IMPERIAL COLLEGE · BOOTSTRAP 2026

Bootstrapping holographic theories in the planar limit

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01

Motivation

01 Why the planar limit?

TREE-LEVEL STRINGS

- Correlation functions in holographic CFTs are dual to string theory scattering in AdS
- The planar limit is dual to **tree-level string scattering**
- The cleanest window on the string spectrum at finite tension

WHICH THEORIES?

$\mathcal{N} = 4$ sYM

- We can study **closed string** scattering in type IIB

USp theory

- 4d $\mathcal{N} = 2\, USp(2N)$ with $SO(8)$ flavour symmetry
- Simplest holographic theory dual to **flavour branes** with a weak string-coupling limit
- $\Rightarrow$ simplest setting for **open string** scattering in holography

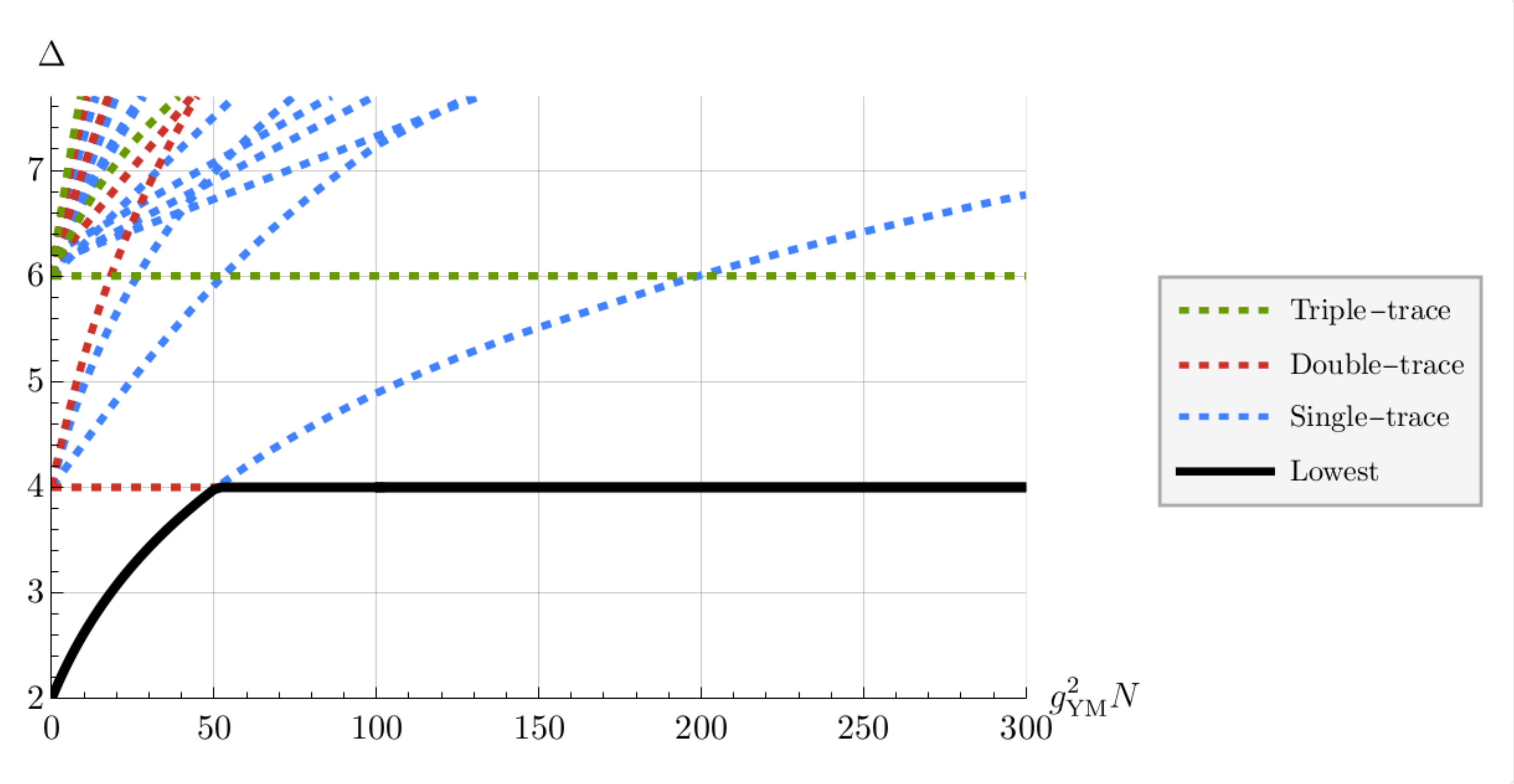
02 Integrability

[Gromov et al., 2013]

The **quantum spectral curve** solves the spectrum of planar $\mathcal{N}=4$ sYM: the scaling dimension of every single-trace operator, at *finite* coupling.

BUT

- OPE coefficients are still out of reach
- Nothing beyond the planar limit
- Not available for the $USp(2N)$ theory

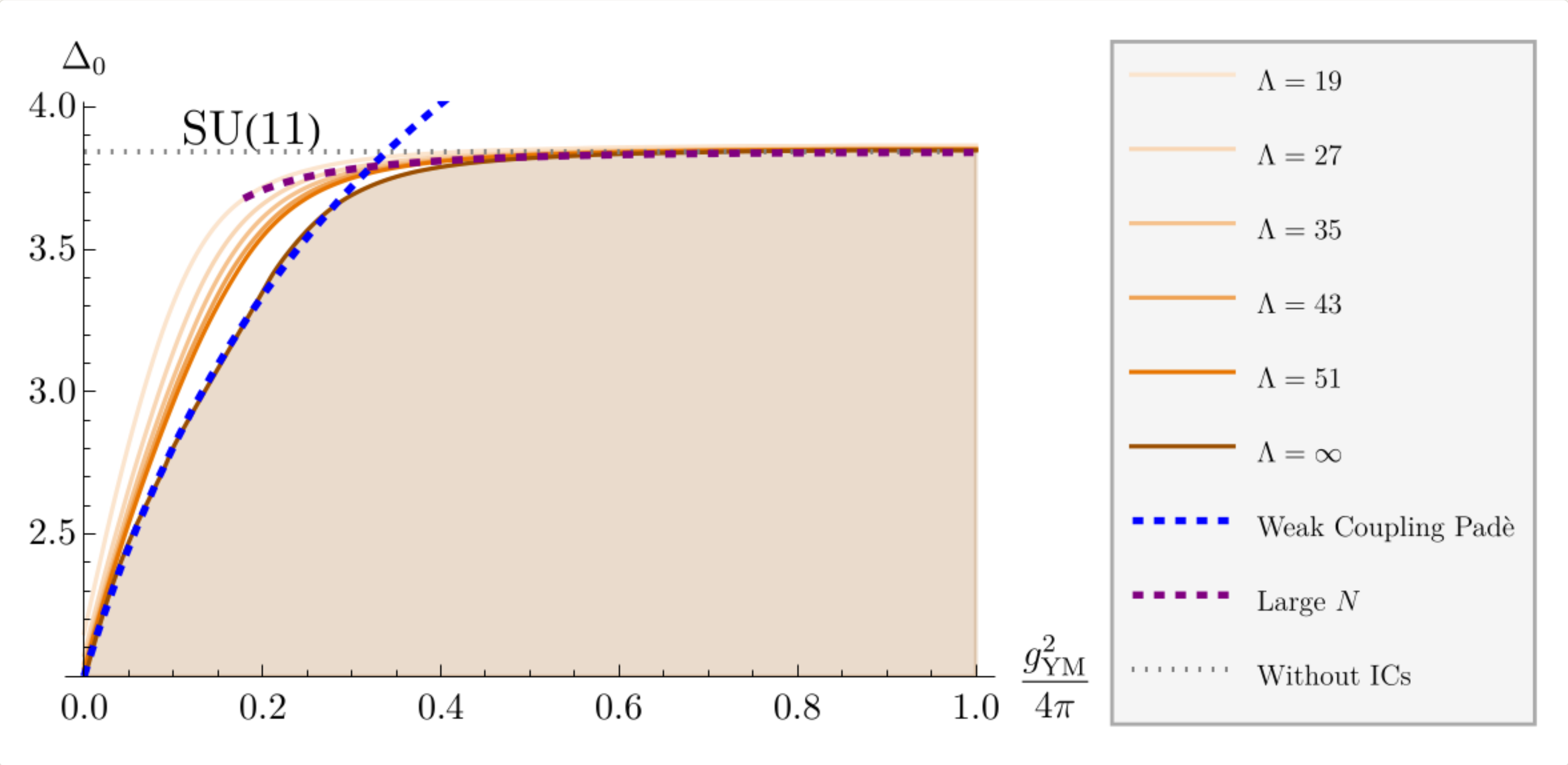


The planar $\mathcal{N}=4$ spectrum at finite coupling.
[Gromov et al., 2023]

We need tools that do not rely on integrability.

03 Adding localization to the bootstrap

- **Localization** — mass derivatives of the sphere free energy give integrated correlators, exact in the coupling
- **Bootstrap** — crossing and unitarity constrain the CFT data
- Far more general than integrability: one can use localization for all lagrangian theories with **enough supersymmetry**



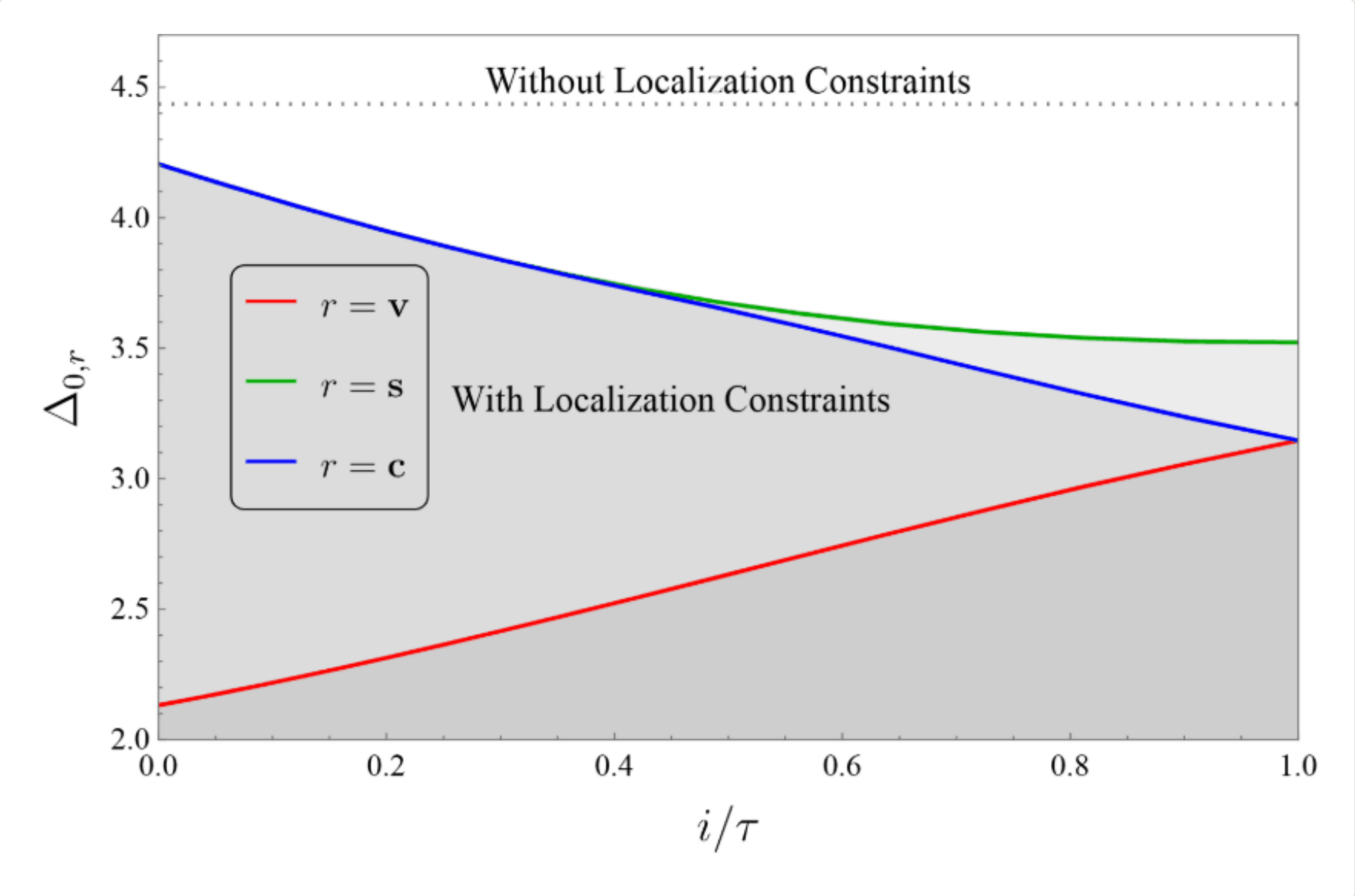
Lowest long scalar Δ_0 at finite N, $SU(11)$.
[Chester et al., 2023]

05 Bootstrap and localization at finite N

$\mathcal{N} = 2$

- Same programme for the $USp(2N)$ theory
- Bounds on the lowest long operator in each $SO(8)$ channel $r = \mathbf{v},\ \mathbf{s},\ \mathbf{c}$
- The integrated constraints cut the allowed region down substantially

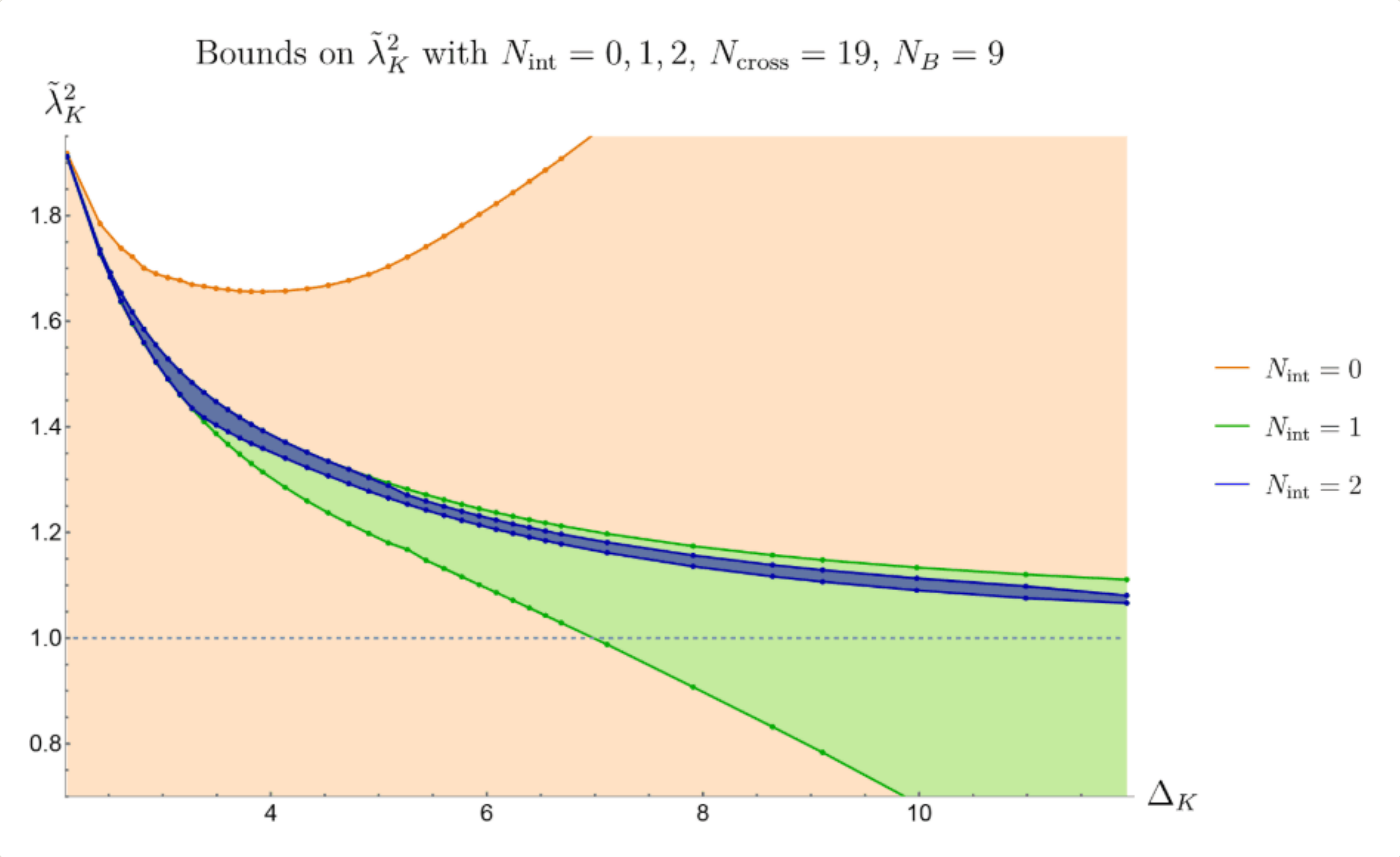
Only $N = 1$ is available — larger N gives very weak bounds.



Bounds on $\Delta_{0,r}$ vs i/τ at $N = 1$, with and without localization constraints.
[Chester, 2023]

04 Planar Bootstrap for OPE coefficients

Localization can be included in the **Bootstrability** program. Doing so one can obtain strong results for the OPE coefficient of the Konishi operator in $\mathcal{N} = 4$ sYM.



Konishi OPE coefficient, planar $\mathcal{N} = 4$ sYM; also uses integrability
[Caron-Huot et al., 2024]

MOTIVATION

06 This work

Can the **planar** bootstrap with localization reach bounds saturated by physical theories with **no input from integrability**?

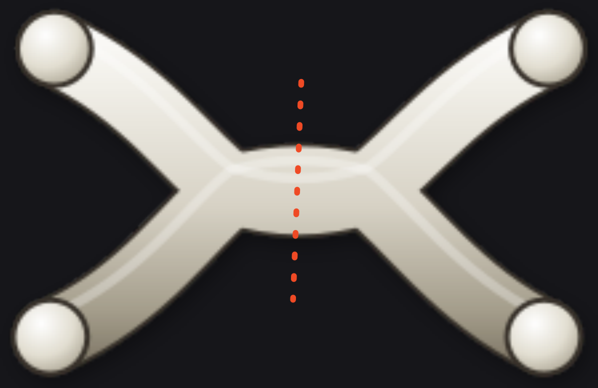
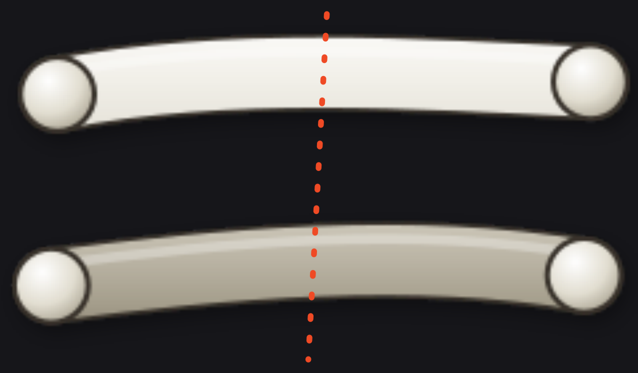
- $\mathcal{N}=2\,USp(2N)$ — use dispersion relations and localization to obtain the first bounds at finite λ
- $\mathcal{N}=4\,\mathrm{sYM}$ — proof of principle: here integrability gives an independent check

CONCLUSION

You can bootstrap holographic theories in the planar limit even when there is no integrability.

02

Planar Bootstrap with localization

$\langle \mathcal{O}_2\, \mathcal{O}_2\, \mathcal{O}_2\, \mathcal{O}_2 \rangle = \sum$  $+ \sum$ 

07 The theory and the correlator

[Aharony et al., 1998]

THE GAUGE THEORY

- 4d $\mathcal{N}=2$ $USp(2N)$, with 4 fundamental $+$ 1 antisymmetric hypermultiplet
- Flavour symmetry $SO(8)$
- Dual to **gluons** on 4 D7-branes; the flavour central charge $k=4N$ controls the planar expansion

THE OBSERVABLE

- $\mathcal{O}_2^a$: the **moment map operator** of the $SO(8)$ flavour symmetry
- Scalar in the adjoint $\mathbf{28}$ of $SO(8)$
- Dual to the **gluons** on the branes

All the dynamics sits in a single function of the cross ratios u, v, with the flavour indices running over the $SO(8)$ channels:

$$\langle \mathcal{O}_2^a \mathcal{O}_2^b \mathcal{O}_2^c \mathcal{O}_2^d \rangle \sim T^{abcd}(u,v)$$

$$\mathbf{28} \times \mathbf{28} = \mathbf{1} \oplus \mathbf{28} \oplus \mathbf{35}_v \oplus \mathbf{35}_s \oplus \mathbf{35}_c \oplus \mathbf{300} \oplus \mathbf{350}$$

08 Mellin space

The **Mellin amplitude** is the natural language for a holographic correlator — the AdS analogue of a flat-space scattering amplitude.

$$T^{abcd}(u,v) = \int \frac{ds\,dt}{(4\pi i)^2}\, u^{\frac{s}{2}-2} v^{\frac{t}{2}-2}\, \Gamma^2(2-\tfrac{s}{2})\Gamma^2(2-\tfrac{t}{2})\Gamma^2(2-\tfrac{u}{2})\, M^{abcd}(s,t)$$

- **Poles** in s, t, u at the twists of the exchanged operators
- **Crossing** acts on s, t, u as for an amplitude ($u = 6 - s - t$)
- The **flat-space limit** returns the string amplitude

09 The OPE expansion

At order $1/k$ only **single-** and **double-trace** operators appear:

$$T^{abcd} = T^{abcd}_{\text{short}} + \frac{1}{k} \sum_{\substack{\text{single trace} \\ r\in\{\mathbf{1},\mathbf{28},\mathbf{35}_v\}}} P_r^{abcd}\, \lambda^2\, g^{\mathcal{N}=2}_{\tau,J} + \sum_{\substack{\text{double trace} \\ r\,\in\,\mathbf{28}\times\mathbf{28}}} P_r^{abcd}\, \lambda'^2\, g^{\mathcal{N}=2}_{\tau,J} + O\left(\frac{1}{k^2}\right)$$

Single-trace — only in $\mathbf{1}$, $\mathbf{28}$, $\mathbf{35}_v$. What we want.

Double-trace — in every channel of $\mathbf{28}\times\mathbf{28}$, and dominant.

- Double traces give the **disconnected** part — dominant, and they **break positivity** at order $1/k$

10 The double-discontinuity

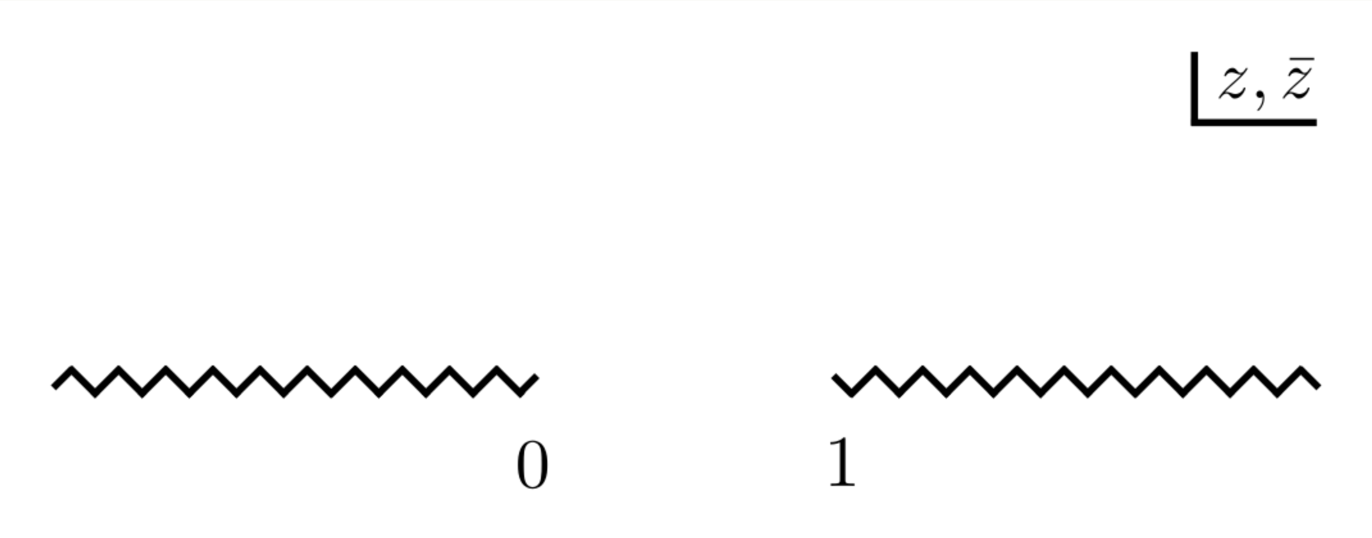
- Remove the double-trace operators with the **double discontinuity**

$$\mathrm{dDisc}\, T(z,\bar z) \equiv T(z^{\curvearrowleft},\bar z^{\curvearrowright}) - \tfrac{1}{2}T(z^{\curvearrowleft},\bar z^{\curvearrowleft}) - \tfrac{1}{2}T(z^{\curvearrowright},\bar z^{\curvearrowright})$$

On a conformal block it is diagonal:

$$\mathrm{dDisc}\; g^{\mathcal{N}=2}_{\tau,J} = 2\sin^2\!\left(\tfrac{\pi}{2}\tau\right)\, g^{\mathcal{N}=2}_{\tau,J}$$

The **double zero** at $\tau = 4+2n$ kills every double trace $\Rightarrow$ the dDisc keeps only **single-trace data**.



Discontinuities in the z plane.

11 The dispersion relation

[Carmi et al., 2020; Caron-Huot et al., 2020]

The dDisc is enough to reconstruct the **whole** correlator:

$$T(u,v) = \int du'\, dv' \; K(u,v;u',v')\; \mathrm{dDisc}\, T(u',v')$$

with K a fixed, theory-independent kernel:

$$K(u,v;u',v') = \frac{u-v+u'-v'}{64\pi(uvu'v')^{3/4}}\, x^{3/2}\, {}_2F_1\!\left(\tfrac{1}{2},\tfrac{3}{2},2,1-x\right)\, \theta\Big(\sqrt{v'} - (\sqrt{u}+\sqrt{v}+\sqrt{u'})\Big)$$
$$+ \frac{1}{4\pi(uvu'v')^{1/4}}\, \frac{1}{\sqrt{u}+\sqrt{u'}}\, \delta\Big(\sqrt{v'} - (\sqrt{u}+\sqrt{v}+\sqrt{u'})\Big)$$

Consequence: in the planar limit T is completely fixed by **single-trace data**.

In **Mellin space** this is just Cauchy's theorem — the Γ functions of the Mellin kernel already supply the double-trace contributions.

$$M(s,t) = \int \frac{ds'}{2\pi i}\, \frac{M(s',t)}{s'-s}$$

12 Polyakov blocks and gluon correlator

Disperse a single block $\Rightarrow$ the **Polyakov block**, closed-form in Mellin space:

$$P_{\tau,J}(s,t) = (-1)^J \sum_{m=0}^{\infty} \frac{\mathcal{Q}^m_{\Delta+2,J}(6-s-t)}{s-(\tau+2m)}$$

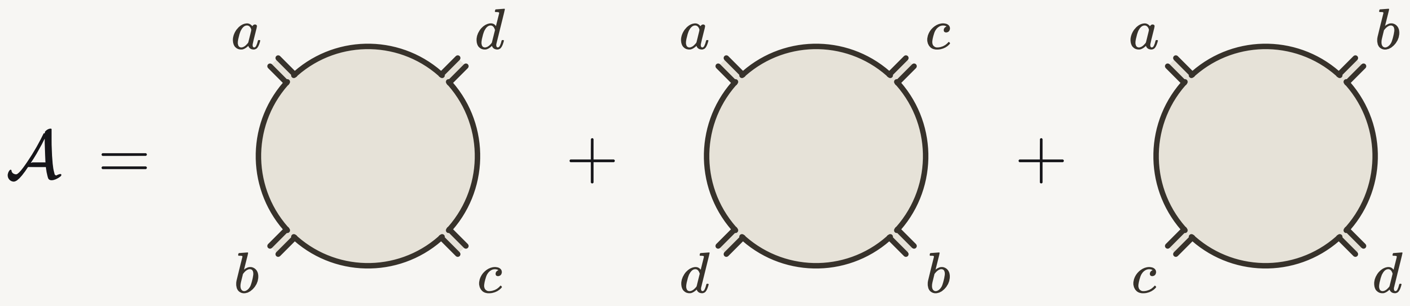
The short multiplet disperses into the gluon amplitude: since $\mathrm{dDisc}\, T^{abcd}_{\text{short}} = \mathrm{dDisc}\, T^{abcd}_{\text{gluon}}$, we conclude

$$T^{abcd}_{\text{gluon}}(u,v) = \int du'\, dv'\;\, K(u,v;u',v')\;\mathrm{dDisc}\, T^{abcd}_{\text{short}}(u',v')$$

Taking the dDisc **broke crossing symmetry** — imposing it back is what constrains the data.

13 Color structure and crossing

The color structure is that of **gluon scattering in AdS**:



$$M^{abcd}(s,t) = \tfrac{1}{k}\left[\mathrm{Tr}(T^aT^bT^cT^d)\,M(s,t) + \mathrm{Tr}(T^aT^dT^bT^c)\,M(t,u) + \mathrm{Tr}(T^aT^cT^dT^b)\,M(u,s)\right]$$

One **color-stripped** amplitude for all three orderings — crossing symmetric becomes:

$$M(s,t) = M(t,s)$$

Its OPE runs over **single-trace operators only** — even spin in the $\mathbf{35}_v$, odd spin in the $\mathbf{28}$:

$$M(s,t) = M^{\text{gluon}}(s,t) + \tfrac{1}{12}\sum_{J\text{ even}}\left[P_{\tau,J}(s,t) + P_{\tau,J}(t,s)\right] + \tfrac{1}{12}\sum_{J\text{ odd}}\left[P_{\tau,J}(s,t) + P_{\tau,J}(t,s)\right]$$

Imposing the color structure on the Polyakov expansion we get the **crossing symmetry equation**.

14 Localization constraint

[Chester, 2022; Behan et al., 2023]

We can **inject the coupling** via localization: in the planar limit exactly one integrated correlator for the USp theory is independent.

$$I\left[M^{\text{gluon}}(s,t) + \sum P(s,t)_{\tau,J}\right] = \frac{4\pi}{\sqrt{\lambda}} \int_0^\infty \frac{dw}{\sinh^2(w)} w^2 J_1\left(\frac{\sqrt{\lambda}w}{\pi}\right) - 3\zeta(3)\,,$$

- $I[\,\cdot\,]$ — a fixed s, t, u-symmetric integration kernel
- $I[M](\lambda)$ known from localization on the **sphere partition function** — the fourth mass derivative $\partial_m^4 F\big|_{m=0}$ of $F = -\log Z_{S^4}$
- Free and strong-coupling pieces cancel $\Rightarrow$ one linear constraint on the **single-trace** data, at finite coupling

15 Normalizing the OPE

To bound CFT data numerically, the functionals must be **normalized on a known quantity**.

ISSUE

M^{gluon} is crossing symmetric, so we have nothing to normalize the OPE with.

15 Soft Regge limit

To bound CFT data numerically, the functionals must be **normalized on a known quantity**.

ISSUE

M^{gluon} is crossing symmetric, so we have nothing to normalize the OPE with.

SOLUTION

String Theory

A string amplitude is **softer in the Regge limit**: at fixed t, M^{gluon} and each Polyakov block are $O(s^{-1})$ but $M(s,t)=o(s^{-1})$, so the leading term must cancel:

$$0=\frac{8}{2-t}+\sum_{J\text{ even}}\frac{\lambda^2_{\mathbf{35}_v,\tau,J}}{12}R^t_{\tau,J}(t)+\sum_{J\text{ odd}}\frac{\lambda^2_{\mathbf{28},\tau,J}}{12}R^t_{\tau,J}(t)\,,\qquad \mathrm{Re}(t)\in(0,2)$$

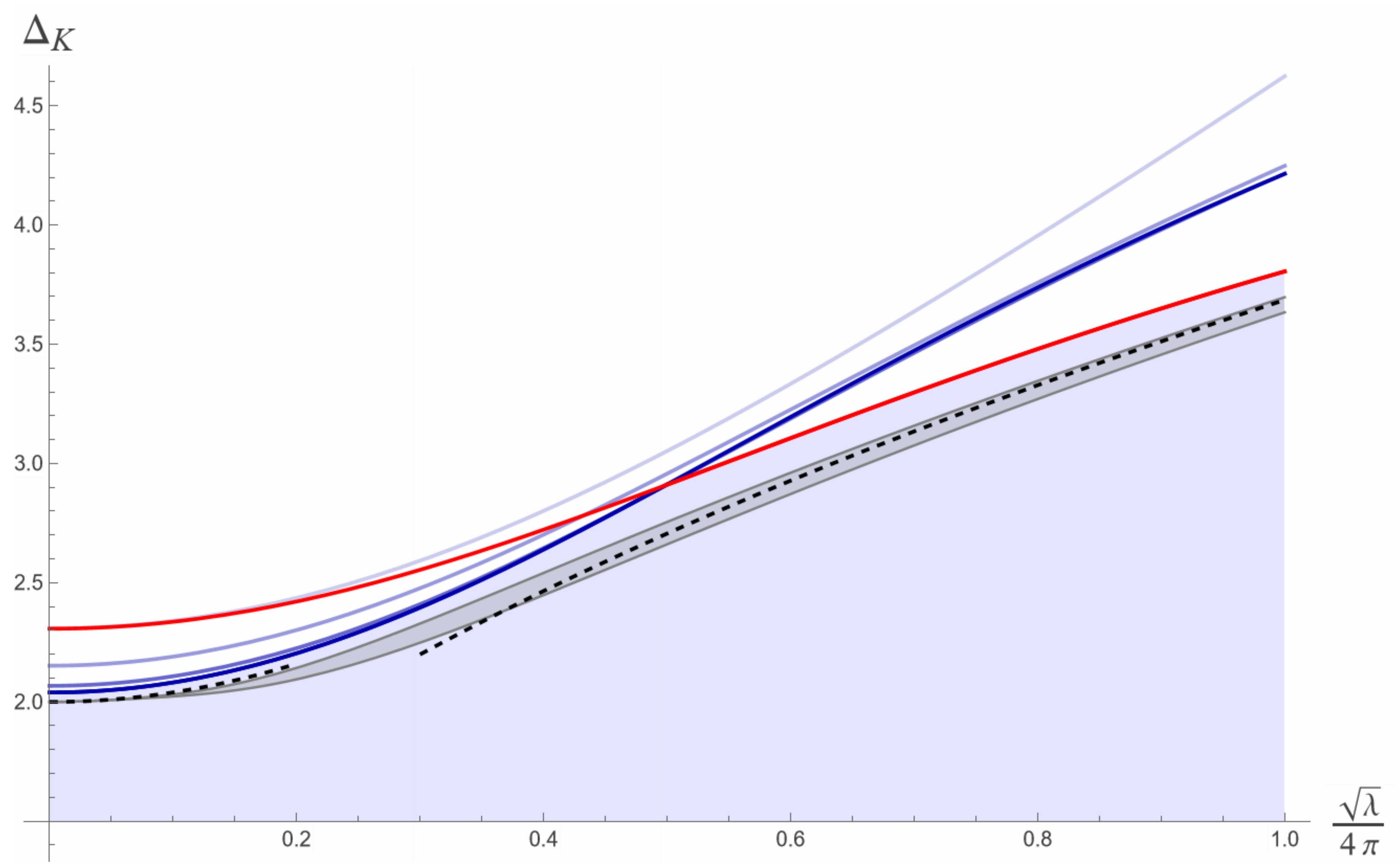
▸ Projection functionals Ψ_l

From R^t we build functionals that **project onto a single spin**:

$$\Psi_l[\tau,J]=\int_{\mathrm{Re}(t)=1}\frac{dt}{2\pi i}\,\Psi_l(t)\,R^t[\tau,J,t]\,,\qquad \Psi_l\left[g^{\mathcal{N}=2}_{2,J}\right]\equiv\delta_{l,J}$$

Ψ_l fixes the OPE coefficients **analytically** in the free-theory limit.

Results



Allowed values for the Konishi scaling dimension $\Delta(\lambda)$ in the planar $USp(2N)$ theory.

- Adding the projection functionals significantly improves the bound at weak coupling resulting a bound that is **better the the corresponding finite N result.**

03

Closed strings: $\mathcal{N} = 4$

17 Setup

[Caron-Huot et al., 2022; 2024]

$\mathcal{N}=4$ sYM with gauge group $SU(N_c)$: the same machinery — dDisc, unitarity and dispersive functionals — applies to **closed string scattering** .

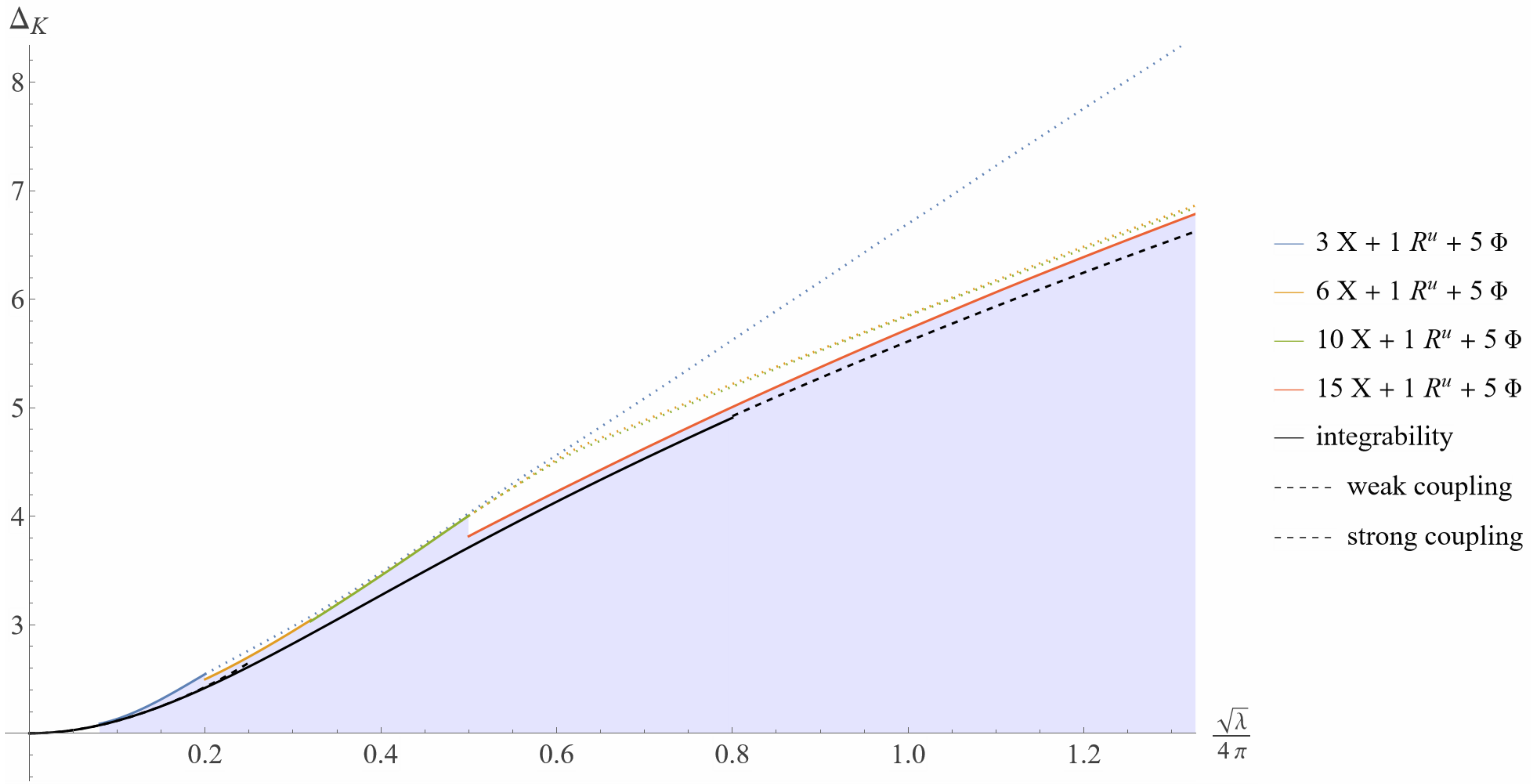
	$\mathcal{N}=2$ · OPEN	$\mathcal{N}=4$ · CLOSED [Caron-Huot et al., 2024]
OPERATOR	moment map $\mathcal{O}_2^a$, **28** of $SO(8)$	stress tensor $\mathcal{O}$, **20′** of $SU(4)_R$
DUAL	gluons on D7, $AdS_5\times S^3$	gravitons, $AdS_5\times S^5$
EXPANSION	$1/k$, $k_{SO(8)}=4N$	$1/c$, $c=\frac{N_c^2-1}{4}$
MELLIN AMPLITUDE	color-ordered, $M(s,t)=M(t,s)$	$M(s,t)$ fully s,t,u-symmetric
PROTECTED TERM	$M^{\text{gluon}}=\frac{8}{(s-2)(t-2)}$	$M^{\text{graviton}}=\frac{8}{(s-6)(t-6)(u-6)}$
REGGE SUM RULE	fixed t — R^t	fixed u — R^u
PROJECTION FUNCTIONALS	Ψ	Φ and Ψ

LOCALIZATION: TWO INTEGRATED CONSTRAINTS

$$I_2(\lambda)=-\frac{1}{2}\int M\,\bar{\Gamma},\qquad I_4(\lambda)=-48\int M\,\bar{\Gamma}\left[\frac{2(u-5)}{(s-6)(t-6)}+\frac{t-s}{u-6}\left(H_{\frac{s}{2}-3}+H_{3-\frac{s}{2}}\right)\right]$$

We bound the dimension of the **Konishi** operator with no gap assumption — and here **integrability** gives an independent check.

Results



Allowed values for the Konishi scaling dimension $\Delta(\lambda)$ in the planar $\mathcal{N}=4$ sYM theory

- Convergence at weak coupling requires imposing positivity for very high spin.

OUTLOOK

Can we do better for $USp(2N)$? Maybe with a **mixed system** of $SO(8)_F$ and $SU(2)_L$ moment maps.

Can we do the same for other theories? Yes — in **planar ABJ**, dual to Type IIA string theory, ▸ we have 2 localization constraints ▸ 2 't Hooft couplings bootstrap.

Can dispersion relations improve the bootstrap at finite N for the $USp(2N)$ theory? suggested by the effectiveness of the Ψ functionals at weak coupling.

Thank You