Regge's Inferno

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This presentation is based on

★ arXiv: 2603.10197 with **with Alessio Miscioscia, Fedor Popov**
★ wip **Alessio Miscioscia, Fedor Popov, and Dmitrii Pavshinkin**

Inspired by recent work:

Semi-universality of CFT$_d$ entropy at large spin

Harsh Anand[a,1] Nathan Benjamin,[b,2] Vipul Kumar,[a,3] Shiraz Minwalla,[a,4] Jyotirmoy Mukherjee,[a,5] Sridip Pal,[c,6] Asikur Rahaman[a,7]

General problem

Determine the operator spectrum

$$\mathcal{O}_{\Delta, J_i, Q_a}$$

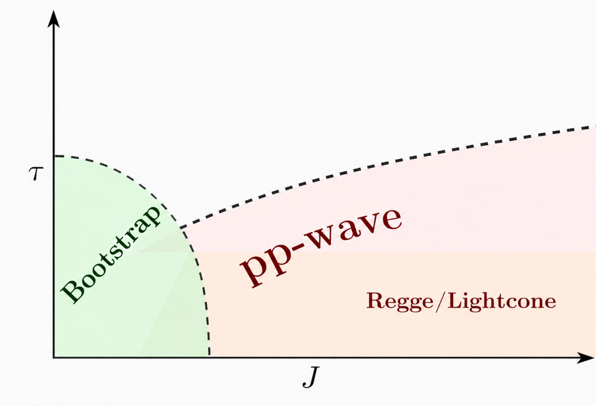
in an interacting CFT.

- Low-lying operators: numerical bootstrap.
- Large charge: EFT/superfluid descriptions.
- $\boxed{\text{Large spin: today.}}$

In 2+1 dimensional theories operators are classified by representations of $SO(3)$ and there is only one Cartan element J, and

$$\tau \equiv \Delta - J \geq \frac{1}{2} \ .$$

We will start from this example as a warm up.
Our goal is to understand operators with very large J.



Asymptotic Freedom at Large J.

Recall: $\tau = \Delta - J$.
At large J one considers multi-twist operators

$$\partial_+^{m_1}\mathcal{O}\cdots\partial_+^{m_n}\mathcal{O}.$$

Their total spin is

$$\sum_i m_i + nJ(\mathcal{O}) \ .$$

Approximate additivity

For $m_i \gg 1$,

$$\Delta \approx n\Delta_0 + \sum_i m_i$$

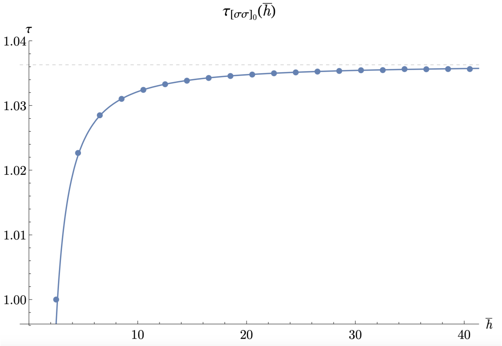
so twists add approximately:

$$\tau \approx n\tau_0 .$$

At sufficiently large spin, composite operators behave as a *nearly-free Fock space of partons* .

[Alday, Maldacena; ZK, Zhiboedov; Fitzpatrick, Kaplan, Poland, Simmons-Duffin...]

The existence of such a Fock space even in strongly coupled theories is essentially confirmed.



Where the Fock-space picture breaks down

$$\partial_+^{m_1}\mathcal{O}\cdots\partial_+^{m_n}\mathcal{O}.$$

Interactions between partons grow with the number of constituents.

Breakdown scale

$$\tau_{\text{strong}} \sim \sqrt{J}$$

At this scale, one should replace the partonic picture by a different description.

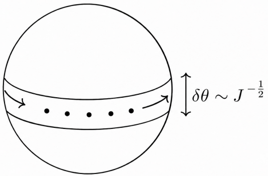
[Cuomo,ZK; Fardelli, Fitzpatrick, Li; Kravchuk, Mann] we will now describe a scaling limit with the properties

$$\boxed{\frac{\tau}{\sqrt{J}} = \text{finite} \ , \quad J \to \infty}$$

The Fock-space picture is only valid for $\frac{\tau}{\sqrt{J}} \ll 1$.

We found the following Brinkmann metric captures these states:

$$\boxed{ds^2 = -(1+y^2)dt^2 + dtdx + dy^2}$$



We obtain this metric as a Penrose limit around the equator of S^2. This metric admits a Heisenberg isometry group H_3 generated by

$$K_1 = \partial_x,$$

$$K_2 = \cos t\, \partial_y + y \sin t\, \partial_x, \quad K_3 = \sin t\, \partial_y - y \cos t\, \partial_x.$$

Heisenberg symmetry organizes the high-spin spectrum.

We do not have to view this geometry as a Penrose limit. We can also view it as time evolution by

$$\underbrace{\Delta + M_{+-}}_{=\tau, \geq 0} + \underbrace{P_{-}}_{\geq 0} \quad .$$

The P_{-} term is necessary to resolve infinite degeneracies and to obtain sensible thermodynamics. (It arises automatically from the Penrose limit.)

In general space dimension d with one large angular momentum we obtain the metric

$$\boxed{ds^2 = -\left(1 + \sum_{i=1}^{d-1} x_i^2\right) dt^2 + dxdt + \sum_{i=1}^{d-1} dx_i^2} \,. \tag{1}$$

with a Heisenberg isometry group $H_{2(d-1)+1}$.

A variant without the $1+$ appears in the literature

$$\boxed{ds^2 = -\left(\cancel{1+}\sum_{i=1}^{d-1} x_i^2\right) dt^2 + dxdt + \sum_{i=1}^{d-1} dx_i^2} . \qquad (2)$$

Now it describes evolution according to the twist

$$\underbrace{\Delta + M_{+-}}_{=\tau, \geq 0}$$

and we can further compactify the null coordinate x to obtain a non-relativistic conformal quantum mechanics in $d-1$ space dimensions (in a Harmonic trap). See for instance recent work of [R. Mouland].

However for our purposes it will be crucial to keep the time evolution according to

$$\underbrace{\Delta + M_{+-}}_{=\tau,\geq 0} + \underbrace{P_-}_{\geq 0}$$

i.e. stick to the metric

$$\boxed{ds^2 = -\left(1 + \sum_{i=1}^{d-1} x_i^2\right) dt^2 + dx dt + \sum_{i=1}^{d-1} dx_i^2} . \qquad (3)$$

An interesting application is to partition functions $\tau = \Delta - J$

$$Z(\beta, \varepsilon) = \sum e^{-\beta\tau - \beta\varepsilon J} = \mathrm{Exp}\left[\frac{2\pi\beta}{\varepsilon} F(\beta) + O(\varepsilon^0)\right].$$

Alternatively,

$$e^S \sim \mathrm{Exp}\left[\sqrt{J}\tilde{F}\left(\frac{\tau}{\sqrt{J}}\right) + O(\varepsilon^0)\right].$$

Interpretation

$$L_x \sim \frac{2\pi}{\varepsilon} \sim \sqrt{J}$$

acts as *emergent volume*. $F(\beta)$ is the pressure of the quantum fields on the pp wave
$ds^2 = -\left(1 + \sum_{i=1}^{d-1} x_i^2\right) dt^2 + dxdt + \sum_{i=1}^{d-1} dx_i^2$.

Emergent Extensivity

The large-spin sector behaves like a thermodynamic system with entropy

$$S(E, V) = V\, s(E/V)$$

The macroscopic variable is *effective volume generated by large angular momentum*.

Locality on the ppwave implies

$$\log Z \propto \beta \times \underbrace{(\text{effective volume})}_{\frac{2\pi}{\epsilon}} \times F(\beta).$$

This is the geometric origin of the universal large-spin density of states.

Due to the large prefactor in the exponent

$$Z \sim \mathrm{Exp}\left[\frac{2\pi\beta}{\varepsilon} F(\beta) + O(\varepsilon^0)\right] \ ,$$

$$e^S \sim \mathrm{Exp}\left[\sqrt{J}\tilde{F}\left(\frac{\tau}{\sqrt{J}}\right) + O(\varepsilon^0)\right]$$

there may be phase transitions, i.e. non-analyticity in $F, \tilde{F}$.
This *does not* require large N.
We see that when considering operators with $\tau \sim \sqrt{J}$, there is a macroscopic amount of them and $\sqrt{J}$ behaves as volume!

For instance, let us consider free scalars in 2+1 dimensions on the pp wave

$$ds^2 = -(1+y^2)dt^2 + dtdx + dy^2$$

We write the equation of motion for the scalar field

$$(1+y^2)\,\partial_x^2\phi + \frac{1}{4}\partial_y^2\phi + \,\partial_t\partial_x\phi = 0. \tag{4}$$

Substituting the plane-wave ansatz $\phi = e^{ikx}\,\varphi(t,y)$ with $k>0$, we find

$$i\,\partial_t\varphi = -\frac{1}{4k}\,\partial_y^2\varphi + \left(1+y^2\right)k\,\varphi, \tag{5}$$

(Stability requires $k>0$, in accordance with the general result $P_- \geq 0$.) The eigenmodes are

$$E_{n,k} = k + \left(n+\frac{1}{2}\right)\ .$$

We obtain for the single particle states in the pp-wave geometry

$$Z_1 = V \int_0^\infty \frac{dk}{2\pi} \sum_n e^{-\beta(k+n+\frac{1}{2})} = \frac{V}{4\pi} \frac{1}{\beta \sinh(\beta/2)} \tag{6}$$

Identifying the volume of the pp-wave with $V = \frac{2\pi}{\epsilon}$, we obtain via the Plethystic exponential

$$\log Z(\beta, \epsilon) = \frac{1}{\epsilon\, \beta} \sum_{m=1}^{\infty} \frac{1}{2m^2 \sinh \frac{m\beta}{2}} , \tag{7}$$

which matches the result from direct operator counting.

There are no phase transitions in this model.

A more complicated model is the $O(N)$ model, which we solve in the large N limit in this high spin regime.
For each longitudinal momentum $-i\frac{\partial}{\partial x} = p$ the problem reduces to the diagonalization of the one-dimensional operator

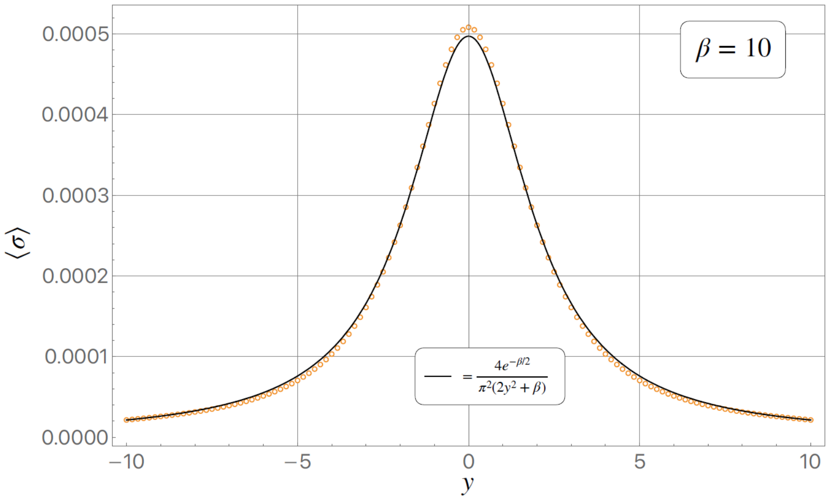
$$H_p[\sigma] = -\partial_y^2 + 4p^2y^2 + \sigma(y),$$

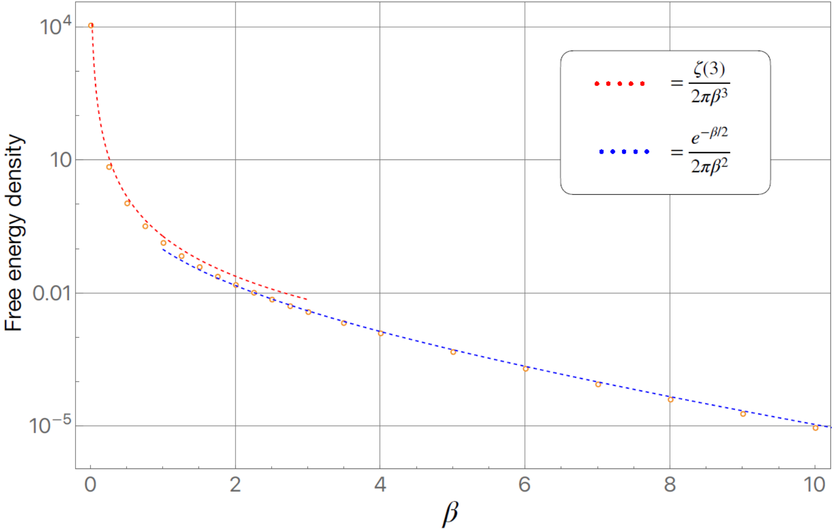
We then calculate the partition function

$$\frac{\log Z}{V_{pp-wave}} = -\int_0^\infty \frac{dp}{2\pi} \sum_a \log\left(1 - e^{-\beta E_a(p)}\right).$$

regualarize, and extremize over $\sigma(y)$.

We do not know how to do this analytically on the pp wave.





Free energy density
10^4
10
0.01
10^{-5}
0
2
4
6
8
10
β
$= \frac{\zeta(3)}{2\pi\beta^3}$
$= \frac{e^{-\beta/2}}{2\pi\beta^2}$

We see a crossover between operators with $\tau/\sqrt{J} \gg 1$ (fluid-like) vs small $\tau/\sqrt{J} \ll 1$ (Fock-space).

A richer example is $SU(2)$ $\mathcal{N} = 4$ SYM theory in 3+1 dimensions.

We can study several variants of the thermal partition function in the large spin limit.

While in flat space the theory is *deconfined* at any temperature, on the pp wave it is *confined* at zero temperature. The confinement is "geometric".

Why do gauge fields confine on the pp wave at zero temperature? The response to a static point charge at $x_1 = x_2 = 0$, $x = x_0$ is described by

$$\Phi(r, v) = q \int_{-\infty}^{\infty} \frac{dk}{2\pi} e^{ik(x-x_0)} g_k(r),$$

where

$$\left[-\frac{1}{r}\frac{d}{dr}\left(\frac{r}{1+r^2}\frac{d}{dr}\right) + k^2\right] g_k(r) = \frac{\delta(r)}{2\pi r}.$$

The electromagnetic field is

$$F = -\left(dt - \frac{dx}{1+r^2}\right) \wedge d\Phi.$$

The stored Killing energy is

$$\mathcal{E} = \frac{1}{2}\int dx_1 dx_2 dx \left[\frac{|\nabla_\perp \Phi|^2}{1+r^2} + (\partial_x \Phi)^2\right].$$

For a point charge this energy is divergent both in the UV and in the IR:

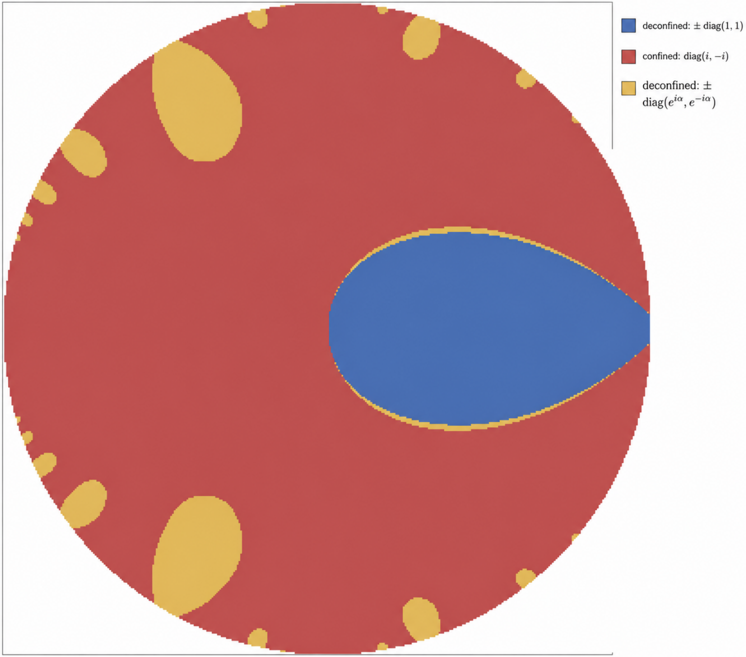
$$\mathcal{E} = \frac{q^2}{8\pi\epsilon} + \frac{q^2}{2\pi^2}\log L + O(1)$$

It is related to to
[Gross,Wilczek; Korchemsky, Radyushkin; Gubser, Klebanov, Polyakov; Alday, Maldacena...]

Let us begin with the large spin limit of the partition function of $SU(2)$ $\mathcal{N}=4$ theory:

$$Z(\beta,\varepsilon)=\sum e^{-\beta\tau-\beta\varepsilon J} \ .$$

We obtain at *weak coupling* the following phase diagram:



deconfined: ± diag(1, 1)
confined: diag(i, $-i$)
deconfined: ± diag($e^{i\alpha}$, $e^{-i\alpha}$)

- We see zero temperature confinement and deconfinement at any nonzero temperature.
- At complexified temperatures other phases are encountered.
- Most of the literature is about the phases at infinite N, e.g. [...Cherman, Dhumunturao ...] here it is just $SU(2)$ SYM.

Since $\Delta - J$ commutes with supersymmetry, we can preserve supersymmetry in the pp wave. We can calculate the usual partition function or a protected version thereof.

$$Z_{protected} = \mathrm{Tr}_{BPS}(-1)^F p^{\Delta - J + \frac{2}{3}r} \ .$$

The theory is placed in a pp wave with a constant chemical potential. We denote $p = e^{-\beta}$.

The chemical potential is needed because we also want to be able to regularize the partition function in a SUSY-preserving fashion.

$$Z_{protected} = \mathrm{Tr}_{BPS}(-1)^F p^{\Delta - J + \frac{2}{3}r} \ .$$

This is regularized by adding $\epsilon(\Delta + J)$ the Hamiltonian. This is identical to how the non-SUSY partition functions are regularized.

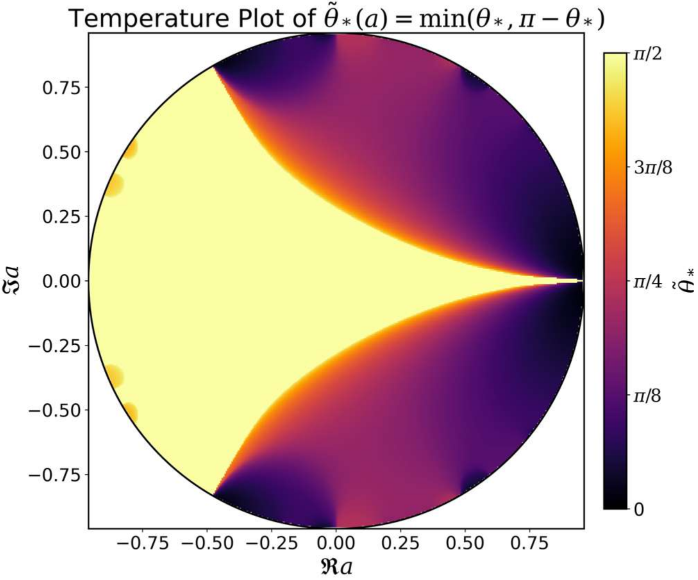
$$Z_{protected} \sim \int d\mu_{SU(2)}(\theta)\ \exp\left[\frac{1}{\epsilon} W_p(\theta)\right] ,$$

with

$$W_p(\theta) = \sum_{n=1}^{\infty} \frac{-1}{n^2} \frac{(1-p^{n/3})^2}{1+p^{n/3}+p^{2n/3}} \left[1 + 2\cos(2n\theta)\right] \ ,$$

and $p = e^{-\beta}$.

In the complex plane $a = p^3$ we find the following phase diagram



We see confinement at low temperatures and deconfined ("fortuitous") phases at complexified temperatures.

We would like to identify this transition in the highly spinning cohomology with **fortuitous** states
[Chang, Lin; Choi, Kim, Lee, Park; ...]

Indeed, this phase diagram alone proves that there are far more elements in the BPS cohomology than just the "gravitons" Fock space.

Consider again the metric

$$ds^2 = -\left(1 + \sum_{i=1}^{d-1} x_i^2\right) dt^2 + dxdt + \sum_{i=1}^{d-1} dx_i^2 \ .$$

It describes time evolution by

$$\underbrace{\Delta + M_{+-}}_{=\tau, \geq 0} + \underbrace{P_-}_{\geq 0} \ .$$

A variant of it,

$$ds^2 = -\sum_{i=1}^{d-1} x_i^2 dt^2 + dxdt + \sum_{i=1}^{d-1} dx_i^2 \ ,$$

describes time evolution by $\Delta + M_{+-}$.
In the absence of the $1+$, there is infinite degeneracy but the spectrum is bounded from below.

Now fix $d = 3$ and pass to a rotating frame in the x_1, x_2 plane. In flat space that means that for $\sqrt{x_1^2 + x_2^2} > 1$ we exceed the speed of light and the theory is sick. The metric in this rotating frame has a corresponding sickness.

Indeed, in flat space, $H - J$ is not bounded from below.

In our case the speed of light "grows" as $\sqrt{x_1^2 + x_2^2} > 1$ and hence the metric does not have any sickness in the rotating frame. Therefore, causality suggests

$$\boxed{H - J = \Delta - J_1 - J_2 \geq 0\ .}$$

This is a new unitarity bound. Previously it was know that $\Delta - max(J_1, J_2) > 0$.
(In special cases stronger bounds can be found in [Cordova,Diab].)

Geometry in $3+1$ dimensions: a family of null geodesics

The physics of the situation with $J_{1,2} \to \infty$ with $J_1/J_2 = \text{finite}$ is interesting.

With two large angular momenta, fast-spinning partons live at *different* latitudes on S^3.

So one cannot zoom around a single null geodesic. Instead one performs a generalized Penrose limit focusing on a family of null geodesics.

$$ds^2 = -(dt' + 2udy)^2 + (dv + 2udy)^2 + du^2 + dy^2 .$$

The isometries include

$$\times(H_3 \rtimes SO(2)),$$

with H_3 preserved by time translations.

$$ds^2 = -(dt' + 2udy)^2 + (dv + 2udy)^2 + du^2 + dy^2.$$

This metric indeed has a global timelike Killing vector and no ergosphere.

If we begin with

$$ds^2 = -\left(1 + \sum_{i=1}^{d-1} x_i^2\right) dt^2 + dxdt + \sum_{i=1}^{d-1} dx_i^2 \ .$$

and transform to a rotating frame we obtain the same result.

Locality implies

$$\log Z = \beta\,\mathrm{Vol}(H_3)\,F(\beta).$$

with the replacement

$$\mathrm{Vol}(H_3) \longleftrightarrow \frac{\pi^2}{2\varepsilon_1\varepsilon_2}.$$

Therefore, In $3+1$ dimensions, with, $\tau = \Delta - J_1 - J_2$

$$Z(\beta,\varepsilon_{1,2}) = \sum_{\mathcal{O}} e^{-\beta\tau-\beta\varepsilon_1 J_1-\beta\varepsilon_2 J_2} = \mathrm{Exp}\left[\frac{\beta\pi^2}{2\varepsilon_1\varepsilon_2}F(\beta)+O(\varepsilon^0)\right].$$

A subtlety is that since we only know that

$$\boxed{H - J = \Delta - J_1 - J_2 \geq 0}$$

but we do not know how to prove that there is a twist gap, the Fock space picture at low temperature may or may not be universal. (In fact the free gauge field saturates the bound.) Either way the Fock space picture breaks down for

$$\tau_{\text{strong}} \sim (J_1 J_2)^{1/3} \qquad (3+1 \text{ d with two large spins}).$$

The appropriate limit that we have been describing in 3+1 dimensions is

$$J_{1,2} \to \infty \ , \quad J_1/J_2 = \text{finite} \ , \quad \frac{\tau}{(J_1 J_2)^{1/3}} = \text{finite} \ .$$

The geometry

$$ds^2 = -(dt' + 2udy)^2 + (dv + 2udy)^2 + du^2 + dy^2 .$$

captures precisely these operators.

- Large spin with fixed $\tau/\sqrt{J}$ and $\tau/(J_1 J_2)^{1/3}$ is captured by pp waves.
- Large spin creates an *effective thermodynamic limit*. This allows phase transitions and we have discussed an example of a phase transition between *monotonous* and *fortuitous* states.
- Locality on pp waves provides a universal explanation of the large-spin free-energy scalings.
- In $3+1$ dimensions, causality leads to a new unitarity bound

$$\Delta - J_1 - J_2 \geq 0 \ .$$

- We can calculate the J dependence of *HLH* OPE coefficients and there are various other applications.