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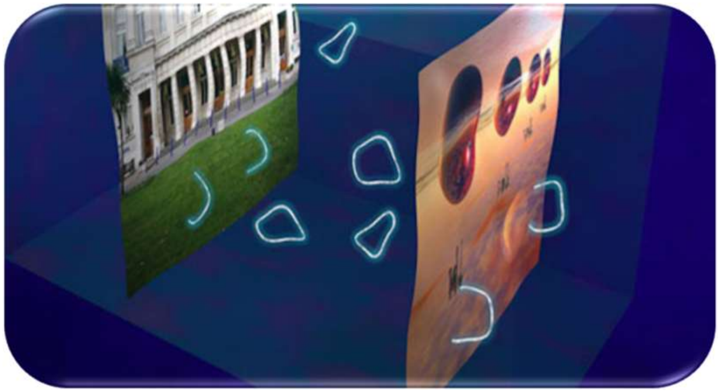

Twisting and reducing the 6d $\mathcal{N}$=(2,0) superconformal bootstrap

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Based on [2506.08560], [2601.15407], [2604.08673],

+ WIP w/ Francesco Aprile and Hector Puerta-Ramisa

6d $\mathcal{N}$=(2,0) theories

[Witten, '95]

The **6d $\mathcal{N}$=(2,0) theories** are strongly interacting SCFTs of maximal dimension.

Their string theory construction **classifies** them by Lie algebra $\mathfrak{g} = \{A_{N\geq 1}, D_{N\geq 4}, E_6, E_7, E_8\}$. They have central charge (6d anomaly polynomial)

$$c_{\mathfrak{g}} = 4d_{\mathfrak{g}}h_{\mathfrak{g}}^{\vee} + r_{\mathfrak{g}} = \begin{cases} 4N^3 - 3N - 1, & \mathfrak{g} = A_{N-1} \\ 16N^3 - 24N^2 + 9N, & \mathfrak{g} = D_N \\ \{3750, 9583, 29768\}, & \mathfrak{g} = \{E_6, E_7, E_8\} \end{cases}$$

Their non-Lagrangian nature make them **inaccessible** using traditional field theory techniques.

And yet, they play a central role by

- **organizing dualities** among QFTs in $d < 6$
- describing the worldvolume of M5 branes, providing a **window into M-theory**

Outline

1. Setup: 4-pt. functions of ½-BPS operators
2. Reduced blocks at next-to-next-to-extremality — 2601.15407, 2604.08673
3. The $\mathcal{W}$-algebra bootstrap of 6d $\mathcal{N}$=(2,0) theories — 2506.08560
4. Application: Composite operators — WIP
5. Summary

6d $\mathcal{N}$=(2,0) theories as CFTs

Despite their mystique, (2,0) theories are CFTs with **local operators** $\{\boldsymbol{\mathcal{O}_i}\}$ with **CFT data** $\{\boldsymbol{\Delta_i, \lambda_{ijk}}\}$**.**

We can study them using the **conformal bootstrap**, which constrains $\{\Delta_i, \lambda_{ijk}\}$ by imposing **crossing symmetry** on 4-pt. functions

$$\langle \mathcal{O}_{k_1}(x_1,t_1)\mathcal{O}_{k_2}(x_2,t_2)\mathcal{O}_{k_3}(x_3,t_3)\mathcal{O}_{k_4}(x_4,t_4)\rangle = \begin{pmatrix}\text{kinematic}\\ \text{factors}\end{pmatrix}\mathcal{G}^{\{k_i\}}(z,\bar{z};\alpha,\bar{\alpha})$$

of scalar operators $\mathcal{O}_k(x,t)$, with $x_i \in \mathbb{R}^6$ and t_i a null $\mathfrak{so}(5)_R$ vector.

- Take $\mathcal{O}_k$ to be ½-BPS superconformal primaries (SCPs).

This is a still a **very hard problem.**

- We should supplement it with any **extra structure** and **exact results** $\mathcal{N} = (2,0)$ supersymmetry can buy
- Lacking a Lagrangian, we **don't** have localization in 6d.

4-pt. functions of ½-BPS operators

Superconformal (SC) symmetry allows us to factorize 4-pt. functions as

$$\left\langle \mathcal{O}_{k_1}(x_1,t_1)\mathcal{O}_{k_2}(x_2,t_2)\mathcal{O}_{k_3}(x_3,t_3)\mathcal{O}_{k_4}(x_4,t_4)\right\rangle = \begin{pmatrix}\text{kinematic}\\ \text{factors}\end{pmatrix} \mathcal{G}^{\{k_i\}}(z,\bar{z};\alpha,\bar{\alpha})$$

$z,\bar{z}$ and $\alpha,\bar{\alpha}$ relate to $\mathfrak{so}(6,2)$ and $\mathfrak{so}(5)_R$ cross-ratios built from $x_i \in \mathbb{R}^6$ and null $\mathfrak{so}(5)_R$ vectors t_i.

Supersymmetry relates conformal multiplets and packages them into superconformal multiplets $\mathcal{X}$.

$\mathcal{G}^{\{k_i\}}$ satisfies SC Ward identities (SCWIs), yielding the superblock decomposition

$$\mathcal{G}^{\{k_i\}}(z,\bar{z};\alpha,\bar{\alpha}) = \sum_{\mathcal{X}} \lambda_{k_1k_2\mathcal{X}}\, \lambda_{k_3k_4\mathcal{X}}\, \mathfrak{G}_{\mathcal{X}}^{k_{12},k_{34}}(z,\bar{z};\alpha,\bar{\alpha})$$

where we sum superblocks $\mathfrak{G}_{\mathcal{X}}^{k_{12},k_{34}}$ over supermultiplets $\mathcal{X}$ in the OPEs $\mathcal{O}_{k_1}\times\mathcal{O}_{k_2} \cap \mathcal{O}_{k_3}\times\mathcal{O}_{k_4}$

DGS solution to the superconformal Ward identities

[Dolan, Gallot, Sokatchev, '04]

The SCWIs in $d = 2(\varepsilon + 1)$ can also be solved with

$$\mathcal{G}^{\{k_i\}}(z,\bar{z};\alpha,\bar{\alpha}) = \mathcal{F}^{\{k_i\}}(z,\bar{z};\alpha,\bar{\alpha}) + \Upsilon^{(\varepsilon)} \circ \mathcal{T}^{\{k_i\}}(z,\bar{z};\alpha,\bar{\alpha})$$

protected contribution

reduced correlator

$2(\varepsilon - 1)^{\text{nd}}$ order operator

2506.08560

2601.15407
2604.08673

Reduced correlator and reduced blocks

[Dolan, Gallot, Sokatchev, '04]

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$$\mathcal{G}^{\{k_i\}}(z,\bar{z};\alpha,\bar{\alpha}) = \mathcal{F}^{\{k_i\}}(z,\bar{z};\alpha,\bar{\alpha}) + \Upsilon^{(\varepsilon)} \circ \mathcal{T}^{\{k_i\}}(z,\bar{z};\alpha,\bar{\alpha})$$

$\mathcal{T}^{\{k_i\}}$ encodes $\mathcal{G}^{\{k_i\}}$ in a much simpler function $\rightarrow$ we should try to bootstrap it directly!

At next-to-next-to-extremality (NNE), e.g. $k_4 = k_1 + k_2 + k_3 - 4$, we have $\mathcal{T}^{\{k_i\}} = \mathcal{T}^{\{k_i\}}(z,\bar{z})$ and $\Upsilon^{(\varepsilon)}$ simplifies to

$$\Upsilon^{(\varepsilon)} = (z\bar{z})^{2\varepsilon}\Delta_\varepsilon(z\alpha-1)(z\bar{\alpha}-1)(\bar{z}\alpha-1)(\bar{z}\bar{\alpha}-1)$$

in terms of $\Delta_\varepsilon = (D_\varepsilon)^{\varepsilon-1}(z\bar{z})^{\varepsilon-1}$ with $D_\varepsilon = \partial_z\partial_{\bar{z}} - \frac{\varepsilon}{z-\bar{z}}(\partial_z - \partial_{\bar{z}})$

- Δ_ε is **trivial** in 4d, a **2nd order operator** in 6d, and **non-local** in 3d and 5d.

Reduced correlator and reduced blocks

earlier studies of $\mathcal{G}^{2222}$ in
[Dolan, Gallot, Sokatchev, '04]
[Chester, Lee, Pufu, Yacoby, '14]

We can compare the reduced correlator and superblock decompositions

$$\mathcal{G}^{\{k_i\}} = \mathcal{F}^{\{k_i\}} + \Upsilon^{(\varepsilon)} \circ \mathcal{T}^{\{k_i\}} = \sum_{\mathcal{X}} \lambda_{k_1 k_2 \mathcal{X}} \lambda_{k_3 k_4 \mathcal{X}} \, \mathfrak{G}_{\mathcal{X}}^{k_{12},k_{34}} = \sum_{m,n} Y_{m,n}^{\{k_i\}} \, A_{m,n}^{\{k_i\}}$$

to see how $\mathcal{T}^{\{k_i\}}$ contributes to each R-symmetry channel. The channel functions $A_{m,n}^{\{k_i\}}$ involve

$$\boxed{A_{m,m}^{\{k_i\}} = \frac{1}{\mathcal{Y}_m^{\{k_i\}}} U^{2\varepsilon} \Delta_\varepsilon \left[\frac{\mathcal{T}^{\{k_i\}}}{U^{3(\varepsilon-1)}} \right],}$$

$$U = z\,\bar{z}, \qquad V = (1-z)(1-\bar{z})$$

$$A_{m,m-1}^{\{k_i\}} = \frac{1}{\mathcal{Y}_{m-1}^{\{k_i\}}} U^{2\varepsilon} \Delta_\varepsilon \left[\left(\frac{\Delta_{12}}{\Delta_{34} - 4\varepsilon} + \frac{V-1}{U} \right) \frac{\mathcal{T}^{\{k_i\}}}{U^{3(\varepsilon-1)}} \right]$$

$$A_{m,m-2}^{\{k_i\}} = U^{2\varepsilon} \Delta_\varepsilon \left[\left(\frac{(\Delta_{12} - \Delta_{34} + 2\varepsilon)(\Delta_{12} + \Delta_{34} - 2\varepsilon)}{4(\Delta_{34} - 2\varepsilon)(\Delta_{34} - 3\varepsilon)} + \frac{\Delta_{12}}{2(\Delta_{34} - 2\varepsilon)} \frac{V-1}{U} + \frac{V+1}{2U} \right) \frac{\mathcal{T}^{\{k_i\}}}{U^{3(\varepsilon-1)}} \right],$$

+ more complicated for $A_{m-1,m-1}^{\{k_i\}}, A_{m-1,m-2}^{\{k_i\}}, A_{m-2,m-2}^{\{k_i\}}$.

Reduced correlator and reduced blocks

For a single superconformal multiplet $\mathcal{X}$, this gives the equations involving a **reduced block** $\mathcal{T}^{\{k_i\}}_{\Delta,\ell}$

$$\boxed{A^{\{k_i\};\mathcal{X}}_{m,m} = \frac{1}{\mathcal{Y}^{\{k_i\}}_{m}} U^{2\varepsilon} \Delta_\varepsilon \left[\frac{\mathcal{T}^{\{k_i\}}_{\Delta,\ell}}{U^{3(\varepsilon-1)}}\right] = U^{\frac{\Delta_{34}}{2}} \mathcal{C}^{\{k_i\}}_{\Delta\prime,\ell\prime;m,m} G^{\Delta_{12},\Delta_{34}}_{\Delta\prime,\ell\prime}\,,}$$

$$A^{\{k_i\};\mathcal{X}}_{m,m-1} = \frac{1}{\mathcal{Y}^{\{k_i\}}_{m-1}} U^{2\varepsilon} \Delta_\varepsilon \left[\left(\frac{\Delta_{12}}{\Delta_{34} - 4\varepsilon} + \frac{V-1}{U}\right)\frac{\mathcal{T}^{\{k_i\}}_{\Delta,\ell}}{U^{3(\varepsilon-1)}}\right] = U^{\frac{\Delta_{34}}{2}} \sum_{\Delta,\ell} \mathcal{C}^{\{k_i\}}_{\Delta,\ell;m,m-1} G^{\Delta_{12},\Delta_{34}}_{\Delta,\ell}$$

$$A^{\{k_i\};\mathcal{X}}_{m,m-2} = U^{2\varepsilon} \Delta_\varepsilon \left[\left(\frac{(\Delta_{12} - \Delta_{34} + 2\varepsilon)(\Delta_{12} + \Delta_{34} - 2\varepsilon)}{4(\Delta_{34} - 2\varepsilon)(\Delta_{34} - 3\varepsilon)} + \frac{\Delta_{12}}{2(\Delta_{34} - 2\varepsilon)}\frac{V-1}{U} + \frac{V+1}{2U}\right)\frac{\mathcal{T}^{\{k_i\}}_{\Delta,\ell}}{U^{3(\varepsilon-1)}}\right] = U^{\frac{\Delta_{34}}{2}} \sum_{\Delta,\ell} \mathcal{C}^{\{k_i\}}_{\Delta,\ell;m,m-2}\, G^{\Delta_{12},\Delta_{34}}_{\Delta,\ell}\,,$$

$$A^{\{k_i\}}_{m-1,m-1} = (\dots), \qquad A^{\{k_i\}}_{m-1,m-2} = (\dots), \qquad A^{\{k_i\}}_{m-2,m-2} = (\dots).$$

Example: $\langle \mathcal{O}_2 \mathcal{O}_2 \mathcal{O}_2 \mathcal{O}_2 \rangle$

earlier studies of $\mathcal{G}^{2222}$ in
[Dolan, Gallot, Sokatchev, '04]
[Chester, Lee, Pufu, Yacoby, '14]

For a single superconformal multiplet $\mathcal{X}$, this gives the equations involving a **reduced block** $\mathcal{T}_{\Delta,\ell}^{\{k_i\}}$:

$$A_{2,2}^{\{2\};\mathcal{X}} = \frac{1}{6} U^{2\varepsilon} \Delta_\varepsilon \left[\frac{\mathcal{T}_{\Delta,\ell}^{\{k_i\}}}{U^{3(\varepsilon-1)}} \right] = \mathcal{C}_{\Delta',\ell';m,m}^{\{k_i\}} G_{\Delta',\ell'}^{0,0} ,$$

$$A_{2,1}^{\{2\};\mathcal{X}} = \frac{1}{2} U^{2\varepsilon} \Delta_\varepsilon \left[\left(\frac{V-1}{U} \right) \frac{\mathcal{T}_{\Delta,\ell}^{\{k_i\}}}{U^{3(\varepsilon-1)}} \right] = \sum_{\Delta,\ell} \mathcal{C}_{\Delta,\ell;m,m-1}^{\{k_i\}} G_{\Delta,\ell}^{0,0}$$

$$A_{2,0}^{\{2\};\mathcal{X}} = U^{2\varepsilon} \Delta_\varepsilon \left[\left(-\frac{1}{6} + \frac{V+1}{2U} \right) \frac{\mathcal{T}_{\Delta,\ell}^{\{k_i\}}}{U^{3(\varepsilon-1)}} \right] = \sum_{\Delta,\ell} \mathcal{C}_{\Delta,\ell;m,m-2}^{\{k_i\}} G_{\Delta,\ell}^{0,0} ,$$

$$A_{1,1}^{\{2\}} = (\dots), \qquad A_{1,0}^{\{2\}} = (\dots), \qquad A_{0,0}^{\{2\}} = (\dots).$$

Example: $\langle \mathcal{O}_2 \mathcal{O}_2 \mathcal{O}_2 \mathcal{O}_2 \rangle$ in 4d $\mathcal{N} = 4$

For a single superconformal multiplet $\mathcal{X}$, this gives the equations involving a **reduced block** $\mathcal{T}_{\Delta,\ell}^{\{k_i\}}$:

$$\boxed{A_{\mathbf{105}}^{\{2\};\mathcal{X}} = \frac{1}{6} U^2 \, \mathcal{T}_{\Delta,\ell}^{\{k_i\}} = \mathcal{C}_{\Delta',\ell';m,m}^{\{k_i\}} G_{\Delta',\ell'}^{0,0} \,,}$$

$$A_{\mathbf{175}}^{\{2\};\mathcal{X}} = \frac{1}{2} U^2 \frac{V-1}{U} \mathcal{T}_{\Delta,\ell}^{\{k_i\}} = \sum_{\Delta,\ell} \mathcal{C}_{\Delta,\ell;m,m-1}^{\{k_i\}} G_{\Delta,\ell}^{0,0}$$

$$A_{\mathbf{84}}^{\{2\};\mathcal{X}} = U^2 \left(-\frac{1}{6} + \frac{V+1}{2U} \right) \mathcal{T}_{\Delta,\ell}^{\{k_i\}} = \sum_{\Delta,\ell} \mathcal{C}_{\Delta,\ell;m,m-2}^{\{k_i\}} \, G_{\Delta,\ell}^{0,0} \,,$$

$$A_{\mathbf{20}}^{\{2\}} = (\ldots), \qquad A_{\mathbf{15}}^{\{2\}} = (\ldots), \qquad A_{\mathbf{1}}^{\{2\}} = (\ldots).$$

It was shown in **[Dolan, Osborn, '02]** that the **reduced block**

$$\mathcal{T}_{\Delta,\ell}^{\{k_i\}}(U,V) = U^{-2} G_{\Delta,\ell}^{0,0}(U,V)$$

Reduced correlator and reduced blocks

For a single superconformal multiplet $\mathcal{X}$, this gives the equations

$$\boxed{A_{m,m}^{\{k_i\};\mathcal{X}} = \frac{1}{\mathcal{Y}_m^{\{k_i\}}} U^{2\varepsilon} \Delta_\varepsilon \left[\frac{\mathcal{T}_{\Delta,\ell}^{\{k_i\}}}{U^{3(\varepsilon-1)}} \right] = U^{\frac{\Delta_{34}}{2}} \mathcal{C}_{\Delta',\ell';m,m}^{\{k_i\}} G_{\Delta',\ell'}^{\Delta_{12},\Delta_{34}} ,}$$

$$A_{m,m-1}^{\{k_i\};\mathcal{X}} = \frac{1}{\mathcal{Y}_{m-1}^{\{k_i\}}} U^{2\varepsilon} \Delta_\varepsilon \left[\left(\frac{\Delta_{12}}{\Delta_{34} - 4\varepsilon} + \frac{V-1}{U} \right) \frac{\mathcal{T}_{\Delta,\ell}^{\{k_i\}}}{U^{3(\varepsilon-1)}} \right] = U^{\frac{\Delta_{34}}{2}} \sum_{\Delta,\ell} \mathcal{C}_{\Delta,\ell;m,m-1}^{\{k_i\}} G_{\Delta,\ell}^{\Delta_{12},\Delta_{34}}$$

$$A_{m,m-2}^{\{k_i\};\mathcal{X}} = U^{2\varepsilon} \Delta_\varepsilon \left[\left(\frac{(\Delta_{12} - \Delta_{34} + 2\varepsilon)(\Delta_{12} + \Delta_{34} - 2\varepsilon)}{4(\Delta_{34} - 2\varepsilon)(\Delta_{34} - 3\varepsilon)} + \frac{\Delta_{12}}{2(\Delta_{34} - 2\varepsilon)} \frac{V-1}{U} + \frac{V+1}{2U} \right) \frac{\mathcal{T}_{\Delta,\ell}^{\{k_i\}}}{U^{3(\varepsilon-1)}} \right] = U^{\frac{\Delta_{34}}{2}} \sum_{\Delta,\ell} \mathcal{C}_{\Delta,\ell;m,m-2}^{\{k_i\}} G_{\Delta,\ell}^{\Delta_{12},\Delta_{34}} ,$$

$$A_{m-1,m-1}^{\{k_i\}} = (\ldots), \qquad A_{m-1,m-2}^{\{k_i\}} = (\ldots), \qquad A_{m-2,m-2}^{\{k_i\}} = (\ldots).$$

To extract **reduced block** $\mathcal{T}_{\Delta,\ell}^{\{k_i\}}$, expand $A_{m,m}^{\{k_i\};\mathcal{X}}$ in **Jack polynomials** $P_{a,b}^{(\varepsilon)}(z,\bar{z})$.

Reduced correlator and reduced blocks

Jack polynomials $P_{a,b}^{(\varepsilon)}$ are

1. symmetric functions in $z, \bar{z}$
2. eigenfunctions of Δ_ε

and we have e.g.

$$G_{\Delta,\ell}^{\Delta_{12},\Delta_{34}}(z,\bar{z}) = \sum_{a,b\geq 0}^{\infty} r_{a,b;\Delta,\ell}^{\Delta_{12},\Delta_{34}} \, P^{(\varepsilon)}_{\frac{\Delta+\ell}{2}+a,\frac{\Delta-\ell}{2}+b}(z,\bar{z})$$

with complicated coefficients $r_{a,b;\Delta,\ell}^{\Delta_{12},\Delta_{34}}$. Now, use properties of $r_{a,b;\Delta,\ell}^{\Delta_{12},\Delta_{34}}$ and $P^{(\varepsilon)}_{\frac{\Delta+\ell}{2}+a,\frac{\Delta-\ell}{2}+b}$ to

1. invert $A_{m,m}^{\{k_i\}} = \frac{1}{y_m^{\{k_i\}}} U^{2\varepsilon} \Delta_\varepsilon \left[\frac{\mathcal{T}^{\{k_i\}}}{U^{3(\varepsilon-1)}}\right]$ to derive $\mathcal{T}_{\Delta,\ell}^{\{k_i\}}$
2. derive recursion relations for $U^{2\varepsilon}\, \Delta_\varepsilon\, f(U,V)\, \Delta_\varepsilon^{-1}\, U^{-2\varepsilon+\frac{\Delta_{34}}{2}}\, G_{\Delta,\ell}^{\Delta_{12},\Delta_{34}}$ to generate blocks in lower channels

Reduced correlator and reduced blocks

The SCWIs in $d = 2(\varepsilon + 1)$ can also be solved with

$$\mathcal{G}^{\{k_i\}}(z, \bar{z}; \alpha, \bar{\alpha}) = \mathcal{F}^{\{k_i\}}(z, \bar{z}; \alpha, \bar{\alpha}) + \Upsilon^{(\varepsilon)} \circ \mathcal{T}^{\{k_i\}}(z, \bar{z}; \alpha, \bar{\alpha})$$

Upshot: At NNE, full supermultiplets are captured by **reduced blocks** $\mathcal{T}^{\{k_i\}}_{\Delta,\ell}$ involving only

$$G^{\Delta_{12},\Delta_{34}-2(\varepsilon-1)}_{\Delta,\ell}(z, \bar{z}) \quad \text{and} \quad g^{\Delta_{12},\Delta_{34}-2(\varepsilon-1)}_{\frac{\Delta+\ell}{2}}(z)$$

$SO(d+1,1)$ block $SL(2,\mathbb{R})$ block

True in all SCFTs with 16 and 8 supercharges, i.e.

- 6d $\mathcal{N} = (2{,}0)$, 4d $\mathcal{N} = 4$, and 3d $\mathcal{N} = 8$
- 6d $\mathcal{N} = (1{,}0)$, 5d $\mathcal{N} = 1$, 4d $\mathcal{N} = 2$, and 3d $\mathcal{N} = 4$

2601.15407

2604.08673

Protected contribution

The SCWIs in $d = 6$ can also be solved with

$$\mathcal{G}^{\{k_i\}}(z,\bar{z};\alpha,\bar{\alpha}) = \boxed{\mathcal{F}^{\{k_i\}}(z,\bar{z};\alpha,\bar{\alpha})} + \Upsilon^{(2)} \circ \mathcal{T}^{\{k_i\}}(z,\bar{z};\alpha,\bar{\alpha})$$

Back to 6d: $\boldsymbol{\varepsilon = 2}$

2506.08560

2601.15407
2604.08673

Protected contribution and chiral algebra twist

The SCWIs in $d = 6$ can also be solved with

[Beem, Lemos, Rastelli, and van Rees , '15]

$$\mathcal{G}^{\{k_i\}}(z,\bar{z};\alpha,\bar{\alpha}) \quad = \quad \boxed{\mathcal{F}^{\{k_i\}}(z,\bar{z};\alpha,\bar{\alpha})} \quad + \quad \Upsilon^{(2)} \circ \mathcal{T}^{\{k_i\}}(z,\bar{z};\alpha,\bar{\alpha})$$

Under the "**chiral algebra twist**" ρ: $\alpha, \bar{\alpha} \to \bar{z}^{-1}$, this decomposition obeys

$$\begin{aligned}\mathcal{G}^{\{k_i\}}\Big|_\rho &= \mathcal{F}^{\{k_i\}}(z,\bar{z};\bar{z}^{-1},\bar{z}^{-1}) + \underbrace{\Upsilon^{(2)}}_{=\,0} \circ \mathcal{T}^{\{k_i\}}(z,\bar{z};\bar{z}^{-1},\bar{z}^{-1}) \\ &= \mathcal{F}^{\{k_i\}}(z)\end{aligned}$$

with $\mathcal{F}^{\{k_i\}}(z)$ meromorphic. The 6d superblock sum reduces to a (restricted) $SL(2,\mathbb{R})$ block sum:

$$\mathcal{G}^{\{k_i\}}\Big|_\rho = \sum_{\mathcal{X}} \lambda_{k_1k_2\mathcal{X}}\lambda_{k_3k_4\mathcal{X}}\, \mathfrak{G}_{\mathcal{X}}^{k_{12},k_{34}}\Bigg|_\rho = \sum_{\mathcal{O},\mathcal{O}'} G_{\mathcal{O}\mathcal{O}'} C_{k_1k_2}^{\mathcal{O}} C_{k_3k_4}^{\mathcal{O}'}\, g_{\frac{\Delta+\ell}{2}}^{\Delta_{12},\Delta_{34}}(z)$$

Infinite chiral symmetry in 6d

[Beem, Lemos, Liendo, Peelaers, Rastelli, and van Rees , '13]
[Beem, Rastelli, and van Rees , '14]

As in 4d $\mathcal{N} \geq 2$ theories, we interpret $\mathcal{F}^{\{k_i\}}(z)$ as a 4-pt. function in a **chiral algebra**.

- $\mathcal{F}^{\{k_i\}}(z)$ is a 4-pt. function of chiral primaries on $\mathbb{R}^2 \subset \mathbb{R}^6$ with holomorphic coordinate z

The twist reduces the 6d superblock decomposition into an $SL(2,\mathbb{R})$ block expansion

$$\mathcal{G}^{\{k_i\}}\Big|_\rho = \sum_{\mathcal{X}} \lambda_{k_1k_2\mathcal{X}}\lambda_{k_3k_4\mathcal{X}}\, \mathfrak{G}_{\mathcal{X}}^{k_{12},k_{34}}\Bigg|_\rho = \sum_{\mathcal{O},\mathcal{O}'} G_{\mathcal{O}\mathcal{O}'} C_{k_1k_2}^{\mathcal{O}} C_{k_3k_4}^{\mathcal{O}'}\, g_{\Delta,\ell}^{k_{12},k_{34}}(z)$$

with chiral algebra CFT data $G_{\mathcal{O}\mathcal{O}'} C_{k_1k_2}^{\mathcal{O}} C_{k_3k_4}^{\mathcal{O}'}$ **TBD**. In addition, for chiral algebra 4-pt. functions we have

$$\mathcal{F}^{\{k_i\}}(z) = G_{k_1k_2}G_{k_3k_4} + \sum_{n=1}^{k_1+k_4} \alpha_n z^n + \sum_{n=1}^{k_2+k_3} \beta_n\left(\frac{1}{(1-z)^n} - 1\right)$$

where α_n and β_n depend on $\{\Delta_i, \lambda_{ijk}\}$.

[Headrick, Maloney, Perlmutter, Zadeh '15]

6d $\mathcal{N}$=(2,0) theory $\mathcal{W}$-algebras

[Beem, Rastelli, and van Rees , '14]

It was conjectured that a 6d (2,0) theory of type $\mathfrak{g}$ has chiral algebra $\mathcal{W}_{\mathfrak{g}}$ with central charge $c_{2d,\mathfrak{g}} = c_{\mathfrak{g}}$

(Informally,) $\mathcal{W}_{\mathfrak{g}}$ is a higher-spin generalization of the Virasoro algebra with $r_{\mathfrak{g}}$ generators $\{W_{k_i}\}$.

6d ½-BPS operators S_{k_i} called **"strong generator operators"** map to $\mathcal{W}_{\mathfrak{g}}$ strong generators as $S_{k_i}\big|_{\rho} = W_{k_i}$.

__**Example:**__ 6d stress tensor superprimary S_2 maps $S_2|_{\rho} = W_2 \equiv T$

- $\mathfrak{g} = A_1$ gives the Virasoro algebra with $c_{A_1} = 25$

$\mathcal{W}$-algebra data $C^{\mathcal{O}}_{k_1 k_2}$ is fixed by demanding **associativity** of the W_{k_i} OPEs, e.g. using `OPEdefs`

$$W_{k_1}(z) \times W_{k_2}(0) \sim \sum_{\mathcal{O}} \frac{C^{\mathcal{O}}_{k_1 k_2}}{z^{k_1+k_1-h_{\mathcal{O}}}} \mathcal{O}(z)$$

[Thielemans , '91]

[Gaberdiel, Gopakumar, '12]
[Procházka, '14]

6d $\mathcal{N}$=(2,0) theory $\mathcal{W}$-algebras: $\mathfrak{g} = A_{N-1}$

$\mathfrak{g} = A_{N-1}$ yields the famous $\mathcal{W}_N$ algebras with generators $\{T, W_3, W_4, \dots W_N\}$ and admit* the central charge

$$c_{2d,A} = 4N^3 - 3N - 1 = c_A$$

The minimal representations of $\mathcal{W}_{1+\infty}$ fix the OPE coefficient C_{33}^4 in

$$W_3(z) \times W_3(0) \sim \frac{1}{z^6}\mathbf{1} + \frac{3\sqrt{2}}{c}\frac{1}{z^4}T(z) + \frac{C_{33}^4}{z^2}W_4(z) + \dots$$

to have the form

$$C_{33}^4[A_{N-1}]^2 = \frac{144}{c_A(5c_A+22)}\frac{(c_A+2)(N-3)(c_A(N+3)+2(4N+3)(N-1))}{(N-2)(c_A(N+2)+(3N+2)(N-1))}$$

[Linshaw, '17]

OPE associativity fixes all remaining $C_{k_1k_2}^{\mathcal{O}}$ in $\mathcal{W}_N$ in terms of c_A and C_{33}^4 in, e.g.

$$C_{44}^4[A_{N-1}] = \frac{3(c_A+3)C_{33}^4}{(c_A+2)} - \frac{288(c_A+10)}{c_A(5c_A+22)C_{33}^4}$$

[Candu, Gaberdiel, Kelm, Vollenweider, '12]
[Procházka, '19]

6d $\mathcal{N}$=(2,0) theory $\mathcal{W}$-algebras: $\mathfrak{g} = D_N$

$\mathcal{W}_{D_N}$ algebras have generators $\{T, W_4, W_6, \ldots W_{2N-2}, p_N\}$ and admit* the central charge

$$c_{2d,D} = 16N^3 - 24N^2 + 9N = c_D$$

$\mathcal{W}_{D_N}$ also have an extra spin-N generator $p_N = P_N|_\rho$, with P_N the ½-BPS **Pfaffian operator** in 6d.

OPE associativity between p_N and W_4, W_6 fixes the coefficient C_{44}^4 in

$$W_4(z) \times W_4(0) \sim \frac{1}{z^8}\mathbf{1} + \frac{4\sqrt{2}}{c}\frac{1}{z^6}T(z) + \frac{C_{44}^4}{z^4}W_4(z) + \ldots$$

to take the form

$$C_{44}^4[D_N]^2 = \frac{144}{c_D(5c_D+22)}\frac{\left({c_D}^2(2(N-1)N-9) + 3c_D(N(N(12N-49)+40)-2) + 2N(N(12N+5)-14)\right)^2}{((c_D-7)N + c_D + 3N^2 + 2)(c_D(N-2)(2N-3) + N(4N-5))(2c_DN + c_D + 2N(8N-5))}$$

OPE associativity determines all remaining $C_{k_1k_2}^{\mathcal{O}}$ in terms of c_D and C_{44}^4.

[Kanade, Linshaw, '19]

6d data extraction

We computed $\mathcal{F}^{\{k_i\}}(z)$ for all 16 non-vanishing $\langle S_{k_1} S_{k_2} S_{k_3} S_{k_4} \rangle$ up to $\{k_i\} = 4$

$\mathcal{E} = 2$: $\langle S_2 S_2 S_2 S_2 \rangle$, $\langle S_2 S_2 S_3 S_3 \rangle$, $\langle S_2 S_2 S_4 S_4 \rangle$, $\langle S_2 S_3 S_3 S_4 \rangle$, $\langle S_2 S_4 S_4 S_4 \rangle$

$\mathcal{E} = 3$: $\langle S_3 S_3 S_3 S_3 \rangle$, $\langle S_3 S_3 S_4 S_4 \rangle$

$\mathcal{E} = 4$: $\langle S_4 S_4 S_4 S_4 \rangle$.

as functions of $c_{\mathfrak{g}}$ and $C_{44}^4(\mathfrak{g})$. We compare

$$\mathcal{F}^{\{k_i\}}(z) = G_{k_1 k_2} G_{k_3 k_4} + \sum_{n=1}^{k_1+k_4} \alpha_n z^n + \sum_{n=1}^{k_2+k_3} \beta_n \left(\frac{1}{(1-z)^n} - 1 \right) = \sum_{\mathcal{O},\mathcal{O}'} G_{\mathcal{O}\mathcal{O}'} C_{k_1 k_2}^{\mathcal{O}} C_{k_3 k_4}^{\mathcal{O}'} \, g_{\Delta,\ell}^{k_{12},k_{34}}(z)$$

to read off* **exact** 6d data $G_{\mathcal{O}\mathcal{O}'} C_{k_1 k_2}^{\mathcal{O}} C_{k_3 k_4}^{\mathcal{O}'} \propto \lambda_{k_1 k_2 \mathcal{X}} \lambda_{k_3 k_4 \mathcal{X}}$

<u>Check:</u> Holography to M-theory on $AdS_7 \times S^4 / \mathbb{Z}_{\boldsymbol{\mathfrak{o}}}$

Supergraviton scattering in M-theory on $AdS_7 \times S^4/\mathbb{Z}_{\mathfrak{o}}$

6d (2,0) of type $\mathfrak{g} = A_{N-1}, D_N$ $\longleftrightarrow$ M-theory on $AdS_7 \times S^4/\mathbb{Z}_{\mathfrak{o}}$ with $\mathfrak{o} = \begin{cases} 1 & \text{if} \quad \mathfrak{g} = A_{N-1} \\ 2 & \text{if} \quad \mathfrak{g} = D_N \end{cases}$

The holographic dictionary relates M-theory parameters to 6d (2,0) parameters with

$$\left(\frac{L_{AdS_7}}{l_{11}}\right)^9 = 16\mathfrak{o}c_{\mathfrak{g}} + O\left({c_{\mathfrak{g}}}^0\right), \qquad c_{\mathfrak{g}} = (2\mathfrak{o})^2 N^3 + O(N^2),$$

[Maldacena, '97]
[Aharony, Oz, Yin , '98]
[Intriligator, '00]

and 6d (2,0) correlators of ½-BPS operators to M-theory amplitudes of graviton KK modes:

$$\mathcal{G}^{\{k_i\}} \longleftrightarrow \mathcal{A}^{\{k_i\}}_{AdS_7 \times S^4/\mathbb{Z}_{\mathfrak{o}}}$$

Supergraviton scattering in M-theory on $AdS_7 \times S^4/\mathbb{Z}_{\mathfrak{o}}$

$$\left(\frac{L_{AdS_7}}{l_{11}}\right)^9 = 16\mathfrak{o}c_{\mathfrak{g}} + O\left(c_{\mathfrak{g}}^{\ 0}\right), \qquad c_{\mathfrak{g}} = (2\mathfrak{o})^2 N^3 + O(N^2),$$

$$\mathcal{G}^{\{k_i\}} \longleftrightarrow \mathcal{A}^{\{k_i\}}_{AdS_7\times S^4/\mathbb{Z}_{\mathfrak{o}}}$$

Holography allows us to organize 6d (2,0) quantities in an M-theory EFT expansion. We can expand $\mathcal{G}^{\{k_i\}}$ in large $c_{\mathfrak{g}}$ to encode M-theory corrections to 11d supergravity

$$\mathcal{G}^{\{k_i\}} = \mathcal{G}^{\{k_i\}}_{(\mathrm{GFF})} + \frac{1}{c_{\mathfrak{g}}}\mathcal{G}^{\{k_i\}}_{(R)} + \frac{1}{\mathfrak{o}^{2/3}c_{\mathfrak{g}}^{\ 5/3}}\mathcal{G}^{\{k_i\}}_{(R^4)} + \frac{1}{\mathfrak{o}^{4/3}c_{\mathfrak{g}}^{\ 7/3}}\mathcal{G}^{\{k_i\}}_{(D^6R^4)} + \sum_{n=4}^{\infty}\frac{1}{\mathfrak{o}^{\frac{2n+6}{9}}c_{\mathfrak{g}}^{\ \frac{2n+15}{9}}}\mathcal{G}^{\{k_i\}}_{(D^{2n}R^4)} + (\text{loops})$$

protected by supersymmetry + known from type II string theory

unprotected + mostly unknown

Scattering on $AdS_7 \times S^4 / \mathbb{Z}_{\mathfrak{o}}$: protected corrections

[Chester, Perlmutter, '18]

Chiral algebra preserves supersymmetry → we expect it to contain **protected** terms

Now including the D-series,

$$\mathcal{F}^{\{k_i\}} = \mathcal{F}^{\{k_i\}}_{(\mathrm{GFF})} + \frac{1}{c_{\mathfrak{g}}}\mathcal{F}^{\{k_i\}}_{(R)} + \frac{1}{\mathfrak{o}^{2/3} c_{\mathfrak{g}}^{5/3}}\mathcal{F}^{\{k_i\}}_{(R^4)} + \frac{1}{\mathfrak{o}^{4/3} c_{\mathfrak{g}}^{7/3}}\mathcal{F}^{\{k_i\}}_{(D^6R^4)} + \sum_{\substack{i,j,k \\ i+j+k>1}}^{\infty} \frac{\mathrm{f}_{i,j,k}(\mathfrak{o})}{c_{\mathfrak{g}}^{\,i+\frac{5}{3}j+\frac{7}{3}k}} \underbrace{\mathcal{F}^{\{k_i\}}_{(\mathrm{loops}_{i,j,k})}}_{\text{loops involving } i\ R,\ j\ R^4,\ \text{and } k\ D^6R^4 \text{ vertices}}$$

<u>Recall:</u> 2d crossing symmetry → $\mathcal{F}^{\{k_i\}}$ can be written in terms of just $c_{\mathfrak{g}}$ and $C_{44}^4(\mathfrak{g})$

- Non-trivial $c_{\mathfrak{g}}$, $\mathfrak{o}$-dependence of any $\mathcal{F}^{\{k_i\}}$ is captured by $C_{44}^4(\mathfrak{g})$ with expansion

Scattering on $AdS_7 \times S^4 / \mathbb{Z}_{\mathfrak{o}}$: protected corrections

$\mathfrak{g} = A_{N-1}$:

$$C^4_{33}[A_{N-1}]^2 = \frac{144}{c_A(5c_A+2\ \ )} \frac{(c_A+2)(N-3)(c_A(N+3)+2(4N+3)(N-1))}{(N-2)(c_A(N+2)+(3N+2)(N-1))}$$

$$C^4_{44}[A_{N-1}] = \frac{3(c_A+3)C^4_{33}}{(c_A+2)} - \frac{288(c_A+10)}{c_A(5c_A+22)C^4_{33}}$$

$\mathfrak{g} = D_N$:

$$C^4_{44}[D_N]^2 = \frac{144}{c_D(5c_D+22)}$$

$$\times \frac{(c_D{}^2(2(N-1)N-9)+3c_D(N(N(12N-49)+40)-2)+2N(N(12N+5)-14))^2}{((c_D-7)N+c_D+3N^2+2)(c_D(N-2)(2N-3)+N(4N-5))(2c_DN+c_D+2N(8N-5))}$$

Scattering on $AdS_7 \times S^4 / \mathbb{Z}_{\mathfrak{o}}$: protected corrections

Chiral algebra preserves supersymmetry → we expect it to contain **protected** terms

- Non-trivial $c_{\mathfrak{g}}$, $\mathfrak{o}$-dependence of any $\mathcal{F}^{\{k_i\}}$ is captured by $C^4_{44}(\mathfrak{g})$

$$\mathcal{F}^{\{k_i\}} = \mathcal{F}^{\{k_i\}}_{(\mathrm{GFF})} + \frac{1}{c_{\mathfrak{g}}}\mathcal{F}^{\{k_i\}}_{(R)} + \frac{1}{\mathfrak{o}^{2/3}c_{\mathfrak{g}}^{5/3}}\mathcal{F}^{\{k_i\}}_{(R^4)} + \frac{1}{\mathfrak{o}^{4/3}c_{\mathfrak{g}}^{7/3}}\mathcal{F}^{\{k_i\}}_{(D^6R^4)} + \sum_{\substack{i,j,k\\ i+j+k>1}}^{\infty} \frac{\mathrm{f}_{i,j,k}(\mathfrak{o})}{c_{\mathfrak{g}}^{i+\frac{5}{3}j+\frac{7}{3}k}}\mathcal{F}^{\{k_i\}}_{(\mathrm{loops}_{i,j,k})}$$

$$C^4_{44}[\mathfrak{g}]^2 = \frac{1}{c_{\mathfrak{g}}}\frac{144}{5} - \frac{1}{\mathfrak{o}^{2/3}c_{\mathfrak{g}}^{5/3}}2^{\frac{4}{3}}540 + \frac{1}{\mathfrak{o}^{4/3}c_{\mathfrak{g}}^{7/3}}2^{\frac{5}{3}}540 + \sum_{\substack{i,j,k\\ i+j+k>1}}^{\infty} \frac{F_{i,j,k}(\mathfrak{o})}{c_{\mathfrak{g}}^{i+\frac{5}{3}j+\frac{7}{3}k}}(C^4_{44})^2_{(\mathrm{loops}_{i,j,k})}$$

✓ Right powers of $\mathfrak{o}$ and $c_{\mathfrak{g}}$

✓ Correct $c_{\mathfrak{g}}$-dependence at supergravity order

Scattering on $AdS_7 \times S^4 / \mathbb{Z}_{\mathfrak{o}}$: 11d supergravity

$$C_{44}^4[\mathfrak{g}]^2 = \boxed{\frac{1}{c_{\mathfrak{g}}}\frac{144}{5}} - \frac{1}{\mathfrak{o}^{2/3} c_{\mathfrak{g}}^{5/3}} 2^{\frac{4}{3}} 540 + \frac{1}{\mathfrak{o}^{4/3} c_{\mathfrak{g}}^{7/3}} 2^{\frac{5}{3}} 540 + \sum_{\substack{i,j,k \\ i+j+k>1}}^{\infty} \frac{F_{i,j,k}(\mathfrak{o})}{c_{\mathfrak{g}}^{i+\frac{5}{3}j+\frac{7}{3}k}} (C_{44}^4)^2_{(\text{loops}_{i,j,k})}$$

Non-extremal ½-BPS couplings $\lambda_{k_1 k_2 p}$ relate to cubic couplings $g_{k_1 k_2 p}$ in the AdS_7 effective action:

$$\frac{1}{c_{\mathfrak{g}}} \lambda_{k_1 k_2 p}{}^{(R)} \lambda_{k_3 k_4 p}{}^{(R)} = \begin{pmatrix} \text{kinematic} \\ \text{factor} \end{pmatrix} g_{k_1 k_2 p}\, g_{k_3 k_4 p} \qquad \text{for} \qquad k_i + k_j > p,$$

with

$$g_{k_1 k_2 p} = \frac{2^{k_1+k_2+p-1}}{(\pi^3 c_{\mathfrak{g}})^{1/2}} \Gamma\left[\frac{k_1 + k_2 + p}{2}\right] \frac{\Gamma\left[\frac{k_2 + p - k_1 + 1}{2}\right] \Gamma\left[\frac{p + k_1 - k_2 + 1}{2}\right] \Gamma\left[\frac{k_1 + k_2 - p + 1}{2}\right]}{\sqrt{\Gamma[2k_1 - 1]\Gamma[2k_2 - 1]\Gamma[2p - 1]}}$$

[Bastianelli, Zucchini, '99]
[Corrado, Florea, McNees, '99]
[Beem, Rastelli, and van Rees , '14]

The 21 examples appearing up to $\mathcal{F}_{4444}$ **agree exactly**.

Scattering on $AdS_7 \times S^4 / \mathbb{Z}_{\mathfrak{o}}$: protected corrections

Chiral algebra preserves supersymmetry → we expect it to contain **protected** terms

- Non-trivial $c_{\mathfrak{g}}, \mathfrak{o}$-dependence of any $\mathcal{F}^{\{k_i\}}$ is captured by $C^4_{44}(\mathfrak{g})$

$$\mathcal{F}_{\{k_i\}} = \mathcal{F}^{(\mathrm{GFF})}_{\{k_i\}} + \frac{1}{c_{\mathfrak{g}}}\mathcal{F}^{(R)}_{\{k_i\}} + \frac{1}{\mathfrak{o}^{2/3} c_{\mathfrak{g}}^{5/3}}\mathcal{F}^{(R^4)}_{\{k_i\}} + \frac{1}{\mathfrak{o}^{4/3} c_{\mathfrak{g}}^{7/3}}\mathcal{F}^{(D^6 R^4)}_{\{k_i\}} + \sum_{\substack{i,j,k \\ i+j+k>1}}^{\infty} \frac{\mathrm{f}_{i,j,k}(\mathfrak{o})}{c_{\mathfrak{g}}^{i+\frac{5}{3}j+\frac{7}{3}k}} \mathcal{F}^{(\mathrm{loops}_{i,j,k})}_{\{k_i\}}$$

$$C^4_{44}[\mathfrak{g}]^2 = \frac{1}{c_{\mathfrak{g}}}\frac{144}{5} - \frac{1}{\mathfrak{o}^{2/3} c_{\mathfrak{g}}^{5/3}} 2^{\frac{4}{3}} 540 + \frac{1}{\mathfrak{o}^{4/3} c_{\mathfrak{g}}^{7/3}} 2^{\frac{5}{3}} 540 + \sum_{\substack{i,j,k \\ i+j+k>1}}^{\infty} \frac{F_{i,j,k}(\mathfrak{o})}{c_{\mathfrak{g}}^{i+\frac{5}{3}j+\frac{7}{3}k}} (C^4_{44})^2_{(\mathrm{loops}_{i,j,k})}$$

✓ Right powers of $\mathfrak{o}$ and $c_{\mathfrak{g}}$

✓ Correct $c_{\mathfrak{g}}$-dependence at supergravity order

✓ Correct c_A-dependence at R^4 order to reproduce

$$f_{R^4}(s,t) = \frac{s\,t\,u}{2^7 \cdot 3}$$

[Chester, Perlmutter, '18]

Summary and outlook

Supersymmetry provides **exact results** and **extra structure** for correlators of ½-BPS operators. We

- derived **reduced blocks** for NNE correlators in SCFTs in $\mathrm{d} > 2$

- examined the solvable sector of 6d $(2{,}0)$ SCFTs by
  - **constructing** 6d $(2{,}0)$ $\mathcal{W}_{\mathfrak{g}}$ algebras for $\mathfrak{g} = A_{N-1}, D_N$ and computing **exact CFT data**
  - show **consistency** with **protected** M-theory corrections to graviton scattering on $AdS_7 \times S^4/\mathbb{Z}_{\mathfrak{o}}$

Future directions:

- More reducing:
  - reduced blocks at higher extremality (need to find a better $\Upsilon^{(\varepsilon)}$)
  - clarify reduced crossing in odd d (**Bonus Slide**)
- More 6d $(2{,}0)$ bootstrap:
  - mixed correlator bootstrap e.g. $\{\langle S_2S_2S_2S_2\rangle,\ \langle S_2S_2S_3S_3\rangle,\ \langle S_3S_3S_3S_3\rangle\}$

Application: Composite operator in 6d $\mathcal{N}$=(2,0) theories

WIP w/ Francesco Aprile and Hector Puerta-Ramisa

Application: Composite operator in 6d $\mathcal{N}$=(2,0) theories

So far, we've studied "strong generator operators" S_k $\longrightarrow$ now consider ½-BPS **composite operators**

Without a Lagrangian, the only definition of ½-BPS operators is using the $\mathcal{W}$-algebra.

Example: $S_2|_\rho = T$ and $S_k|_\rho = W_k$ so there is a unique composite quasi-primary

$$[S_2 S_p]\Big|_\rho (z) = \mathcal{N}_{2,k}\left(:TW_k:(z) - \frac{3}{2(2k+1)} W_k{}''(z) \right)$$

- $[S_2 S_p]$ is **orthogonal** to S_k

WIP: bootstrap (NNE) $\langle [S_2 S_p]\ S_2\ S_p\ S_{p+q-2} \rangle$ at large $c_{\mathfrak{g}}$

$$\mathcal{G}^{[2,p],2,p,p+q-2} = \underbrace{\mathcal{F}^{[2,p],2,p,p+q-2}}_{\mathcal{W}_{\mathfrak{g}}} + \Upsilon^{(2)} \circ \mathcal{T}^{[2,p],2,p,p+q-2}$$

reduced blocks
+
see Francesco's talk

WIP w/ Francesco Aprile and Hector Puerta-Ramisa

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Example: $S_2|_\rho = T$ and $S_k|_\rho = W_k$ so there is a unique composite quasi-primary

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$$\mathcal{G}^{[2,p],2,p,p+q-2} = \underbrace{\mathcal{F}^{[2,p],2,p,p+q-2}}_{\mathcal{W}_{\mathfrak{g}}} + \Upsilon^{(2)} \circ \underbrace{\mathcal{T}^{[2,p],2,p,p+q-2}}_{\text{reduced blocks}}$$

Thank you for 𝔅𝔬𝔬𝔱𝔰𝔱𝔯𝔞𝔭 2026!

Bonus: 6d $\mathcal{N}$=(2,0) theory $c_{\mathfrak{g}}$ from truncations of $\mathcal{W}_{1+\infty}$

The N-dependence of $c_{\mathfrak{g}}$ (e.g. N^3 scaling) in the 6d (2,0) theories is mysterious

- Producing this in 6d requires hard holography calculations in M-theory + anomaly considerations
- The $\mathcal{W}$-algebra gives a simpler prescription

Start with a truncation of $\mathcal{W}_{1+\infty}$ labeled by roots $(\lambda_1, \lambda_2, \lambda_3)$:

1. Triality symmetry fixes $\quad \lambda_3 = -\frac{\lambda_1\lambda_2}{\lambda_1+\lambda_2} \;\rightarrow\; (\lambda_1, \lambda_2, \lambda_3) = \left(\lambda_1, \lambda_2, -\frac{\lambda_1\lambda_2}{\lambda_1+\lambda_2}\right)$
2. Specialization to $\mathcal{W}_N$ or $\mathcal{W}_{D_N}$ fixes $\quad \lambda_3 = N \;\rightarrow\; (\lambda_1, \lambda_2, \lambda_3) = \left(\lambda_1, -\frac{\lambda_1 N}{\lambda_1+N}, N\right)$
3. Specialization to **known 6d $\mathcal{N}$=(2,0) theories** with

$$c_{2d,A} = 4N^3 - 3N - 1 = c_A$$

$$c_{2d,D} = 16N^3 - 24N^2 + 9N = c_D\,,$$

fixes $\quad \lambda_1 = -2N \;\rightarrow\; (\lambda_1, \lambda_2, \lambda_3) = (-2N, -2N, N)$

Bonus: Crossing symmetry

The numerical bootstrap makes use of crossing equations obtained by exchanging operators $1 \leftrightarrow 3$.

Focus again on NNE with $k_4 = k_1 + k_2 + k_3 - 4$

With 16 supercharges, we have

$$\mathcal{G}^{k_1 k_2 k_3 k_4}(U, V; \sigma, \tau) = \frac{U^{2\varepsilon}}{V^{2\varepsilon}} \, \tau^2 \, \mathcal{G}^{k_3 k_2 k_1 k_4}\left(V, U; \frac{\sigma}{\tau}, \frac{1}{\tau}\right)$$

while with 8 supercharges, we have

$$\mathcal{G}^{k_1 k_2 k_3 k_4}(U, V; \alpha) = \frac{U^{2\varepsilon}}{V^{2\varepsilon}} (\alpha - 1)^2 \, \mathcal{G}^{k_3 k_2 k_1 k_4}\left(V, U; \frac{\alpha}{\alpha - 1}\right)$$

Bonus: Crossing symmetry for reduced correlators

$$\mathcal{G}^{k_1k_2k_3k_4}(U,V;\sigma,\tau) = \frac{U^{2\varepsilon}}{V^{2\varepsilon}}\tau^2\,\mathcal{G}^{k_3k_2k_1k_4}\left(V,U;\frac{\sigma}{\tau},\frac{1}{\tau}\right)$$

$$\mathcal{G}^{k_1k_2k_3k_4}(U,V;\alpha) = \frac{U^{2\varepsilon}}{V^{2\varepsilon}}(\alpha-1)^2\,\mathcal{G}^{k_3k_2k_1k_4}\left(V,U;\frac{\alpha}{\alpha-1}\right)$$

Ideally, we should be able to translate these equations into constraints on $\mathcal{T}^{\{k_i\}}(z,\bar{z})$ and $h^{\{k_i\}}(z)$.

To do this, we note that

$$\Delta_\varepsilon\Big|_{\substack{U\to V\\ V\to U}} = \Delta_\varepsilon\,\frac{U^{1-\varepsilon}}{V^{1-\varepsilon}}$$

Bonus: Crossing symmetry for reduced correlators

8 supercharges:

$$\mathcal{G}^{k_1k_2k_3k_4}(U,V;\alpha)=\frac{U^{2\varepsilon}}{V^{2\varepsilon}}(\alpha-1)^2\,\mathcal{G}^{k_3k_2k_1k_4}\left(V,U;\frac{\alpha}{\alpha-1}\right),\qquad \Delta_\varepsilon\Big|_{\substack{U\to V\\ V\to U}}=\Delta_\varepsilon\,\frac{U^{1-\varepsilon}}{V^{1-\varepsilon}}$$

Now by applying crossing and examining coefficients of α, we find **3 independent equations**:

$$U^\varepsilon\Delta_\varepsilon\left[b^{\{k_i\}}(U,V)-\left(\frac{U}{V}\right)^{1-\varepsilon}b^{\{k_i\}}(V,U)\right]=\Delta_\varepsilon\left[U^{1-\varepsilon}\frac{f^{\{k_i\}}(z)-f^{\{k_i\}}(\bar{z})}{z-\bar{z}}\right]+\left(\frac{U}{V}\right)^{\varepsilon}\Delta_\varepsilon\left[U^{1-\varepsilon}\frac{f^{\{k_i\}}(1-z)-f^{\{k_i\}}(1-\bar{z})}{z-\bar{z}}\right]$$

$$U^\varepsilon\Delta_\varepsilon\left[U\left(b^{\{k_i\}}(U,V)-\left(\frac{U}{V}\right)^{1-\varepsilon}b^{\{k_i\}}(V,U)\right)\right]=\left(\frac{U}{V}\right)^{\varepsilon}\Delta_\varepsilon\left[U^{1-\varepsilon}\frac{z\,f^{\{k_i\}}(1-z)-\bar{z}\,f^{\{k_i\}}(1-\bar{z})}{z-\bar{z}}\right]$$

& $U\leftrightarrow V$

Bonus: Crossing symmetry for reduced correlators

8 supercharges:

$$\mathcal{G}^{k_1 k_2 k_3 k_4}(U,V;\alpha) = \frac{U^{2\varepsilon}}{V^{2\varepsilon}}(\alpha-1)^2\, \mathcal{G}^{k_3 k_2 k_1 k_4}\left(V,U;\frac{\alpha}{\alpha-1}\right), \qquad \Delta_\varepsilon\Big|_{\substack{U\to V\\ V\to U}} = \Delta_\varepsilon\, \frac{U^{1-\varepsilon}}{V^{1-\varepsilon}}$$

Now by applying crossing and examining coefficients of α, we find **3 independent equations**:
If $\boldsymbol{f^{\{k_i\}}}$ can be absorbed into $\boldsymbol{b^{\{k_i\}}}$ (for $\boldsymbol{d \neq 4}$):

$$U^\varepsilon \Delta_\varepsilon\left[b^{\{k_i\}}(U,V) - \left(\frac{U}{V}\right)^{1-\varepsilon} b^{\{k_i\}}(V,U)\right] = 0$$

$$U^\varepsilon \Delta_\varepsilon\left[U\left(b^{\{k_i\}}(U,V) - \left(\frac{U}{V}\right)^{1-\varepsilon} b^{\{k_i\}}(V,U)\right)\right] = 0$$

& $U \leftrightarrow V$