# Strong coupling limit in YM matrix models

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# Holography

Large $N$ gauge theories

$\mathcal{N} = 4$ SYM in $d = 4$

$\longleftrightarrow$

Quantum gravity
(String theory)

$AdS_5 \times S^5$ ($D = 10$)

# Gravity emergence in holography ?

Many things are understood in $\mathcal{N}=4$. One can use techniques such as bootstrap, integrability, localization, …

But the emergence of dynamical gravity is still poorly understood.

What is special about a gauge theory that has (in some limit) a classical gravitational description?

How would one derive the gravitational background of the dual theory, or the equation of motion for fluctuations around this background?

# Where to ask such questions ?

What is the simplest model (with minimal content) to address the question of gravity emergence ?

Let's search for a holographic gauge theory where:

1. There is no quantum field.
2. There is no Hilbert space.
3. There is no time.
4. The path integral is a product of finitely many integrals.
5. All observables are well-defined and non-trivial functions of a coupling.
6. There is evidence that it is dual to a Euclidean geometry in 10d.

# The polarized IKKT model

Gauge theory in 0 dimension

$$Z = \int [dX_I][d\psi_\alpha] e^{-S}$$

$$\langle f(X,\psi) \rangle = \frac{1}{Z} \int [dX_I][d\psi_\alpha] f(X,\psi) e^{-S}$$

$\longleftrightarrow$

Gravity around some Euclidean geometry

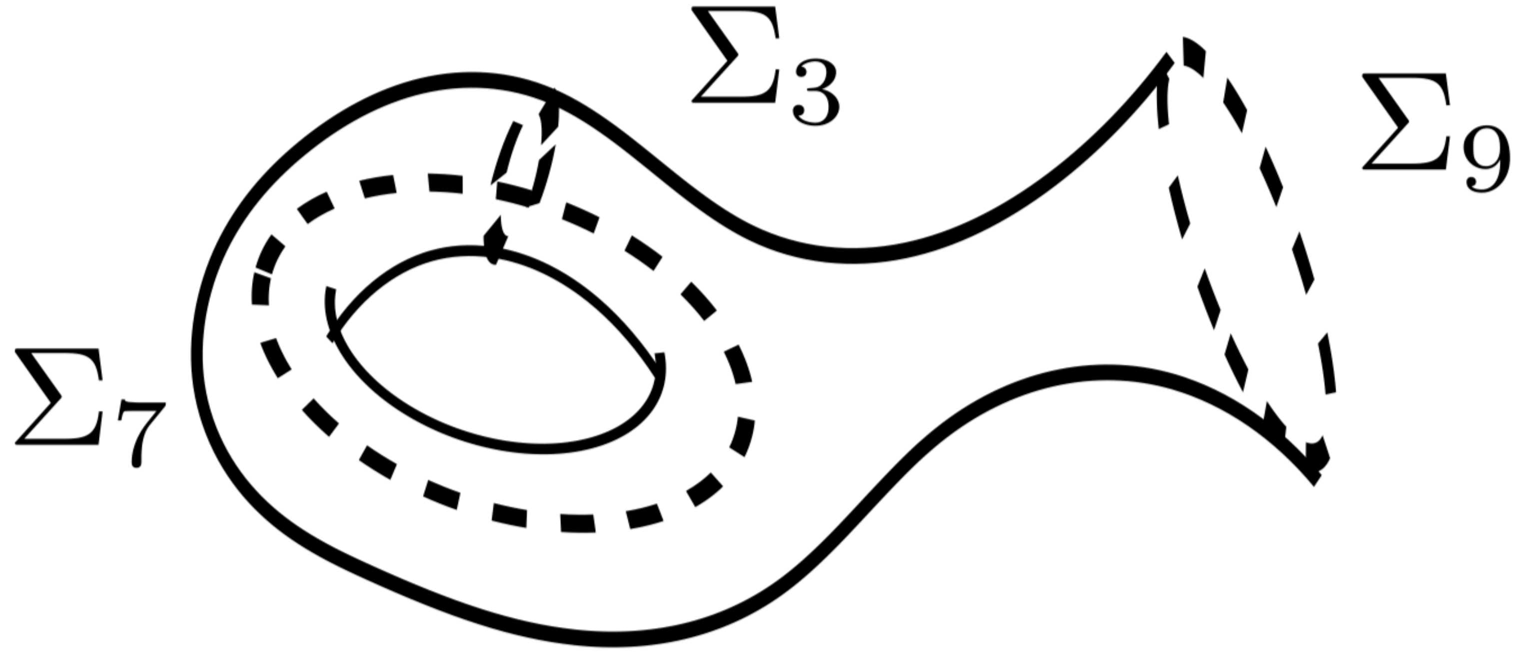



Image from [Hartnoll, Liu ’25]

# The polarized IKKT model

[Bonelli '02]

[Hartnoll, Liu '24]

$$S = \frac{N}{\lambda}\,\mathrm{Tr}\Bigg[-\frac{1}{4}[X_I, X_J]^2 - \frac{i}{2}\bar{\psi}_\alpha \Gamma^I_{\alpha\beta}[X_I, \psi_\beta]$$

$$+\frac{3}{4^3}X_i X_i + \frac{1}{4^3}X_p X_p + i\frac{1}{3}\epsilon^{ijk}X_i X_j X_k - \frac{1}{8}\bar{\psi}_\alpha \Gamma^{123}_{\alpha\beta}\psi_\beta\Bigg]$$

$i, j, \ldots = 1,2,3$ $\qquad$ $p, q, \ldots = 4,5,\ldots,10$

$SO(3) \times SO(7)$, $\;SU(N)$, 16 supersymmetries

# Supergravity regime

- Each classical saddle of matrix integral $=$ a specific geometry.

- The trivial saddle $X_I = 0$ has a well-behaved supergravity dual in the **strong 't Hooft coupling regime**

$$N^{5/3} \gg \lambda \gg 1$$

- Protected observables can be matched between the two sides in this limit.

[Komatsu, AM, Penedones, Vuignier, Zhao '24]

# Today’s question

A proposed proxy for gravity emergence is **commutativity** of the matrices. If matrices commute in the holographic strong-coupling limit, they may potentially admit an interpretation as space coordinates.

[Berenstein, Hanada, Hartnoll ’08]

So the question we will address today is

$\boxed{\text{Do matrices commute in the ’t Hooft limit at strong coupling ?}}$

# Massive YM matrix models

Polarized IKKT is numerically complicated (sign problem, multiple saddles). We will thus study an easier toy example :

$$S = \frac{N}{\lambda} \mathrm{Tr} \left[ -\frac{1}{4}[X_I, X_J]^2 - \frac{i}{2}\bar{\psi}_\alpha \Gamma^I_{\alpha\beta}[X_I, \psi_\beta] + X_I X_I + \bar{\psi}_\alpha \psi_\alpha \right]$$

$$I = 1, \ldots, D \qquad \alpha = 1, \ldots, \mathcal{N}_f$$

# More explicitly

$$Z = \int [dX][d\psi] e^{-S_{\text{bos}}(X) - S_{\text{fer}}(X,\psi)} = \int [dX] \text{Pf} \mathscr{M}(X) e^{-S_{\text{bos}}}$$

Properties of these models:

- Supersymmetric in $D = 3{,}4$ only. [AM '25]
- $\text{Pf}\mathscr{M}(X) \geq 0$ in $D = 4$ only.

We will study various $D$ and $\mathcal{N}_f$ and we replace

$$\text{Pf}\mathscr{M}(X) \to |\text{Pf}\mathscr{M}(X)| \text{ in } D \neq 4.$$

# The method

We use Hybrid Monte-Carlo numerics and estimate

$$R(\lambda) \equiv \lim_{N\to\infty} \left\langle \frac{-\mathrm{Tr}[X_I, X_J]^2}{\mathrm{Tr}\{X_I, X_J\}^2} \right\rangle$$

At large $\lambda$, we will find either:

$$R(\lambda) \sim \lambda^{-\alpha} \implies \text{commuting}$$

$$R(\lambda) \sim \mathcal{O}(1) \implies \text{noncommuting}$$

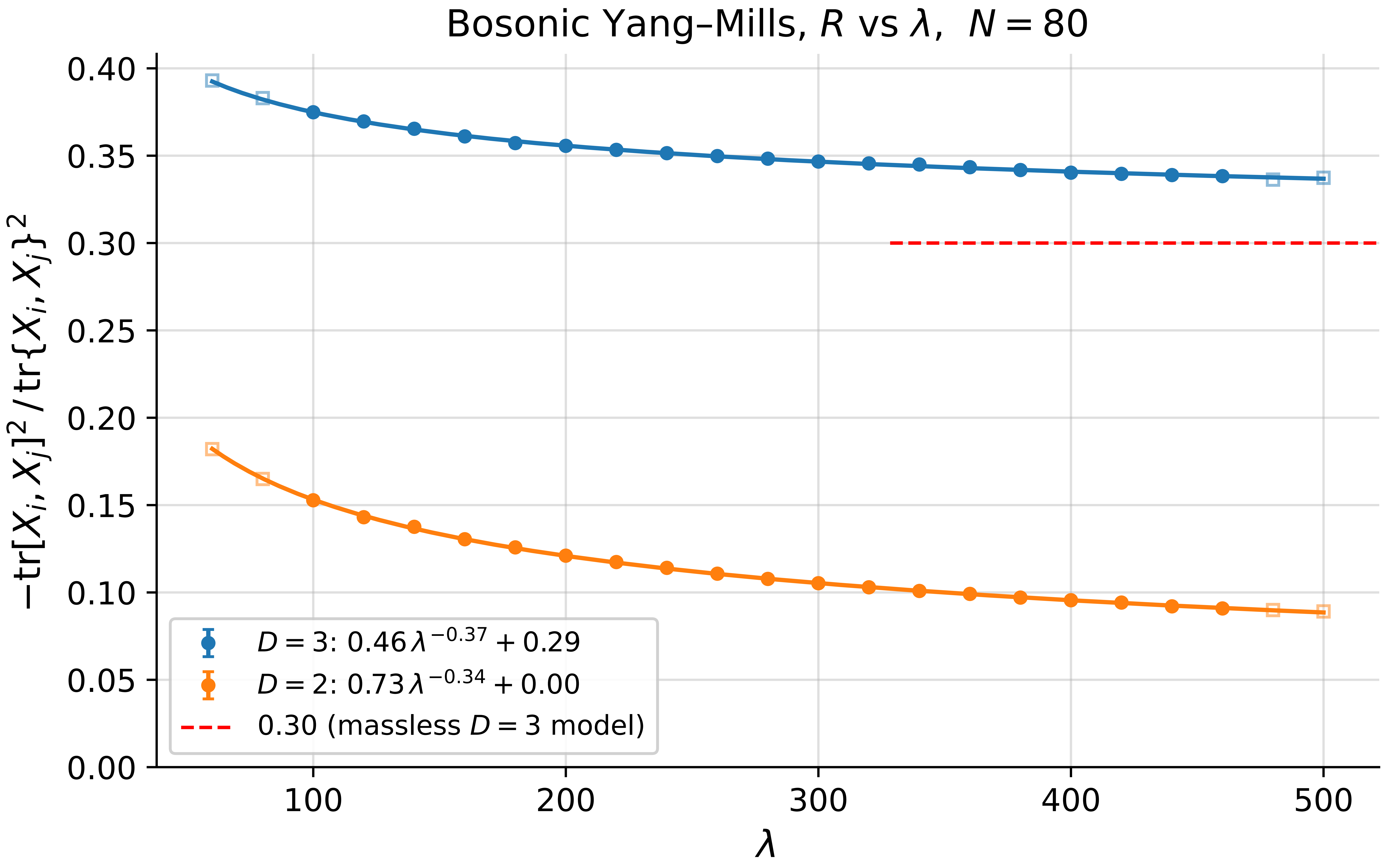
# Outline

- Bosonic models
- Adding fermions, various $\mathcal{N}_f$ regimes.
- Analytical method: When we can integrate out the off-diagonal elements…

# Bosonic models

$$S = \frac{N}{\lambda} \operatorname{Tr}\left[ -\frac{1}{4}[X_I, X_J]^2 + X_I X_I \right]$$

- The commutator squared potential prefers **commuting** configurations.
- The dimensionality of the space of commuting matrices is small, entropy favours **non-commuting** configurations.



Bosonic Yang–Mills, $R$ vs $\lambda$, $N = 80$
$-\mathrm{tr}[X_i, X_j]^2 / \mathrm{tr}\{X_i, X_j\}^2$
$\lambda$
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
100
200
300
400
500
$D = 3$: $0.46\,\lambda^{-0.37} + 0.29$
$D = 2$: $0.73\,\lambda^{-0.34} + 0.00$
0.30 (massless $D = 3$ model)

# Bosonic conclusion

- The $D = 2$ model commutes. In fact, it is solvable. Moreover, it is known to be divergent without the mass term.
- The $D \geq 3$ models do **not** commute. They go to the (convergent) massless models

What happens when we add fermions ?

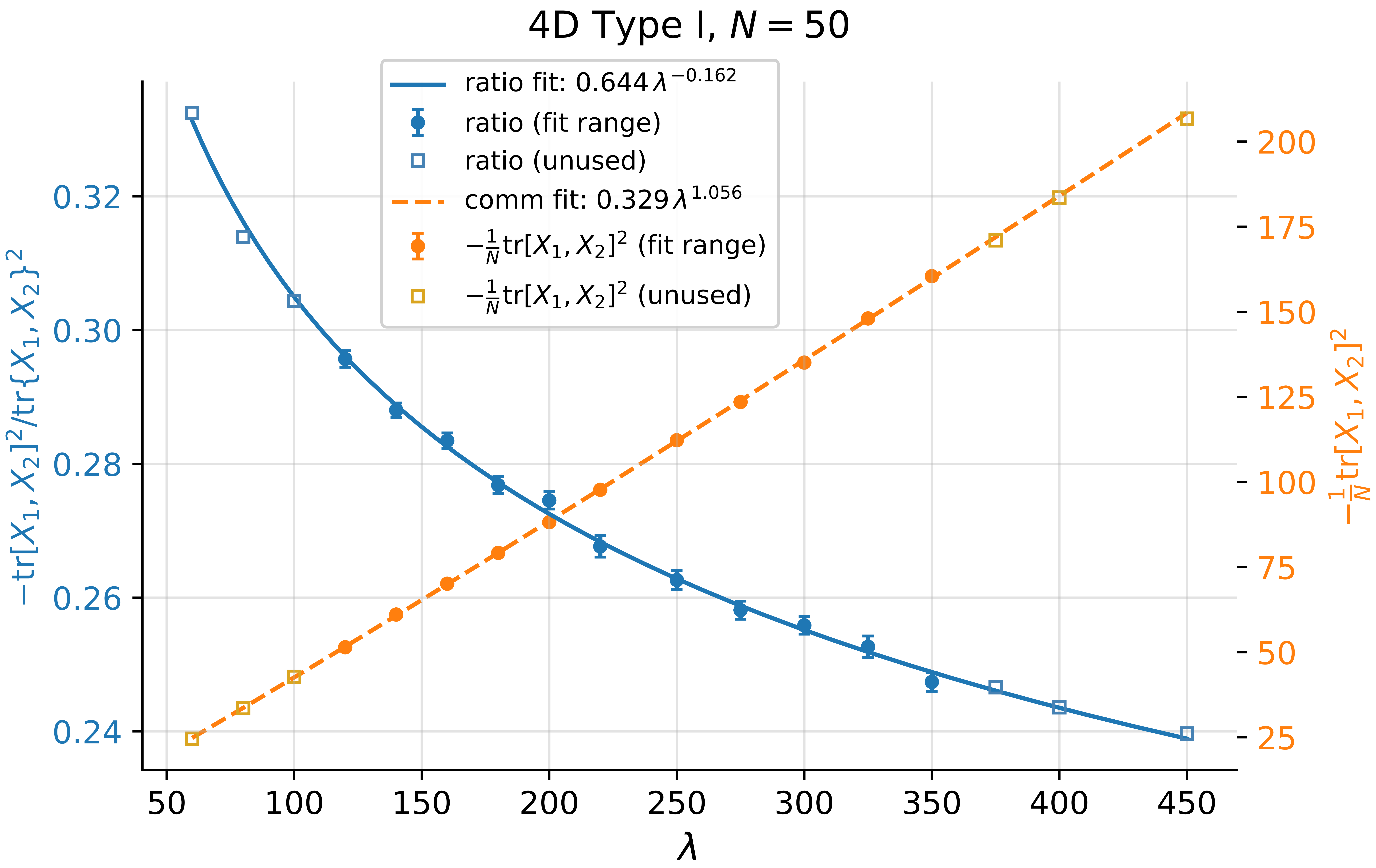
# Fermionic models

To see how the number of fermions affect the picture, we distinguish three cases:

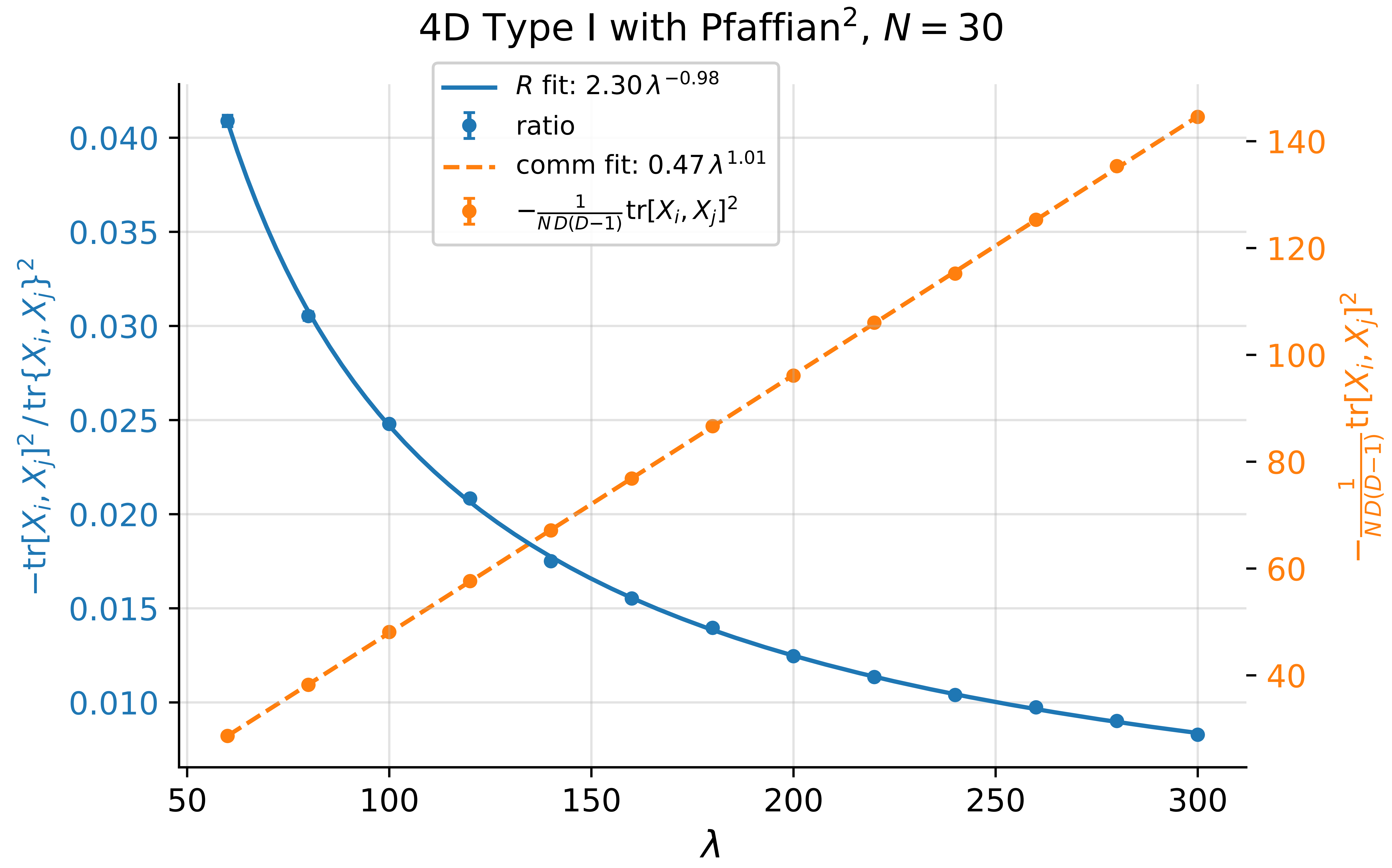
$$\mathcal{N}_c = 2(D-2)$$

| Subcritical | Critical | Supercritical |
| --- | --- | --- |
| $\mathcal{N}_f < \mathcal{N}_c$ | $\mathcal{N}_f = \mathcal{N}_c$ | $\mathcal{N}_f > \mathcal{N}_c$ |

# Critical in $D = 4$



# Supercritical in $D = 4$



# Summary of the numerics

| Subcritical | Critical | Supercritical |
|---|---|---|
| $\mathcal{N}_f < \mathcal{N}_c$ | $\mathcal{N}_f = \mathcal{N}_c$ | $\mathcal{N}_f > \mathcal{N}_c$ |
| $R \sim \text{const}$ | $R \sim \lambda^{-\alpha}$ | $R \sim \lambda^{-1}$ |

| $D$ | $\alpha$ |
|---|---|
| 2 | 1/3 |
| 3 | 0.25 |
| 4 | 0.16 |
| 5 | 0.11 |

$$\mathcal{N}_c = 2(D-2)$$

# Analytical method: Integrate out off-diagonals

Split the matrices into diagonal + off-diagonal terms, as

$$X^I = \sqrt{\lambda} x^I + q^I \qquad \psi_\alpha = \sqrt{\lambda}\eta_\alpha + \lambda^{1/4}\chi_\alpha$$

diagonal off-diagonal diagonal off-diagonal

$$S = S_0(x_I, \eta_\alpha) + \frac{1}{2}|x_a - x_b|^2 |q_{ab}|^2 - \frac{i}{2}(x_a^I - x_b^I)\bar{\chi}_{ba}\Gamma^I \chi_{ab} + \frac{1}{\sqrt{\lambda}} x q^3 + \frac{1}{\lambda} q^4 + \dots$$

# Effective action for the diagonals and validity

$$S_{\text{eff}} = N|x_a|^2 - (\mathcal{N}_f - \mathcal{N}_c) \sum_{a<b} \log |x_a - x_b|$$

| Subcritical | Critical | Supercritical |
|---|---|---|
| $\mathcal{N}_f < \mathcal{N}_c$ | $\mathcal{N}_f = \mathcal{N}_c$ | $\mathcal{N}_f > \mathcal{N}_c$ |
| Divergent,<br>not self-consistent | No repulsion among eigenvalues, $N$ cannot be too large | Eigenvalues are repelled, $N$ can be taken to infinity |
| | $\lambda \gg N^{\#}$ | $\lambda \gg 1$ |

# Summary

| Subcritical | Critical | Supercritical |
|---|---|---|
| $\mathcal{N}_f < \mathcal{N}_c$ | $\mathcal{N}_f = \mathcal{N}_c$ | $\mathcal{N}_f > \mathcal{N}_c$ |
| Non-commuting | Commuting | Commuting |
| $R \sim \mathcal{O}(1)$ | $R \sim \lambda^{-\alpha}$ | $R \sim \lambda^{-1}$ |
| Goes to the massless model | Not tractable analytically except in a ultra-strong coupling regime (outside of ’t Hooft) | Tractable at large N in terms of joint densities. Analytical predictions match numerical results. |

# Future directions

- Study the polarized IKKT model. The saddle structure is very rich but subtle. Can we isolate holographic saddles ? Is the trivial vacuum metastable ?

  [WIP with H. Murali]

- Study the matrix quantum mechanics analogs. Are the supercritical models too trivial to describe any nice gravitational system?

Other related questions

- The holographic dictionary in polarized IKKT

  [WIP with M. Guica, M. Ligorio, and J. Penedones]

- Can we find a direct way to derive the gravitational background / equations of motion ? For example study some D(-1) probe and write consistency equations that reduce to the supergravity equations of motion ?

  [WIP with S. Komatsu, J. Penedones, B. Tafreshi, A. Vuignier and X. Zhao]

# The difference between critical and supercritical

$$\frac{1}{N}\left\langle \mathrm{Tr}\frac{X_1^{2k}}{\lambda^k} \right\rangle \sim \left(\lambda^{(\alpha-1)/2}\right)^k$$

| Critical | Supercritical |
|---|---|
| $0 < \alpha < 1$ | $\alpha = 1$ |
| $\ldots \ll \langle \mathrm{Tr}X^4/\lambda^2 \rangle \ll \langle \mathrm{Tr}X^2/\lambda \rangle$ | $\ldots \sim \langle \mathrm{Tr}X^4/\lambda^2 \rangle \sim \langle \mathrm{Tr}X^2/\lambda \rangle$ |
| The strong coupling theory is **insensitive** to deformations $\delta V(x/\sqrt{\lambda})$ | The strong coupling theory is **sensitive** to deformations |