Heavy CFT Data From Thermal Correlators

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Bootstrap 2026

Imperial College, London, Aug 13, 2026

CLUSTER OF EXCELLENCE
QUANTUM UNIVERSE

HELMHOLTZ RESEARCH FOR GRAND CHALLENGES

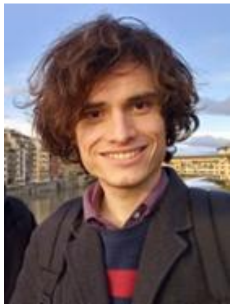
CRC 1624
HIGHER STRUCTURES, MODULI
SPACES AND INTEGRABILITY

UHH

WOLFGANG PAULI CENTRE

DESY.

Based on work with

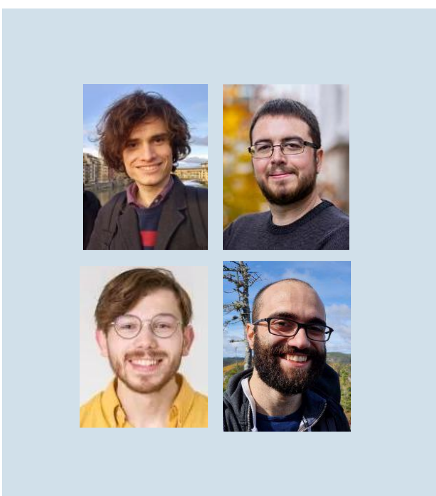


Ilija Buric

Francesco Mangialardi

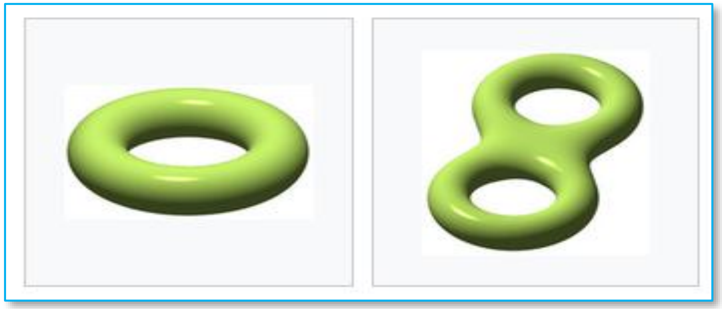
Alessandro Vichi

Francesco Russo



CFT$_d$ in Non-trivial Geometries

Thermal Geometry S^{d-1} x S^1 and beyond

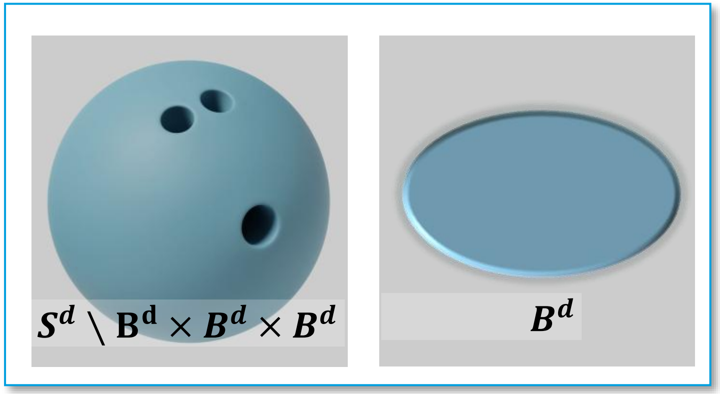


Lesson from 2D: Interesting to consider CFT on surfaces other than S^2, i.p. torus $S^1 \times S^1_\beta$

→ **Consider higher dimensional CFT on geometries that can be conformally glued from 3-holed spheres and (punctured) discs.**

$S^d \setminus \mathrm{B}^d \times B^d \times B^d$ B^d $(\setminus \{pt\})$

[Benjamin et al] *[Buric,Mangialardi,Russo,VS]*



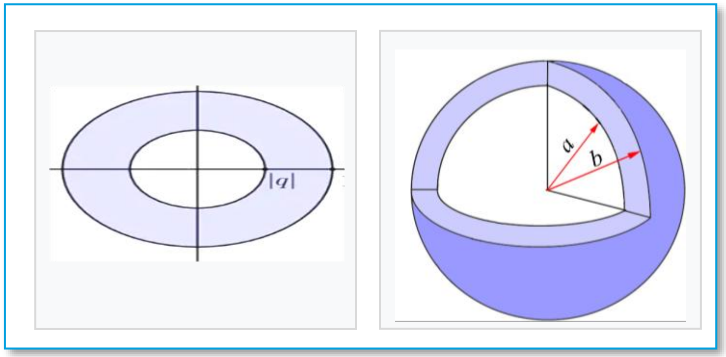
Same CFT data as on the sphere stitched together with new conformal blocks

Examples: Genus-2 maniflds HHH OPE data

[Benjamin, Lee, Ooguri, Simmons-Duffin]

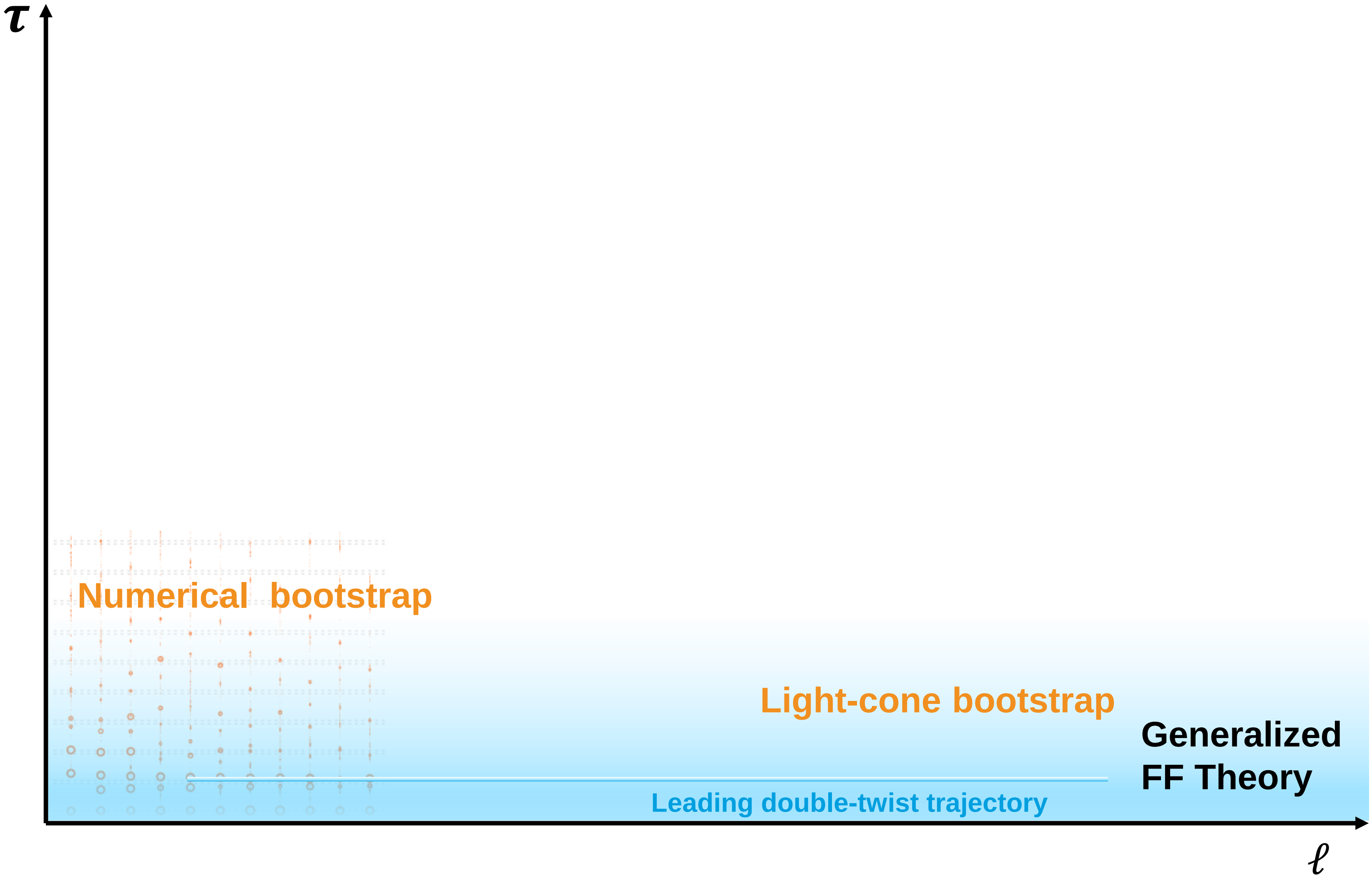
Thermal Geometry $S^{d-1} \times S^1_\beta$ HHL OPE data

[Cardy], [Gobeil, Maloney, Ng, Wu] …



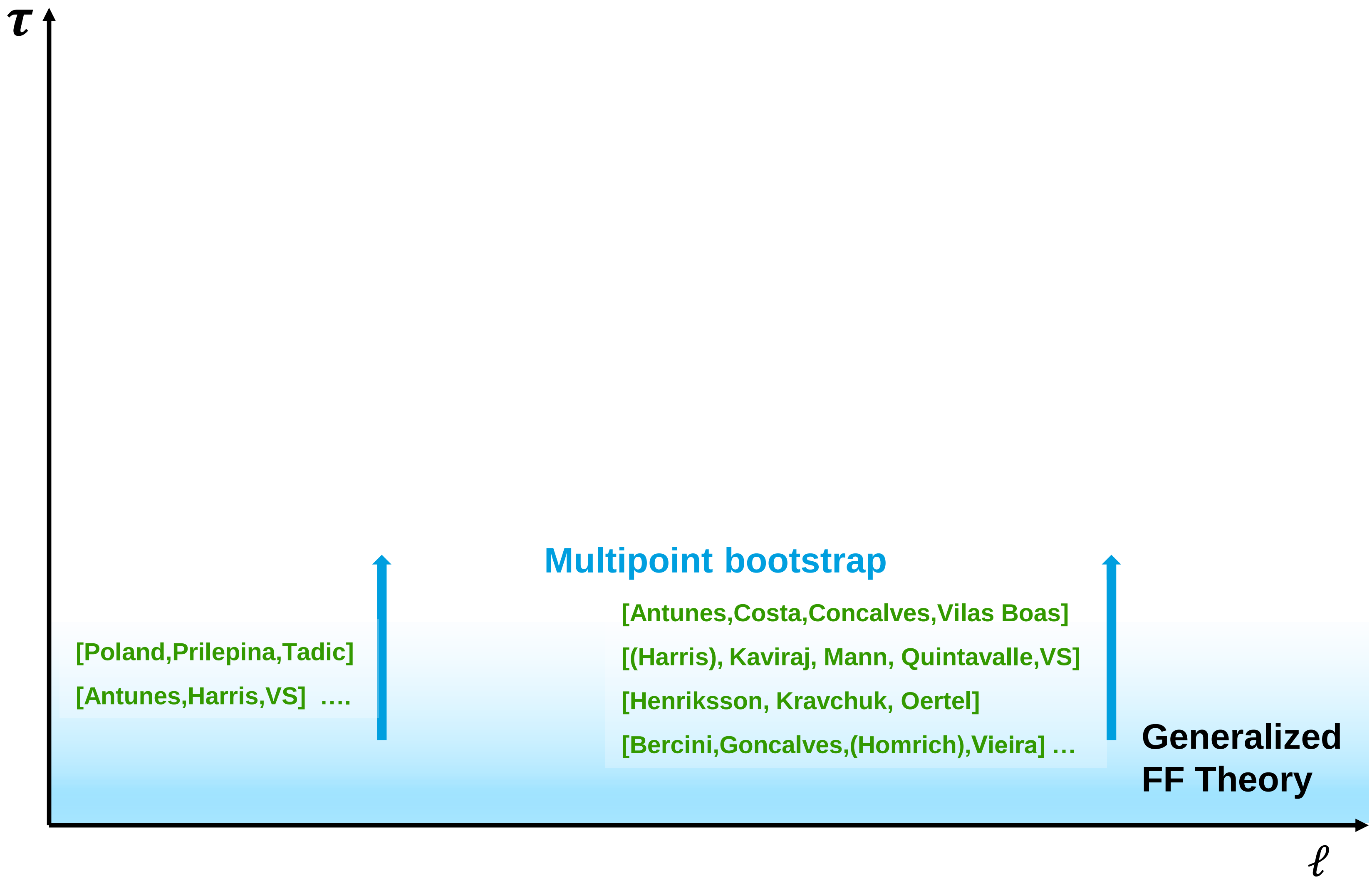
CFT in the Twist-Spin Plane

Highlights from the Bootstrap



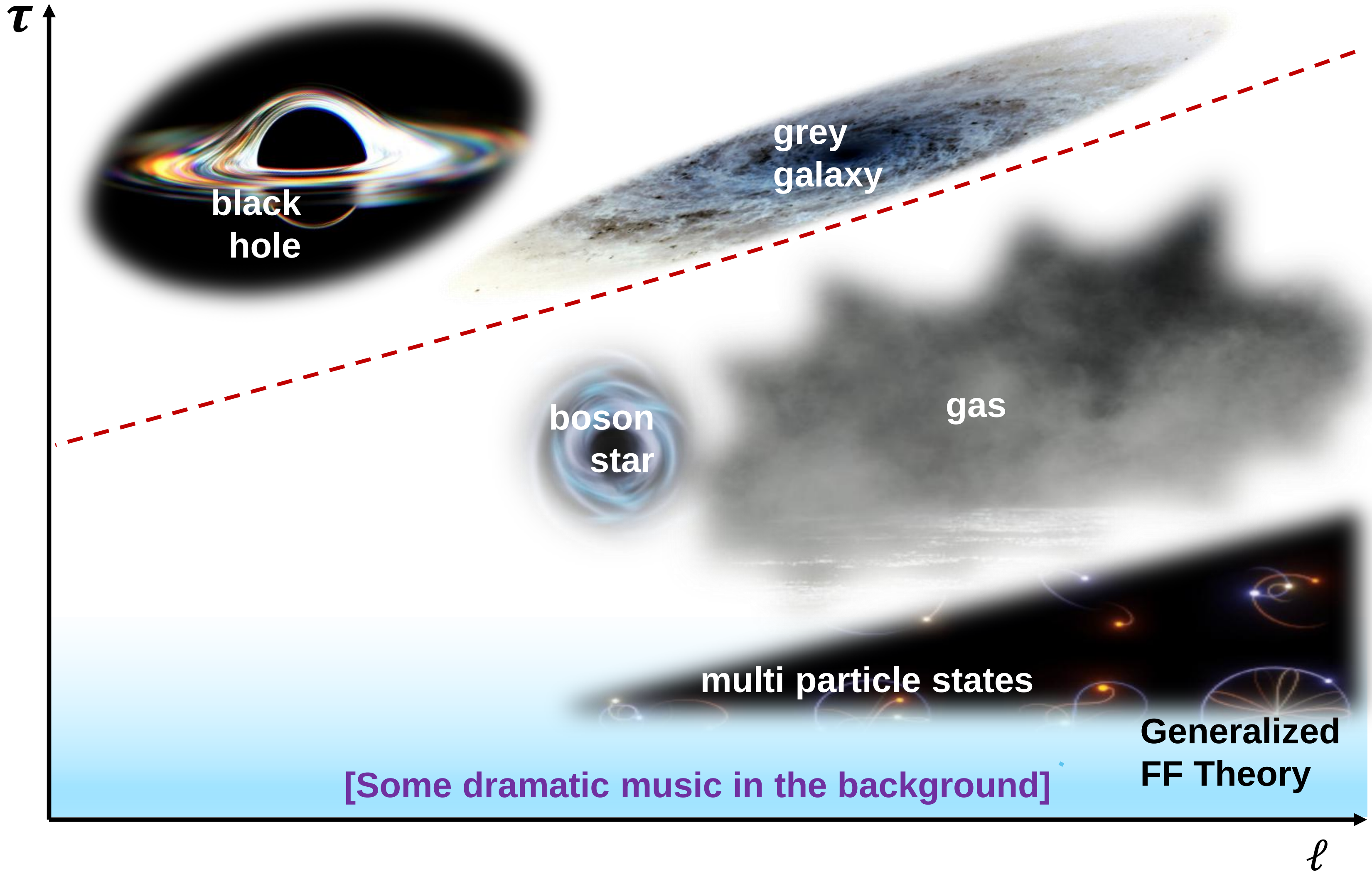
CFT in the Twist-Spin Plane

The multi-point bootstrap



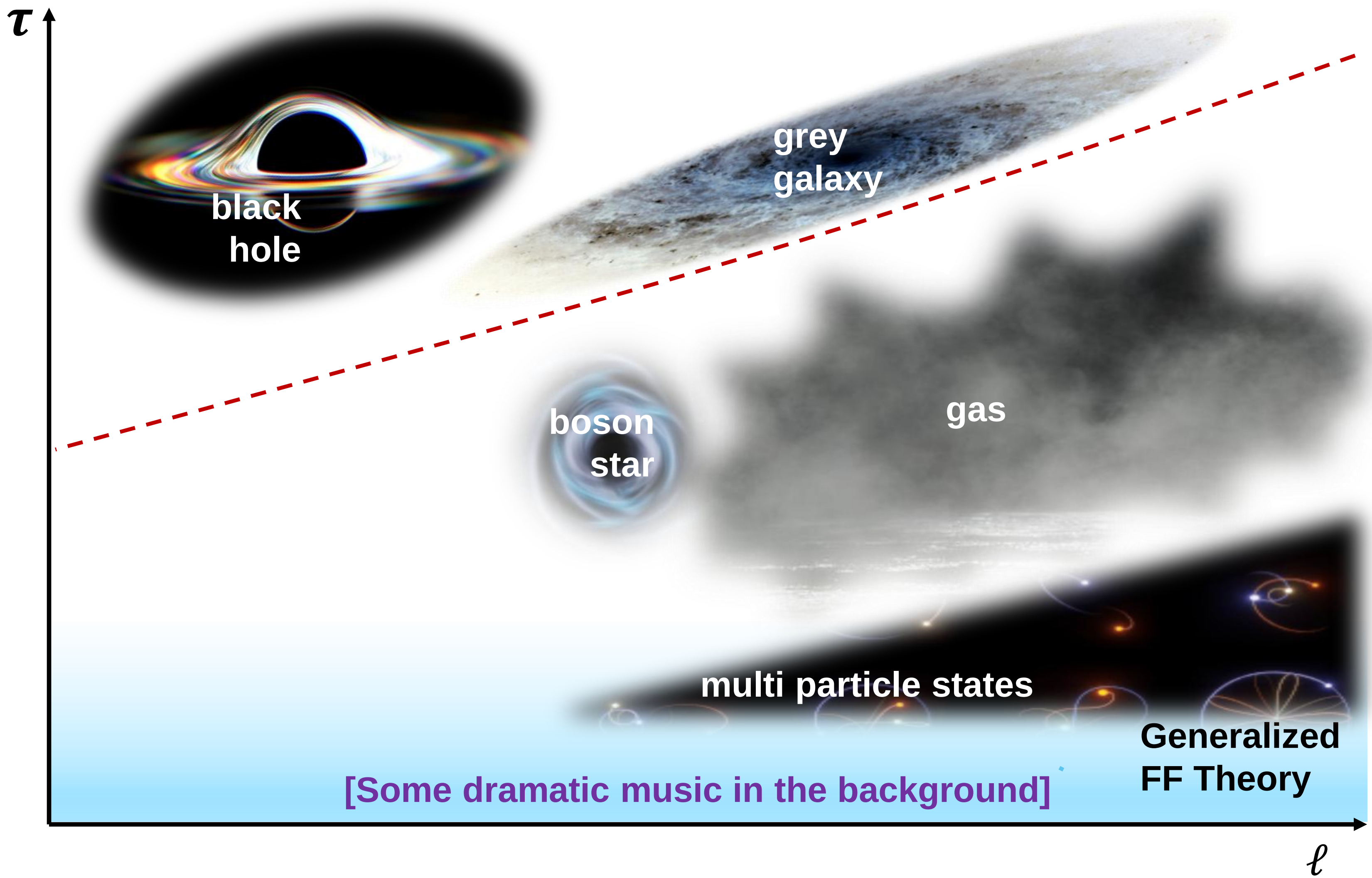
CFT in the Twist-Spin Plane

From few particle states to black holes



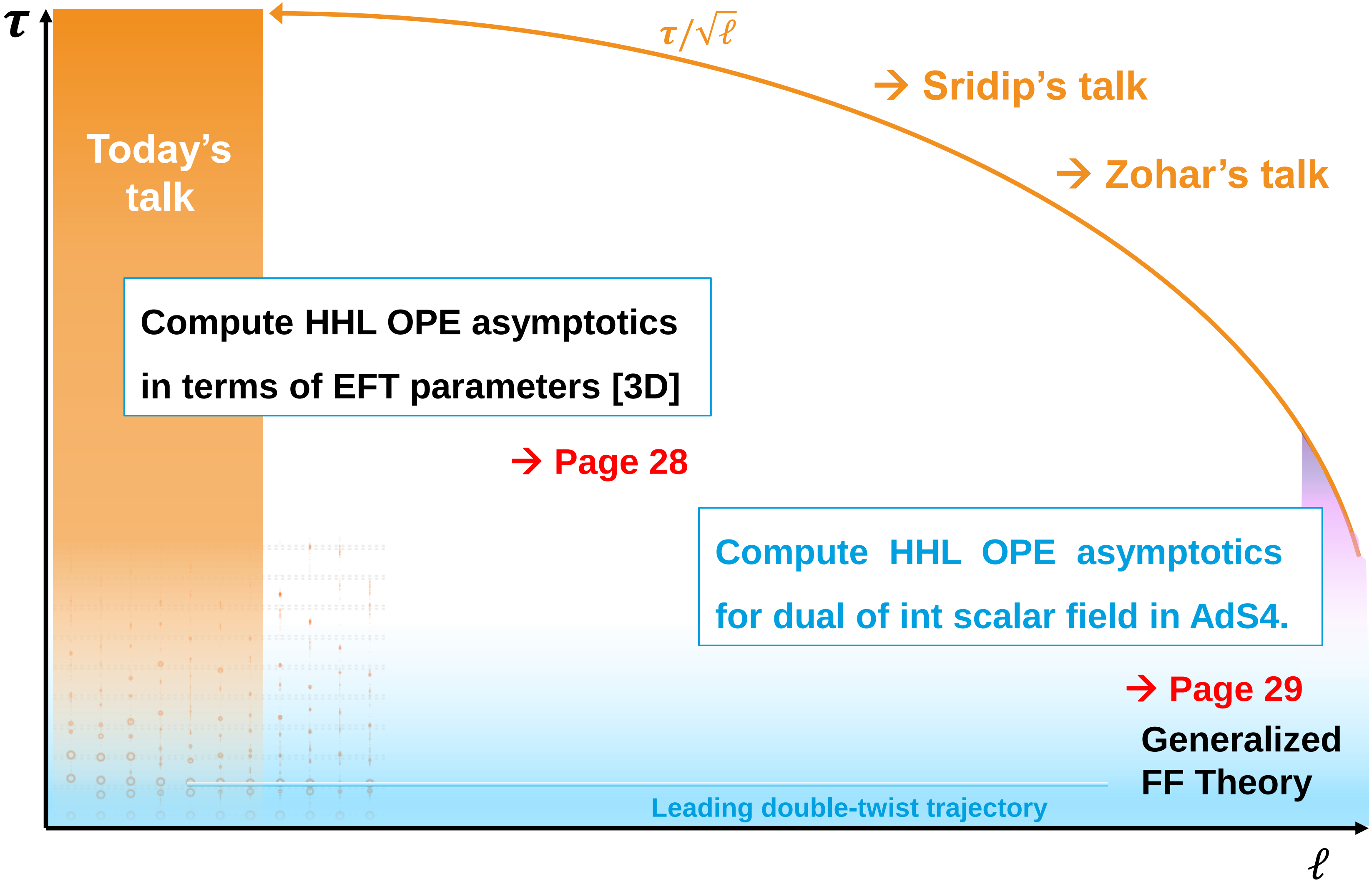
CFT in the Twist-Spin Plane

From few particle states to black holes



CFT in the Twist-Spin Plane

Thermal CFT and heavy CFT data



Introduction: Thermal Correlators

Definition and some features

Multi-point correlators of local fields in the geometry $S^1_\beta \times S^{d-1}$ given by

$$\langle \phi_1(x_1)\dots\phi_n(x_n)\rangle_{q,y_a} = \frac{1}{\mathcal{Z}(q,y)} \mathrm{tr}_{\mathcal{H}}\left(\phi_1(x_1)\dots\phi_n(x_n) q^D y_2^{H_2}\dots y_r^{H_r}\right)$$

$$\mathcal{Z}(q,y_a) = \mathrm{tr}_{\mathcal{H}}\left(q^D y_2^{H_2}\dots y_r^{H_r}\right) \qquad \phi(x) = e^{-\Delta_\phi \tau}\Phi(\tau,\Omega)$$

Comment 1: Obtained by partial fixing of conformal frame from g-twisted

$$\langle \phi_1(x_1)\dots\phi_n(x_n)\rangle_g = \frac{1}{\mathcal{Z}(q,y)} tr_{\mathcal{H}}\left(\phi_1(x_1)\dots\phi_n(x_n)\ g\ \right) \qquad g \in SO(1,d+1)$$

Comment 2: Our thermal correlators are closely related to the correlation functions in the geometry $S^1_\beta \times \mathbb{R}^{d-1}$ $= \lim_{R\to\infty} S^1_\beta \times S^{d-1}_R$

Latter involves new CFT data b_ϕ in the 1-point functions of primary fields

$$\langle \phi(x)\rangle_\beta = \langle \phi(x)\rangle_{S^1_\beta\times\mathbb{R}^{d-1}} = \frac{b_\phi}{\beta^{\Delta_\phi}}$$

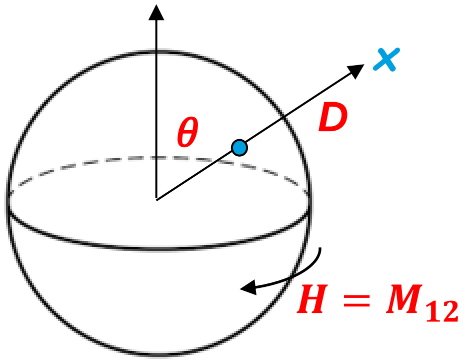
(for scalar ϕ)

[Iliesiu,Kologlu,Mahajan,Perlmutter, Simmons-Duffin]

Introduction: Example of Thermal Correlator

Thermal 1-point function of free boson in 3D

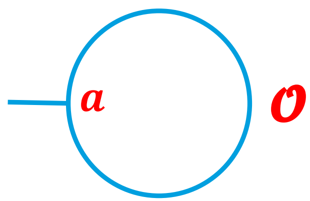
$$\langle \phi^2(x) \rangle_{q,y} = \frac{\sqrt{2}}{r} \sum_{l,m} \frac{(l-m)!}{(l+m)!} \frac{q^{l+1/2} y^m}{1 - q^{l+1/2} y^m} P_l^m (\cos\theta)^2$$

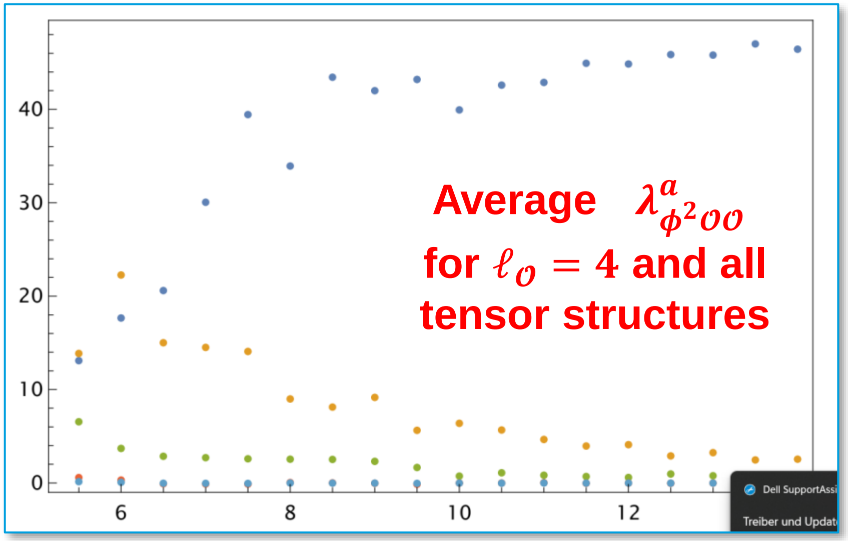


scalar field of weight $\Delta = 1$

Block expansion gives access to OPE coefficients $\lambda^a_{\phi^2 \mathcal{O}\mathcal{O}}$

$$\langle \phi^2(x) \rangle_{q,\beta} \sim \sum_{\mathcal{O}} \lambda^a_{\phi^2 \mathcal{O}\mathcal{O}}$$





Can we predict/model the asymptotic behavior of the CFT data for large $\Delta_{\mathcal{O}}$?

Plan

1. **Kinematics: Thermal 1-point Blocks**

   Small weight expansions and large weight asymptotics

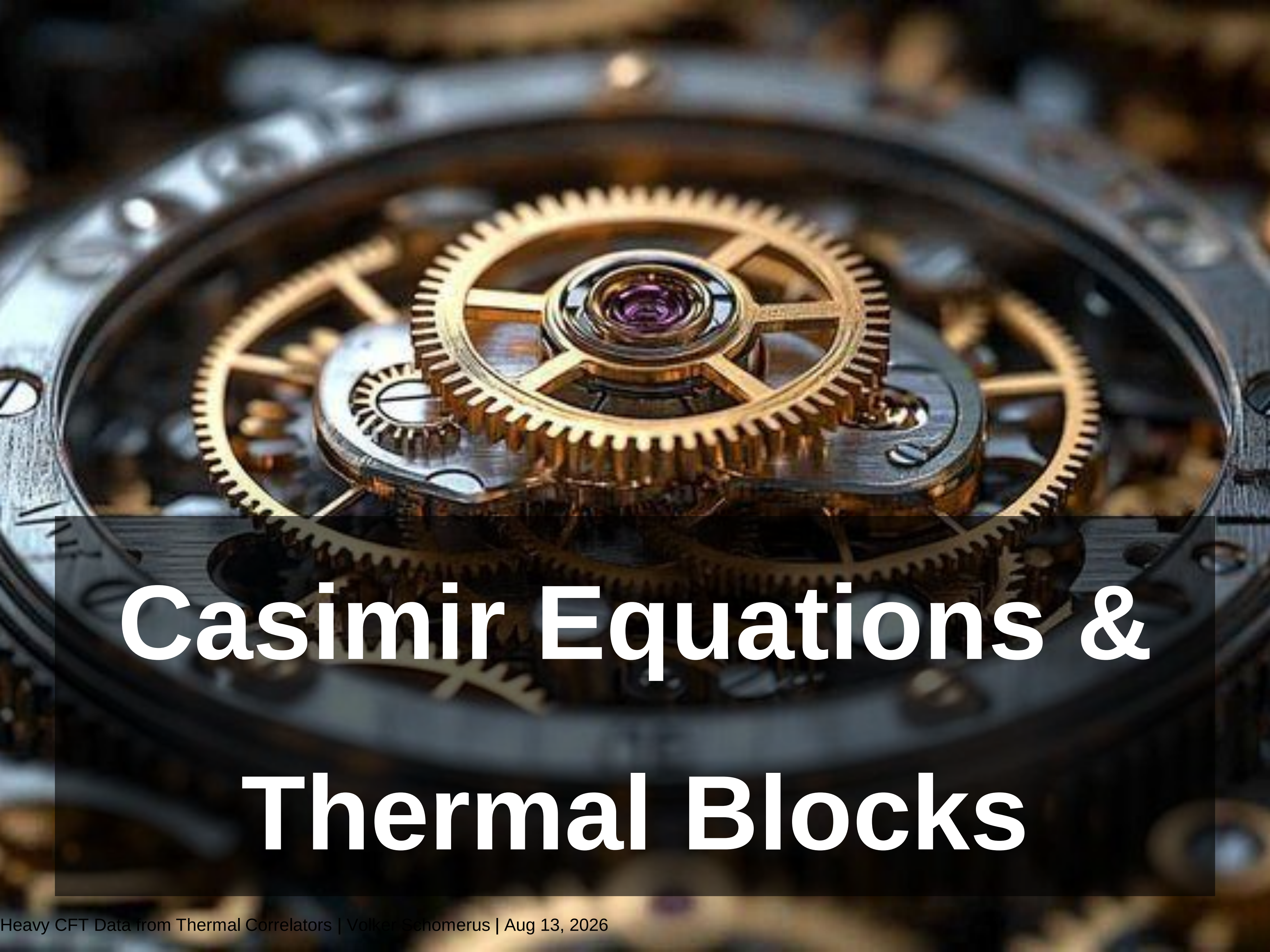
2. **Interlude: Integrability Does Not Melt**

   Hitchin integrability of thermal conformal blocks

3. **Commissioning: The Free Bosonic Field**

   How well do asymptotics capture conformal data ?

4. **Application: Heavy CFT data in Gapped CFTs and beyond**



Casimir Equations & Thermal Blocks

Thermal Conformal 1-point Blocks

A fully constructive definition

Casimir operators $\mathcal{C}^{\Delta_\phi,\ell_\phi}$ for 1-point functions of spinning primaries known

and are effective

[Buric, Russo, Schomerus, Vichi]

Focus on scalar external operator, for simplicity: $\boxed{\mathcal{C}^{\Delta_\phi} = \mathcal{C}^{\Delta_\phi}(\partial_q, \partial_u, \partial_s)}$

[Gobeil, Maloney, Ng, Wu]

multiplicity index

$q = e^{-\beta}$ $\quad y = e^{i\mu} = e^{i\beta\Omega}$ $\quad a = 0, \dots, \ell$

$$\boxed{G_1(q, y; x)|_{\mathcal{O}} = r^{-\Delta_\phi} g^{\Delta_\phi, a}_{\Delta,\ell}(q, u, s) = r^{-\Delta_\phi} q^{\Delta} \sum_{n=0} q^n f^{\ell,a}_n(u, s)}$$

$\mathcal{O} = \mathcal{O}_{\Delta,\ell}$ $\quad u = \frac{1}{4} sin^2 \frac{\mu}{2}$ $\quad s = sin^2\, \theta$

polynomials (of order $n + \ell$ in u)

Casimir equation solved recursively with *initial condition*:

$$\boxed{\lim_{q\to 0} q^{-\Delta} g^{\Delta_\phi, a}_{\Delta,\ell}(q, u, s) = f^{\ell,a}_0(u, s) \sim u^a P^{\left(2a+\frac{1}{2}, -\frac{1}{2}\right)}_{\ell-a}\left(1 + \frac{u}{2}\right) P^{\left(0,-\frac{1}{2}\right)}_a(1 - 2s)}$$

Jacobi polynomials

Implementation

A code that computes the thermal blocks recursively is available.

Thermal_blocks · GitLab

Similar to [Hogervorst, Rychkov] [Costa, Hansen, Penedones,Trevisani]

for scalar and spinning 4-point blocks

A code that relates basis of our tensor structures labeled by a to conventional tensor structures is also available.

Orthogonality and Inversion Formula

The Casimir operator $\mathcal{C}^{\Delta_\phi}$ is symmetric with respect to following pairing

$$\langle g_1, g_2 \rangle = \frac{1}{2} \iiint d\beta d\mu d\theta \; \omega(\beta,\mu) \sin\theta \; \tilde{g}_1(\beta,\mu,\theta) \; g_2(\beta,\mu,\theta)$$

$$\omega(\beta,\mu) = \left(\frac{8}{\sqrt{\pi}} \sinh\frac{\beta}{2} \sin\frac{\mu}{2} \sinh\frac{\beta+i\mu}{2} \sinh\frac{\beta-i\mu}{2} \right)^2$$

Claim: Thermal blocks $g^{\Delta_\phi,a}_{\Delta,l}(q,u,s)$ orthogonal with respect to pairing

Euclidean inversion formula for thermal 1-point functions takes the form

$$\rho(\Delta,\ell) \, \overline{\lambda^a_{\phi\mathcal{O}\mathcal{O}}}(\Delta,\ell) = \frac{r^{\Delta_\phi}}{4\pi i} \iiint \underbrace{d\beta d\mu d_\theta \; \omega(\beta,\mu) \; sin\;\theta}_{\text{measure}} \; g^{3-\Delta_\phi,a}_{3-\Delta,\ell}(\beta,\mu,\theta) \; G_1(\beta,\mu;x)$$

Averaged only (!) over $\mathcal{O}$ with same Δ,ℓ

measure

block

1-pt fct

With spectral density $\rho(\Delta,\ell) = \frac{1}{4\pi i} \int_{\gamma+i\mathbb{R}} d\beta \int_{-\pi}^{\pi} d\mu \omega(\beta,\mu) \, \chi_{3-\Delta,\ell}(\beta,\mu) \mathcal{Z}(\beta,\mu)$

Asymptotics of Thermal Blocks

In the limit of large weights Δ

Observation: Casimir equation for thermal blocks can be brought into form

independent of Δ !

$$2\ \Delta\left(q\,\partial_q+\frac{q(3q^2-2q(u+3)+u+3)}{(q-1)^3-(q-1)qu}\right)F+\mathcal{D}F=0$$

$$F^{\Delta_\phi,a}_{\Delta,l}(q,u,s)=q^{-\Delta}g^{\Delta_\phi,a}_{\Delta,l}(q,u,s)$$

Can be solved order-by-order in power series in $^1/_\Delta$ through integration in q

$$F(q,u,s)=\sum_{n=0}F^{(n)}(q,u,s)\Delta^{-n}\qquad \textit{with}\qquad F^{\Delta_\phi,a(0)}_{\Delta,l}(q,u,s)=\frac{f_0^{\ell,a}(u,s)}{(1-q)^3-(1-q)qu}$$

f_0 given above

Note: Generalization to spinning external fields is also known *(to appear)*

Integrability Approach to Thermal Blocks

Integrability does not melt !

In my talk at Bootstrap 2016 I showed:

4-point Casimir

$$D^2_\epsilon := D^2 + \overline{D}^2 + \epsilon\left[\frac{z\bar{z}}{\bar{z}-z}\left(\bar{\partial}-\partial\right) + \left(z^2\partial - \bar{z}^2\bar{\partial}\right)\right]$$

$$D^2 = z^2(1-z)\partial^2 - (a+b+1)z^2\partial \; - abz.$$

[Dolan, Osborn]

$$z = -\sinh^{-2}\frac{x}{2}$$

[Isachenkov,VS]

Calogero-Sutherland Hamiltonian

$$H^{CS} = \partial^2_{x_1} + \partial^2_{x_2} + V^{CS}_{a,b,\epsilon}\,(x_1, x_2)$$

2-particle Poeschl-Teller

solvable [Heckman, Opdam]…

Thermal Casimir

$$\mathcal{C}_{\Delta_\phi} = -2q^2\partial_q^2 - 2y^2\partial_y^2 + \frac{4q^2}{1-q}\partial_q + \frac{4y^2}{1-y}\partial_y - 2\frac{q+y}{q-y}(q\partial_q - y\partial_y) - 2\frac{qy+1}{qy-1}(q\partial_q + y\partial_y)$$
$$+ \frac{q}{(q-1)^2}\mathcal{D}^{(1)}_{\Delta_\phi}(s) + \frac{y}{(y-1)^2}\mathcal{D}^{(2)}_{\Delta_\phi}(s) + \left(\frac{qy}{(q-y)^2} + \frac{qy}{(qy-1)^2}\right)\mathcal{D}^{(3)}_{\Delta_\phi}(s)\,,$$

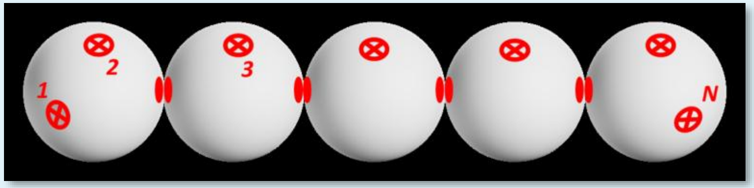
$$\mathcal{D}^{(1)}_{\Delta_\phi} = 8s^2(1-s)\partial_s^2 + 4\big(1 + 2\Delta_\phi - 2s(\Delta_\phi+1)\big)s\partial_s - 2\Delta_\phi\big(1 + (s-1)\Delta_\phi\big)\,,$$

$$\mathcal{D}^{(2)}_{\Delta_\phi} = 8s(s-1)\partial_s^2 + 4(3s-2)\partial_s\,, \quad \mathcal{D}^{(3)}_{\Delta_\phi} = -\frac{\mathcal{D}^{(1)}_{\Delta_\phi} + \mathcal{D}^{(2)}_{\Delta_\phi}}{2} + \Delta_\phi(\Delta_\phi - 3)\,.$$

?

[Gobeil, Maloney, Ng, Wu],[Buric,Russo,VS,Vichi] $_{\ell\neq 0}$

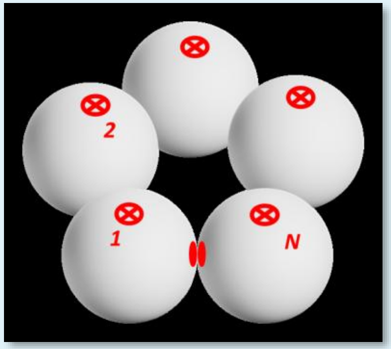
Claim T=0: Multi-point conformal blocks are wave functions of integrable Gaudin model in some limit on $\mathcal{M}_{0,N}$ [Buric,Lacroix,Mann,Quintavalle,VS]



Caterpillar limit for comb channel

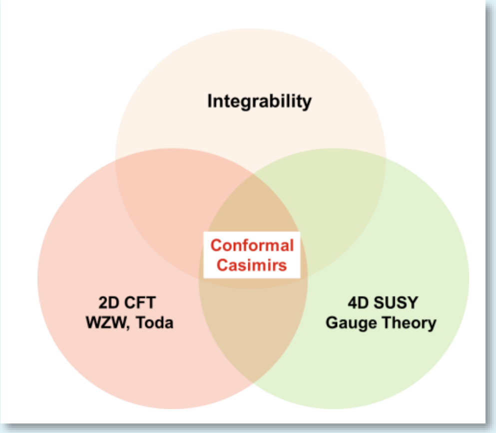
Gaundin ↔ critical level limit of KZ connection for 2D SO(1,d+1) WZW.

Claim T>0: Thermal conformal blocks are wave functions of integrable Hitchin model in some limit on $\mathcal{M}_{g,N}$ *[Buric,Mangialardi,Russo,VS,Vichi]*



Necklace limit

Also valid for CFTs on genus-g manifolds g>1 and in all OPE channels



Commissioning in

Free Field Theory

Partition function and 1-point function

Of the Free Bosonic Field in 3D

Logarithm of the partition function of the 3D free bosonic field is given by

$$\log \mathcal{Z}^{FB}(q,y) = \sum_{n=1} \frac{1}{n}\, z^{FB}(q^n, y^n) \qquad z^{FB} = \chi_{\frac{1}{2},0} - \chi_{\frac{5}{2},0} = \frac{q^{\frac{1}{2}}(1+q)}{(1-qy)(1-qy^{-1})}$$

sum over particle # $\quad (\Delta_\phi, \ell_\phi) = \left(\frac{1}{2}, 0\right) \quad$ eom

We computed character decomposition up to $\Delta \sim 200$ → spectrum of FB

Thermal 1-point function of ϕ^2 computed with method of images & OPE

$$\langle \phi^2(x) \rangle_{q,y} = \sum_{n\neq 0} \frac{q^{\frac{n}{2}}}{(1+q^{2n} - 2q^n(\cos^2\theta + \sin^2\theta\cos(n\mu)))^{1/2}}$$

sum over images

We computed thermal block decomposition to $\Delta \sim 50$ → OPE coefficients

any spin ℓ

Computing Heavy Asymptotics

A mathematical tool [Zagier]

Given: $g(\beta) = \sum_{n=1}^{\infty} f(n\beta)$

with $f(\beta) = f_- + f_+ \sim \sum_{m<-1} a_m \beta^m + \sum_{m\geq -1} b_m \beta^m$ rapidly decaying at large β

finite sum

Then $g(\beta)$ has the following asymptotic expansion at $\beta \sim 0$:

$$g(\beta) = \sum_{m<-1} a_m \zeta(-m)\beta^m - b_{-1}\frac{\log\beta}{\beta} + \frac{I_f}{\beta} + \sum_{m\geq -1} b_m \zeta(-m)(-\beta)^m$$

with $I_f = \int_0^\infty d\beta\,(f_+(\beta) - \frac{b_{-1}e^{-\beta}}{\beta})$

applies to both, partition function and 1-point function of free boson

The Spectral Density

Exact spectrum and asymptotics

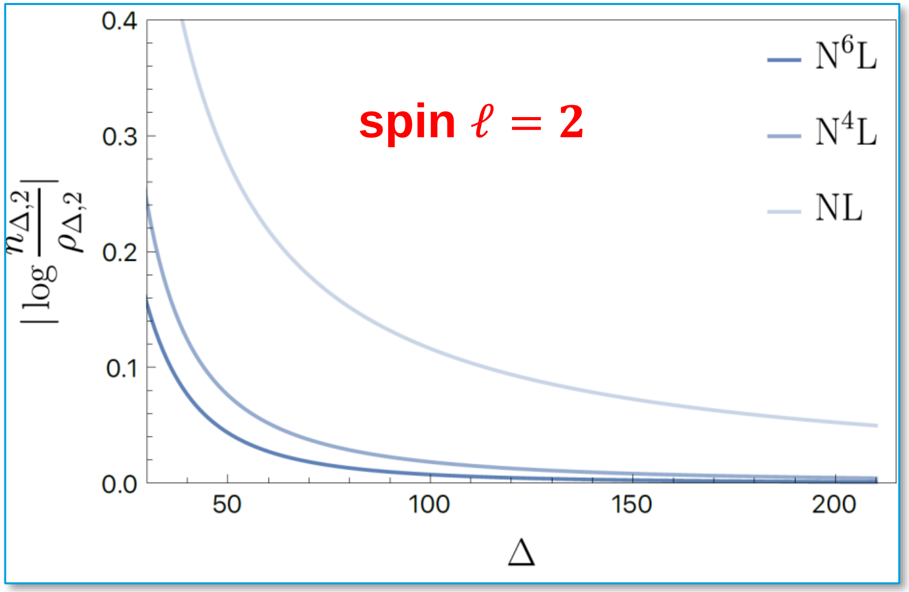
contain only $\log^n \Delta$

$$\boxed{\rho^{FB}(\Delta,\ell) = \frac{e^{-\zeta'(-1)}(2\ell+1)}{\sqrt{3}}\Delta^{-\frac{131}{36}} e^{3\pi^{\frac{1}{3}} f^{\frac{1}{3}} \Delta^{\frac{2}{3}}} \sum_{n=0} \rho_n(\Delta,\ell)\Delta^{-n/3}}$$

$$f = \frac{\zeta(3)}{2\pi}$$

computable, see eqs. (D.6-11) in 2506.21671

Summing terms up to $n = 6$ capture exact spectrum well down to $\Delta \sim 100$



Accuracy for scalar primaries at $\Delta = 210$:

L	(n=0)	40 %
NL	(n≤1)	5.5 %
N^4L	(n≤4)	0.14 %
N^6L	(n≤6)	0,06 %

The HHL OPE coefficients

Exact $\mathcal{O}\mathcal{O}\phi^2$ coefficients and asymptotics

contain only $\log^n \Delta$

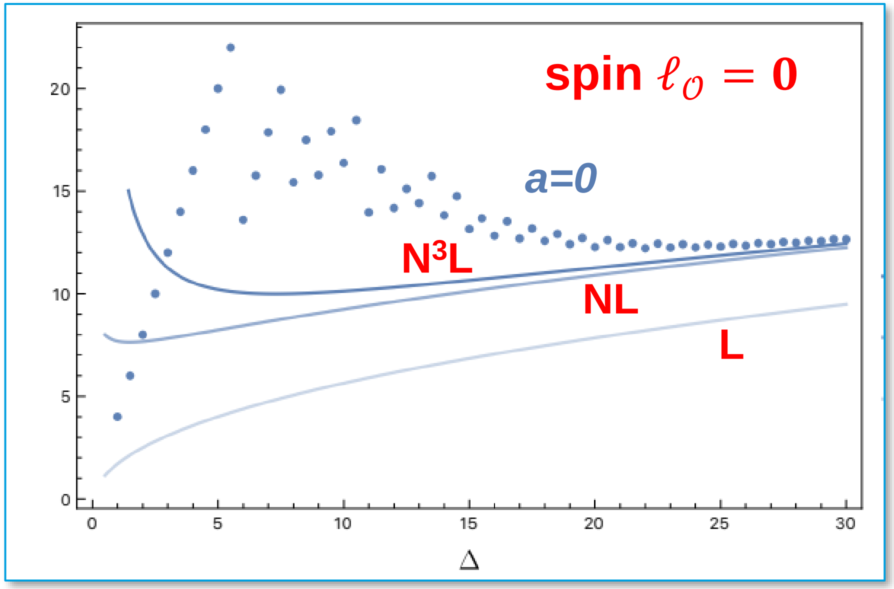
$$\boxed{\overline{\lambda^a_{\mathcal{O}\mathcal{O}\phi^2}}(\Delta,\ell) = \left(\frac{\Delta}{\pi f}\right)^{\frac{1}{3}} \Delta^{-2a} \sum_{n=0} \overline{\lambda^a_n}(\Delta,\ell)\Delta^{-\frac{n}{3}}}$$

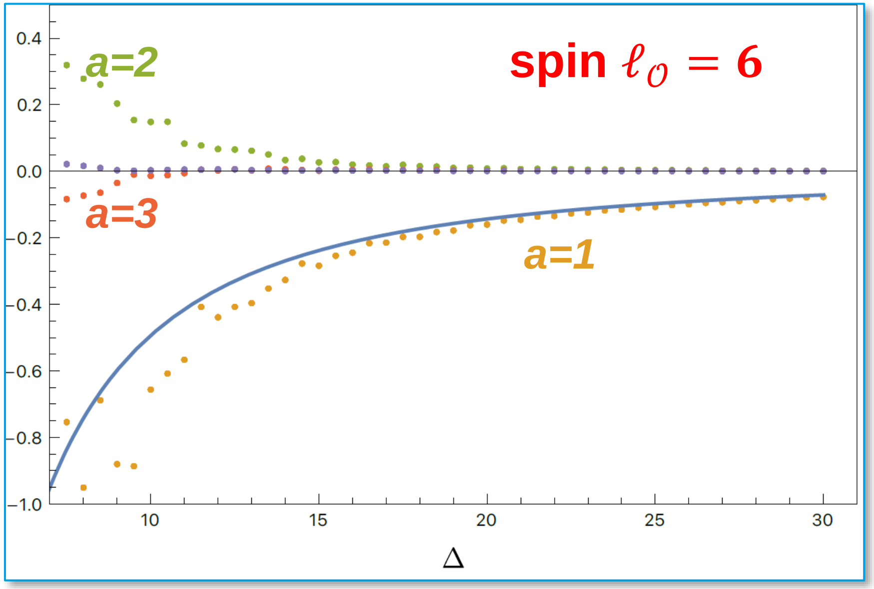
$f = \dfrac{\zeta(3)}{2\pi}$

TS suppression

computable, see eqs. (D.13-37) in 2506.21671

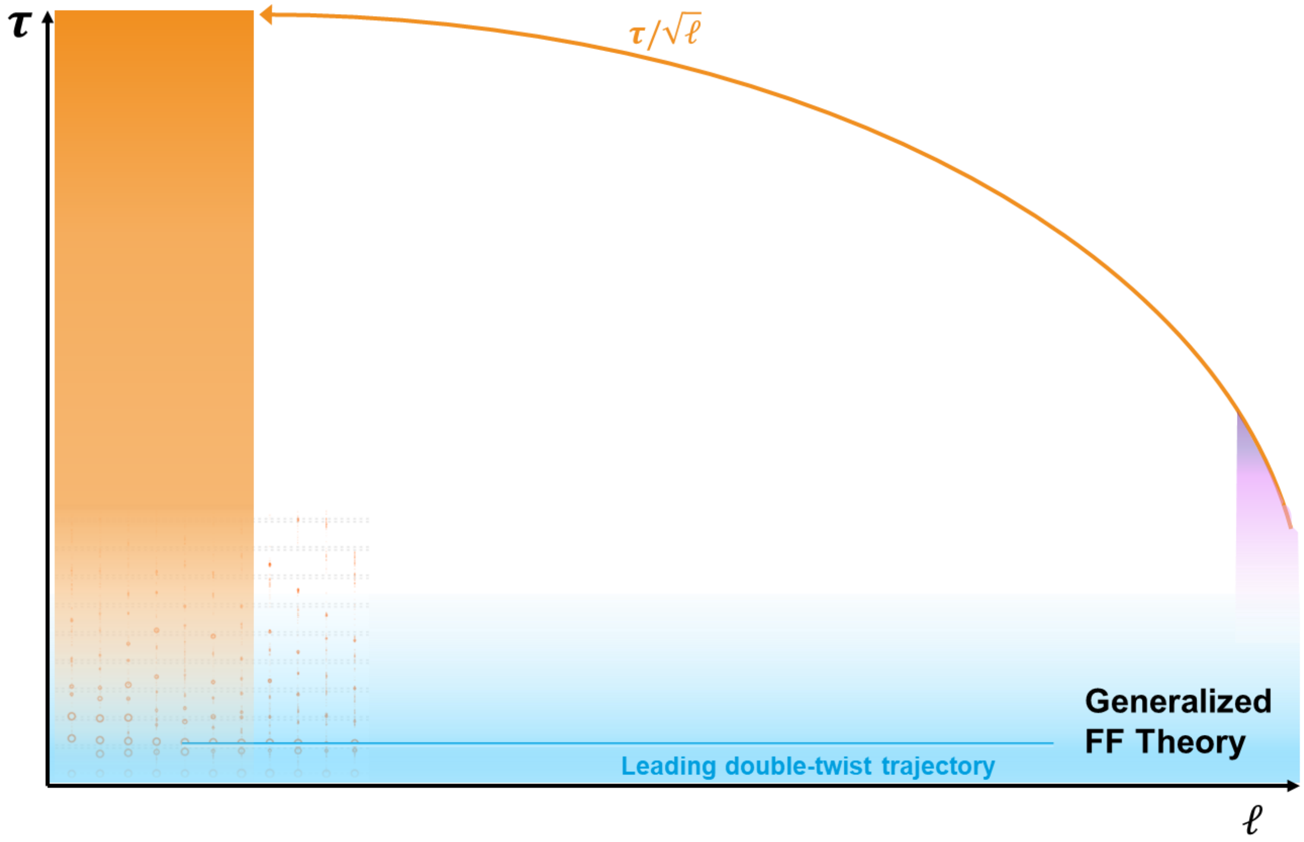
Summing first few terms captures OPE coefficients very well down to $\Delta \sim 25$





Applications: Heavy CFT Data

In gapped CFTs and dual to AdS4



Thermal Effective Actions

For gapped theories

Dimensional reduction of a *generic* CFT_d along the thermal circle gives a gapped QFT_{d-1} which can be captured by the thermal effective action.

[Braaten et al.] . . . . [Bhattacharya et al.] . . . . [Benjamin et al.]

$$\mathcal{Z}^{CFT}_{S^{d-1}\times S^1}(\underset{\downarrow}{\overset{\text{d-dim metric}}{G}}, \dots) \sim \mathcal{Z}^{GP}_{S^{d-1}}(\overbrace{g, A, \varphi,}^{\text{KK modes}} \dots) =: e^{-S_{th}(g,A,\varphi,\dots)}$$

with $$S_{th} = \int d^{d-1}x \sqrt{\hat{g}}\left(-f + c_1 \hat{R} + c_2 F^2 + \cdots\right) + \cdots$$

Thermal effective action replaces 2D modular invariance for CFT in d>2

[Benjamin, Lee, Ooguri, Simmons-Duffin]

For computation of λ^{2D}_{HHL} from modular invariance see [Kraus, Maloney]

Asymptotics of Spectral Density

In the case of gapped theories

Leading terms in expansion of $\log \mathcal{Z}^{GP}$ [Bhattacharya et al.],[Benjamin et al.]

$$\log \mathcal{Z}^{GP}(\beta,\Omega) = \frac{4\pi f}{\beta^2(1+\Omega^2)} - 8\pi c_1 - \frac{32\Omega^2\pi c_2}{3(1+\Omega^2)} + \beta^2\left(4\pi c_3 + O(\Omega^2)\right) + o(\beta^2)$$

$\log \beta$ term absent

EFT parameters

Insertion into inversion formula and saddle point analysis gives expansion:

$$\rho^{GP}(\Delta,\ell) = \frac{8\pi^{\frac{2}{3}} f^{\frac{5}{3}}(2\ell+1)}{\sqrt{3}} e^{-8\pi c_1} \Delta^{-\frac{11}{3}} e^{3\pi^{\frac{1}{3}}\Delta^{\frac{2}{3}}} \times$$

← given in [Benjamin et al.] $_{\Delta \gg \ell}$

$$\times\left(1 - \frac{3\pi^{\frac{1}{3}} f^{\frac{1}{3}}}{\Delta^{\frac{1}{3}}} - \frac{\pi^{-\frac{1}{3}} f^{-\frac{1}{3}}\left(16\pi c_2 + \frac{35}{9}\right) - \pi^{\frac{2}{3}} f^{\frac{2}{3}}(16\pi c_3 + 5)}{\Delta^{\frac{2}{3}}} + O(\Delta^{-1})\right)$$

Important correction in FB – depends on f

HHL Asymptotics of OPE coefficients

In the case of gapped theories

Leading terms in expansion of $\langle\phi(x)\rangle_{\beta,\Omega}^{GP}$

$$\langle\phi(x)\rangle_{\beta,\Omega}^{GP} = \frac{1}{\beta^{\Delta_\phi}}\left(b_{0,0} + b_{0,2}(s)\,\Omega^2 + O(\Omega^4)\right) + \frac{1}{\beta^{\Delta_\phi - 1}}\left(\mathbf{b}_{1,0} + O(\Omega^2)\right) + O\left(\beta^{2-\Delta_\phi}\right)$$

$$= b_{0,2,0} + b_{0,2,1}s$$

Insertion into inversion formula and saddle point analysis gives expansion:

$$\overline{\lambda^a_{\phi\mathcal{O}\mathcal{O}}}(\Delta,\ell) = b_{0,2a,a}\left(\frac{\Delta}{8\pi f}\right)^{\frac{\Delta_\phi}{3}} \mathcal{N}_{a,\ell}\,\Delta^{-2a}\left(1 + O(\Delta^{-\frac{1}{3}})\right)$$

Note: hierarchy tensor structures

Consistent with [Delacretaz]

$$\mathcal{N}_{a,\ell} = \frac{\Gamma(1+2a)}{2^{1+4a}\,\Gamma(\frac{3}{2}+2a)}\sqrt{\frac{\pi(4a+1)\Gamma(2(1+\ell+a))}{(2\ell+1)\Gamma(1+2(\ell-a))}}\,,$$

$$\overline{\lambda^0_{\phi\mathcal{O}\mathcal{O}}}(\Delta,\ell) = \left(\frac{\Delta}{8\pi f}\right)^{\frac{\Delta_\phi}{3}}\left(b_{0,0} + \frac{2\pi^{\frac{1}{3}} f^{\frac{1}{3}} b_{1,0}}{\Delta^{\frac{1}{3}}} + O(\Delta^{-\frac{2}{3}})\right)$$

L NL + …

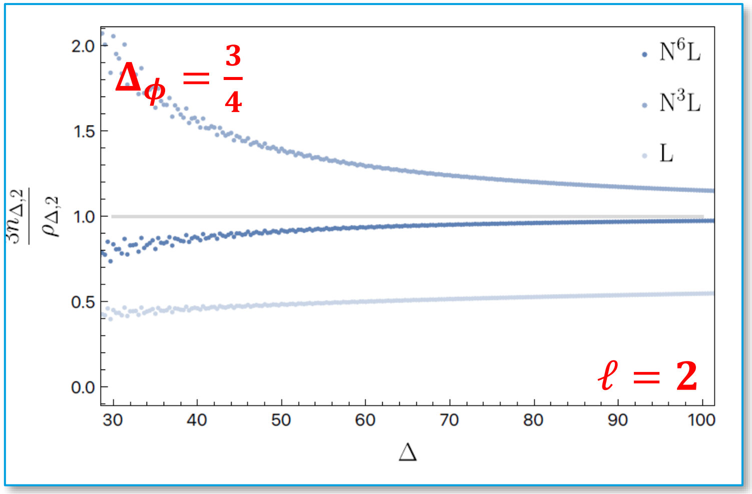
L agrees with prediction from ETH

[Lashkari,Dymarsky,Liu]

Dual of massive Klein-Gordon Field in AdS4

Asymptotics in weakly interacting particle regime

$$S = \int_{AdS4} d^4x\,\sqrt{g}\left(\Phi\left(\nabla^2 + m^2\right)\Phi + \lambda_3\Phi^3 + \lambda_4\Phi^4\right)$$



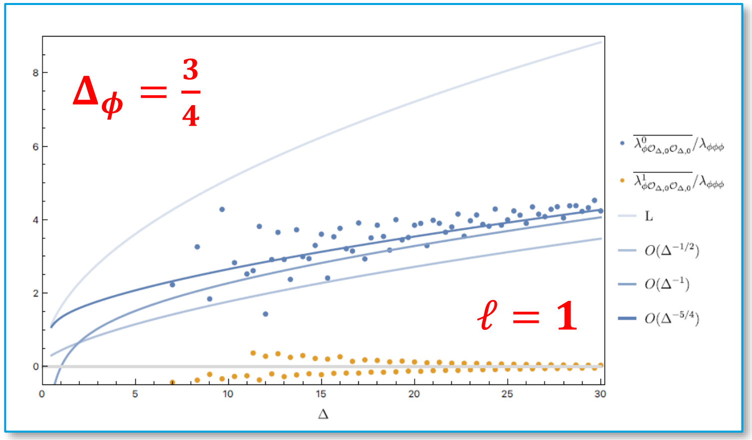
Zeroth order is GFF on boundary ∂AdS_4:

Asympototic expansions of density and

OPE work to intermediate Δ.

with exponents of 4D theory [2/3→3/4, 1/3→1/4]

Computed spectrum and OPE coefficients in first order perturbation theory for small Δ, ℓ and in asymptotic regime. **good match**



See 2606.17167 for more

Conclusions and Outlook

Conformal block decompositions for thermal 1-point functions on $S_\mu^{d-1} \times S_\beta^1$ of scalar primary fields ϕ in 3D work well. Thermal blocks · GitLab

- Maximal resolution of $\lambda_{\phi\mathcal{O}\mathcal{O}}^a$ - except for degeneracies in spectrum
- Hierarchy $\overline{\lambda_{\phi\mathcal{O}\mathcal{O}}^a} \sim \Delta_\mathcal{O}^{-2a}$ of tensor structures for large $\Delta_\mathcal{O}$ in GFF & gapped theory

Extension to thermal 1-point functions of spinning primary fields possible

Casimir operator known & blocks computable

Casimir operators for any number of fields and higher genus-g manifolds can be identified as Hitchin Hamiltonians.

- Can one fit EFT parameters in asymptotics from numerical Data (Ising) ?
- Can one do bootstrap for heavy data ?
  - 2-point functions on $S^2 \times S^1$ or
  - partition function on genus-2
- OPE coefficients in regime of large twist and large spin ?