

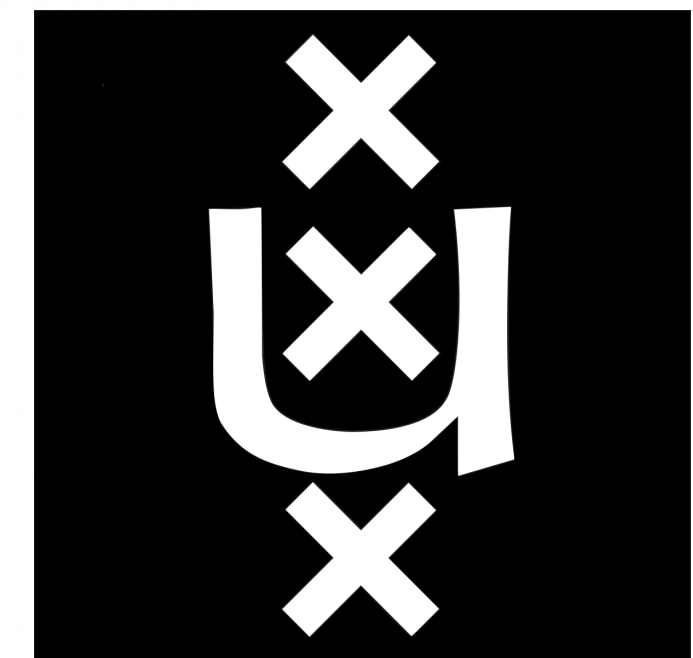
Bootstrapping de Sitter boundary

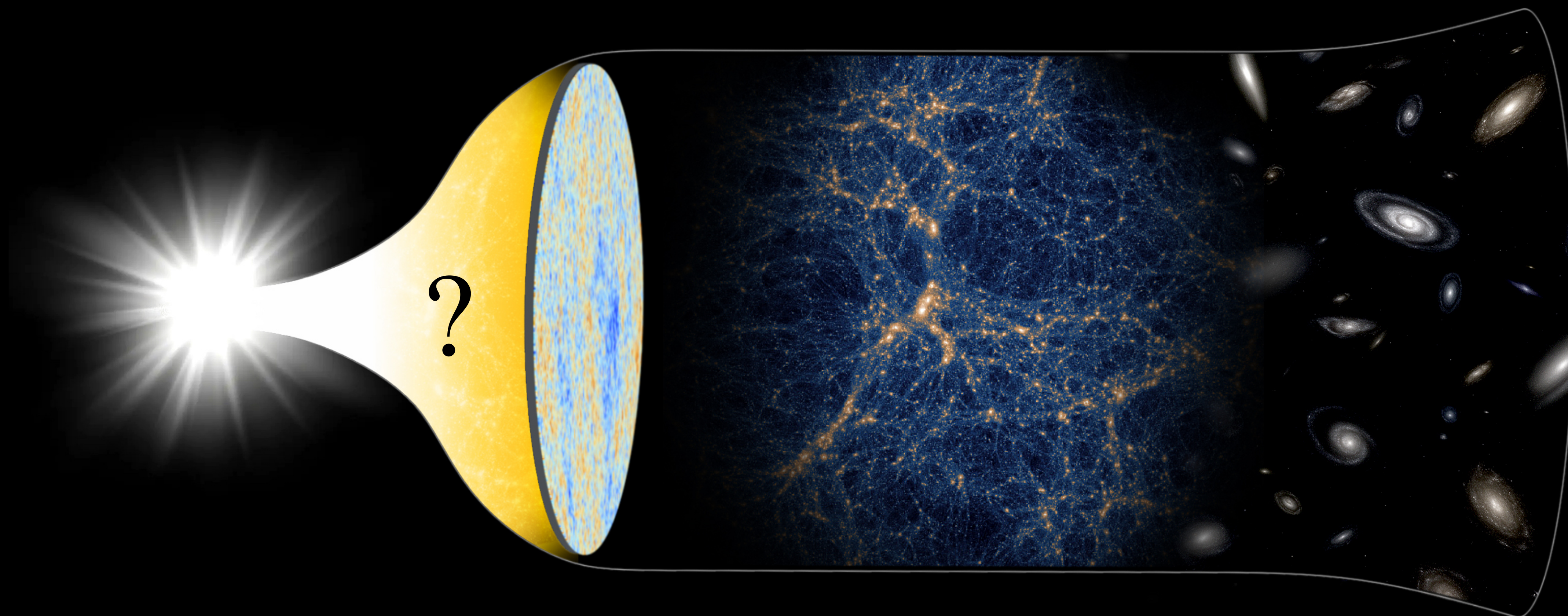
Kamran Salehi Vaziri

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- [WP] Dalimil Mazac, KSV, Xiang Zhao
- [2412.00183] KSV
- [2306.00090] Manuel Loparco, João Penedones, KSV, Zimo Sun
- [2301.04146] João Penedones, KSV, Zimo Sun
- [2107.13871] Matthijs Hogervorst, João Penedones, KSV





Why de Sitter?

- **Cosmology:** A good approximation of the early universe (inflation) and the future!
- **Particle Physics:** Access to high energies 10^{15} GeV: the Cosmological Collider
- **QFT:** A simple example of QFT in the curved background, simplest after flat and AdS
- **Quantum Gravity:** A simple playground for quantum gravity other than AdS

Why non-perturbative?

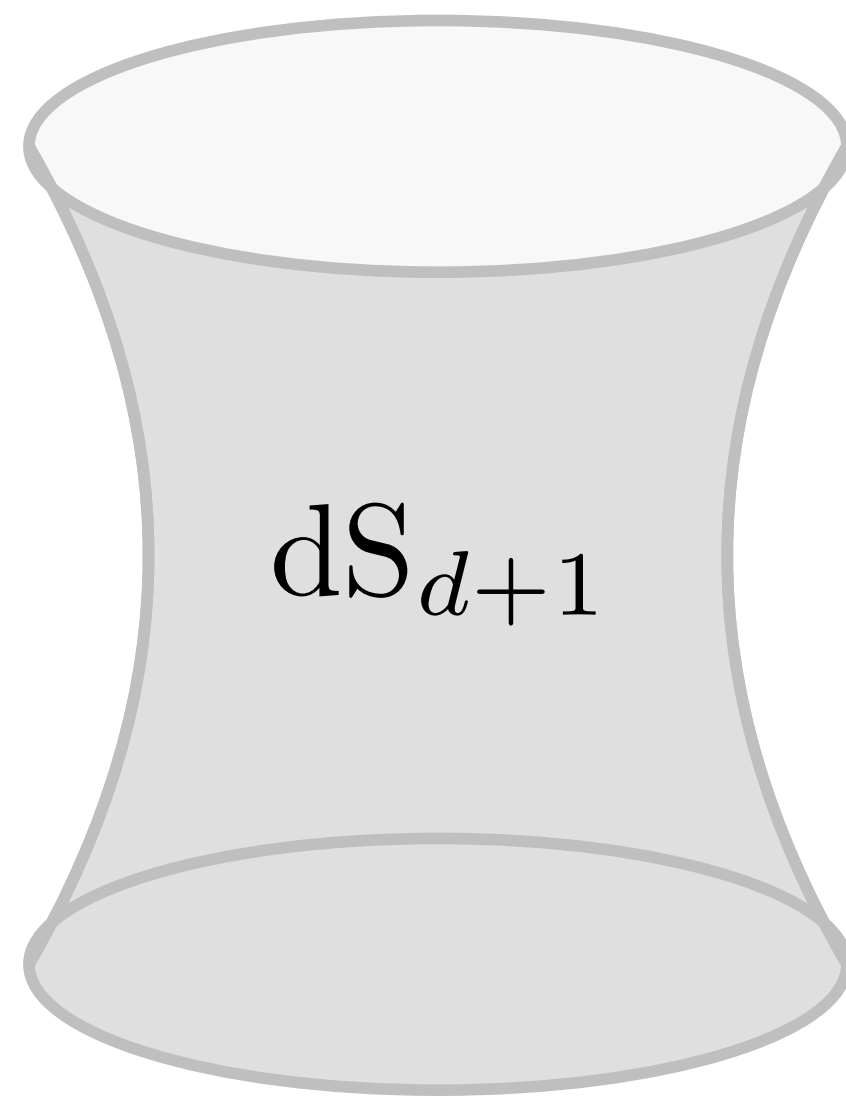
- We **should**
- **IR/Secular divergences:**
Perturbative calculations lead to IR divergences and late-time growth

Sharp Questions?

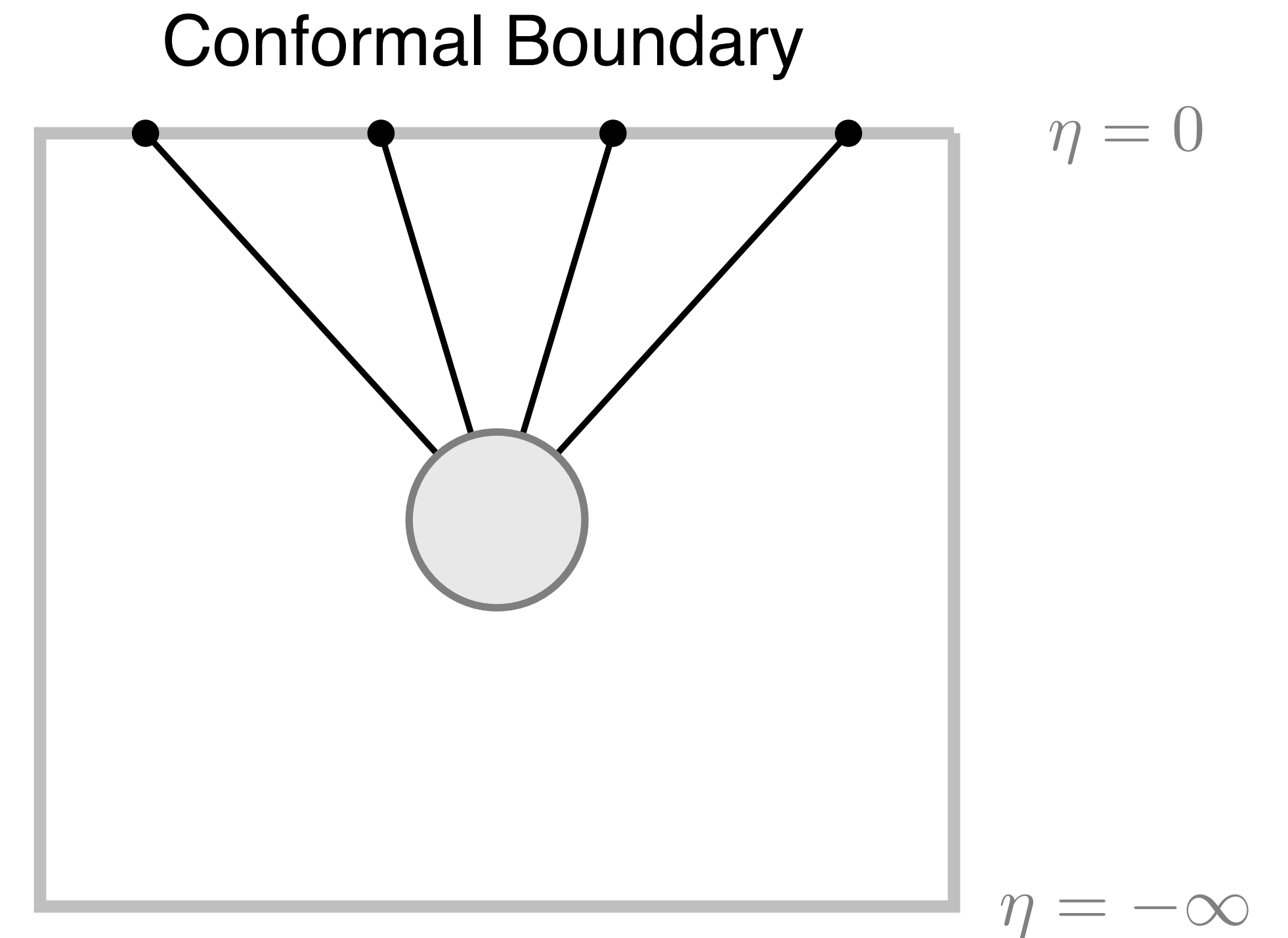
- Is single field inflation compatible with **unitarity**?
- Is there any **holographic** description for quantum gravity in the bulk of de Sitter?

de Sitter

$$-Y_0^2 + Y_1^2 + \cdots + Y_{d+1}^2 = R^2$$



Isometry Group: $SO(d+1, 1)$
 D , P_i , K_i , M_{ij}



$$ds^2 = \frac{-d\eta^2 + d\vec{y}^2}{\eta^2}$$

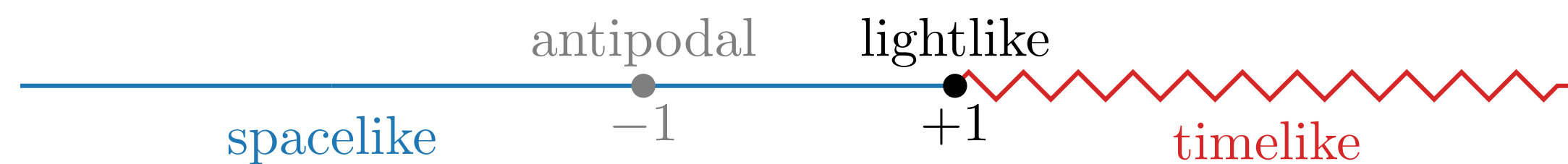
Free scalar field in de Sitter

- Casimir: $C = \Delta(d - \Delta) = m^2 R^2 \quad \Delta = \frac{d}{2} \pm \sqrt{\frac{d^2}{4} - m^2 R^2} = \frac{d}{2} \pm i\lambda$

- Bulk-to-bulk propagator:

$$G_{\text{free}}(\sigma) = {}_2F_1 \left[\Delta, d - \Delta, \frac{d+1}{2}, \frac{1+\sigma}{2} \right]$$

$$\sigma = Y_1 \cdot Y_2 = \frac{\eta_1^2 + \eta_2^2 - y_{12}^2}{2\eta_1\eta_2}$$



- Boundary operators: $\Delta \notin$ Unitary bounds

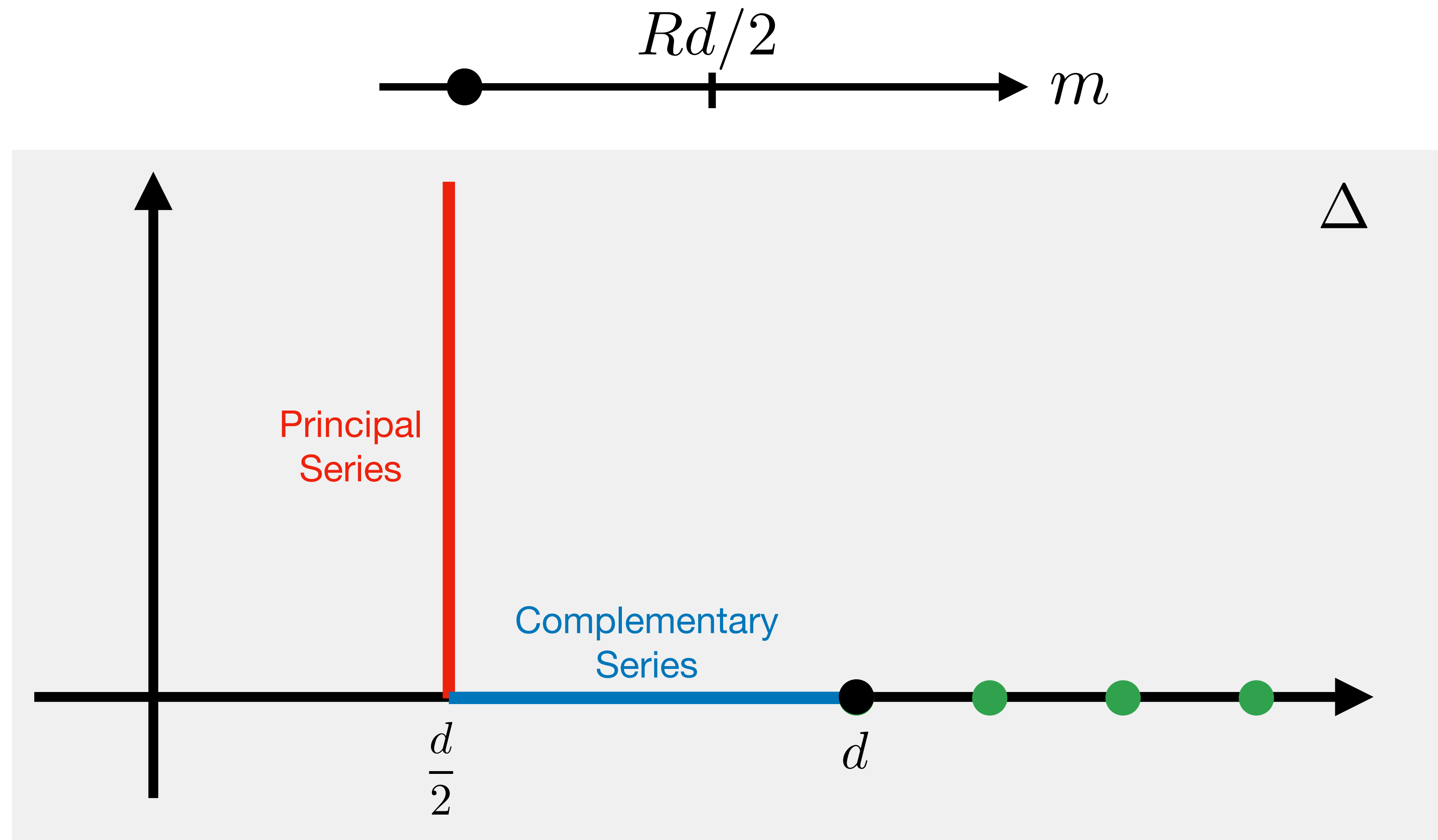
$$\langle \mathcal{O}_\Delta(\vec{y}_1) \mathcal{O}_\Delta(\vec{y}_2) \rangle = \frac{1}{y_{12}^{2\Delta}}$$

$$\langle \mathcal{O}_\Delta(\vec{y}_1) \mathcal{O}_{\bar{\Delta}}(\vec{y}_2) \rangle \propto \delta^{(d)}(\vec{y}_{12})$$

$$\phi \sim (-\eta)^\Delta \mathcal{O}_\Delta(\vec{y}) + (-\eta)^{\bar{\Delta}} \mathcal{O}_{\bar{\Delta}}(\vec{y})$$

$$G_{\text{free}}(\sigma)$$

Unitary irreducible representations

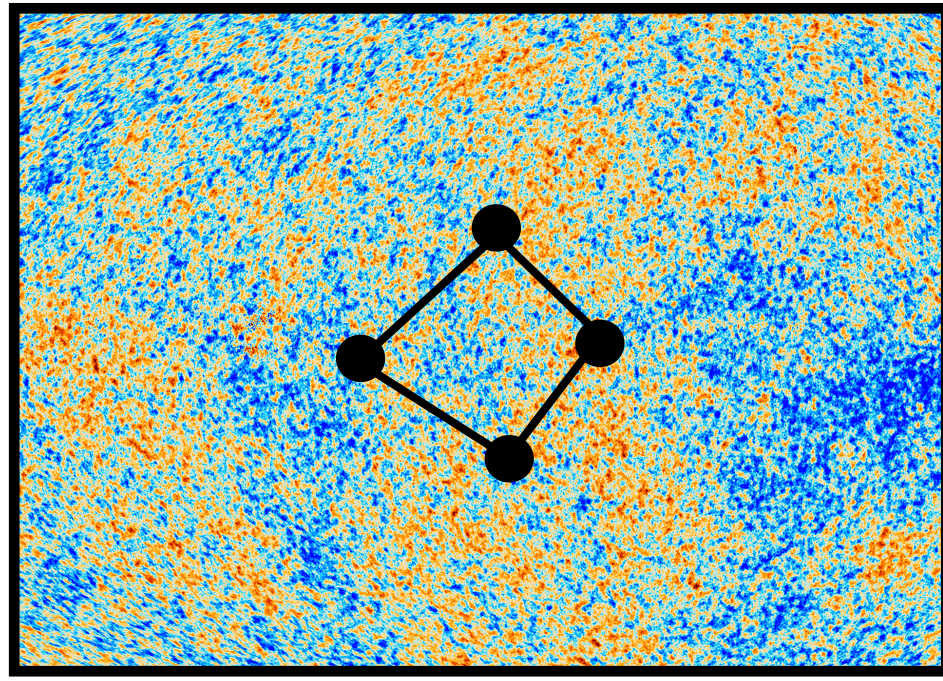


Bulk Hilbert space: $|\Delta, \vec{y}\rangle \longrightarrow \mathbb{1} = \int_{\Delta \in \text{irreps}} \int_{\vec{y}} |\Delta, \vec{y}\rangle \langle \Delta, \vec{y}|$

How do we study QFT in dS Non-perturbatively?

One way: Conformal boundary $\longrightarrow$ Conformal bootstrap

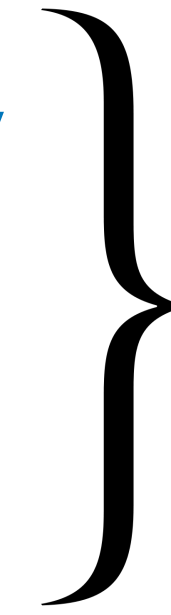
Non-perturbative Cosmological Bootstrap



Conformal Symmetry

Bulk Unitarity

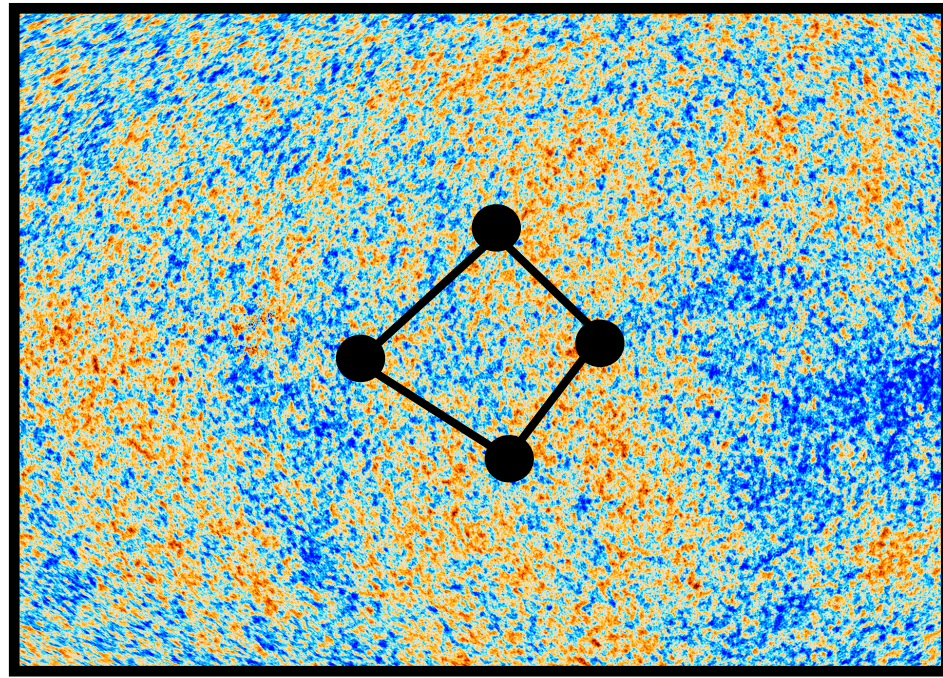
Crossing Symmetry



**Bounds on
Cosmological Observables**

$$\begin{aligned}
 \langle \mathcal{O}(x_1) \dots \mathcal{O}(x_4) \rangle_{\text{CFT}} &= \sum_{\Delta, \ell} \lambda_{\Delta, \ell}^2 g_{\Delta, \ell}(z, \bar{z}) \\
 \uparrow & \quad \quad \quad \uparrow \\
 \text{OPE} \sim \text{resolution} & \quad \quad \quad \text{Unitarity} \quad \quad \text{Conformal} \\
 \text{of identity} & \quad \quad \quad \geq 0 \quad \quad \text{symmetry}
 \end{aligned}
 \qquad
 \begin{aligned}
 \langle \mathcal{O}(x_1) \dots \mathcal{O}(x_4) \rangle_{\text{dS}} &= \sum_{\ell} \int_{\Delta} I_{\Delta, \ell} \Psi_{\Delta, \ell}(z, \bar{z}) \\
 \uparrow & \quad \quad \quad \uparrow \\
 \mathbb{1} = \int_{\Delta, \vec{y}} |\Delta, \vec{y}\rangle \langle \Delta, \vec{y}| & \quad \quad \quad \text{Unitarity} \quad \quad \text{dS} \\
 & \quad \quad \quad \geq 0 \quad \quad \text{Symmetry}
 \end{aligned}$$

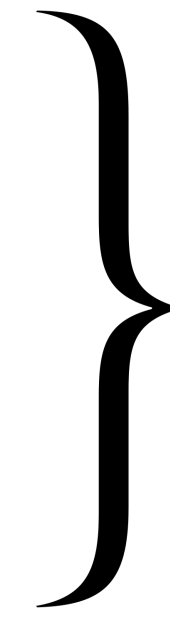
Non-perturbative Cosmological Bootstrap



Conformal Symmetry

Bulk Unitarity

Crossing Symmetry



**Bounds on
Cosmological Observables**

$$\langle \mathcal{O}(x_1) \dots \mathcal{O}(x_4) \rangle_{\text{CFT}} = \sum_{\Delta, \ell} \lambda_{\Delta, \ell}^2 g_{\Delta, \ell}(z, \bar{z}) \quad \langle \mathcal{O}(x_1) \dots \mathcal{O}(x_4) \rangle_{\text{dS}} = \sum_{\ell} \int_{\Delta} I_{\Delta, \ell} \Psi_{\Delta, \ell}(z, \bar{z})$$

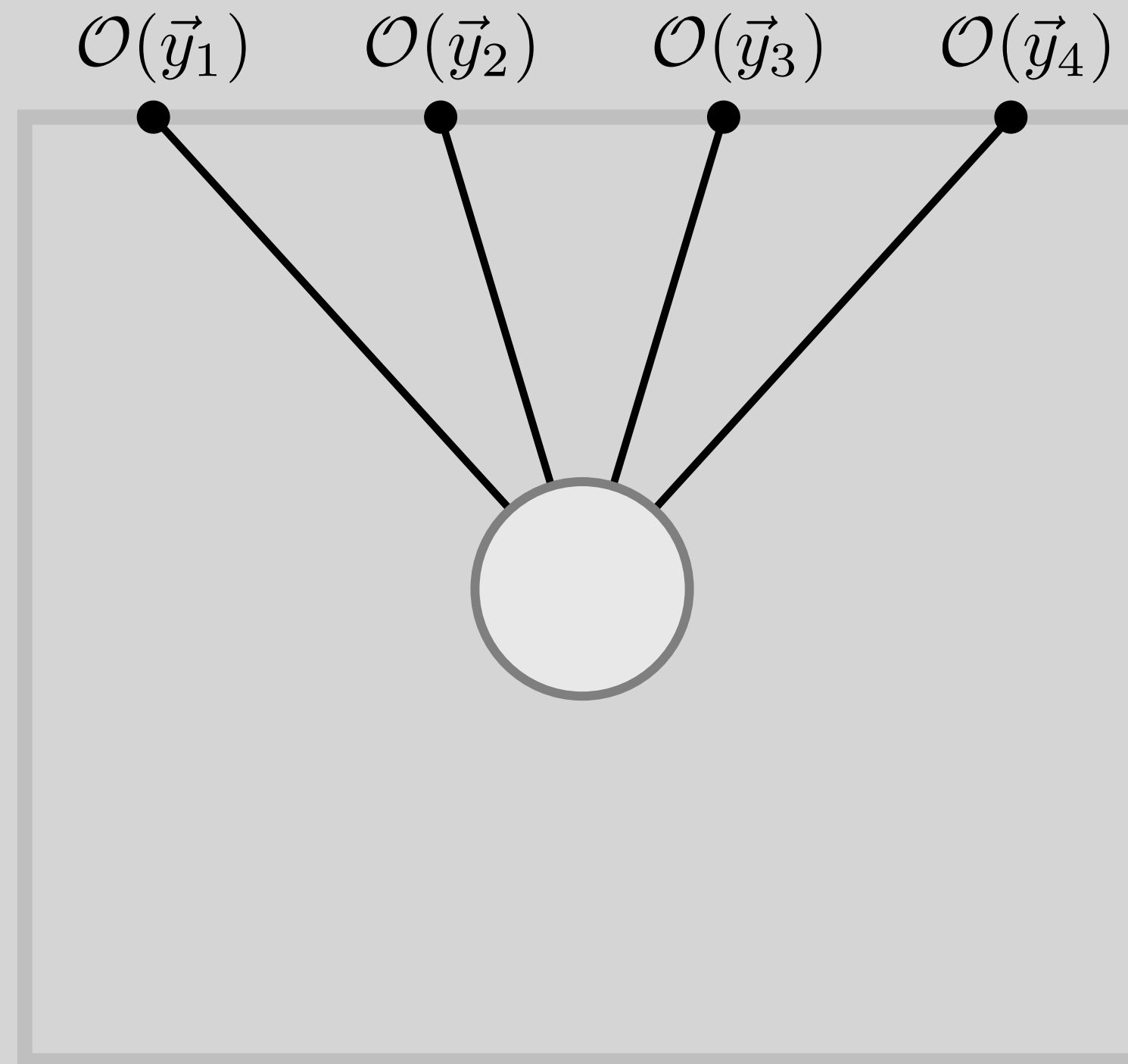
- Conceptual challenges: *What are the boundary operators?*
- Technical challenges: *Numerical bootstrap setup, Convergence issues*

Proof of concept bounds for 2d:
[Hogervorst, Penedones, KSV '21]

$$0 < I_{\Delta, \ell} < \#$$

[WP; Mazac, KSV, Zhao]
[Bonifacio, Kravchuk, Mazac, Pal 23]

What are these boundary operators?
What is their spectrum?



Given a bulk theory, how do we find them?

Bulk Spectrum: Kallen Lehmann

A spectral decomposition of the two-point function into a sum/integral over free propagators

$$\langle \phi(x_1) \phi(x_2) \rangle = \int_{\mu} \rho(\mu) G_{\mu}^{\text{free}}(x_{12})$$

Minkowski

$$\langle \phi(Y_1) \phi(Y_2) \rangle = \int_{\Delta} \rho(\Delta) G_{\Delta}^{\text{free}}(\sigma)$$

$\mathbb{1} = \int_{\Delta, \vec{y}} |\Delta, \vec{y}\rangle \langle \Delta, \vec{y}|$

Unitarity ≥ 0

dS Symmetry

[Bros, Moschella, 95]

[Hogervorst, Penedones, KSV 21]

[Di Pietro, Gorbenko, Komatsu 21]

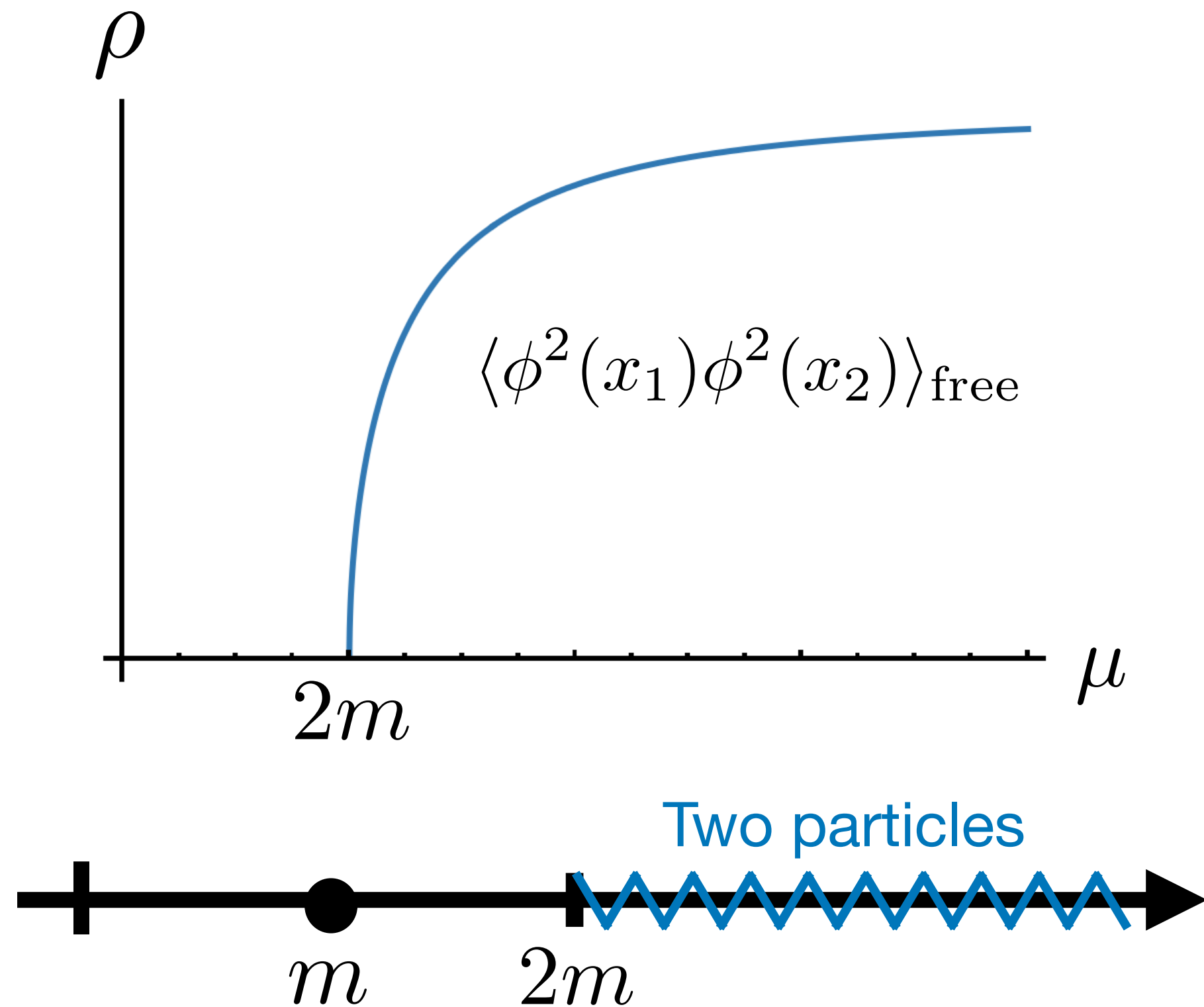
[Loparco, Penedones, KSV, Sun 23]

de Sitter

Bulk Spectrum: Kallen Lehmann

A spectral decomposition of the two-point function into a sum/integral over free propagators

$$\langle \phi(x_1) \phi(x_2) \rangle = \int_{\mu} \rho(\mu) G_{\mu}^{\text{free}}(x_{12})$$



$$\langle \phi(Y_1) \phi(Y_2) \rangle = \int_{\Delta} \rho(\Delta) G_{\Delta}^{\text{free}}(\sigma)$$

?

How to calculate de Sitter spectral density?

Spectrum: Inversion formula

- Analytic continuation from sphere:

Sommerfeld-Watson transformation + Froissart-Gribov formula type analysis

$$\rho(\Delta) \propto \int_1^\infty d\sigma {}_2F_1 \left[1 - \Delta, 1 - \bar{\Delta}, \frac{3-d}{2}, \frac{1-\sigma}{2} \right] \text{Disc} [\langle \phi \phi \rangle]$$



[Hogervorst, Penedones, KSV 21]

- Analytic continuation from EAdS:

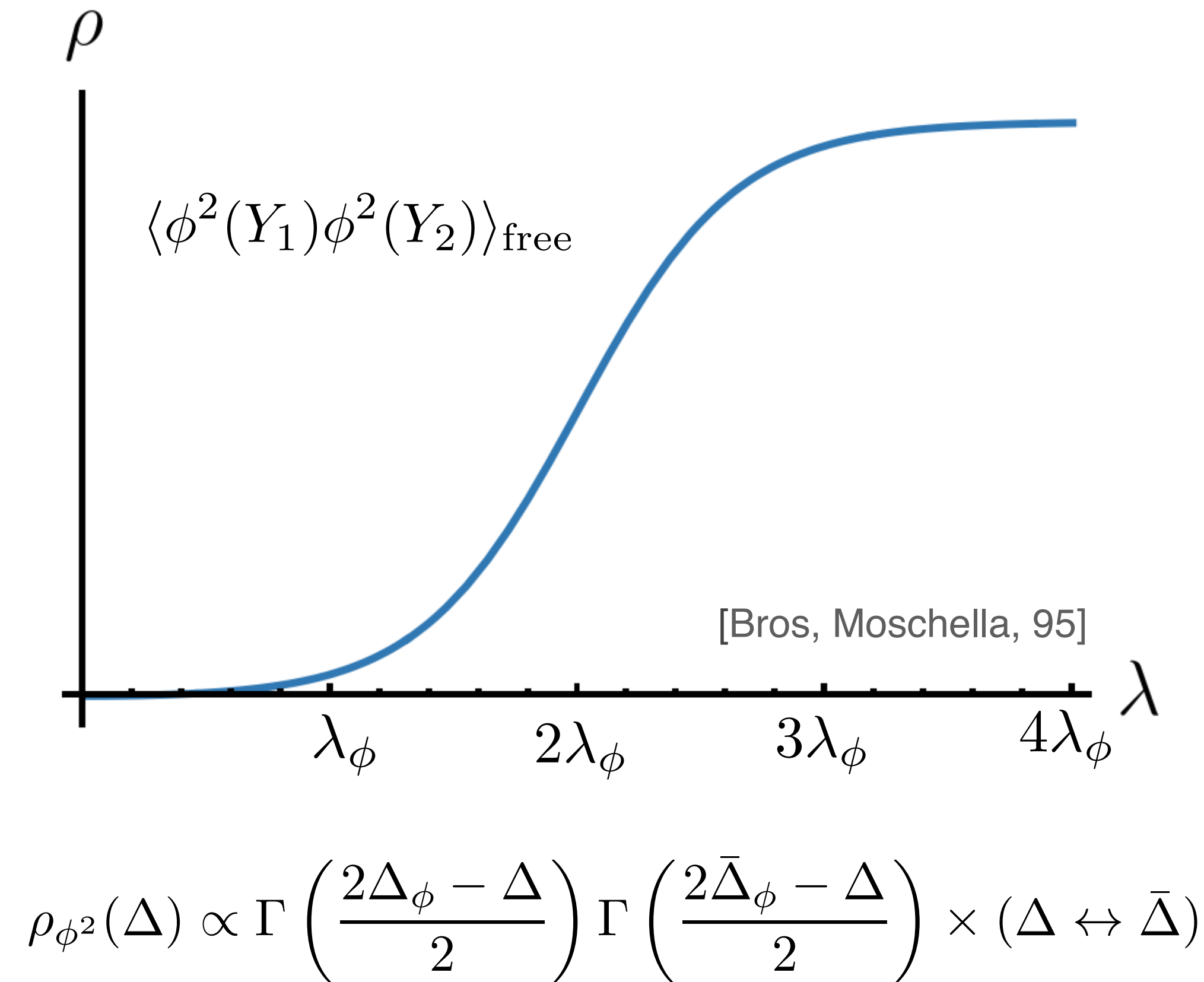
Harmonic functions orthogonality: Works for **any spin**

$$\rho(\Delta) \propto \int_{-\infty}^{-1} d\sigma (\sigma^2 - 1)^{\frac{d-1}{2}} {}_2F_1 \left[\Delta, \bar{\Delta}, \frac{d+1}{2}, \frac{1+\sigma}{2} \right] \langle \phi \phi \rangle$$

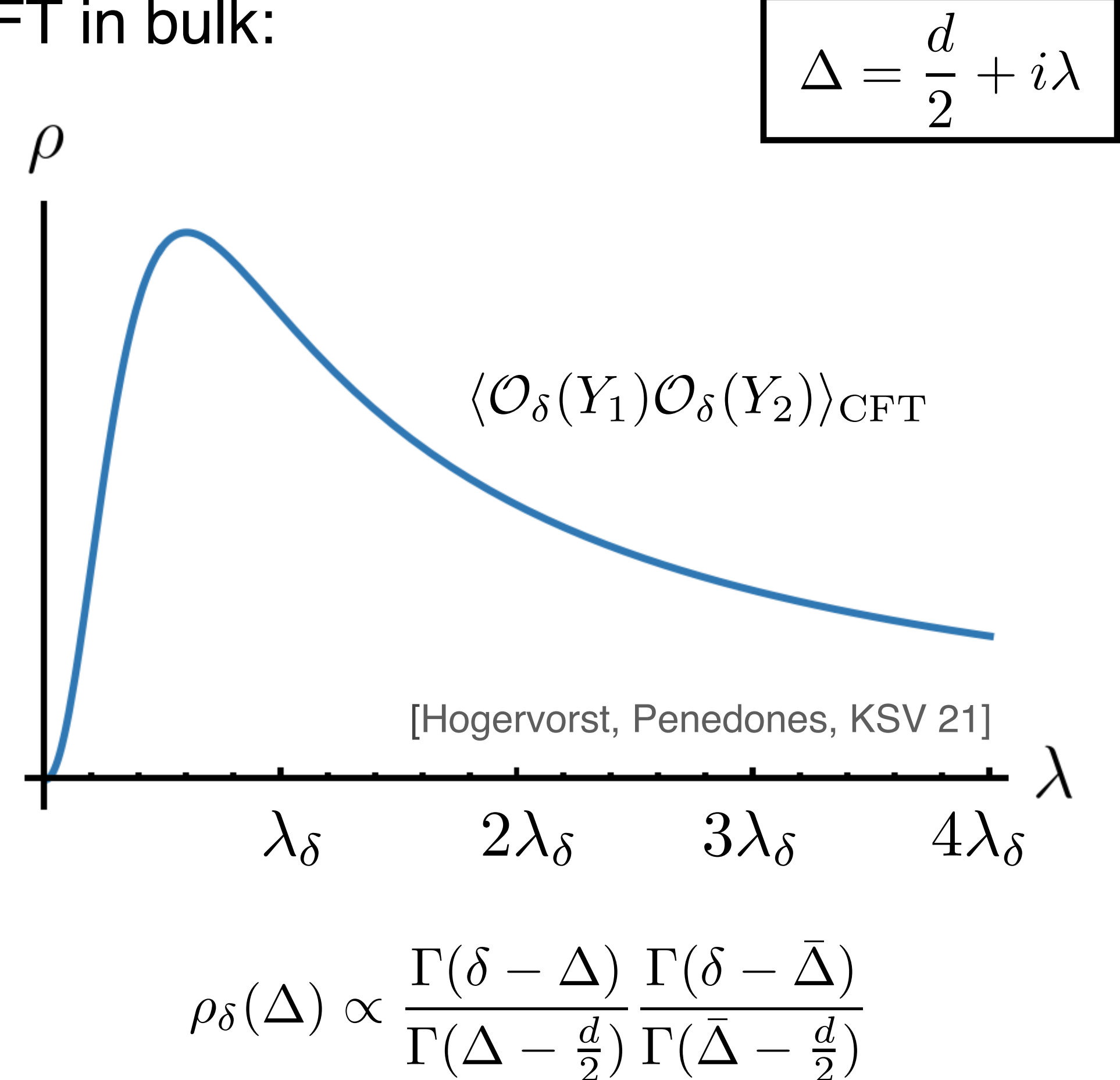
[Loparco, Penedones, KSV, Sun 23]

Spectrum: Examples

Free theory composite operator:



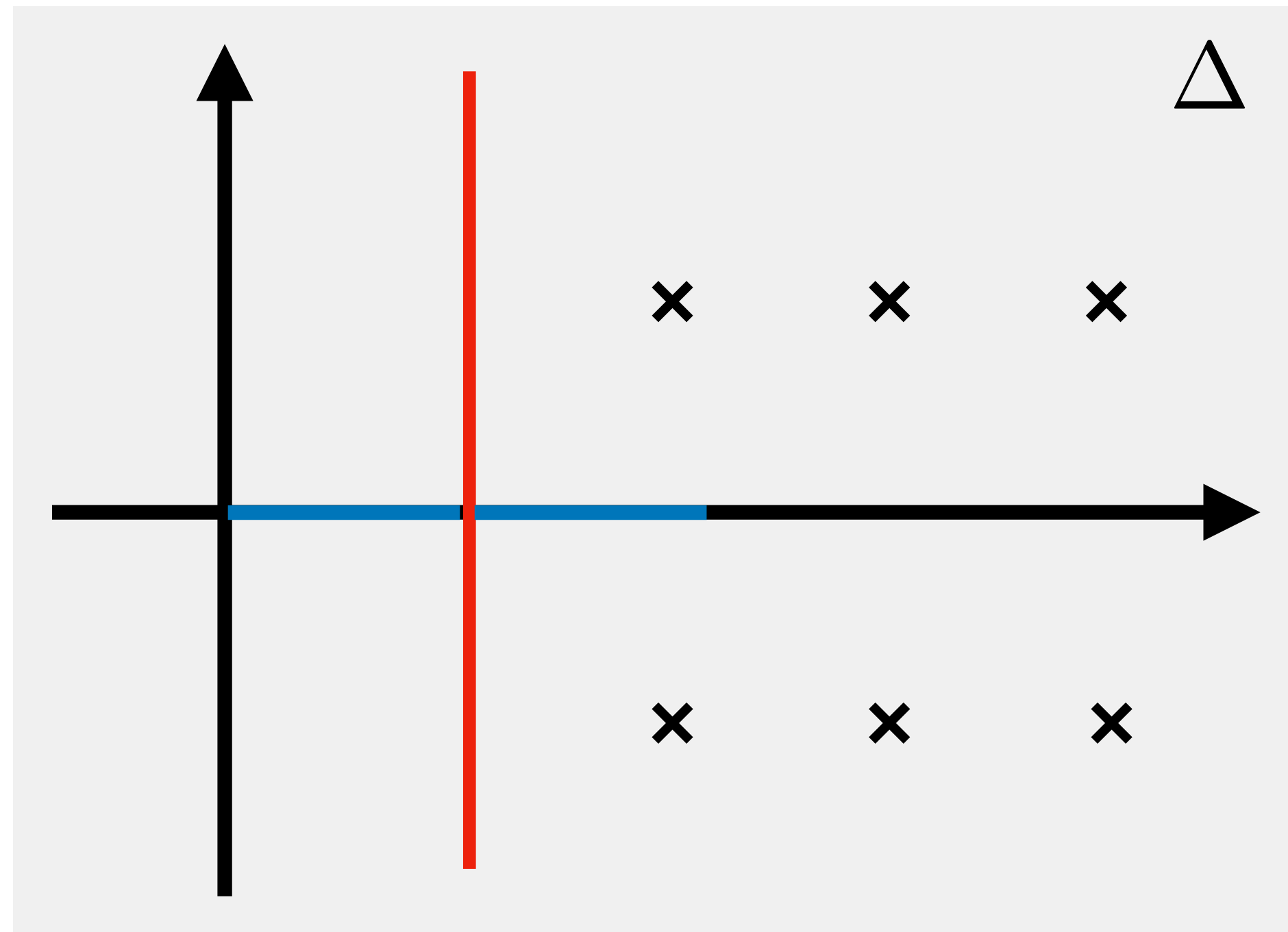
CFT in bulk:



Principal series are continuously furnished with no gap!

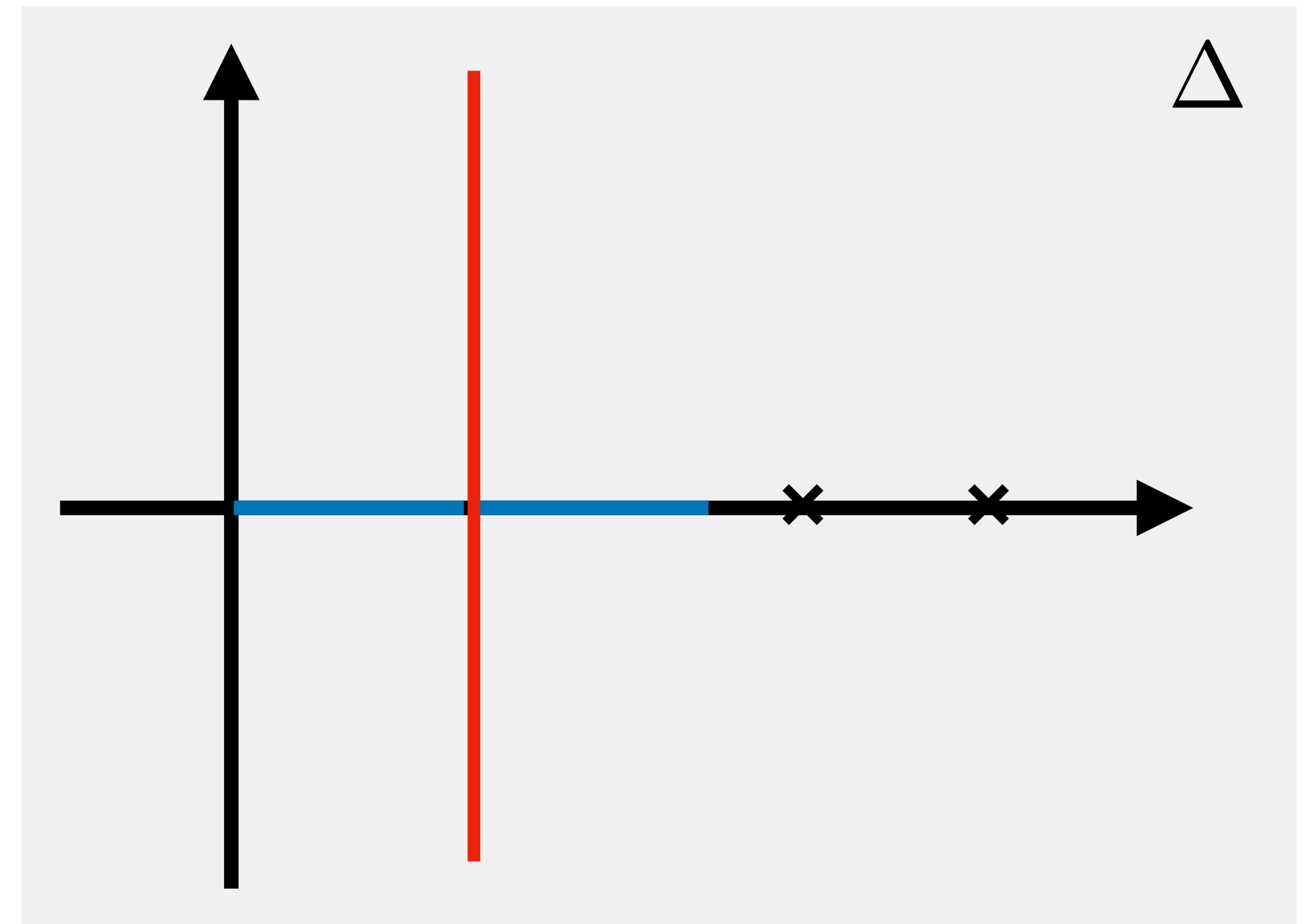
Kallen Lehmann: Analytic Structure

Free theory composite operator:



$$\rho_{\phi^2}(\Delta) \propto \Gamma\left(\frac{2\Delta_\phi - \Delta}{2}\right) \Gamma\left(\frac{2\bar{\Delta}_\phi - \Delta}{2}\right) \times (\Delta \leftrightarrow \bar{\Delta})$$

CFT in bulk:

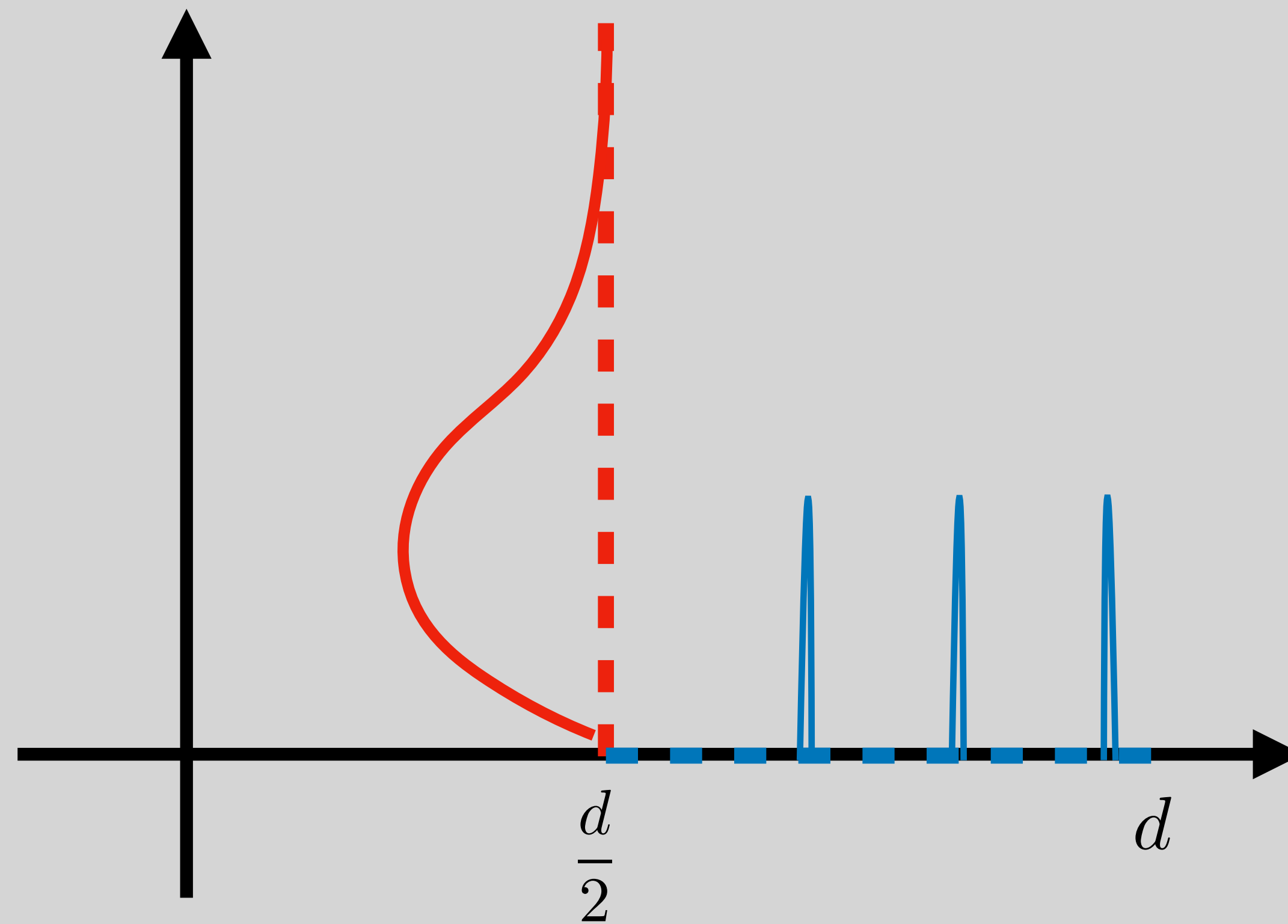


$$\rho_\delta(\Delta) \propto \frac{\Gamma(\delta - \Delta)}{\Gamma(\Delta - \frac{d}{2})} \frac{\Gamma(\delta - \bar{\Delta})}{\Gamma(\bar{\Delta} - \frac{d}{2})}$$

Principal series are continuously furnished with no gap!

Bulk spectrum

Complementary series: Discrete
Principal series: Continuous with no gap



What about the boundary spectrum?

AdS vs dS Boundary

Bulk state

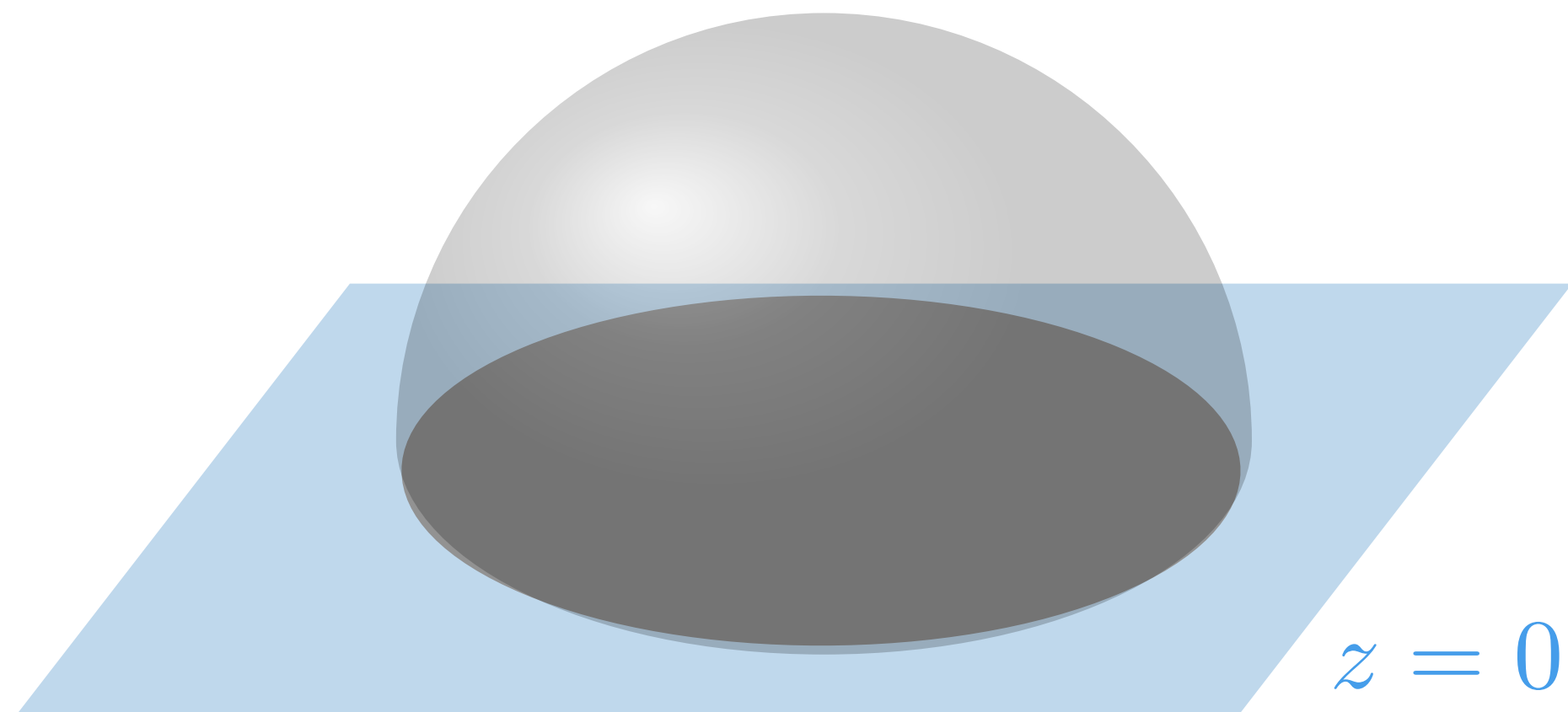
bulk unitarity
discrete spectrum

Boundary operator

reflection positive
discrete set

$$\phi(\vec{x}, z) = \sum_i b_i z^{\Delta_i} \mathcal{O}_i(\vec{x}) + \text{des}$$

[Paulos, Penedones, Toledo, van Rees, Viera; 16]



AdS vs dS Boundary

Bulk state

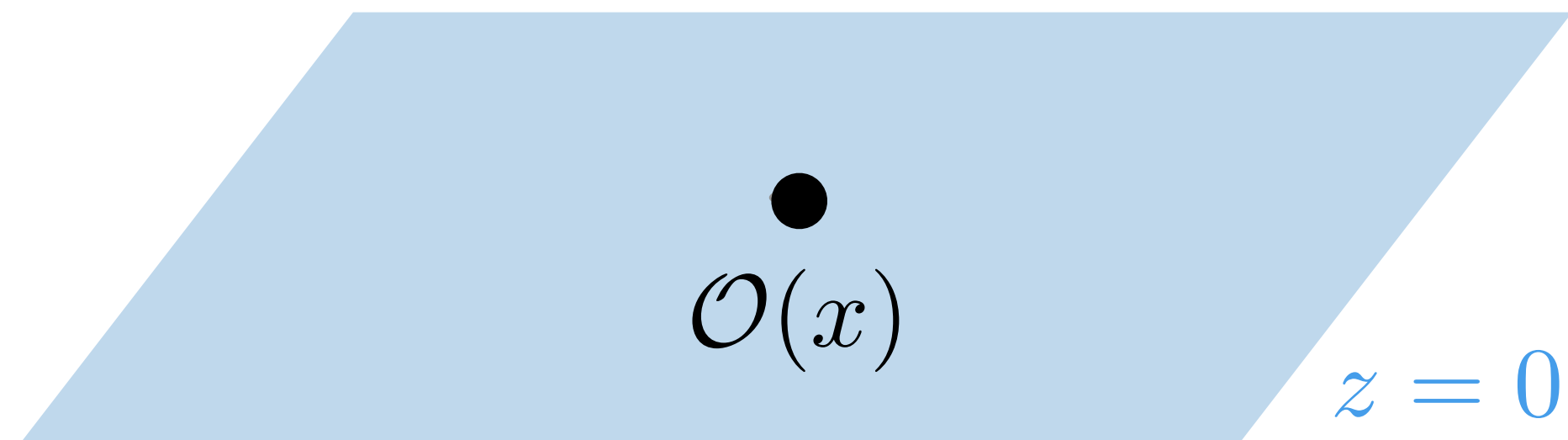
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[Paulos, Penedones, Toledo, van Rees, Viera; 16]



Naive copy of AdS

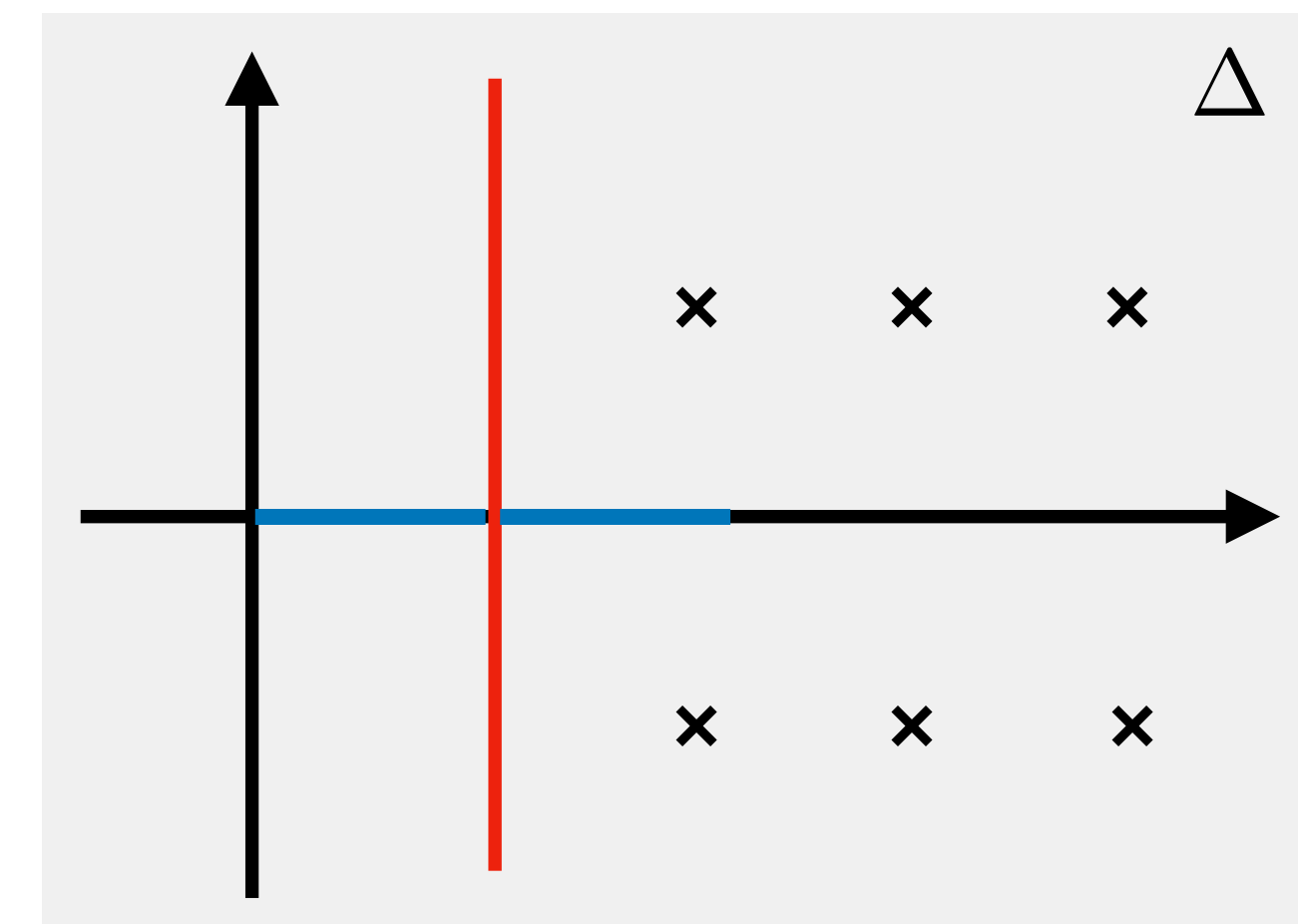
$$\phi(\vec{y}, \eta) = \sum_i b_i (-\eta)^{\Delta_i} \mathcal{O}_i(\vec{y}) + \text{des}$$



Matching with boundary expansion
of Kallen Lehmann



$\Delta_i \longleftrightarrow$ Poles of $\rho(\Delta)$



AdS vs dS Boundary

Naive copy of AdS:

Creating states with dimension different from UIRs

$$|\psi_i\rangle \equiv \mathcal{O}_i(\vec{y}')|0\rangle \xrightarrow{\Delta \neq \Delta_i} \langle \Delta, \vec{y} | \psi_i \rangle = 0$$

- **Paradox:** Zero overlap with resolution of identity

$$\int_{\Delta, \vec{y}} \langle \phi(Y_1) | \mathbb{1} | \phi(Y_2) \rangle = \int_{\Delta, \vec{y}} \langle \phi(Y_1) | \Delta, \vec{y} \rangle \langle \Delta, \vec{y} | \phi(Y_2) \rangle = \int_{\Delta, \vec{y}} \sum_{ij} b_i b_j (-\eta)^{\Delta_i + \Delta_j} \underbrace{\langle 0 | \mathcal{O}_i | \Delta, \vec{y} \rangle \langle \Delta, \vec{y} | \mathcal{O}_j | 0 \rangle}_0$$

Not a suitable bulk-to-boundary expansion for spectral decomposition like partial waves!

What is a suitable bulk-to-boundary expansion?

Boundary dS

Principal series contribution:

$$\left. \begin{array}{l} 1. \text{ Continuous spectrum} \\ 2. \text{ Operators living on Principal series} \end{array} \right\} \phi(\vec{y}, \eta) = \int_{-\infty}^{\infty} d\lambda \, a_{\Delta} \hat{D}_{\Delta}(\vec{y}, \eta) \mathcal{O}_{\Delta}(\vec{y})$$

[KSV, 25]

- $\hat{D}_{\Delta}(\vec{y}, \eta)$ is an operator that produces the descendants and is fixed by symmetries:

$$\hat{D}_{\Delta}(\vec{y}, \eta) \equiv (-\eta)^{\Delta} {}_0F_1(\Delta - d/2 + 1, \frac{1}{4}\eta^2 \partial_{\vec{y}}^2)$$

- The bulk-to-boundary coefficient is given by KL **spectral density**

$$a_{\Delta}^2 \propto \rho(\Delta)$$

Given a bulk theory, what are the boundary operators?

Boundary dS

Given a bulk field there is a **continuous family** of operators living at the boundary

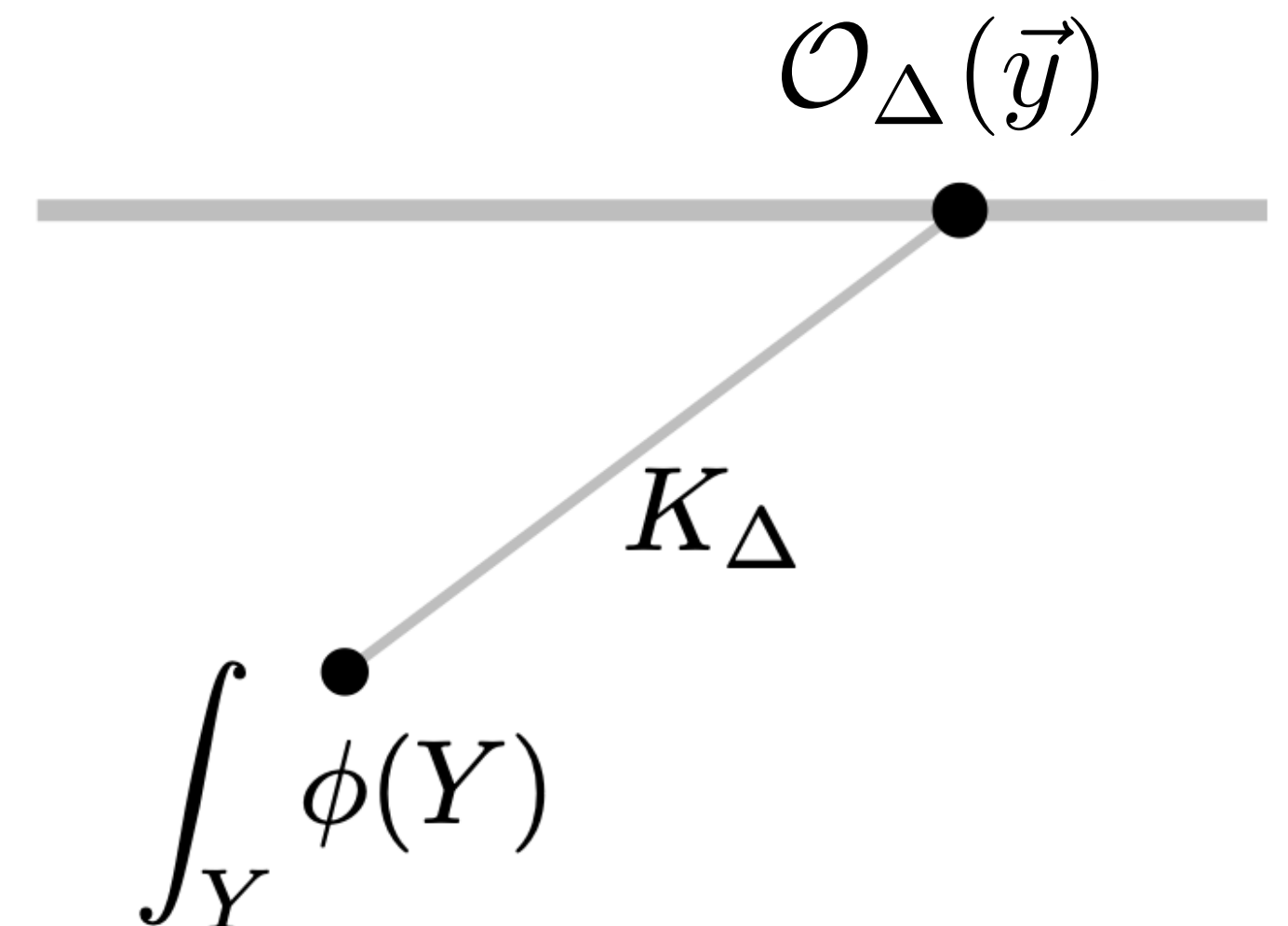
$$\mathcal{O}_\Delta(\vec{y}) = \frac{1}{N_\Delta} \int_{\text{bulk}} K_\Delta(Y, \vec{y}) \phi(Y)$$

[KSV, 25]

- Sketch of the proof: Going to Fourier space and using completeness/orthogonality of Hankel functions (Kontorovich–Lebedev transform)

- Produces the contact term
- Hermitian conjugation: shadow transform
- Recovers perturbation theory

$$\langle \mathcal{O}_{\Delta_1}(\vec{y}_1) \mathcal{O}_{\Delta_2}(\vec{y}_2) \rangle = \frac{\delta_{\lambda_1 \lambda_2}}{y_{12}^{2\Delta_1}} + \gamma_{\Delta_1} \delta_{\lambda_1, -\lambda_2} \delta_{\vec{y}_1, \vec{y}_2}$$



Summary

- Do we have a KL decomposition for dS? **For any spin and any spacetime dimensions**
- Can we find explicit expression for spectral densities? **The inversion formula**

$$\rho(\Delta) \propto \int_{-\infty}^{-1} d\sigma (\sigma^2 - 1)^{\frac{d-1}{2}} {}_2F_1 \left[\Delta, \bar{\Delta}, \frac{d+1}{2}, \frac{1+\sigma}{2} \right] \langle \phi \phi \rangle$$

- Can we define boundary operators from bulk theory? **a continuous family of boundary operators**

$$\mathcal{O}_{\Delta}(\vec{y}, \eta) = \frac{1}{N_{\Delta}} \int_{\text{bulk}} dY K(Y; \vec{y}, \eta) \phi(Y)$$

Future directions

- **Numerical** bootstrap bounds on inflation
- **Bounds on EFT coefficient** in dS. Role of the Hubble scale?
- **Boundary OPE**, its relation to the [interaction in] bulk theory and its convergence.

Simple playground: CFT in bulk

[WP; Arundine, Fumagalli, KSV]

- Boundary operator for gauge fields? **Diffeomorphism?**

Thank You!