Long Heavy Objects in CFTs: Defects meet Effective Field Theory

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FONDO ITALIANO PER LA SCIENZA

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Motivation

Energy

Defect CFT: Captures the low-energy interaction of massive degrees of freedom with gapless excitations.

[Wilson; 1975][Cardy; 1984][Billò, Goncalves, Lauria, Meineri; 2016]

Defect CFT

- p-dim. flat defect in d-dim. CFT explicitly breaks part of the conformal symmetry

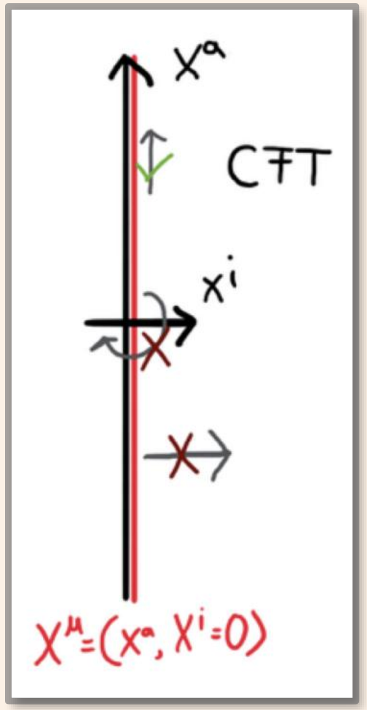
- Breaking of translational invariance induces the existence of the displacement operator

$$\partial_\mu T^{\mu i}\left(x^a, x^j\right) = \delta^{d-p}(x_\perp)\, \mathrm{D}_i\left(x^a\right)$$

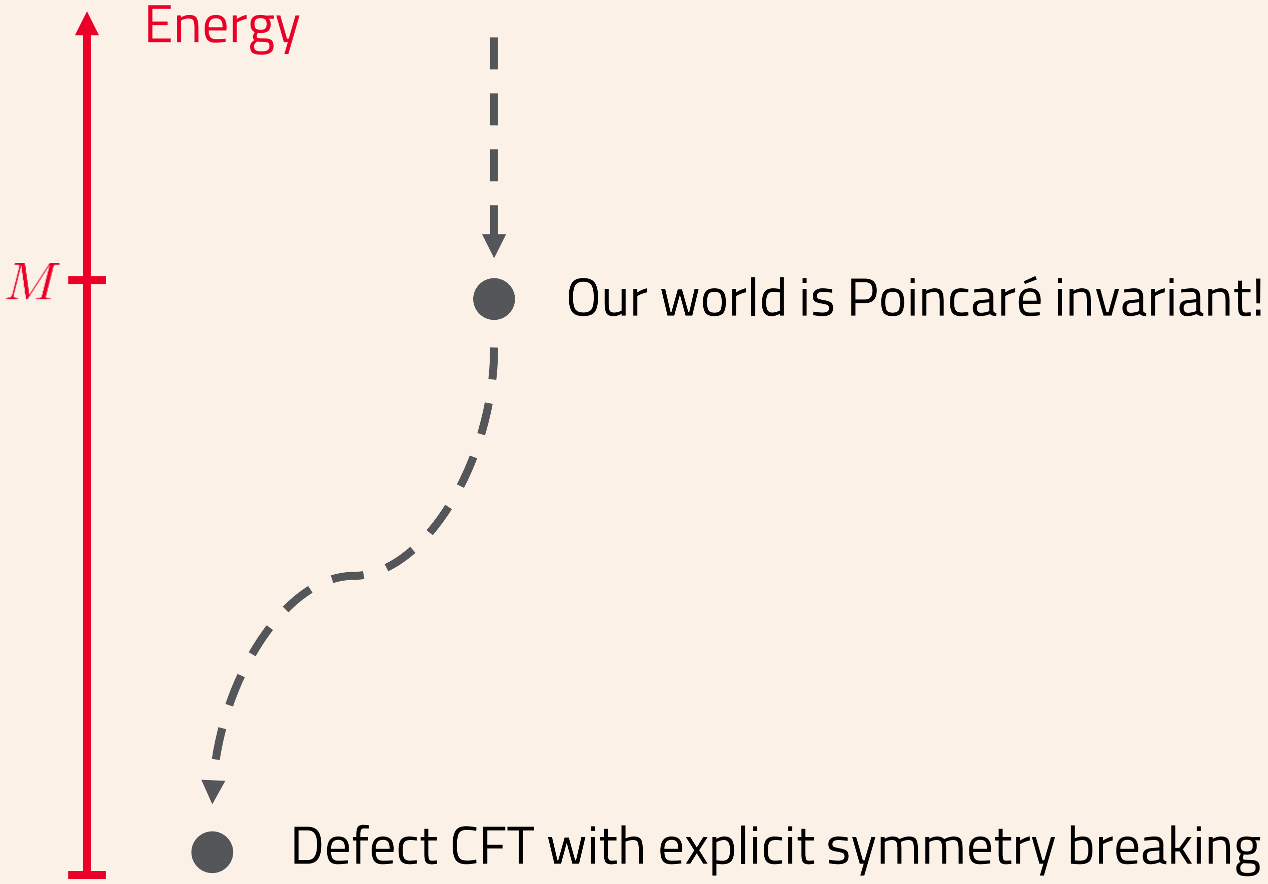
→ fixes $\langle \mathrm{D}_i \mathrm{D}_j \rangle = \dfrac{\delta_{ij} c_D}{(\text{distance})^{2(p+1)}}$

- D_i determines the variation of the path integral under an infinitesimal conformal transformation that does not preserve the shape of the defect

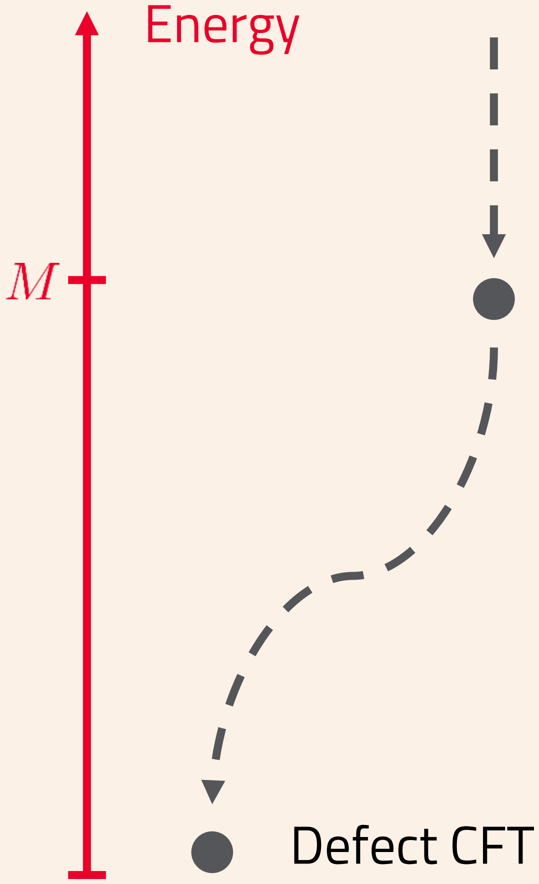
$$\delta_\xi S_{\mathrm{DCFT}} = -\int \mathrm{D}_i \xi^i\,, \qquad \text{for } \xi^\mu = (\xi^a, \xi^i) \text{ conf. killing vector}$$



Motivation


Energy
M
Our world is Poincaré invariant!
Defect CFT with explicit symmetry breaking

Motivation



Our world is Poincaré invariant
→ Symmetry breaking must be spontaneous

Defect CFT with explicit symmetry breaking

QFT with Spontaneous Symmetry Breaking

- SSB leads to the existence of massless Goldstone bosons.

  [Goldstone, Salam, Weinberg; 1962]

- Long, heavy object that spontaneously break the Poincaré symmetry support the propagation of Goldstone bosons on their worldvolume.
  → Effective String Theory

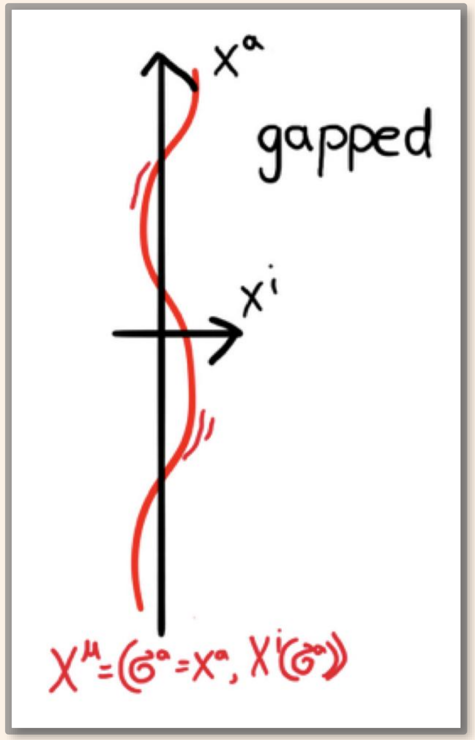
  [Luscher; 1981] [Luscher, Symanzik, Weisz; 1991] [Polchinski, Strominger;1991]

- Leading term is universal and given by the Nambu-Goto action

  [Nambu; 1974] [Goto; 1971]

$$S_{\text{brane}} = T \int_{\mathcal{D}} \sqrt{\det\left(\partial_a X^\mu(\sigma) \partial_b X_\mu(\sigma)\right)} + \ldots$$

- Dependence on the system enters only through $T = M^p$.

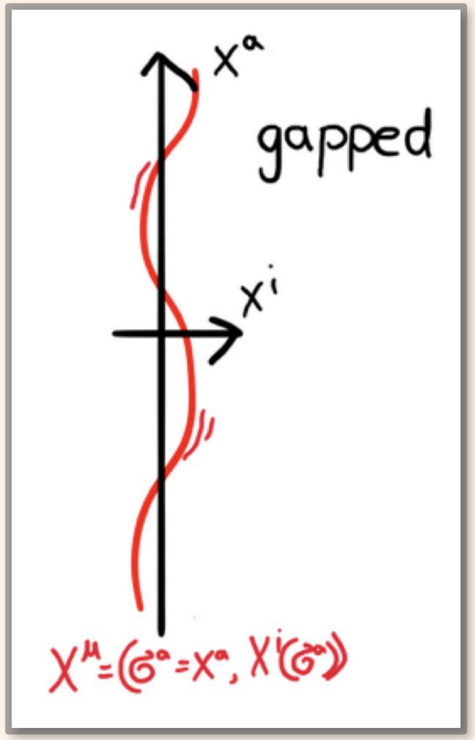


QFT with Spontaneous Symmetry Breaking

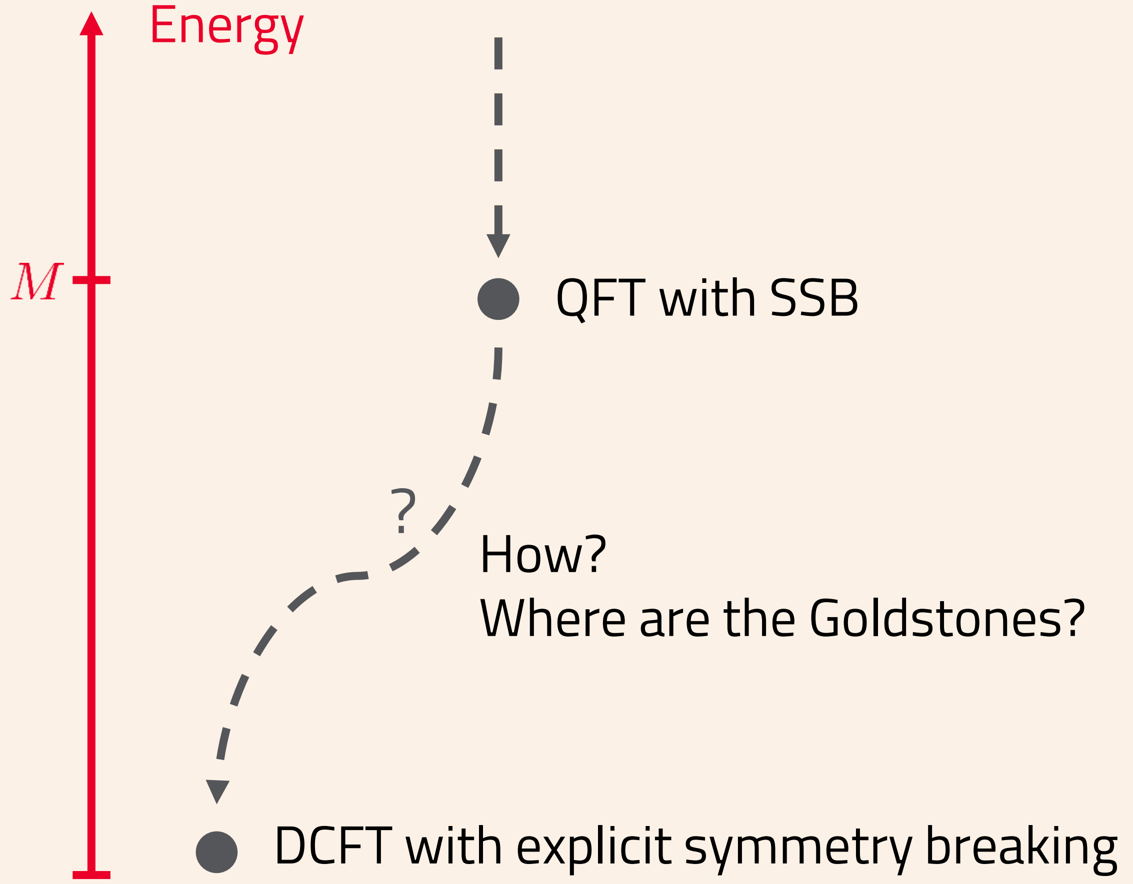
- $$S_{\mathrm{brane}} = M^p \int_{\mathcal{D}} \sqrt{\det\left(\partial_a X^\mu(\sigma)\partial_b X_\mu(\sigma)\right)} + \dots$$
  $$\approx M^p \int \left(1 + \frac{1}{2}\partial_a X^i \partial^a X_i + \dots\right)$$

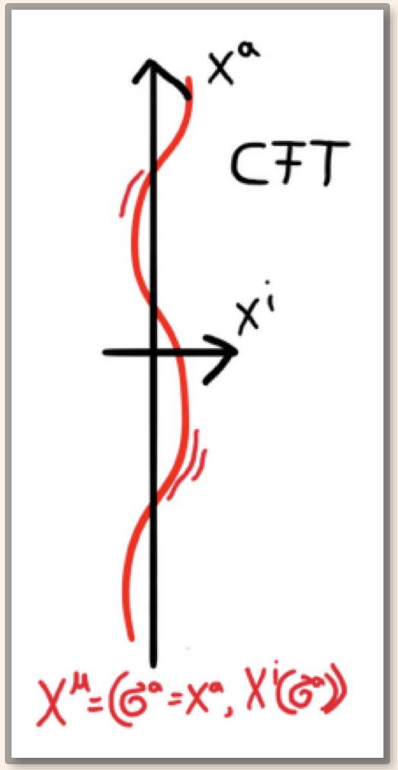
- Poincaré symmetry is realized non-linearly as:

$$P_k : \quad X^i \to X'^i = X^i + \epsilon^k \delta^i_k \ ,$$
$$L_{bk} : \quad X^i \to X'^i = X^i - \omega^{bk}\delta^i_k x_b - \omega^{bk} X_k \partial_b X^i \ .$$

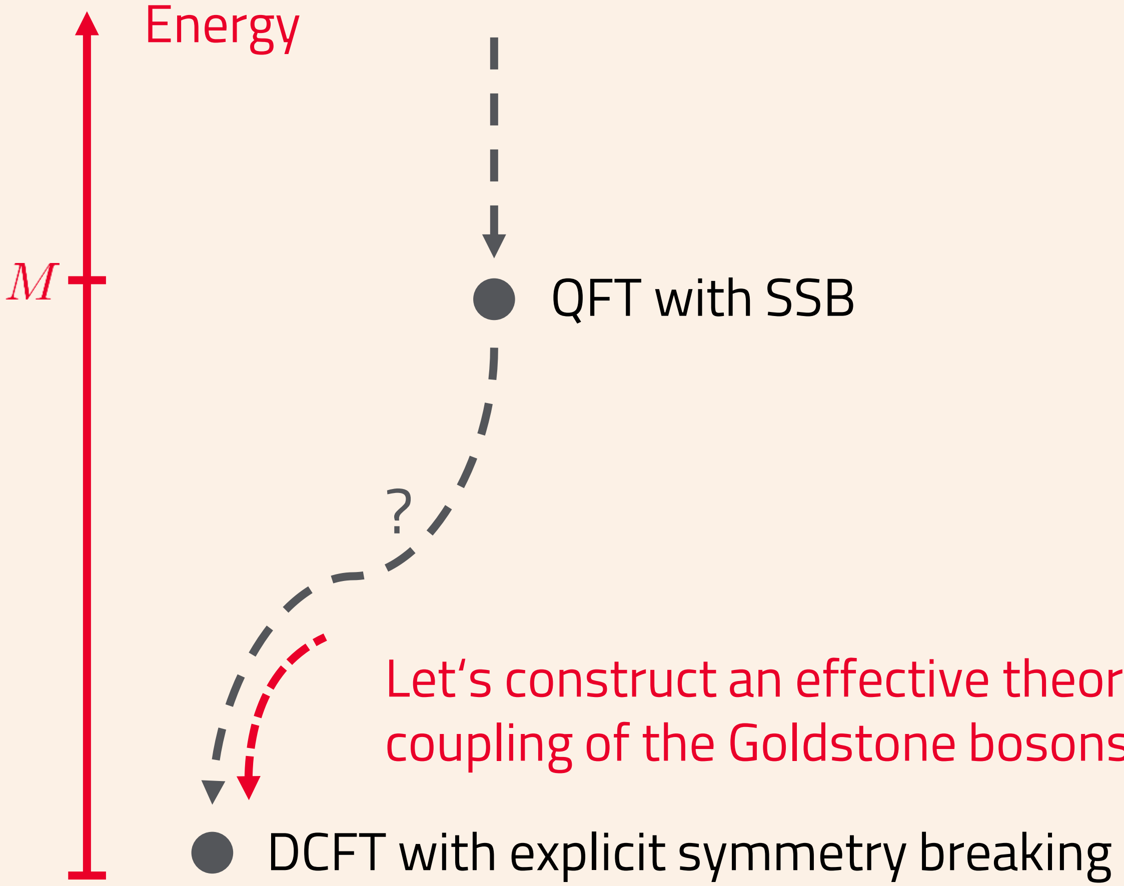


Motivation

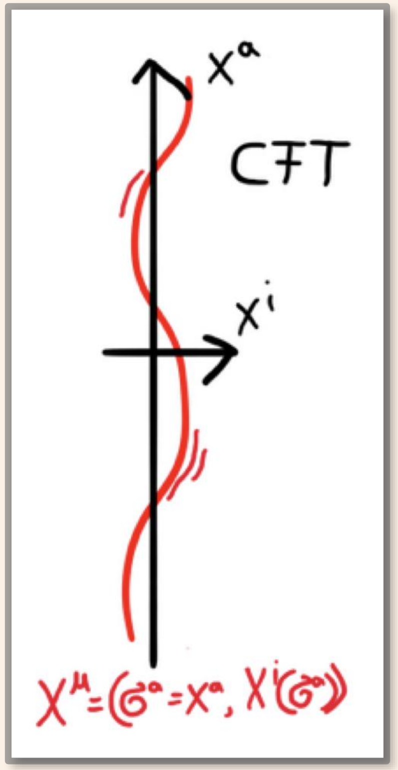

Energy
M
QFT with SSB
?
How?
Where are the Goldstones?
DCFT with explicit symmetry breaking


X^a
CFT
X^i
$X^\mu = (\sigma^a = X^a, X^i(\sigma^a))$

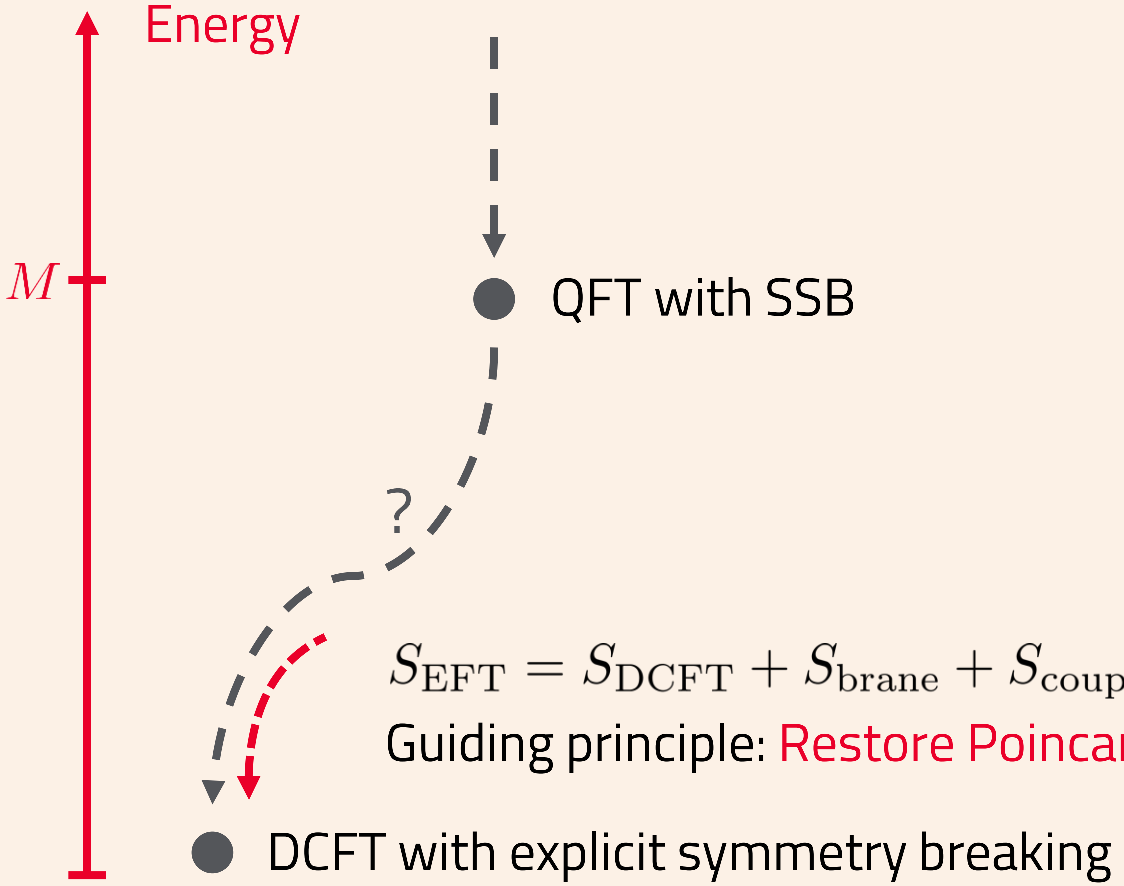
Motivation



Let's construct an effective theory that describes the coupling of the Goldstone bosons to a generic DCFT!



Motivation



$$S_{\text{EFT}} = S_{\text{DCFT}} + S_{\text{brane}} + S_{\text{coupling}}$$

Guiding principle: Restore Poincaré invariance!

Leading Order EFT

- $S_{\rm EFT} = S_{\rm DCFT} + S_{\rm brane} + S_{\rm coupling}$
- Recall: $\delta_{\epsilon^k} S_{\rm DCFT} = -\int \mathrm{D}_k \epsilon^k$

  $P_k : \quad X^i \to X'^i = X^i + \epsilon^k \delta^i_k$

Restoration of translational symmetry mandates:

$$S_{\rm EFT} = S_{\rm DCFT} + S_{\rm brane} + \int \mathrm{D}_i X^i + \dots$$

Leading Order EFT

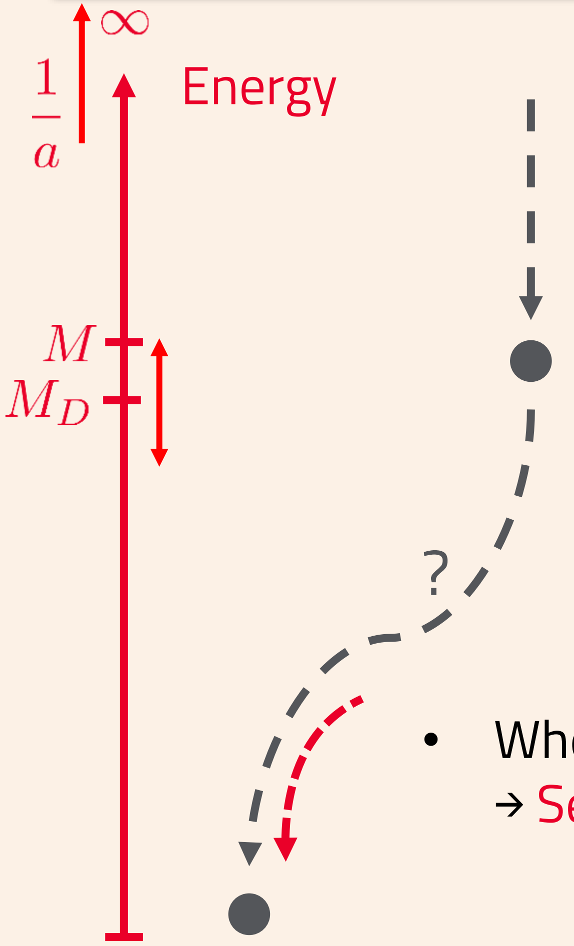
- Leading order EFT: $$S_{\text{EFT}} = S_{\text{DCFT}} + S_{\text{brane}} + \int \mathrm{D}_i X^i + \ldots$$

- Define canonically normalized field $\mathcal{X}^i = M^{\frac{p}{2}} X^i$ and normalized operator $\hat{\mathrm{D}}_i = c_D^{-\frac{1}{2}} \mathrm{D}_i$

$$S_{\text{EFT}} = S_{\text{DCFT}} + \int M^p + \int \frac{1}{2} \partial^a \mathcal{X}^j \partial_a \mathcal{X}_j + \underbrace{\left(c_D^{\frac{1}{p}} M^{-1}\right)^{\frac{p}{2}}}_{= M_D^{-\frac{p}{2}}} \int \hat{\mathrm{D}}_i \mathcal{X}^i + \ldots$$

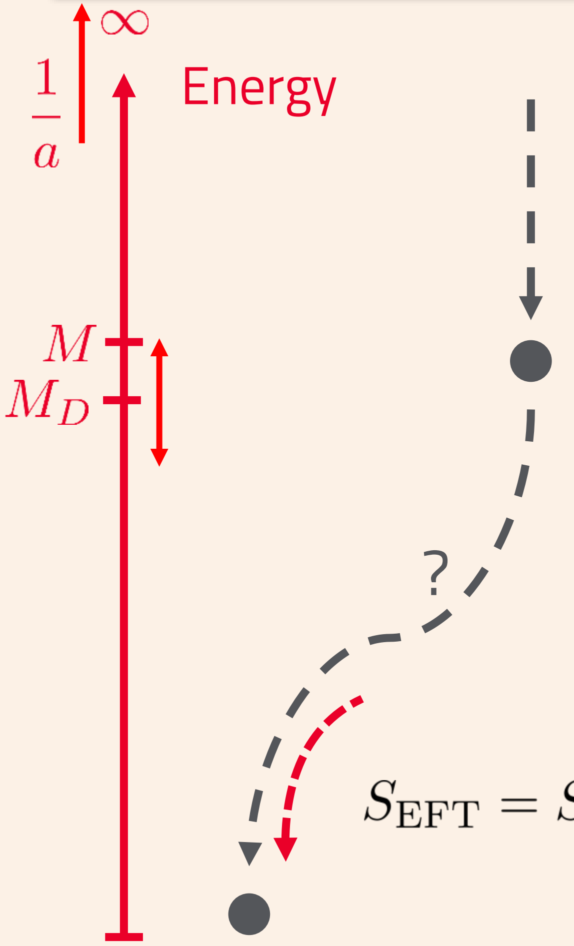
- Coupling is irrelevant: $\hat{\Delta}_D + \hat{\Delta}_{\mathcal{X}} = \dfrac{3p}{2}$
→ The Goldstones decouple in the IR!

Leading Order EFT



- When is the EFT valid?
  → Separation of scales $\frac{1}{a} \gg \min(M, M_D)$

Leading Order EFT



$$S_{\text{EFT}} = S_{\text{DCFT}} + S_{\text{brane}} + S_{\text{coupling}}$$

Related Work:

[Metlitski; 2022], [Krishnan, Metlitski; 2024]
[Horn, Nicolis, Penco; 2015] ,[Goon, Hinterbichler, Trodden; 2011] [de Rahm, Tolley; 2011],....
[Murakami, Watanabe; 2014] ,[Punk, Sachdev; 2012],...
[Damia, Galati, Rizi; 2026], [Soto, Valls; 2026],...

Example: Soliton in 2-dimensional ϕ^4-Theory

$$S = \int dt dx \, \frac{1}{2}(\partial_\mu \phi)^2 + \frac{\lambda}{4}\left(\phi^2 - \frac{m^2}{\lambda}\right)^2 + \frac{1}{2}(\partial_\mu \psi)^2 - \frac{u}{2}\frac{\sqrt{\lambda}}{m}(\partial_\mu \psi)^2 \phi$$

- Classical kink solution $\phi_{\rm cl}(x)$ is localized at $x_0 = 0$

  → SSB and existence of a Goldstone boson
- The kink has finite static energy setting the scale $M = E[\phi_{\rm cl}] = \frac{2\sqrt{2}}{3}\frac{m^3}{\lambda}$
- Taking the zero-energy limit $\phi(t,x) \approx \phi_{\rm cl}(x) \xrightarrow{m|x| \gg 1} \frac{m}{\sqrt{\lambda}} \mathrm{sign}(x)$ gives CFT with an interface between two free scalar theories with different normalization of the kinetic term.

  [Bachas, de Boer, Dijkgraaf, Ooguri; 2002]

$$S_{\rm DCFT}[\psi] = \frac{1+u}{2}\int_{x<0} d^2x \, (\partial_\mu \psi)^2 + \frac{1-u}{2}\int_{x>0} d^2x \, (\partial_\mu \psi)^2$$

→ Displacement operator:

$$\mathrm{D} = T^+_{xx}\big|_{x=0} - T^-_{xx}\big|_{x=0} \,, \qquad c_D = \frac{u^2}{\pi^2}$$

Example: Soliton in 2-dimensional ϕ^4-Theory

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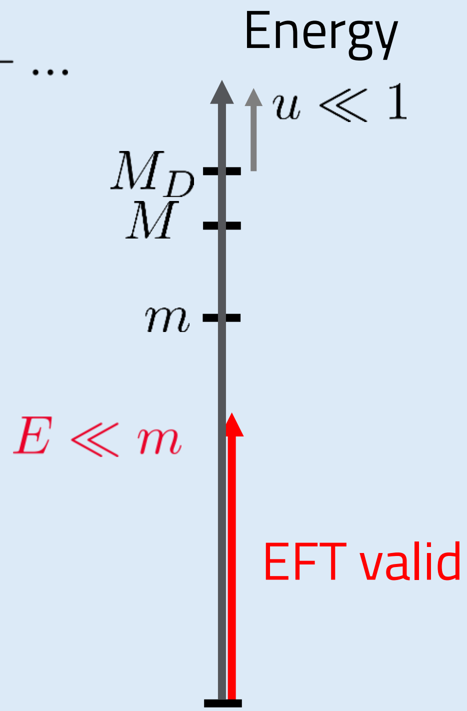
- Taking $\phi(t,x) \approx \phi_{\rm cl}(x - X(t))$ reproduces the EFT

$$S_{\rm EFT} = S_{\rm DCFT} + M \int dt + \int dt \, \frac{1}{2} M \left(\partial_t X(t)\right)^2 + \int dt \, \mathrm{D}(t) X(t) + \ldots$$

- However, corrections are suppressed only by $m \ll M$



What is beyond Leading Order ?

$$S_{\mathrm{EFT}} = S_{\mathrm{DCFT}} + S_{\mathrm{brane}} + \int \mathrm{D}_i X^i + ?$$

What is beyond Leading Order ?

$$S_{\text{coupling}} = \int \mathrm{D}_i\, X^i + \kappa_0^{(3)} \int \cancel{\mathrm{D}_j\, X^i} X^2 + \kappa_1^{(3)} \int \mathrm{D}_i\, X^i \partial_a X^j \partial^a X_j + \ldots \kappa_{\hat{O}} \int \sqrt{g_{ab}} \hat{O} + \ldots$$

What is beyond Leading Order ?

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- The spectrum of defect operators does not contain special operators.
- D_i is chosen to be the operator defined at $X^i(x_\parallel) = 0$.

What is beyond Leading Order ?

$$S_{\text{coupling}} = \int \mathrm{D}_i\, X^i + \kappa_0^{(3)} \int \cancel{\mathrm{D}_i\, X^i} X^2 + \kappa_1^{(3)} \int \mathrm{D}_i\, X^i \partial_a X^j \partial^a X_j + \ldots \kappa_{\hat{O}} \int \sqrt{g_{ab}} \hat{O} + \ldots$$

- $X^i(x_\parallel)$ is chosen such that $X^i_{\text{cl}}(x_\parallel)$ describes the transverse profile of the defect

  → The partition function describing a defect of arbitrary smooth shape is invariant under rescaling $X^i \to \lambda X^i \,,\; x^a \to \lambda x^a$

  → Scale invariance in the classical limit requires $\kappa_0^{(3)} = 0$

What is beyond Leading Order ?

$$S_{\text{coupling}} = \int \mathrm{D}_i\, X^i + \kappa_0^{(3)} \int \cancel{\mathrm{D}_j X^i} X^2 + \kappa_1^{(3)} \int \mathrm{D}_i\, X^i \partial_a X^j \partial^a X_j + \ldots \kappa_{\hat{O}} \int \sqrt{g_{ab}} \hat{O} + \ldots$$

- Completion such that $\delta_{L_{bk}} \left(S_{\text{DCFT}} + \int \mathrm{D}_i\, X^i + \kappa_1^{(3)} \int \mathrm{D}_i\, X^i \partial_a X^j \partial^a X_j + \ldots \right) \overset{!}{=} 0$

  → Wilson coefficients unconstrained by studying the non-linear action of the broken generators on the displacement operator

  $$L_{bk}\, \mathrm{D}_i = -\alpha_1^{(1)} \partial_b X_k\, \mathrm{D}_i - \alpha_2^{(1)} \partial_b X_i\, \mathrm{D}_k + \ldots$$

  → Wilson coefficients constrained by reproducing defect of arbitrary smooth profile in the classical limit.

  [Gabai, Sever, Zhong; 2025][Drukker, Kong, Kravchuk;2025]

What is beyond Leading Order ?

$$S_{\text{coupling}} = \int \mathrm{D}_i\, X^i + \kappa_0^{(3)} \int \cancel{\mathrm{D}_j\, X^i} X^2 + \kappa_1^{(3)} \int \mathrm{D}_i\, X^i \partial_a X^j \partial^a X_j + \dots \kappa_{\hat{O}} \int \sqrt{g_{ab}} \hat{O} + \dots$$

- Satisfy $\delta_{L_{bk}} \left(\kappa_{\hat{O}} \int \sqrt{g_{ab}} + \dots \right) = 0$ independently.

  [Coleman, Wess, Zumino; 1969],[Callan, Coleman, Wess, Zumino; 1969],[Volkov; 1973],[Ogievetsky; 1974]

  → Wilson coefficients depend on the UV

  → Renormalize

What is beyond Leading Order ?

$$S_{\text{EFT}} = S_{\text{DCFT}} + S_{\text{brane}} + \int \mathrm{D}_i X^i + \,!$$

... we developed a systematic framework for describing the low-energy dynamics of long heavy objects in a CFT !

... the leading interaction between the massless Goldstones and bulk excitations is universal !

Final EFT

$$S_{\text{EFT}} = S_{\text{DCFT}} + S_{\text{brane}} + \int \mathrm{D}_i X^i + \, !$$

- Definition of conserved Noether charges for the restored Poincaré symmetry.
  [Girault, Paulos, van Vliet; 2025]

- The EFT extends both: DCFT and effective brane action !
  → It gives corrections to DCFT, Goldstone and mixed observables.
- Identify relevant examples and test predictions, e.g., against lattice results.

- Study more sophisticated examples: supersymmetry, holography…

Thank You !