# A Bootstrap Study of Confinement in AdS

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# Introduction

Based on the following work:

- Lorenzo Di Pietro, **Stefanos Robert Kousvos**, Marco Meineri, Alessandro Piazza, Marco Serone and Alessandro Vichi **2512.00150 [hep-th]**

# Outline

- Introduction and Motivation: Confinement, QFT in AdS, the Conformal Bootstrap
- The Three Scenarios
- The Bootstrap Set-up
- Results
- Conclusion and Outlook

# Confinement

We would like to understand Confinement, however the fact that the IR is strongly coupled has made this historically hard. Today we will place our QFT of interest ($4D\ SU(n)$ Yang-Mills) in AdS as a form of regulation.

- Contrary to the lattice, this is a continuum quantum field theory
- In the large radius limit, $L \to \infty$, $ds^2 = \frac{L^2}{z^2}(dz^2 + \delta_{ij}dx^i dx^j)$, we expect to recover flat space physics.
- The limit $L \to 0$ is perturbative.
- The boundary CFT can be treated with the conformal bootstrap bridging the perturbative limit with flat space.

Note: This is **not AdS/CFT!** In the boundary CFT there is **no conserved stress tensor**.

# QFT (Yang-Mills) in AdS

Let us discuss the weak ($L\Lambda_{YM} << 1$) and strong ($L\Lambda_{YM} >> 1$) coupling limits.

**At weak coupling (small radius)**: We can identify two boundary conditions for the gauge field $A^a_\mu(z, x)$.

- Dirichlet: $A^a_i(z, x) \to zJ^a_i(x)$, the group acts as a global symmetry on the boundary
- Neumann: $A^a_i(z, x) \to a^a_i(x)$, which transforms as a gauge field

The boundary CFT related to the **Dirichlet** boundary condition **is a GFV theory generated by products of conserved global symmetry currents** $J^a_i(x)$.

**At strong coupling (large radius)**: The boundary spectrum should reproduce asymptotic scattering states. $\Delta_i \sim m_i L$. These shouldn't be charged.

**At intermediate radii**: The Dirichlet boundary condition would lead to coloured states, hence we would like it to disappear before reaching flat space. It would thus be nice to **start from the weak coupling limit**, where we know the boundary conditions and their spectra, and show via the conformal bootstrap (or other tools) that only Neumann survives to flat space.

**Comment**: Away from $L \to 0$, a charged Lorentz scalar (in Dirichlet) could become marginally relevant, e.g. $O_R \sim J^2$ ($\Delta \to 4$ as $L \to 0$), adding this to the boundary action would lead to a new boundary CFT. This is analogous to generating a new CFT via (conformal) perturbation theory by adding a marginally relevant operator.

$$\delta S \sim \int d^3x O_R \tag{1}$$

# Moving away from small radius

Let us now discuss the proposals for **what might happen to Dirichlet as we approach flat space** [Aharony et al., 2013] [Copetti et al., 2024].

**Scenario 1**: "Decoupling", here $J_i^a$ decouples from the spectrum by becoming a null state.

$$\langle J_i^a J_j^b \rangle \sim C_J(L\Lambda_{YM}) \tag{2}$$

with

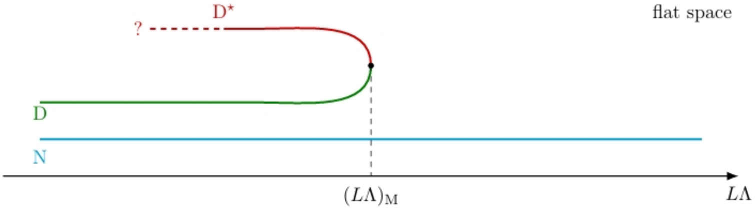
$$C_J(L\Lambda_{YM}) \to 0 \tag{3}$$

for some $L \to L^\star$.

**Scenario 2**: "Merger", the Dirichlet boundary CFT merges with another boundary CFT, which we will call $D^\star$. **The smoking gun in this case would be a scalar singlet approaching marginality** $\Delta_S \to d = 3$ for some $L \to L^\star$ (since we have a three-dimensional boundary).

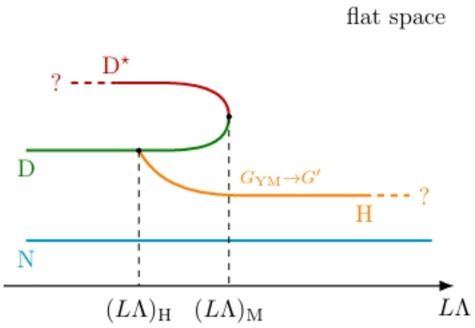
**Scenario 3**: "Higgsing", before $\Delta_S \to 3$, a charged operator becomes marginally relevant $\Delta_R \to 3^-$ giving birth to a new boundary CFT.

# Pictures: Merging



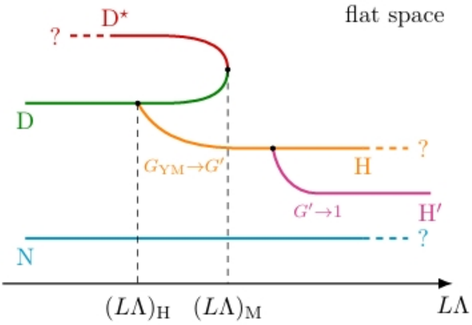

Smoking gun: $\Delta_S \to 3$

# Pictures: Higgsing




Smoking gun: $\Delta_R \to 3^-$

# Pictures: Higgsing (More)




Smoking gun: $\Delta_R \to 3^-$

# The Machinery: The Numerical Conformal Bootstrap

The tool with which we will try to interpolate away from the weak coupling limit is the **numerical conformal bootstrap**, with which we will **study the boundary CFT**.

# Sum Rules

Remember that $J_i^a$ is the **operator that generates the spectrum of the Dirichlet boundary CFT**.

The crossing equation for $\langle J_i^a J_j^b J_k^c J_l^d \rangle$ leads to the following sum rule:

$$\vec{\mathcal{V}}_1 + \frac{1}{C_J}\theta \cdot \vec{\mathcal{V}}_J \cdot \theta + \sum_O \lambda_{JJO}^T \cdot \vec{\mathcal{V}}_O \cdot \lambda_{JJO} = 0 \quad \theta = \begin{pmatrix} 3 - 5a_{JJJ} \\ -a_{JJJ} \end{pmatrix} \quad (4)$$

Then we can ask for which values of $C_J$ as a function of $a_{JJJ}$ (**Decoupling**) and of the $\Delta$'s (**Merging vs Higgsing**) this sum rule can be satisfied.

Correlators of currents have been bootstrapped in [Dymarsky et al., 2019] for Abelian, and [He et al., 2024] for Non Abelian. We are going to use the same set-up as the second (implemented in simpleboot).

# Results

Since in the numerical bootstrap **we need to plug in explicitly the value of** $n$ of $SU(n)$ we are considering, I will show results for $n = 2, 3$. Perturbation theory for the anomalous dimensions of operators of Yang-Mills in AdS4 has been carried out in [Ciccone et al., 2024].

Let us start by seeing what **bounds** can be placed **on** $C_J$ **from the conformal bootstrap**:

# Decoupling $SU(2)$

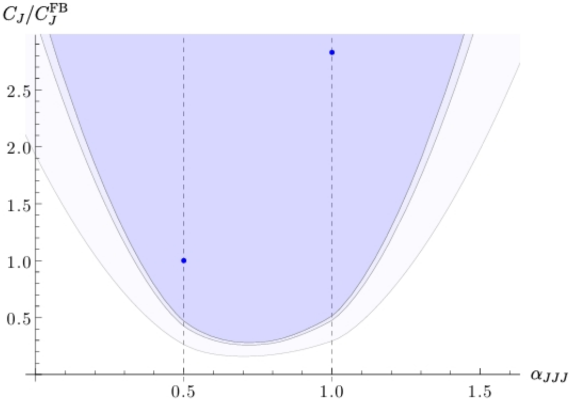



Figure 3: Lower bound on $C_J$ in the case of $SU(2)$ as a function of $\alpha_{JJJ}$. The shaded area is allowed, darker colour corresponding to higher $\Lambda$, we have $\Lambda = 11, 19$ and $23$ respectively. The dashed vertical lines and the blue dots at $\alpha_{JJJ} = 1/2, 1$ denote the free boson and free fermion theory, respectively. We see that $C_J = 0$ is excluded for any choice of $\alpha_{JJJ}$.

# Decoupling $SU(3)$

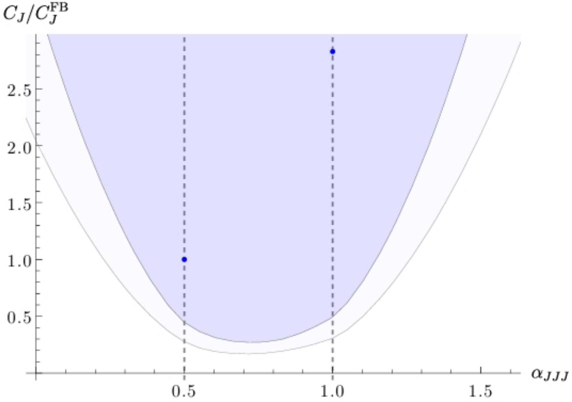



Figure 4: Lower bound on $C_J$ in the case of $SU(3)$ as a function of $\alpha_{JJJ}$. The shaded area is allowed, the darker area corresponding to $\Lambda = 19$ (and the rest to $\Lambda = 11$). The dashed vertical lines and the blue dots at $\alpha_{JJJ} = 1/2, 1$ denote the free boson and free fermion theory, respectively. As in the $SU(2)$ case, $C_J = 0$ is excluded for any choice of $\alpha_{JJJ}$.

# Decoupling

We thus see that $C_J$ is bounded to be **strictly positive** for any value of $a_{JJJ}$, $C_J > 0$. **Hence, we rigorously exclude the decoupling scenario.**

Let us now see what we can say about the "merging" and "Higgsing" scenarios. For these we will concentrate on the values of the **three lightest Lorentz scalars**: $(O_s, O_a, O_{s\bar{s}}) \sim J^2$ ("smoking guns"). The subscripts $s$, $a$ and $s\bar{s}$ denote the symmetry irrep of each operator. (For $SU(2)$ $O_a$ is much heavier.)

Hence, we will scan over the scaling dimensions of these three operators, to see **which and when is allowed to become marginal**.

# Bounds without assuming anything about the spectrum

Let us see what bounds can be obtained on the operators of interest, **first, without assuming anything on the spectrum.** For $SU(2)$ we have:

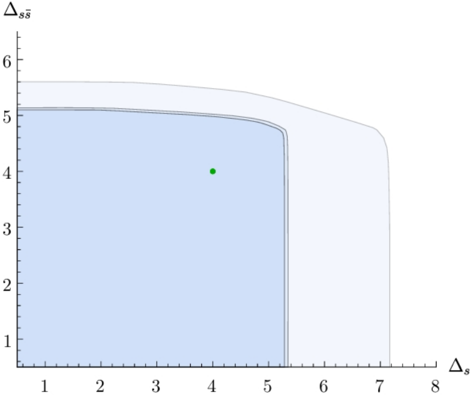



Figure 7: Gap maximisation bound for $(\Delta_s, \Delta_{s\bar{s}})$ in the case of $SU(2)$. The shaded area is allowed, darker colour means higher $\Lambda = 11, 19, 23$. The green dot at $(\Delta_s, \Delta_{s\bar{s}}) = (4, 4)$ denotes the generalised free vector current theory. The precise spectrum assumptions are reported in (D.3).

# $SU(3)$ bounds

Whereas for $SU(3)$ we get:

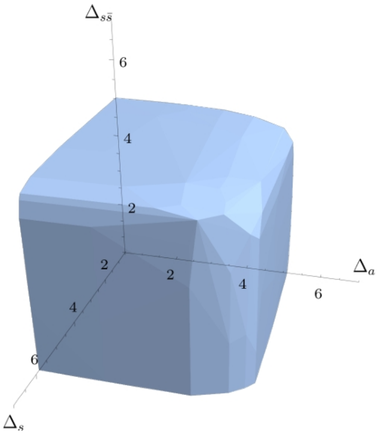



Figure 8: Gap maximisation bound for $(\Delta_s, \Delta_a, \Delta_{s\bar{s}})$ in the case of $SU(3)$ for $\Lambda = 23$. The shaded coloured region is allowed. The precise spectrum assumptions are reported in (D.4).

# $SU(3)$ bounds

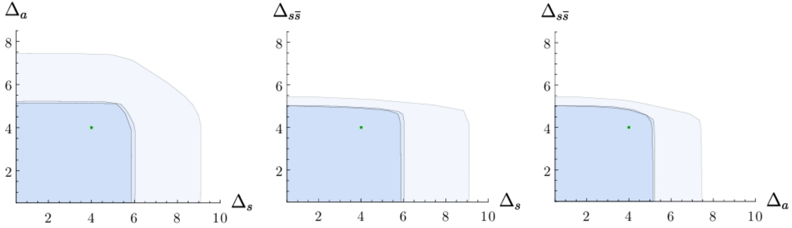



Figure 9: Projections of the gap maximisation bound in Figure 8 for $(\Delta_s, \Delta_a, \Delta_{s\bar{s}})$ in the case of $SU(3)$, to the three planes $(\Delta_s, \Delta_a)$, $(\Delta_s, \Delta_{s\bar{s}})$ and $(\Delta_a, \Delta_{s\bar{s}})$ respectively. The shaded area is allowed; darker colours indicate higher values of $\Lambda = 11, 19, 23$. The green dot at $(\Delta_s, \Delta_a, \Delta_{s\bar{s}}) = (4, 4, 4)$ denotes the generalised free vector current theory. The precise spectrum assumptions are reported in (D.4).

# Less General Bounds

It is clear we cannot say much with these generic bounds. Since we do not assume anything on the spectrum (other than unitarity), **they are too general, because they need to be satisfied by all theories** with an $SU(n)$ current.

In order to make the bootstrap say something specifically about Yang-Mills in AdS, we will **start injecting information about the theory's spectrum**.

Since the **spectrum is a deformation of the GFV** we will demand that the spectrum of the theory we are studying is that of a GFV up to a tolerance $\delta$. The tolerance will account for anomalous dimensions as the flow progresses. We **expect $\delta$ to behave as an indirect probe of the AdS radius** (bigger $\delta$ allows for bigger radii).

# GFV-like Assumptions

We will thus assume that our theory has the following spectrum:

$$\Delta_{l,p,r} \in [\Delta_{GFV} - \delta/l, \Delta_{GFV} + \delta/l] \cup \left[\Delta'_{GFV} - \delta, \infty\right)$$

where the operators corresponding to $\Delta_{GFV}$ are of the double-twist form $[J, J]_{n,l}$.

For the three "smoking gun" operators we will assume

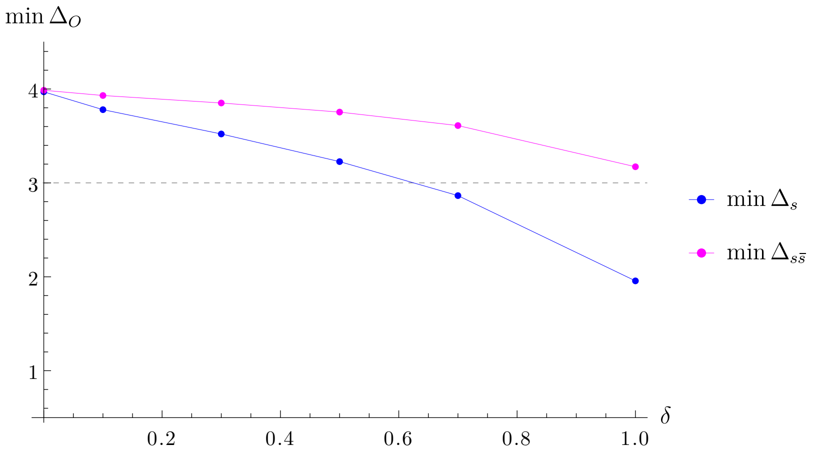
$$\Delta_{0,+,s} \in \Delta_s \cup \left[\Delta'_{GFV} - \delta, \infty\right)$$

$$\Delta_{0,+,a} \in \Delta_a \cup \left[\Delta'_{GFV} - \delta, \infty\right)$$

$$\Delta_{0,+,s\bar{s}} \in \Delta_{s\bar{s}} \cup \left[\Delta'_{GFV} - \delta, \infty\right)$$

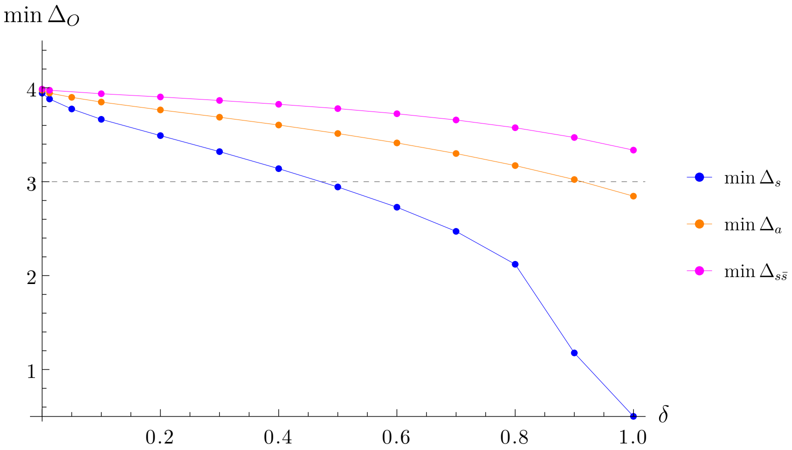
Using the "Navigator" method [Reehorst et al., 2021], we can scan over the vector $(\Delta_s, \Delta_a, \Delta_{s\bar{s}}, a_{JJJ})$, for $\delta = 0.0125, 0.05, 0.1, 0.2, ..., 1$.

# $SU(2)$ Results



$\min \Delta_O$
4
3
2
1
0.2
0.4
0.6
0.8
1.0
$\delta$
$\min \Delta_s$
$\min \Delta_{s\bar{s}}$

# $SU(3)$ Results




These figures seem to provide evidence in favour of (but not prove) the merging scenario.

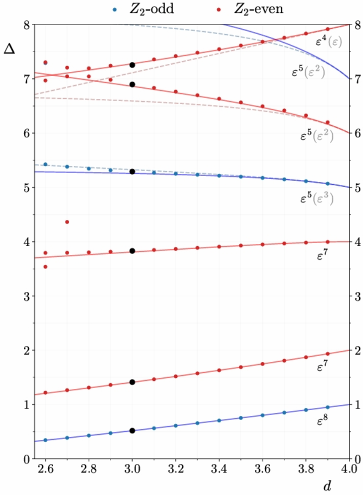
# Conclusions and Outlook

- We saw that **confinement** (in $\mathrm{AdS}_4$, i.e. the "real deal", modulo matter) can be **quantitatively addressed using existing conformal bootstrap technology**.
- We **rigorously excluded** the decoupling scenario for the Dirichlet boundary condition.
- We gave evidence in favour of "merging".

Future directions:

- Quantitative understanding of $D^\star$.
- More complicated boundary correlator systems may allow to strictly exclude $\Delta_a$ and $\Delta_{s\bar{s}}$ ever becoming marginal.
- Get a more precise handle on $L$ in the bootstrap, instead of indirect probes like $\delta$

Taken from [Henriksson, Herzog, Kousvos, Roosmale-Nepveu, 2025] and [Henriksson, Kousvos, Reehorst, 2021]



# A Hope

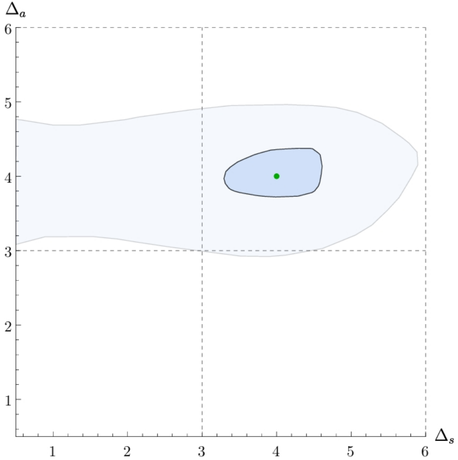



Figure 10: Allowed region for $(\Delta_s, \Delta_a)$ in the case of $SU(3)$ assuming $\Delta_{s'} \geq 6$, $\Delta_{a'} \geq 6$ and $\Delta_{T'_{\mu\nu}} \geq 4$ (see (D.5)). The shaded areas correspond to $\Lambda = 11$ (lighter) and $\Lambda = 19$ (darker). The green dot $(\Delta_s, \Delta_a) = (4, 4)$ corresponds to the GFV. The dashed lines denote either marginality or the value of the gap imposed on the sub-leading scalar operators.

# Acknowledgements

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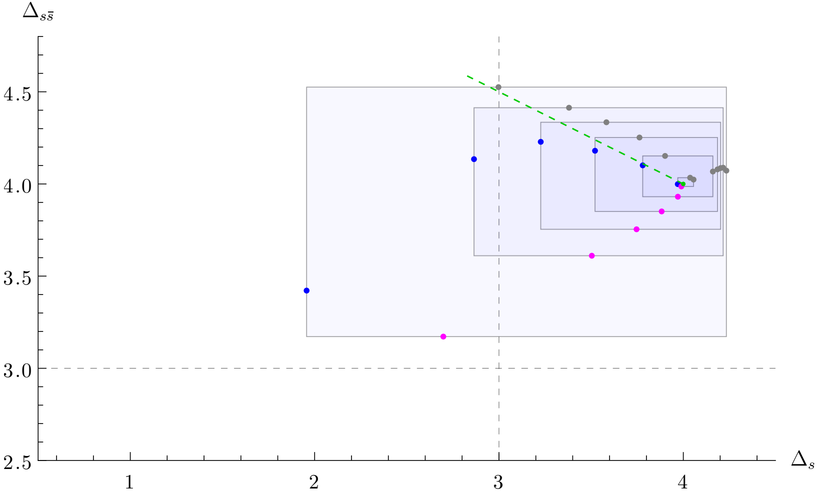
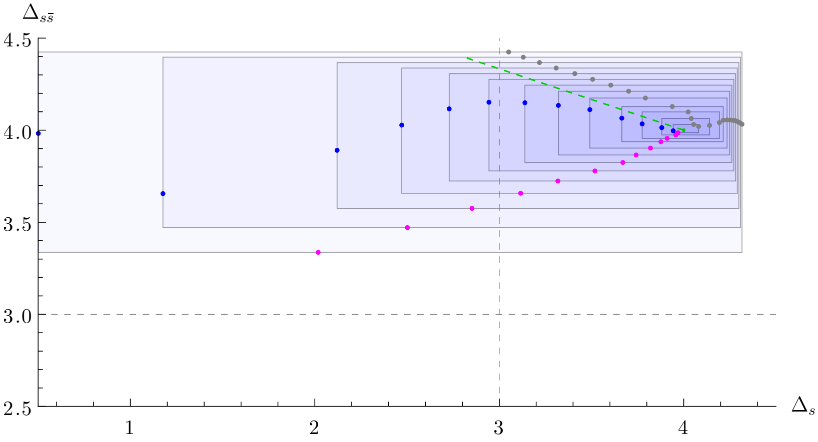
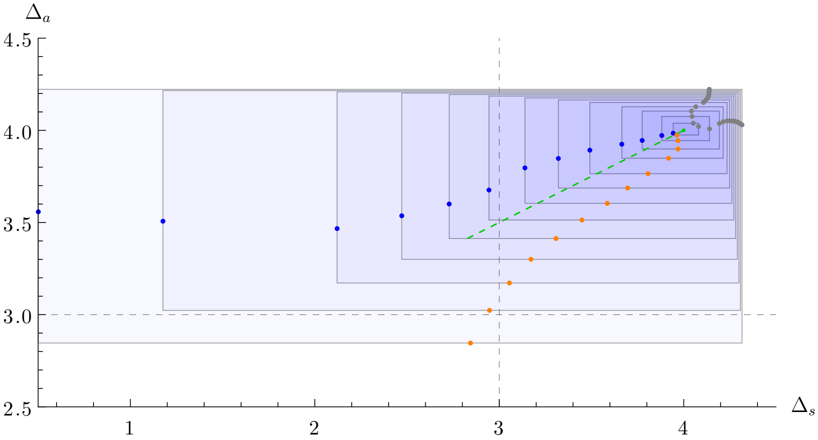
# Additional Bounds $SU(2)$



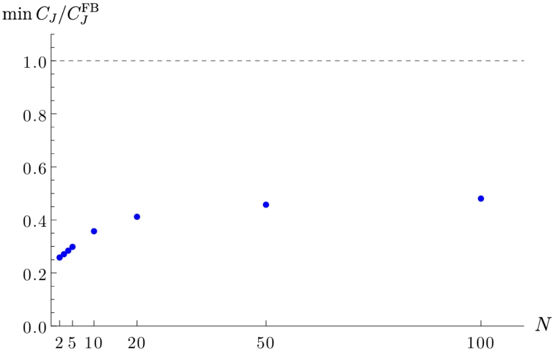
# Additional Bounds $SU(3)$



# Additional Bounds $SU(3)$



# $C_J$ Minima for various $n$




| $N$ | $\alpha^*_{JJJ}$ |
|---|---|
| 2 | 0.717 |
| 3 | 0.727 |
| 4 | 0.736 |
| 5 | 0.743 |
| 10 | 0.744 |
| 20 | 0.731 |
| 50 | 0.697 |
| 100 | 0.673 |

Figure 6: Absolute lower bound of $C_J$ as a function of $N$ for $SU(N)$ non-Abelian conserved currents. The bound is obtained by scanning over $\alpha_{JJJ}$. The minimum is reached for $\alpha^*_{JJJ}$, whose value is reported on the table to the right of the plot. The bound was obtained at $\Lambda = 19$.