Beyond the Bootstrap: Gluons in AdS

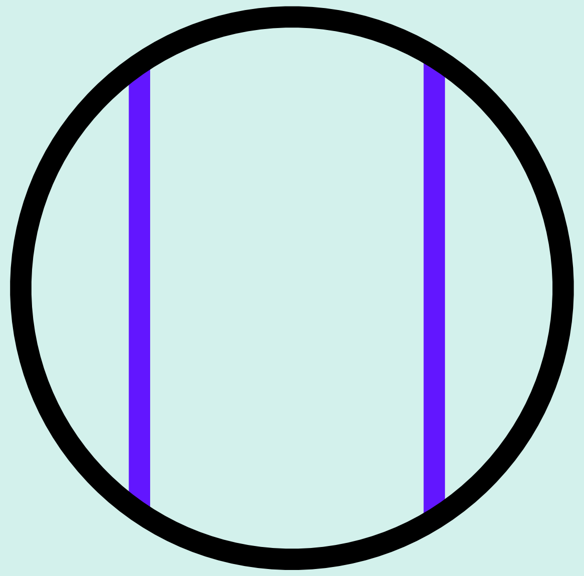
Based on:

- *Extremal couplings, graviton exchange, and gluon scattering in AdS*
  Chester, Mouland, van Muiden, May ’25
- *Extremal couplings and gluon scattering in M-theory*
  Chester, Mouland, van Muiden, Virally, Dec ’25
- *Something something 1-loop gluons*
  Chester, Mouland, Virally, Autumn ’26

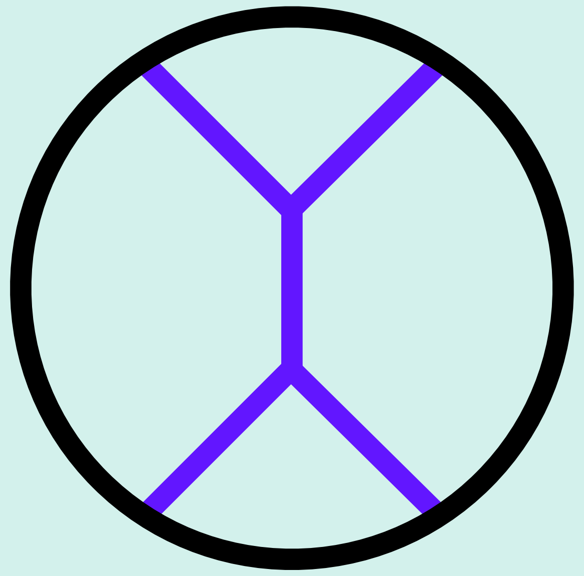
Rishi Mouland (Imperial), Bootstrap 2026

The large N analytic bootstrap

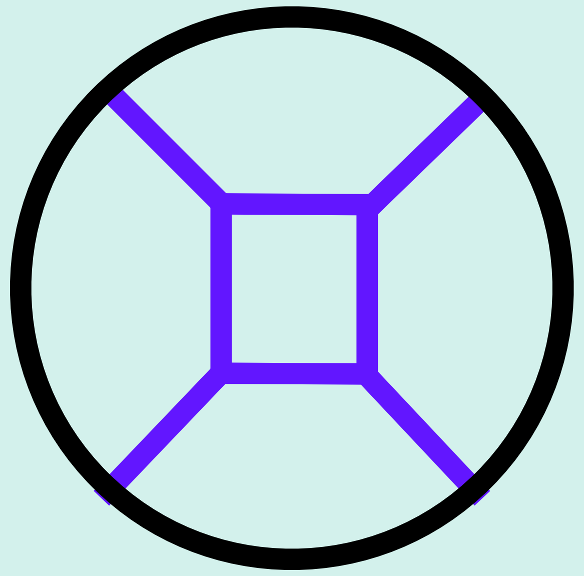
$\langle\phi\phi\phi\phi\rangle =$



$+ \frac{1}{c}$



$+ \frac{1}{c^2}$



$+ \dots$

Conceptually clear what to do, but technically **totally prohibitive**

$$\sum_{\mathcal{O}} a_{\mathcal{O}} g_{\Delta,l} = a^{(0)}_{\mathcal{O}} g_{\Delta_0,l} + \frac{1}{c}\left(a^{(1)}_{\mathcal{O}} + (a^{(0)}_{\mathcal{O}}\gamma^{(1)}_{\mathcal{O}})\partial_\Delta\right) g_{\Delta_0,l} + \dots$$

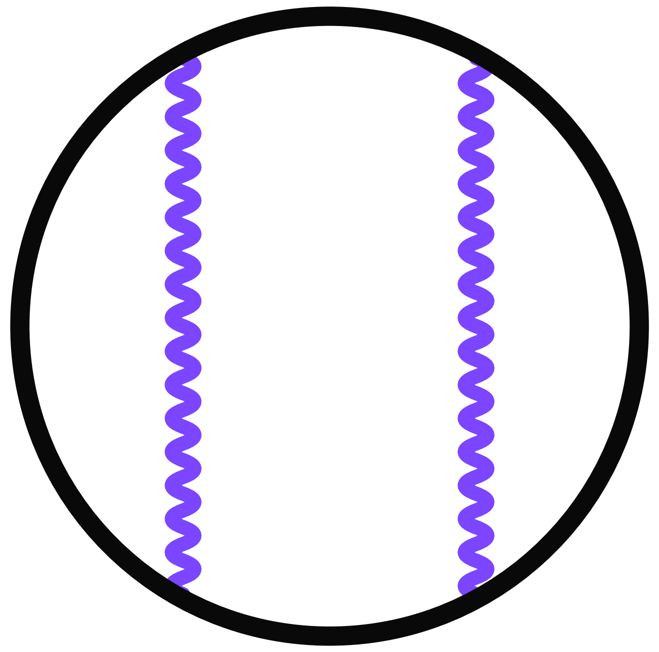
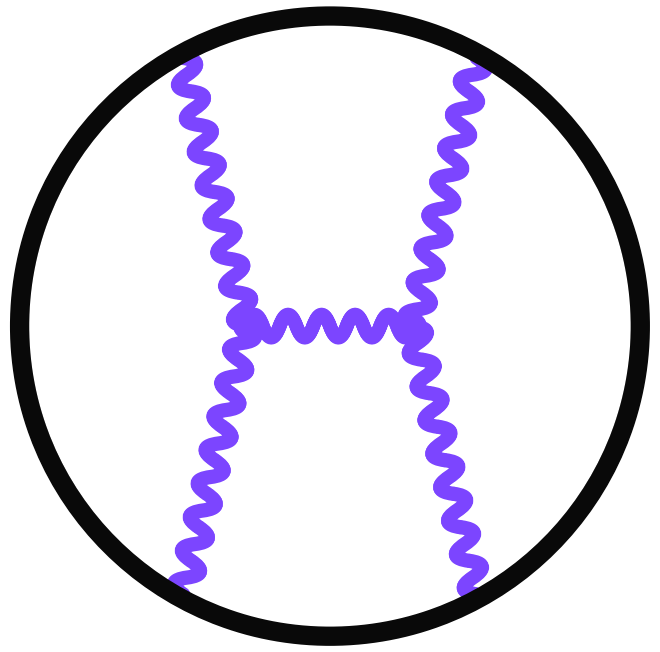
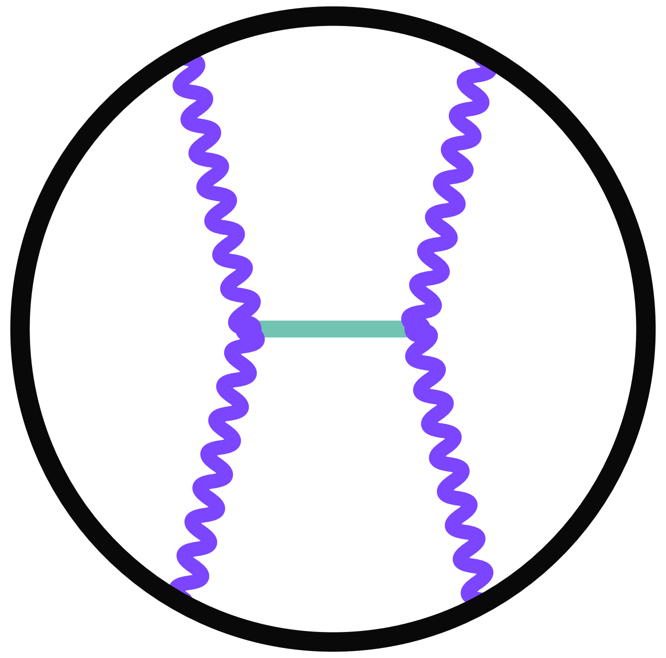
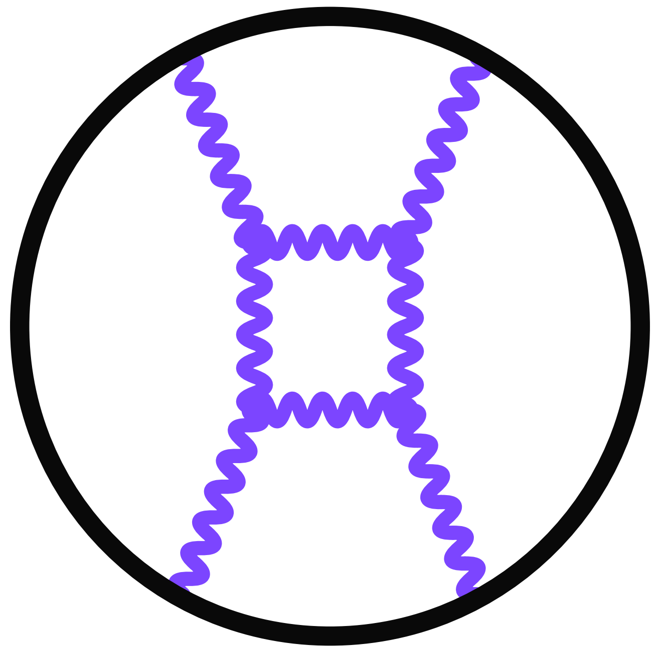
Bootstrap: (super-)conformal symmetry, crossing, analyticity, flat space limit (+ SUSY localisation) $\longrightarrow$ **Try to indirectly resum Witten diagrams**

- Many elements of the story **extremely generic** (e.g. Mellin space, flat space limit)
- Especially effective in string/M-theory setups with **maximal SUSY**
  [See: many people in this room]

What we computed

String coupling $\tau \sim g_s^{-1} \sim N/\lambda$ is fixed

- **Half-maximal** (8 SUSYs) SCFTs in $d = 3,4,5,6$, labelled by rank $N \gg 1$
- $\mathrm{AdS}_{d+1} \times (S^n/\Gamma)$ bulk, with **flavour branes** at $\mathrm{AdS}_{d+1} \times S^3$ fixed point locus
- KK reduce to AdS_{d+1}
  - **Gravitons:** stress tensor multiplet (short) along with a tower (long)
  - **Gluons:** tower ϕ_p, $p = 2,3,\ldots$ (all short) in adjoint of flavour symm G_F

$\langle \phi_2^A \phi_2^B \phi_2^C \phi_2^D \rangle =$  $+ \frac{1}{c_J}$  $+ \frac{1}{c_T}$  $+ \frac{1}{c_J^2}$  $+ \ldots$

Generalised free fields
Easy peasy

Short multiplet exchange
Bootstrap-able
[Alday, Behan, Ferrero, Zhou]

Old school bulk calculation
$d = 3,4,6$

Unitarity cut
[Aharony et al.]
$d = 3,4,5,6$

$d = 4$ in [Alday, Bissi, Zhou]

What I want to tell you about it

- **A new feature:** **Extremal couplings**, which induce $\mathcal{O}(1)$ mixing between single- and double-traces

- **A new tool:** **Reduced correctors and blocks** in Mellin and position space

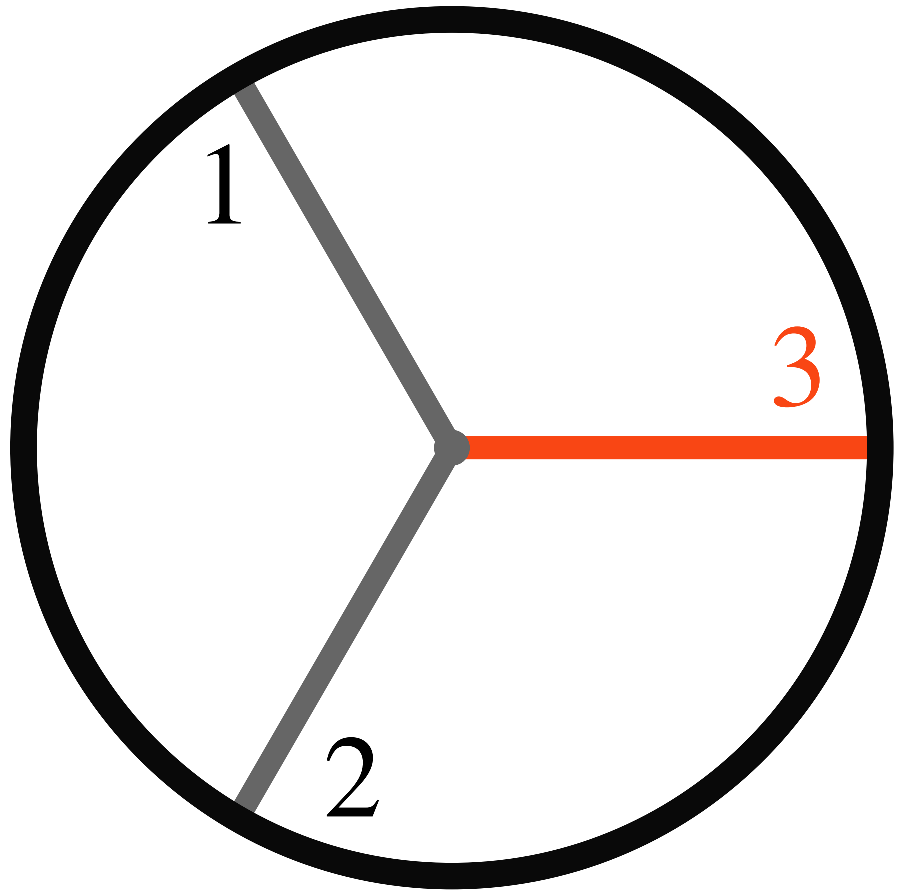
- **A non-trivial constraint/match:** Integrated constraints from **SUSY localisation**

I: Extremal couplings

The general mechanism

Extremal Witten diagrams

- OPE coefficients diverge at "extremal" arrangements $\Delta_3 \to \Delta_1 + \Delta_2$

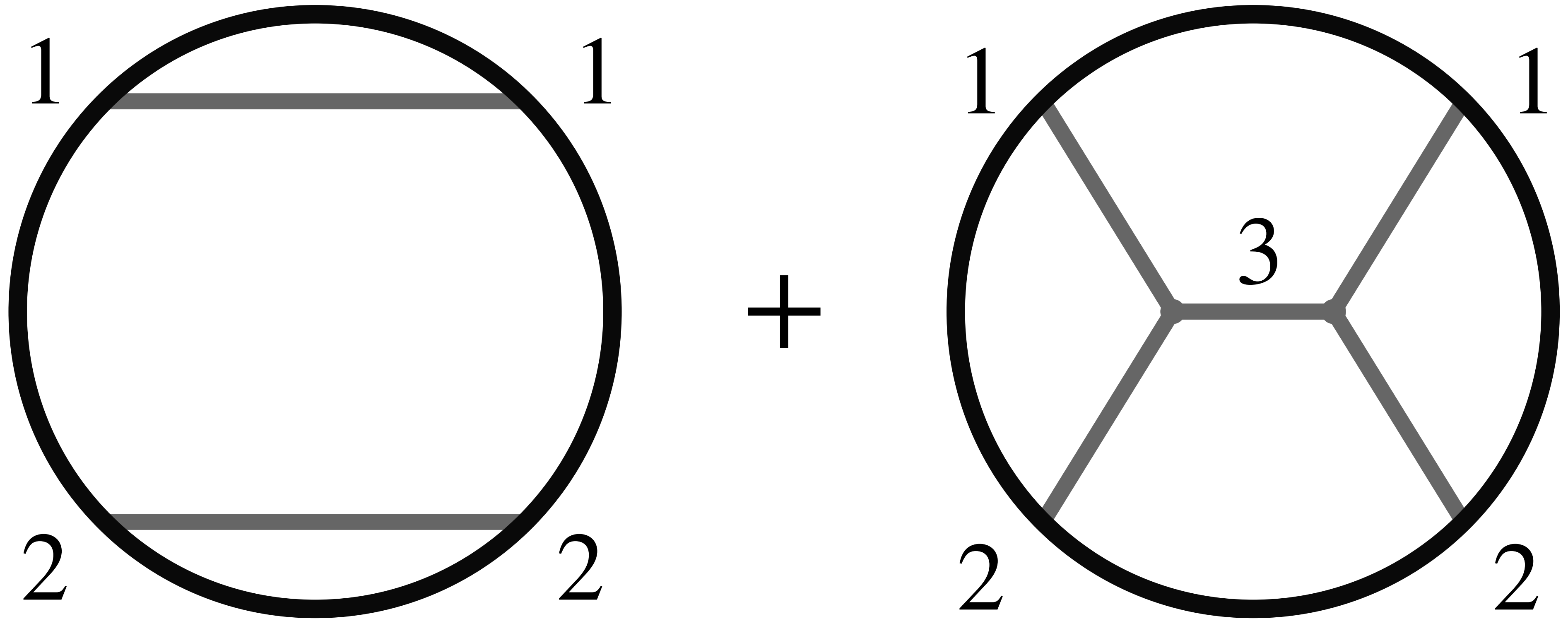


$$\sim \Gamma\left(\tfrac{\Delta_1+\Delta_2-\Delta_3}{2}\right)\Gamma\left(\tfrac{\Delta_2+\Delta_3-\Delta_1}{2}\right)\Gamma\left(\tfrac{\Delta_3+\Delta_1-\Delta_2}{2}\right)\Gamma\left(\tfrac{\Delta_1+\Delta_2+\Delta_3-d}{2}\right) \sim \frac{1}{\Delta_3 - \Delta_1 - \Delta_2}$$

- Such couplings **absent** with max SUSY. Suspected **forbidden?** [Bobev et al.]
- **No!** Divergence just an artefact of **mixing** between $\mathcal{O}_3$ and $\mathcal{O}_1\mathcal{O}_2$
- One way to see this: **holographic renormalisation** [Castro, Martinez]

The general mechanism

Another way: just compute something finite



$$\boxed{\Delta_3 \neq \Delta_1 + \Delta_2}$$

$$= \left(A + \frac{1}{c}\left(B\partial_\Delta + \dots\right)\right) g_{\Delta,0}(U,V)\Big|_{\Delta=\Delta_1+\Delta_2} + \dots$$

Expand $\mathcal{O}_1\mathcal{O}_2$ CFT data...

$$\lambda = \lambda^{(0)} + \mathcal{O}(1/c) \qquad \Delta = \Delta_1 + \Delta_2 + \frac{\gamma}{c} + \mathcal{O}(1/c^2)$$

$$\lambda^2 g_{\Delta,0}(U,V) = \left(\underbrace{(\lambda^{(0)})^2}_{A} + \frac{1}{c}\left((\lambda^{(0)})^2\gamma\,\partial_\Delta + \dots \right)\right) g_{\Delta,0}(U,V)\Big|_{\Delta=\Delta_1+\Delta_2}$$

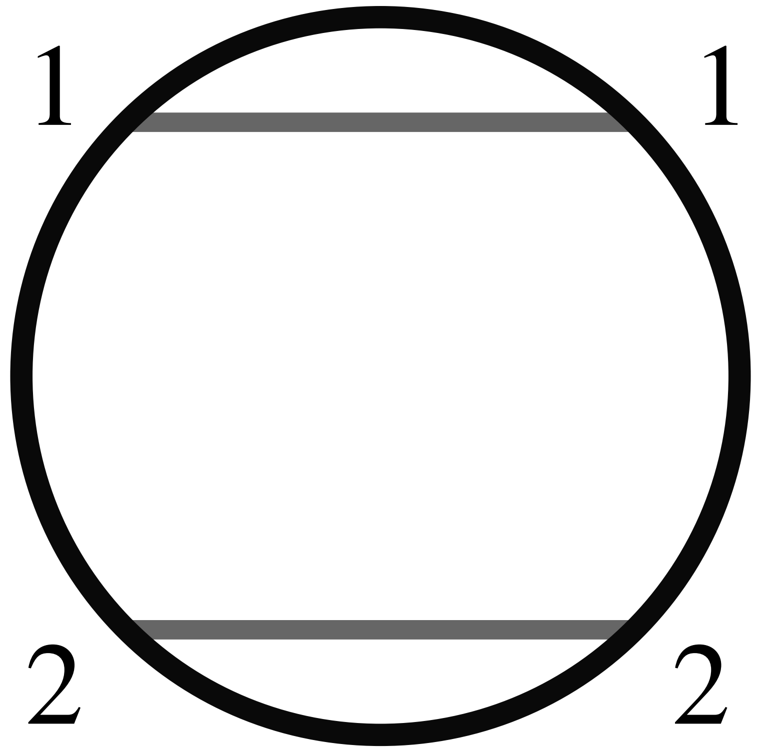
B **contributes to this**

The general mechanism

Another way: just compute something finite

In Mellin space:
double pole $\longrightarrow$ triple pole

$$\boxed{\Delta_3 = \Delta_1 + \Delta_2}$$



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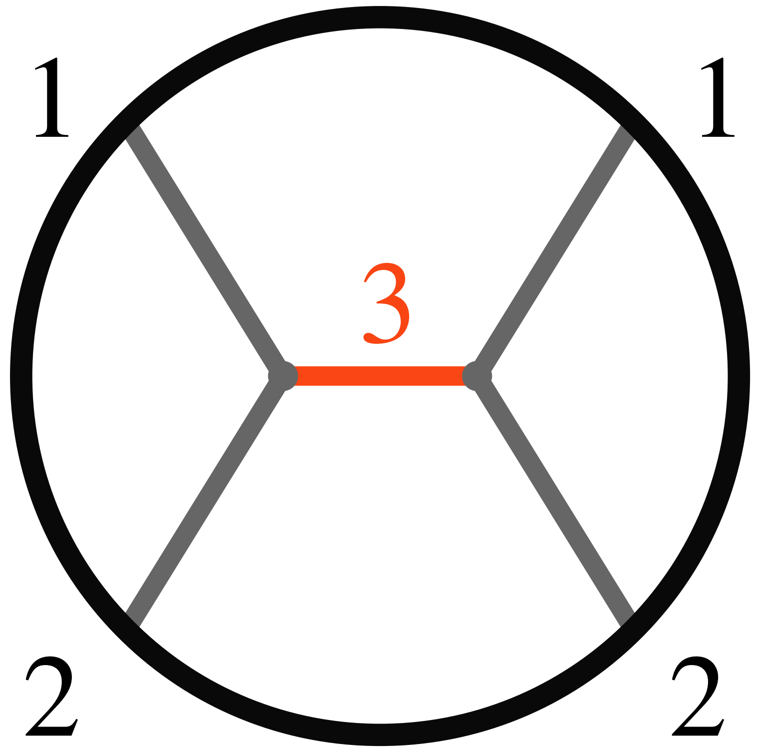


$$= \left(A + \frac{1}{c}\left(B\partial_\Delta^2 + \ldots\right)\right) g_{\Delta,0}(U,V)\Big|_{\Delta=\Delta_1+\Delta_2} + \ldots$$

Explained by $\mathcal{O}_3$ mixing with $\mathcal{O}_1\mathcal{O}_2$

Upshot: Unmixed operators

$$\mathcal{O}_\pm = \frac{1}{\sqrt{2}}\left(\mathcal{O}_3 \pm \mathcal{O}_1\mathcal{O}_2\right)$$

$$\lambda_\pm^2 = \frac{A}{2} + \mathcal{O}(1/c)$$

$$\Delta_\pm = \Delta_1 + \Delta_2 \pm \frac{1}{\sqrt{c}}\sqrt{\frac{B}{A}} + \mathcal{O}(1/c)$$

Manifestation for half-maximal SUSY

- More bells and whistles:
  - At each engineering dimension, **one graviton ST** and **many gluon DTs** mixing
  - **Two couplings** $1/c_J$ and $1/c_T$ scaling differently with $1/N$
- Mixing **qualitatively different** in each dimension:

Dominates in $d = 5{,}6$

$$\begin{pmatrix} \langle \mathcal{O}_{\mathrm{ST}}(x)\mathcal{O}_{\mathrm{ST}}(y)\rangle & \langle \mathcal{O}_{\mathrm{ST}}(x)\mathcal{O}^j_{\mathrm{DT}}(y)\rangle \\ \langle \mathcal{O}^i_{\mathrm{DT}}(x)\mathcal{O}_{\mathrm{ST}}(y)\rangle & \langle \mathcal{O}^i_{\mathrm{DT}}(x)\mathcal{O}^j_{\mathrm{DT}}(y)\rangle \end{pmatrix} = \frac{1}{|x-y|^\Delta}\left(1 - \begin{pmatrix} \frac{A}{c_T} & \frac{B}{\sqrt{c_T}} \\ \frac{B}{\sqrt{c_T}} & \frac{C}{c_J} \end{pmatrix} \log|x-y|^2 + \ldots\right)$$

Dominates in $d = 3$

- $d = 4$ more interesting as $c_J^2 \sim c_T \sim N^2$

II: Reduced correlators

Imposing superconformal symmetry

Conformal cross ratios

Projectors onto flavour irreps

$$\langle \phi_2^A \phi_2^B \phi_2^C \phi_2^D \rangle = (\dots) \sum_{\mathbf{r}\in\mathbf{Adj}^2} G_{\mathbf{r}}(\overbrace{U,V};\alpha) P_{\mathbf{r}}^{ABCD}$$

$SU(2)_R$ cross ratio

$$U = z\bar{z}, \qquad V = (1-z)(1-\bar{z})$$

- Each flavour channel independently constrained by **superconformal Ward identities**

$$(2z\partial_z - (d-2)\alpha\partial_\alpha)G_{\mathbf{r}}(U,V;\alpha)\Big|_{\alpha=1/z} = 0 \qquad + \qquad (z \leftrightarrow \bar{z})$$

- **Brute force:** expand in **superconformal blocks**

$$G_{\mathbf{r}}(U,V;\alpha) = \sum_{\mathfrak{M}} \lambda_{\mathfrak{M}} \mathfrak{G}_{\mathfrak{M}}(U,V;\alpha)$$

Superconformal multiplets

(Short and long) **superconformal blocks,** each a finite sum of **conformal blocks**

Conceptually fine, but **technically cumbersome**

Transforms and reductions

Expand **full correlator** $G(U, V; \alpha)$ in **superconformal blocks**

$\xrightarrow{\text{Mellin transform}}$

Expand **Mellin amplitude** $M(s, t; \alpha)$ in (sums of) **exchange diagrams**

Reduce
Seek differential operator R such that
$$G(U, V; \alpha) = R \circ H(U, V)$$
[Dolan, Gallot, Sokatchev]

Reduce
Seek difference operator D such that
$$M(s, t; \alpha) = D \circ K(s, t)$$

Expand **reduced correlator** $H(U, V)$ in **conformal blocks**

$\xleftarrow{\text{Transform back}}$

Expand **reduced Mellin amplitude** $K(s, t)$ in **exchange diagrams**

- Even dim well-understood [Beem et al], [Chang, Lin], See also Mitchell's talk! Extra term in 4d to capture protected subsector **(also in 3d!)**

- Odd dim: R non-local. Some special correlators understood, but **crossing subtle** [Woolley]

$$\text{Mellin}^{-1}\left(D \circ K(s,t)\right) = \tilde{R} \circ \left(\underbrace{\text{Mellin}^{-1} K(s,t)}_{\text{New position space reduced correlator}}\right)$$

Local in all dimensions!

III: Comparing to localisation

Integrated constraints from SUSY localisation

- Compute **supersymmetric sphere free energy** $F(m_i)$ using **SUSY localisation**
- Take derivatives to find **integrated constraints** on 4-point functions

$$\partial^4_{m_i} F\big|_{m_i=0} \sim \int d^d x_1 \dots d^d x_4 \, \langle \phi_2(x_1)\phi_2(x_2)\phi_2(x_3)\phi_2(x_4)\rangle \;\sim \int dU\, dV\; f(U,V)G(U,V)$$

- Used these in **two places…**
  - **In 4d**, graviton exchange enough to fix $\langle \phi_2\phi_2\phi_2\phi_2\rangle$ **up to order** $1/N^2$ only up to a τ-dependent **contact term**. For the theory $G_F = D_4$ we **fixed these completely** using integrated constraints [see also Behan, Chester, Ferrero]
  - **In 3d**, **no contact term ambiguities** at gluon 1-loop order $1/c_J^2$. So can seek **non-trivial match** with localisation…

A match with 3d $\mathcal{N} = 4$ localisation

[Chester, Pufu, Wang, Yin]

Find **squashed sphere free energy** as matrix integral via **SUSY localisation**

Take derivatives to get **integrated constraints**

Extract $1/c_J^2$ term in large $N \sim c_J^2$ expansion to find

$$\lambda^2_{\text{1-loop}} = \frac{1}{12\pi^2} + \frac{1}{720} - \frac{\pi^2}{60480} + \frac{\pi^4}{1814400} + \dots$$

Resum

Want: 1-loop OPE coeff $\lambda^2_{\text{1-loop}}$ of a particular **protected multiplet** $\mathcal{D}[4]$

$$\lambda^2_{\text{1-loop}} = \frac{1}{4\pi^2} - \frac{1}{64}$$

Compute **1-loop gluon amplitude** using **reduced correlators** and **AdS unitarity method**

Compute relevant **double discontinuities** and apply **Lorentzian inversion formula**

$\log^2 V$-type: $\frac{1}{16\pi^2}$

$V^{n/2}$-type: $\frac{3}{16\pi^2} - \frac{1}{64}$

Sum

Some final words

What we did, and what we found

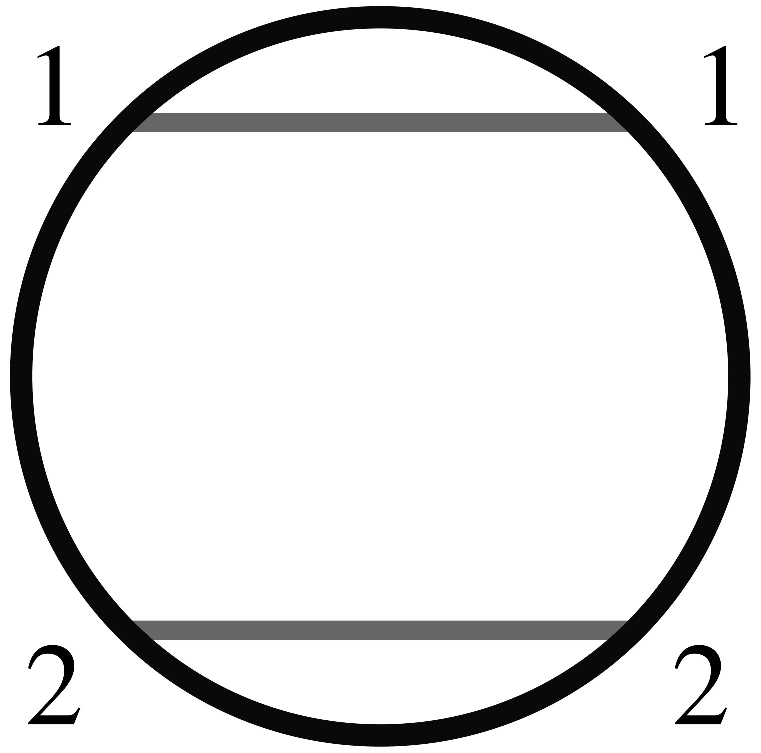
- In large N expansion of $\langle \phi_2\phi_2\phi_2\phi_2 \rangle$ we computed the contributions from:
    - **Graviton exchange**, by a bulk computation
    - **Gluon 1-loop**, using the AdS unitarity method
- Both computations leveraged new **reduced correlators** to impose superconformal symmetry
- Uncovered and resolved a **new feature: extremal couplings** and **ST/DT mixing**
- Used **SUSY localisation** to **fix** (4d) and **check** (3d) results
- Glossed over **flat space limit: non-trivial matches**, and **a new prediction**

Thanks!

Backup slides

Rishi Mouland

Unmixing



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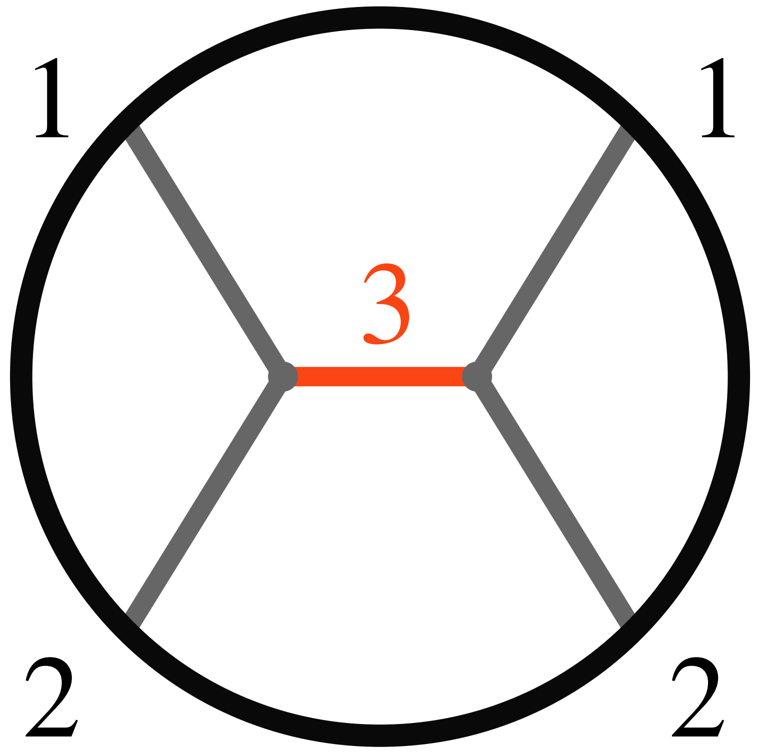

$$= \left(A + \frac{1}{c}\left(B\partial_\Delta^2 + \dots\right)\right) g_{\Delta,0}(U,V)\Big|_{\Delta=\Delta_1+\Delta_2} + \dots$$

In Mellin space:
double pole $\longrightarrow$ triple pole

Two-dimensional subspace of operators of engineering dimension $\Delta = \Delta_1 + \Delta_2$

$$\lambda_{[i]} = \lambda_{[i]}^{(0)} + \mathcal{O}(1/c) \qquad \Delta_{[i]} = \Delta_1 + \Delta_2 + \frac{\gamma_{[i]}}{\sqrt{c}} + \mathcal{O}(1/c) \qquad i = 1, 2$$

$$\lambda^2 g_{\Delta,0}(U,V) = \Bigg(\underbrace{\langle(\lambda^{(0)})^2\rangle}_{A} + \frac{1}{\sqrt{c}}\Big(\underbrace{\langle(\lambda^{(0)})^2\gamma\rangle}_{0}\partial_\Delta + \dots\Big) + \frac{1}{c}\Big(\underbrace{\langle(\lambda^{(0)})^2\gamma^2\rangle}_{B}\partial_\Delta^2 + \dots\Big)\Bigg) g_{\Delta,0}(U,V)\Big|_{\Delta=\Delta_1+\Delta_2}$$

Corrections to $\langle\mathcal{O}_3\mathcal{O}_3\rangle$ scale like $1/c$ $\Longrightarrow$ **Fourth constraint** $\lambda_{[1]}^{(0)}\gamma_{[2]} + \lambda_{[2]}^{(0)}\gamma_{[1]} = 0$