More strings attached: Bootstrapping the confining flux tube

Philine van Vliet

ENS Paris

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Based on:
Work in progress with B. Gabai, V. Gorbenko, B. Offertaler, J. Qiao

Recap of setup: Yang-Mills flux tube in AdS_3

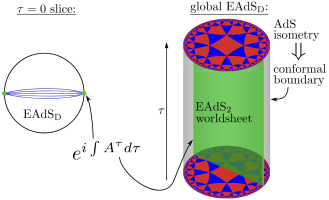

Figure: [Gabai, Gorbenko, Qiao '25]

- On the boundary: 1d dCFT with displacement operator $\mathbb{D} \sim F_{\tau i}$ (weak) or $\mathbb{D} \sim X_i$ (EST) [Gabai, Gorbenko, Qiao '25], see also [Qiao '26][Bianchi, de Sabbata, Meineri '26]
- Bulk is a 2d nonlocal CFT, no rotation symmetry around the defect:

$$SO(2+1,1) \to SO(1+1,1)\,.$$

- Simplest case, but therefore difficult.

Recap of setup: Yang-Mills flux tube in AdS_3

- Two perturbative descriptions:
  - R_{AdS} small: expansion in weak coupling $\lambda \equiv \Lambda_{QCD} R_{AdS}$.
  - R_{AdS} large: Effective String Theory.

- Interpolate through Padé approximation[Gabai, Gorbenko, Offertaler '26].

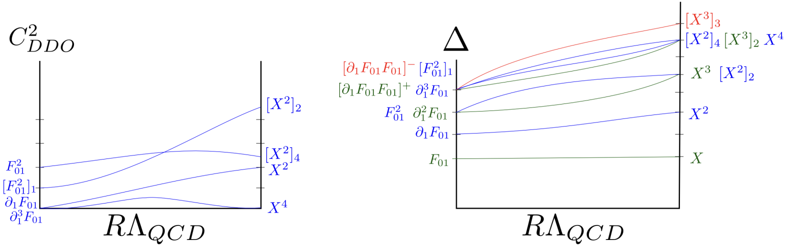

Figure: Artist interpretation [Gabai, Gorbenko, Qiao '25].

The spectrum

- Displacement operator $\mathbb{D}$ with dimension $\Delta_{\mathbb{D}} = 2$ is lowest-lying operator (pseudovector).
- Second operator: "$\mathbb{D}^2$" $\sim \partial F_{\tau 1}$ or $\sim X^2$ with dimension $3 \leq \Delta_{\mathbb{D}^2} \leq 4$ $\rightarrow$ substitute for coupling.
- OPEs:

$$\mathbb{D} \times \mathbb{D}\,, \mathbb{D}^2 \times \mathbb{D}^2 = \mathbb{D}^2 + \sum_{\Delta_{++} \geq 4} \mathcal{O}_{++} + \sum_{\Delta_{+-} \geq 6} \mathcal{O}_{+-}\,,$$

$$\mathbb{D} \times \mathbb{D}^2 = \mathbb{D} + \sum_{\Delta_{-+} \geq 4} \mathcal{O}_{-+} + \sum_{\Delta_{--} \geq 5} \mathcal{O}_{--}\,.$$

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$$\mathbb{D} \times \mathbb{D}^2 = \mathbb{D} + \sum_{\Delta_{-+} \geq 4} \mathcal{O}_{-+} + \sum_{\Delta_{--} \geq 5} \mathcal{O}_{--}\,.$$

- Displacement operator obeys integrated correlator constraints/sum rules [Gabai, Sever, Zhong '22,'23,'25, Cavaglià, Gromov, Julius, Preti '22, Girault, Paulos, PvV '25, Drukker, Kong, Kravchuk '25]
  See also [Kutasov '89, Friedan, Konechy '12, Billò, Goncalves, Lauria, Meineri '16, Behan '17, Drukker, Kong, Sakkas '22, Belton, Kong '25]

Functional sum rules

$$\sum_{\Delta} c^2_{\mathbb{D}\mathbb{D}O} f_I^{D,\text{soft1}}[\Delta_O] = 0\,.$$

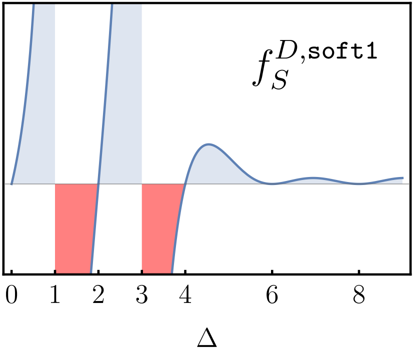

Figure: [Girault, Paulos, PvV '25]

In this setup: $3 \leq \mathbb{D}^2 \leq 4$ and no operator appearing in $\mathbb{D} \times \mathbb{D}$ below.

Outline

- Bootstrapping the Yang-Mills flux tube in AdS_3

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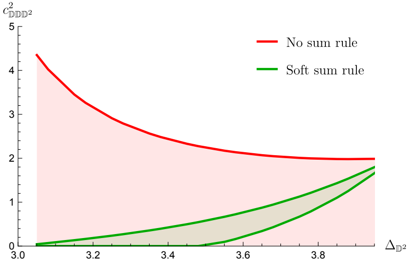
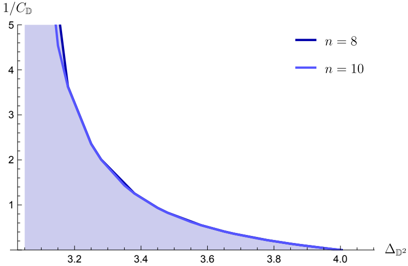
- Outlook

Single correlator

- Consider $\langle \mathbb{D}(x_1)\mathbb{D}(x_2)\mathbb{D}(x_3)\mathbb{D}(x_4)\rangle$ with $\mathbb{D} \times \mathbb{D} \sim$ "$\mathbb{D}^2$" $+ \ldots$.
- Use analytic functional basis [Mazac '17, Paulos, Mazac '18, ...] and SDPB [Simmons-Duffin '15, Landry and Simmons-Duffin '19]. See also Zechuan's talk
- Add two sum rules:
  - Homogeneous/soft: $\int_0^1 dt\,(1+t+t^2)\mathcal{G}^{\mathbb{D}}(t) = 0$.
  - Inhomogeneous/double-soft:
    $\int_0^1 dz \left[(t^2-2t-2)\log t + (t^2+4t+1)\log(1+t)\right]\mathcal{G}^{\mathbb{D}}(t) = 4/C_{\mathbb{D}}$.
- where $t = \frac{(x_1-x_2)(x_3-x_4)}{(x_1-x_4)(x_2-x_3)}$, see also [Cavaglià, Gromov, Julius, Preti '21].
- Bootstrap $\Delta_O, c^2_{\mathbb{D}\mathbb{D}O}, C_{\mathbb{D}} = \langle \mathbb{D}(0)\mathbb{D}(1)\rangle$.
- Compare and use 2-sided Padé approximations as Barak discussed.

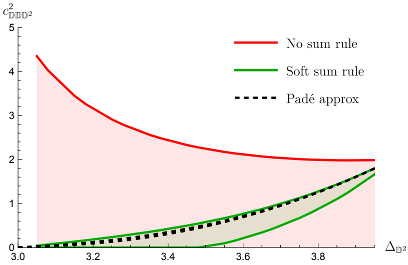
Single correlator

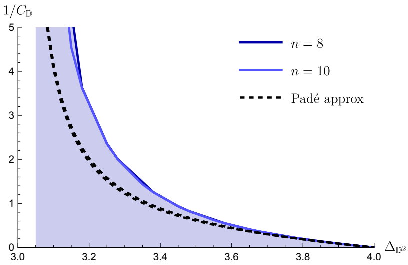




- Gap assumption: $\Delta_{++} = \Delta_{\mathbb{D}^2} + 0.5$ (extremely conservative).

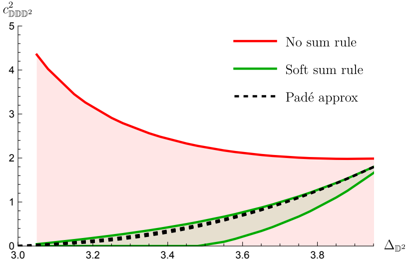
Single correlator

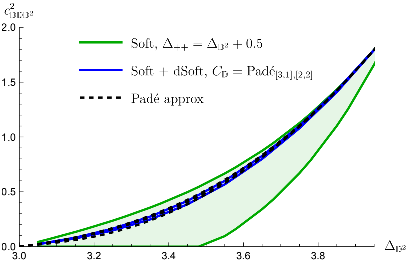




- Gap assumption: $\Delta_{++} = \Delta_{\mathbb{D}^2} + 0.5$ (extremely conservative).
- Compared to Padé$_{[4,0],[3,1],[1,3],[2,2]}$ for $c^2_{\mathbb{D}\mathbb{D}\mathbb{D}^2}$ and Padé$_{[3,1],[1,3],[2,2]}$ for $C_{\mathbb{D}}$

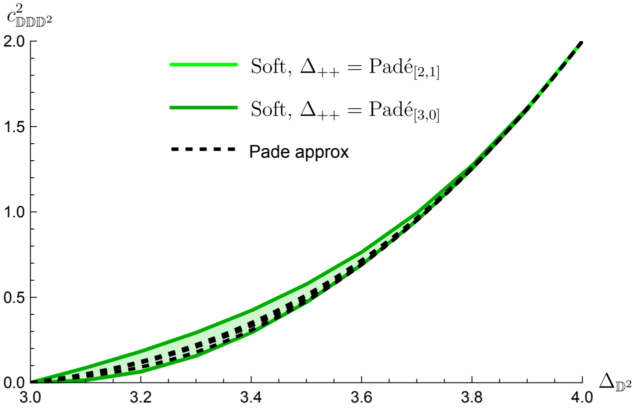
Single correlator





- Gap assumption: $\Delta_{++} = \Delta_{\mathbb{D}^2} + 0.5$ (extremely conservative).
- Use Padé prediction as an input for inhomogeneous/double-soft sum rules.

Single correlator



- Gap assumption: Δ_{++} given by Padé estimate.
- Compared to Padé$_{[4,0],[3,1],[1,3],[2,2]}$ for $c^2_{\mathbb{D}\mathbb{D}\mathbb{D}^2}$.

Single correlator

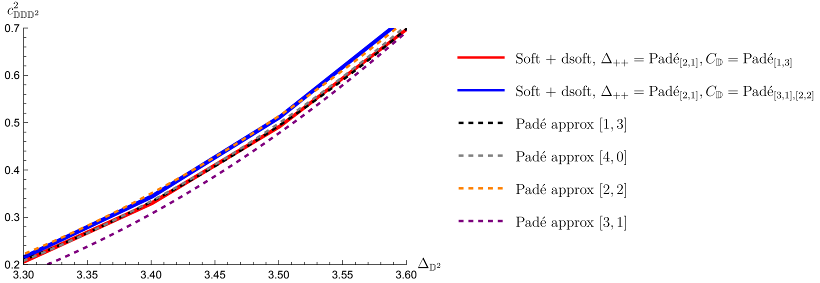
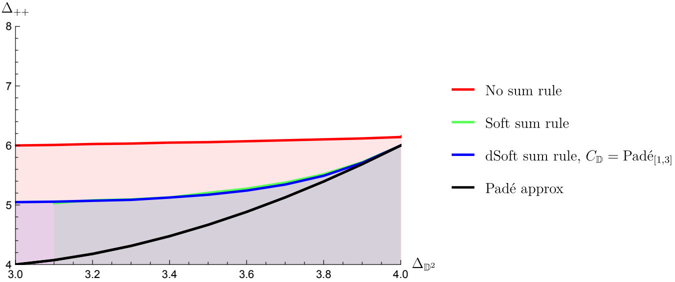


Figure: Bounds on c^2_{DDO++} with gap assumption from Pade

- Gap assumption: Δ_{++} given by Padé estimate.
- Compared to Padé$_{[4,0],[3,1],[1,3],[2,2]}$ for $c^2_{\mathbb{DDD}^2}$.

Single correlator

- Bound Δ_{++}:



- Compared to Padés for Δ_{++}: Padé $_{[3,0],[2,1],[0,3]}$.
- Padé$_{[1,2]}$ develops poles.

Multi-correlator setup

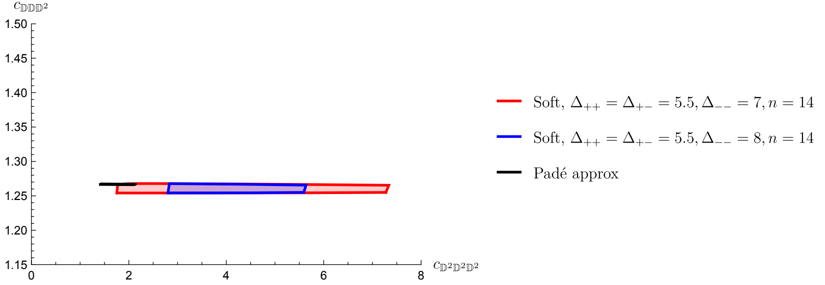
- Now include $\mathbb{D}^2$ as second external operator:

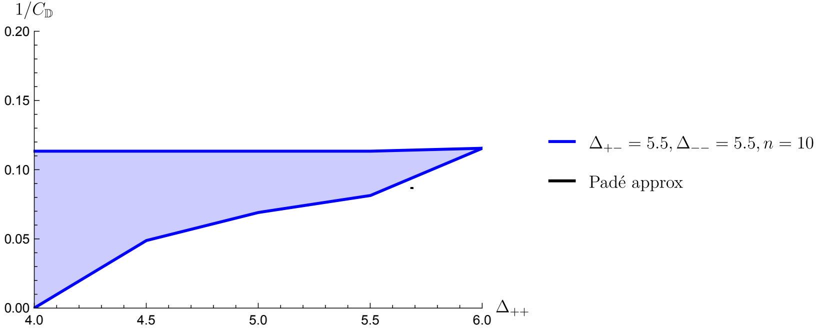
$$\langle \mathbb{D}\mathbb{D}\mathbb{D}\mathbb{D}\rangle\,, \quad \langle \mathbb{D}^2\mathbb{D}^2\mathbb{D}^2\mathbb{D}^2\rangle\,, \quad \langle \mathbb{D}\mathbb{D}\mathbb{D}^2\mathbb{D}^2\rangle + \text{ perms}\,.$$

- Impose that $\mathbb{D}\times\mathbb{D}\sim\mathbb{D}^2+\dots\,,\quad \mathbb{D}^2\times\mathbb{D}^2\sim\mathbb{D}^2+\dots$ and $\mathbb{D}\times\mathbb{D}^2\sim\mathbb{D}+\dots$
- Use analytic functionals for multi-correlator [Ghosh, Kaviraj, Paulos '23].
- Add 4pt function displacement sum rules.

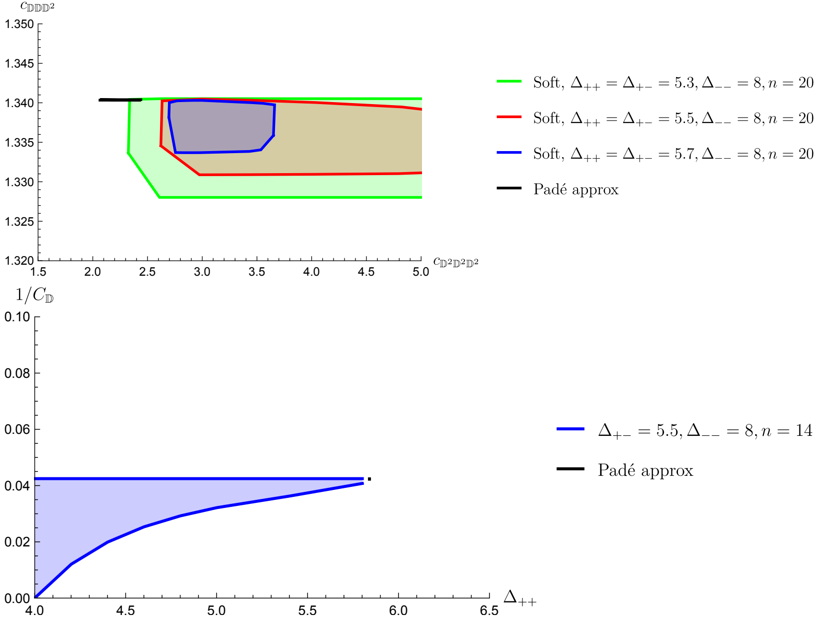
- Add mixed correlator sum rules with $n=0,1,2$:
  - $\int_0^1 dz\left[z^n((z-1)^{2-n}+1)\mathcal{G}_{\mathbb{D}\mathbb{D}\mathbb{D}^2\mathbb{D}^2}(z)+z^{2-n}\mathcal{G}_{\mathbb{D}\mathbb{D}^2\mathbb{D}\mathbb{D}^2}(z)\right]=0\,,$
  - $\int_0^1 dz\left[2(z^2-1)\log z\,\mathcal{G}_{\mathbb{D}^2\mathbb{D}\mathbb{D}\mathbb{D}^2}(z)\right.$ $\left.+(2z-1)\log\left(\frac{z}{1-z}\right)\mathcal{G}_{\mathbb{D}\mathbb{D}^2\mathbb{D}\mathbb{D}^2}(z)\right]=8/C_{\mathbb{D}}\,.$

Multi-correlator: $\Delta_{\mathbb{D}^2} = 3.9$





Multi-correlator: $\Delta_{\mathbb{D}^2} = 3.95$



Multi-correlator: closing in on strong coupling

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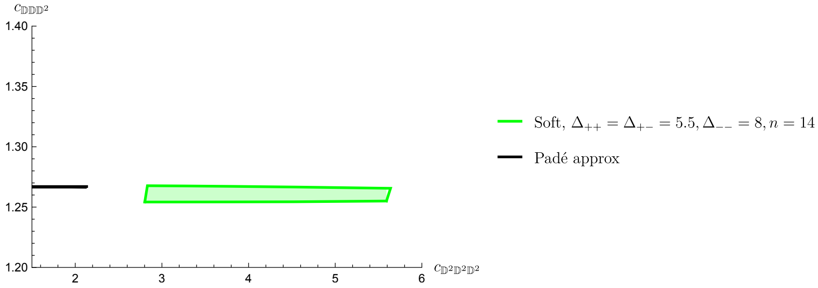

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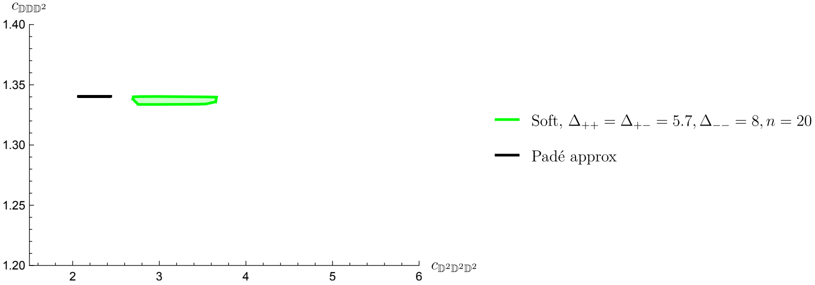
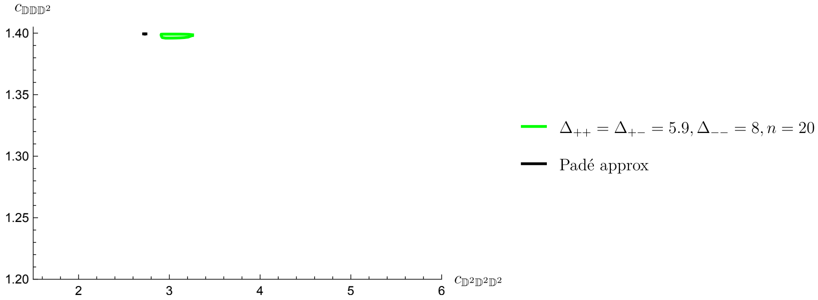


Multi-correlator: closing in on strong coupling

- $\Delta_{\mathbb{D}^2} = 3.99$.



Open questions/opportunities

- Why is there a discrepancy with results for $c_{\mathbb{D}^2\mathbb{D}^2\mathbb{D}^2}$?
  - Something wrong with numerics?
  - Something wrong with perturbative calculation for $c_{\mathbb{D}^2\mathbb{D}^2\mathbb{D}^2}$?
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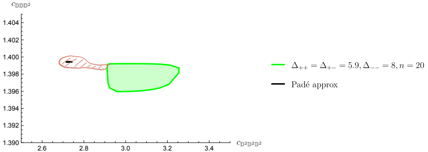

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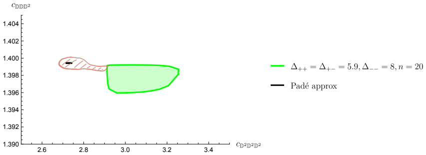
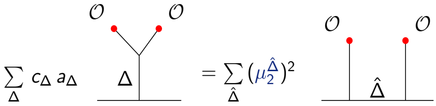


- Implement mixed correlator sum rules $\rightarrow$ in what direction does allowed region shrink?
- Constrain higher-dimensional exchanged operators?
- AdS_4?

Outlook: Bootstrapping defects

- For defects there are technical difficulties for numerical bootstrap:
    - No information from the bulk when considering only defect correlators.
    - No positivity for bulk operators.



- Recent progress [Cavaglià, Gromov, Julius, Preti '21, Lanzetta, Liu, Metlitski '25, Meineri, Radhakrishnan '25]

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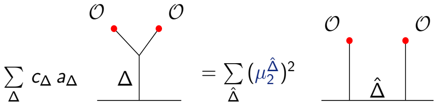

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\+ Combine bootstrap with defect soft sum rules, also without integrability/SUSY.

- e.g. pinning defect in 3d Ising wip with Bastien Girault, Stefanos Kousvos, Miguel Paulos
- See also other methods such as truncated bootstrap [Gliozzi '13], [Padayasi, Krishnan, Metlitski, Gruzberg, Meineri '21, Hu, Li '26, ...] and see Wenliang's talk.

Thank you!

Backup

A Padé crash course

- Approach to make perturbative series better behaved.

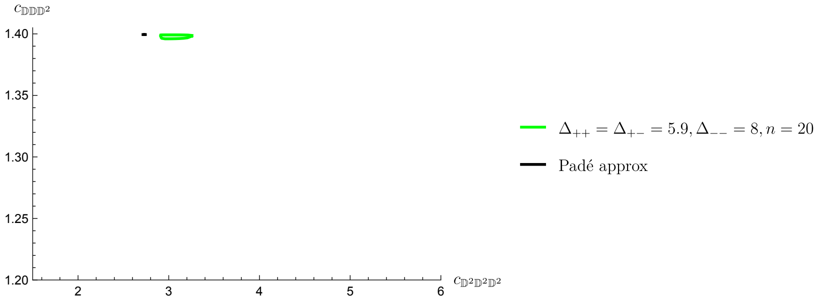
$$f_{[M/N]}(x) = \frac{\sum_{m=0}^{M} p_m x^m}{1 + \sum_{n=1}^{N} q_n x^n}$$

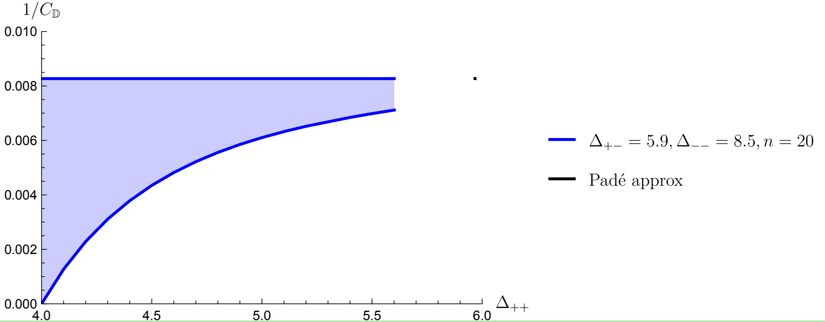
- Match to perturbative series in x up to order $M + N$: usual Padé.

- Match to perturbative series in $\tilde{x}$ and x' with $\tilde{x} \leq x \leq x'$: double-sided Padé.

- Good and bad Padés: throw out results that give poles in the desired range of x.

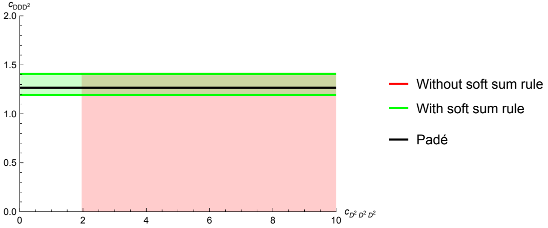
Multi-correlator: $\Delta_{\mathbb{D}^2} = 3.95$





The importance of putting gaps

- What if we lower the gaps? $\rightarrow \Delta_{++} = 4.5,\ \Delta_{+-} = 4.5,\ \Delta_{--} = 5.$



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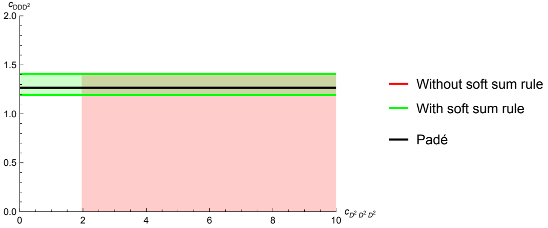
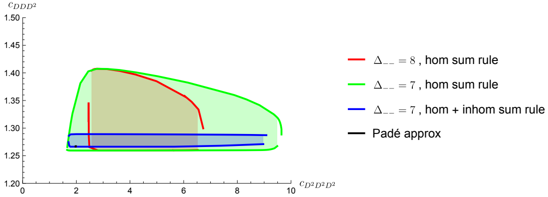
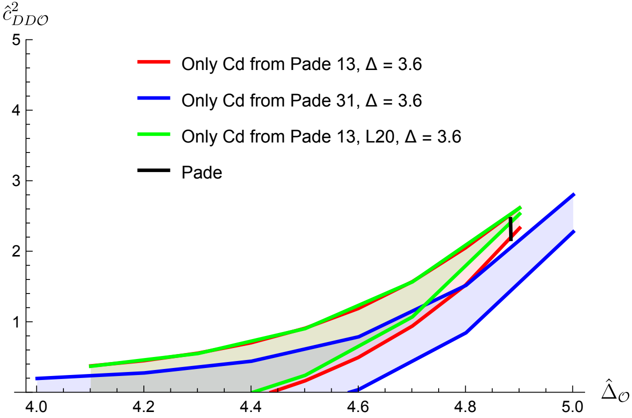


- And if we raise the gaps? $\rightarrow \Delta_{++} = 5.5,\ \Delta_{+-} = 5.5,\ \Delta_{--} = 8.$



Single correlator, $\Delta_{\mathbb{D}^2} = 3.6$



Ward identities

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- Lead to broken Ward identities

$$
\begin{aligned}
\text{flavour}: \quad & Q^{\tilde{s}}_{\Sigma} |0\rangle_{\mathcal{D}} = \int_{\Sigma} d^p x\, d^q y\, \partial_\mu \mathcal{J}^{\tilde{s},\mu} |0\rangle_{\mathcal{D}} = \int_{\Sigma} d^p x\, t^{\tilde{s}} |0\rangle_{\mathcal{D}} \\
\text{conformal}: \quad & Q^{a}_{\Sigma} |0\rangle_{\mathcal{D}} = \int_{\Sigma} d^p x\, d^q y\, \partial_\mu T^{\mu,a} |0\rangle_{\mathcal{D}} = \int_{\Sigma} d^p x\, \xi^a_m D^m |0\rangle_{\mathcal{D}} \\
\text{SUSY}: \quad & Q_{\alpha,\Sigma} |0\rangle_{\mathcal{D}} = \int_{\Sigma} d^p x\, d^q y\, P_{\alpha\beta} \partial_\mu J^{\mu\beta} |0\rangle_{\mathcal{D}} = \int_{\Sigma} d^p x\, \psi_\alpha |0\rangle_{\mathcal{D}}
\end{aligned}
$$

- with three protected operators:

$$
\begin{aligned}
\text{tilt } t: \quad & \Delta_t = p\,, \\
\text{displacement } D: \quad & \Delta_D = p+1\,, \\
\text{displacino } \psi: \quad & \Delta_\psi = p + \frac{1}{2}\,,
\end{aligned}
$$

Functional sum rules

- Decompose 4-point correlator in conformal blocks:

$$\langle t(x_1)t(x_2)t(x_3)t(x_4)\rangle = \frac{\mathcal{G}(z)}{x_{12}^2 x_{34}^2} = \frac{1}{x_{12}^2 x_{34}^2}\sum_{\hat{\Delta}} \hat{c}^2_{tt\hat{\Delta}}\, G_{\hat{\Delta}}(z)\,.$$

with $G_{\hat{\Delta}}(z) = z^{\hat{\Delta}}\ {}_2F_1(\hat{\Delta},\hat{\Delta},2\hat{\Delta};z)$.

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- Improve: replace $\mathcal{G}(z)$ by dispersion relation [Paulos '21]

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$$\mathcal{G}(z) = -\int_0^1 d\omega \, h(z,\omega) \, \mathrm{dDisc}\, \mathcal{G}(\omega)$$

  leading to

$$\int_0^1 dz \int_0^1 d\omega \, \mathcal{K}(z,\omega) \; \mathrm{dDisc}\, \mathcal{G}(\omega) = \mathrm{cst} \implies \sum_{\hat{\Delta}} \hat{c}^2_{tt\hat{\Delta}} \, f^t[\hat{\Delta}] = \mathrm{cst} \, .$$

Conformal bootstrap + Integrability = Bootstrability

- At $N \to \infty$ use Quantum Spectral Curve to obtain spectrum [Gromov, Kazakov, Leurent, Volin, '13 '14].

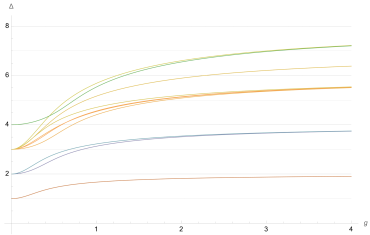

Figure: [Cavaglià, Gromov, Julius, Preti, '22]

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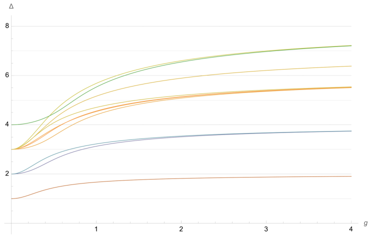

Figure: [Cavaglià, Gromov, Julius, Preti, '22]

- Add integrated 4pt correlators [Cavaglià, Gromov, Julius, Preti, '21 '22]

$$\int_0^1 dz\,(1+\log z)\frac{\delta G(z)}{z^2} = \frac{3\mathbb{C}-\mathbb{B}}{8\mathbb{B}^2}\,,\;\int_0^1 dz\,\frac{\delta f(z)}{z} = \frac{\mathbb{C}}{4\mathbb{B}^2}+\mathbb{F}-3\,.$$

Conformal bootstrap + Integrability = Bootstrability

- At $N \to \infty$ use Quantum Spectral Curve to obtain spectrum [Gromov, Kazakov, Leurent, Volin, '13 '14].

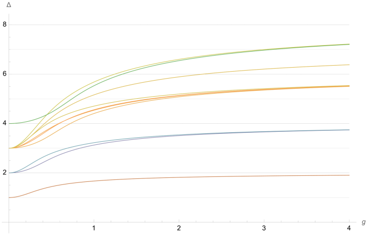

Figure: [Cavaglià, Gromov, Julius, Preti, '22]

- Add integrated 4pt correlators [Cavaglià, Gromov, Julius, Preti, '21 '22]

$$\int_0^1 dz\,(1+\log z)\frac{\delta G(z)}{z^2} = \frac{3\mathbb{C}-\mathbb{B}}{8\mathbb{B}^2}\,, \int_0^1 dz\,\frac{\delta f(z)}{z} = \frac{\mathbb{C}}{4\mathbb{B}^2} + \mathbb{F} - 3\,.$$

- RHS can be computed exactly for any coupling with integrability ($\mathbb{C}$) [Gromov, Levkovich-Maslyuk '15] or localization ($\mathbb{B}$) [Correa, Henn, Maldacena, Sever '12, Fiol, Garolera, Lewcowycz '12].

Bounds on OPE coefficients

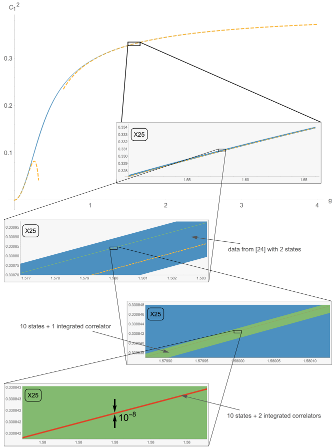

Figure: Bounds on OPE coefficient $\hat{c}_{\Delta_0}$ [Cavaglià, Gromov, Julius, Preti, '22]

Compared with analytic bootstrap results at

- Strong coupling [Ferrero, Meneghelli '21]
- Weak coupling [Cavaglià, Gromov, Julius, Preti, '22]

Generalization to higher p

- 3-point bulk-defect correlators:

$$\boxed{\int_0^1 \frac{dw}{w^{1+\frac{p}{2}}(1-w)^{1-\frac{p}{2}}}\mathcal{H}_{tt}^{\Phi}(w) = -\mu_t^{\Phi}\,\frac{\Gamma\left(\frac{p}{2}\right)}{\pi^{\frac{p}{2}}}\,.}$$

- Tilt soft sum rule

$$\boxed{\iint_{\mathrm{Im}\, z>0} \frac{dz\, d\bar{z}}{|z|^{p-\Delta_{\hat{\mathcal{O}}}}}\,(\mathrm{Im}\, z)^{p-2}\,\mathcal{G}^{JijI}(z,\bar{z}) = 0\ .}$$

- Tilt double-soft sum rule

$$\boxed{\iint_{\mathrm{Im}\, z>0,\, |z|<1} \frac{dz\, d\bar{z}}{|z|^{p-\Delta_{\hat{\mathcal{O}}}}}(\mathrm{Im}\, z)^{p-2}\log|z|\,\mathcal{G}^{I[ij]J}(z,\bar{z}) = Q_{IJ}^{[ij]}\,\frac{\Gamma(p-1)}{(2\pi)^{p-1}}\ .}$$