Giant gravitons as zero-dimensional defects

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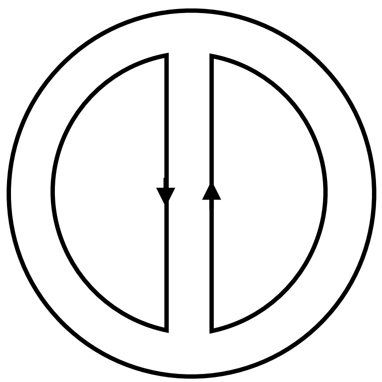
with Junding Chen, Xi-Er Du, Yunfeng Jiang, and Hynek Paul

Bootstrap 2026, London

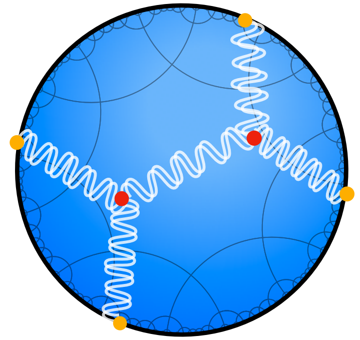
$\mathcal{N} = 4$ SYM is well controlled in the planar regime

Planar limit: $N \to \infty$

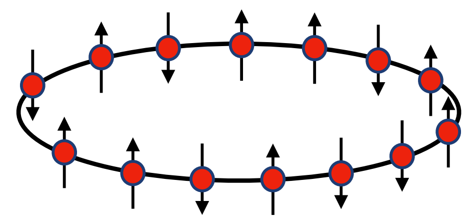
$\lambda \ll 1$: planar diagrams



$\lambda \gg 1$: classical AdS SUGRA
+ bootstrap method

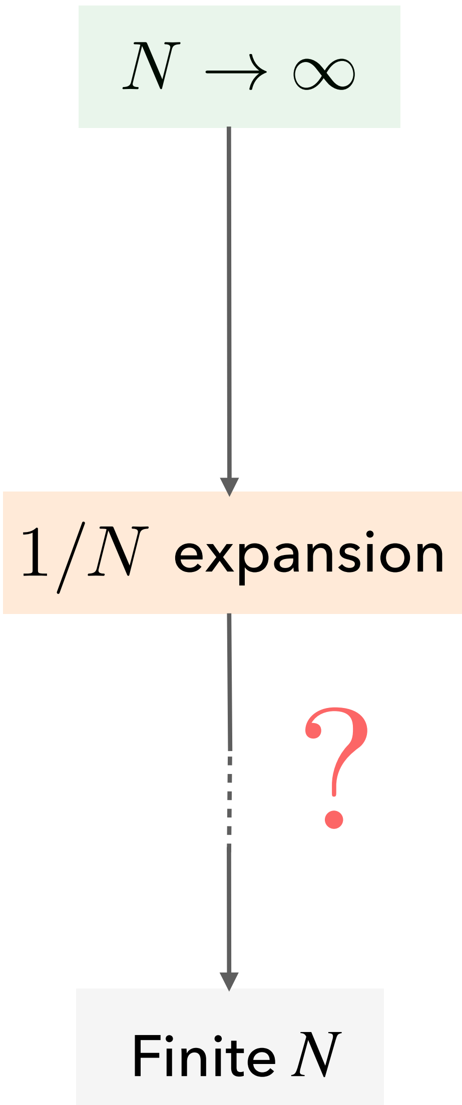


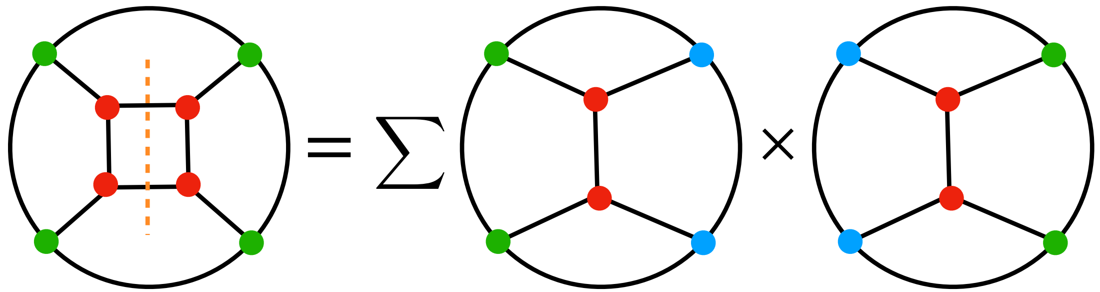
λ finite: integrability



The planar regime admits many complementary exact tools.

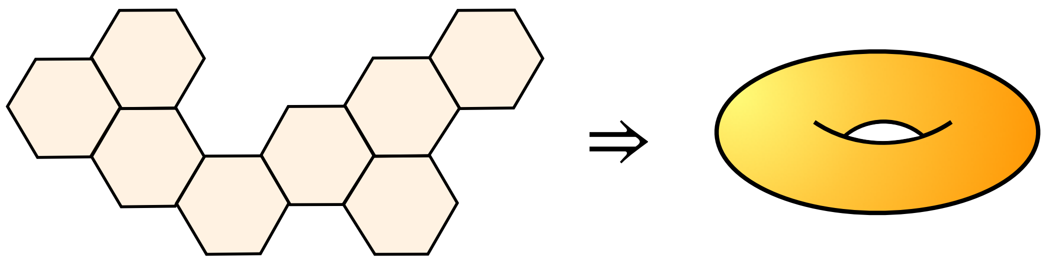
Perturbative $1/N$ corrections are partly under control





AdS unitarity method

Aharony Alday Bissi Perlmutter '16, Alday Bissi '17
Aprile, Drummond, Heslop, Paul '17



Tessellation of hexagons

Bargheer Caetano Fleury Komatsu Vieira '17

But

- Most systematic progress is restricted to "light" single-trace probes.

  $\frac{1}{2}$-BPS operators: $\mathcal{O}_k^{I_1 \dots I_k} = \mathrm{tr}(\Phi^{\{I_1} \dots \Phi^{I_k\}})$ $\quad \Delta_k = k \sim \mathcal{O}(N^0)$

- Truly finite N physics is still difficult to access directly.

Giant gravitons sit between $1/N$ expansion and finite N

Giant gravitons McGreevy Susskind Toumbas '00

$$\mathcal{D} = \det(\Phi \cdot t) \qquad t \cdot t = 0$$

- They are also $\frac{1}{2}$-BPS operators.
- Dimension $\Delta = N$ scales with N. Not light excitations
- Determinant color structure invalidates the ribbon graph counting

$$\langle \mathcal{O}\,\mathcal{D}\,\mathcal{D} \rangle \sim \mathcal{O}(1) \qquad\qquad \langle \mathcal{O}\,\mathcal{O}\,\mathcal{O} \rangle \sim \mathcal{O}(1/N)$$

Good intermediate target: BPS enough to control, heavy enough to retain non-planar physics.

What is known about giant graviton correlators?

Standard view in the literature: correlators of light and heavy local operators

$\langle \mathcal{O}\,\mathcal{D}\,\mathcal{D} \rangle$

Bissi Kristjansen Young Zoubos '11, Caputa de Mello Koch Zoubos '12, Jiang Komatsu Vescovi '19, Yang Jiang Komatsu, Wu '21, ...

Fixed by symmetry up to a constant. Protected for $\frac{1}{2}$-BPS operators.

$\langle \mathcal{O}_1 \mathcal{O}_2\,\mathcal{D}\,\mathcal{D} \rangle$

First object with nontrivial spacetime dependence.

Weak coupling: up to 3 loops. Jiang Komatsu Vescovi '19; Jiang Wu Zhang '23, '24, He Shi Tang Wen '26

Strong coupling: almost none, only limited integrated data. Brown Galvagno Wen '24

Local operator perspective: correct but may not be most useful.

It turns out that the more natural perspective is to view giant gravitons as zero-dimensional defects.

Why is the defect viewpoint natural?

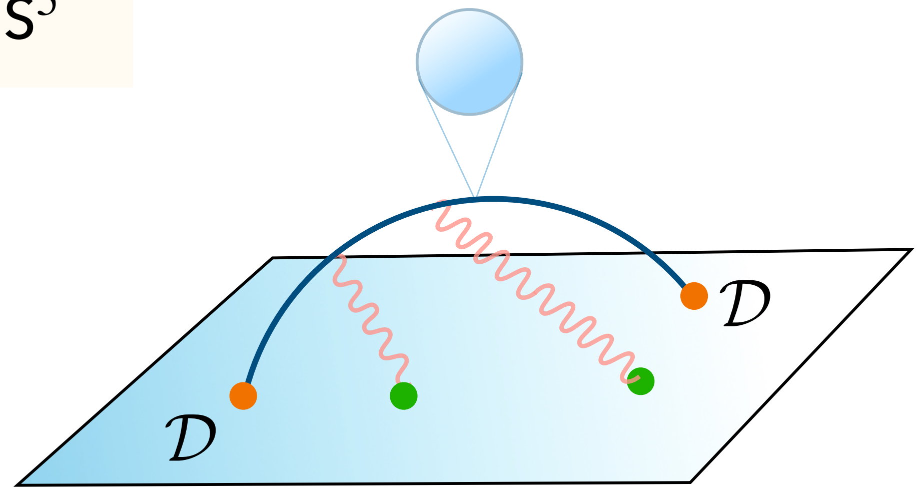
Holography: Giant graviton $\mathcal{D}$ $\Longleftrightarrow$ D3 wrapping $S^3 \subset S^5$

Bulk: worldline in AdS_5 (1d)

$\Downarrow$

Boundary: 0d defect

(AdS objects of dim $p+1 \Rightarrow$ boundary dim p)



Symmetry: a dim p defect preserves $SO(d+1,1) \to SO(p+1,1) \times SO(d-p)$

For a giant graviton pair: $p=0$ Defect kinematic results are still valid.

The 0d defect viewpoint follows from both brane geometry and symmetry kinematics.

What is the defect perspective good for?

Computation: uses recent bootstrap techniques for strong coupling

Holographic correlators from symmetry and consistency $\longrightarrow$ Extended to CFTs with ordinary $p \geq 1$ defects $\longrightarrow$ Giant gravitons as $p = 0$ defects

Rastelli XZ '16

Gimenez-Grau '23, Chen Gimenez-Grau XZ '23

Structure: hidden 10d symmetry becomes manifest

strong coupling correlators + weak coupling integrands

Generality: a natural language for light-light-heavy-heavy (LLHH) correlators.

via 0d defect conformal blocks

independent of susy

The defect perspective is not only natural; it is computationally useful.

Defect vs no defect

Operators:

$$\mathcal{O}_k(x,t) = \mathrm{tr}\left((\Phi(x)\cdot t)^k\right) \qquad \mathcal{D}(x,t) = \det(\Phi(x)\cdot t)$$

Null vector t: $SO(6)$ R-symmetry polarization

Null vector of $SO(5,1)$: lifting x into embedding space $\quad x \to P = (\overset{-}{\tfrac{1+x^2}{2}}, \overset{+}{\tfrac{1-x^2}{2}}, \overset{+}{x})$

Defect correlators are defined by taking the ratio

$$\langle\langle \mathcal{O}_k \rangle\rangle = \frac{\langle \mathcal{O}_k\, \mathcal{D}\, \mathcal{D}\rangle}{\langle \mathcal{D}\, \mathcal{D}\rangle} \qquad G_{k_1k_2} = \frac{\langle \mathcal{O}_{k_1}\mathcal{O}_{k_2}\mathcal{D}\mathcal{D}\rangle}{\langle \mathcal{D}\,\mathcal{D}\rangle} = \langle\langle \mathcal{O}_{k_1}\mathcal{O}_{k_2}\rangle\rangle$$

Convention: the heavy operators are inserted at 3 and 4.

The giant graviton defect enters through projectors

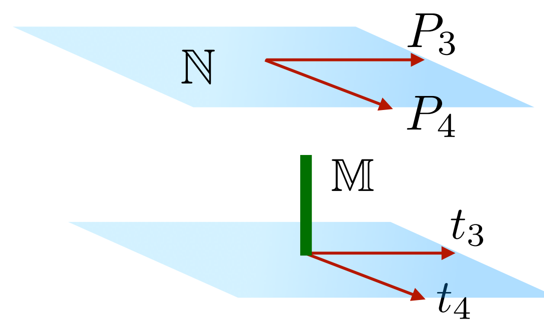
Spacetime:

$$\mathbb{N}_{AB} = \frac{P_{3,A}P_{4,B} + P_{4,A}P_{3,B}}{P_3 \cdot P_4}$$

R-symmetry:

$$\mathbb{M}_{IJ} = \delta_{IJ} - \frac{t_{3,I}t_{4,J} + t_{4,I}t_{3,J}}{t_3 \cdot t_4}$$

Projectors



1pt function is fixed:

$$\langle\langle \mathcal{O}_k(P_i,t_i)\rangle\rangle = a_k \frac{(\frac{1}{2}t_i\cdot\mathbb{M}\cdot t_i)^{\frac{k}{2}}}{(P_i\cdot\mathbb{N}\cdot P_i)^{\frac{k}{2}}} = a_k\left(\frac{(t_i\cdot t_3)(t_i\cdot t_4)x_{34}^2}{(t_3\cdot t_4)x_{3i}^2 x_{4i}^2}\right)^{\frac{k}{2}} = \frac{\langle \mathcal{O}_k(P_i,t_i)\mathcal{D}(P_3,t_3)\mathcal{D}(P_4,t_4)\rangle}{\langle \mathcal{D}(P_3,t_3)\mathcal{D}(P_4,t_4)\rangle}$$

2pt function has 4 cross ratios:

$$G_{k_1k_2} = \prod_{i=1}^{2}\left(\frac{(t_i\cdot t_3)(t_i\cdot t_4)x_{34}^2}{(t_3\cdot t_4)x_{3i}^2 x_{4i}^2}\right)^{\frac{k_i}{2}} \mathcal{G}_{k_1k_2}(U,V;\sigma,\tau)$$

$$U = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2}\ , \quad V = \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2}\ ,$$

$$\sigma = \frac{(t_1\cdot t_3)(t_2\cdot t_4)}{(t_1\cdot t_2)(t_3\cdot t_4)}\ , \quad \tau = \frac{(t_1\cdot t_4)(t_2\cdot t_3)}{(t_1\cdot t_2)(t_3\cdot t_4)}$$

The defect correlator inherits the 4pt superconformal kinematics

Superconformal Ward identities of $\frac{1}{2}$-BPS 4pt function Eden Petkou Schubert Sokatchev '00, Nirchl Osborn '04

$$\partial_{\bar{z}}[\mathcal{G}(z,\bar{z};\alpha,\bar{\alpha})|_{\bar{\alpha}=1/\bar{z}}]=0$$

where we have made the change of variables $U=z\bar{z}\,,\quad V=(1-z)(1-\bar{z})\,,\quad \sigma=\alpha\bar{\alpha}\,,\quad \tau=(1-\alpha)(1-\bar{\alpha})$

The solution is known and for defect 2pt functions it takes the form

$$G_{k_1k_2}=G_{k_1k_2,\text{free}}+\frac{t_{12}^2x_{13}^4x_{24}^2}{x_{34}^2}R\,H_{k_1k_2}$$

$$R=(1-z\alpha)(1-z\bar{\alpha})(1-\bar{z}\alpha)(1-\bar{z}\bar{\alpha})$$

The reduced correlator $H_{k_1k_2}$ contains all the dynamical information and has shifted quantum numbers

Conformal dimension: $\{k_1,k_2\}\to\{k_1+2,k_2+2\}$ R-symmetry charge: $\{k_1,k_2\}\to\{k_1-2,k_2-2\}$

SUGRA gives a defect effective theory

$$\text{D3 brane action} \xrightarrow{\text{integrating out } S^3} {}^{*}\ \text{1d effective action}$$

$*$ Also need to integrate over the moduli of the supergravity solution.

Expanded in bulk and defect fluctuations

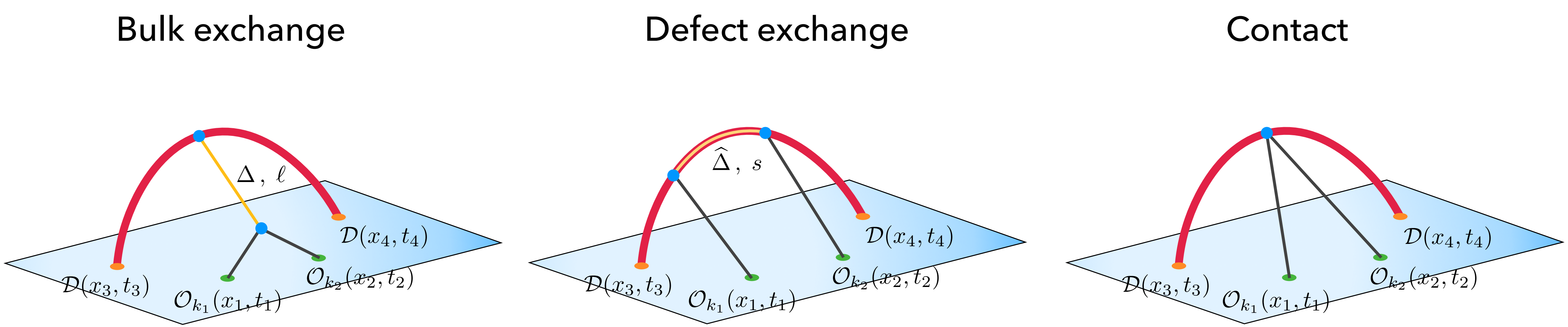
$$S_{\mathrm{D3}} = -\frac{N}{2\pi^2}\int d\tau \left(1 + L_{\mathrm{D3}}^{(2,0)} + L_{\mathrm{D3}}^{(0,1)} + L_{\mathrm{D3}}^{(1,1)} + L_{\mathrm{D3}}^{(0,2)} + \ldots\right)$$

$L_{\mathrm{D3}}^{(m,n)}$ — m → defect, n → bulk

- Defect field spectrum can be read off from the $(2,0)$ term, and is organized into short multiplets of the preserved superconformal symmetry.
- Bulk spectrum is unaffected and is just the light SUGRA KK modes.

In principle the giant graviton correlators can be computed by brute force as a sum of diagrams.

Tree-level Witten diagrams



The procedure is standard. But the implementation is cumbersome.

Nontrivial to extract all the needed vertices.

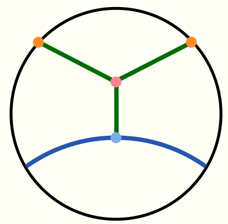
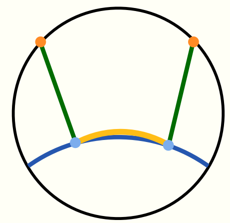
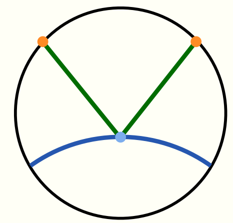
Tedious to keep track of all the combinatorics.

Bootstrap algorithm

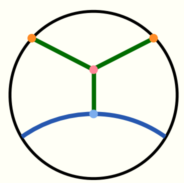
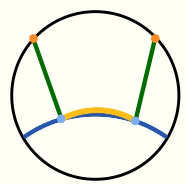
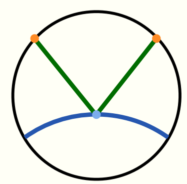
The better strategy is to fix the correlators directly using symmetry and consistency conditions

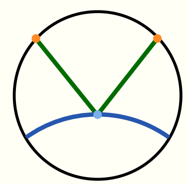
1. Ansatz
2. Evaluate
3. Solve

Sum over all possible diagrams with **unfixed** coefficients

$$G = \sum \mu_b$$  $$+ \sum \mu_d$$  $$+ \sum \mu_c$$ 

Reduce exchanges to contacts

  $$\longrightarrow \sum_{\text{finite}}$$ 

 $$\sim {}_2F_1(\ldots) \longrightarrow \log V$$

Ansatz ⇒ an elementary function of cross ratios

Impose Ward identities $\Longrightarrow$ Solves all the unknowns μ_b μ_d μ_c

Result: bootstrap fixes all defect 2pt functions

For example, for $k_1 = k_2 = 2$

$$G_{22,\text{free}} = \frac{2t_{12}^2(\tau - \sigma\tau U + \sigma V)}{N(P_1 \cdot \mathbb{N} \cdot P_1)(P_2 \cdot \mathbb{N} \cdot P_2)U} \qquad H_{22} = \frac{2V\left(V^2 - 2V\log V - 1\right)}{N(P_1 \cdot \mathbb{N} \cdot P_1)^2(P_2 \cdot \mathbb{N} \cdot P_2)^2 U(1-V)^3}$$

Similar but lengthier expressions for higher k_1, k_2.

A few crosschecks:

- ✓ Free theory correlators
- ✓ One-point functions
- ✓ Integrated correlator from localization

A hidden 10d structure emerges

For all light $\frac{1}{2}$-BPS operators, hidden symmetry was already found in 4pt functions Caron-Huot Trinh '18

The lowest 4pt function can be turned into a generating function

$$Z_i = (P_i, t_i) \qquad \mathbf{T}: \; P_i \cdot P_j \to Z_i \cdot Z_j = P_i \cdot P_j + t_i \cdot t_j$$

(4d distances $\to$ 10d distances)

$$H_{2222}(x_{ij}^2) \xrightarrow{\mathbf{T}} \mathbf{H}(x_i, t_i) = H_{2222}(x_{ij}^2 - 2t_{ij}) \xrightarrow{\text{Taylor}} \text{all} \quad H_{k_1k_2k_3k_4}$$

$-2P_i \cdot P_j = x_{ij}^2$

(generating function)

Intuitive picture:

$$\mathrm{AdS}_5 \times \mathrm{S}^5 \xrightarrow{\text{Weyl}} \mathbb{R}^{10}$$

"a single amplitude in 10d"

The naive uplift fails for giant gravitons

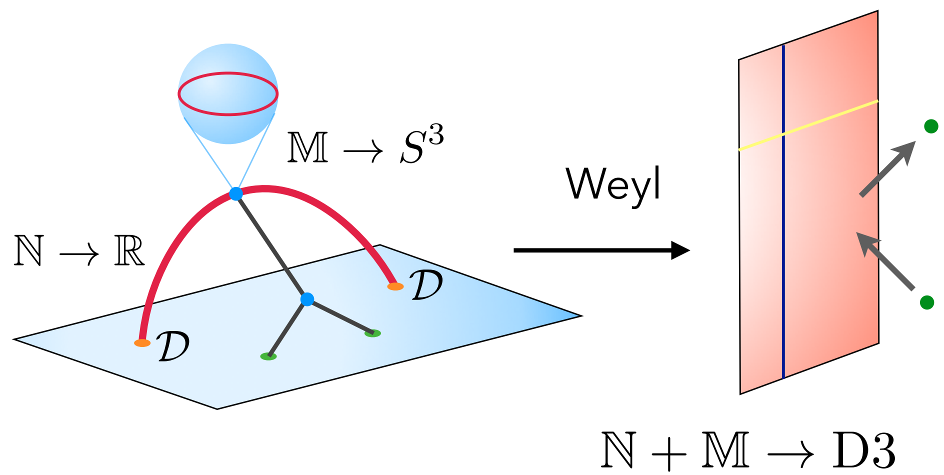
It fails because the Taylor expansion changes the charges of the heavy operators.

The correct prescription:

$$\mathbf{T}' : \begin{array}{l} P_1 \cdot P_2 \to Z_1 \cdot Z_2 \\ P_a \cdot \mathbb{N} \cdot P_b \to Z_a \cdot (\mathbb{N} + \mathbb{M}) \cdot Z_b \qquad a, b = 1, 2 \end{array}$$

$H_{22} \xrightarrow{\mathbf{T}'}$ generating function $\mathbf{H} \xrightarrow{\text{Taylor}} H_{k_1 k_2}$

Uplift only the projectors of the heavy operators



"a single scattering form factor"

The defect perspective is not only natural but also necessary: the hidden 10d symmetry is **partially broken**.

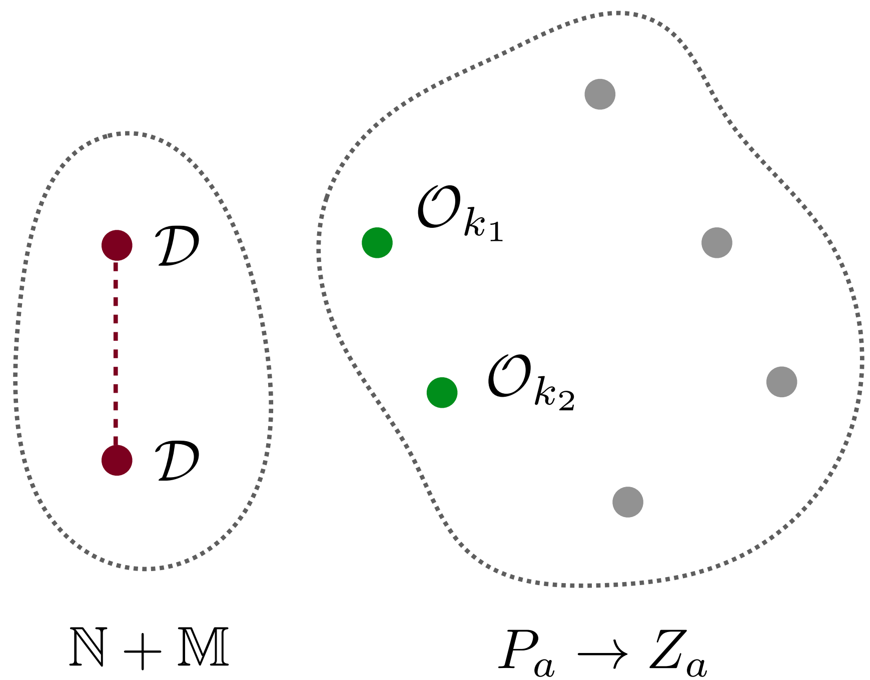
Same hidden structure also exists at weak coupling

However, it now acts at the level of the Lagrangian insertion integrands of the reduced correlators

$$H^{(\ell)}_{k_1 k_2} = \int \prod_{p=5}^{\ell+4} dP_p I^{(\ell)}_{k_1 k_2}$$

$$Z_p = (P_p, 0) \qquad P_a \cdot \mathbb{N} \cdot P_b \to Z_a \cdot (\mathbb{N} + \mathbb{M}) \cdot Z_b$$

$$a, b \in \{1, 2, 5, \ldots, \ell + 4\}$$



Lagrangian insertion integrand at ℓ loop: defect $(\ell + 2)$-pt correlator

The structure packages KK integrands into a single generating function (explicitly checked at 1 and 2 loops)

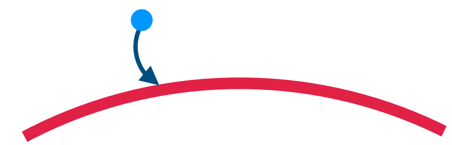
$$I^{(\ell)}_{22} \longrightarrow \text{generating function } \mathbf{I}^{(\ell)} \longrightarrow I^{(\ell)}_{k_1 k_2}$$

A similar story was also found for correlators of all light operators. Caron-Huot Coronado '21

Extracting data from giant gravitons correlators

$\log V$ **coefficient:**

Anomalous Dimension of "double-particle" operators in the defect channel $\Longleftrightarrow$ **Binding Energy** of a light and giant graviton pair



$$\widehat{\Delta} = k + 2n + s + \frac{2}{N}\widehat{\gamma} + \ldots$$

The complete spectrum is obtained from solving an operator mixing problem using all KK correlators

$$\boxed{\widehat{\gamma}^{(1)}_p = -\frac{2M_t^{(2)} M_{t+s+1}^{(2)}}{(s+p-r_1-1)_3}}$$

$$M_t^{(2)} = (t-1)(t+r_1) \qquad t = \frac{\widehat{\tau}-r_1-r_2}{2}$$

Further implications of hidden symmetry:

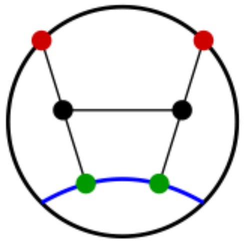
- The use of 10d defect blocks automatically diagonalizes the mixing problem.
- Leads to a 4th order differential operator whose eigenvalue is the $\widehat{\gamma}$ numerator. The operator simplifies the leading log singularities to all AdS loop orders

$$\widehat{\mathbf{\Delta}}^{(4)} = \frac{x\bar{x}\beta\bar{\beta}}{(x-\bar{x})(\beta-\bar{\beta})}(x\partial_x - \beta\partial_\beta)(x\partial_x - \bar{\beta}\partial_{\bar{\beta}})(\bar{x}\partial_{\bar{x}} - \beta\partial_\beta)(\bar{x}\partial_{\bar{x}} - \bar{\beta}\partial_{\bar{\beta}})\frac{(x-\bar{x})(\beta-\bar{\beta})}{x\bar{x}\beta\bar{\beta}}$$

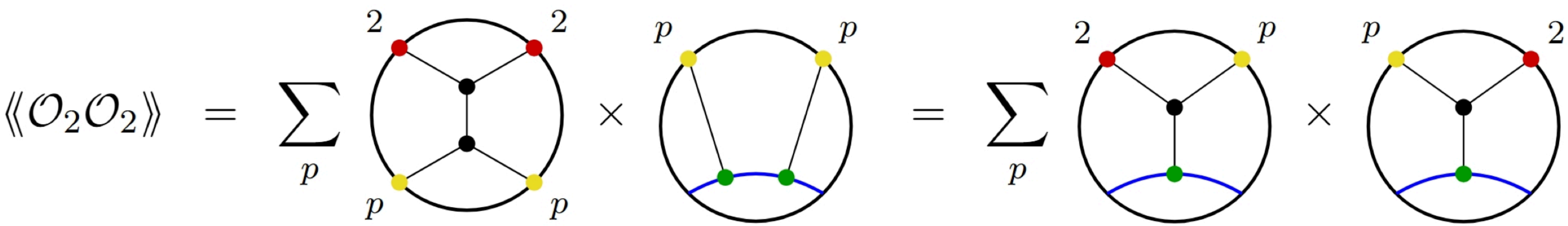
$$x = 1-z\,, \quad \bar{x} = 1-\bar{z}\,, \quad \beta = \frac{\alpha}{1-\alpha}\,, \quad \bar{\beta} = \frac{\bar{\alpha}}{1-\bar{\alpha}}$$

Quantum gravity corrections

Chen Du Paul XZ '26



one-loop correction



Defect unitarity method: the one-loop leading logarithmic singularities in both defect and bulk channels are obtained from "gluing" the lower order correlators. We then complete the singularities into the full correlator. Chen Gimenez-Grau Paul XZ '24

- Mellin space

$$\widetilde{\mathcal{M}}^{(2)}_{22}(\delta,\gamma) = \sum_{m,n=0}^{\infty} \frac{c_{mn}}{(\delta+n)(\gamma-3-2m)}$$

$$c_{mn} = \frac{2^{2m-5}\sqrt{\pi}\,\Gamma(n+m-1)}{3\,\Gamma\left(\frac{5}{2}-m\right)\Gamma(n+2m-2)}\left[\alpha_{m,n}\,\mathfrak{h}_{-1}(m,n)+\beta_m\,\mathfrak{h}_2(m,n)+\gamma_{m,n}\,\mathfrak{h}_0(m,n)\right]$$

$$\mathfrak{h}_a(m,n) = {}_3F_2\left(\begin{matrix} a-m\,,\ a-m-\frac{n+1}{2}\,,\ a-m-\frac{n}{2} \\ \frac{1}{2}+a-m\,,\ a-m-n \end{matrix}\,\middle|\,1\right)$$

similar to surface defects in 6d (2,0) theory with the same ${}_3F_2$ functions

- Position space

$$\mathcal{H}^{(2)}_{22}(z,\bar{z}) = \widehat{\Delta}^{(4)}\mathcal{P}^{(2)}(z,\bar{z}) + \frac{1}{2}\mathcal{H}^{(1)}(z,\bar{z})$$

$$\mathcal{Q}_1 = \log V\ \phi^{(1)}(z,\bar{z})\,,\quad \mathcal{Q}_{3,4} = \log U\ \log V\,,\ \log^2 V\,,\quad \mathcal{Q}_7 = 1\,,$$
$$\mathcal{Q}_2 = \phi^{(1)}(x,\bar{x})\,,\quad \mathcal{Q}_{5,6} = \log U\,,\ \log V\,,\quad \mathcal{Q}_8 = \mathrm{Li}_2(V) - \frac{1}{4}\log^2 V + \log(1-V)\log V - \frac{\pi^2}{6}\,,$$

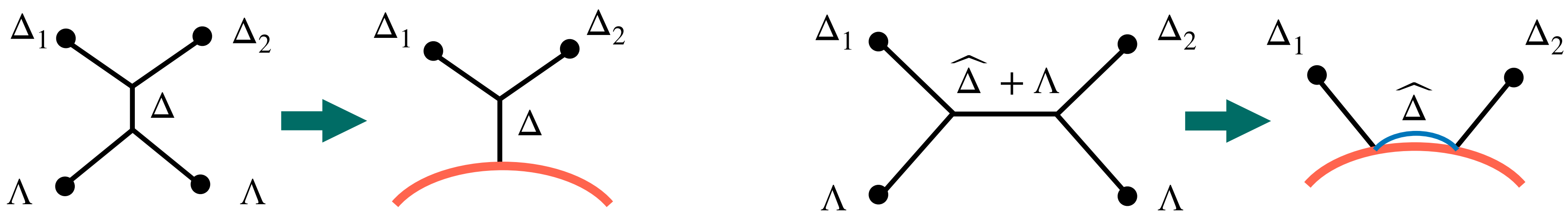
new basis function, "defect 1-loop diagram"

0d defect is a natural language for general LLHH

Heavy limit of LLHH: $\langle \mathcal{O}_{\Delta_1}\mathcal{O}_{\Delta_2}\mathcal{O}_{\Lambda}\mathcal{O}_{\Lambda}\rangle \qquad \Lambda \to \infty$

Not: at correlator level, due to order of limit issues $\qquad \langle k_1\, k_2\, p\, p\rangle\big|_{p\to\infty} \not\approx \langle\langle k_1\, k_2\rangle\rangle$

But: at the level of conformal blocks



s-channel 4pt conformal block | bulk-channel conformal block | t/u-channel 4pt conformal block | defect-channel conformal block

Kinematic and nonperturbative: in the infinitely **heavy** limit, all LLHH correlators admit a defect conformal block decomposition.

Next step: defect version of "analytic functionals" $\Rightarrow$ nonperturbative sum rules for LLHH

Mazac Paulos '18, Mazac Rastelli XZ '18 '19, Kaviraj Paulos '18, Caron-Huot Mazac Rastelli Simmons-Duffin '20

(equivalent to Mellin-Polyakov bootstrap and the dispersion relation)

Gopakumar Kaviraj, Sen, Sinha '16 $\qquad$ Caron-Huot '17, Carmi Caron-Huot '19

Some future directions

- Non-maximal and dual giant gravitons, *in progress* Chen Huang Jiang XZ

  Does the hidden conformal structure persist?

  Large charge of limit of the dual giant graviton is related to Coulomb branch.

- Stringy corrections: $1/R$ expansion of AdS Veneziano amplitude, *in progress* Alday Virally XZ

- Other backgrounds, e.g., $\mathrm{AdS}_7 \times \mathrm{S}^4$, *to appear* Chen Jiang Liu XZ

- More giant graviton operators.

- Analytic bootstrap for LLHH.