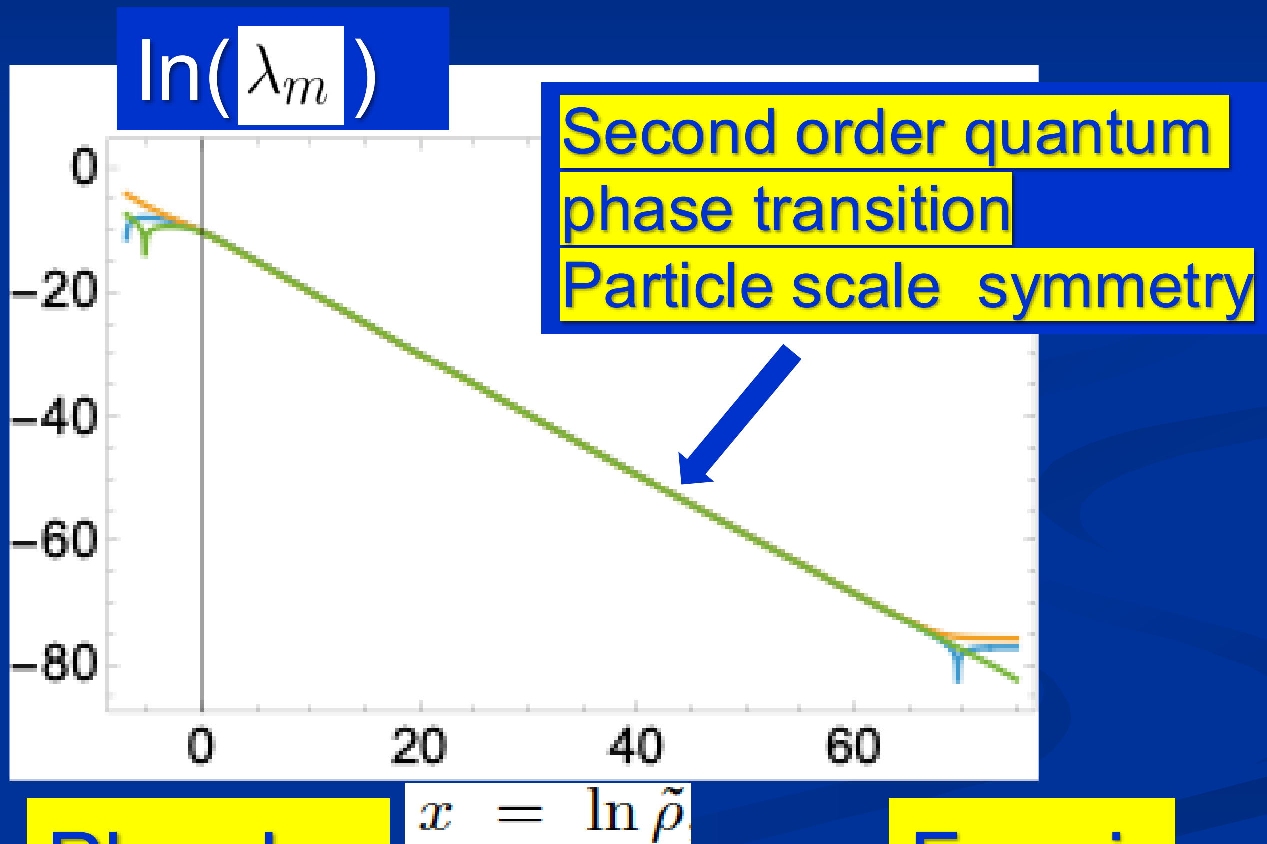
Fermi scale from quantum gravity scaling solution

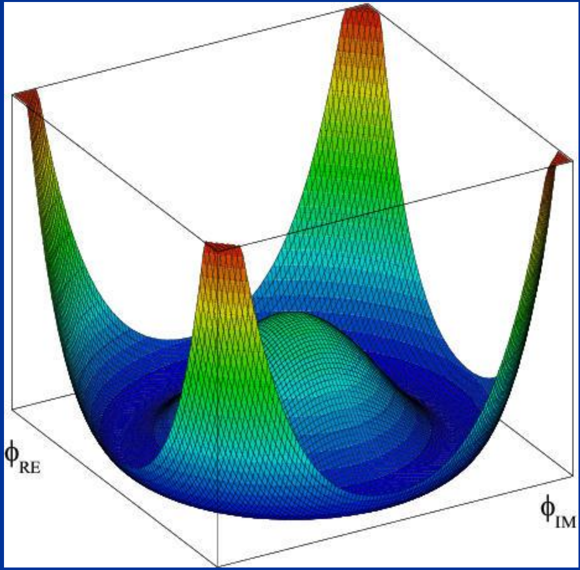


Planck scale

Fermi scale

Fermi scale

Effective potential for Higgs scalar



$$V(\varphi) = -\mu^2 \varphi^\dagger \varphi + \frac{1}{2}\lambda(\varphi^\dagger\varphi)^2$$

$$= \frac{1}{2}\lambda(\varphi^\dagger\varphi - \varphi_0^2)^2 + \text{const.})$$

Fermi scale

$$\varphi_0 = 175 \text{ GeV}$$

Scale for weak interactions and masses of quarks and leptons

Mass scales

- Fermi scale φ_0 ~ 100 GeV
- Planck mass M ~ 10^{18} GeV

- Gauge hierarchy

$$\varepsilon = \frac{\varphi_0^2}{M^2} = 5 \cdot 10^{-33}$$

- Key problem in particle physics

Can quantum gravity predict the gauge hierarchy ?

- Gravity at Fermi scale is very weak : How can it influence the effective potential for the Higgs scalar and the mass of the Higgs boson ?
- Quantum gravity as quantum field theory (asymptotic safety)
- Flow equation from UV to IR
- UV – completeness
- UV – fixed point

Scaling solution

- At fixed point: all (infinitely many) dimensionless couplings take fixed values
- Full momentum dependence of graviton propagator (no polynomial expansion)
- Whole scalar potential is fixed, for arbitrary values of scalar field
- Functional flow equations are needed

Relevant parameters

Relevant parameters describe flow away from scaling solution – cannot be predicted

- M relevant parameter : no predictivity for gauge hierarchy
- φ_0 relevant parameter : no predictivity for gauge hierarchy
- LIMS: largest intrinsic mass scale generated by flow away from scaling solution
- LIMS << φ_0 : gauge hierarchy determined by scaling solution

Scaling solutions are rare

Only discrete number : Predictivity

Condensed matter :

scaling solution = Widom scaling EOS for $T=T_c$

LIMS = $T-T_c$

Scale symmetric standard model

LIMS << Λ_{QCD} (scale of strong interactions)

predicts scale invariant standard model

all scales given by scalar field χ

$$\mathrm{M} \sim \chi$$

$$\varphi_0 \sim \chi$$

$$\Lambda_{\mathrm{QCD}} \sim \chi$$

Scale symmetric standard model

- Replace all mass scales by scalar field χ

(1) Higgs potential $U = \frac{\lambda_H}{2}(\varphi^\dagger\varphi - \epsilon\chi^2)^2 \Rightarrow \varphi_0^2 = \epsilon\chi^2$

Fujii, Zee, CW

(2) Strong gauge coupling, normalized at $\mu = \chi$, is independent of χ

$$g(\chi) = \bar{g} \Rightarrow \Lambda_{QCD} = \chi \exp\left(-\frac{1}{b_0\bar{g}^2}\right) \qquad b_0 = \frac{1}{16\pi^2}\left(22 - \frac{4}{3}N_f\right)$$

(3) Similar for all dimensionless couplings

Quantum effective action for standard model does

not involve intrinsic mass or length

Quantum scale symmetry CW'87, Shaposhnikov et al

Effective action

$$\Gamma_k = \int_x \sqrt{g'}\Big\{ -\frac{F}{2}R' + U + \frac{K}{2}D^\mu\chi D_\mu\chi + Z_H D^\mu H^\dagger D_\mu H + \dots \Big\}.$$

$$F = \xi\chi^2 + \xi_H H^\dagger H + f_\infty k^2$$

H : Higgs doublet, χ : scalar singlet

$$u = \frac{U}{k^4}\,, \quad \tilde\rho = \frac{\chi^2}{2k^2}\,, \quad \tilde h = \frac{H^\dagger H}{k^2}\,, \quad h = \frac{\tilde h}{\tilde\rho} = \frac{2H^\dagger H}{\chi^2}$$

gauge hierarchy

$$h_0 = \frac{2\varphi_0^2}{\chi^2} = 2\xi\frac{\varphi_{0E}^2}{M^2}$$

$$\frac{\partial u}{\partial h}(\tilde\rho, h_0) = 0$$

Scaling solution for effective potential

$$k\partial_k u = -4u + 2\tilde{\rho}\partial_{\tilde{\rho}} u + 4c_U$$

at fixed h

effective standard model below M

$$c_U = \frac{1}{128\pi^2}\left(\bar{N}_S + 2\bar{N}_V - 2\bar{N}_F + 2\right)$$

$$\bar{N}_F = \sum_f \left(1 + \frac{m_f^2}{k^2}\right)^{-1} + 3$$

$$= \sum_f \left(1 + y_f^2 h\tilde{\rho}\right)^{-1} + 3$$

$$k\partial_k u = 0$$

$$\tilde{\rho}\partial_{\tilde{\rho}} u = 2\left(u - c_U\right)$$

flow equation for effective potential

here: based on simplified flow equation

quantum gravity coupled to a scalar field

Henz, Pawlowski, Rodigast, Yamada, Reichert, Maitiniyazi

Eichhorn, Pauly, Laporte, Pereira, Saueressig,

Wang, Knorr, …

for low order polynomial expansion of potential : Percacci, Narain, …

Scaling solutions are restrictive

- scaling solutions are solutions of

  non-linear differential equations $\tilde{\rho}\partial_{\tilde{\rho}}u = 2\,(u - c_U)$
- scaling potential needs to extend over whole range of scalar field
- analyticity for $\tilde{\rho} \rightarrow 0$
- predictivity !

in presence of gravitational fluctuations:

scalar effective potential no longer approximated by polynomial

Cosmon – Higgs coupling

- Dimensionless cosmon – Higgs coupling λ_m determines gauge hierarchy

scaling solution
for k → 0

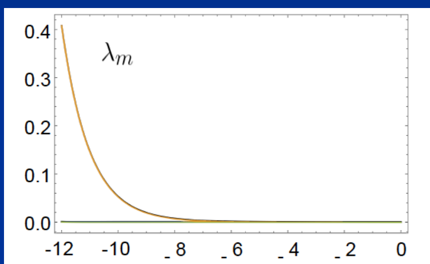
$$U = \frac{1}{8}\bar{\lambda}_\chi \chi^4 + \frac{1}{2}\bar{\lambda}_m \chi^2 H^\dagger H + \frac{1}{2}\bar{\lambda}_h (H^\dagger H)^2 \left(1 + \Delta\left(\frac{2H^\dagger H}{\chi^2}\right)\right)$$

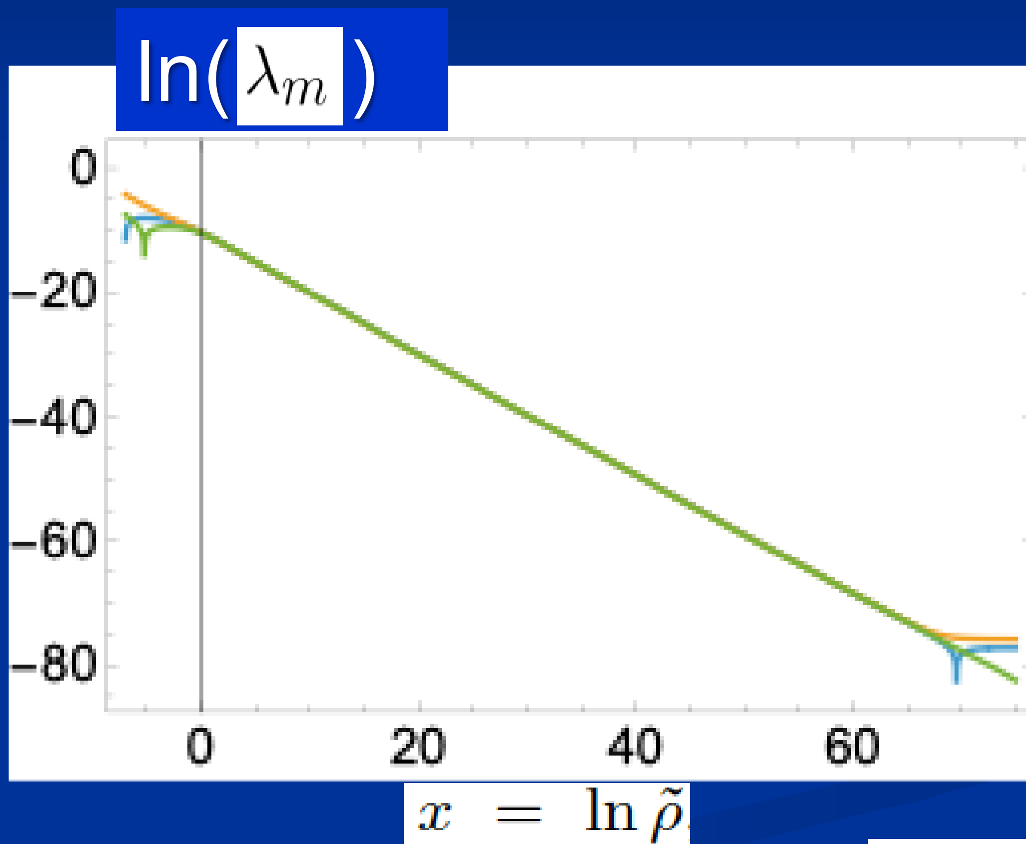
$$h_0 = \frac{2\varphi_0^2}{\chi^2} = -\frac{\bar{\lambda}_m}{\bar{\lambda}_h}$$

$$\lambda_m(\tilde{\rho}, \tilde{h}) = \partial_{\tilde{\rho}} \partial_{\tilde{h}} u(\tilde{\rho}, \tilde{h})$$

$$\lambda_m(\tilde{\rho} \to \infty, h = 0) = \bar{\lambda}_m$$

Numerical solution cosmon-Higgs coupling





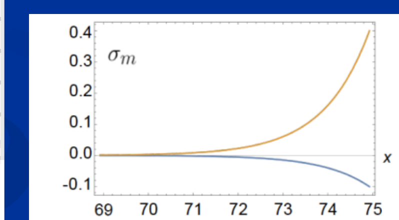


FIG. 2. Infrared flow of cosmon-Higgs coupling. We plot $\sigma_m = \tilde{\rho}\lambda_m$ as a function of $x = \ln \tilde{\rho}$. The two curves correspond to initial values $\sigma_m(x_F) = -0.1$ (blue) and $\sigma_m(x_F) = 0.4$ (orange).

Second order quantum electroweak phase transition

For constant gauge and Yukawa couplings:

scaling solution λ_m =0

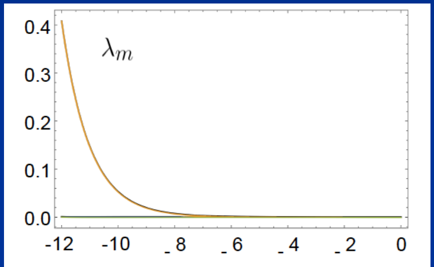
quantum electroweak phase transition is of second order in this approximation

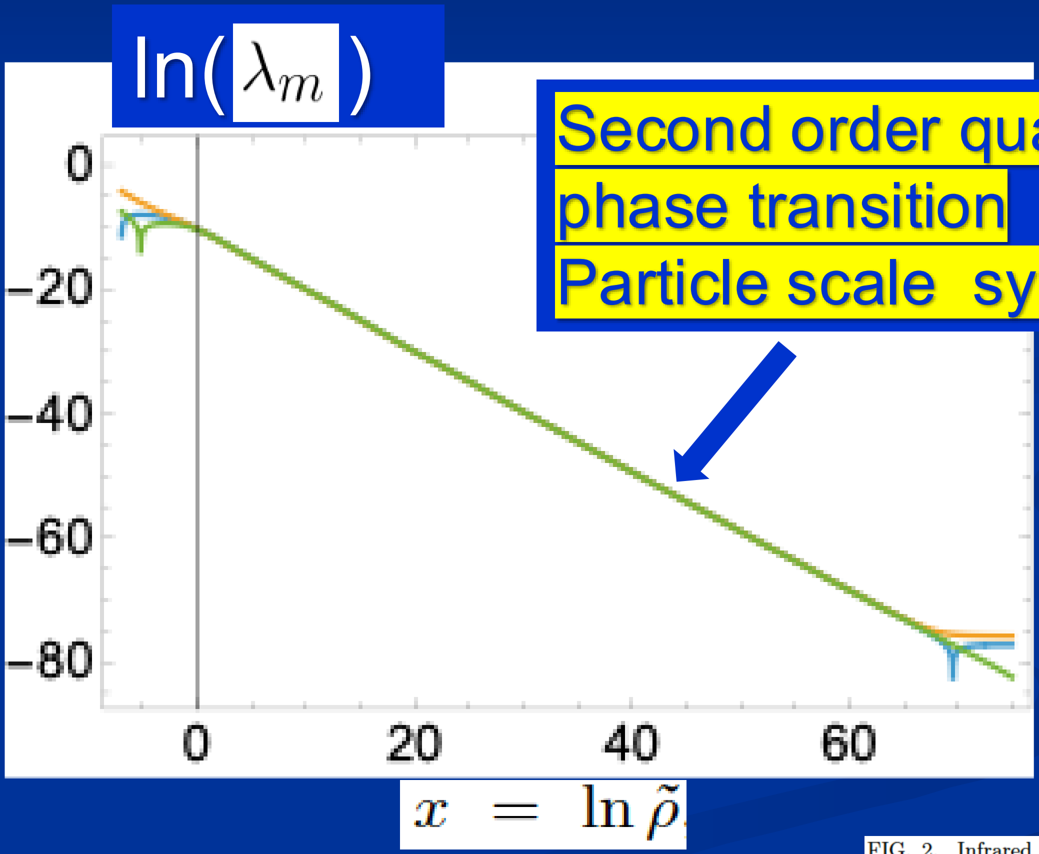
particle scale symmetry

flowing couplings violate particle scale symmetry

critical surface : $\lambda_m^{(\mathrm{cr})} = \frac{1}{128\pi^2\tilde{\rho}} \sum_i \hat{n}_i \beta_i$

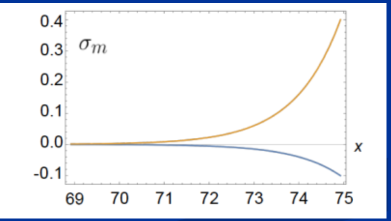
Numerical solution cosmon-Higgs coupling





Second order quantum phase transition

Particle scale symmetry



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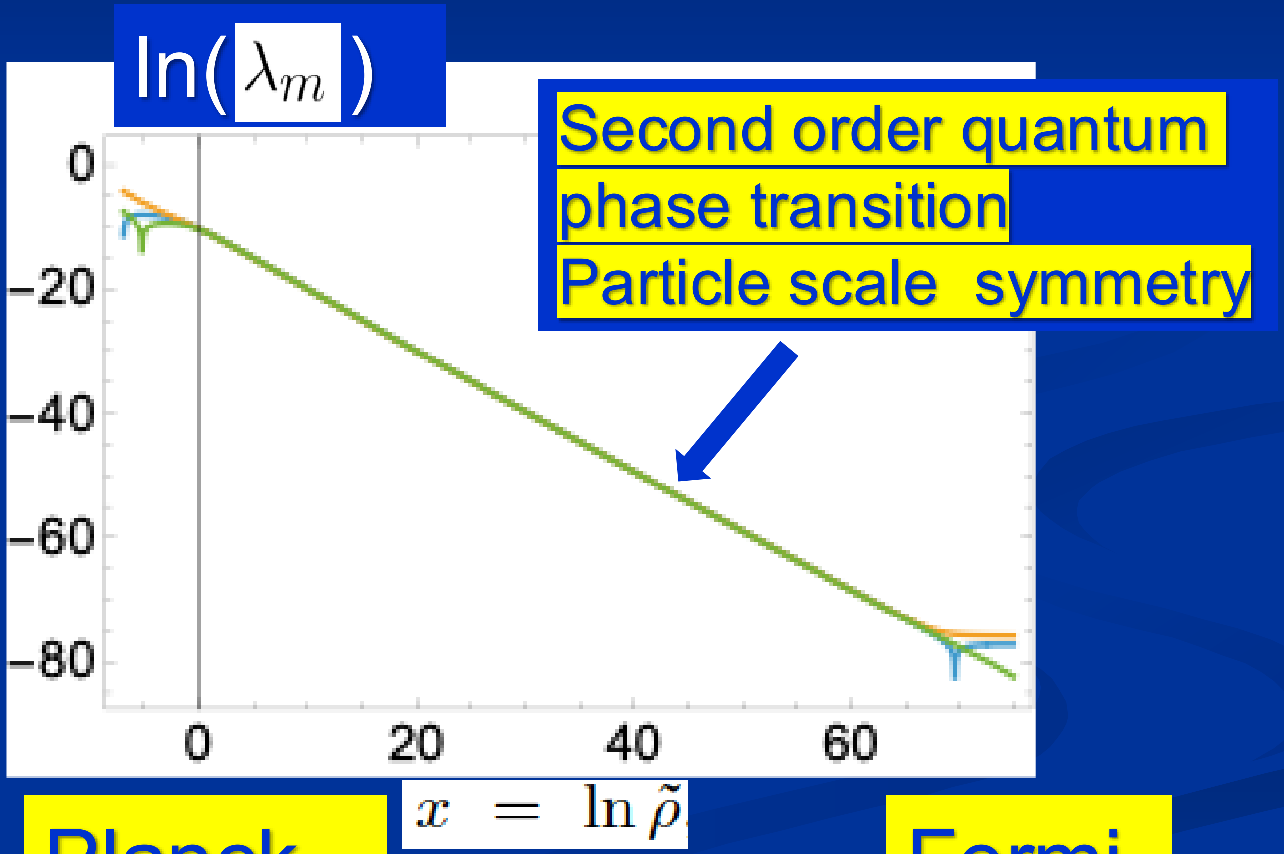
Fermi scale from quantum gravity scaling solution



Planck scale

Fermi scale

Scale transformations

- Scale invariance : scaling solution is scale invariant
- Fundamental scale invariance : LIMS =0
- Quantum scale symmetry : There exists choice of fields such that the effective action does not involve parameters with dimension mass of length, even not k

  global symmetry , SSB by χ : massless Goldstone boson, scale invariant primordial fluctuation spectrum
- Particle scale symmetry : For fixed χ =M: no scale in effective low energy theory, even not ~ χ

  realized for second order electroweak phase transition

Approximate scale symmetry near fixed points

- UV : approximate scale invariance of primordial fluctuation spectrum from inflation

- IR : cosmon is pseudo Goldstone boson of
  spontaneously broken scale symmetry,
  tiny mass,
  responsible for dynamical Dark Energy

Conclusions

- New quantum gravity fixed point , scaling solution different from extended Reuter fixed point with flat potential and constant coefficient of curvature scalar, dilaton quantum gravity
- LIMS << Λ_{QCD} : predictive for all parameters of standard model
- gauge hierarchy is predicted for given model
- prediction depends on short distance model
- predictive for cosmology: inflation and dynamical dark energy

end

Simplified flow equation

- Flow equation for effective average action is sometimes difficult to handle in practice…
- Simplified flow equation

$$k\partial_k\Gamma_k[\varphi] = k^{2n}\mathrm{tr}\Big\{\Big(1 - \frac{1}{2}E[\varphi]\Big)\Big(\Gamma_R^{(2)}[\varphi] + k^2\Big)^{-n}\Big\}$$

$$\Gamma_R^{(2)}[\varphi] = g^{-1/2}N^{-1}[\varphi]\Gamma_k^{(2)}[\varphi]\big(N^T\big)^{-1}[\varphi]$$

$$E[\varphi] = -N^{-1}\partial_t\big(NN^T\big)\big(N^T\big)^{-1}[\varphi]$$

Simplified flow equation

- Particular field-dependent cutoff function is used

$$k\partial_k \Gamma_k[\varphi] = k^{2n} \mathrm{tr}\left\{ \left(1 - \frac{1}{2} E[\varphi]\right) \left(\Gamma_R^{(2)}[\varphi] + k^2\right)^{-n} \right\}$$

- Field-dependent root of wave function renormalization N

$$\Gamma_R^{(2)}[\varphi] = g^{-1/2} N^{-1}[\varphi] \Gamma_k^{(2)}[\varphi] \left(N^T\right)^{-1}[\varphi]$$

- Field-dependent anomalous dimension E

$$E[\varphi] = -N^{-1} \partial_t \left(N N^T\right) \left(N^T\right)^{-1}[\varphi]$$

- n needs to be large enough for UV- finiteness and decoupling of massive modes, d=4 : n=3

Local gauge symmetry

- second functional derivative of gauge invariant effective action involves covariant derivatives and transforms as tensor
- sufficient that N is chosen to transform as a tensor
- add universal measure term from regularized Faddeev-Popov determinant

$$k\partial_k \Gamma_k[\varphi] = k^{2n}\mathrm{tr}\left\{\left(1-\frac{1}{2}E[\varphi]\right)\left(\Gamma_R^{(2)}[\varphi]+k^2\right)^{-n}\right\}$$

$$\Gamma_R^{(2)}[\varphi] = g^{-1/2}N^{-1}[\varphi]\Gamma_k^{(2)}[\varphi]\left(N^T\right)^{-1}[\varphi]$$

$$E[\varphi] = -N^{-1}\partial_t\left(NN^T\right)\left(N^T\right)^{-1}[\varphi]$$

Arbitrary metric and signature

- g= det $g_{\mu\nu}$
- all factors of i for Minkowski signature from g<0

$$k\partial_k\Gamma_k[\varphi] = k^{2n}\mathrm{tr}\Big\{\big(1-\frac{1}{2}E[\varphi]\big)\big(\Gamma_R^{(2)}[\varphi]+k^2\big)^{-n}\Big\}$$

$$\Gamma_R^{(2)}[\varphi] = g^{-1/2}N^{-1}[\varphi]\Gamma_k^{(2)}[\varphi]\big(N^T\big)^{-1}[\varphi]$$

Analytic continuation

- for analytic E: all poles are given by zeros of

$$\Gamma_R^{(2)}[\varphi] + k^2$$

$$k\partial_k \Gamma_k[\varphi] = k^{2n} \mathrm{tr}\Big\{\big(1 - \frac{1}{2}E[\varphi]\big)\big(\Gamma_R^{(2)}[\varphi] + k^2\big)^{-n}\Big\}$$

$$\Gamma_R^{(2)}[\varphi] = g^{-1/2} N^{-1}[\varphi]\Gamma_k^{(2)}[\varphi]\big(N^T\big)^{-1}[\varphi] \qquad E[\varphi] = -N^{-1}\partial_t\big(NN^T\big)\big(N^T\big)^{-1}[\varphi]$$

Automatic cutoff of dangerous modes with large effects in perturbation theory

- IR –cutoff for massless modes
- cutoff of momenta near Fermi surface

$$k\partial_k\Gamma_k[\varphi] = k^{2n}\mathrm{tr}\left\{\left(1-\frac{1}{2}E[\varphi]\right)\left(\Gamma_R^{(2)}[\varphi]+k^2\right)^{-n}\right\}$$

$$\Gamma_R^{(2)}[\varphi] = g^{-1/2}N^{-1}[\varphi]\Gamma_k^{(2)}[\varphi]\left(N^T\right)^{-1}[\varphi]$$

Derivative expansion of effective action at minimum of Higgs potential

$$\Gamma = \int_\chi \sqrt{g}\Big\{ -\frac{1}{2}F(\chi)R + \frac{1}{2}K(\chi)\partial^\mu\chi\partial_\mu\chi + U(\chi)\Big\}$$

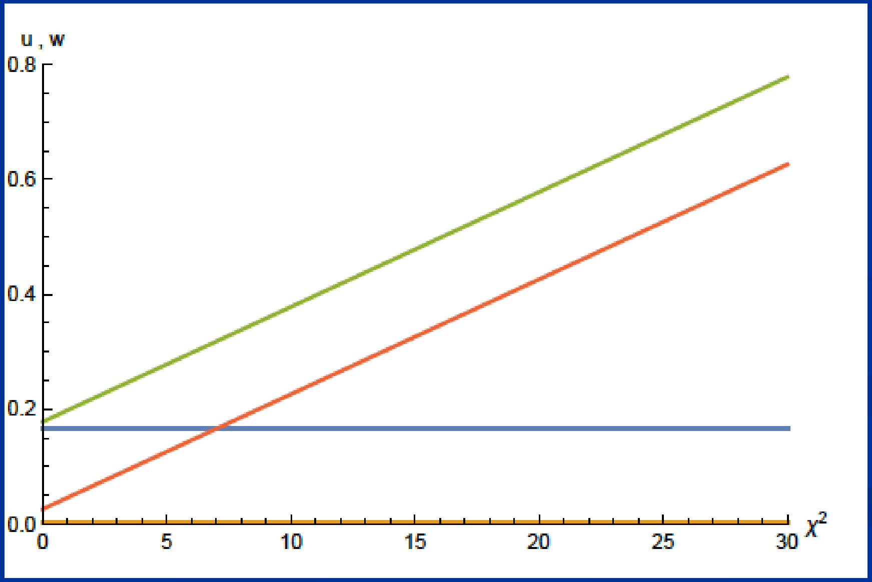
variable gravity

F : field dependent squared Planck mass

Approximate scaling solution

- flat potential: u constant
- non-minimal scalar- gravity coupling $w=F/k^2$:

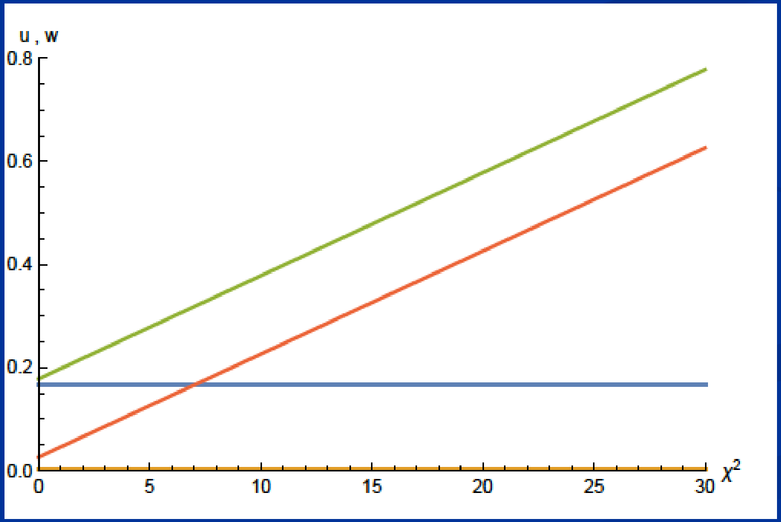
  for large scalar field w increases proportional χ^2



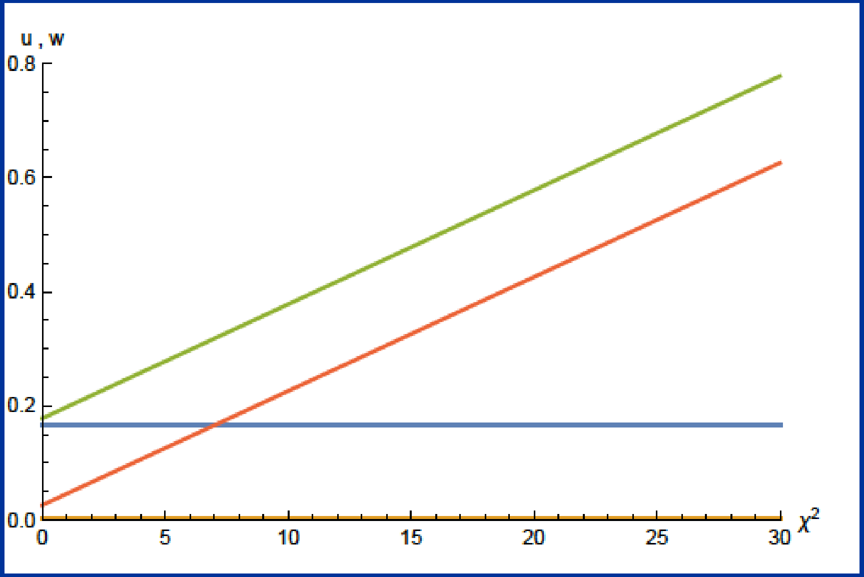
looks natural

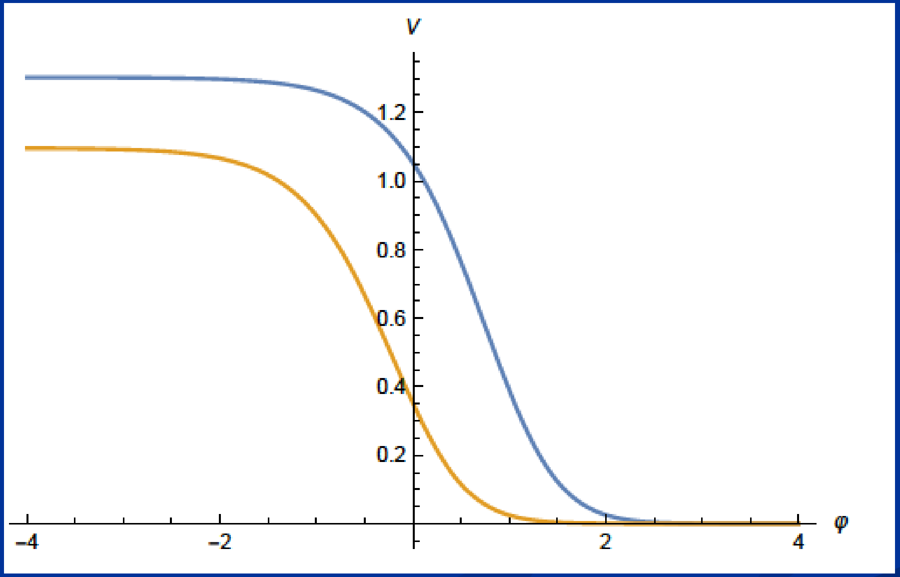
no small parameter

no tuning

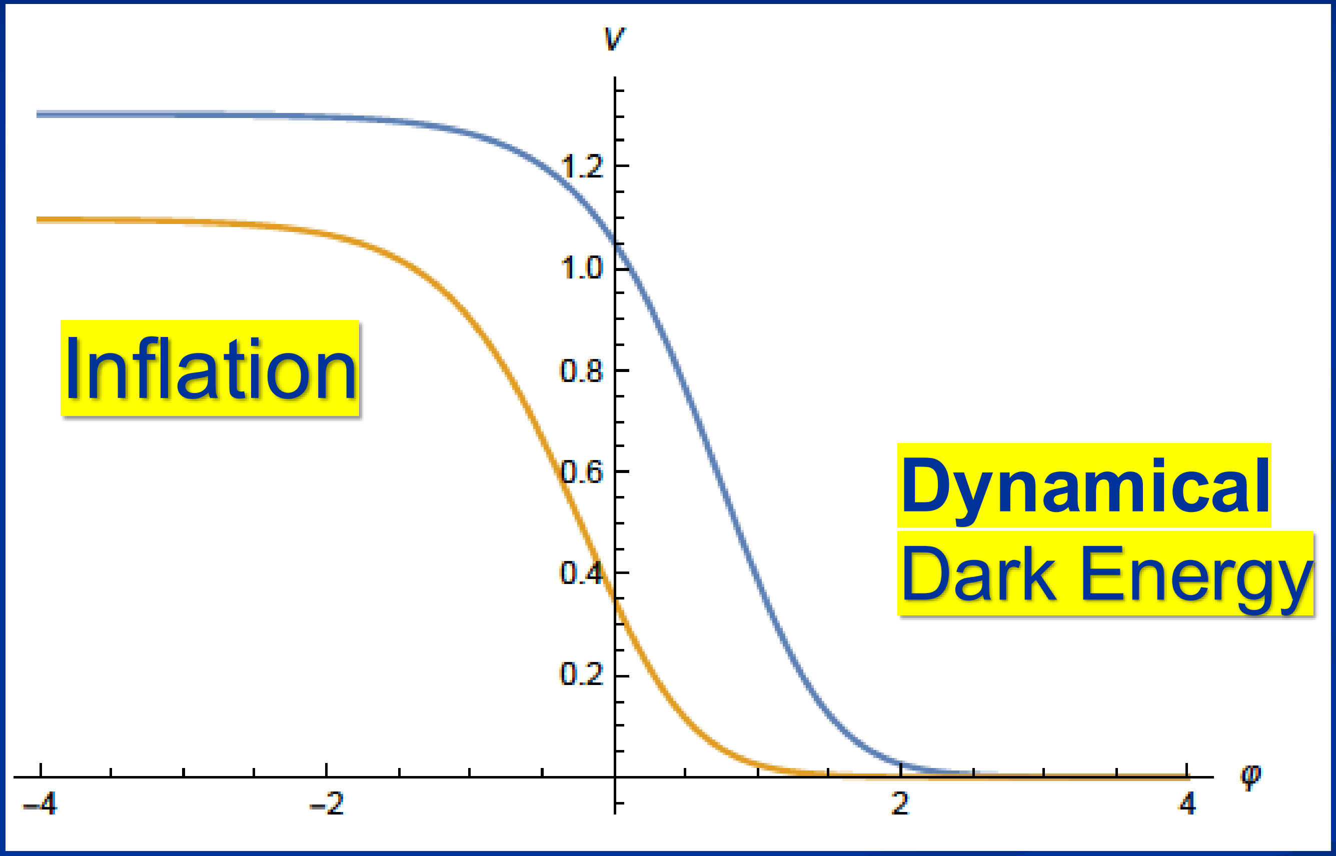


Scaling solution in Einstein frame





Quintessential inflation



Spokoiny, Peebles, Vilenkin, Peloso, Rosati, Dimopoulos, Valle, Giovannini, Brax, Martin, Hossain, Myrzakulov, Sami, Saridakis, de Haro, Salo, Bettoni, Rubio…

Weyl transformation for variable gravity

$$g_{\mu\nu} = (M^2/F)g'_{\mu\nu}$$

$$\varphi = 4M\ln(\chi/k)$$

$$\Gamma = \int_x \sqrt{g}\left\{ -\frac{1}{2}F(\chi)R + \frac{1}{2}K(\chi)\partial^\mu\chi\partial_\mu\chi + U(\chi)\right\}$$

$$\Gamma = \int_x \sqrt{g}\left\{ -\frac{M^2}{2}R' + \frac{1}{2}Z(\varphi)\partial^\mu\varphi\partial_\mu\varphi + V(\varphi)\right\}$$

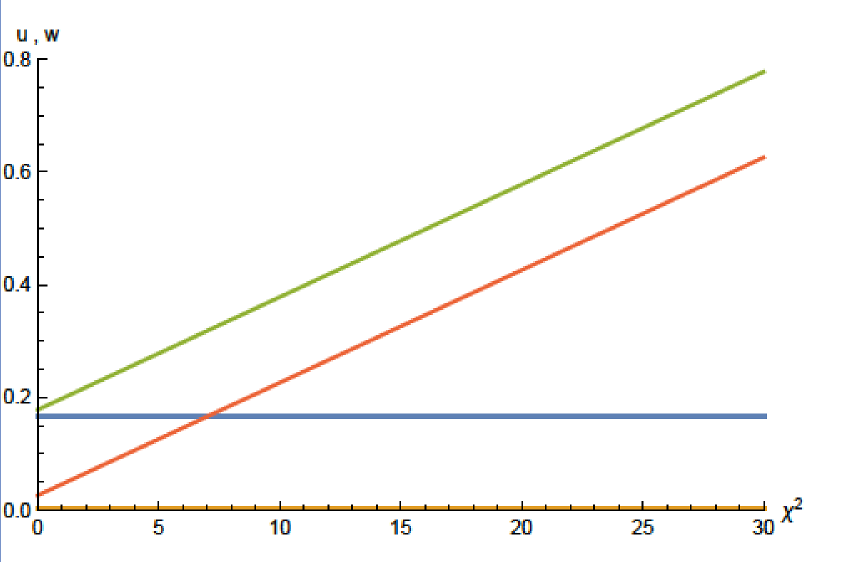
$$V(\varphi) = \frac{UM^4}{F^2}$$

$$Z(\varphi) = \frac{1}{16}\left\{\frac{\chi^2 K}{F} + \frac{3}{2}\left(\frac{\partial\ln F}{\partial\ln\chi}\right)^2\right\}$$

Scaling solution

$$U = u_0 k^4$$

$$F = 2 w_0 k^2 + \xi \chi^2$$



For low energy

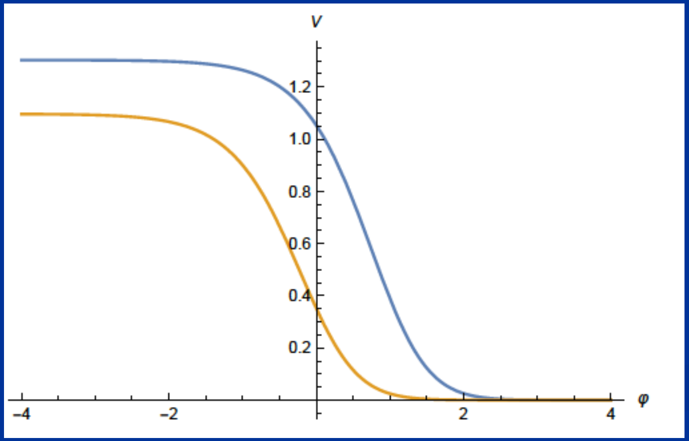
standard model :

$$u_\infty = \frac{7}{256\pi^2}$$

Asymptotic solution of cosmological constant problem

$$V = \frac{u_0 M^4}{\left(2w_0 + \xi \exp\left(\frac{\varphi}{2M}\right)\right)^2}$$

$$= \frac{u_0 M^4}{\xi^2}\left[1 + \frac{2w_0}{\xi}\exp\left(-\frac{\varphi}{2M}\right)\right]^{-2}\exp\left(-\frac{\varphi}{M}\right)$$



no tiny parameter !