The Confining String in AdS

dCFT for the confining string in planar pure YM

based on work with:

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2508.08250, 2602.16694

Question: What is the UV completion of the confining string?

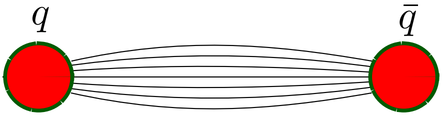
TLDR

- To make life easier: no quarks and yes planar (Planar pure Yang-Mills theory)
- The long confining string: one of the simplest probes of confinement.
- In flat space, effective string theory (EST) and lattice data already give a fairly detailed picture of low energy physics.
- Even though we have asymptotic freedom, no understanding of the UV completion of the EST.

Working proposal: Put it in AdS. The geometry gives controls over the scale and uniformizes on-shell observables across scales. Conformal symmetry makes bootstrap applicable!

Why focus on the flux tube?

- Separate a heavy quark-antiquark pair in planar Yang-Mills.
- The color flux does not spread; it organizes into a long flux tube.
- At large separation the object admits a systematic low-energy effective string theory (EST) description.
- This emergence is one of the most ubiquitous features of confinement.



The flat-space story

IR description of a long string

$$S_{\text{eff}} = -\int d^2\sigma \sqrt{-g} \left(\ell_s^{-2} + \dots\right), \qquad g_{ab} = \partial_a X_\mu \partial_b X^\mu .$$

$$K_{ab} \equiv n_\mu \partial_a X^\nu \nabla_\nu \partial_b X^\mu .$$

- The light fields are the transverse modes $X^i(\sigma, \tau)$.
- The scale is set by $\ell_s^{-1} \sim \Lambda_{\text{QCD}}$.
- These modes are Goldstones of

$$ISO(1, D-1)/ISO(1,1) \times O(D-2).$$

Effective String Theory, Lattice, and S-matrix Bootstrap

Effective string theory

$$S = \int d^2\sigma\sqrt{h}\left[\ell_s^{-2} + \kappa(\ell_s^2 K_{\alpha\beta}K^{\alpha\beta})^2 + \ldots\right]$$

[Luscher, Polchinski, Strominger, Weisz, Aharony, Komargodski, Dubovsky, Gorbenko, …]

$$\longleftrightarrow$$

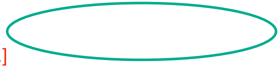
Scattering excitations on a long open string

[Guerrieri, Miro, Hebbar, Penedones, Vieira, Homrich, Albert …]

$$e^{2i\delta(s)}, \qquad 2\delta(s) = \frac{1}{4}s\ell_s^2 + \frac{\gamma_S}{768}\ell_s^6 s^3 + \ldots$$

Spectrum on a winding closed string

[Caselle, Teper, Athenodorou, …]



$$E_n(R) = \frac{R}{\ell_s^2} + \frac{\#}{R} + \frac{\#\ell_s^2}{R^3} + \frac{\#\ell_s^4}{R^5} + \ldots$$

[Plagiarized from Bendeguz Offertaler's slides]

A useful parametrization

- A practical way to (partially) organize the EST is through the two-particle worldsheet phase shift.
- EST coefficients appear as higher energy corrections to the phase. In 3d,

$$2\delta(s) = \ell_s^2 s/4 + \gamma_3 \ell_s^6 s^3 + \gamma_5 \ell_s^{10} s^5 + \cdots + i\gamma_8 \ell_s^{16} s^8 + \dots$$

- This is a very useful parametrization. To an order to which the scattering is elastic, we can also use TBA to get the finite-volume spectrum.

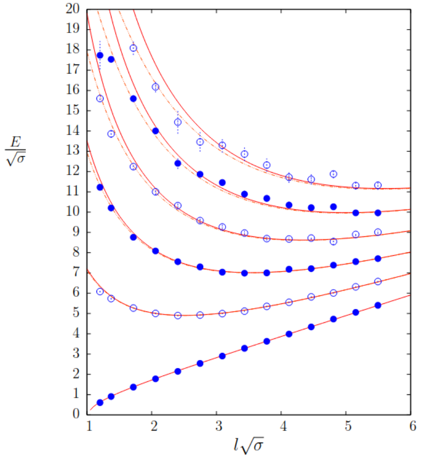
$$\text{worldsheet EFT} \longrightarrow \delta(p) \longrightarrow \text{TBA} \longrightarrow E(R)$$

- Inspired by critical ST, we will refer to the phase shift with $\gamma_n = 0$ as the NG phase shift, and the resulting TBA spectrum as NG spectrum

In $3d$ the NG spectrum is very close to the lattice data

- Much of the $3d$ spectrum already lies close to the prediction of this NG integrable theory.
- The interpretation is that the EST works well to quite high energies. There is no clear signal of a breakdown.

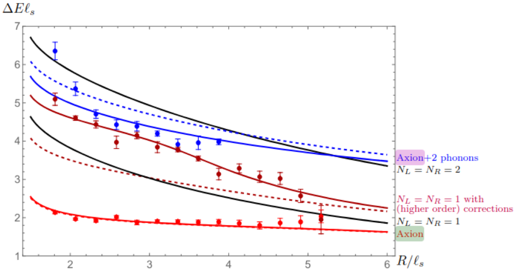
Lattice data by Athenodorou, Bringoltz, Teper, and collaborators.



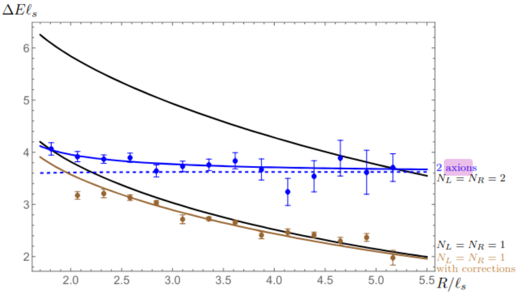
In 4d: lattice spectrum predicts a new excitation

Much of the $4d$ spectrum is still close to the NG baseline. The clearest anomaly is the 0^{--} channel, suggestive of a worldsheet pseudoscalar.

Energy levels in the 0^{--} channel



Energy levels in the 0^{++} channel



Black = NG. Dashed = “naive" NG. Color lines = NG+corrections+axion

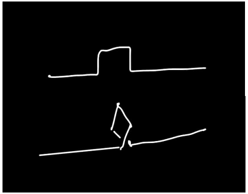
Punchline from the lattice

- Once the worldsheet axion is included, the EST continues to work well to surprisingly high scales.
- In other words, even in 4d there's no signal of a breakdown of EST.

This leads to the hope that the UV and IR are not very “far" from each other. This project is part of the attempt to make this notion concrete.

A tentative UV/IR dictionary

Counting D.O.F suggests an intuitive matching of lattice/UV operators to IR particles

Gauge-side insertion	Worldsheet description	Figure
Wilson line	long string	
$F_{\parallel j}$	X_j	
F_{12}	axion	

Question: Can this matching be made precise?

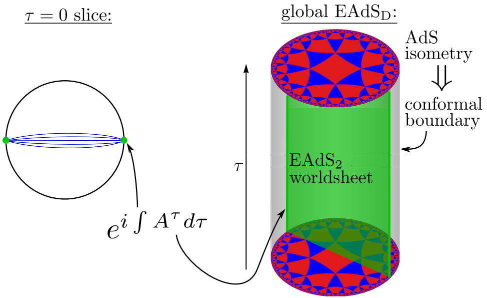
Part I

Set-up in AdS

The radius R controls the RG scale.

Yang-Mills Flux Tube in AdS

- Put planar pure YM on rigid AdS with magnetic (Neumann) boundary conditions.
- (Dirichlet has no gauge fields or Wilson lines on the boundary. It also believed to undergo a deconfinement phase transition at some finite radius.)
- At small $R\Lambda_{\mathrm{QCD}}$, defect correlators can be computed perturbatively.
- At large $R\Lambda_{\mathrm{QCD}}$, the flux tube approaches the flat-space confining string, and EST can be used.



$$ds^2 = R^2\left(d\rho^2 + \cosh^2\rho\, d\tau^2 + \sinh^2\rho\, d\Omega_{d-1}^2\right)$$

Callan-Wilczek; Aharony et al.; Gabai-Gorbenko-Qiao; Gabai-Gorbenko-Offertaler.

Two perturbative expansions

Small radius: use Yang-Mills

$$S_{\mathrm{YM}}[A] = \frac{1}{4R^{d-3}} \int_{\mathrm{AdS}_{d+1}} \sqrt{G}\, \mathrm{Tr}\, F_{\mu\nu} F^{\mu\nu}.$$

Use perturbation theory in AdS$_{d+1}$.

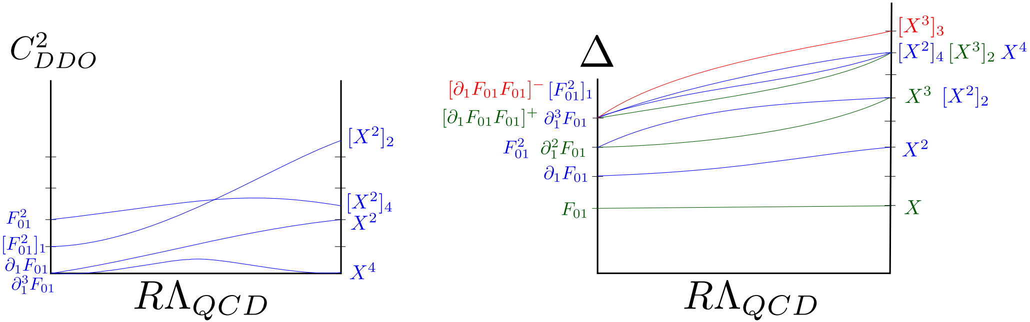
Large radius: use the effective string

$$S[z] = \int d^2\sigma \sqrt{g} \left[\ell_s^{-2} + \kappa \ell_s^2 K^4 + O(\ell_s^4)\right].$$

Use the EST: a theory on AdS$_2$.

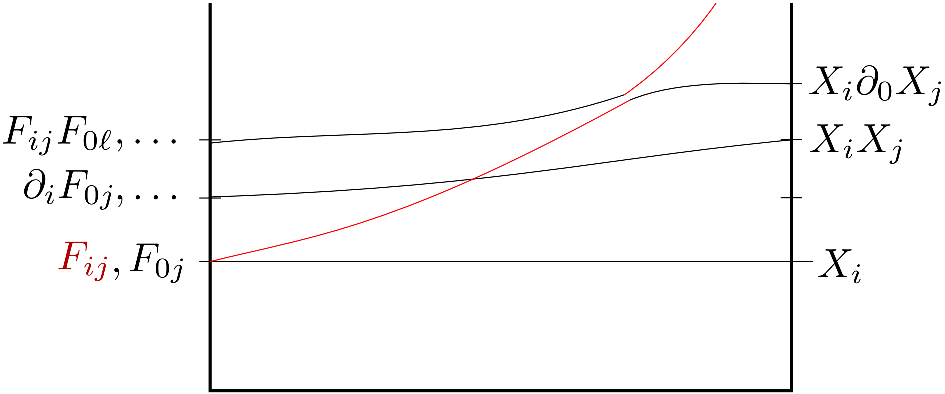
Dimensionless knob: $R\Lambda_{\mathrm{QCD}} \sim R/\ell_s$

How is the answer expected to look

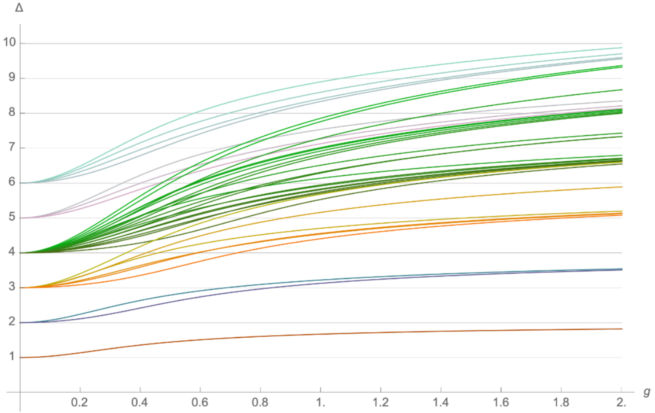


Schematic goal: track how defect operators at small $R\Lambda_{\rm QCD}$ reorganize into worldsheet excitations at large $R\Lambda_{\rm QCD}$.

A 4d version of the same plot



Inspiration: the Maldacena-Wilson line



Weak coupling | EST regime

whole trajectory from integrability

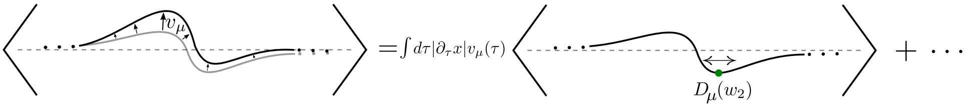
Cavaglià, Gromov, Julius, Preti, and collaborators.

Part II

Defect-CFT tools

Leveraging locality and the AdS isometries.

The displacement operator

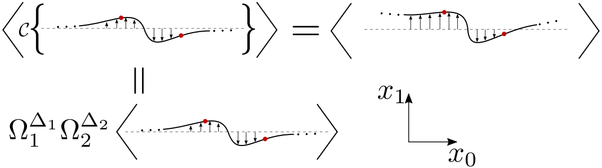


- Deforming the line is equivalent to an insertion of the displacement operator $\mathbb{D}_i$.
- Dimensional analysis fixes its dimension: $\Delta_{\mathbb{D}} = 2$.
- At weak coupling, $\mathbb{D}_i \sim F_{\tau i}$.
- At large radius, $\mathbb{D}_i \sim X_i$.

[Existence in AdS case: Bianchi, de Sabbata, Meineri 26', Qiao 26']

What CFT methods add

- Analytic and numerical bootstrap techniques become available.
- The line insertion breaks AdS isometries, making them nonlinearly realized.
- These broken symmetries become integrated constraints on correlators involving $\mathbb{D}$.
- While the same constraints are implemented by the form of the effective action, implementation on on-shell observables is directly applicable to bootstrap



Refs: B.G., Sever, Zhong 22', 23', 25' (2501.06900); Cavaglià, Gromov, Julius, Preti 22'; Drukker, Kong, Sakkas 22'; Drukker, Kravchuk, Kong 25'; Kong 25'; Girault, Paulos, van Vliet 25'.

Part III

Some concrete outputs

Three approaches to obtain dCFT data.

Three approaches

- Small-R perturbation theory.
- Large-R worldsheet perturbation theory.
- Bootstrap/nonperturbative constraints (see Philine's talk!)
- The flat-space motivation is especially sharp in $4d$, but the explicit calculations so far are in planar $3d$ Yang-Mills.

Weak-coupling constraints

- In planar $3d$ YM, the light defect operators are built from adjoint insertions,

$$F_{\tau 1}, \quad \partial_1 F_{\tau 1}, \quad F_{\tau 1}^2, \quad \dots$$

- Define notation for single letter operators,

$$a^{(n)} \equiv \partial_1^{n-1} F_{\tau 1}$$

- Integrated constraints have an enhancement phenomenon at weak couplings - contributions from different perturbative orders mix. For example,

$$\frac{4c_{112}^2}{\gamma_2} = \frac{4c^2_{F_{\tau 1}, F_{\tau 1}, \partial_1 F_{\tau 1}}}{\gamma_{\partial_1 F_{\tau 1}}} = \int_0^\infty (1 + 2t + 3t^2) G_c^{(\text{free})}(t) dt = \frac{5}{3}$$

Weak-coupling data

- The integrated constraints reduce to a solvable recursion relation. For the single-letter data,

$$C_{1,1,2}C_{n,n,2} = \frac{5\left(n^2+n+3\right)}{72n(2n-1)\pi}\lambda + O(\lambda^2),$$

$$C^2_{1,n-1,n} = \frac{\left(n^2+1\right)^2}{4n(2n-1)\left(n^2-1\right)\pi}\lambda + O(\lambda^2),$$

$$\gamma_n = \frac{\lambda}{2\pi}\left[\frac{2}{n+1} + 2\big(\psi(n)+\gamma\big) - 1\right] + O(\lambda^2).$$

Weak-coupling data

- Tree-level perturbation theory fixes the generic three-point coefficient of single-letter operators:

$$C_{n_1,n_2,n_3} = \left[\sum_{j=1}^{3} n_j(1+n_j)\right] \frac{\left(\frac{n_1+n_2+n_3}{2}\right)!}{\pi^2} \times \frac{\Gamma\left(\frac{n_{123}+1}{2}\right)\Gamma\left(\frac{n_{132}+1}{2}\right)\Gamma\left(\frac{n_{231}+1}{2}\right)}{\prod_{j=1}^{3}\sqrt{2n_j(n_j+1)n_j!\left(\frac{1}{2}\right)_{n_j}}}\lambda + O(\lambda^2).$$

$n_{ijk} \equiv n_i + n_j - n_k$. This vanishes unless the triangle inequalities hold and $n_1+n_2+n_3$ is even.

Large radius: the worldsheet description

Effective string on AdS_2

$$S_{\text{eff.}} = \int d^2\sigma \left[\frac{1}{2}(\partial X)^2 + X^2 - \frac{\ell_s^2}{8R^2}(\partial X)^4 + \frac{\ell_s^2}{3R^2}X^4 + \dots\right]$$

- At large radius the relevant degrees of freedom are the branons on the AdS_2 worldsheet.
- Up to the order shown here, the action depends on the AdS radius (in unites of string length) and one non-universal Wilson coefficient - same as flat space.
- The goal is again to extract defect dimensions and OPE coefficients.

Large radius: transcendentality Ansatz bootstrap

- Study the four-point function of the displacement/branon sector.
- Witten diagram computations are notoriously hard
- Guess that the correlator is a linear combination of functions of bounded transcendentality
- Impose the OPE, crossing, integrated symmetry constraints, and flat-space limit.
- This fixes the correlator to two loops, up to one flat-space Wilson coefficient.
- From that correlator one reads off low-lying anomalous dimensions and OPE coefficients.

Large radius: transcendentality Ansatz bootstrap

$$
\begin{aligned}
&\text{Transc. } 1: \quad \log(1-\chi), \\
&\text{Transc. } 2: \quad \log^2(1-\chi), \quad \mathrm{Li}_2(\chi), \\
&\text{Transc. } 3: \quad \log^3(1-\chi), \quad \log(1-\chi)\mathrm{Li}_2(\chi), \quad \mathrm{Li}_3(\chi), \quad S_{1,2}(\chi).
\end{aligned}
$$

$$S_{1,2}(\chi) \equiv \frac{1}{2}\int_0^1 \frac{\mathrm{d}t}{t}\log^2(1-\chi t) \quad \text{(Nielsen polylogarithm)}$$

- Modified “bose" symmetry

$$\langle O(x)O(y)O(z)O(w)\rangle = \langle O(y)O(z)O(w)O(x)\rangle \text{``} = \text{''} \langle O(y)O(x)O(z)O(w)\rangle$$

- Regge/flat-space limit:

$$z^{-4}G^{(L)}(z)\Big|_{z=\frac{1}{2}+is} \overset{s\to\infty}{=} O(s^L), \quad \Leftarrow \quad 2\delta(s) = \frac{1}{4}\ell_s^2 s + \frac{\gamma_{\mathrm{SM}}}{768}\ell_s^6 s^3 + O(\ell_s^8 s^4).$$

Large radius: sample conformal data

Dimensions

$$\Delta_X = 2, \qquad \Delta_{X^2} = 4 - \varsigma$$
$$\Delta_{[X^2]_2} = 6 - \frac{13}{4}\varsigma + \frac{9787}{8960}\varsigma^2 + O(\varsigma^3)$$
$$\Delta_{[X^2]_4} = 8 - \frac{13}{2}\varsigma + \frac{32713}{6720}\varsigma^2 + O(\varsigma^3)$$

OPE coefficients

$$a_{[X^2]} = 2 - \frac{251}{60}\varsigma + \frac{11011}{4800}\varsigma^2 + O(\varsigma^3)$$
$$a_{[X^2]_2} = \frac{40}{9} - \frac{11455}{2268}\varsigma - \frac{1219193}{326592}\varsigma^2 + O(\varsigma^3)$$

Gabai, Gorbenko, Offertaler, *Yang-Mills Flux Tube in AdS II*.

Padé

Pade is an approach that aims to extend the validity of perturbative expansions. Take,

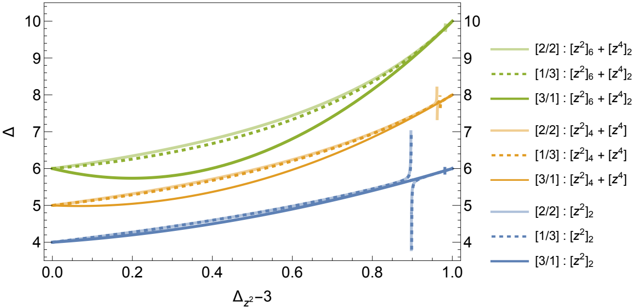
$$f_{[M/N]}(x) \equiv \frac{\sum_{m=0}^{M} p_m x^m}{1 + \sum_{n=1}^{N} q_n x^n},$$

and fix the parameters to match the known orders.

- There have been recent implementations in our field [Henriksson, Kousvos, Nepveu 25'][Alday, Armanini, Häring, Zhiboedov 25'] [Dempsey, Karlsson, Pufu, Zahraee, Zhiboedov 25']
- **Notably** [Julius, Sokolova 25']
- There's plenty of mathematical theory about this (e.g. [Nuttall 70', . . .])

A first interpolation attempt

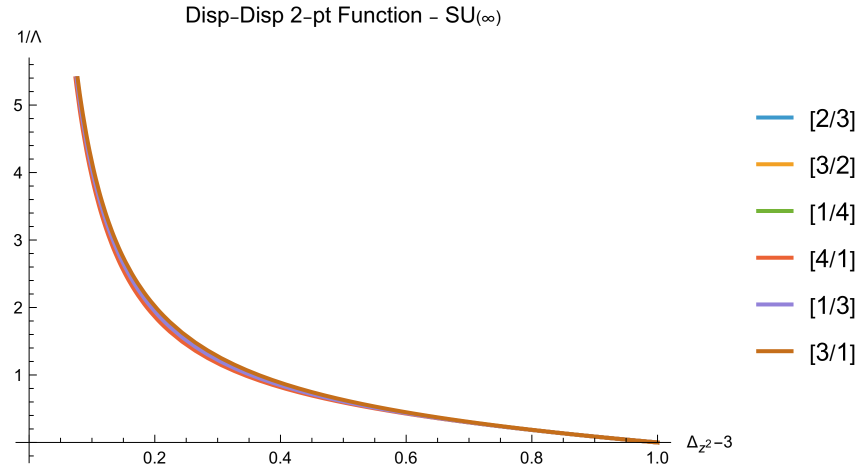
- As a simple check, combine perturbative data from both ends with Padé interpolation.
- This is only exploratory, but it is consistent with a smooth interpolation.



- $\mathcal{N}=4$ experiments $\rightarrow$ one more order at weak couplings $\rightarrow$ $\sim 1\%$ approximates

[Julius, Sokolova 25']

C_D



Disp-Disp 2-pt Function - SU(∞)
1/Λ
5
4
3
2
1
0.2
0.4
0.6
0.8
1.0
Δ_{z²}−3
[2/3]
[3/2]
[1/4]
[4/1]
[1/3]
[3/1]

Takeaway

What we know

- In AdS, the flux tube defines a line defect with a protected displacement operator.
- The same defect observables admit weak-coupling and large-radius expansions.
- In $3d$, one can already compute concrete dimensions and OPE coefficients on both sides.

What we hope to know

- Smooth interpolation across all $R\Lambda_{\mathrm{QCD}}$?
- Constrain Wilson coefficients beyond S-matrix bootstrap?
- How to “unitarize" NG ($+$corrections?) in AdS?

Possible next steps

- Can we control a finite/high flat space limit?
- Push the weak-coupling side to higher orders.
- Bootstrap?
- Extend the analysis to $4d$, where we hope to see the axion via avoided level crossing.
- Study other observables and related theories: adjoint QCD, rotating strings, pions, and supersymmetric versions.

Thank you

Questions?