Analytic results in Asymptotically Safe Quantum Gravity

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Why Quantum Gravity?

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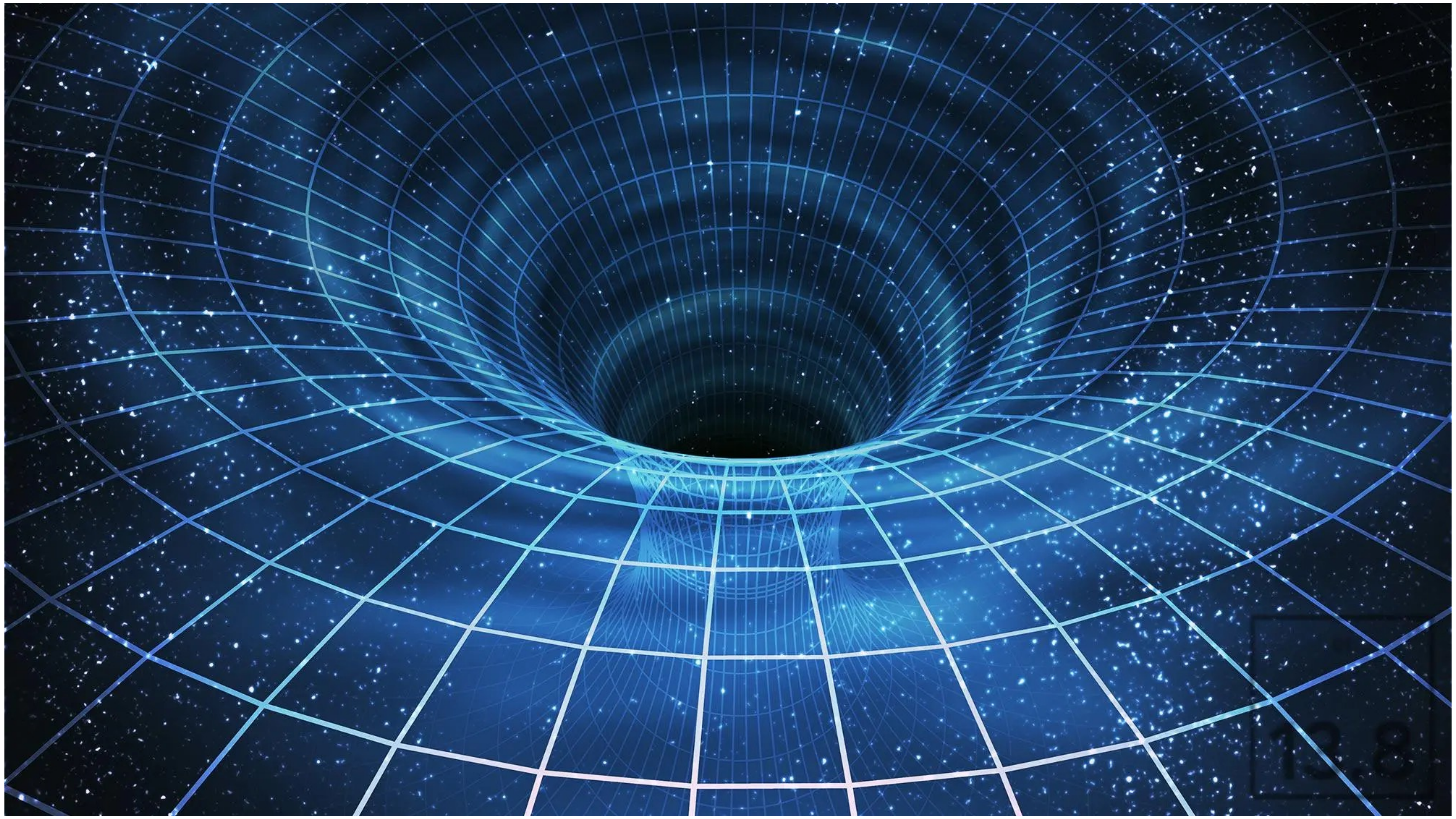

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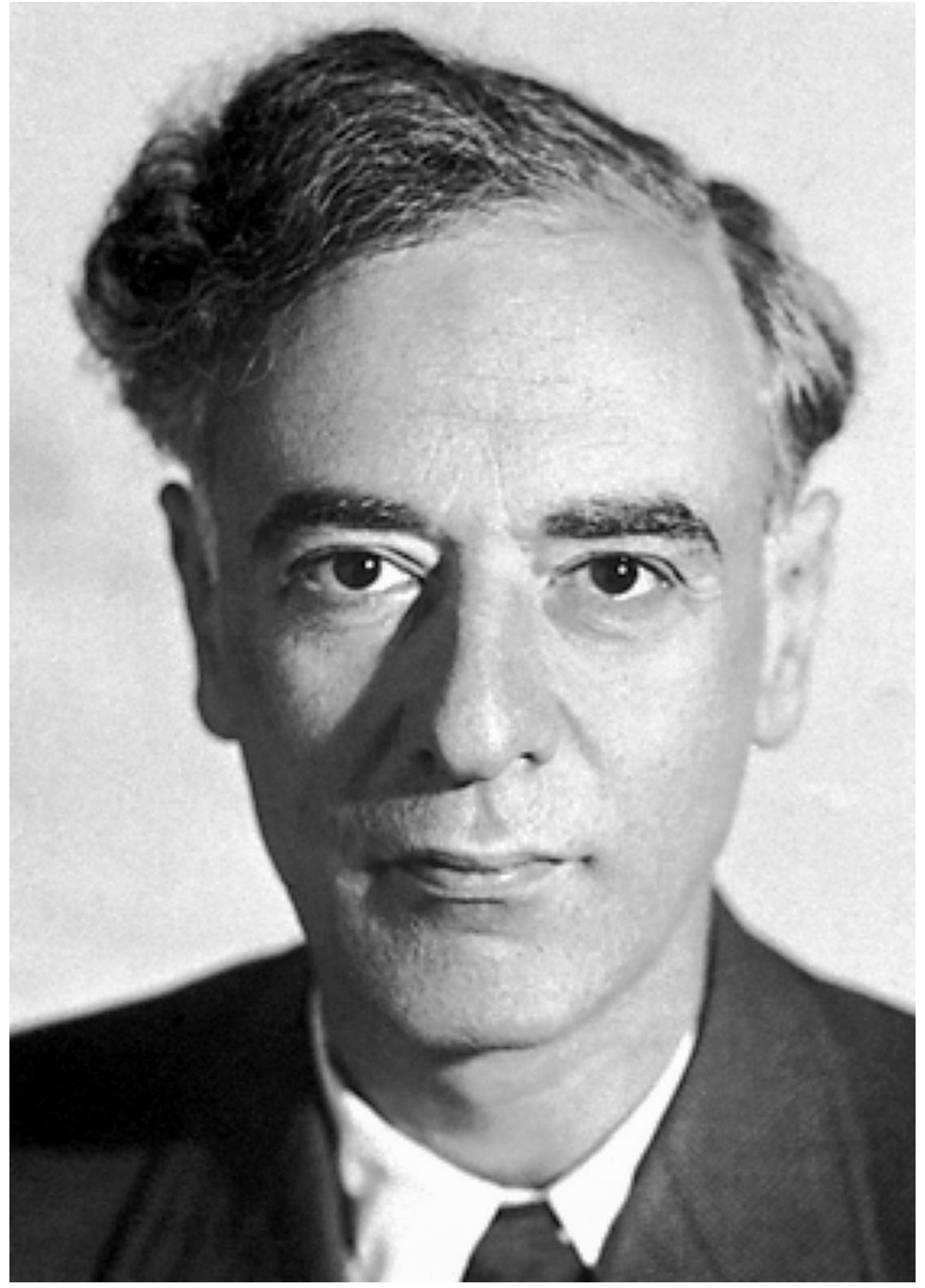

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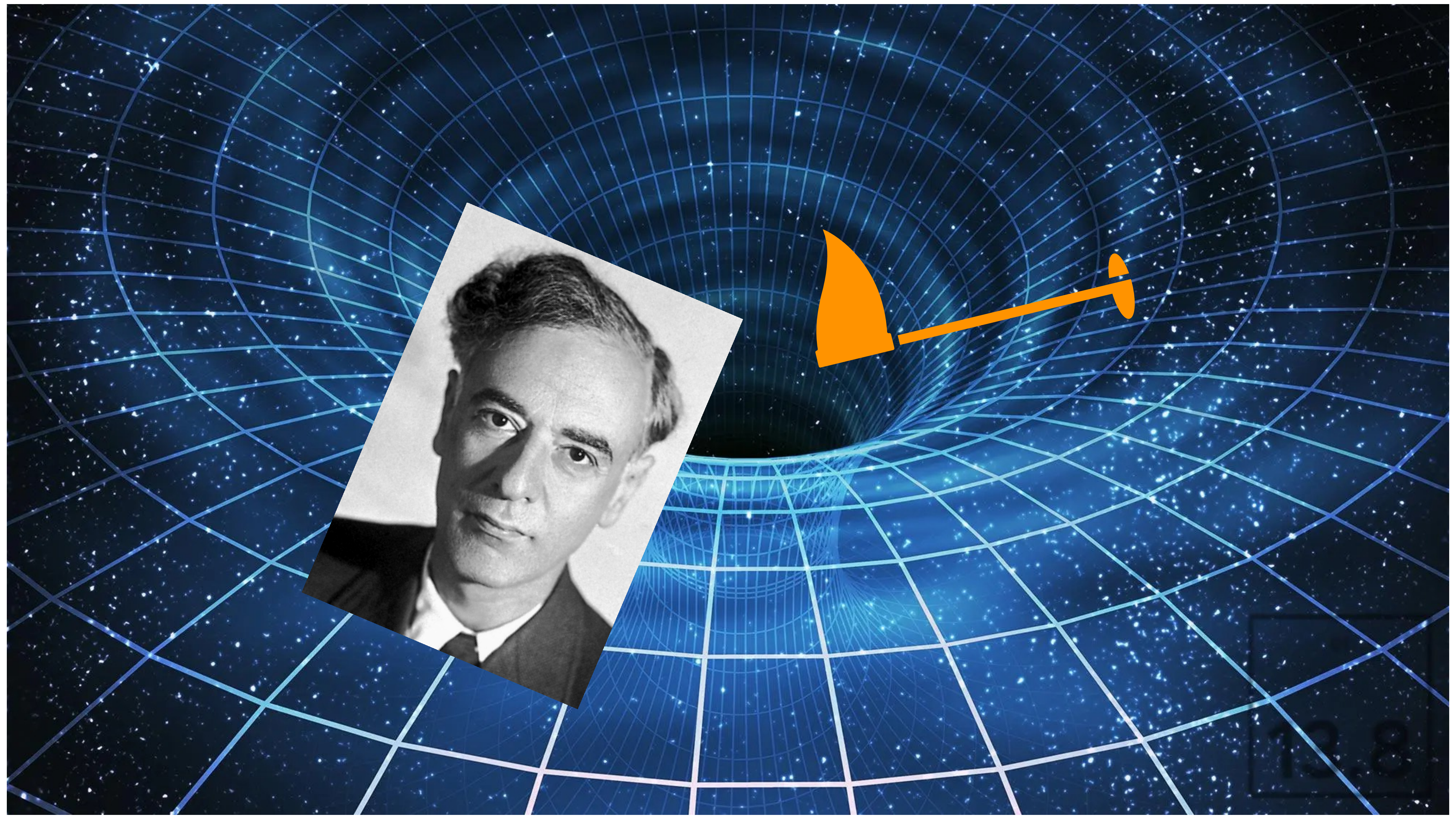

Why not perturbative Quantum Gravity?

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  2) violation of **unitarity** at high energies

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= predict almost all c_n from a finite number of free coefficients

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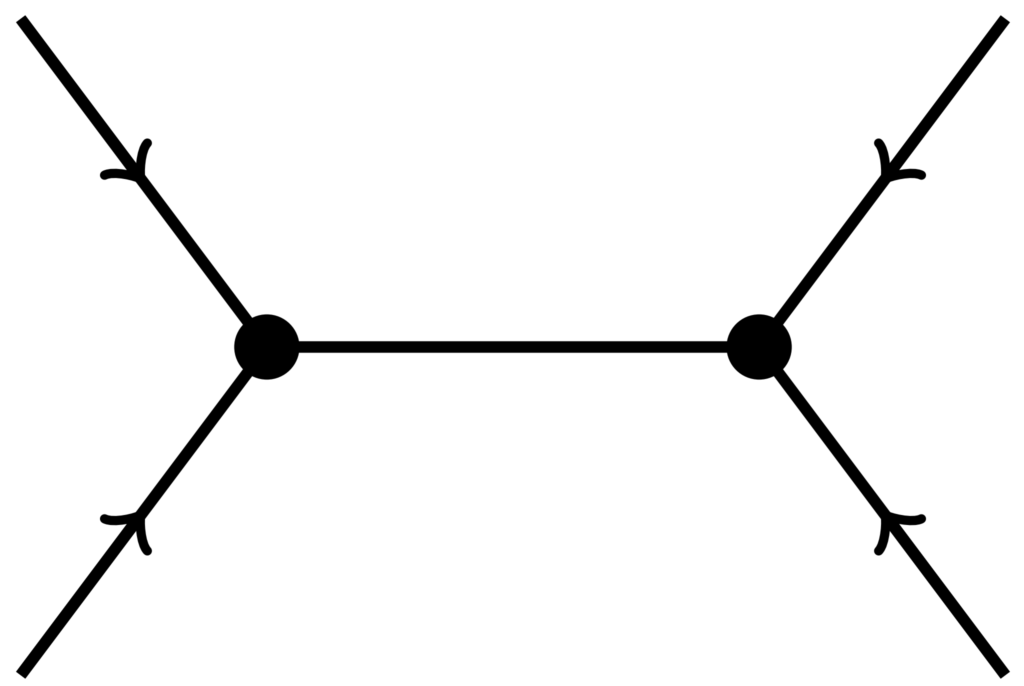

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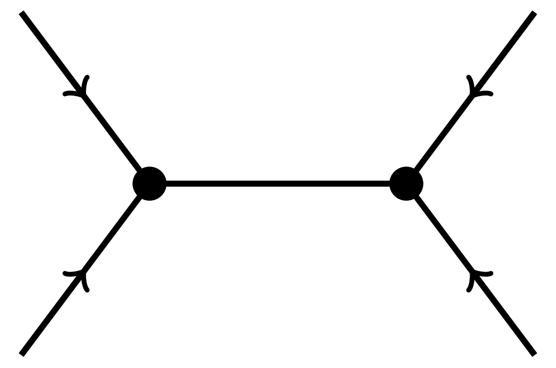

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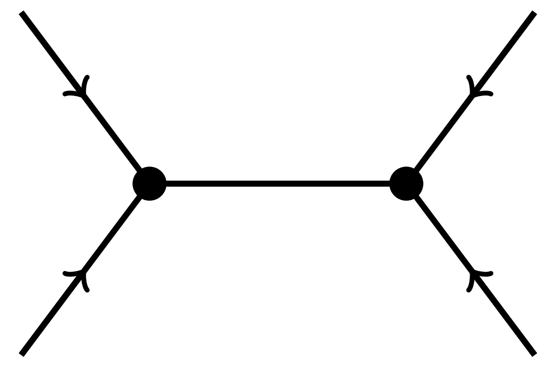

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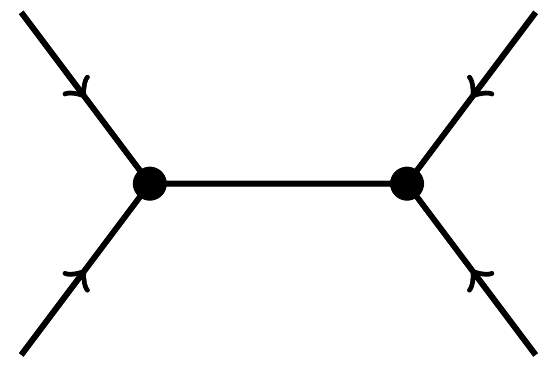

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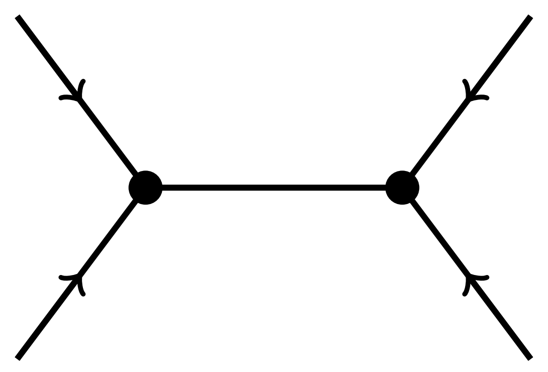

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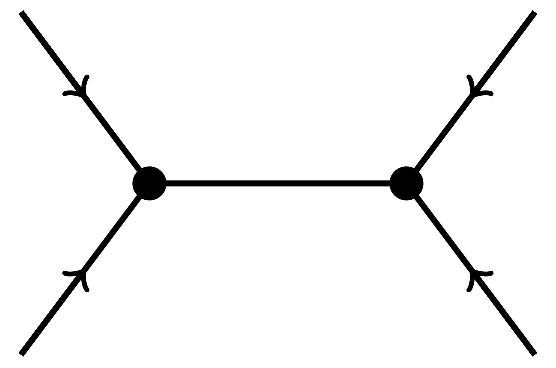

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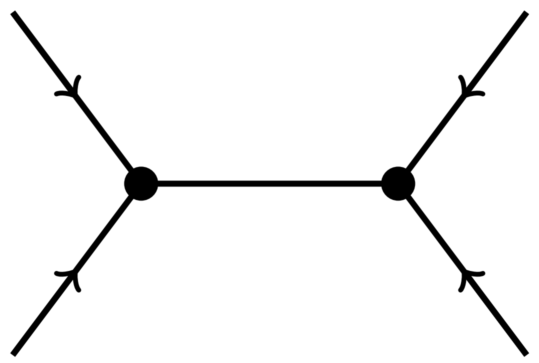

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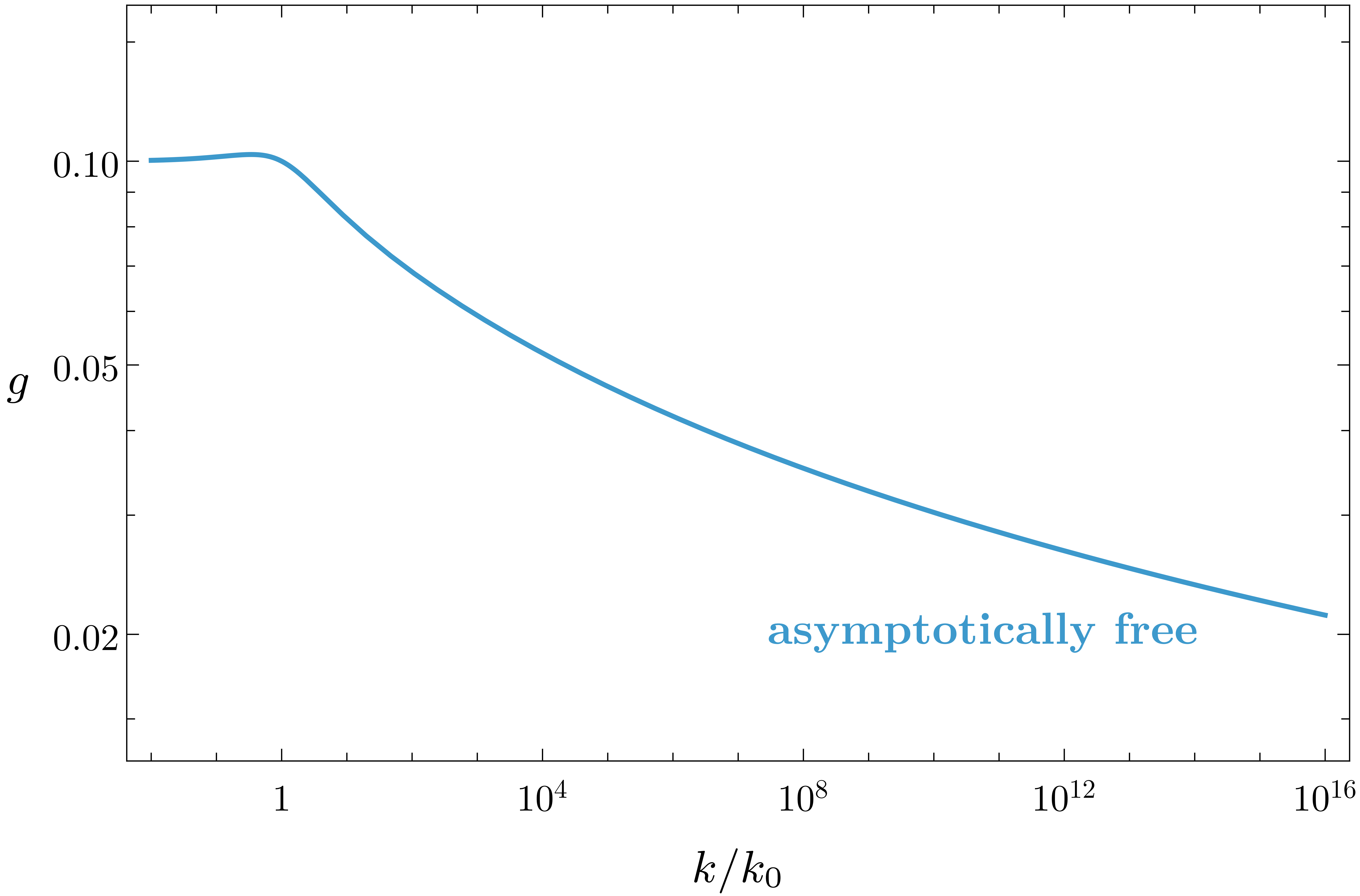
= c_n need to combine so that amplitude is bounded

Asymptotic Safety
to the rescue

Asymptotic Safety



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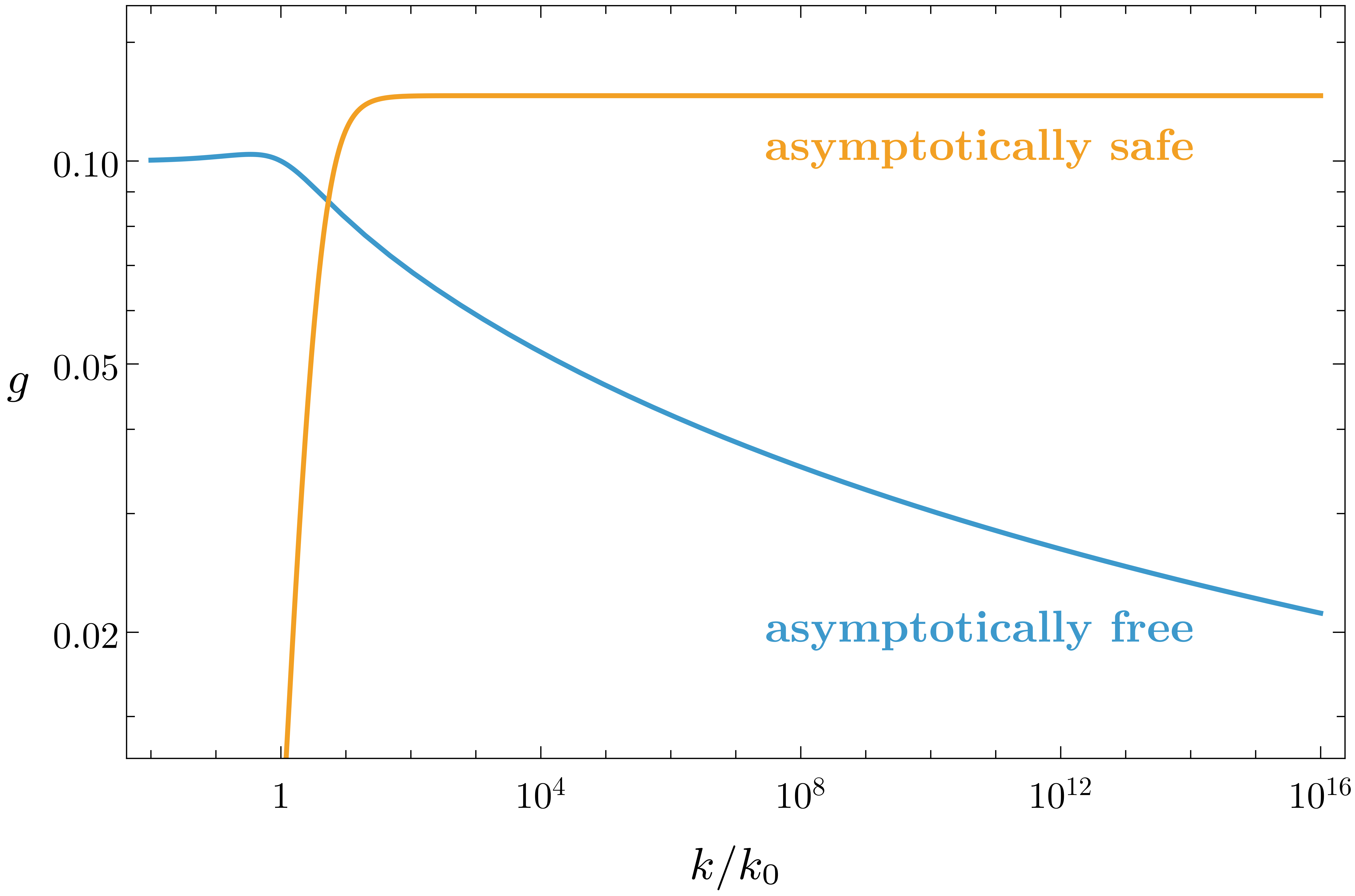

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2210.11356, 2210.13910, 2210.16072,
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recent review: 2606.21522

Eichhorn

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ASQG posters:
- Moritz Gessner (fermions)
- Paul P. Sprenger (diff invariance)
- Lidia Marino (black holes)
- ...

ASQG talks:
- Marc Schiffer (Wed, lattice)
- Fabian Wagner (Wed, cosmology)
- Zois Gyftopoulos (Thu, SM neutrino sector)
- Gabriel Assant (Thu, graviton propagator)
- Diego Buccio (Thu, IR divergences)
- ...

Is Asymptotic Safety unitary?

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focus on gravity-mediated scalar scattering

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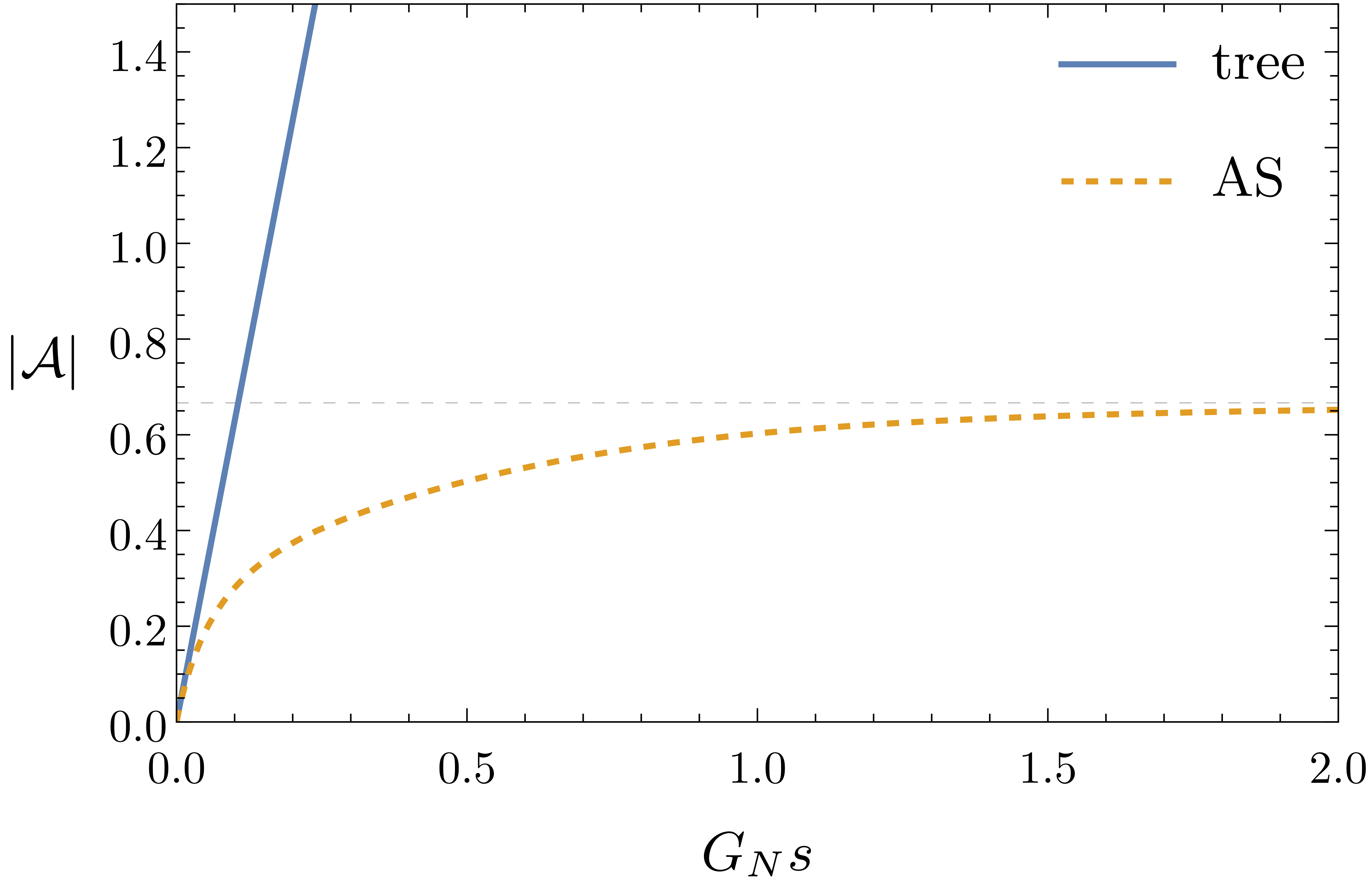

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2007.00733, 2007.04396, 2111.12365, 2205.13558, 2210.16072

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Scalar-scalar scattering

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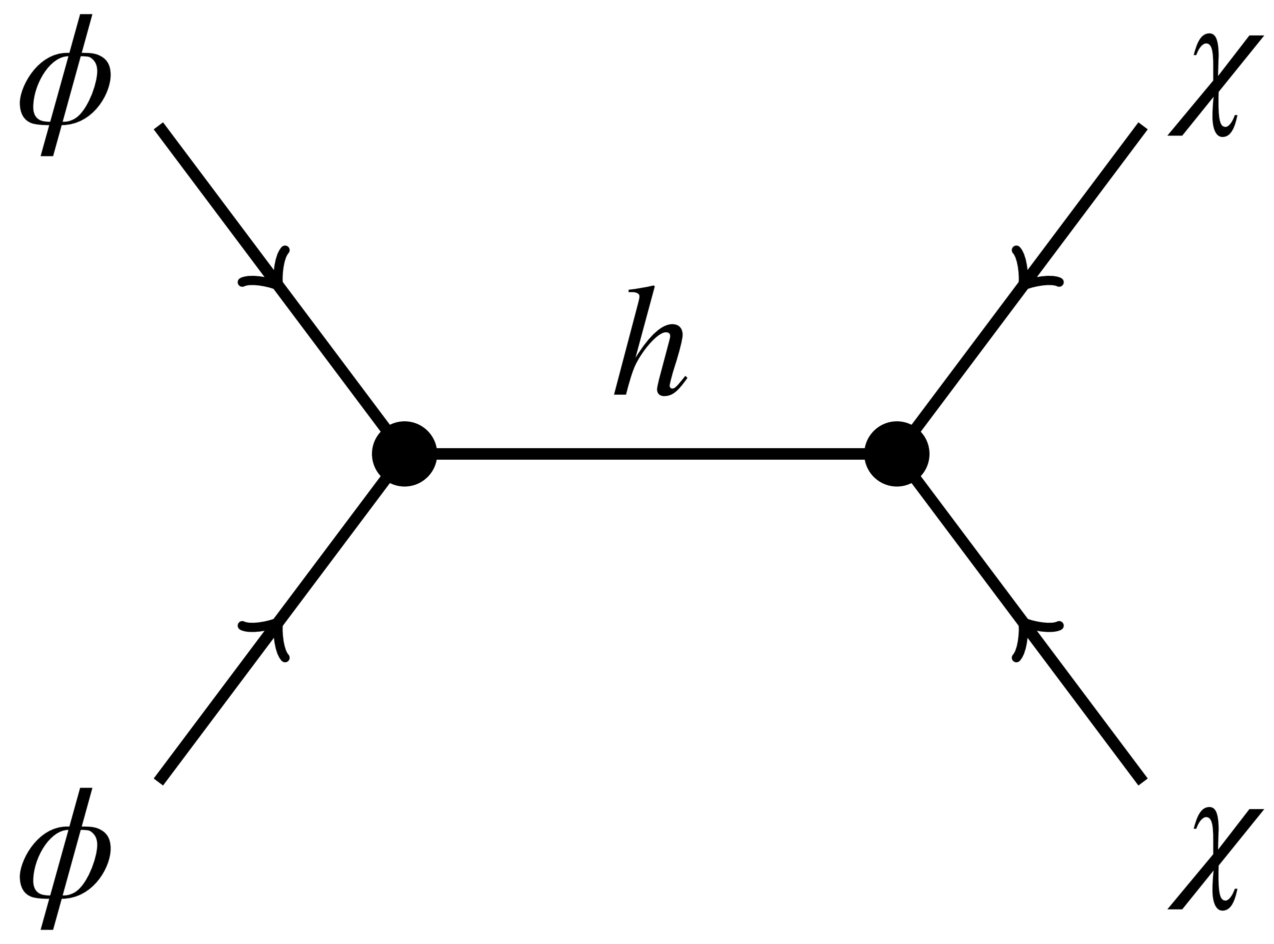

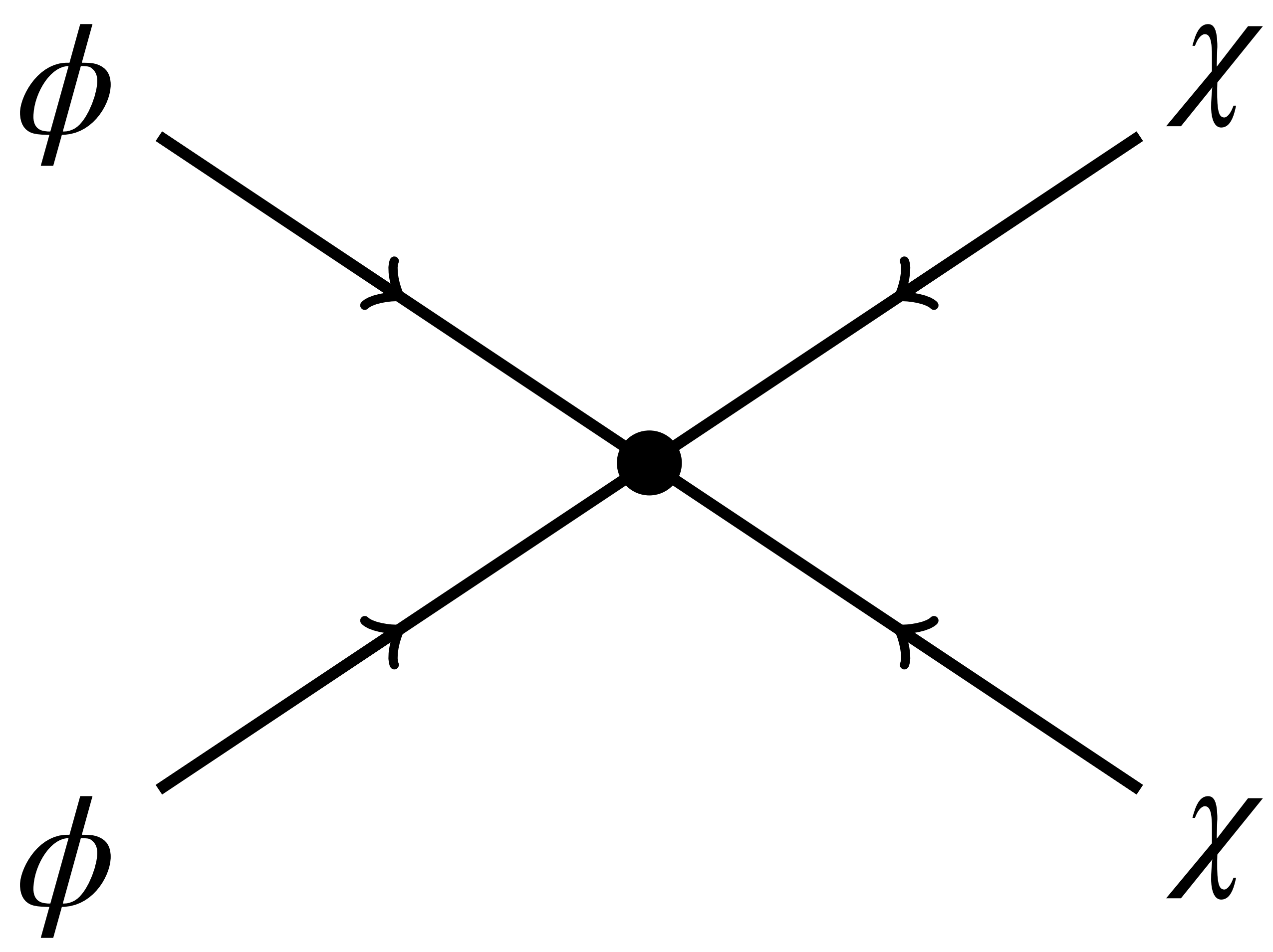

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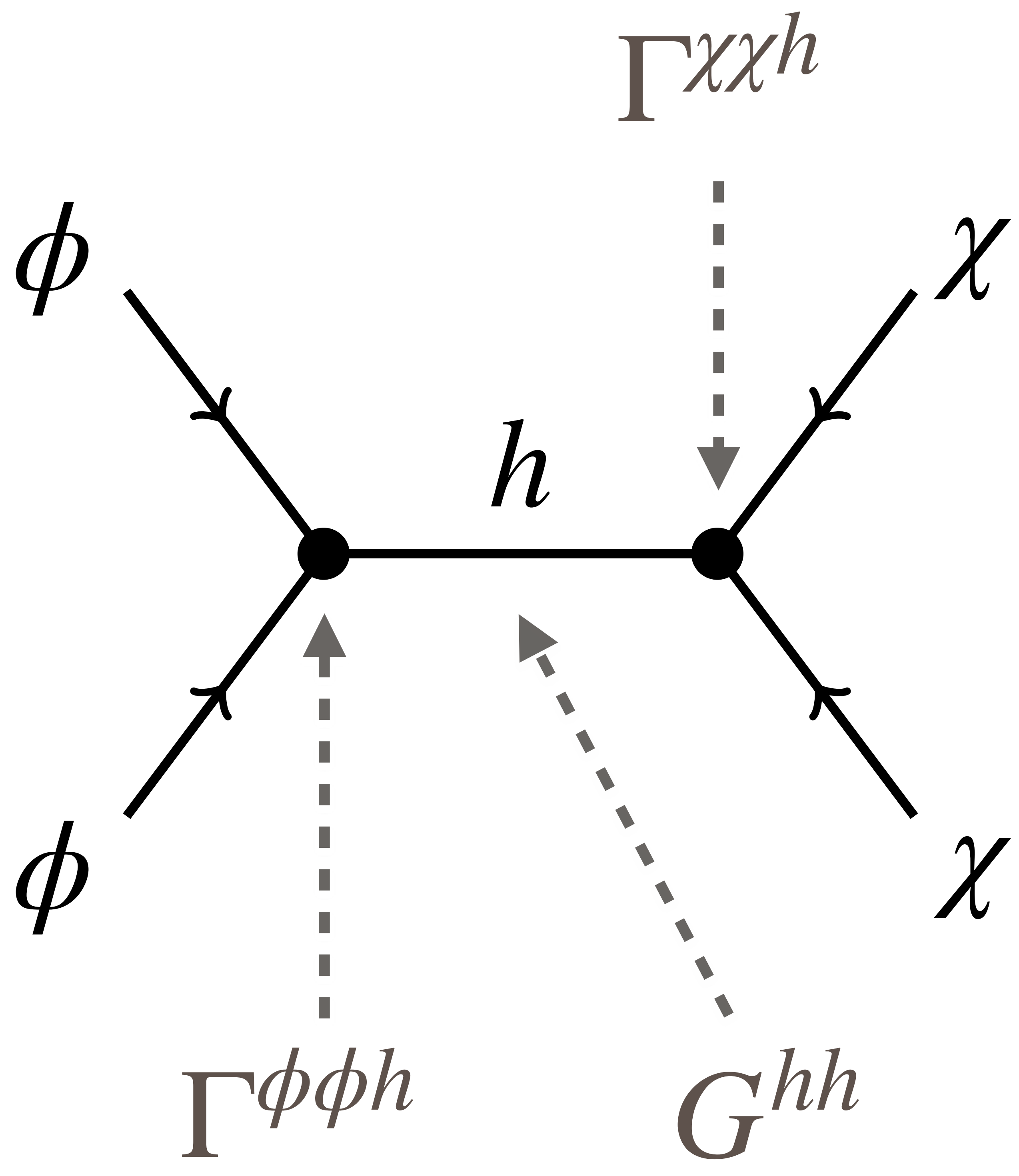

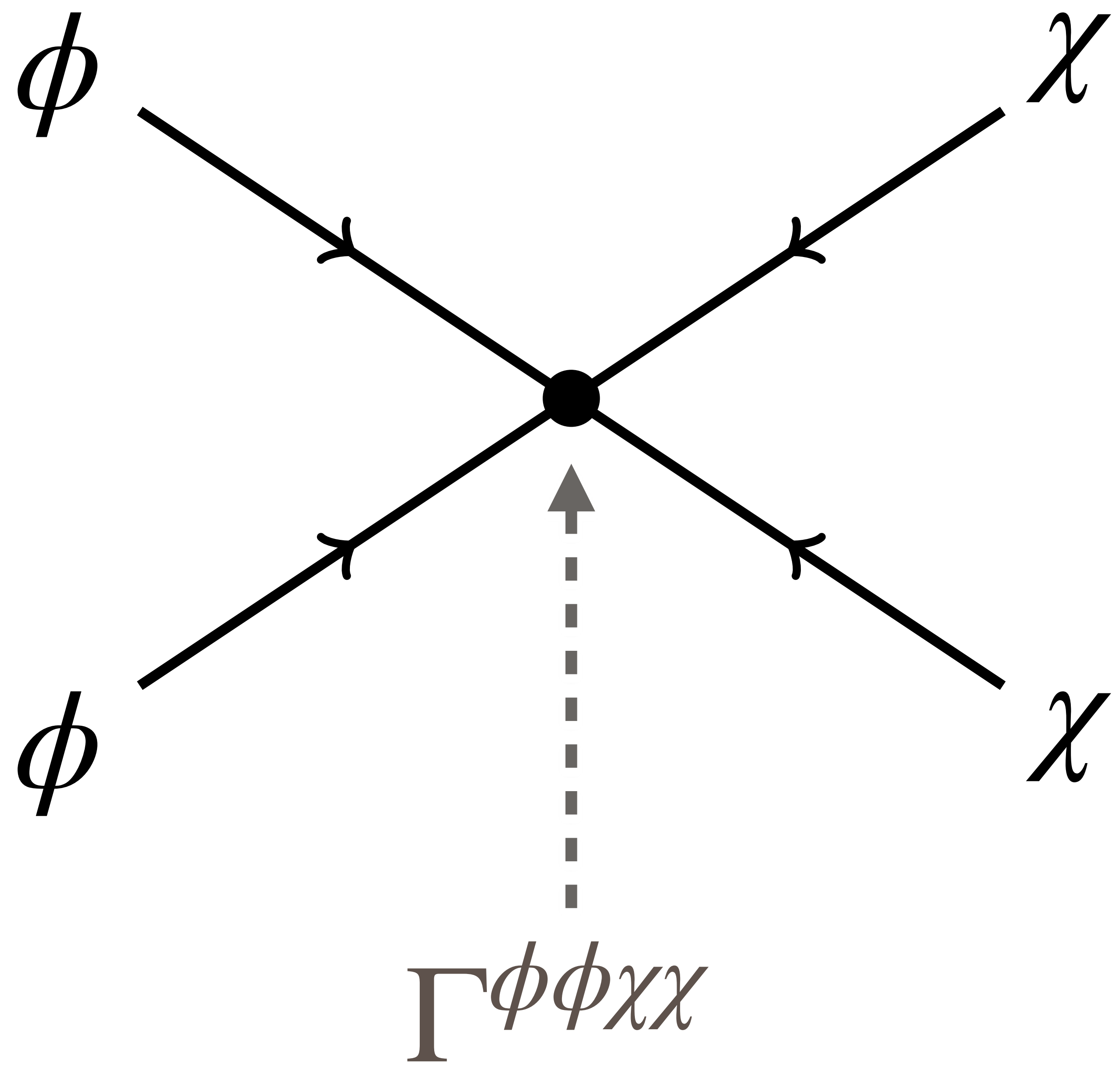

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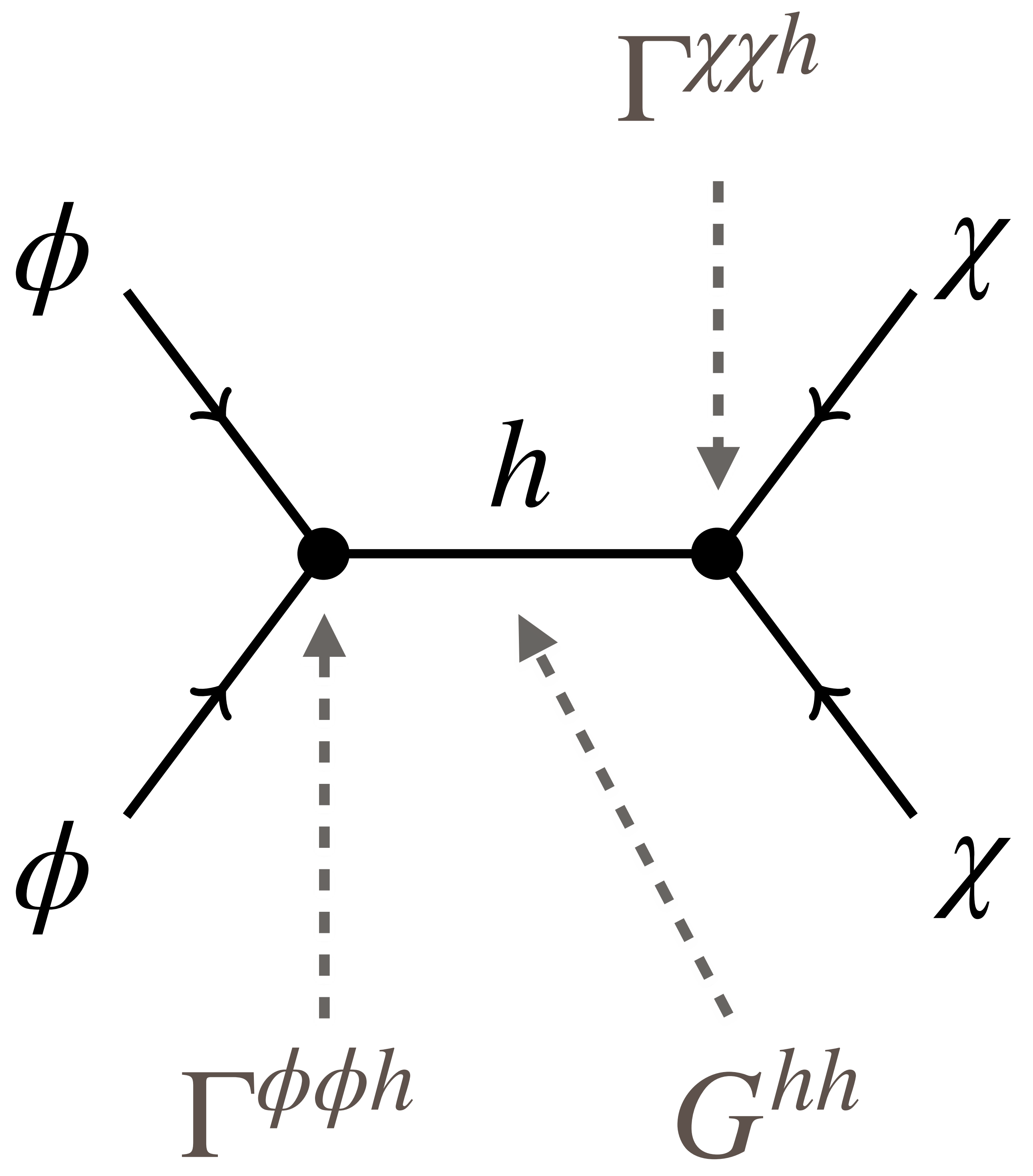

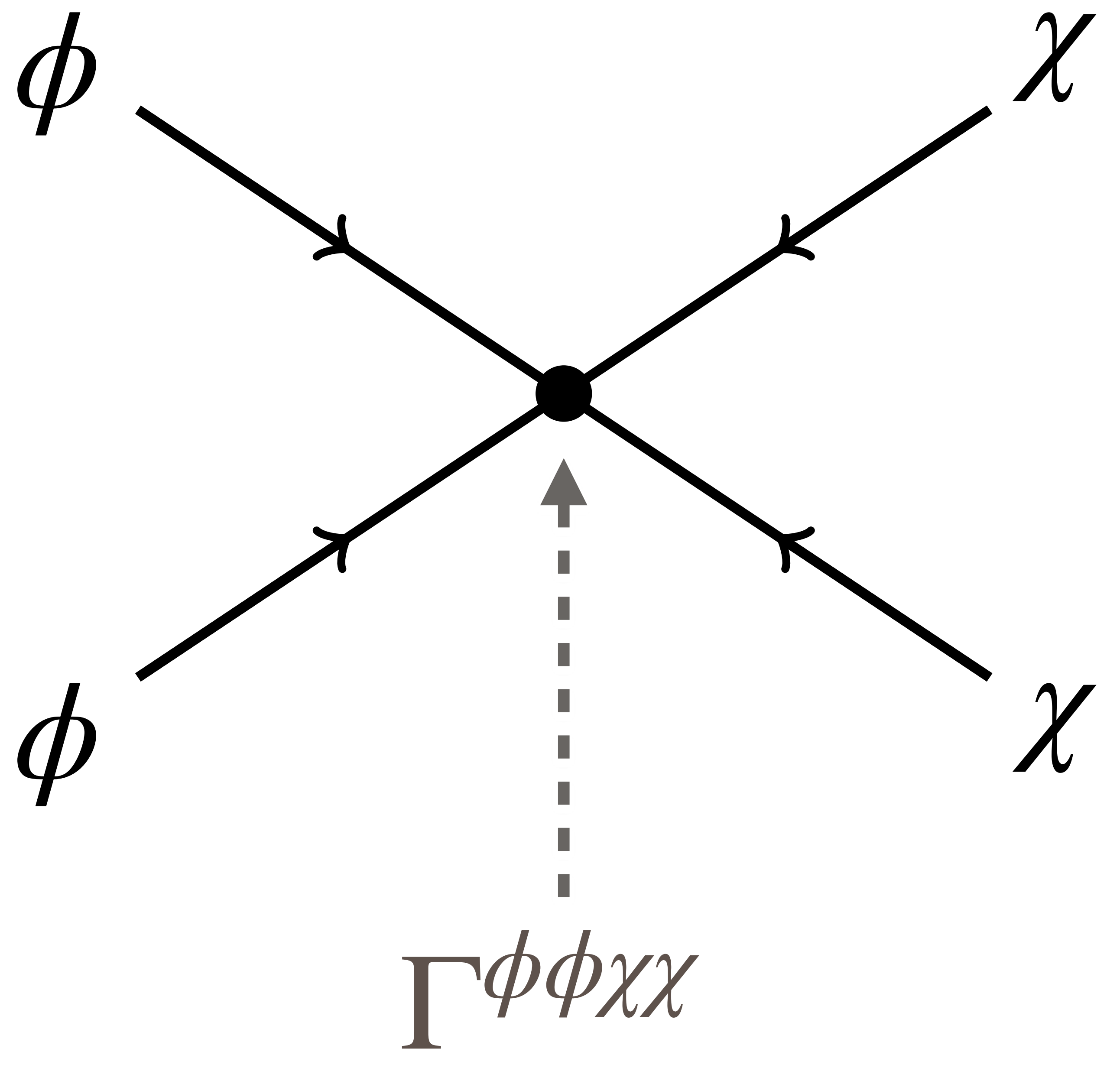

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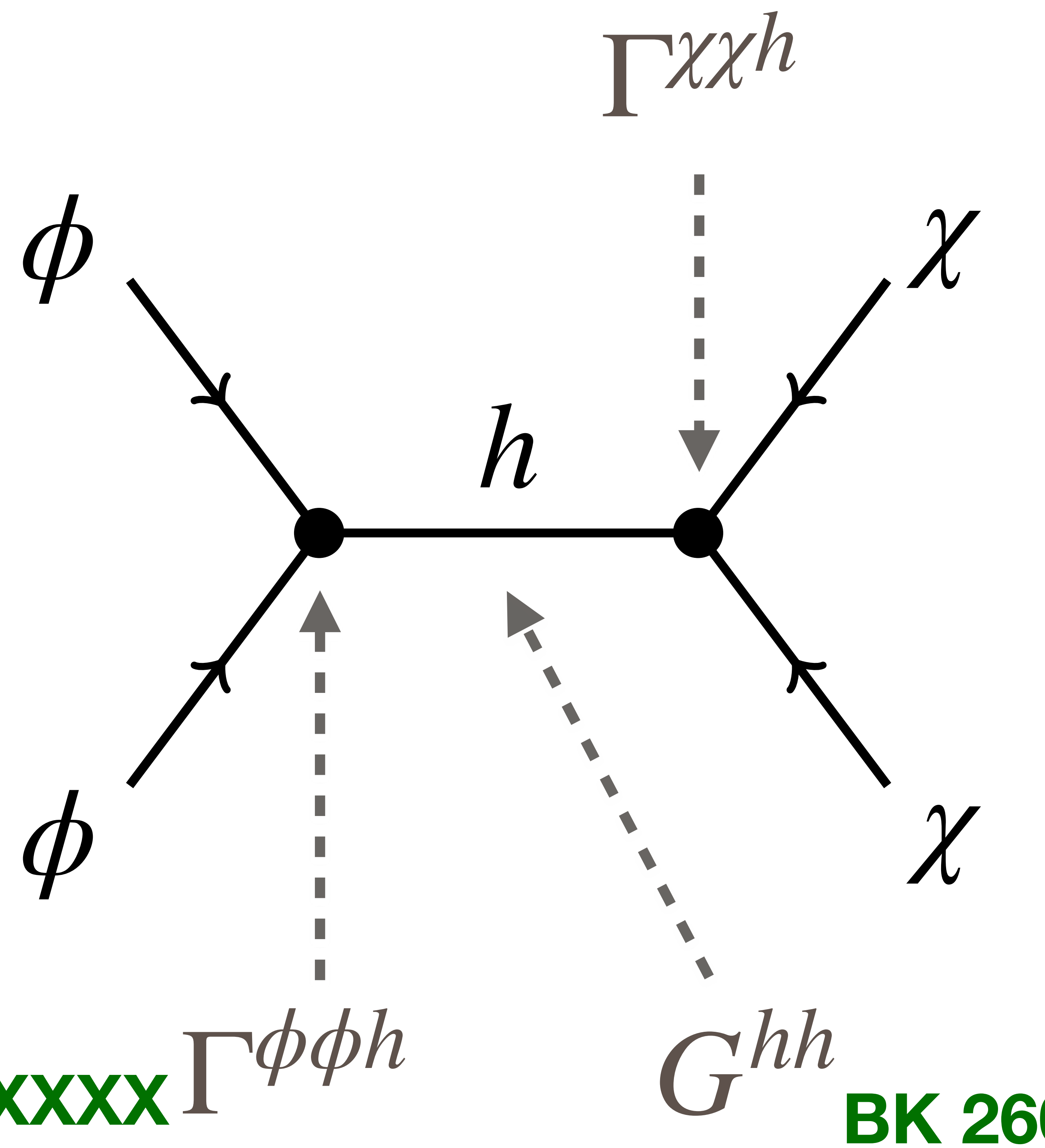
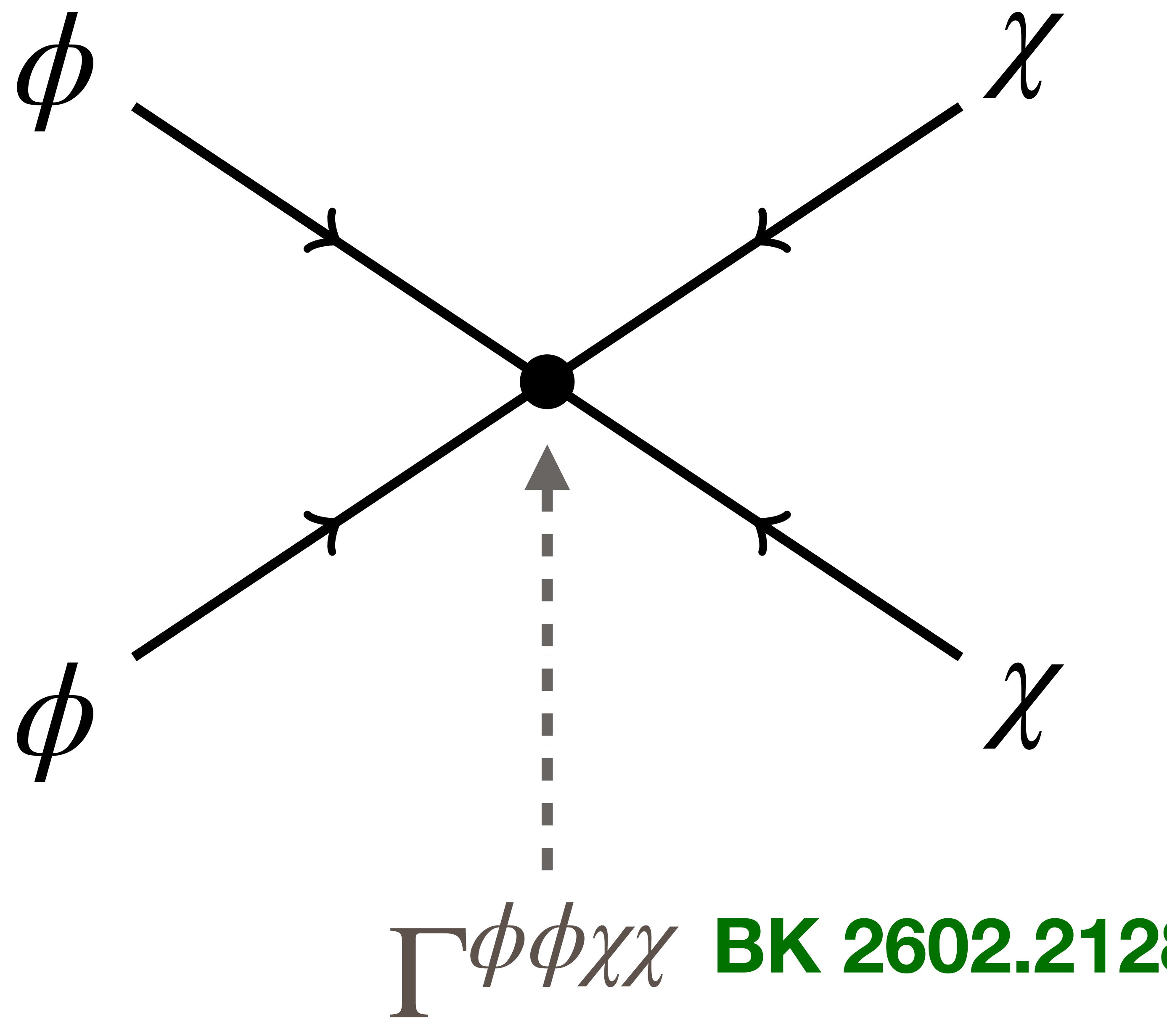


BK, T. Eckert 2609.XXXXX

BK 2606.18343



BK 2602.21285

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Analytic momentum dependence in Asymptotic Safety

Analytic ingredients

- throw out cosmological constant and masses
- specific regulator shape
- approximate vertices by classical vertices — no/limited self-feedback
- model running Newton’s constant

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$\Rightarrow$ ***ask me about the gory details!***
(maybe in private…)

Graviton propagator

- propagator in Minkowski background:

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gauge garbage

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  - Euclidean signature

    Christiansen, BK, Pawlowski, Rodigast 1403.1232
    BK, Schiffer 2105.04566

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    Bonanno, Denz, Pawlowski, Reichert 2102.02217
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numerical story and details:
Gabriel Assant (Thu)

Assant, Litim, Reichert 2606.19321

Analytic graviton propagator

key input from
BK, Schiffer 2105.04566

$$\ln Z_a(p^2) = \int_0^\infty dk' \ldots = g_* \sum_{n=1}^{7} c_a^{(n)}(X) f^{(n)}(X)\,, \qquad X = \frac{G_N p^2}{g_*}$$

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BK 2606.18343

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$$\begin{aligned} f^{(1)}(X) = {} & X\left(4\,{}_3F_3(1,1,1;2,2,2;X) - {}_3F_3\left(1,1,1;2,2,2;\frac{X}{2}\right)\right) + \frac{1}{4}\left(-8\gamma\mathrm{Ei}(\mathrm{X}) - \pi^2 + 2\gamma^2\right) - \frac{21}{X^4} - \frac{4}{X^3} - \frac{1}{X^2} \\ & + e^{X/2}\left(\frac{96}{X^4} + \frac{16}{X^3} + \frac{4}{X^2} + \frac{2}{X}\right)\Gamma\left(0,\frac{X}{2}\right) + e^X\left(-\frac{48}{X^5} - \frac{12}{X^4} - \frac{4}{X^3} - \frac{2}{X^2} - \frac{2}{X}\right)\Gamma(0,X) \\ & - \frac{1}{2}\left[\ln\left(\frac{X}{2}\right)\right]^2 - \left[\ln(X)\right]^2 + (\ln(X) - \gamma)\ln\left(\frac{X}{2}\right) + 3\gamma\ln(X) + \ln(-X)\left(-\ln\left(\frac{X}{2}\right) + 2\ln(X) - \gamma\right) \\ & + \left(-\ln\left(\frac{X}{2}\right) - \gamma\right)\Gamma\left(0,-\frac{X}{2}\right) + 2\ln(X)\Gamma(0,-X) + {}_1F_1^{(2,0,0)}\left(0;1\middle|X\right) - \frac{1}{2}\,{}_1F_1^{(2,0,0)}\left(0;1\middle|\frac{X}{2}\right) \end{aligned}$$

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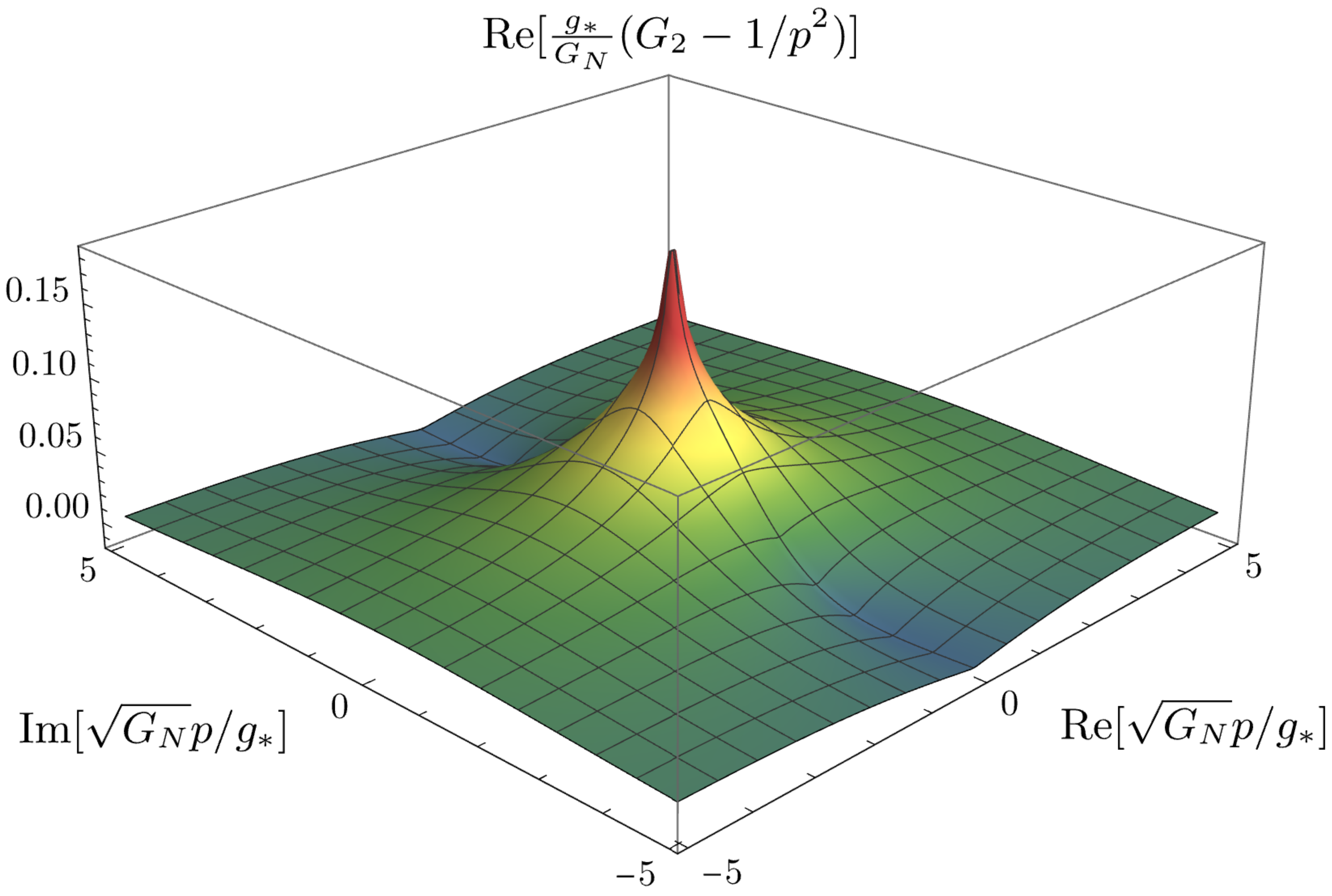
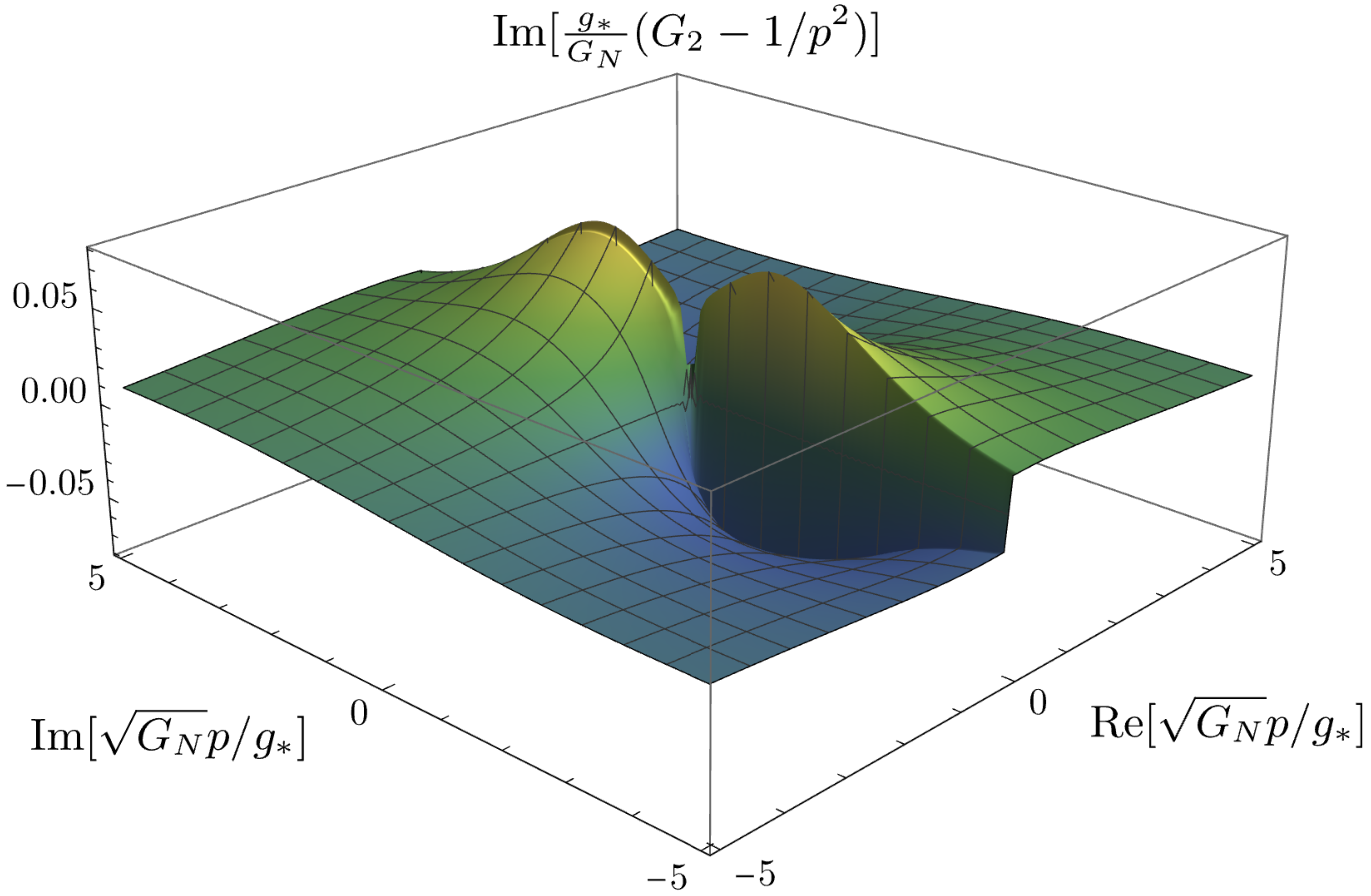
Analytic graviton propagator

spin two





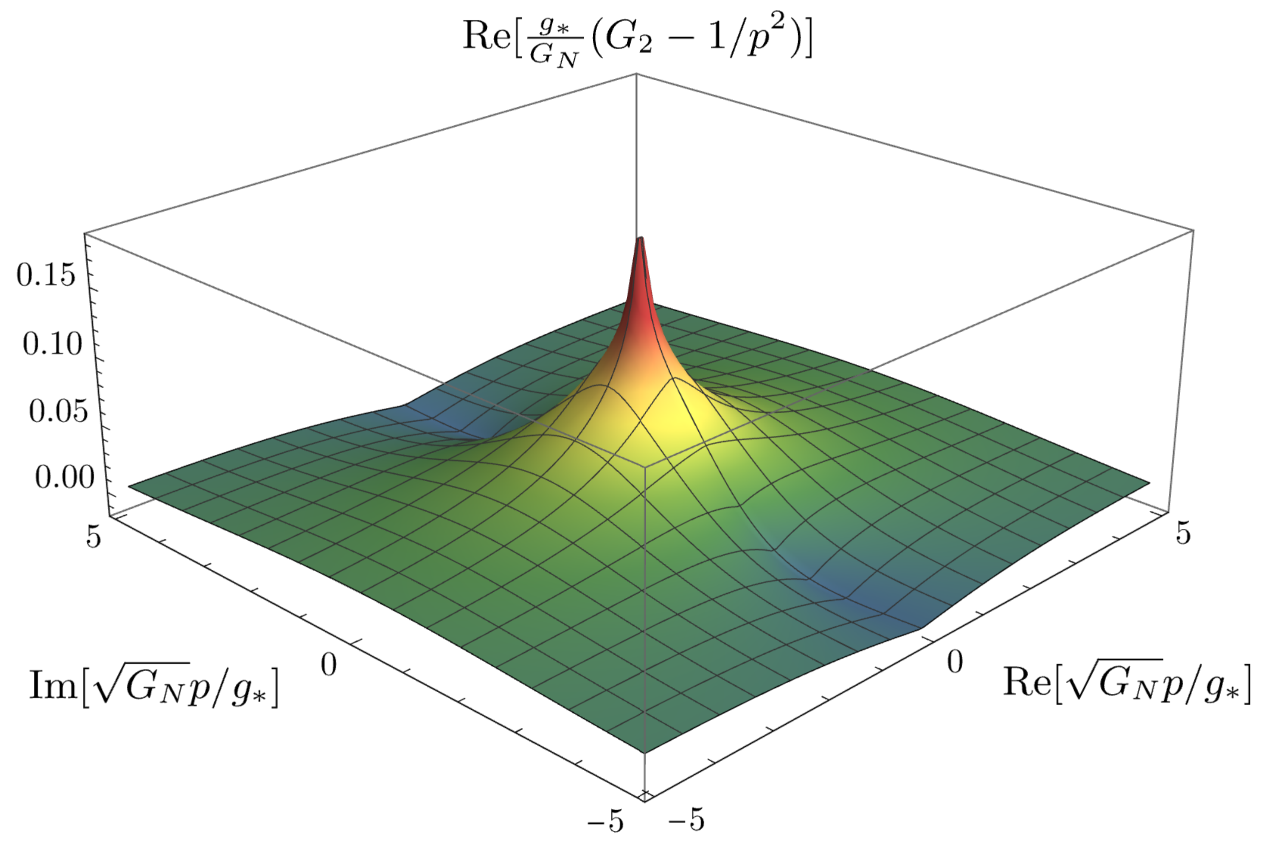
qualitatively close to numerical results of

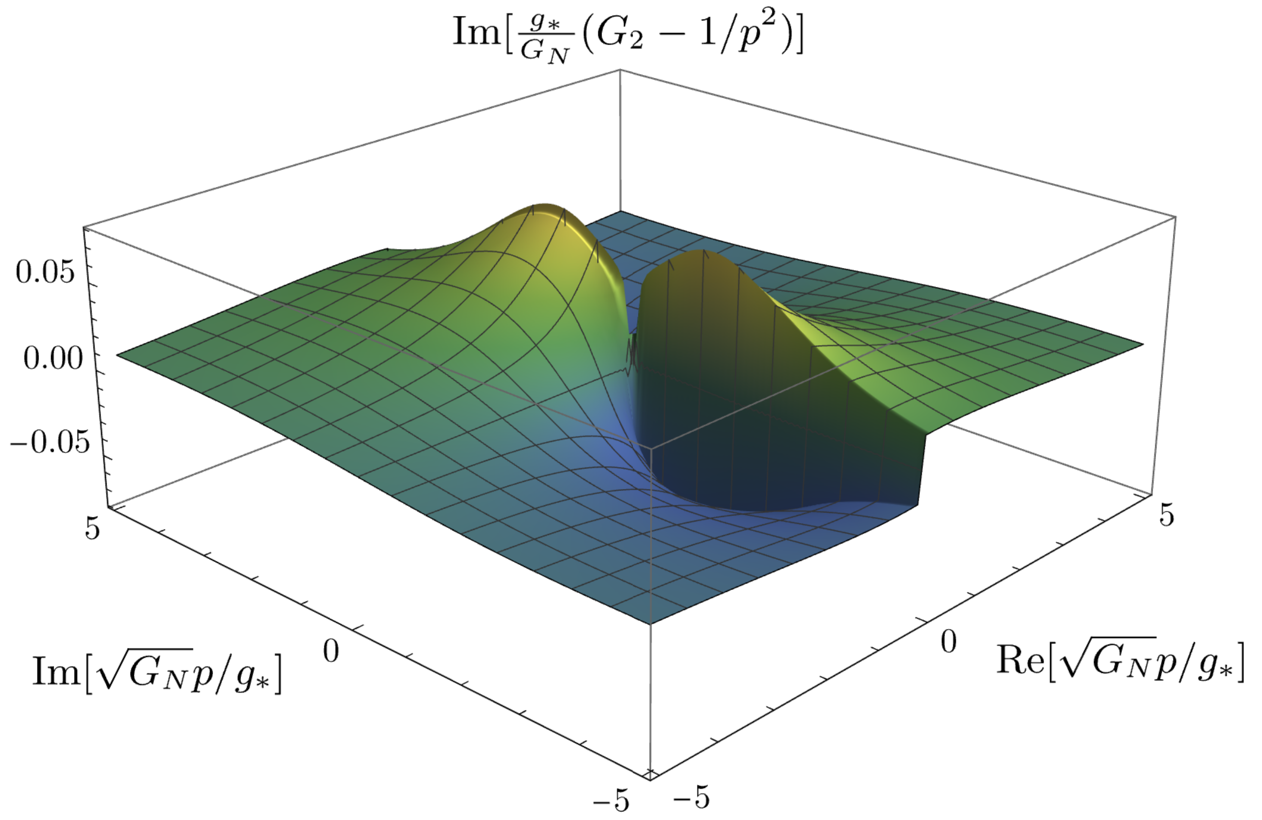
Bonanno, Denz, Pawlowski, Reichert 2102.02217
Fehre, Litim, Pawlowski, Reichert 2111.13232
Pawlowski, Reichert, Wessely 2507.22169
Assant, Litim, Reichert 2606.19321

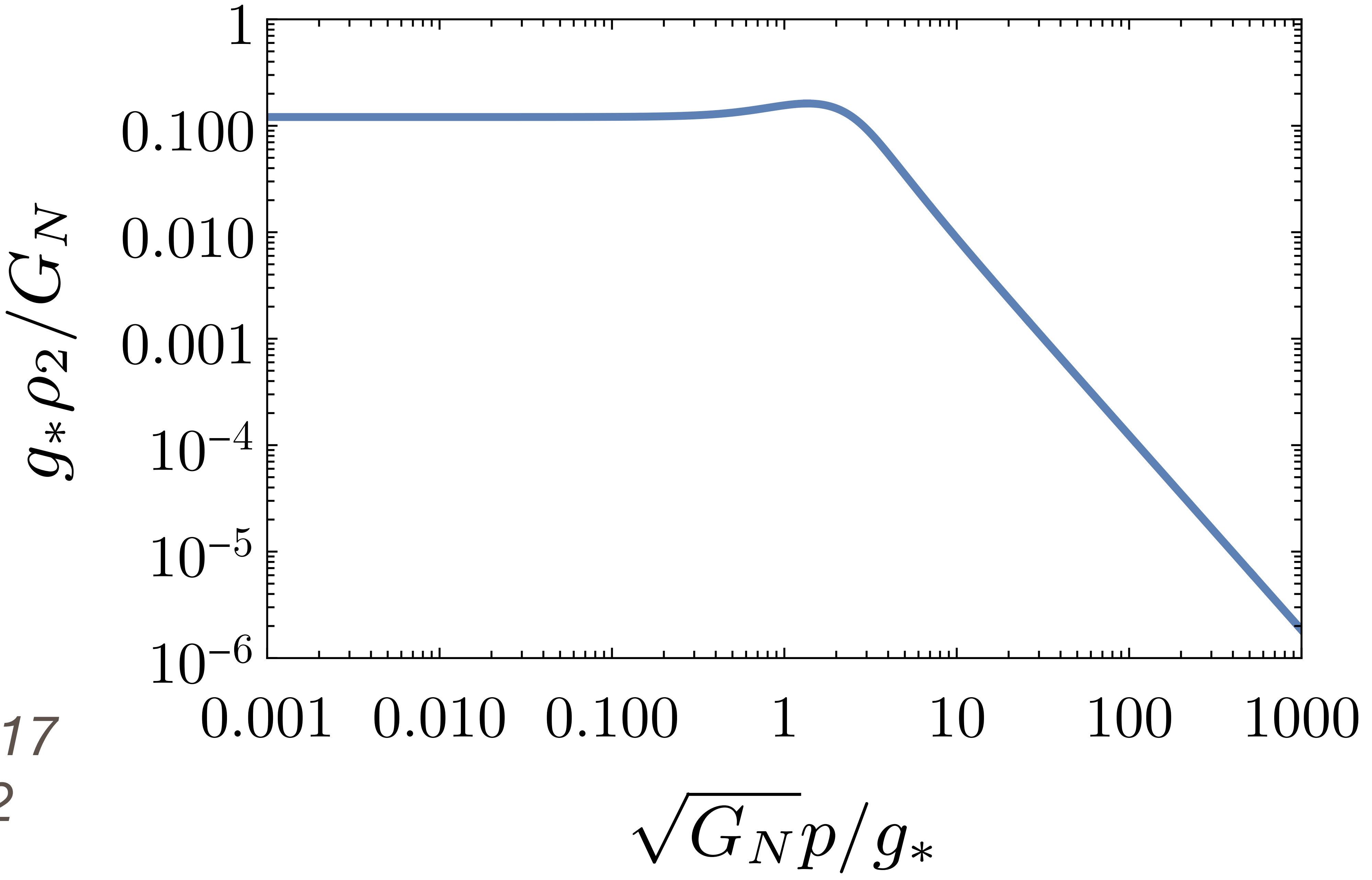
BK 2606.18343

Analytic graviton propagator

spin two







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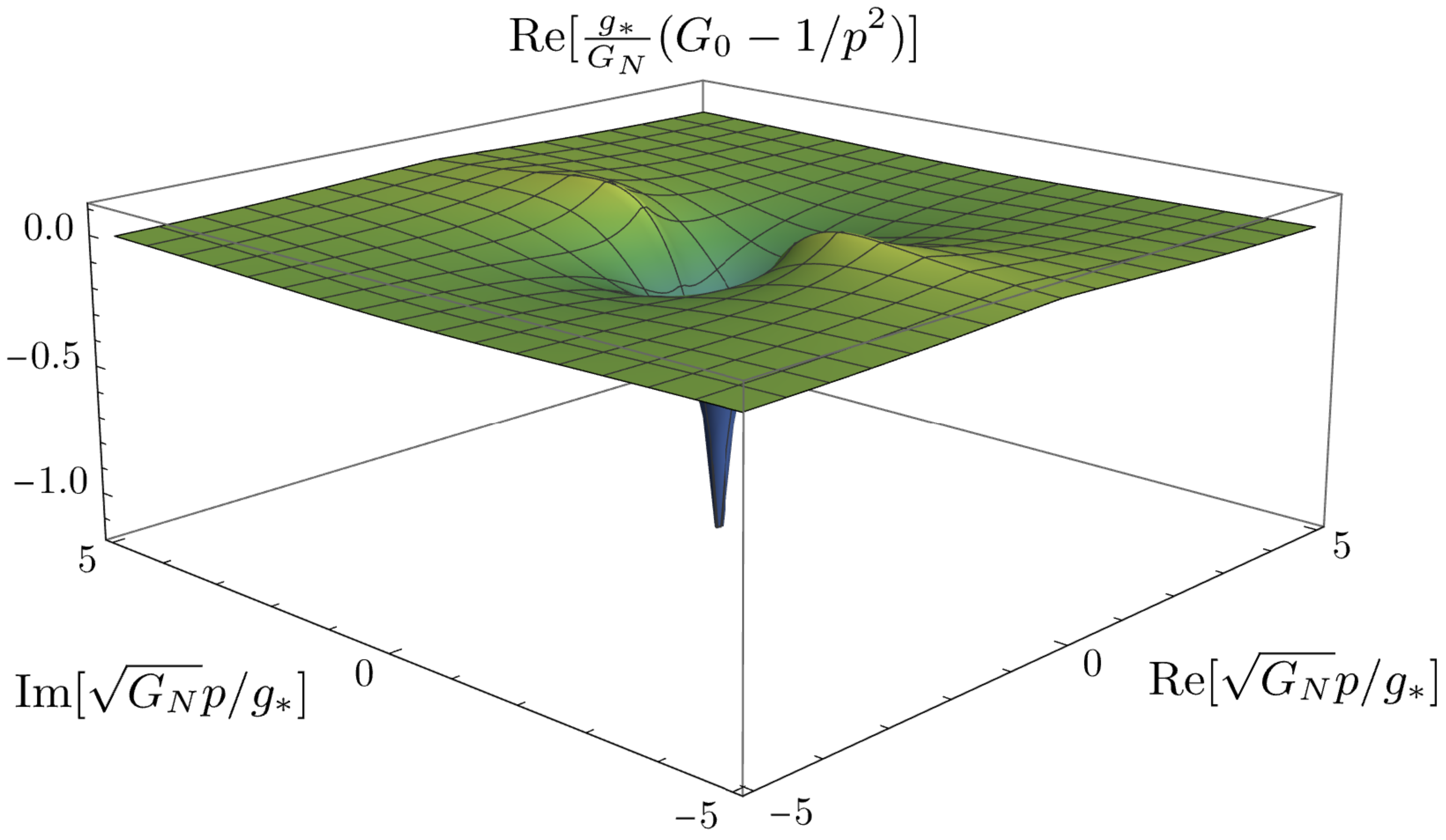

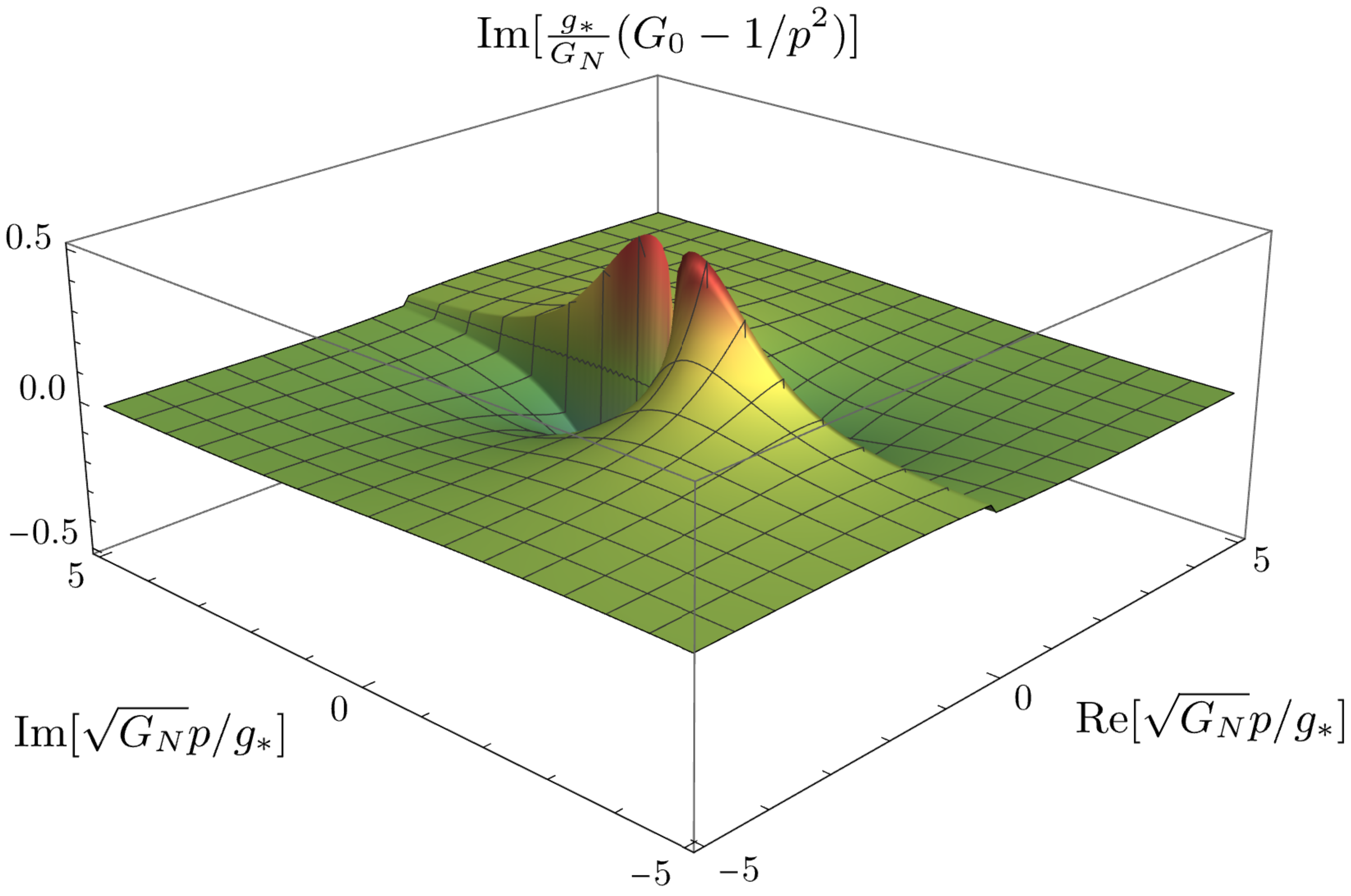

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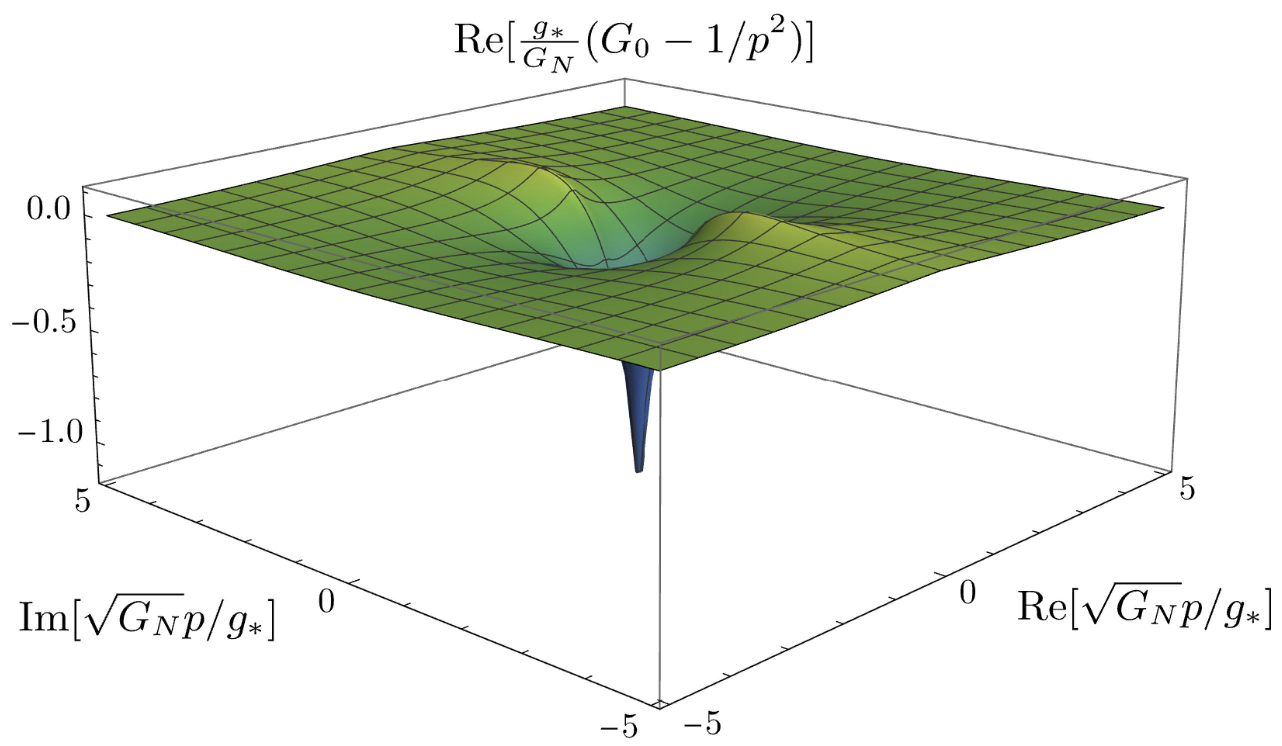

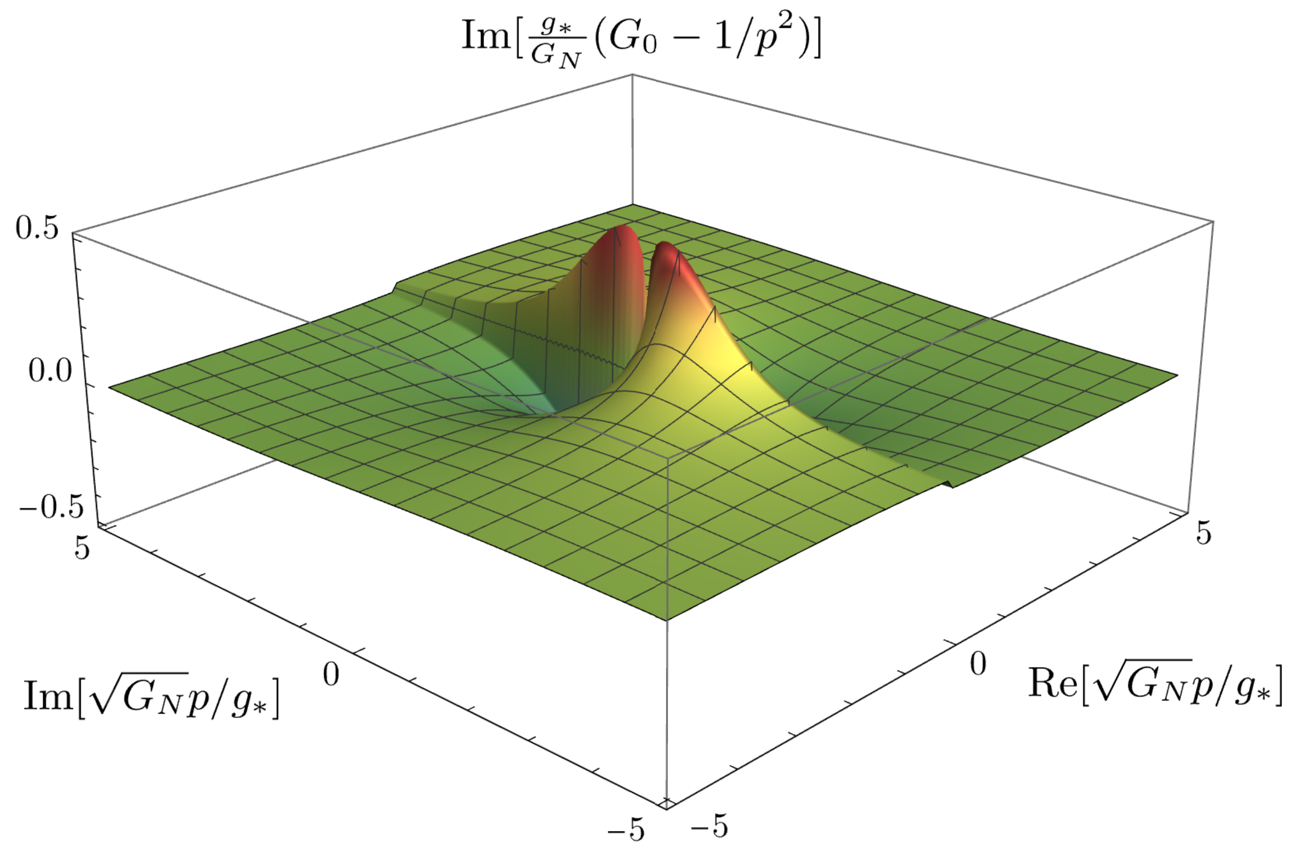

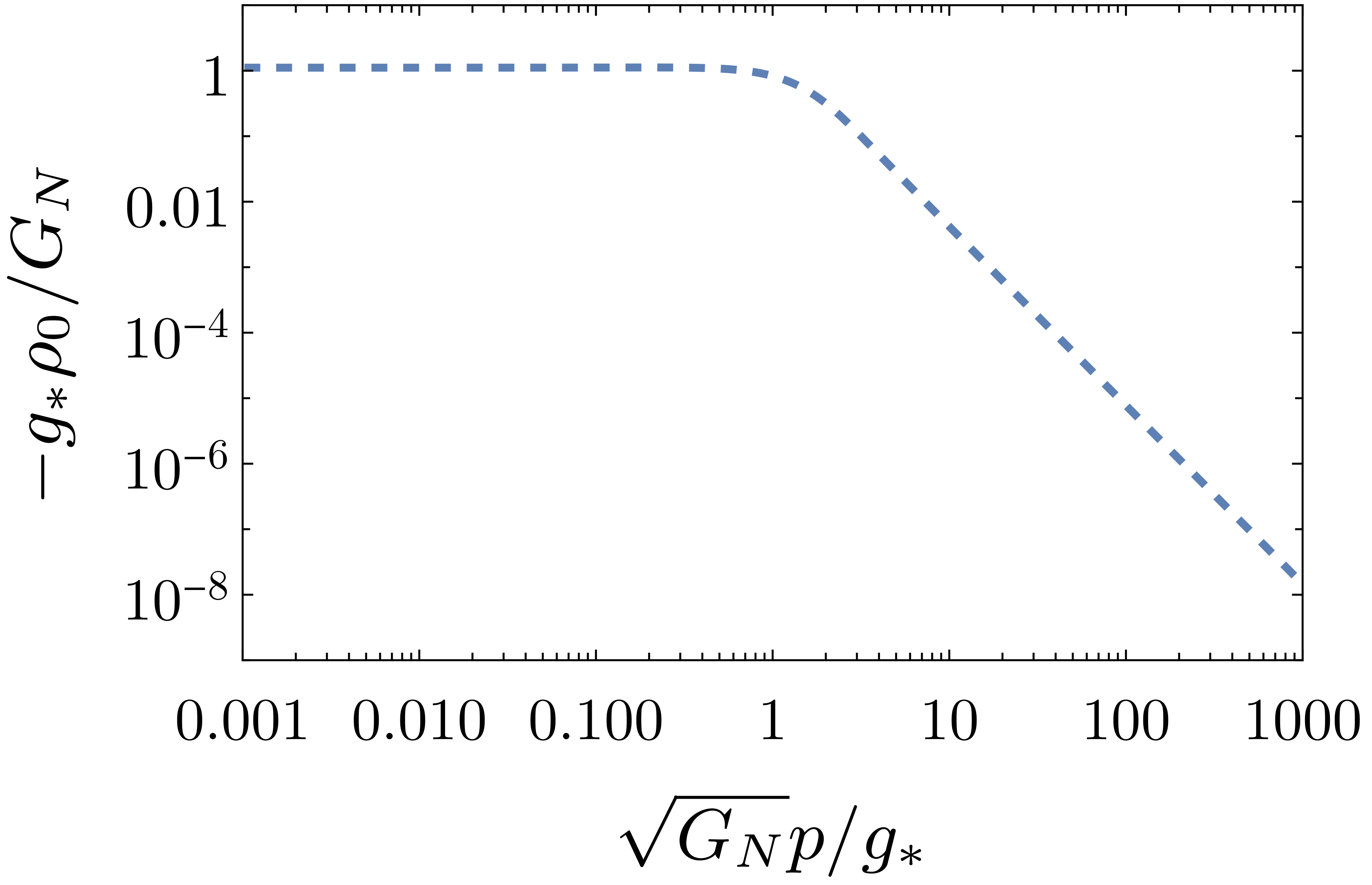

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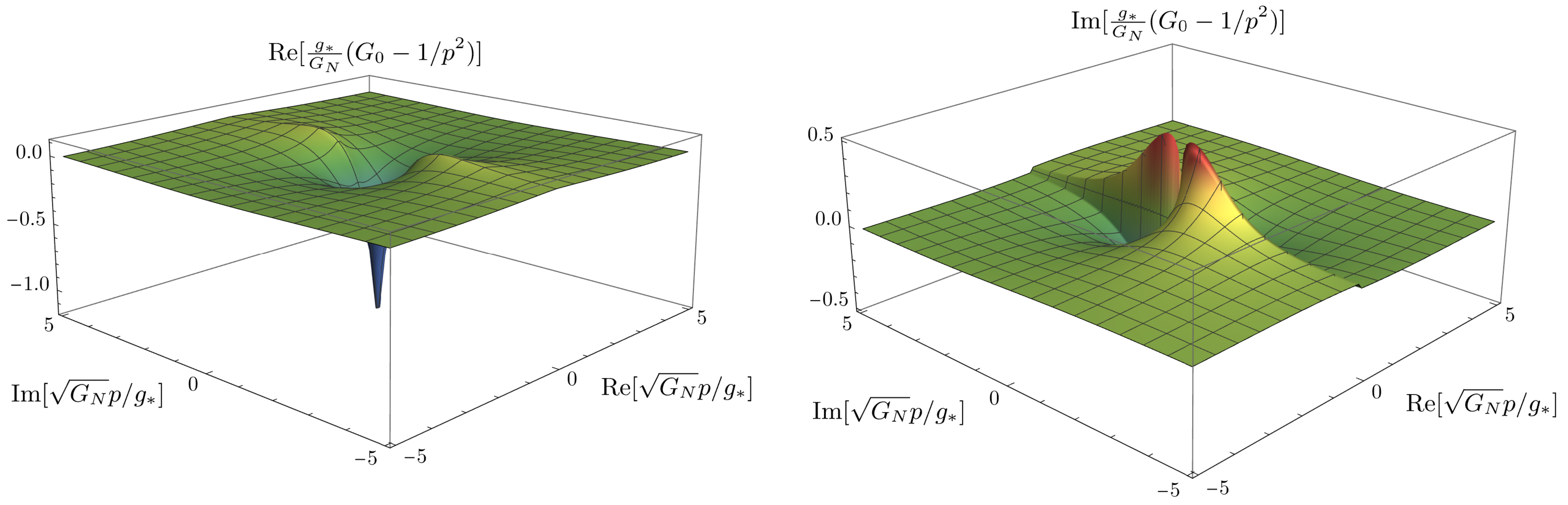

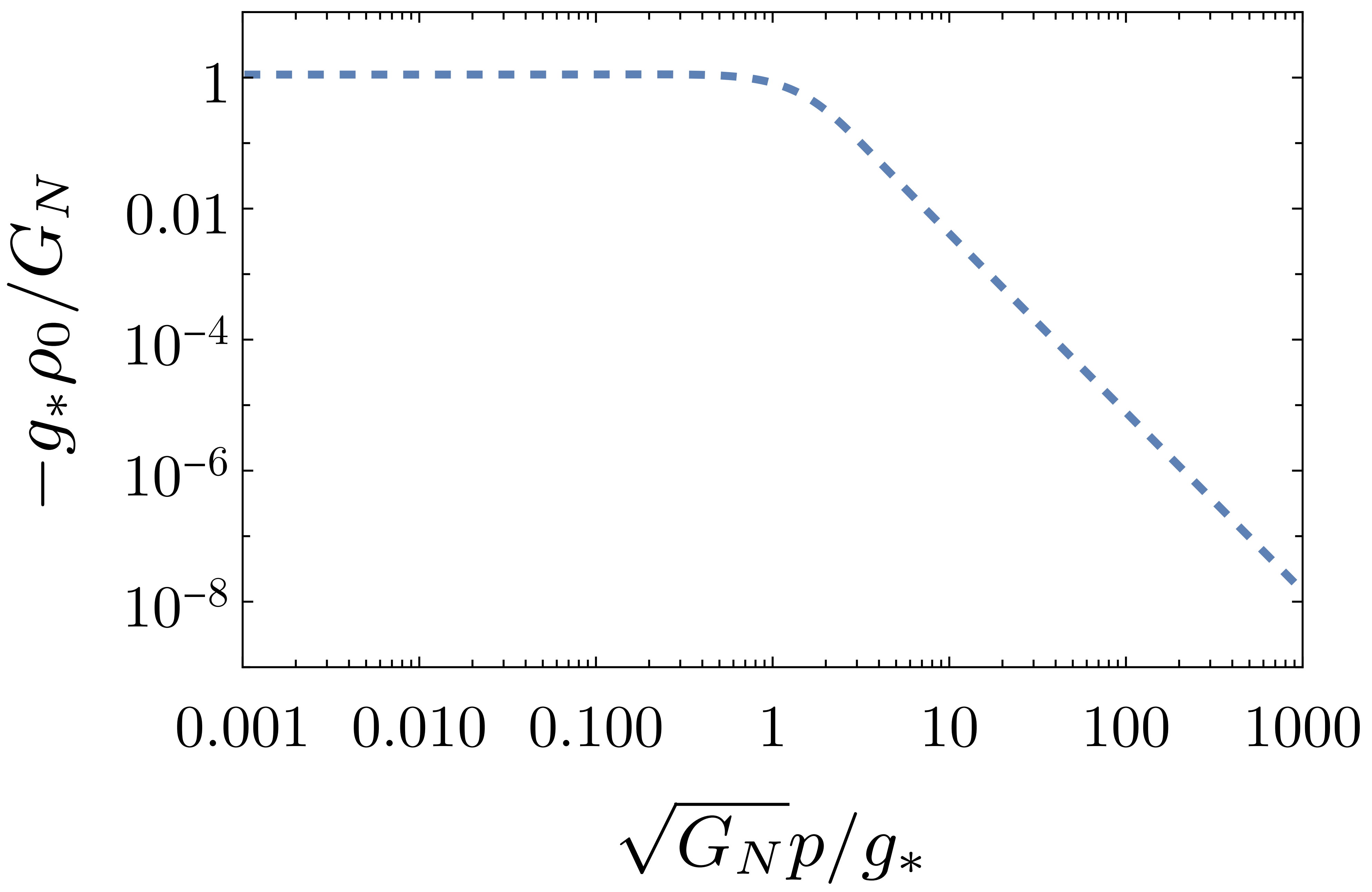

numerical story and details:
Gabriel Assant (Thu)

Assant, Litim, Reichert 2606.19321

qualitatively close to numerical results of
Assant, Litim, Reichert 2606.19321

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Analytic graviton propagator

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BK 2606.18343

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Platania, Wetterich 2009.06637

- analytic in upper half plane — related to causality

***analytic* evidence that AS might be unitary and causal!**

BK 2606.18343

Graviton-scalar-scalar vertex

- vertex in Minkowski background:

BK, T. Eckert WIP

Graviton-scalar-scalar vertex

- vertex in Minkowski background:

$$\Gamma_k|_{(h\phi\phi)} = \sqrt{32\pi G_N}\int \mathrm{d}^4x \sqrt{Z_h(-\partial_1^2)Z_\phi(-\partial_2^2)Z_\phi(-\partial_3^2)}\ \Bigg[\frac{1}{2}F_g(-\partial_1^2, -\partial_2^2, -\partial_3^2)h\phi_1\phi_2$$

$$-\frac{1}{2}F_{\partial\partial\phi}(-\partial_1^2, -\partial_2^2, -\partial_3^2)h^{\mu\nu}(\partial_\mu\partial_\nu\phi_1)\phi_2 - \frac{1}{2}F_{\partial\phi\partial\phi}(-\partial_1^2, -\partial_2^2, -\partial_3^2)h^{\mu\nu}(\partial_\mu\phi_1)(\partial_\nu\phi_2)\Bigg]$$

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worried about diffeomorphism invariance?
visit Paul P. Sprenger at his poster!

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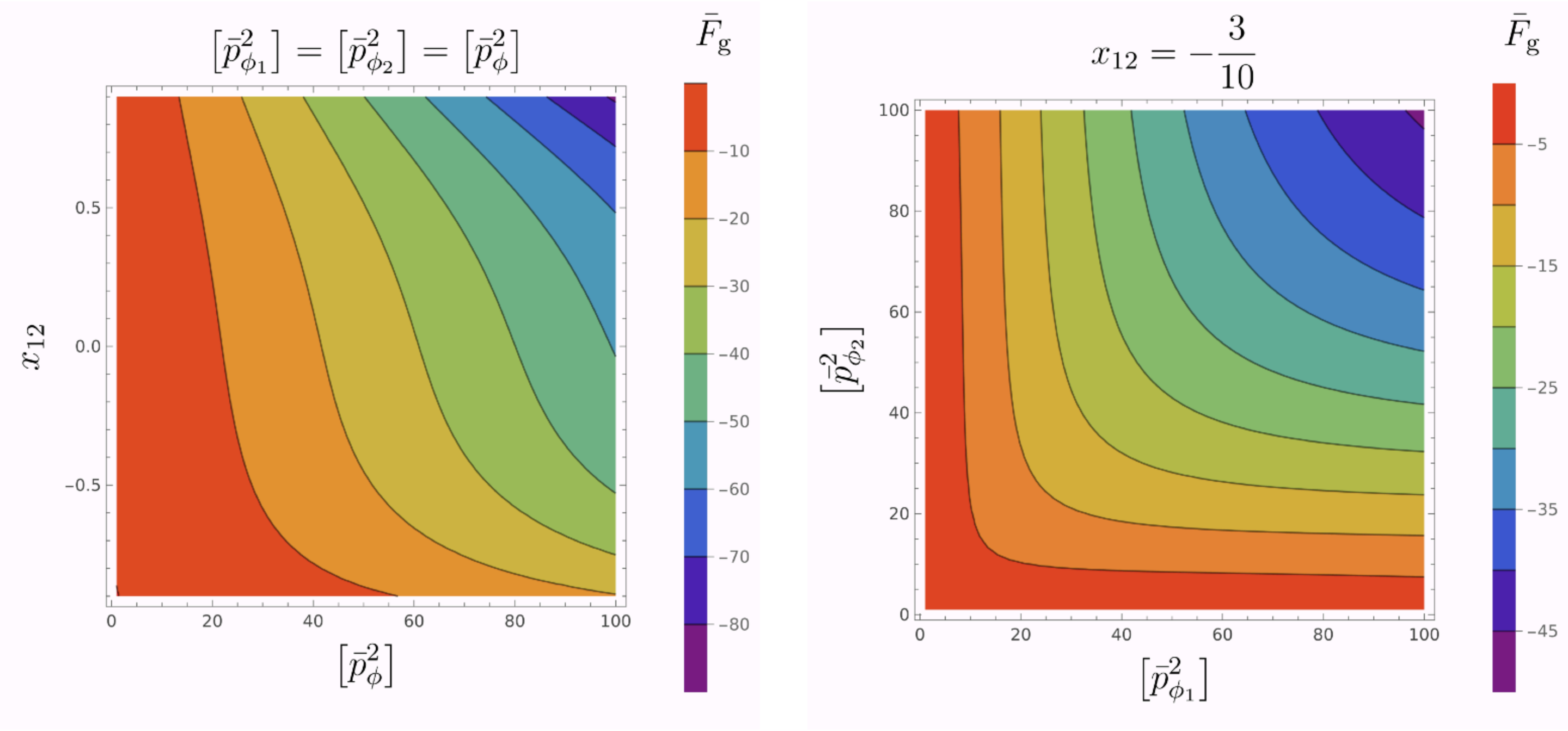
(Almost) analytic graviton-scalar-scalar vertex



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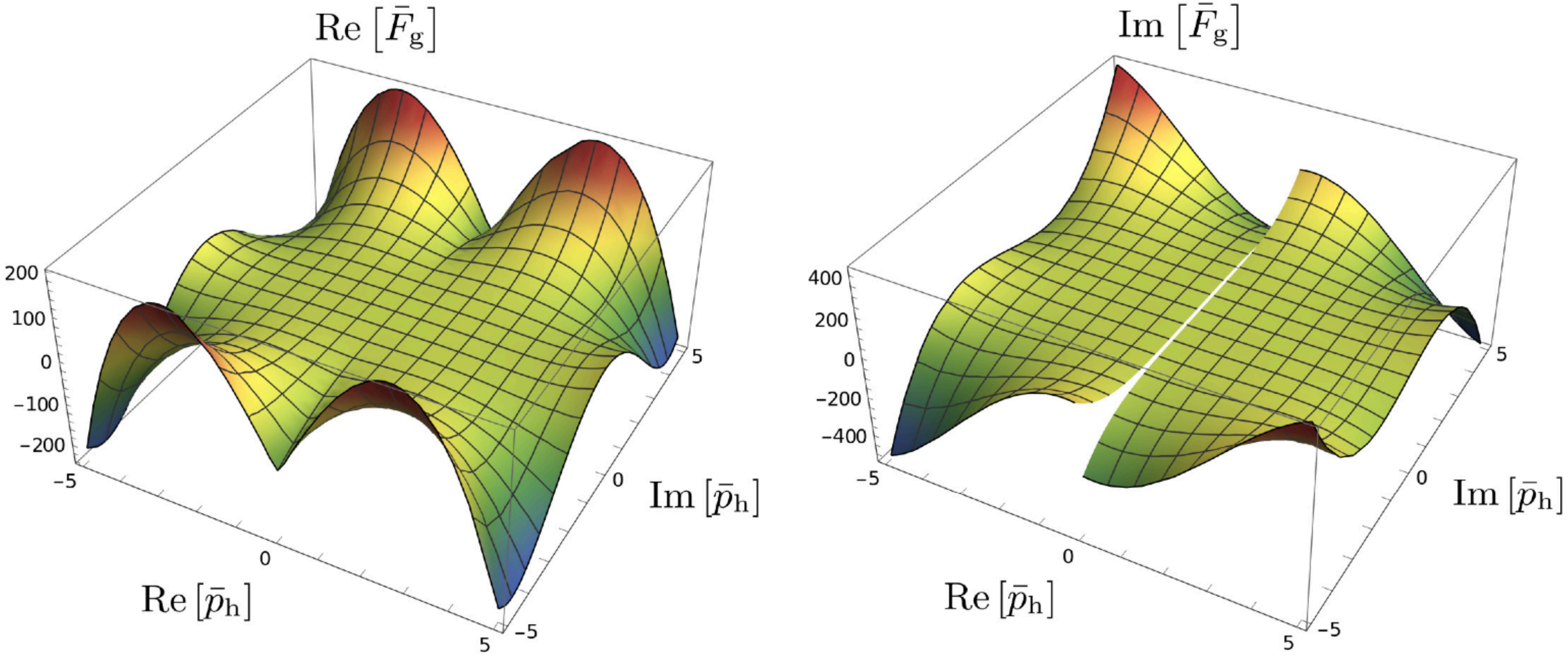

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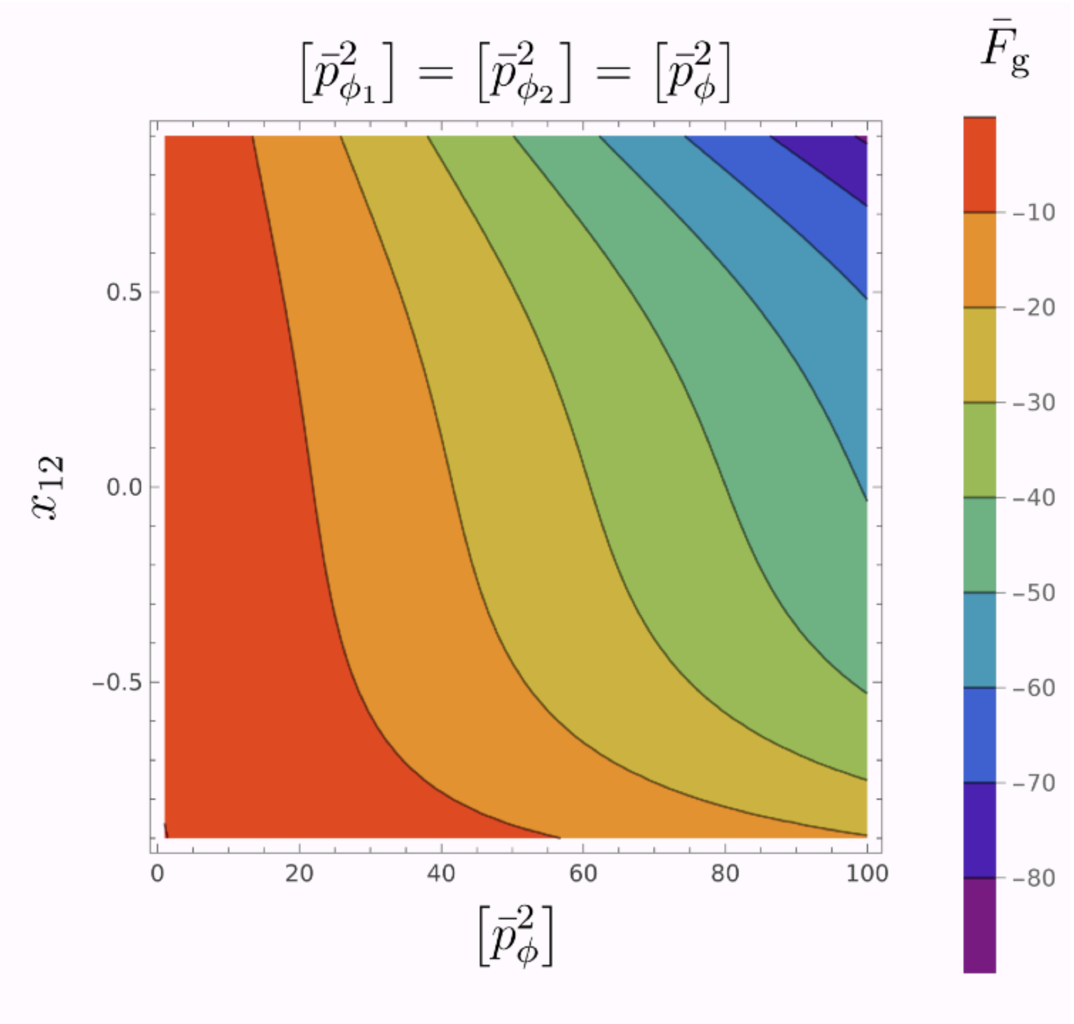

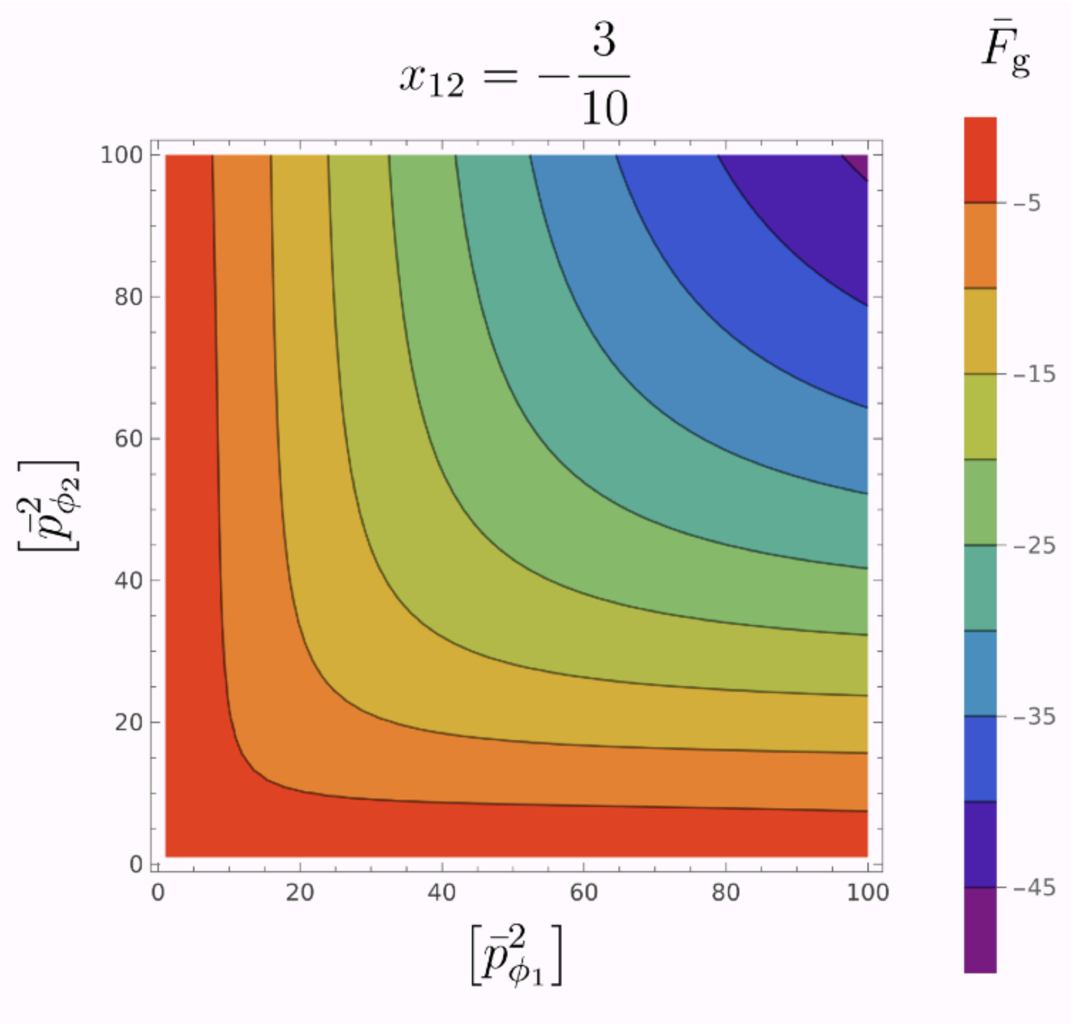

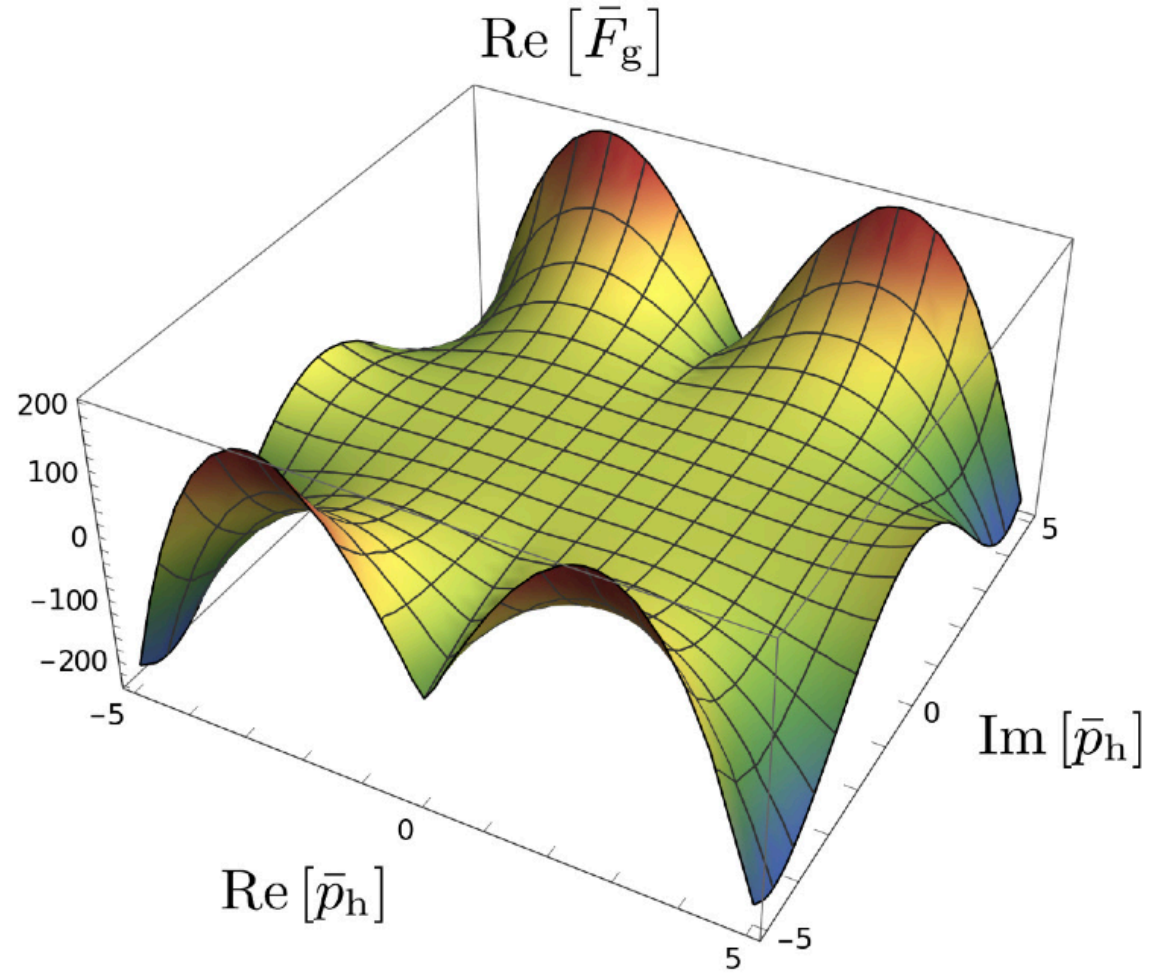

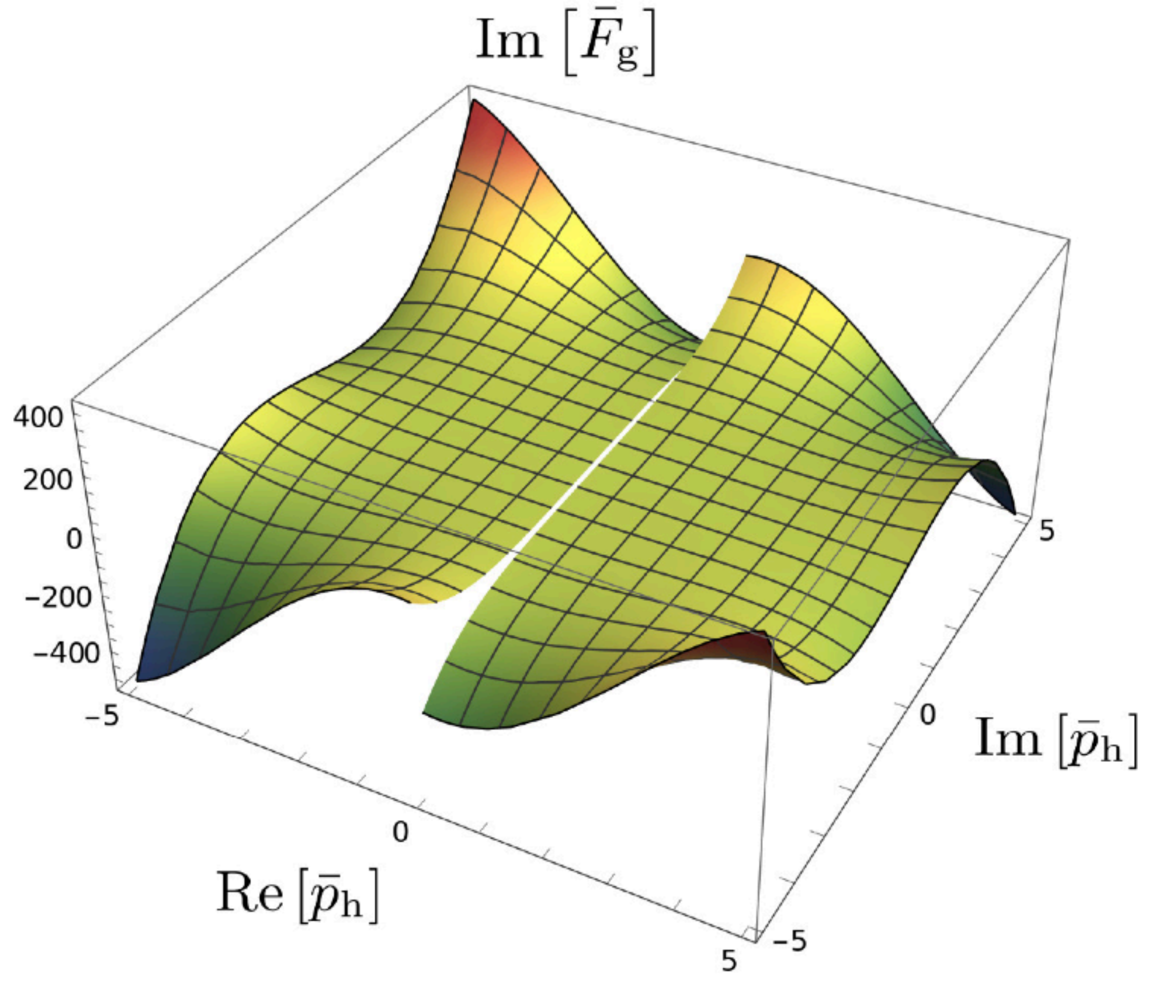

⇒ ask me about Weinberg’s soft theorem, positivity bounds, and other infrared shenanigans

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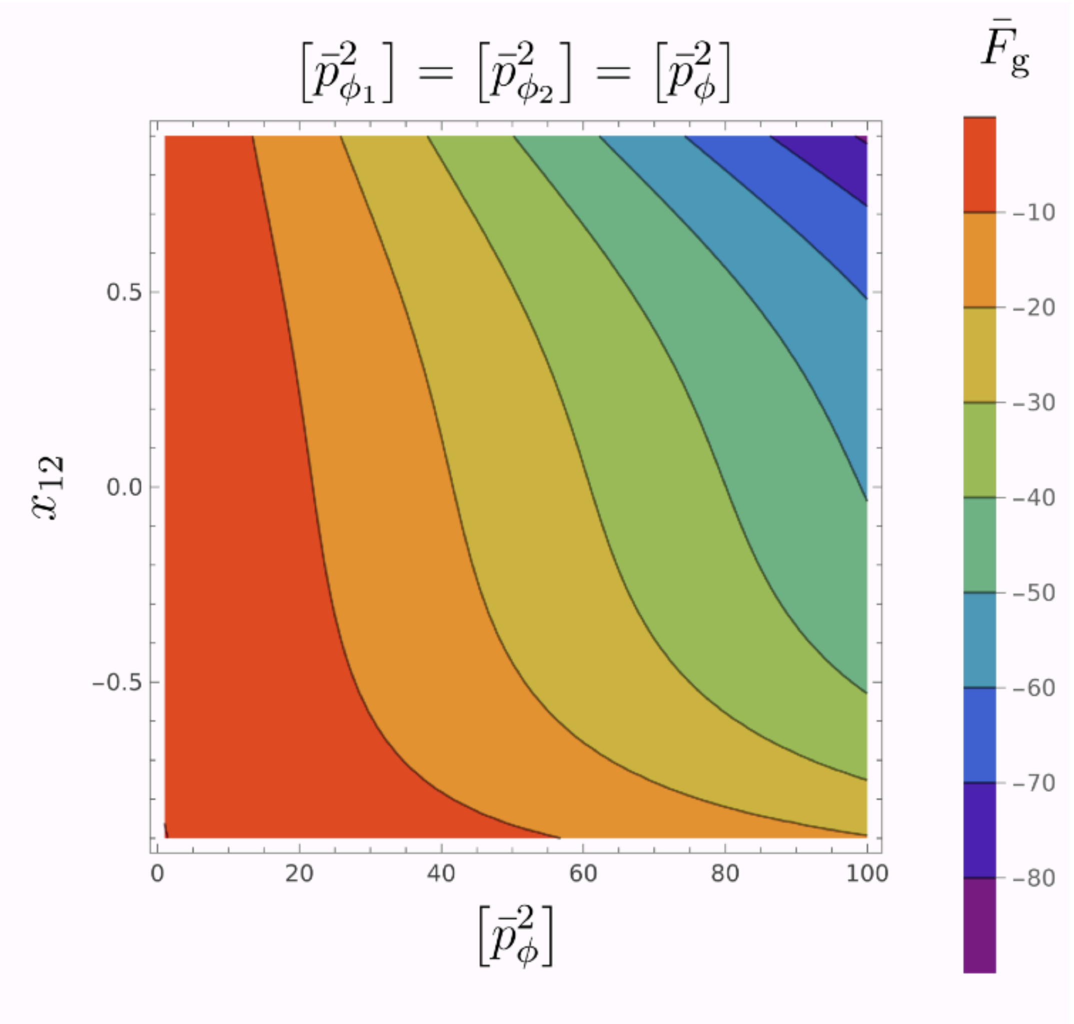

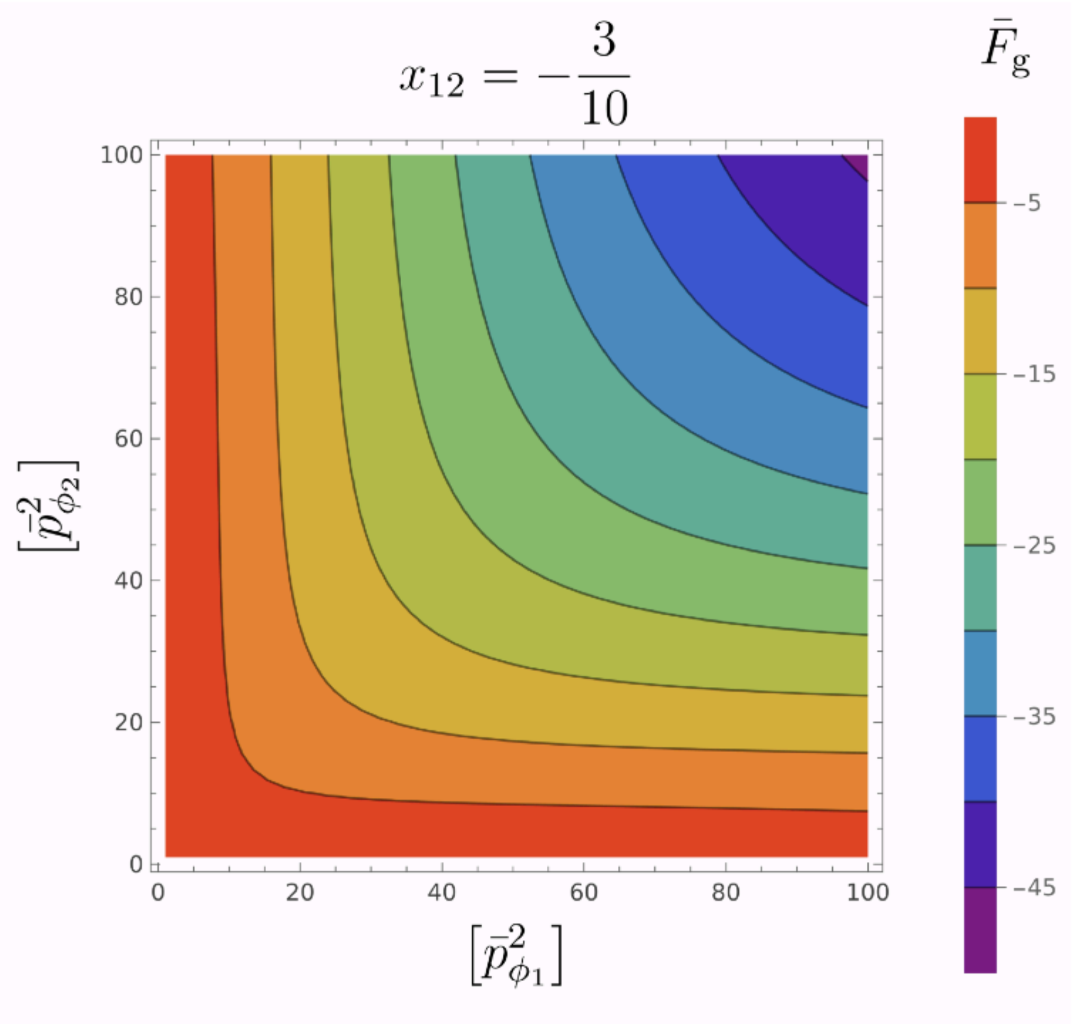

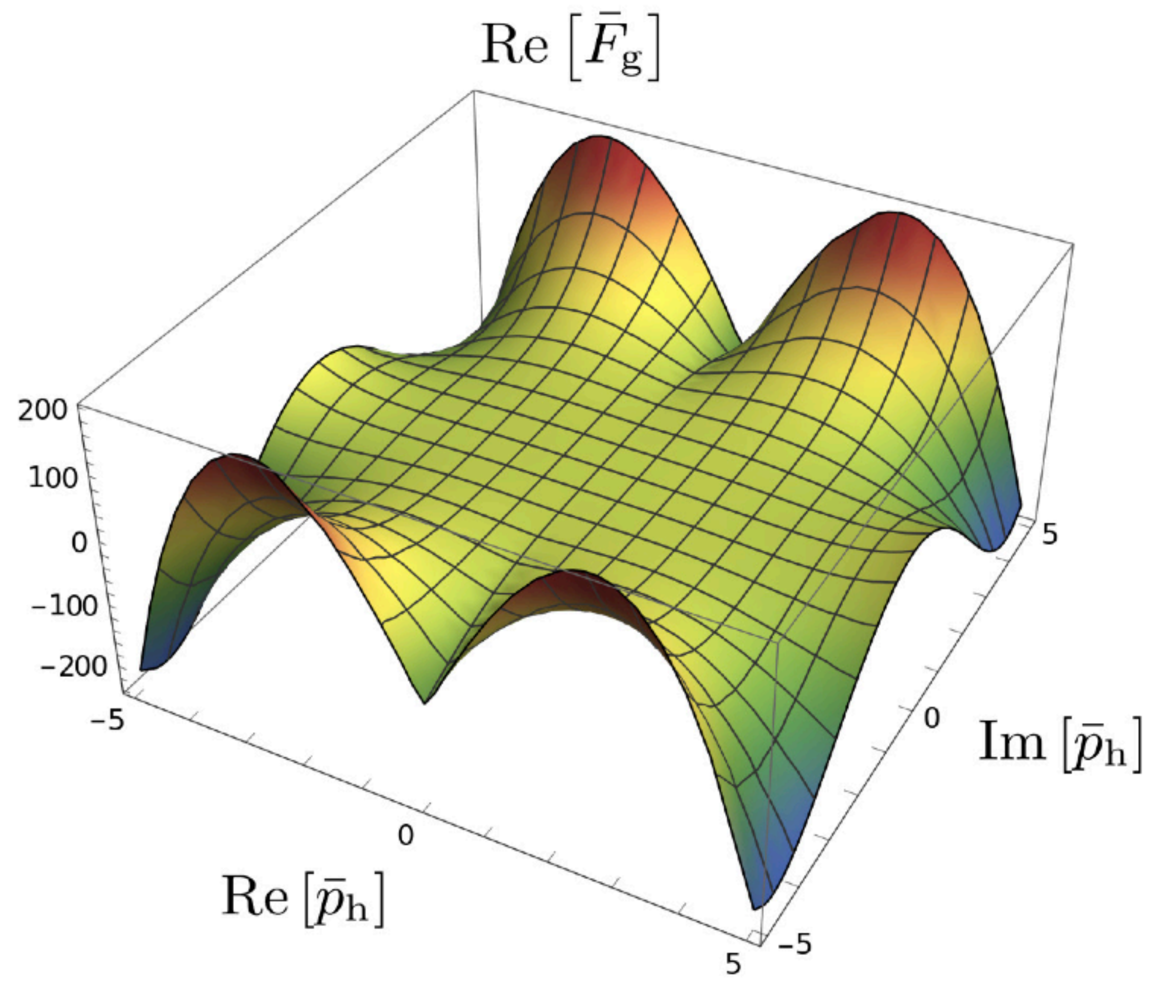

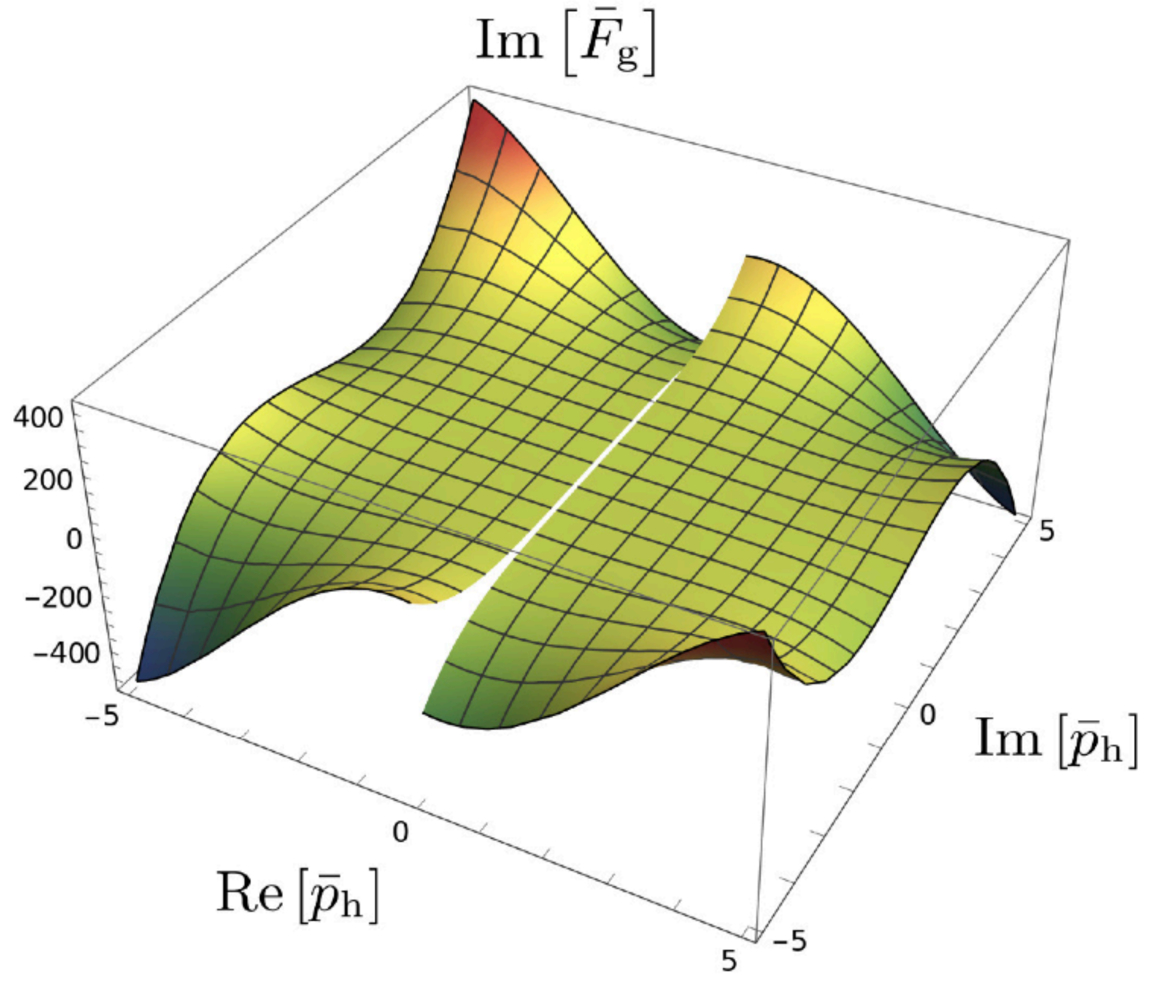

numerical story:
Angelo Chiesa (Thu)

Chiesa, Pawlowski, Reichert 2603.10168

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- numerical Wick rotation is painful (I heard)

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- analytic input as guidance for even more sophisticated numerical computations:
  - learn about complex-analytic structure: branch cuts, poles, …
  - learn which analytic continuations work, and which do not

Summary

- existence of fixed point in ASQG is settled (IMO)
- hope to answer ***soon***: is it unitary, causal, …
- computing graviton correlation functions and scattering amplitudes can help answering these questions
- analytic approximations give guidance on complex-analytic features of correlation functions

some gory details

there is waaaay more…

Analytic graviton propagator

- results of *BK, Schiffer 2105.04566*

$$\eta(p^2) = -\,k\partial_k \ln Z_k(p^2)$$

  - 4 integral equations for momentum-dependent anomalous dimensions

$$\eta(p^2) = f(p^2) + I[\eta](p^2)$$

approximation 1: neglect $I[\eta]$

  - integrate along RG trajectory to get wave function renormalisation = propagator dressing

approximation 2: model flow of gravitational coupling

$$g_k = \frac{G_N k^2}{1 + \frac{G_N k^2}{g_*}}$$

Bonanno, Reuter hep-th/0002196
Fehre, Litim, Pawlowski, Reichert 2111.13232

BK 2606.18343

Analytic anomalous dimensions

$$\eta_2 = g_k \sum_{n=0}^{4} \eta_2^{(i)} \tilde{\beta}^i \,, \qquad \eta_0 = g_k \sum_{n=0}^{4} \eta_0^{(i)} \tilde{\beta}^i \,, \qquad \eta_{c^T} = g_k \sum_{n=0}^{3} \eta_{c^T}^{(i)} \tilde{\beta}^i \,, \qquad \eta_{c^L} = g_k \sum_{n=0}^{4} \eta_{c^L}^{(i)} \tilde{\beta}^i$$

$$\eta_2^{(0)} = \frac{\left(p^6 + 27p^4 + 142p^2 + 58\right) p^4 \Gamma\left(0, \frac{p^2}{2}\right)}{240\pi} - \frac{\left(p^6 + 27p^4 + 135p^2 + 29\right) p^4 \Gamma\left(0, p^2\right)}{120\pi}$$
$$- \frac{255p^6 - 1093p^4 + 1424p^2 + 160}{20\pi p^8} + \frac{e^{-p^2}\left(p^{14} + 26p^{12} + 110p^{10} - 58p^8 + 103p^6 + 1936p^4 + 2706p^2 - 8544\right)}{120\pi p^6}$$
$$- \frac{e^{-\frac{p^2}{2}}\left(p^{16} + 25p^{14} + 96p^{12} - 58p^{10} - 372p^8 + 176p^6 + 9144p^4 - 17568p^2 - 960\right)}{120\pi p^8}$$

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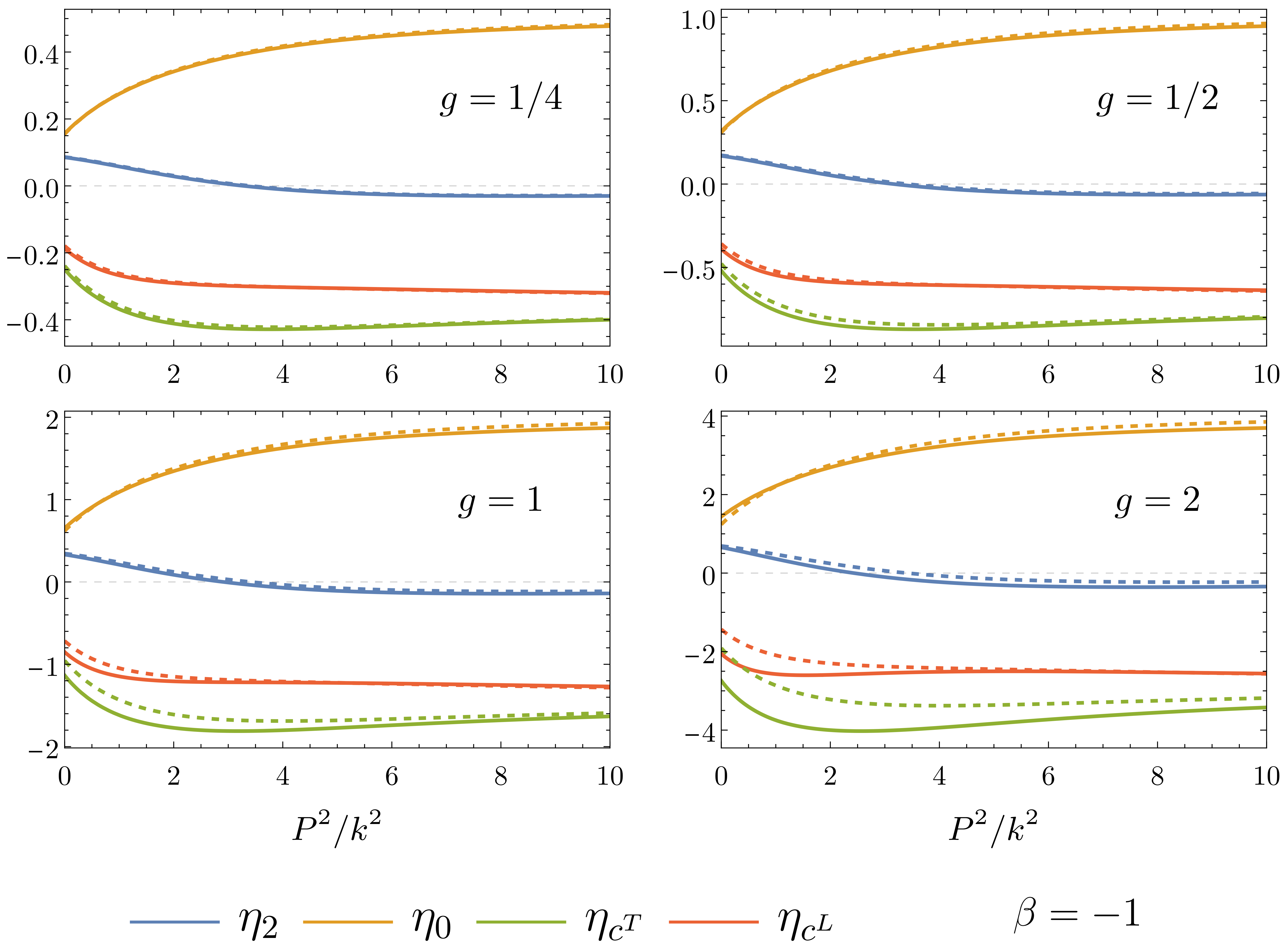
Analytic anomalous dimensions



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Three-point function

- scattering amplitudes that include massless fluctuations are tricky
- soft divergences — Weinberg soft theorem
- we get the correct doubly-eikonal soft logarithms

$$\Gamma^{(3)}_{\text{soft}} \sim \Gamma^{(3)}_{\text{kin}} \times \# \times \ln \frac{p^2_{\phi_1}}{p^2_h} \times \ln \frac{p^2_{\phi_2}}{p^2_h} + \ldots$$

we get it right — tensor structure, logarithms, AND prefactor!

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Other insights

- derivative expansion works well at FP, but fails qualitatively in IR

$$\int_0^\infty \mathrm{d}x \, \frac{e^{-x}}{x+a} = e^a \Gamma(0,a) \sim -\ln a - \gamma$$

- RGI can fail miserably