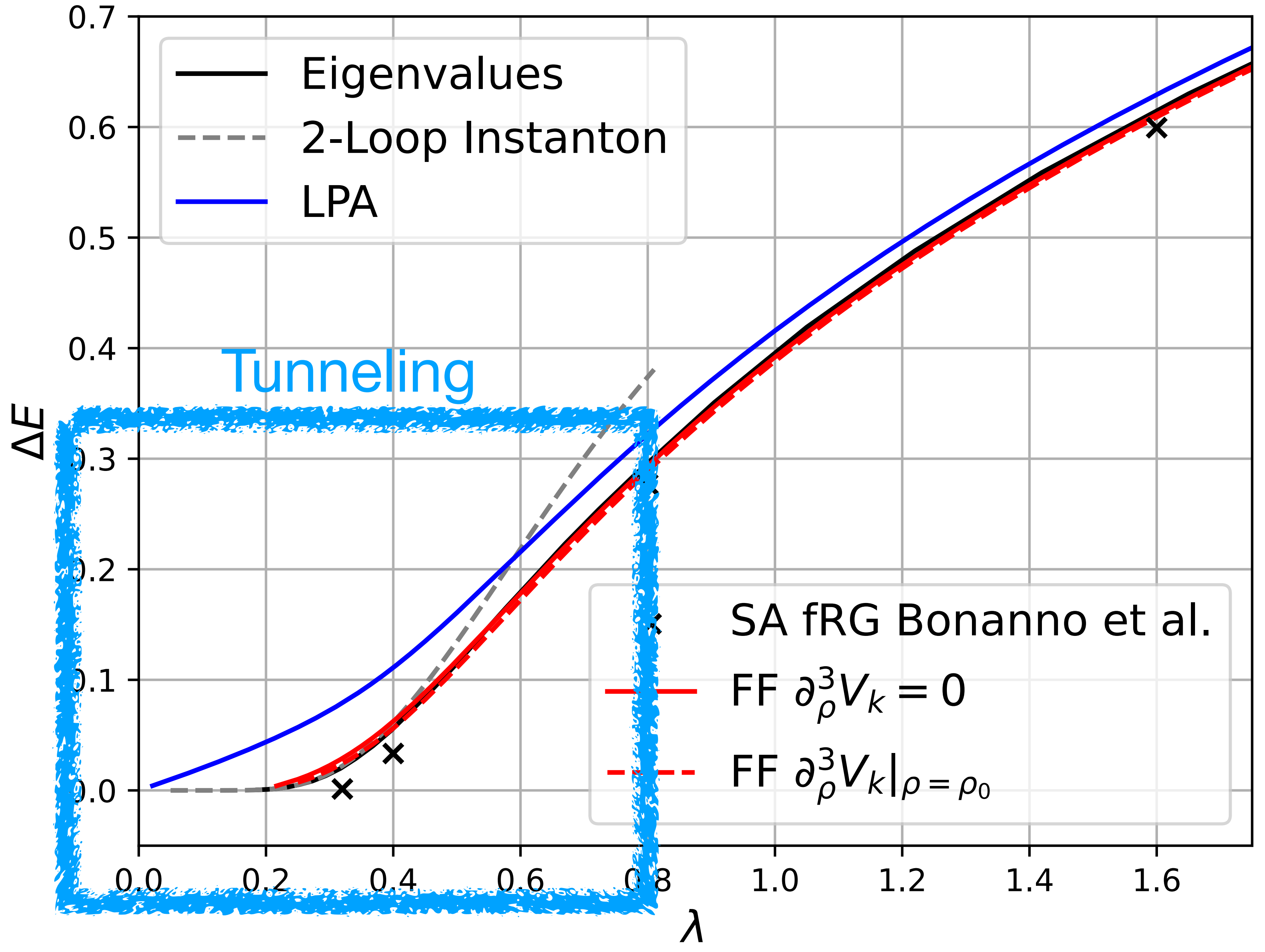

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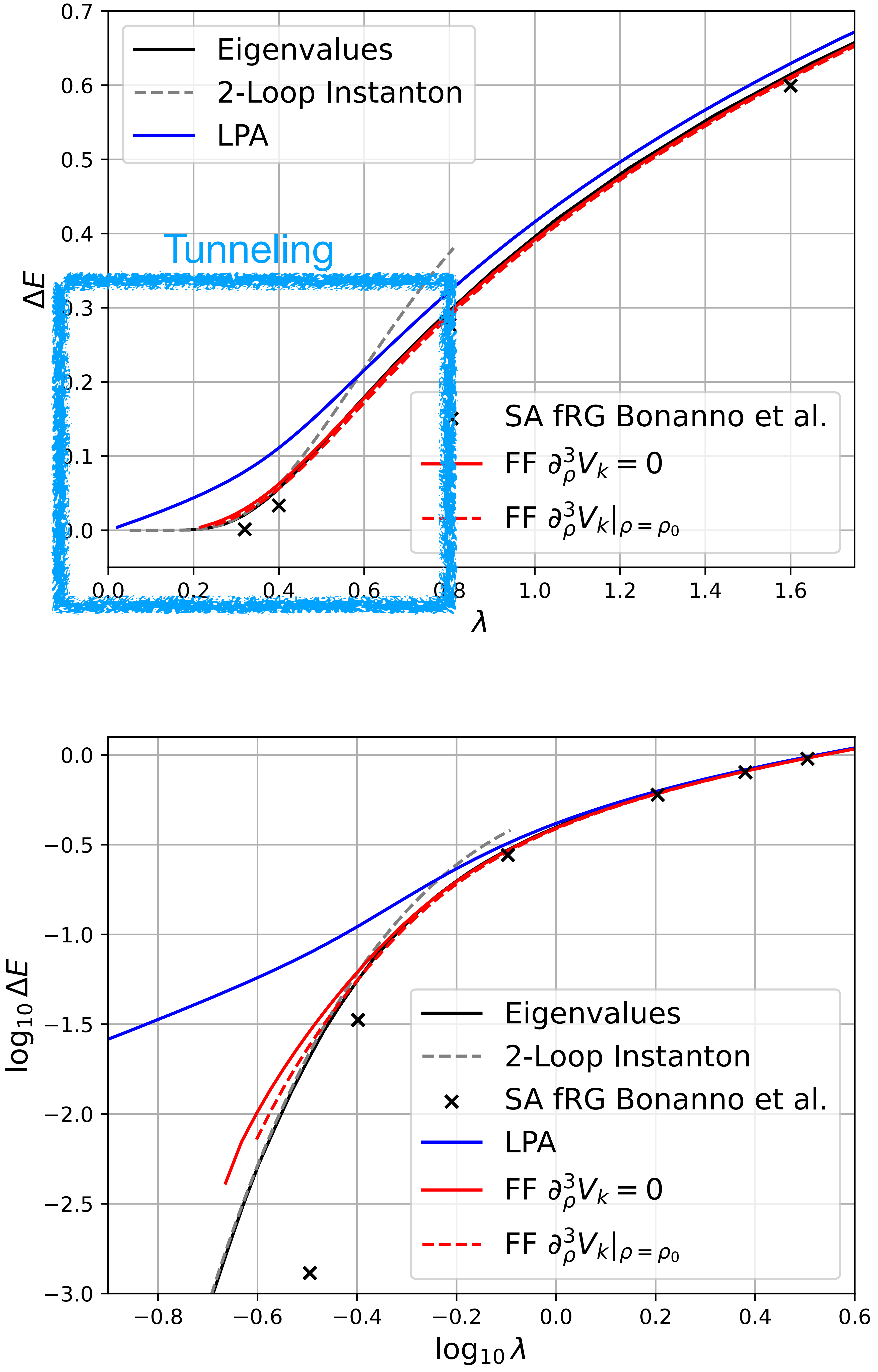

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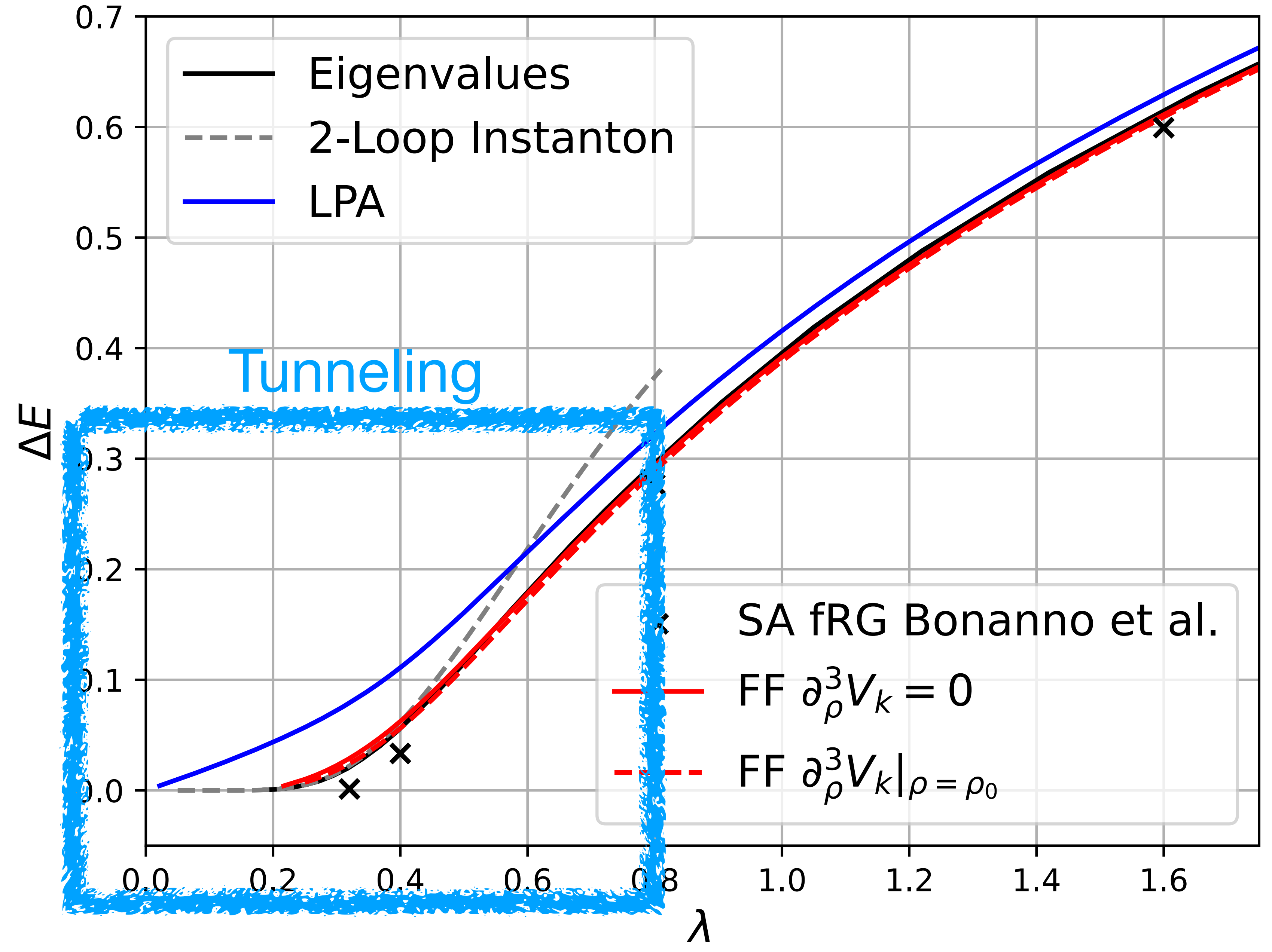
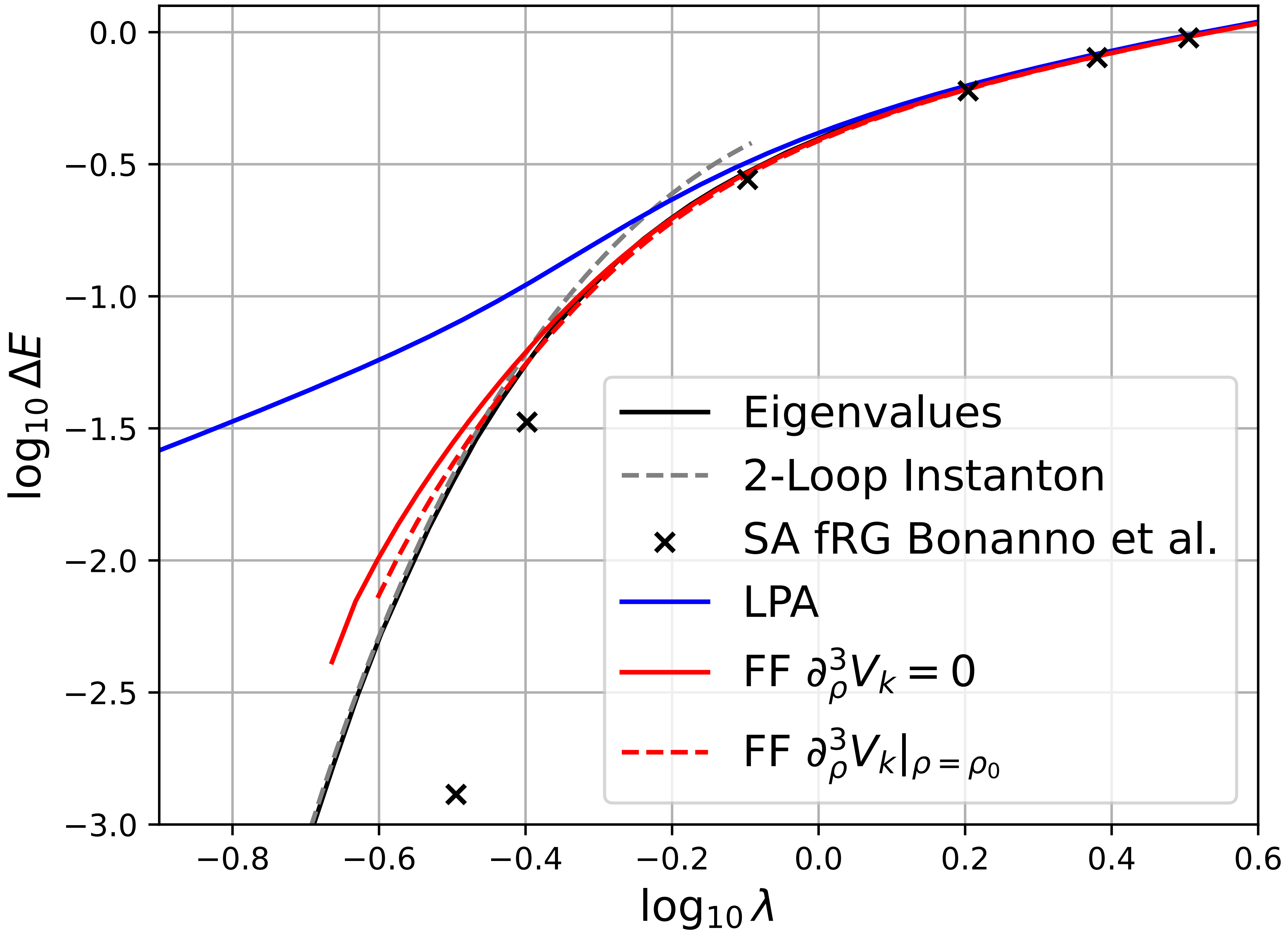
Solve the generalized equation in the same truncation for Γ_k

With an **optimized kernel**:
(Ground state expansion)

$$\dot{\phi}(\phi) \to \partial_t(\partial_{p^2}\Gamma_k^{(2)}[\phi]) = 0$$





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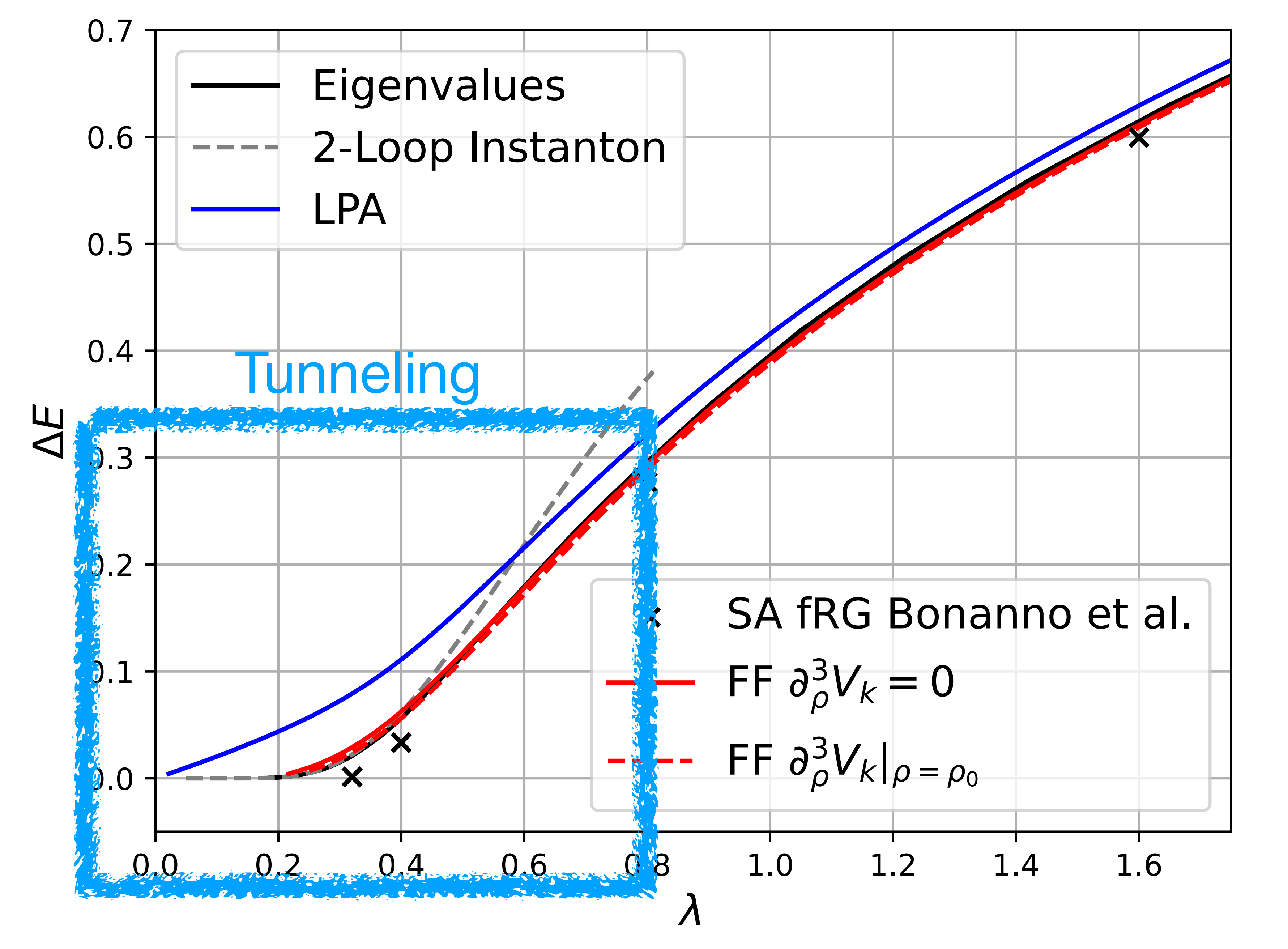
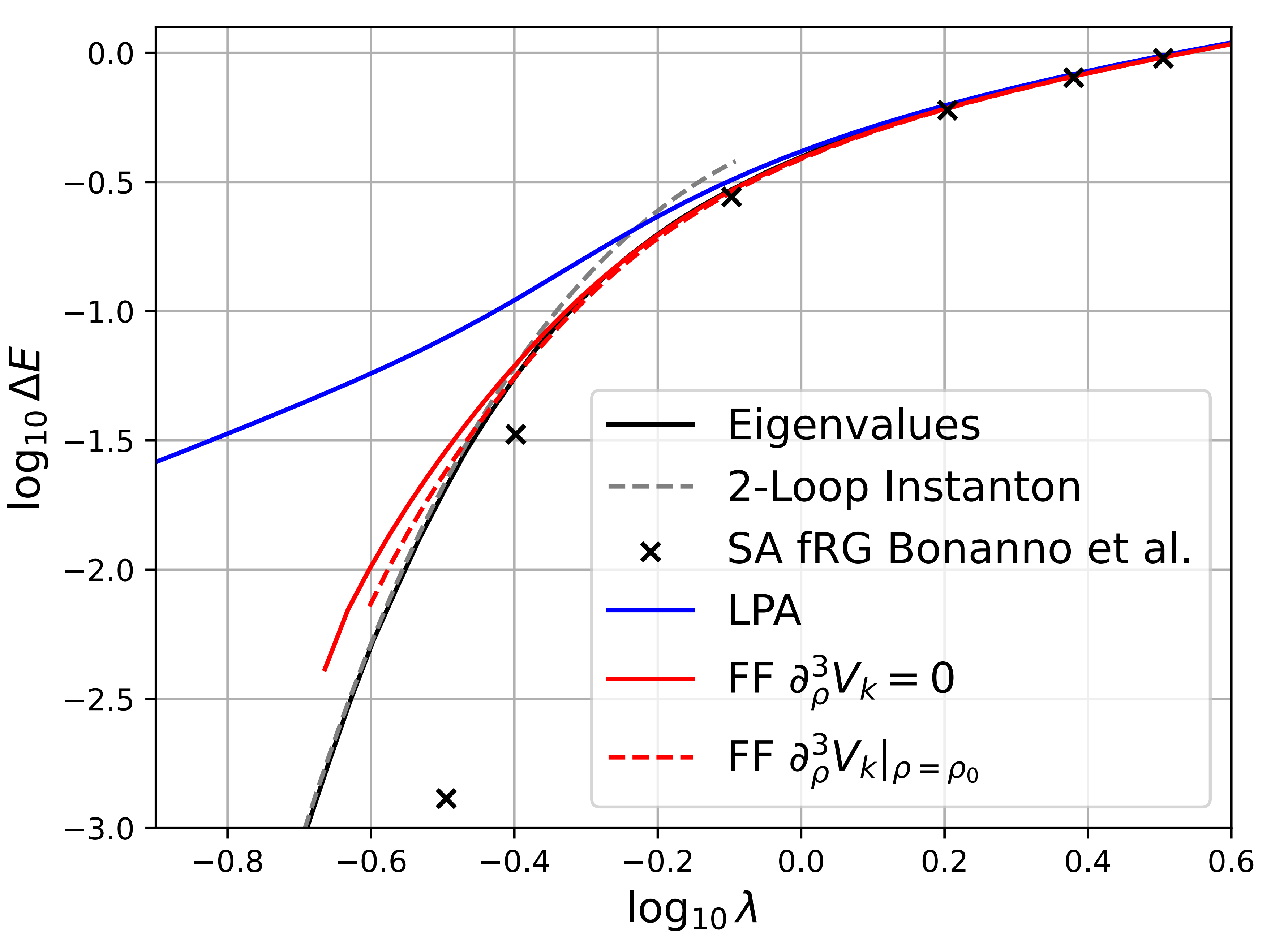
Results in a field dependent anomalous dimension:





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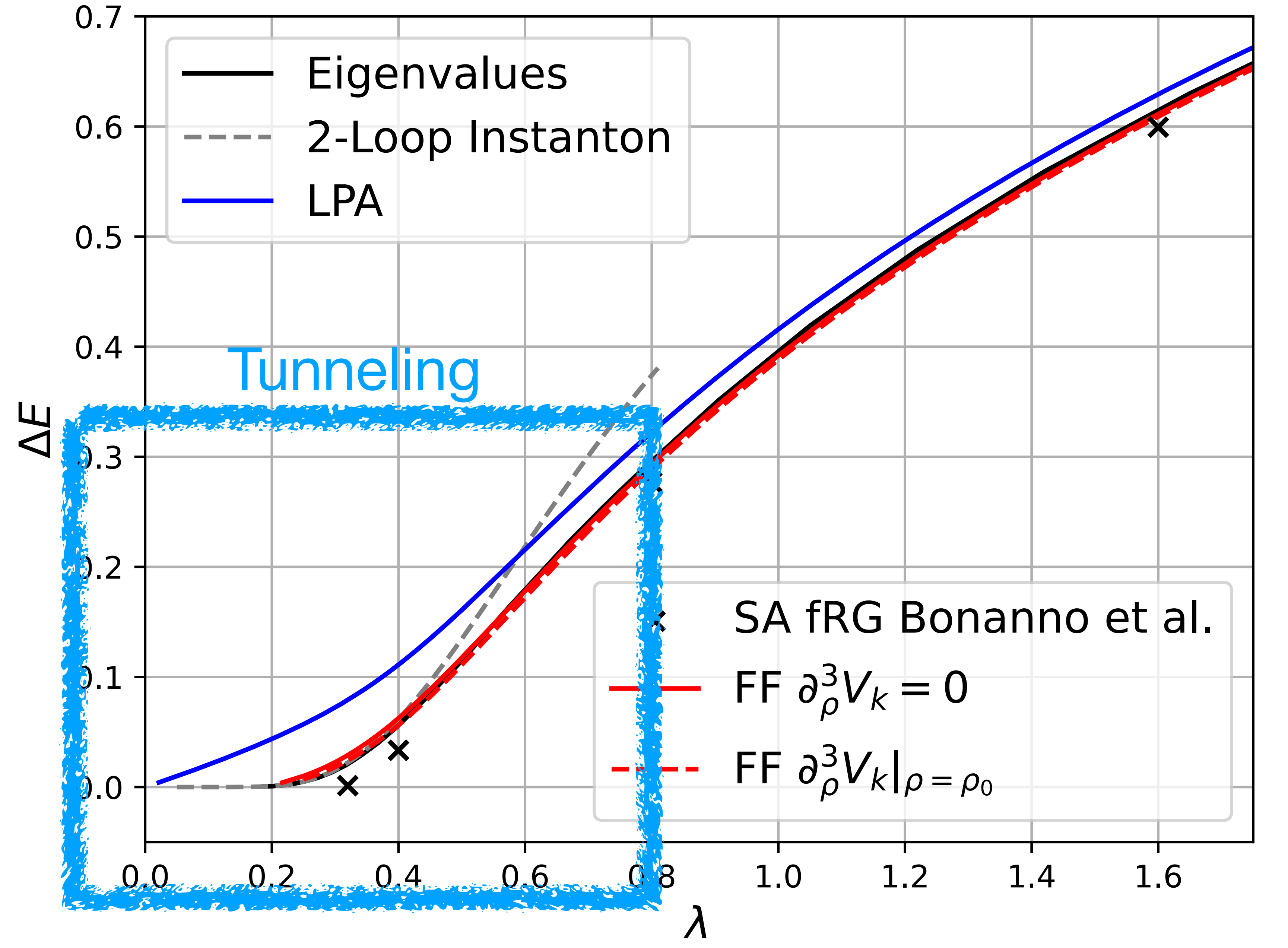
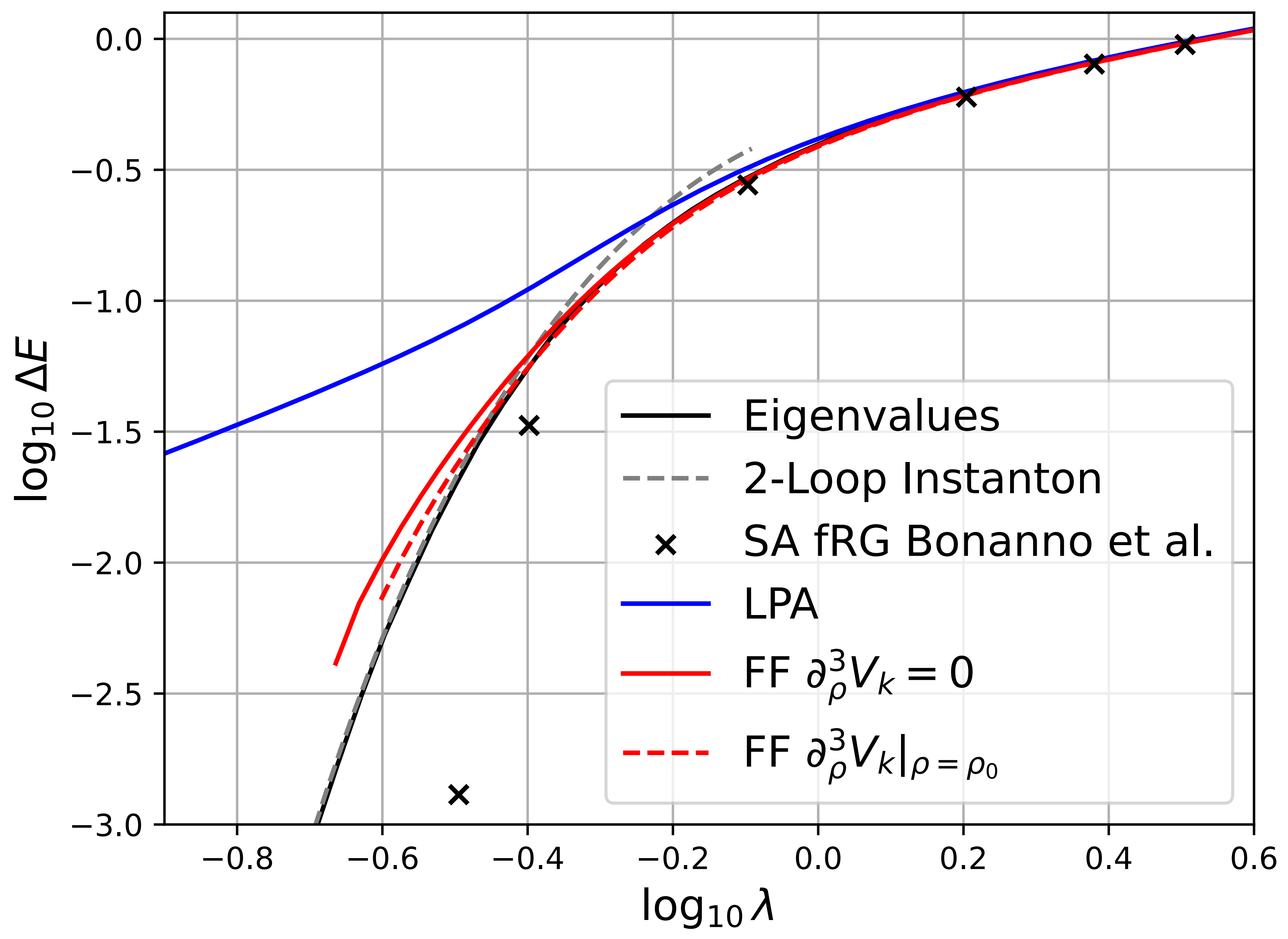
Results in a field dependent anomalous dimension:

This allows the extraction of the **instanton scaling** up to 1-2% exactness!

$a_{inst,ex} = 1.886....$ $a_{inst,FF} = 1.910(2)$





Bonanno, **FI,** Pawlowski, SciPost Phys. Core 9, 005 (2026)

The ground-state expansion

Baldazzi, Ben Ali Zinati, Falls, *SciPost Phys.* 13 (2022) 4, 085
FI, Pawlowski, arXiv:2305.00816
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Consider the standard derivative expansion:

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First-order expansion

RG-flow now accounts for fluctuation physics of order p^2

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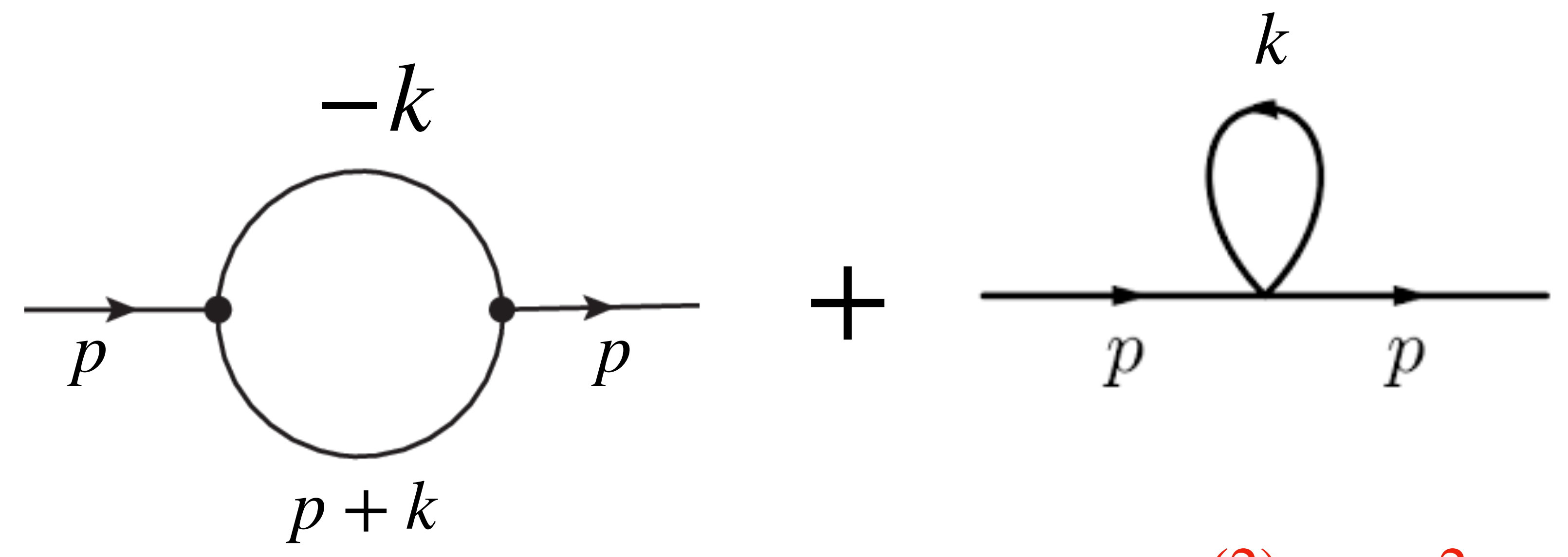
In this formulation the self-energy diagrams get momentum dependent corrections:

→It creates e.g. a momentum dependent Tadpole diagram for all n-point functions

$$\partial_t\Gamma_k^{(2)}[\phi] \propto$$



$\propto Z^{(2)}(\phi)p^2$

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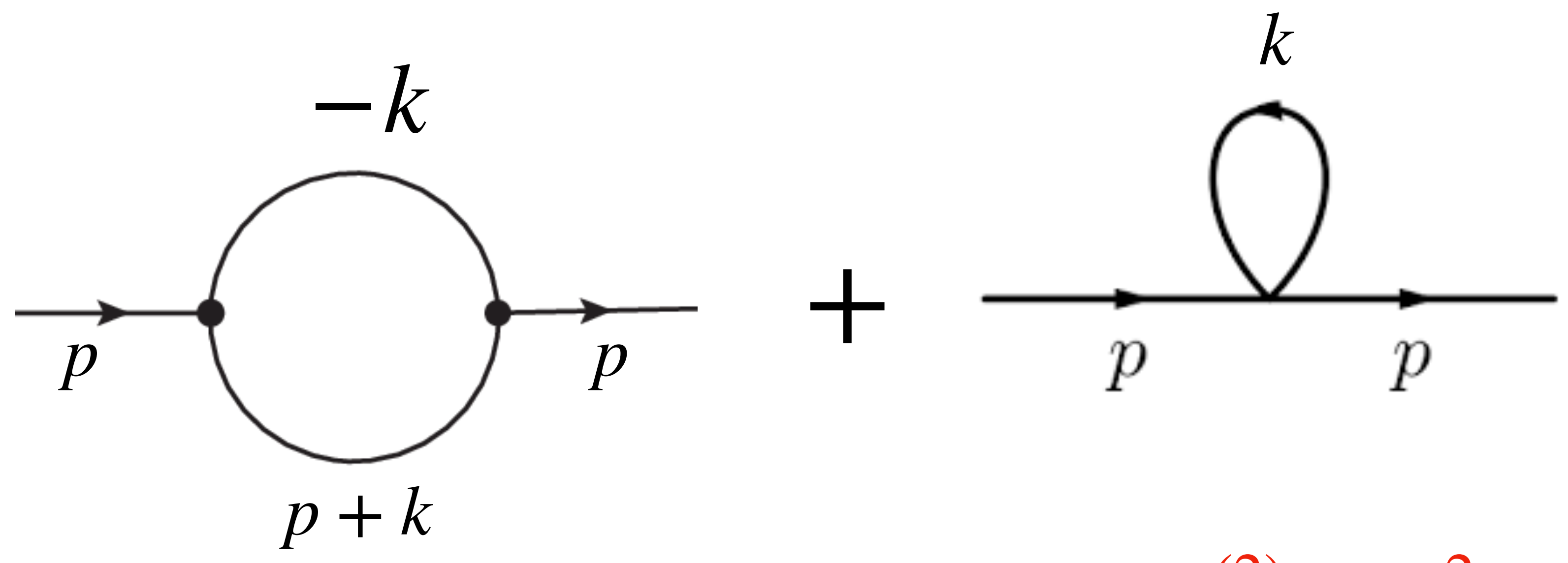

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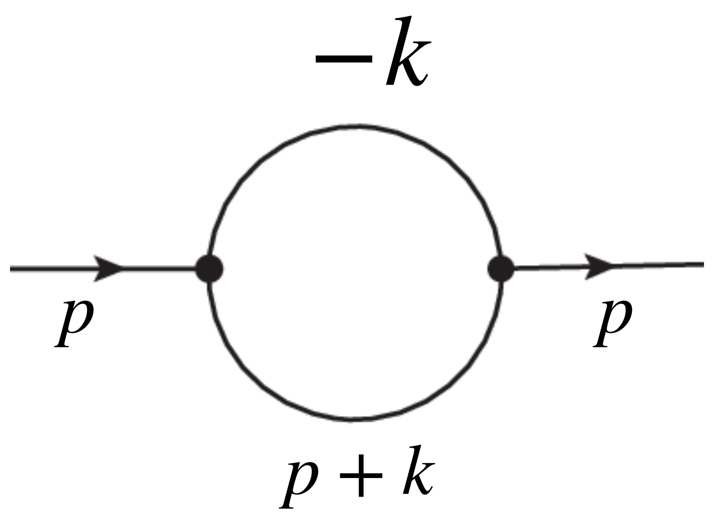

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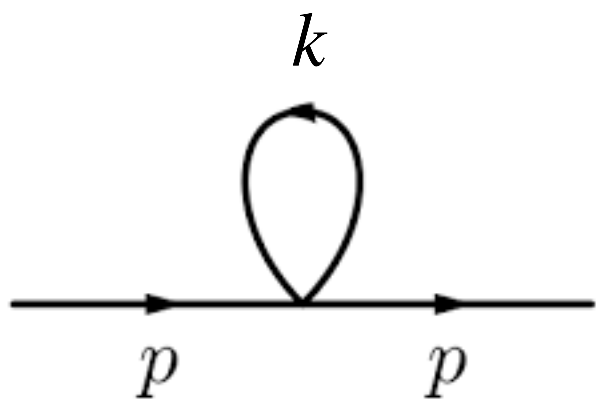

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$$\Gamma_k^{(2)} = p^2 + \tilde{m}^2(\phi) \longrightarrow \dot{\phi}(\phi) \to \partial_t Z(\phi) \equiv 0$$

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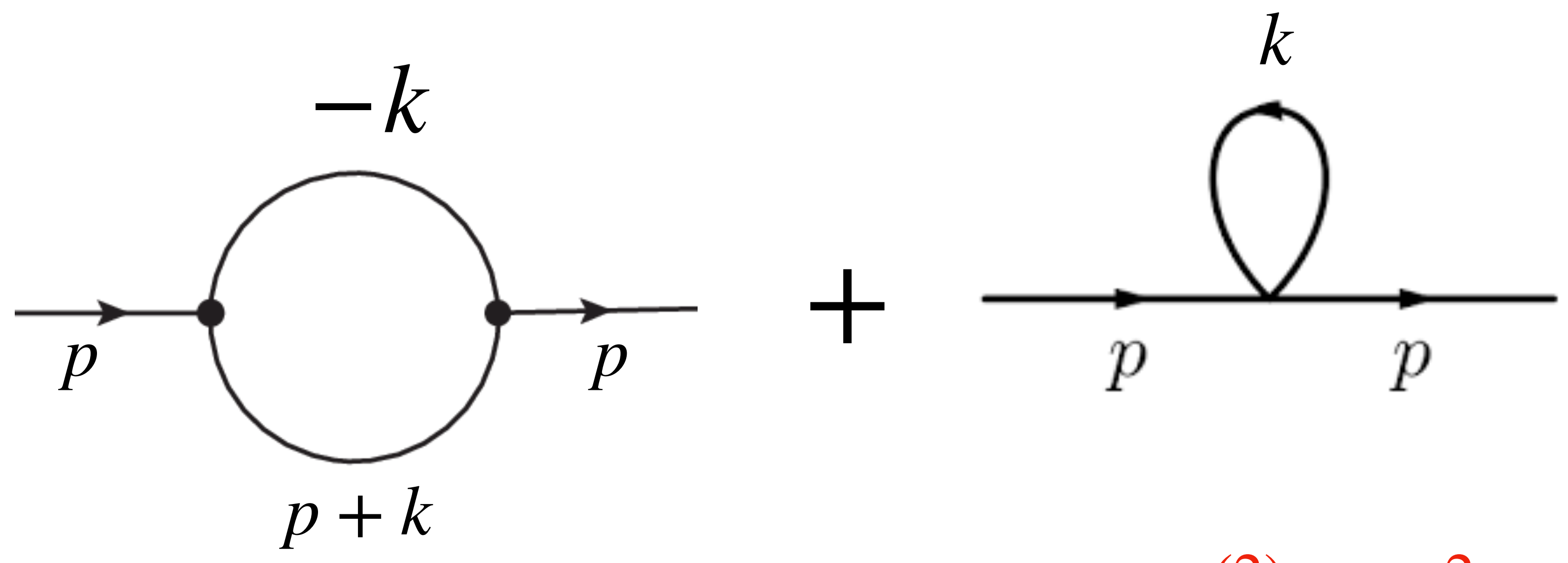

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Vertex has no field dependence →
No momentum dependent tadpole

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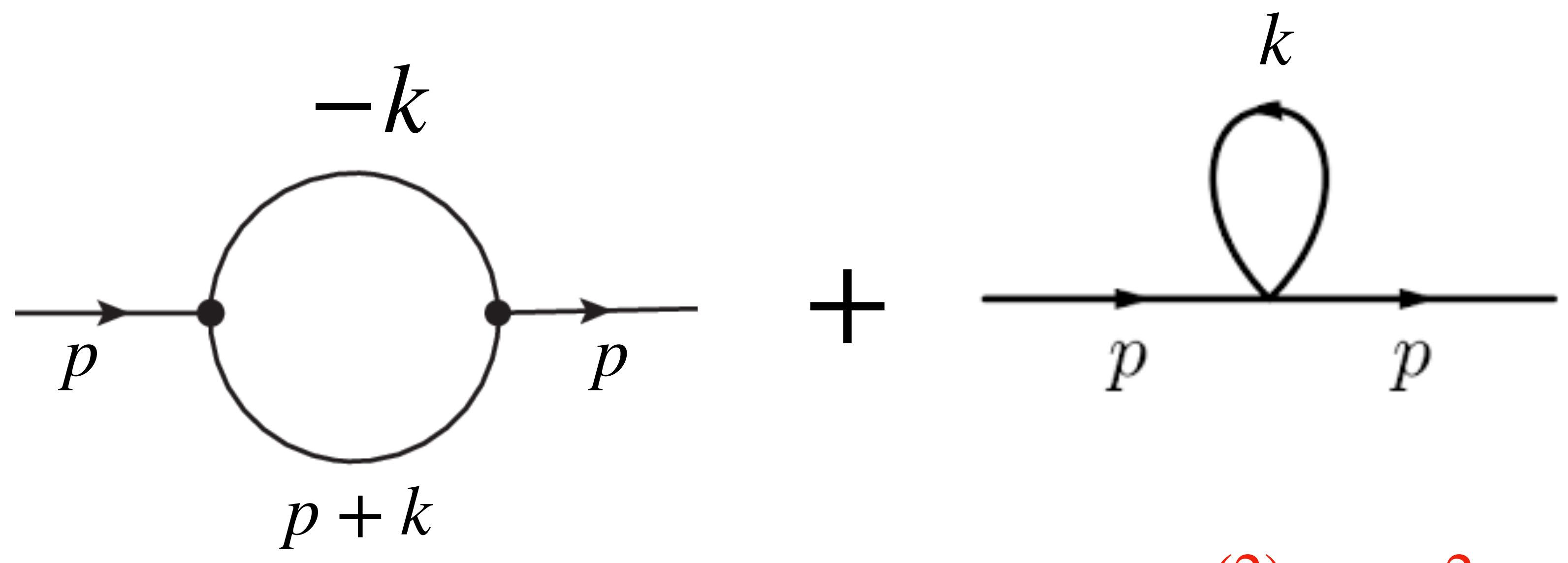

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Effective action looks zeroth-order again, but **includes momentum dependent physics of all orders of tadpole diagrams** via $\dot{\phi}$

$$Z(\phi) \equiv 1$$

Vertex has no field dependence → No momentum dependent tadpole

The ‚absorption strategy‘

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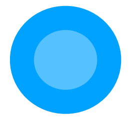
The ground-state expansion can be thought of as a **multi-loop scheme** (to infinite order):

The ‚absorption strategy‘

Loop-correction

 $=$  $\partial_t R_k$ $+ \dfrac{\delta \dot{\phi}}{\delta \phi} R_k$

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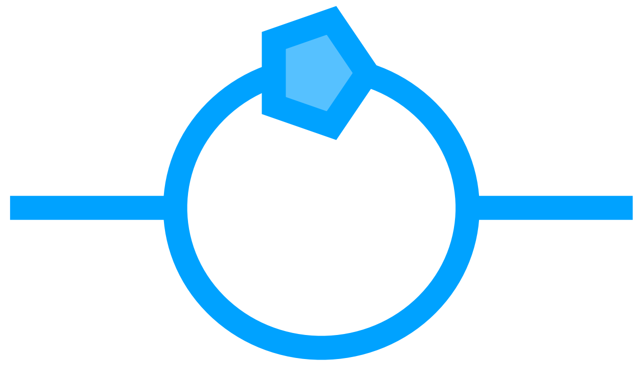
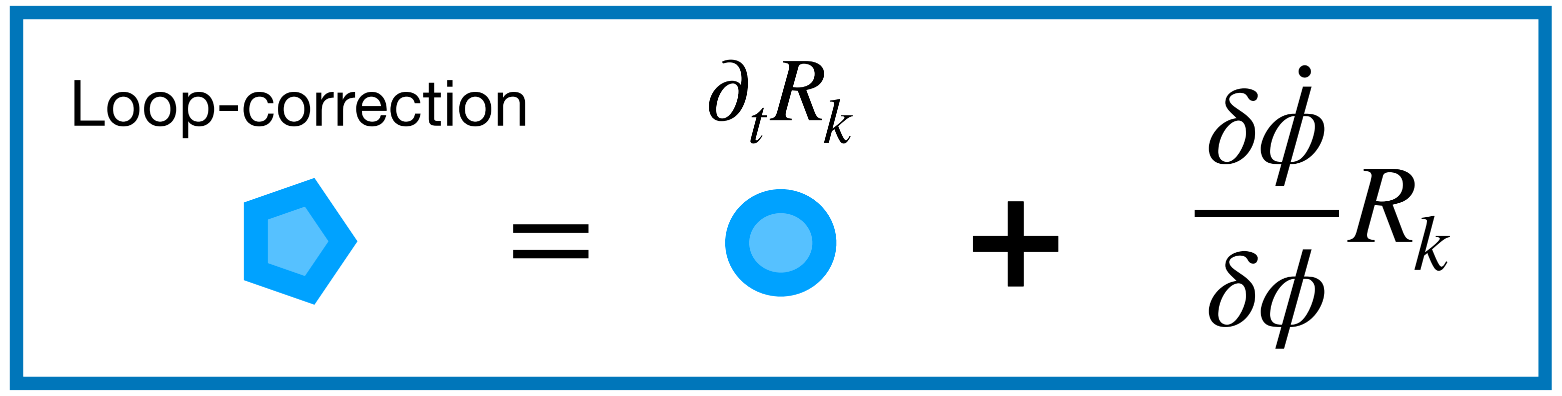


$\sim p^2$

p^2-correction to the flow

The ‚absorption strategy‘



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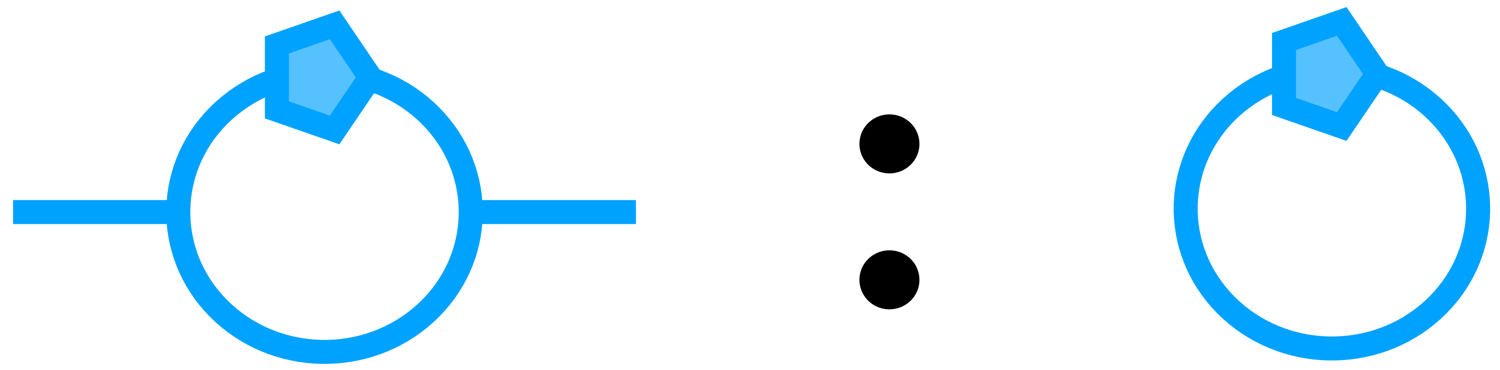
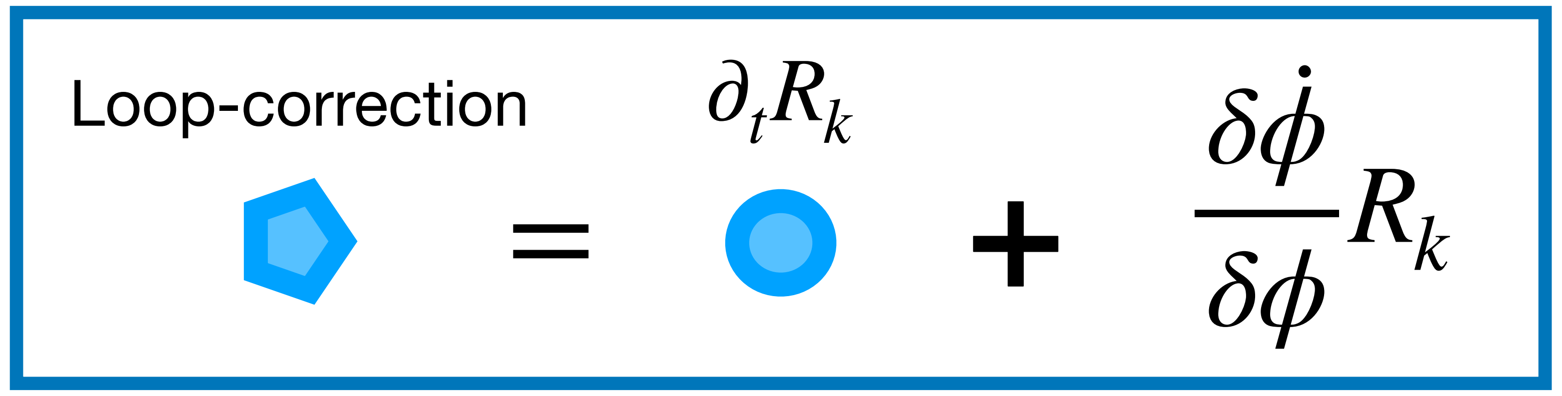


$\sim p^2$

p^2-correction to the flow

Appears in the loop-correction

The ‚absorption strategy‘



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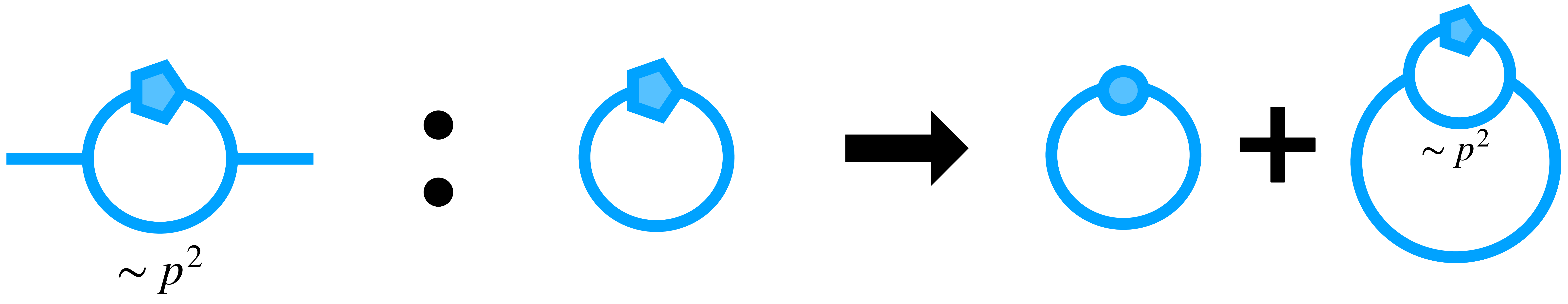

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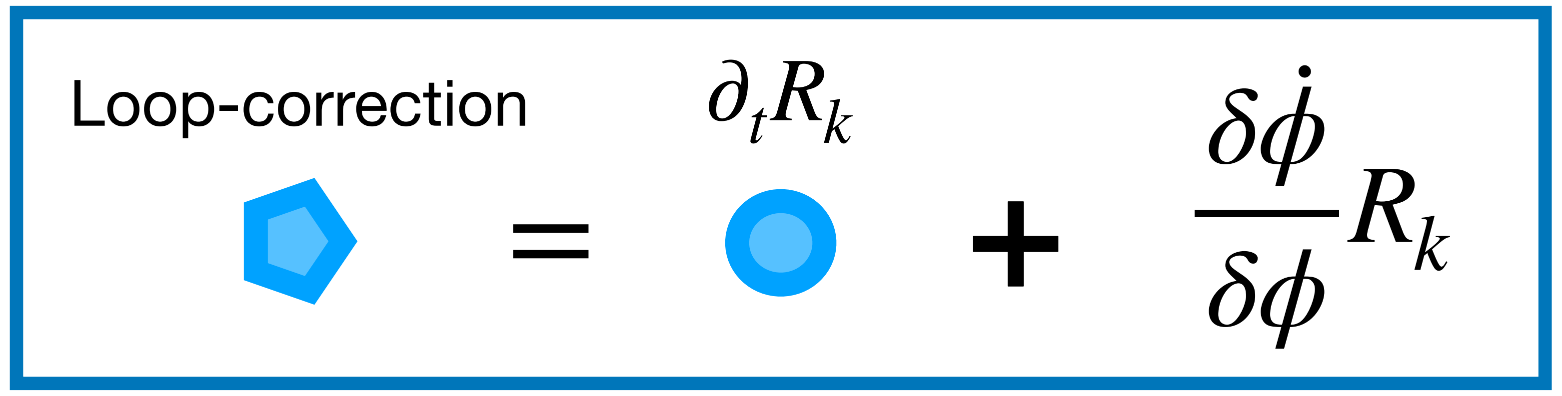

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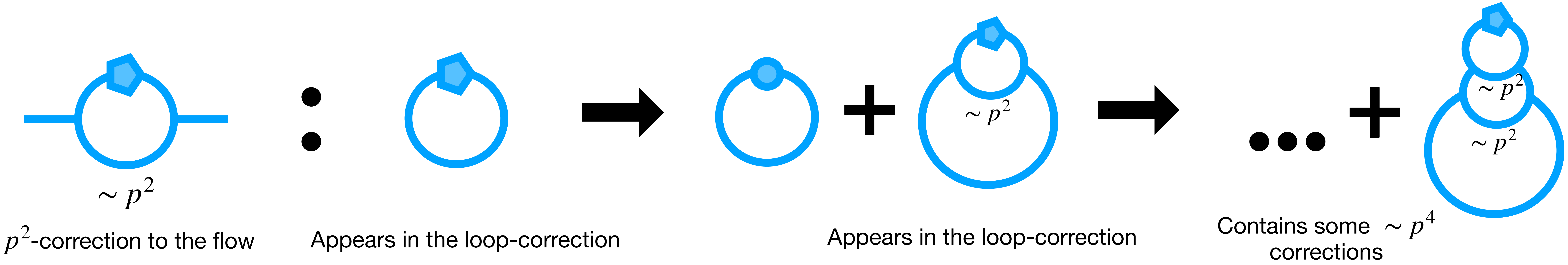

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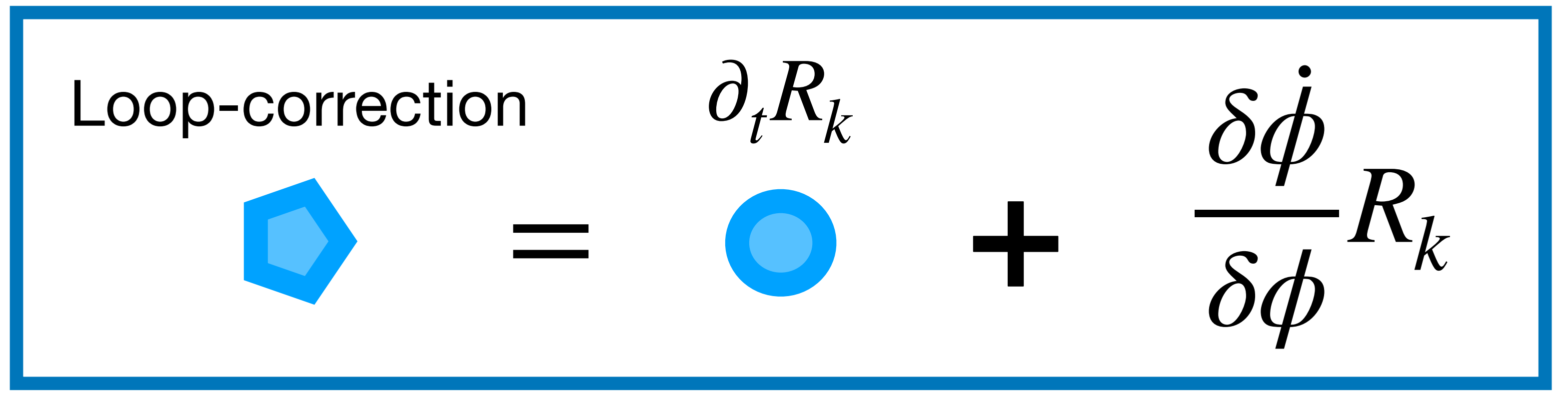

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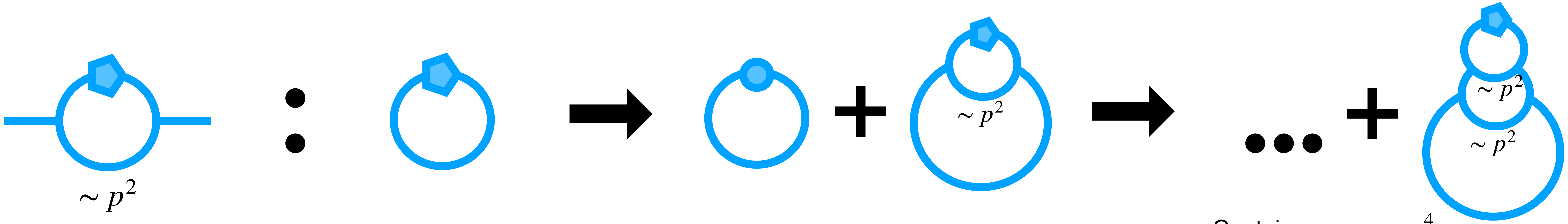


p^2-correction to the flow

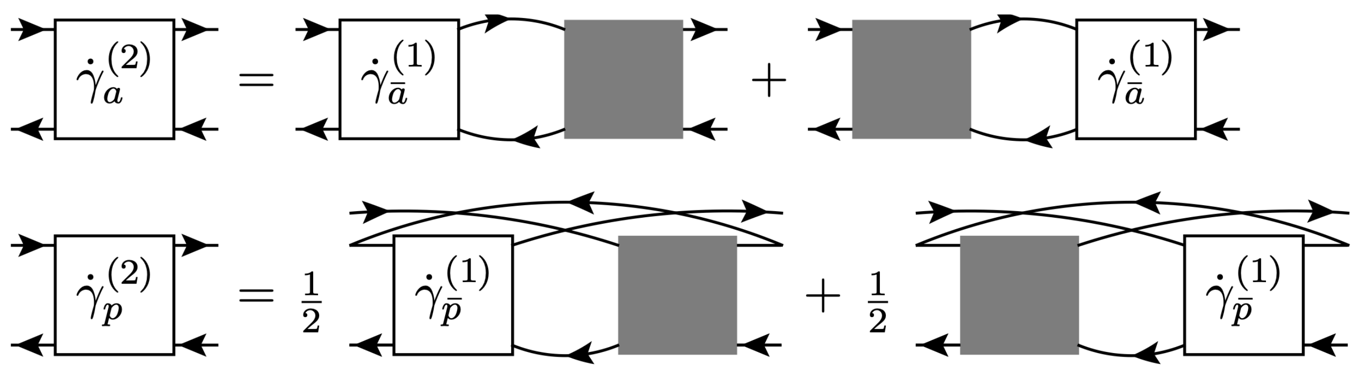
Appears in the loop-correction

Appears in the loop-correction

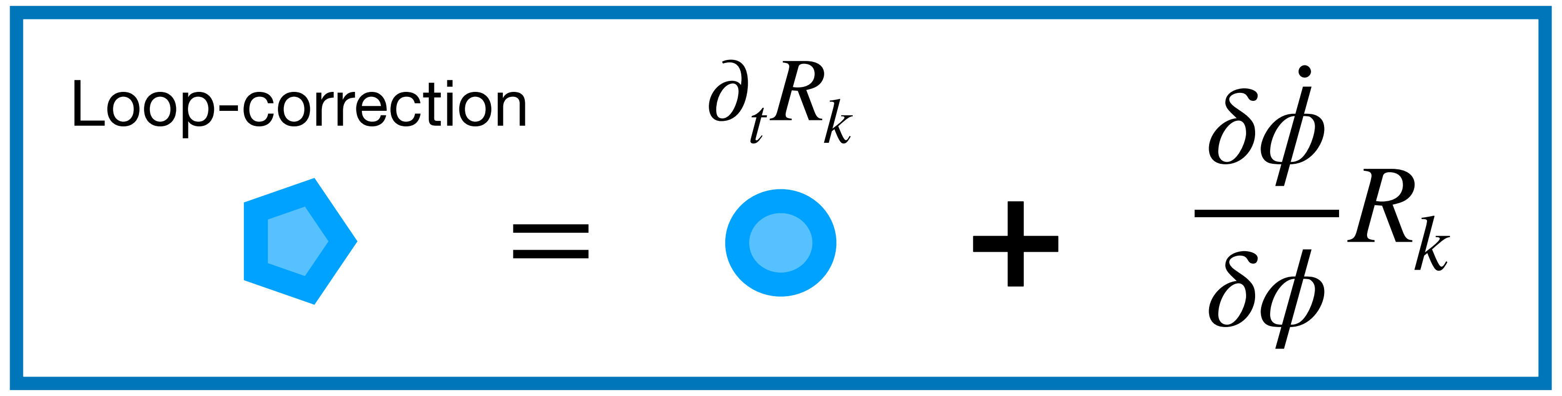
Contains some $\sim p^4$ corrections

In this sense it is related to the very successful **multi-loop fRG** (Fermionic 6-pt vertex is absorbed)

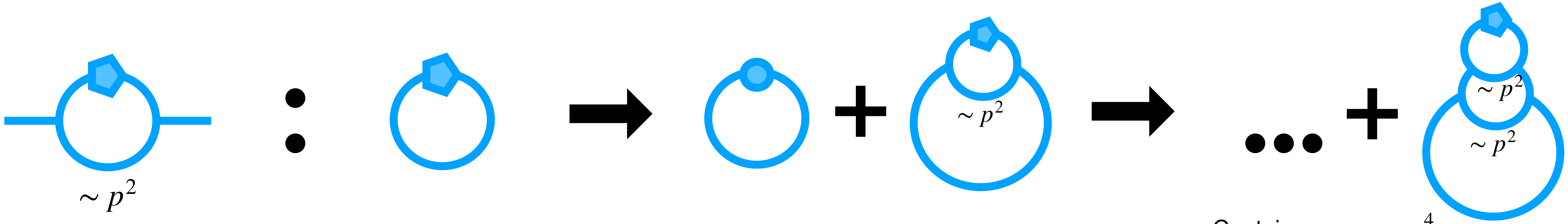
Kugler, von Delft, PRL 120, 057403 (2018)
Kugler, von Delft, PRB 97, 035162 (2018)



The ‚absorption strategy‘



The ground-state expansion can be thought of as a **multi-loop scheme** (to infinite order):



p^2-correction to the flow

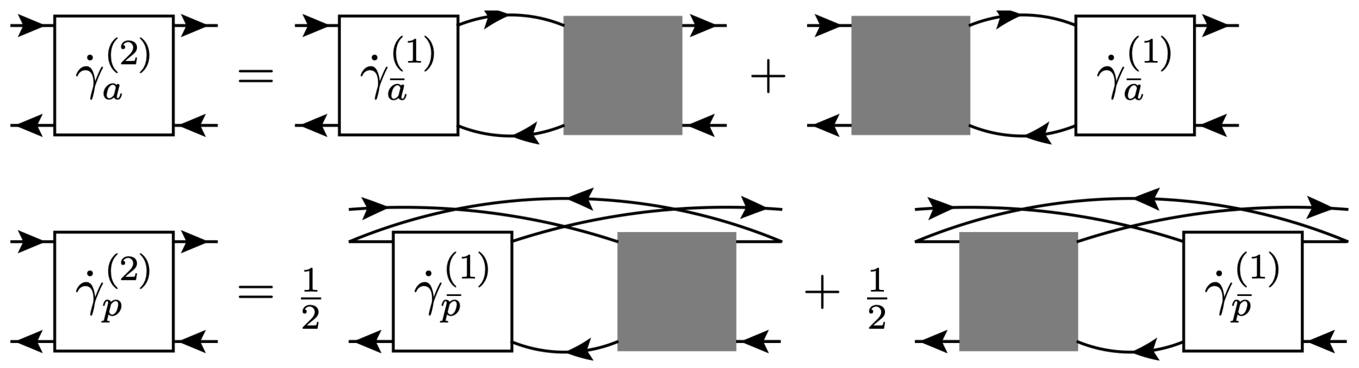
Appears in the loop-correction

Appears in the loop-correction

Contains some $\sim p^4$ corrections

In this sense it is related to the very successful **multi-loop fRG** (Fermionic 6-pt vertex is absorbed)

Kugler, von Delft, PRL 120, 057403 (2018)
Kugler, von Delft, PRB 97, 035162 (2018)



→ But automatically **includes all orders** (resummed scheme)

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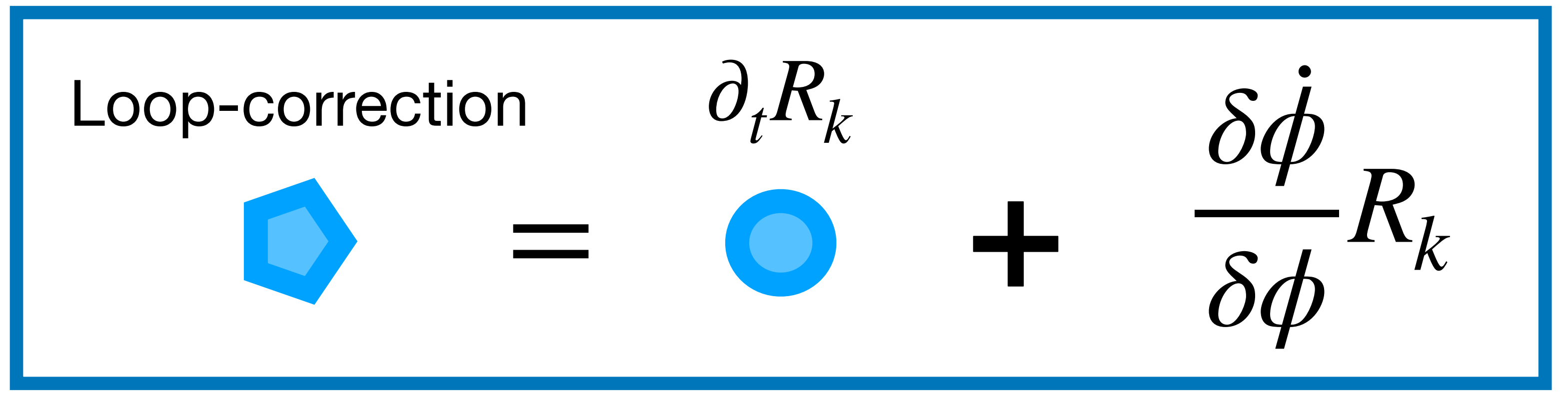

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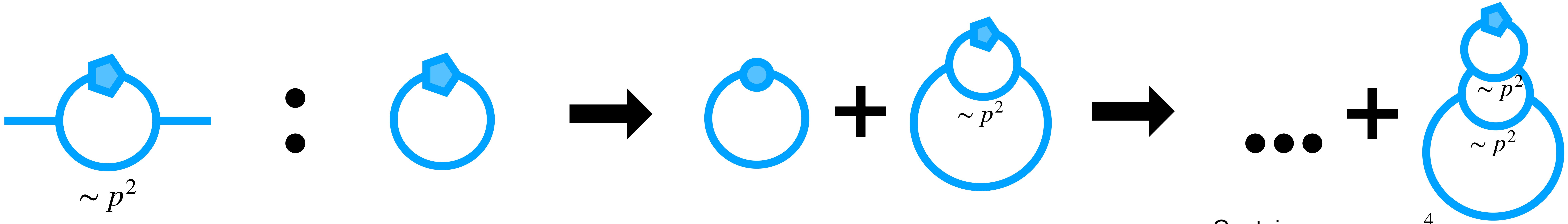

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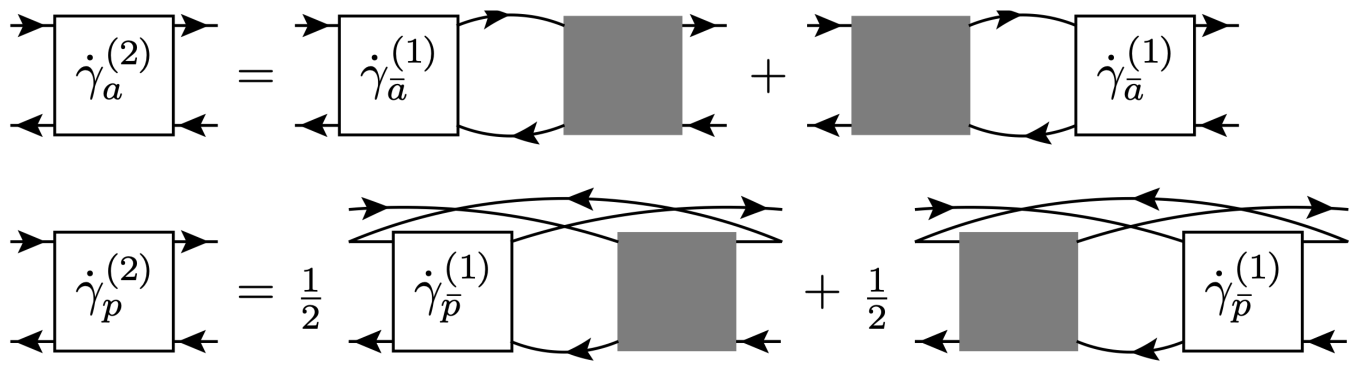

Application to Gauge theories (and other symmetry constraints): **Absorb terms that break the gauge invariance of the scheme!**

→while feeding back their flow via implicit higher loop-corrections

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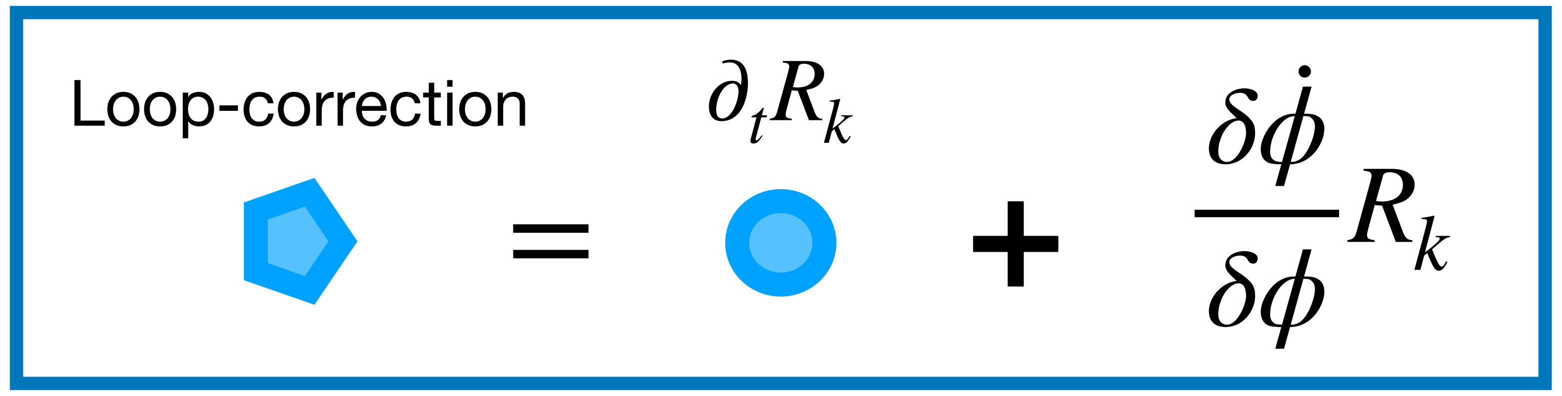

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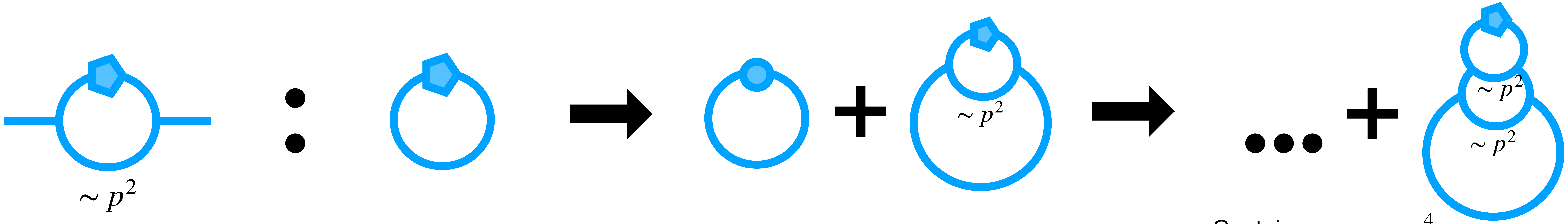

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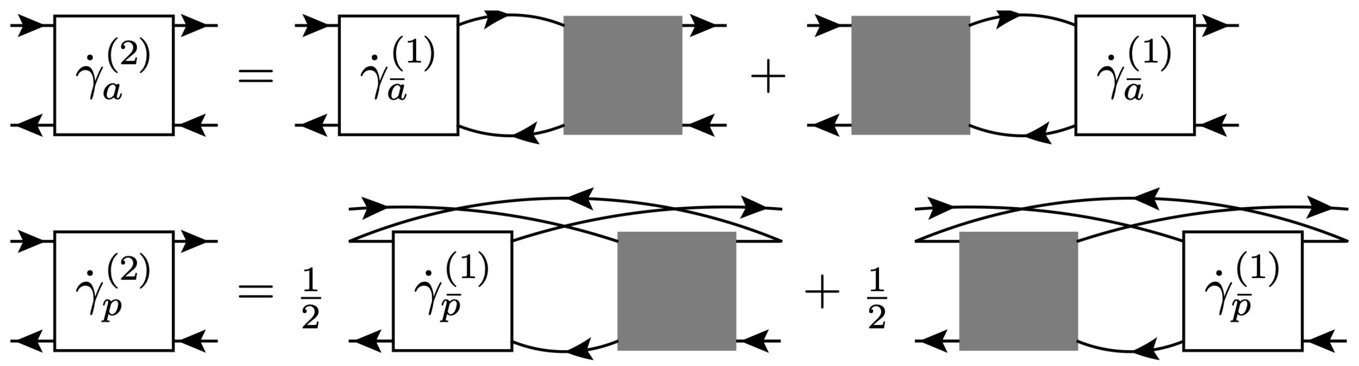

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E.g. Hard-wiring the modified Nielsen-Identities

FI, Pawlowski, PRD 112 (2025) 10, 105005

Diffeomorphism-invariant Approach to Asymptotically Safe Quantum Gravity

Friederike Ihssen,[1, 2] Benjamin Knorr,[2] Silas Mezger,[3, 2] Jan M. Pawlowski,[2, 4] and Paul P. Sprenger[2]

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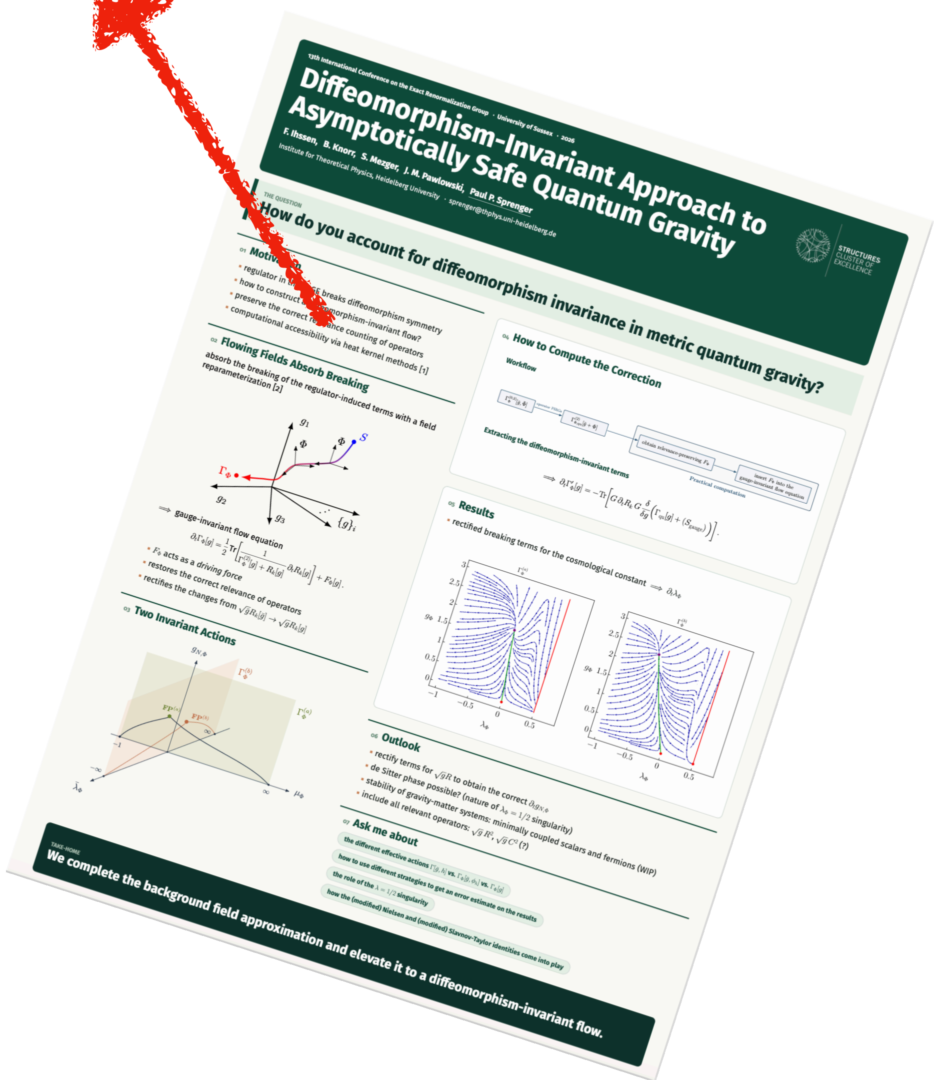

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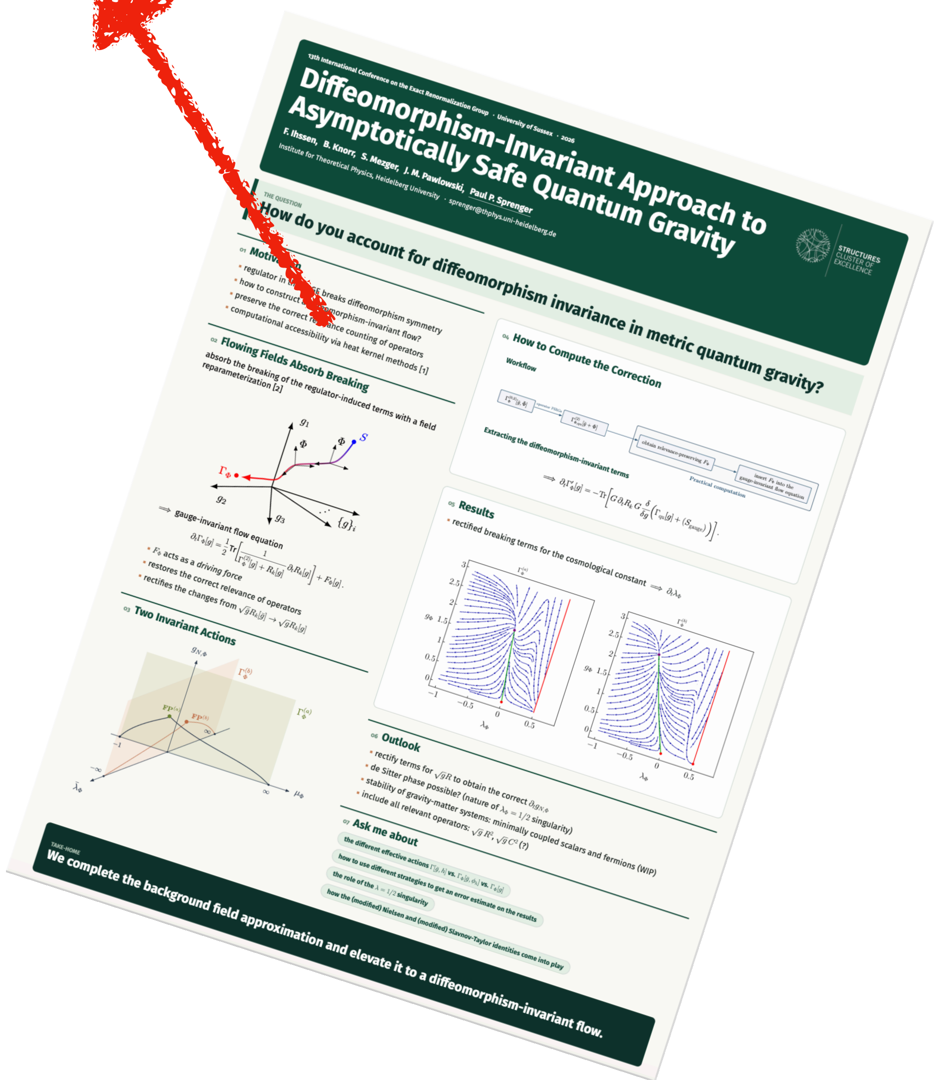

The regulator breaks gauge invariance.

Exact relations are around, e.g. modified Ward identities, Slavnov-Taylor, modified Nielsen Identities.

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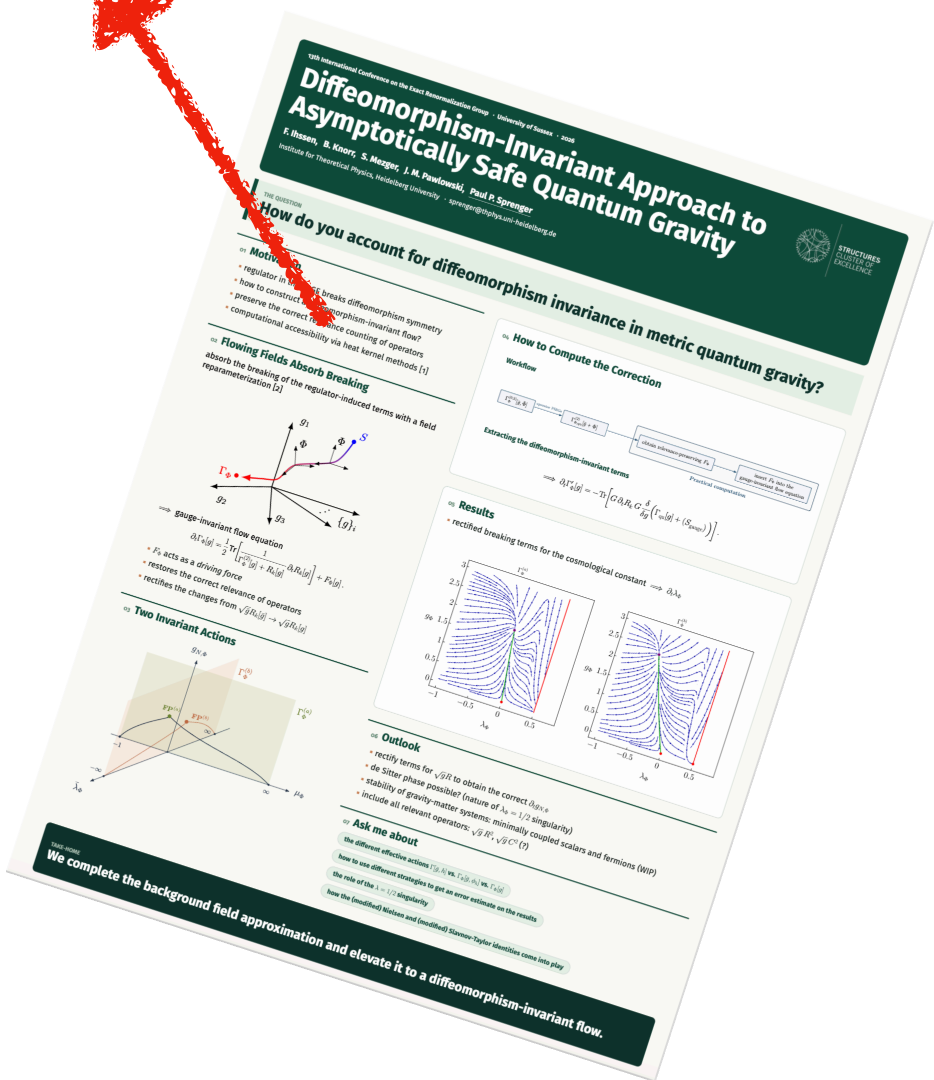

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Constraint fixes Γ_k

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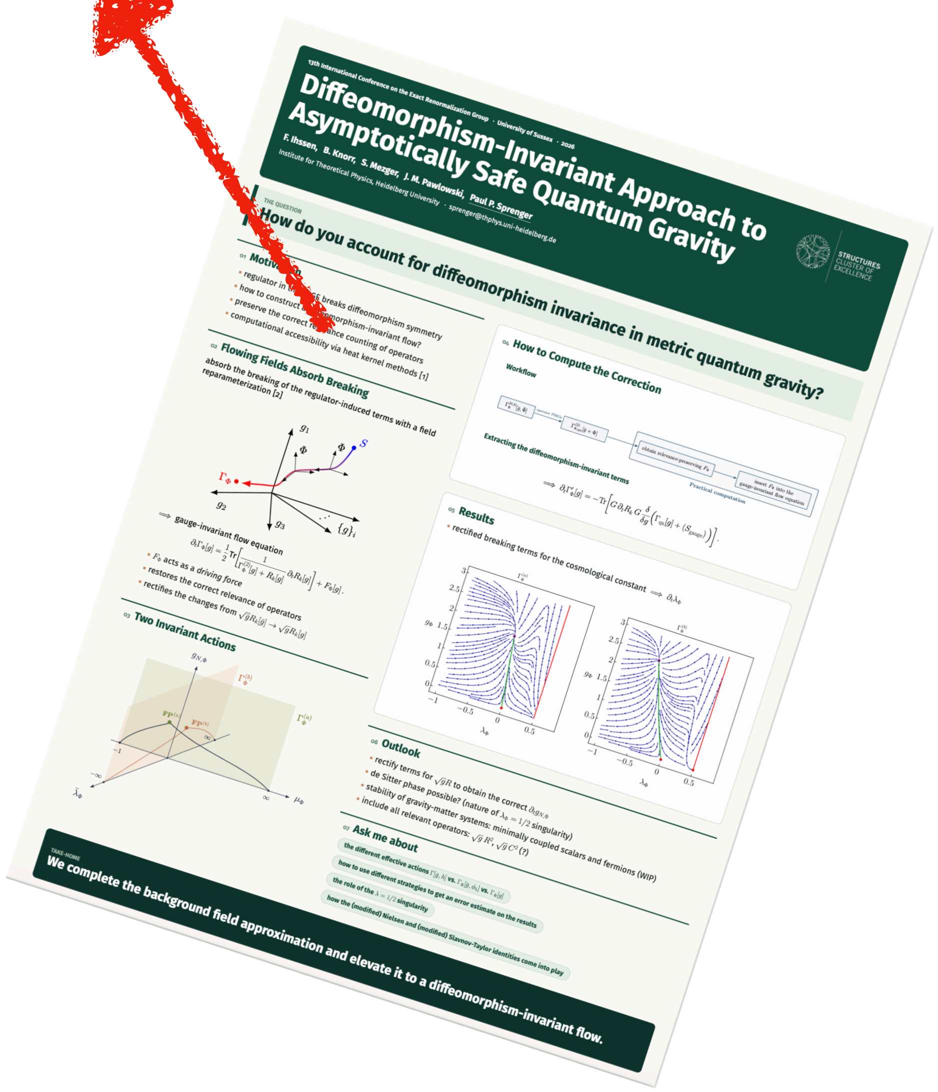

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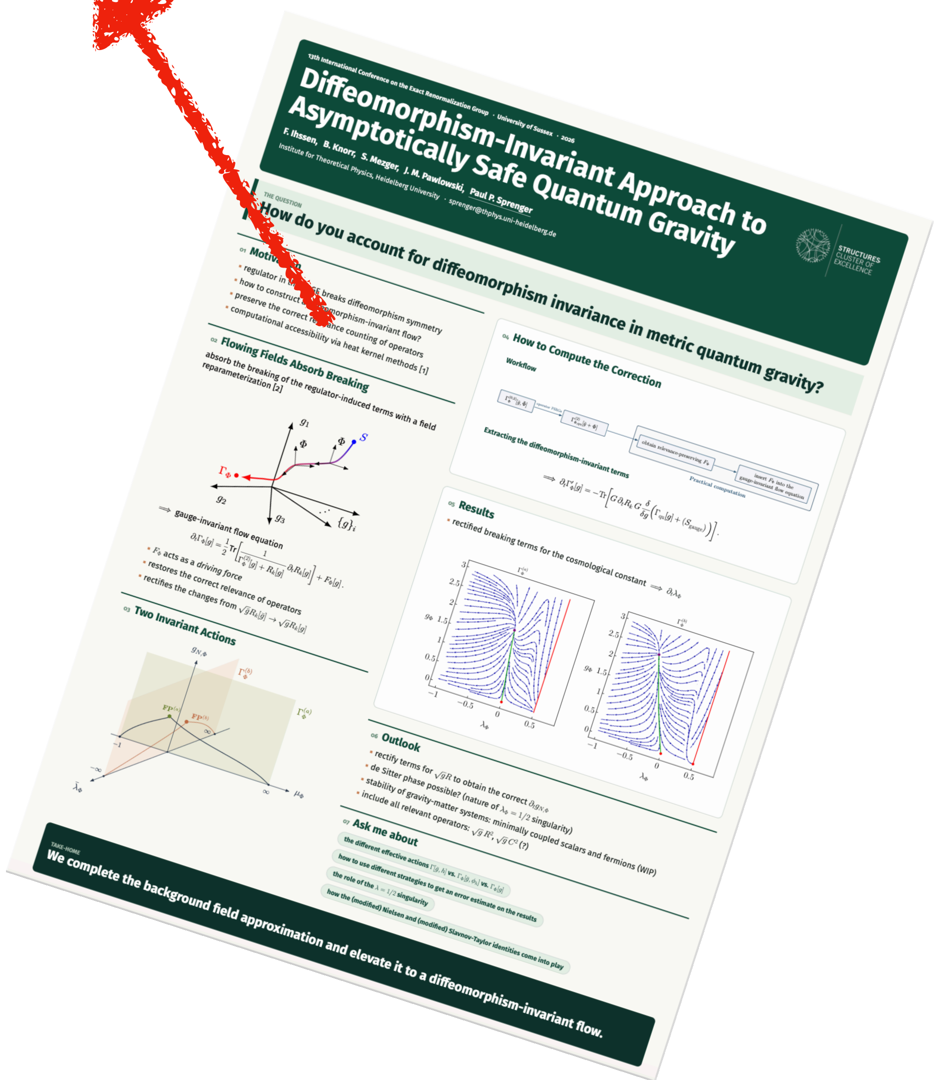

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Identify

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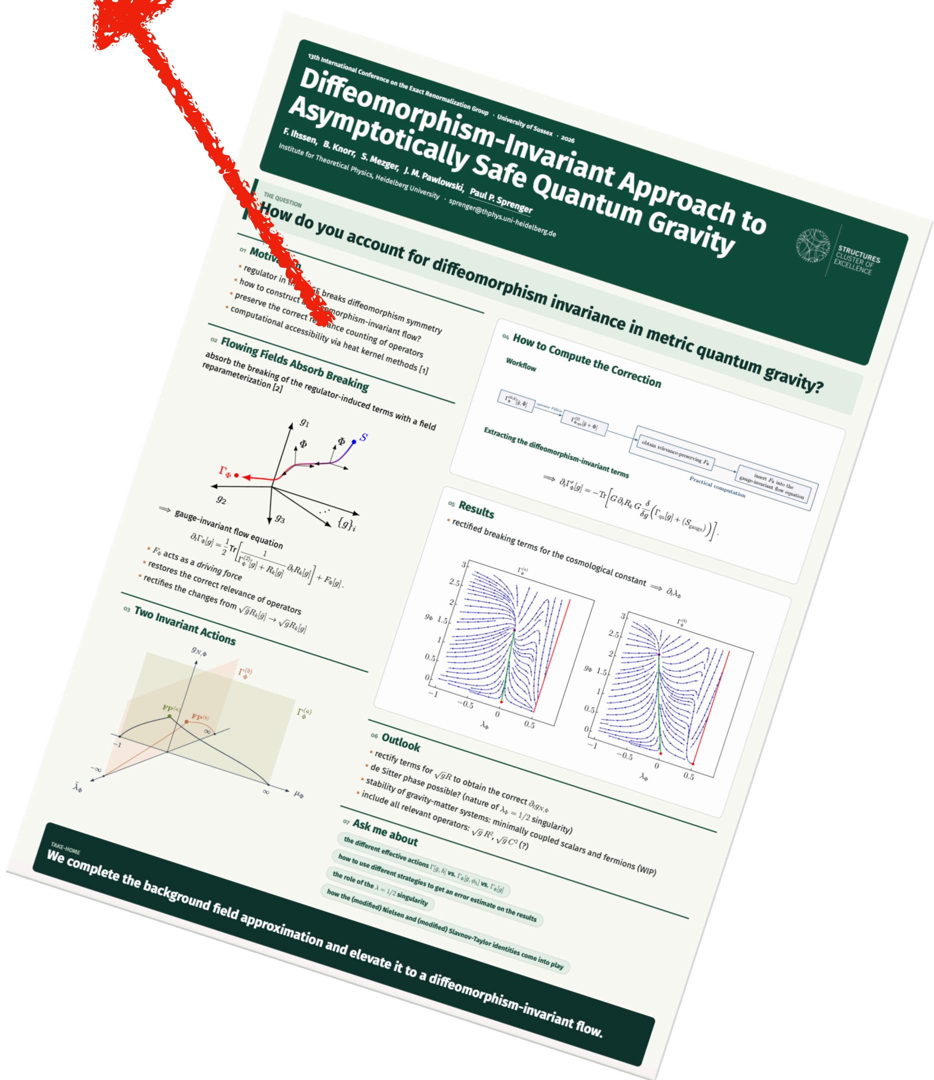

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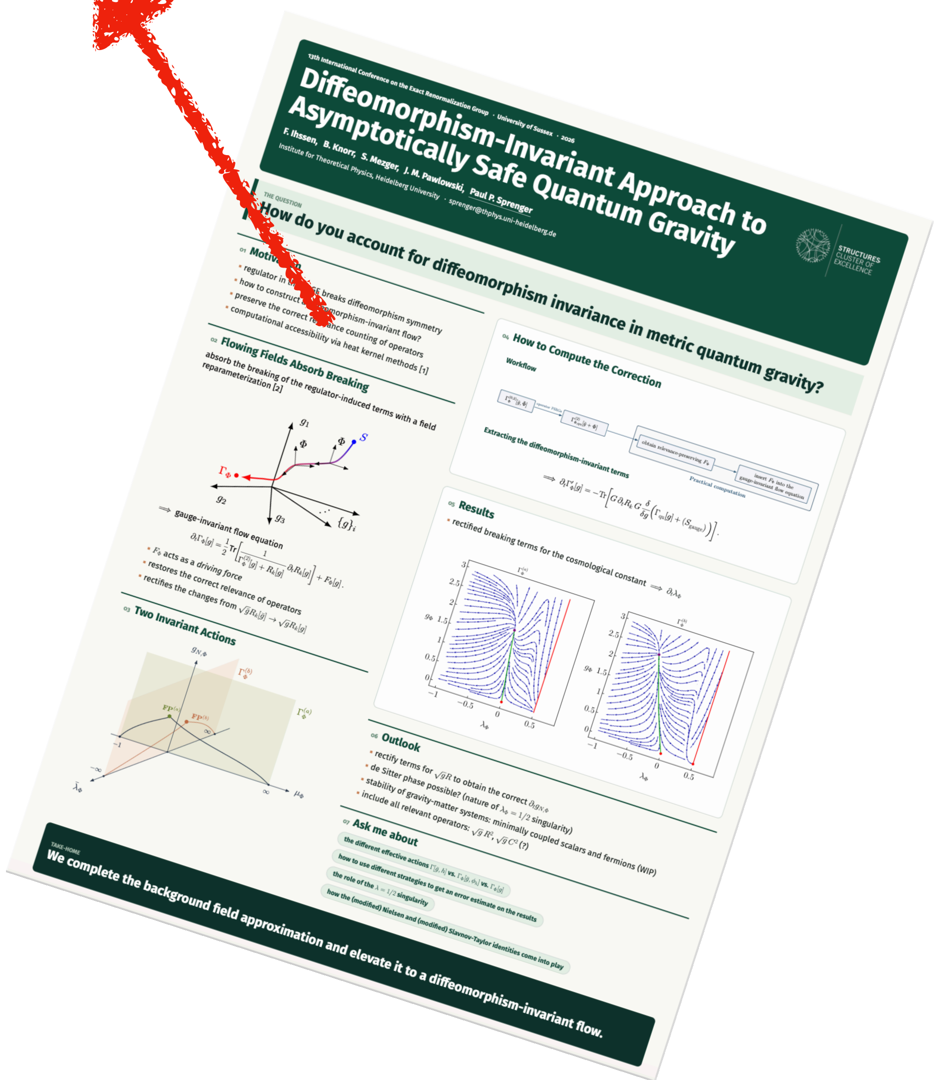

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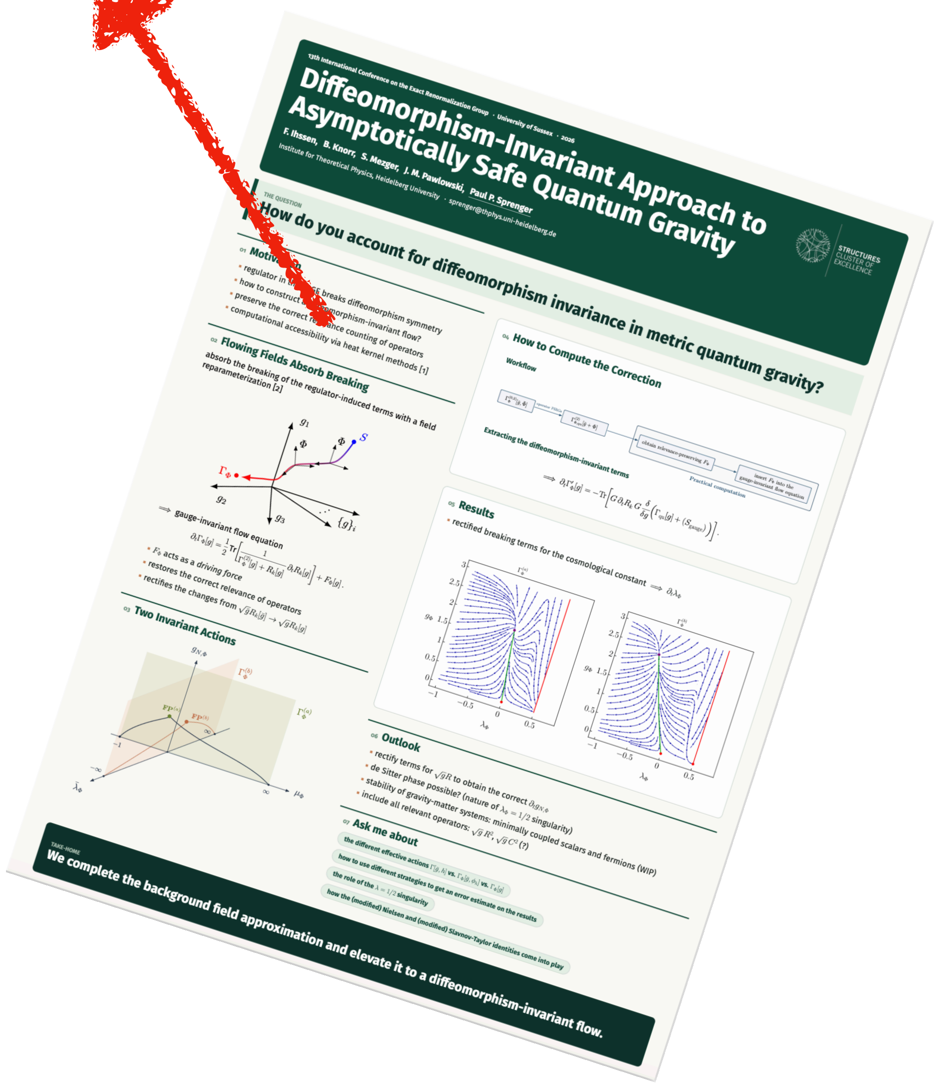

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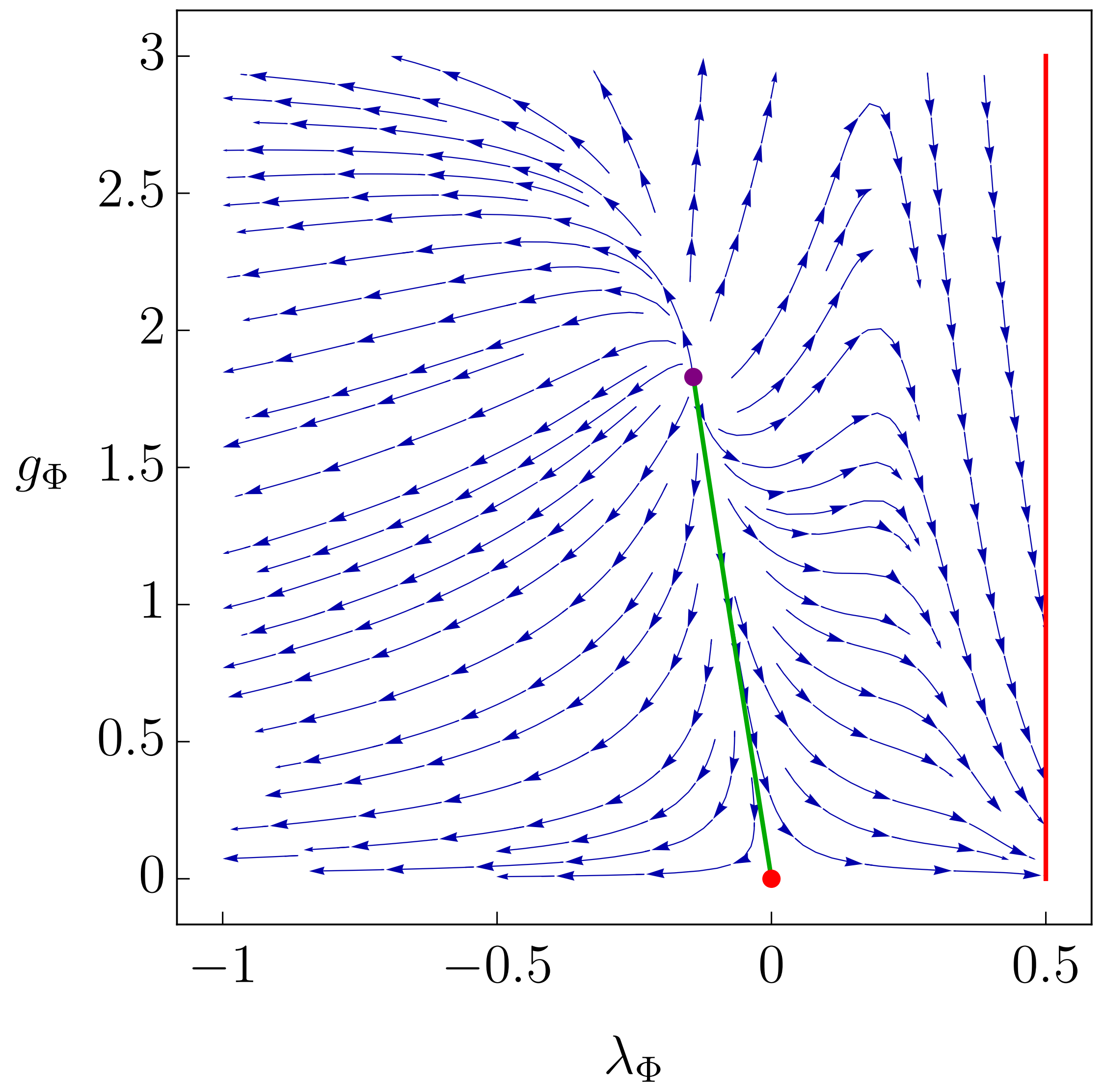

$猪_k^{(a,b)}$	$(\lambda^*_\Phi, g^*_\Phi)$	$\{\theta_i\}$
$\Gamma^{(a)}_{\Phi,\mathrm{qu}}[g]$	$(-0.142,\ 1.830)$	$\{3.86,\ 1.95\}$
$\Gamma^{(b)}_{\Phi,\mathrm{qu}}[g]$	$(-0.473,\ 2.369)$	$\{4.66\,,\ 1.89\}$

Identify

Extract

Reinsert

$$\partial_t \Gamma_{\Phi,rp}[\bar{g}+\Phi] = \frac{1}{2}\mathrm{Tr}\, G_\Phi[\bar{g}+\Phi]\partial_t R_k[g] + F_\Phi[\bar{g}+\Phi]$$

Conclusion

- Generalised flows connect many of the topics of the conference!

  → Sampling strategies

  → Flows of the 1PI effective action

- **Analytical access** is very interesting to optimize the sampling task and understand AI algorithms

  → Complex actions + sign problem

- **Optimisation** in the Wetterich equation

  → Test approximations

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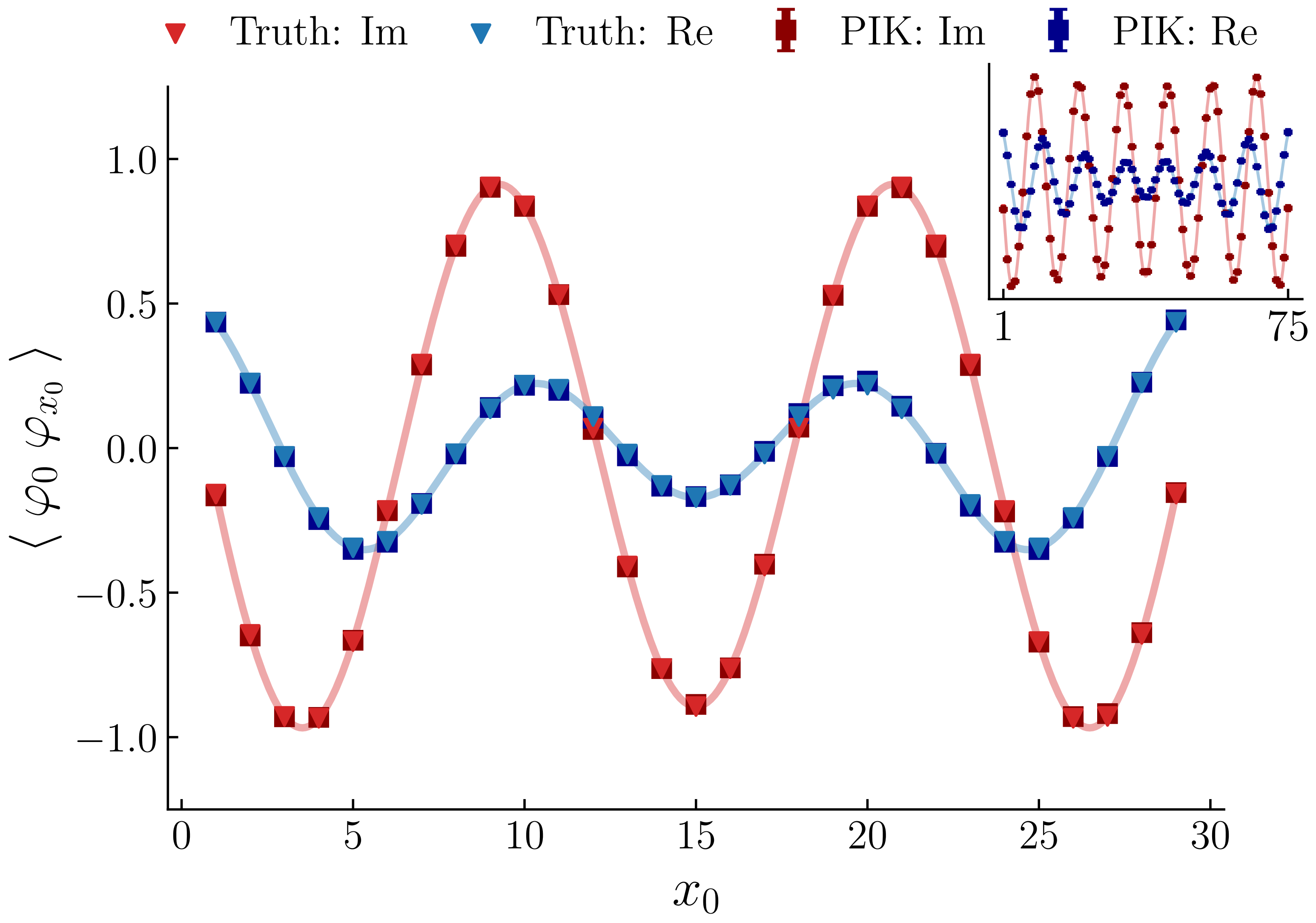

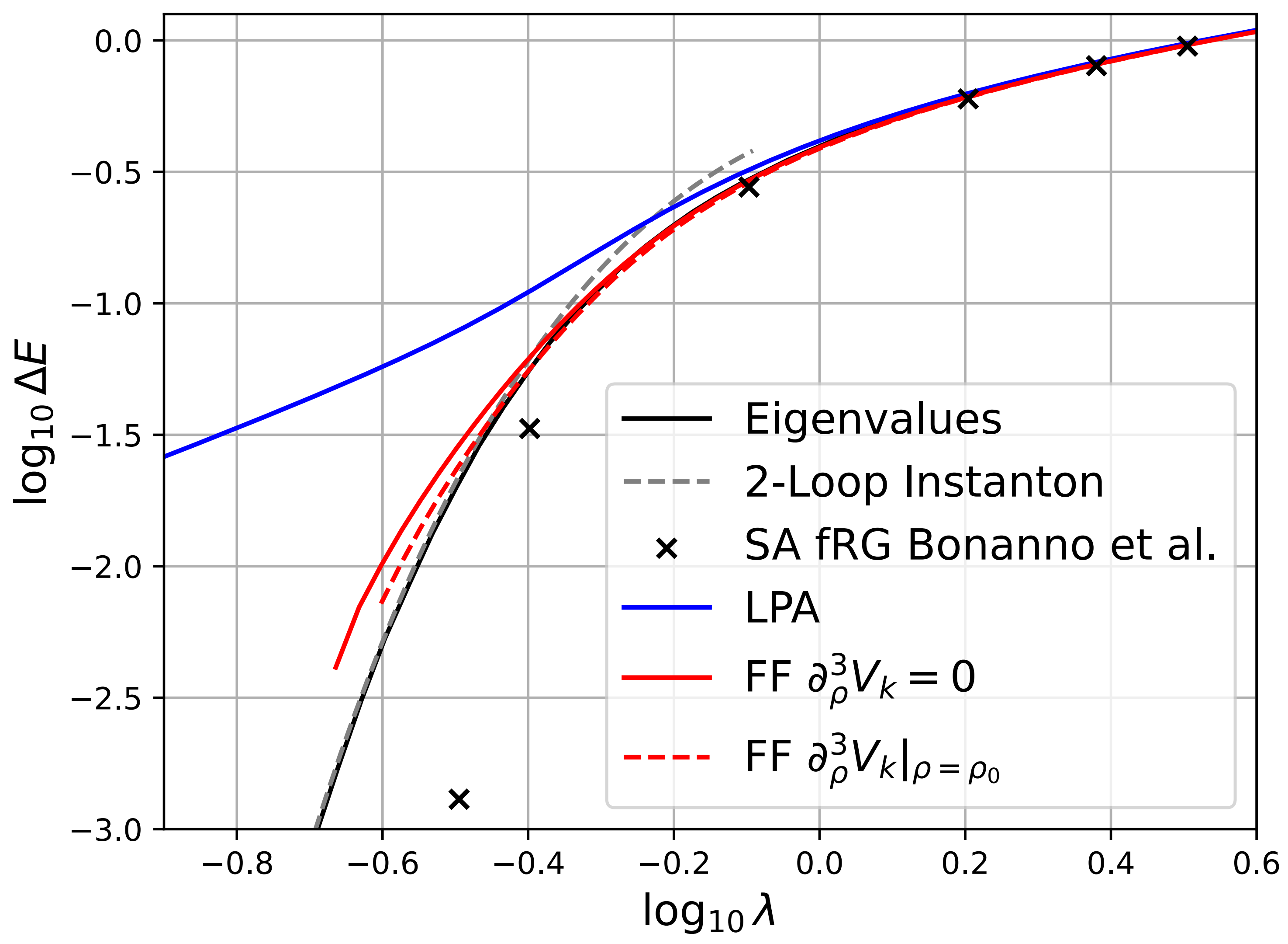

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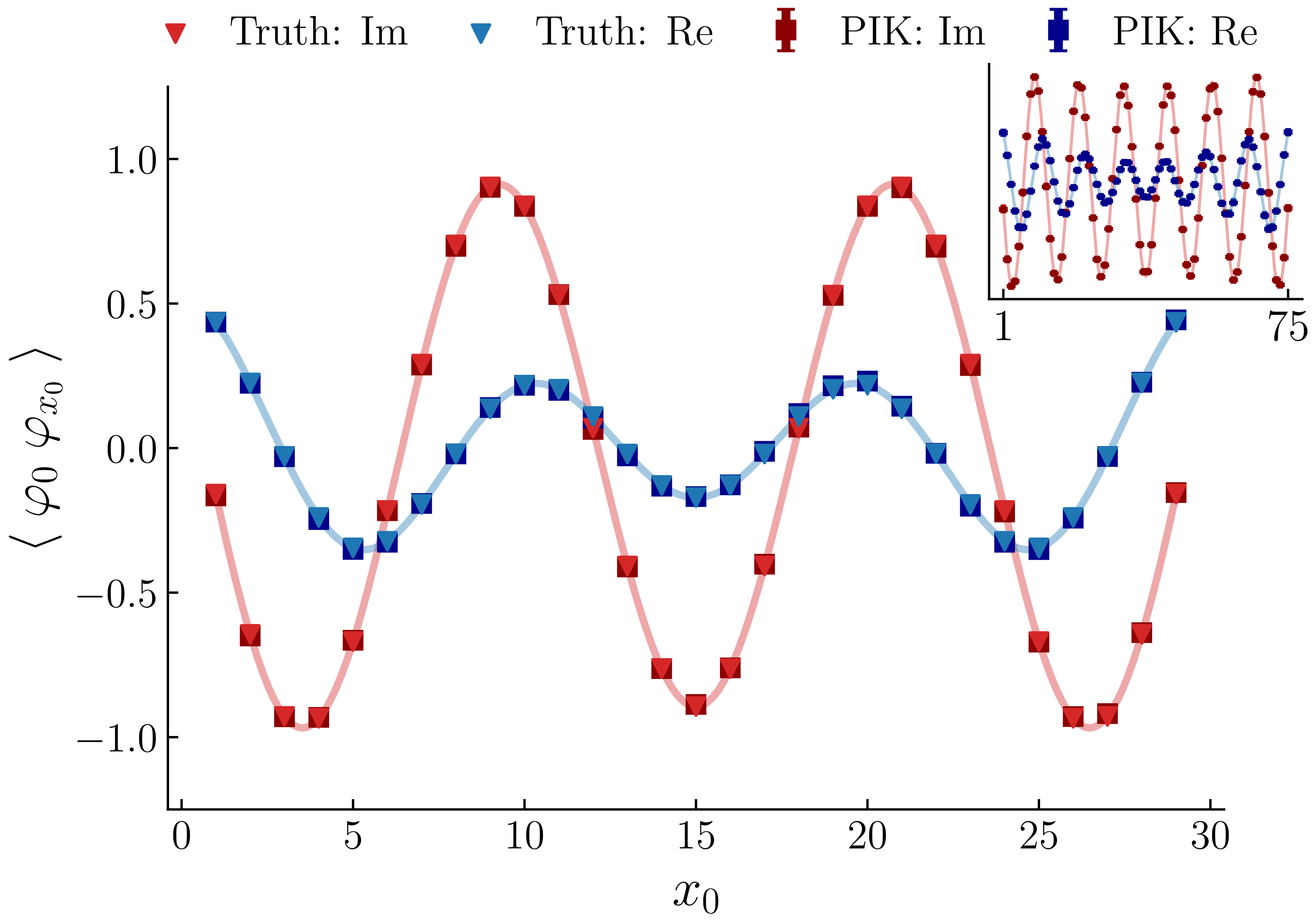

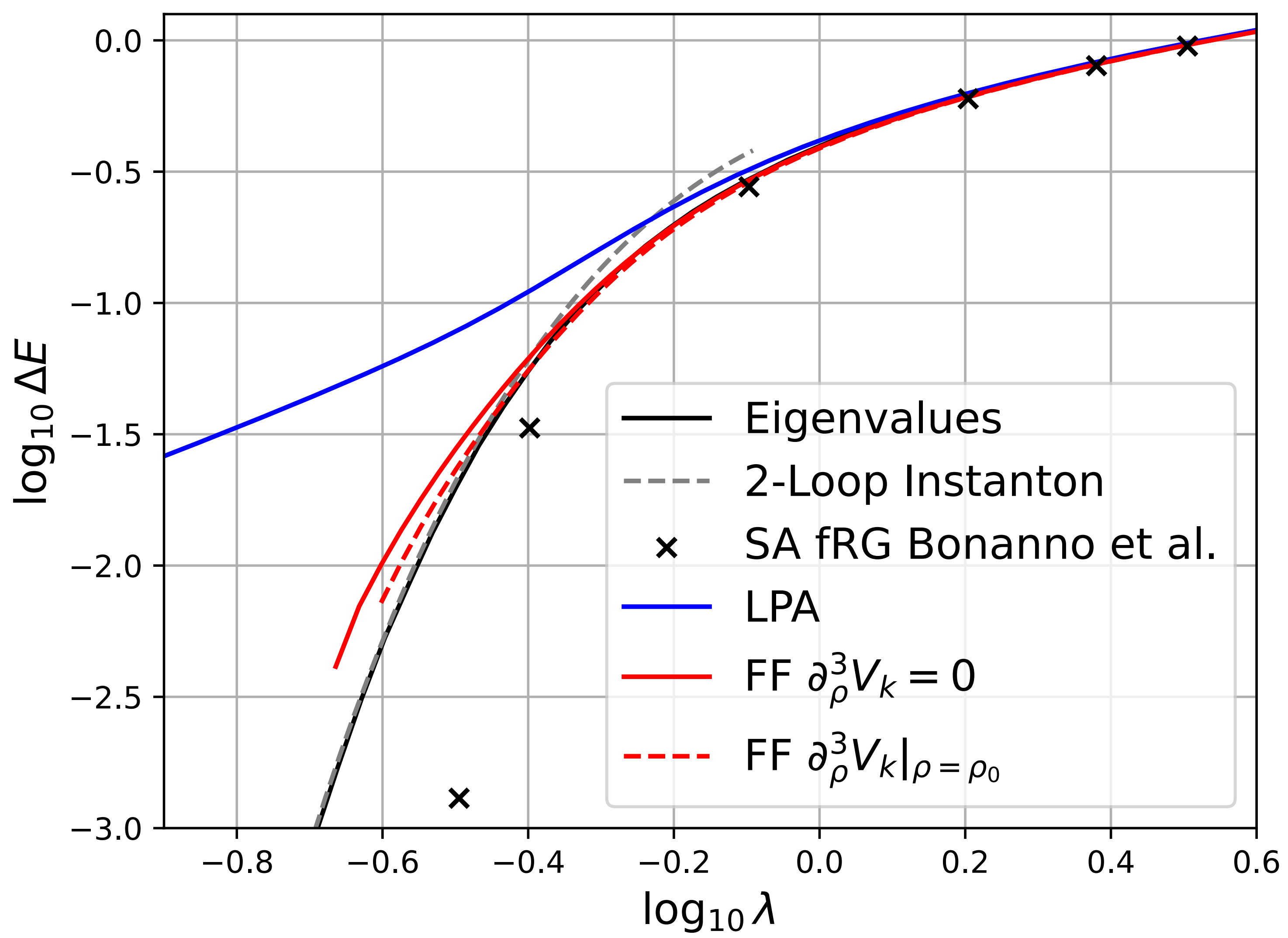

Thank you for your attention!