Bootstrapping Open and Closed Strings with Maximal Supersymmetry and Peculiar Parity

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Open string [arXiv:2601.11705]
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Closed string [arXiv:2607.14230]
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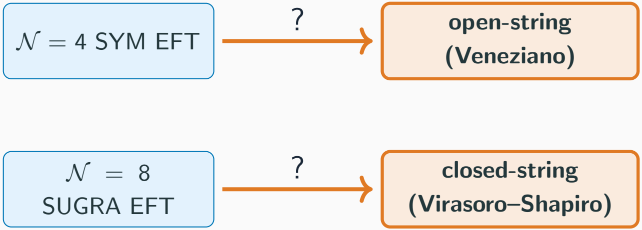
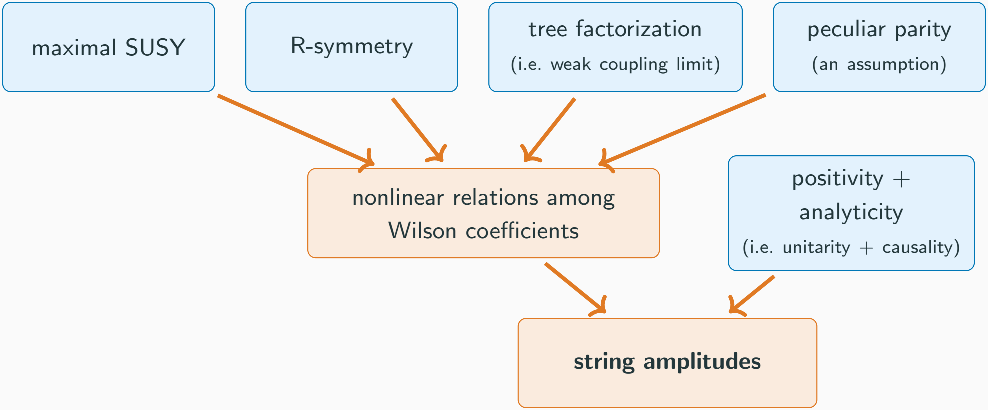
Can consistency alone single out string theory?

Start with a maximally supersymmetric low-energy EFT. Assume a **weak-coupling limit** exists. Demand consistent higher-point factorization and a positive UV spectrum. Could the only survivors be Veneziano and Virasoro–Shapiro?



Additional inputs are deliberately field-theoretic



No worldsheet, monodromy, double copy, zeros, or special UV behavior assumed.

Part 1: Gauge Theory

planar $\mathcal{N} = 4$ SYM and the open string

Four-point freedom looks enormous before higher points

$$\mathcal{L} = \underbrace{-\frac{1}{4}\mathrm{Tr}F^2}_{\text{leading SYM}} + \underbrace{\mathrm{Tr}F^3}_{\text{forbidden by SUSY}} + \underbrace{\mathrm{Tr}F^4 + \mathrm{Tr}D^2F^4 + \mathrm{Tr}D^4F^4 + \mathrm{Tr}D^6F^4 + \dots}_{\text{higher-derivative EFT}}$$

	$\mathrm{Tr}F^3$	$\mathrm{Tr}F^4$	$\mathrm{Tr}D^2F^4$	$\mathrm{Tr}D^4F^4$	$\mathrm{Tr}D^6F^4$
number of operators	1	3	5	8	10
with lin. $\mathcal{N}=4$ SUSY	0	1	1	2	2
effective couplings		$a_{0,0}$	$a_{1,0}$	$a_{2,0}$ $a_{2,1}$	$a_{3,0}$ $a_{3,1}$

Linearized $\mathcal{N}=4$ SUSY compresses the basis, but leaves the $a_{k,q}$ **independent**.

One function determines every planar four-point amplitude

Instead of working with the Lagrangian, we bootstrap the amplitudes directly.

$$\underbrace{A_4[--++]}_{\text{component amplitude}} = \underbrace{\langle 12\rangle^2[34]^2}_{\text{fixed kinematics}} \underbrace{F(s,u)}_{\text{all EFT data}}$$

$$F(s,u) = \underbrace{-\frac{g^2}{su}}_{\text{pure SYM}} + \underbrace{f(s,u)}_{\text{higher derivatives}}$$

$$\underbrace{f(s,u)}_{\text{Wilson expansion}} = a_{0,0} + a_{1,0}(s+u) + a_{2,0}(s^2+u^2) + a_{2,1}su + \dots$$

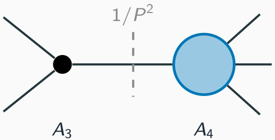
At four points, maximal SUSY imposes only the crossing relation $F(u,s) = F(s,u)$.

Higher points from factorization and locality

Reconstruct general EFT amplitudes from factorization on physical poles plus local contact terms:

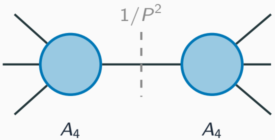
$$\lim_{P^2\to 0} A_n \;=\; A_L(\ldots, P)\,\frac{1}{P^2}\,A_R(-P,\ldots)$$

5-point: $A_3 \times A_4$



one $A_4 \Rightarrow$ linear in $a_{k,q}$

6-point: $A_4 \times A_4$



two $A_4 \Rightarrow$ **quadratic** in $a_{k,q}$

Poles are fixed by lower-point amplitudes; locality leaves only free local contact terms.

Six points expose information four points cannot see

$$\underbrace{\mathbb{Z}_1}_{\text{minimal-pole seed}} \equiv \underbrace{A_6[z_1 z_2 z_3 \bar{z}_3 \bar{z}_1 \bar{z}_2]}_{\text{six-scalar NMHV component}}$$

$$\underbrace{\mathbb{Z}_6}_{\text{sixth cyclic image}} = \underbrace{c_1\mathbb{Z}_1 + c_2\mathbb{Z}_2 + c_3\mathbb{Z}_3 + c_4\mathbb{Z}_4 + c_5\mathbb{Z}_5}_{\text{SUSY Ward identity}}$$

- Same kinematic c_i at every EFT order
- The Ward identity imposes a linear relation among amplitudes related by cyclic permutations
- First overconstraint at cubic Mandelstam order

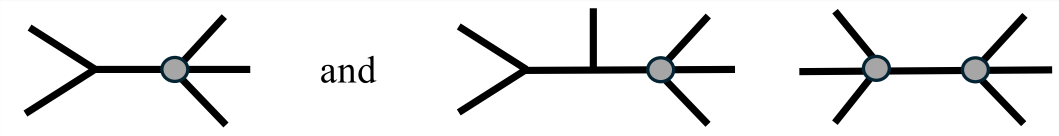
Peculiar parity: the extra assumption

$\underbrace{\text{no } \epsilon_{\mu\nu\rho\sigma}}_{\text{local contact terms}}$ $\longleftrightarrow$ $\overline{\mathbb{Z}}_1 = \mathbb{Z}_1$ $\longleftrightarrow$ $\underbrace{SO(6) \to O(6)}_{\text{pure scalar sector}}$

- Not implied by $SU(4)$ R-symmetry
- Applied only to the six-scalar amplitude
- Broken by fermion loops beyond tree level

Every gauge-theory result that follows requires this parity assumption.

Quadratic constraints come from 3-particle factorization channels



5-point pole · 6-point 2-particle pole · 6-point 3-particle channel — the last carries $A_4 \times A_4$

$$\underbrace{\operatorname*{Res}_{s_{ijk}=0} \mathbb{Z}_1}_{\text{six-point pole}} \sim \underbrace{A_4}_{\text{first EFT vertex}} \times \underbrace{A_4}_{\text{second EFT vertex}} \supset \underbrace{f \times f}_{\text{quadratic in } a_{k,q}}$$

$$\underbrace{g^2 a_{2,0}}_{\mathrm{Tr} D^4 F^4 \;\; (q=0)} = \frac{2}{5} a_{0,0}^2 \,, \qquad \underbrace{g^2 a_{2,1}}_{\mathrm{Tr} D^4 F^4 \;\; (q=1)} = \frac{1}{10} a_{0,0}^2$$

Purely tree-level factorization at $\mathcal{O}(s^3)$: turning on $\mathrm{Tr} F^4$ forces both $\mathrm{Tr} D^4 F^4$ structures.

The constraints have teeth

one massive
$\mathcal{N} = 4$ multiplet

infinite spin tower
in gauge theory

one-loop
Coulomb-branch
box amplitude

- Four-point SUSY allows each candidate
- Six-point parity + SUSY rejects each candidate
- Their six-scalar amplitudes must contain parity-odd contact terms

Ruled out under the parity condition, not inconsistent as quantum theories.

Veneziano satisfies every constraint

The 4-gluon open-superstring (Veneziano) amplitude is a product of Gamma functions, i.e. an Euler Beta function:

$$A_4^{\text{str}}[--++] = -g^2\,\alpha'^2\,\langle 12\rangle^2[34]^2\,\frac{\Gamma(-\alpha' s)\,\Gamma(-\alpha' u)}{\Gamma\big(1-\alpha'(s+u)\big)}\,.$$

Its low-energy α'-expansion gives Wilson coefficients that are **Riemann zeta values**:

$$a_{0,0} = g^2\alpha'^2\zeta_2, \quad a_{1,0} = g^2\alpha'^3\zeta_3, \quad a_{2,0} = g^2\alpha'^4\zeta_4, \quad a_{2,1} = \frac{g^2}{4}\alpha'^4\zeta_4, \quad a_{3,0} = g^2\alpha'^5\zeta_5, \ \ldots$$

(→ full coefficient list)

These obey the nonlinear constraints *automatically*, because of relations among the zeta values. The first constraint $g^2 a_{2,0} = \frac{2}{5}a_{0,0}^2$ is nothing but

$$\zeta_4 = \frac{2}{5}\,\zeta_2^2\,, \qquad \left(\frac{\pi^4}{90} = \frac{2}{5}\left(\frac{\pi^2}{6}\right)^2\right). \quad \checkmark$$

Beyond the overall scale, only the odd zetas stay free

The unfixed coefficients are the overall scale $a_{0,0}$ and the odd-zeta slots:

Parameter	Veneziano value
$a_{0,0}$	$g^2\alpha'^2\zeta_2 = g^2\frac{\pi^2}{6}\alpha'^2$
$a_{1,0}$	$g^2\alpha'^3\zeta_3$
$a_{3,0}$	$g^2\alpha'^5\zeta_5$
$a_{5,0}$	$g^2\alpha'^7\zeta_7$
$\vdots$	$\vdots$
$a_{2m-1,0}$ $(m \geq 1)$	$g^2\alpha'^{2m+1}\zeta_{2m+1}$

Constraints fix everything that could be fixed. Remaining freedom: the overall scale $a_{0,0}$ and the odd-zeta slots $a_{2m-1,0}$.

($\rightarrow$ all nonlinear relations)

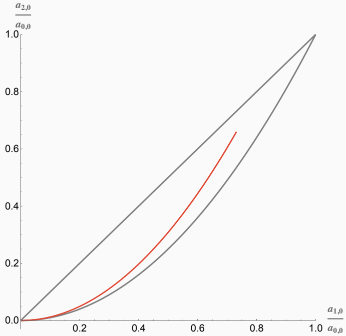
The other input: an unsubtracted dispersion relation

$$a_{k,q} = \sum_{\ell=0}^{\infty} \int_1^{\infty} dy\, \rho_\ell(y)\, y^{-k-1} v_{\ell,q}\,,$$

$$P_\ell(1+2\delta) = \sum_{q=0}^{\ell} v_{\ell,q}\delta^q$$

- $y = s'/M^2_{\mathrm{gap}}$, all $a_{k,q}$ dimensionless
- Unitarity enters only as $\rho_\ell \geq 0$; also $v_{\ell,q} \geq 0$
- $\mathcal{N} = 4$ SUSY gives the s^2 prefactor, hence **no subtractions**

$$0 \leq \left(\frac{a_{1,0}}{a_{0,0}}\right)^2 \leq \frac{a_{2,0}}{a_{0,0}} \leq \frac{a_{1,0}}{a_{0,0}} \leq 1$$



Positivity alone: a convex wedge containing the open string.

[Arkani-Hamed, T.-C. Huang, Y.-t. Huang, 2012.15849; Caron-Huot, Van Duong, 2011.02957]

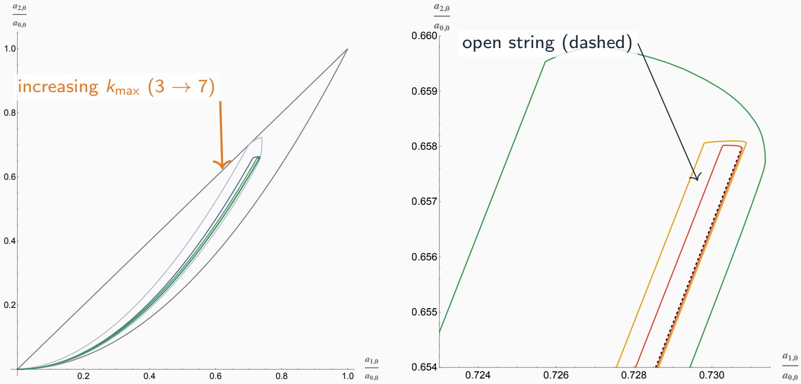
Higher-point constraints destroy convexity

convex allowed region for four-point data $\longrightarrow$ **non-convex locus after quadratic relations**

$$\underbrace{\rho_1, \rho_2 \geq 0, \;\; c_1, c_2 \geq 0 \implies c_1\rho_1 + c_2\rho_2 \geq 0}_{\text{positivity preserves convex combinations}} \qquad \underbrace{a_{2,0} \propto a_{0,0}^2}_{\text{nonlinearity does not}}$$

A line between two allowed theories **need not** be allowed.

The SYM region collapses onto the open-string curve



Non-convex regions shrink with $k_{\max}$; dashed black curve = open string. The bootstrap returns a curve, not an island: the surviving freedom is $\alpha' M^2_{\text{gap}} \le 1$. ($\to$ the $a_{2,0}$ vs $a_{2,1}$ plane)

$k_{\max} = 7$: endpoint 0.65802 versus $\zeta_4/\zeta_2 = 0.657974\ldots$, a 4.6×10^{-5} separation.

Independent route: monodromy + positivity ⇒ Veneziano

Monodromy relations: linear identities among differently ordered open-string amplitudes, from worldsheet contour deformations [Plahte, 1970; Bjerrum-Bohr, Damgaard, Vanhove, 0907.1425; Stieberger, 0907.2211]

$$A(1,2,3,4) + e^{i\pi\alpha' s_{12}} A(2,1,3,4) + e^{i\pi\alpha'(s_{12}+s_{13})} A(2,3,1,4) = 0$$

- Veneziano is the unique solution of dispersion relations + unitarity + monodromy, to all orders in α' [Wan, Zhou, 2605.11084]
- Numerical precursors [Huang, Liu, Rodina, Wang, 2008.02293; Berman, Elvang, Herderschee, 2310.10729]

Here monodromy follows from the six-point SUSY relations: derived, not assumed.

Part 2: Gravity

$\mathcal{N} = 8$ supergravity and the closed string

Too much R-symmetry leaves only pure supergravity

Why the gravity analysis must break maximal R-symmetry before it can begin:

unbroken $SU(8)$
$+$ peculiar parity

$\xrightarrow{+\text{ positivity}}$

all 4-point corrections vanish:
pure $\mathcal{N} = 8$ SUGRA at 4 points

$$\underbrace{g_0 = g_2 = g_3 = \cdots = 0}$$

with unbroken $SU(8)$, parity gives the first zeros; positivity then forces every 4-point Wilson coefficient to vanish

To admit stringy corrections, maximal R-symmetry must break to $SU(4) \times SU(4)$.

Same mechanism applies in gravity

$$\underbrace{F(s,t,u)}_{\text{crossing-symmetric dynamics}} = \underbrace{\frac{\kappa^2}{stu}}_{\text{pure SUGRA}} + \underbrace{g_0 + g_2(s^2+t^2+u^2) + g_3 stu + \dots}_{D^{2k}R^4 \text{ corrections}}$$

	Gauge theory	Gravity
Maximal SUSY	$\mathcal{N}=4$	$\mathcal{N}=8$
R-symmetry	$SU(4)$	$SU(4)\times SU(4)$
Kinematics	color ordered	fully crossing symmetric
Seed residue	$A_4 \times A_4$	$M_4 \times M_4$

No planarity or color ordering, but the higher-point mechanism **survives** in gravity.

Peculiar parity in the $(6,6)$ scalars, and its tower

2 singlets axio-dilaton	32 scalars $(\mathbf{4},\overline{\mathbf{4}}) + \text{c.c.}$	**36 scalars** $(\mathbf{6},\mathbf{6})$

$$\underbrace{\mathbb{Z}_{1,1} = \overline{\mathbb{Z}}_{1,1}}_{\text{no Levi–Civita terms in the chosen six-scalar amplitude}} \qquad \Longleftrightarrow \qquad \underbrace{O(6) \times O(6)}_{\text{enhanced scalar symmetry}}$$

$$\underbrace{\kappa^2 g_3 = \frac{1}{2} g_0^2}_{\mathcal{O}(s^7)}\,, \qquad \underbrace{\kappa^2 g_5 = g_2 g_0}_{\mathcal{O}(s^9)}\,, \qquad \underbrace{\kappa^4 g_6' = \frac{8}{3}\kappa^4 g_6 + \frac{1}{6} g_0^3}_{\mathcal{O}(s^{10})}$$

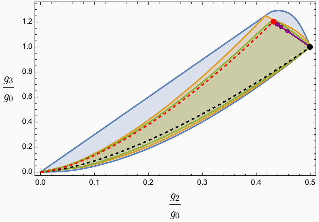
The g_{2m} (coefficients of the even crossing-symmetric structures) remain free; all other coefficients are fixed nonlinearly in terms of them, with this pattern verified through $\mathcal{O}(s^{13})$.

A tree-level bosonic condition, violated by fermion loops.

Without an extra spectral input, gravity has two corners

$$M_4^{\rm str}(--++) = \langle 12\rangle^4[34]^4\,\frac{\kappa^2}{stu}\,\frac{\Gamma(1-\alpha' s)\Gamma(1-\alpha' t)\Gamma(1-\alpha' u)}{\Gamma(1+\alpha' s)\Gamma(1+\alpha' t)\Gamma(1+\alpha' u)}$$

$$M_4^{\rm IST}(--++) = \langle 12\rangle^4[34]^4\left(\frac{\kappa^2}{stu} + \frac{\lambda^2}{(m^2-s)(m^2-t)(m^2-u)}\right)$$



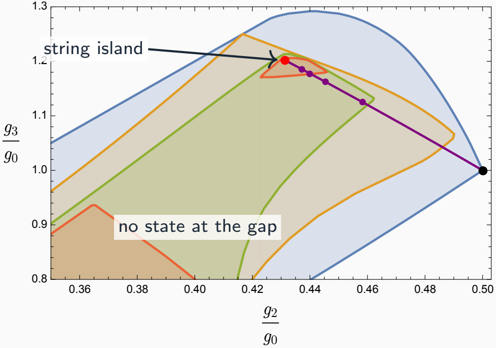
Red: Virasoro–Shapiro. Black: IST.

Purple: generalized towers between them. Units: $\Lambda = 1$.

Both solve every nonlinear relation; the IST needs $\lambda^2 = 2\kappa^2$.

Odd zetas permit the formal replacement $\zeta_{2k+1} \to 1$: string $\to$ IST — possible because the Γ-ratio contains only (algebraically independent) odd zetas.

Finitely many spins at the gap isolate the string



Only scalars at Λ^2; arbitrary positive spectrum above $\mu_c \Lambda^2$ with $\mu_c = 1.1$. Units: $\Lambda = 1$. ($\rightarrow$ full region)

The region bifurcates: a string island + a rump with no state at the assumed gap.

Analyticity explains the linear string spectrum

nonlinear relations → exponential in the free g_{2m} → analyticity $\Rightarrow$ product → finitely many spins at m_1 → **linear spectrum Virasoro–Shapiro**

$$\underbrace{M_4^{\{m\}}}_{\text{meromorphic amplitude}} = \underbrace{\frac{\kappa^2 s^4}{stu} \prod_n \frac{(m_n^2+s)(m_n^2+t)(m_n^2+u)}{(m_n^2-s)(m_n^2-t)(m_n^2-u)}}_{\text{positive product form}} \Longrightarrow \underbrace{m_n^2 = n\, m_1^2}_{\text{polynomial lowest-level residue}}$$

(→ derivation)

Discussion

What we showed, said carefully

Results

- Higher points constrain four-point EFT data
- Gauge region approaches the open-string curve
- Gravity has string and IST corners
- Finitely many spins at the gap isolate Virasoro–Shapiro

Caveats

- Peculiar parity assumed, not derived
- Weak-coupling tree level throughout
- Assumes all-orders exponentiation

Consistency points to strings – and to the missing principle

Higher points

Factorization converts symmetries into nonlinear relations among lower-point data.

Non-convex bootstrap

Distinguished amplitudes appear as curves, corners, and islands.

Open problems

"Derive" peculiar parity; generalize to AdS/CFT; reduce SUSY.

Collaborators & Thanks

Joint work with

arXiv:2601.11705

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Thank you. Questions?