Bootstrapping massive *spin-2* amplitudes

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Why study massive spin-2's?

- Appear in compactification of higher-dimensional general relativity (Kaluza-Klein).
  $\rightarrow$ do all massive spin-2's (w/o spin>2) arise this way?

- Can an *isolated* massive spin-2 exist?
  Special ghost-free theory (dRGT) claimed to be non-positive unless cutoff $\Lambda \lesssim \mathcal{O}(30m)$ $\rightarrow$quantify?
  [Bellazzini,Isabella,Ricossa& Riva '23]

- Technology: similar to off-shell stress-tensors $T^{\mu\nu}(q)$ in any QFT $\rightarrow$possible future applications?

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- Spin is good !!

A long history…

- Fierz-Pauli ('39): linearized graviton with 5 modes
- 70's: vDZ discontinuity as $m \to 0$, Vainshtein screening mechanism, Boulware-Deser ghosts
- Basic issue like SM without Higgs, longitudinal modes $\sim p^\mu/m$ $$\frac{1}{p^2+m^2}\left(\eta^{\mu\nu}+\frac{p^\mu p^\nu}{m^2}\right)$$

  For gravity the issue is squared: $\mathscr{M}_{LLLL} \supset \frac{G_N stu}{m^4}$

  EFT power-counting seems OK below $\Lambda \lesssim (M_{\rm pl} m)^{2/3}$

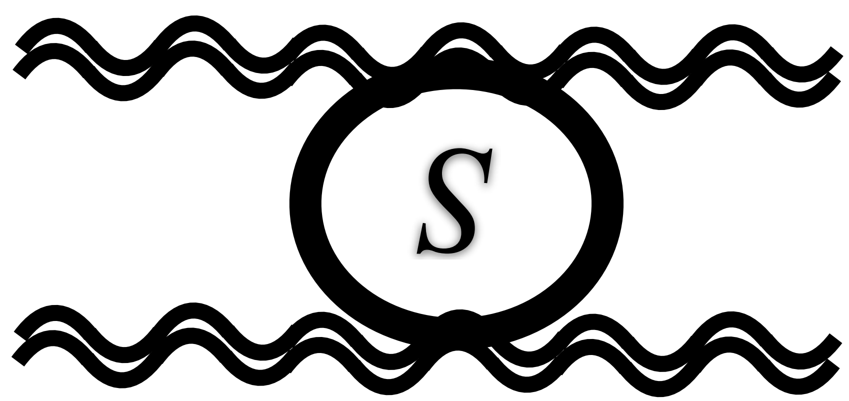
  [Arkani-Hamed& Schwartz '02, …]
- dRGT: explicit theory with $\sim E^6$ behavior, no ghosts

  [deRham, Gabadadze& Tolley '10]

Which effective theories of massive spin-2 particles are consistent with causality and unitarity?

What we do

- We scatter spin-2 particles of mass m^{KK} in a flat Minkowski background



- Assume numerical hierarchy: $m^{\mathrm{KK}} \ll M^{\mathrm{higher-spin}}$
- Test whether a causal UV completion can exist by imposing analyticity and positivity of $2 \to 2$ amplitude
- Preliminary conclusions: a *tower* of spin-2's required; also a light scalar ('radion'?).

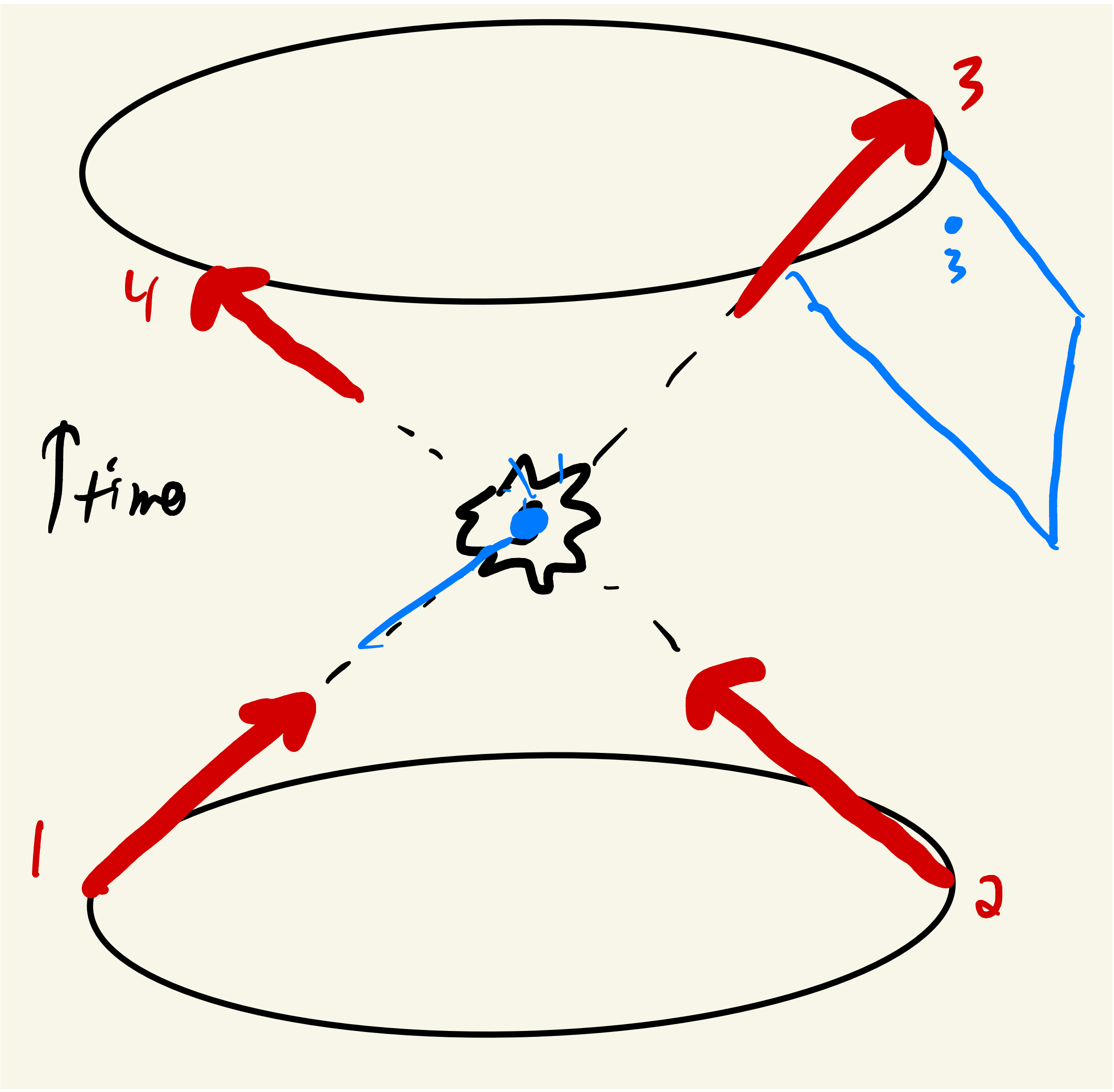
Outline

- Review of dual S-matrix bootstrap setup
  -massless graviton constraints

- Massive spin-2's
  -Technical challenges & fixed-angle analyses
  -New: Superconvergence & dual amplitudes

- Preliminary bounds on light scalar& tensor masses and couplings

Causality for 2->2 scattering



- Fixed-angle scattering
  not directly constrained
  [think hard spheres; cf Mizera 2306.05395]

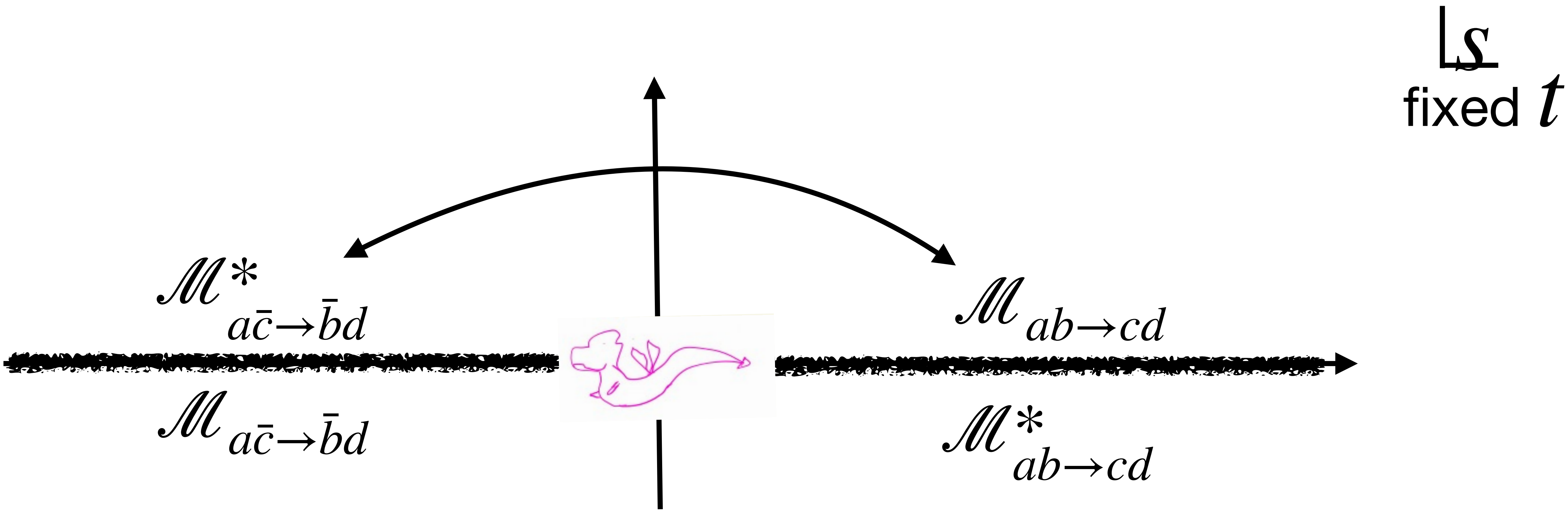
- Key is ultrarelativistic, near-forward kinematics:

  $s \to \infty$
  t or b fixed
  "Regge limit"

- Related to crossing:

particle $1 \to 3 \simeq$ antiparticle $3 \to 1$

Causality $\simeq$ analyticity in upper-half-plane



Unitarity bounds amplitudes:

$$\langle \text{out} \,|\, \text{in} \rangle \leq |\langle \text{out} \,|\, \text{out} \rangle \langle \text{in} \,|\, \text{in} \rangle |^{1/2}$$

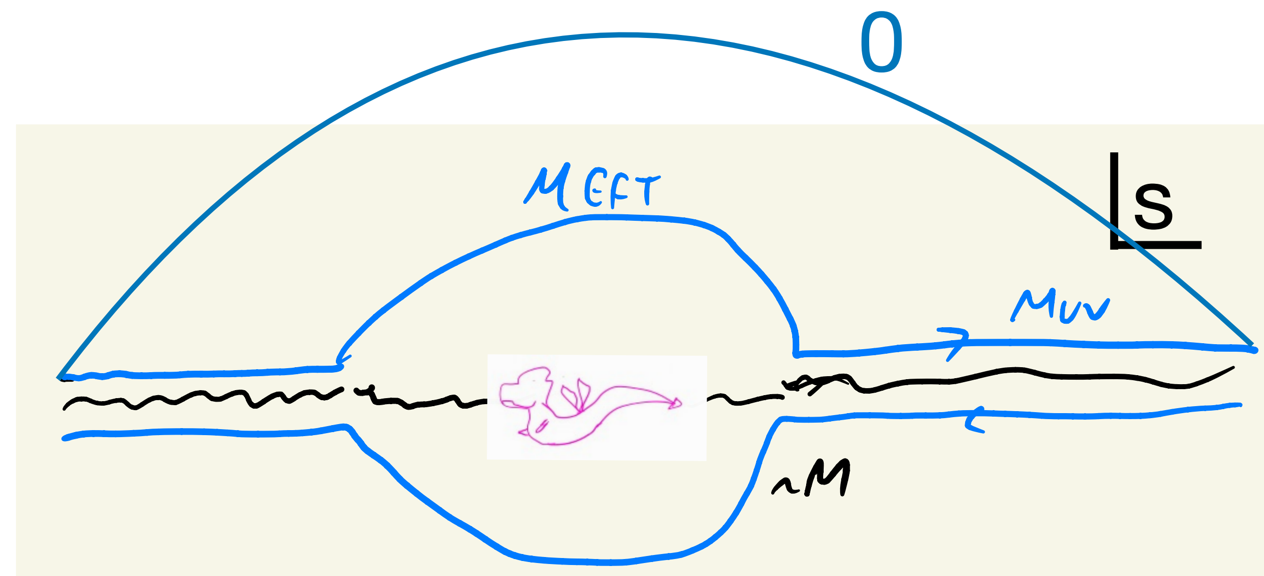
$(S^\dagger S \leq 1)$

$\Rightarrow$ "Froissart Lemma": $\displaystyle\int_t \psi(t)\mathcal{M}(s,t) \leq |s| \times \text{const}$

through upper-half-plane
(mass gap not needed)

[SCH, Li, Parra-Martinez& Simmons-Duffin '22;
Häring+Zhiboedov '22; SCH 2025 TASI (recorded)]

Dispersive sum rules
à la Kramers-Kronig



Relation between IR and UV:

$$0 = \oint_{|s|=\infty} ds \frac{M_\psi(s)}{s^{k+1}} \quad \Rightarrow \quad \oint_{\text{EFT}} (\cdots) = \int_{\text{heavy}} \frac{ds}{s^{k+1}} \mathrm{Im} M_\psi(s) \qquad (k > 1)$$

$$\mathrm{Im} M_\psi(s) = \sum_J |c_J|^2 P_J(1 - 2p^2/s)_\psi$$

positive

(Im M = sum of Legendre's with positive coefficients)

(low-energy couplings) = (sums of high-energy unknowns)

Crossing-symmetric disp. rel.

- Fix: $\dfrac{1}{s}+\dfrac{1}{t}+\dfrac{1}{u}\equiv\dfrac{-1}{p^2}$

[Auberson& Khuri '72]
[Sinha& Zahed '20], …

- p=momentum transfer in Regge limit
s-plane at fixed-p unites s, t, u channels

- Automatically isolates *finite* sets of couplings in tree-level EFTs

- Integrating against $\psi(p)$ avoids forward limit expansions: stable against loop corrections

[Beadle, Isabella, Perrone, Ricossa, Riva & Serra '25; Chang& Parra-Martinez '25, …

- Note: $\mathcal{M}(s = \text{large}, t = \text{small})$ is IR-sensitive!

[SCH & Tokuda '24]

Example: single scalar

Assume massless& weakly coupled for $s, t, u \ll M^2$

[SCH& van Duong '20]
[Tolley, Wang& Zhou '20]
[Bellazzini,Miró,Rattazzi, Riembau& Riva'20] $\cdots$

$$\mathcal{M}_{\text{low}}(s,t) = -\,g^2\left[\frac{1}{s}+\frac{1}{t}+\frac{1}{u}\right] - \lambda + 8\pi G(\frac{st}{u} + \text{perm})$$

$$(\partial\phi)^4 \rightarrow + g_2(s^2+t^2+u^2) + g_3(stu) + g_4(s^2+t^2+u^2)^2 + \quad \cdots$$

$$k=2: \quad \frac{8\pi G}{p^2} + 2g_2 + p^2 g_3 = \left\langle \frac{2m^2+3p^2}{m^6}\mathcal{P}_J\left(\frac{\sqrt{m^2-3p^2}}{\sqrt{m^2+p^2}}\right)\right\rangle$$

$$k=4: \quad 4g_4 + 2p^2 g_5 + p^4 g_6 = \left\langle \frac{(m^2+p^2)(2m^2+3p^2)}{m^{12}}\mathcal{P}_J\left(\frac{\sqrt{m^2-3p^2}}{\sqrt{m^2+p^2}}\right)\right\rangle$$

[to be integrated against $\psi(p)$]

[cf Pasiecznik '25]

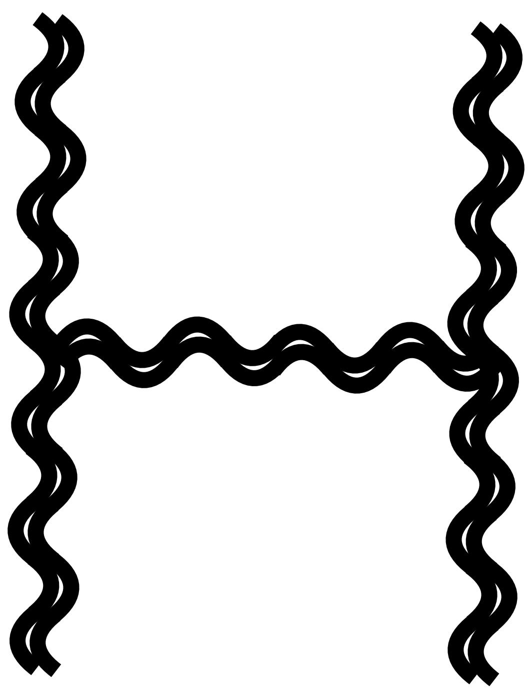
Sums over heavy states with $m > M$

In general, any EFT term with fixed-angle growth > E^4 (eg. stu or s^3) is bounded by $1/M^{\#}$ times a combin. of g_2/M^2 or G_N/M^4

[Bellazzini,Pomarol,Romano & Sciotti '25]

Massless gravitons

Can constrain modifications to gravity:



scalar with $\phi\mathrm{Riem}^2$/dCS

$$M^{+--+} = [14]^4\langle 23\rangle^4 \times 8\pi G\left[\frac{1}{stu} + \frac{|g_3|^2 su}{4t} + \frac{|g_s|^2}{-t} + g_4 + g_5 t + \dots\right]$$

Einstein!

Riem^3 at vertices

contacts: Riem^4 and derivatives

Thought experiment: scattering of gravity waves at frequencies E $\ll M_{\rm pl}$ and momentum transfer $p \lesssim E$

Superconvergence “spin is good”

$$M^{+--+} = \underbrace{[14]^4\langle 23\rangle^4}_{\sim t^4} \times \underbrace{f(s,u)}_{\leq Cs^{1-4}}$$ @ fixed-u

Fixed-u sum rule for $|h_1 - h_3| > 2$ kill all contacts!

$$\frac{G_N}{p^2} + \ldots = \sum_{J,m\geq M_{\text{higher-spin}}} c^2_{J,m} P_J(1-2p^2/m^2),$$

$$\oint s ds f(s,u) = 0$$

$$G_N|g_s|^2 = \sum_{J,m\geq M_{\text{higher-spin}}} \frac{\text{similar}}{m^4}$$

1. Gravity is attractive!!!!
2. Any correction bounded by $G_N/M^{\#}_{\text{higher-spin}}$
3. Graviton-graviton scattering must be weak below M_{pl}

Constraints on GR modifications

[Camanho,Edelstein,Maldacena& Zhiboedov '14]
[SCH, Li, Parra-Martinez& Simmons-Duffin '22][2]

$$S = \int \frac{\sqrt{-g}}{G_N}(R + \lambda \mathrm{Riem}^2 + \ldots + (\partial\phi)^2 + g\phi\mathrm{Riem}^2 + \ldots)$$

- In d=5 flat space: $|\lambda_{\mathrm{Riem}^2}| \leq \approx 11.5/M^2_{\mathrm{higher-spin}}$

$\leftrightarrow$ In $\mathrm{AdS}_5/\mathrm{CFT}_4$: $|a - c|/c \leq \approx 23/(\Delta^{\mathrm{gap}}_{\mathrm{higher-spin}})^2$

Mass of first fundamental* particle of spin ≥ 4

*decays to two-gravitons with strength $> \# M^2 \sqrt{G_N}$

- In d=4: $\lambda_{\mathrm{Riem}^3} \leq 25/M^8_{\mathrm{higher-spin}} \log(M/\mu_{\mathrm{IR}})$

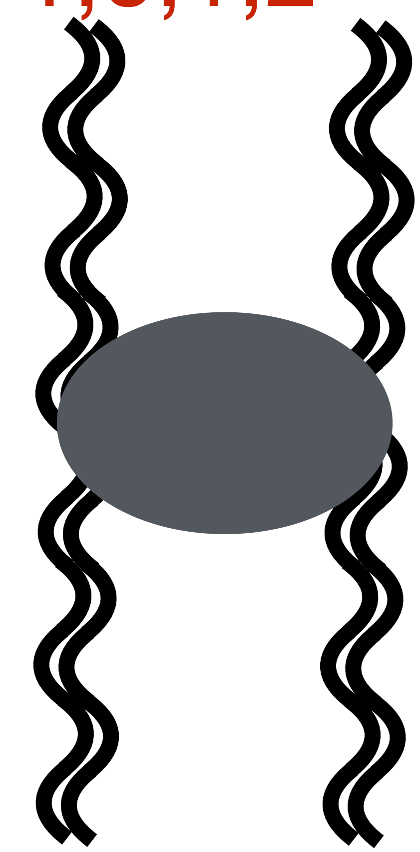
$$g_{\phi\mathrm{Riem}^2} \leq \#/M^4_{\mathrm{higher-spin}} \log(M/\mu_{\mathrm{IR}})$$

$\ldots$

Massive spin-2: the problem

- LOTS of helicity amplitudes
- All mix under crossing
- MANY EFT couplings:
  - 3pt vertices
  - 4pt vertices

good high-energy behavior



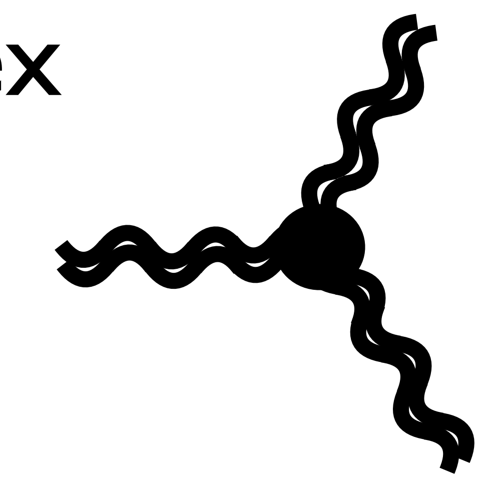
A huge mixed system? Need better way!

Lessons from previous work

Tree amplitudes with $\sim E^2$ high-energy growth at fixed angles fully characterized:

[Bonifacio & Hinterbichler '19]

- Only one tensor structure at each vertex can *potentially* lead to $\sim E^2$ at 4pt (Einstein-Hilbert!)



- Even then, $\sim E^2$ is only possible if three-point coefficients satisfy *sum rules*

Bonifacio-Hinterbichler sum rules

['19]

$\sim E^2$ growth $\Rightarrow$ constraints on 3pt-couplings to massless graviton& scalar, massive scalars, and massive spin 2's

massive tensors

Massless

$$\underline{a_2^2+4b_1^2}+\frac{1}{4}\sum_a\left(4-\frac{3m_a^2}{m_\star^2}\right)\underline{e_{5,a}^2}=0,$$

$$\underline{a_2^2+4b_1^2}-24\sum_{\mathcal{I}}\underline{c_{1,\mathcal{I}}^2}+\frac{1}{4}\sum_a\left(\frac{5m_a^2}{m_\star^2}-4\right)\frac{m_a^2}{m_\star^2}\underline{e_{5,a}^2}=0,$$

massive scalars

$$6\sum_{\mathcal{I}}\underline{m_{\mathcal{I}}^2c_{1,\mathcal{I}}^2}+\frac{1}{4}\sum_a\left(\frac{m_a^2}{m_\star^2}-4\right)\left(\frac{m_a^2}{m_\star^2}-1\right)m_a^2\underline{e_{5,a}^2}=0.$$

$\Rightarrow$Need another spin-2 particle with $m'^2 \in [4/3,4]\, m_{\text{ext}}^2$

$\Rightarrow$Need a scalar with $m_0^2 < 4/3 m_{\text{ext}}^2$

[see also Mirbabayi& Villadoro '26]

Assuming $\sim E^2$ (or E^4) fixed-angle growth eliminates any theory of an isolated spin-2 particle (including dRGT).

But is it required by causality+unitarity?

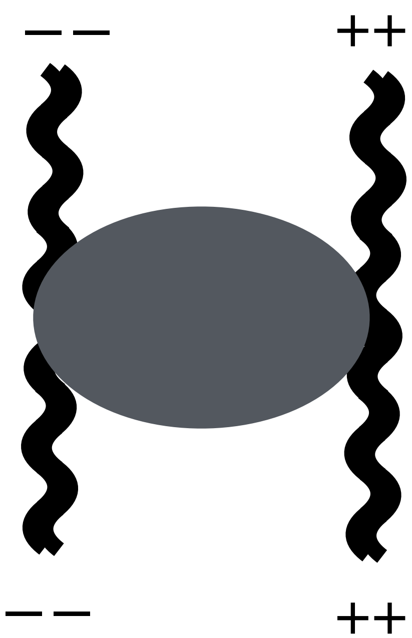
[Bellazzini,Isabella,Ricossa& Riva ’23]

[Bellazzini,Pomarol,Romano & Sciotti ’25]

We use: superconvergence relations and (new!) dual amplitudes to bound other structures, and derive B-H sum rules, *directly from unitarity* + *causality*.

Superconvergence “spin is good”

Amplitudes with helicity changes enjoy improved high-energy behavior



Recall massless example:

$$M^{+--+} = \underbrace{[14]^4\langle 23\rangle^4}_{\sim\, t^4} \times f(s,u)$$

$\leq Cs^{1-4}$

@fixed-u

Massive version: fix helicities in u-channel rest frame to improve large-s @ fixed-u.

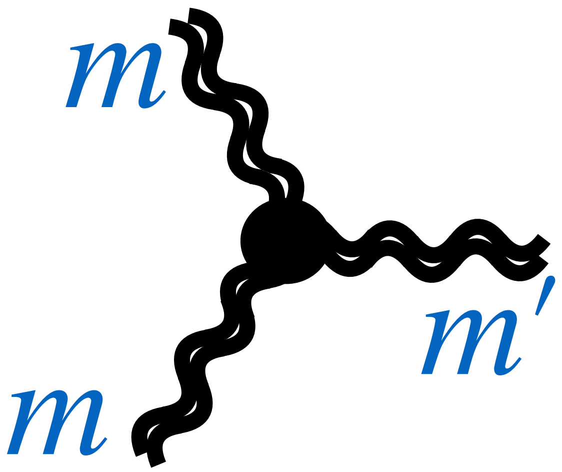
(Group-theory: Froissart-Gribov for $J_u < |h_1 - h_3|$)

Only pain is conversion between rest frames

[Hebbar, Karateev & Penedones ’20]

Fixed-u sum rules with $|h_1 - h_3| > 2$ kill all contacts! $\Rightarrow$ Cleanly constrain 3pt couplings.

Ex: parametrize 3-pt couplings to light spin 2 as



$$= c_1\,(e1\cdot p2\; e2\cdot e3 - e1\cdot e3\; e2\cdot p1 + e1\cdot e2\; e3\cdot p1)$$
$$+ c_2\; e1\cdot e2\; e1\cdot e3\; e2\cdot e3\; m[1]\; m[3]^2$$
$$+ \ldots$$
$$+ c_7\,(e1\cdot p2)^2\,(e2\cdot p1)^2\,(e3\cdot p1)^2$$
$$+ \text{parity odd}$$

Einstein-Hilbert

junk

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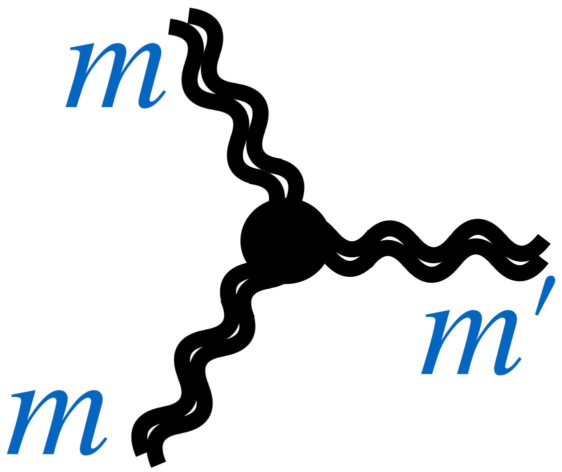

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Einstein-Hilbert

Sum rule $\mathscr{M}^{2,-2,-2,2}$ [in (13)-frame] for $|u| = p^2 \gg m^2$:

$$\frac{1}{p^2}\overbrace{\sum_{m'}|c_1(m')|^2}^{G_N^{\text{eff}}} + \ldots + \sum_{m'} p^6 |c_7(m')|^2 = \langle(\cdots)\rangle_{J>3}$$

Implies* $\sum_m |c_7|^2 < \#G_N^{\text{eff}}/M^8_{\text{higher-spin}}\cdots$ all $c_2, \ldots c_{12}$ small!

Preliminary

Like in massless case, sum rules

$$G_N^{\text{eff}} = \langle \text{positive} \rangle_{\text{heavy}}$$

imply that scattering of massive spin-2's below the Planck scale can never be strongly coupled

(even for energies $E > M^{\text{higher-spin}}$)

This justifies a weakly coupled EFT ansatz

Dual amplitudes

- Still need to study 4pt amplitudes including contacts
- Project amplitude onto specific polarizations in crossing-symmetric way by contracting with tensor:

$$f_1(s,t) = \sum_{\{e_i\}} \left((e_1 \cdot e_2)^2 (e_3 \cdot e_4)^2 + \text{perm.}\right) \mathcal{M}(\{p_i, e_i\})$$

… to which we apply crossing-symmetric disp. rel.

- Main idea: 2-3 dual amplitudes suffice, not $O(100)$!!
- 'Random' choices seem ok, but good to have one 'magic' $\mathcal{T}$ s.t. $\mathcal{T}/stu$ have strictly >0 partial waves

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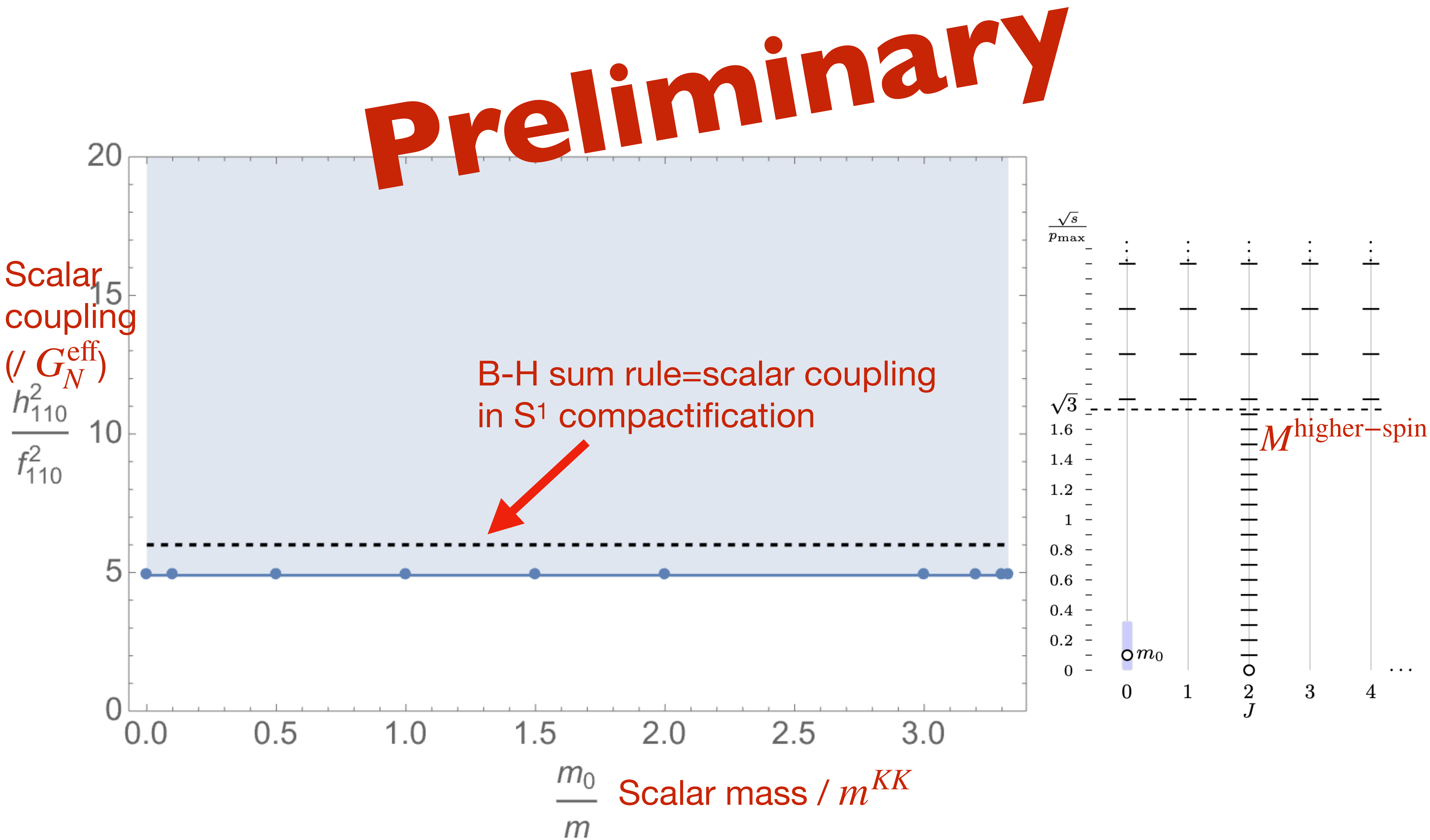
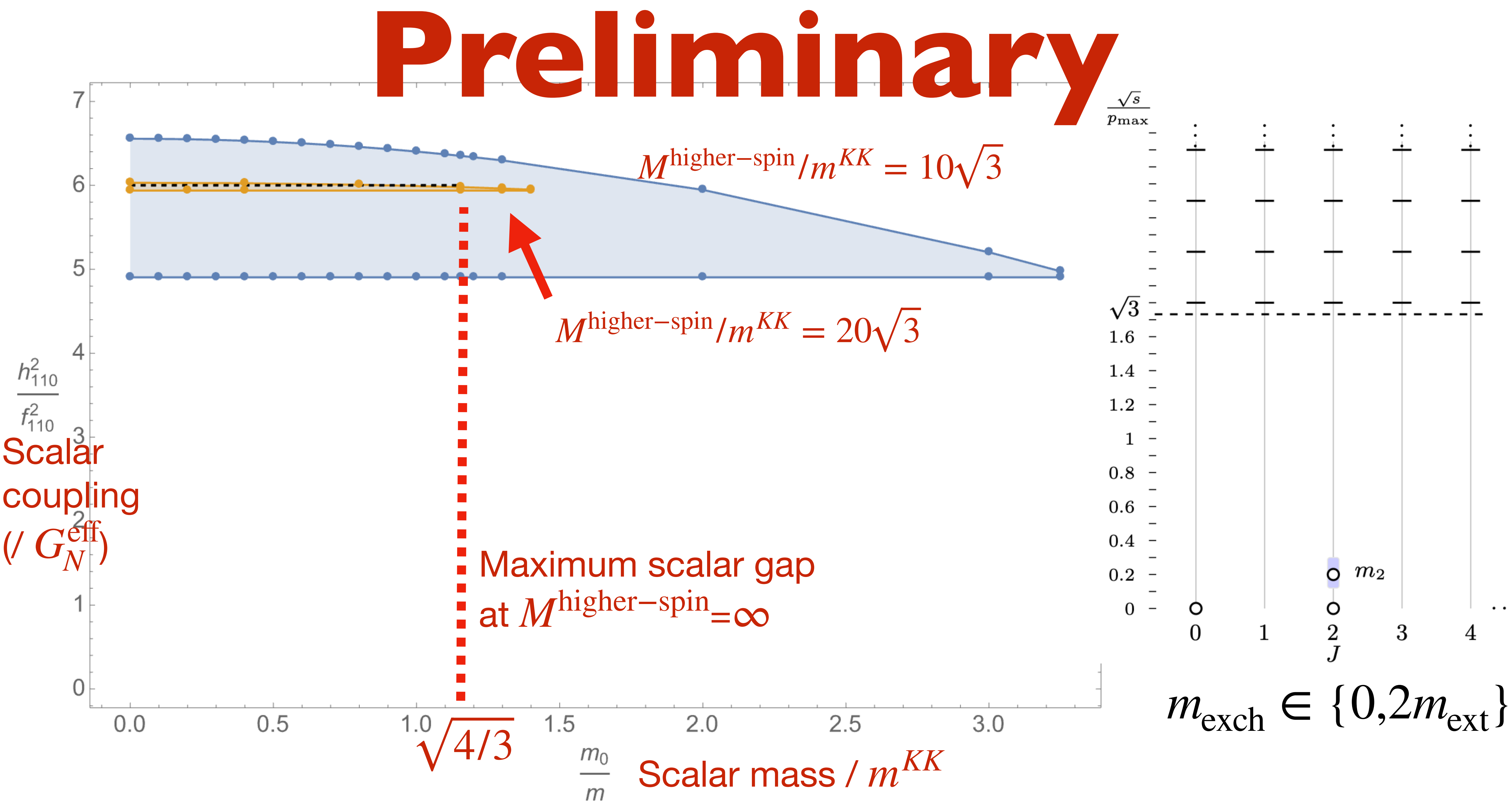
Ex: some light scalar must exist with coupling > 0



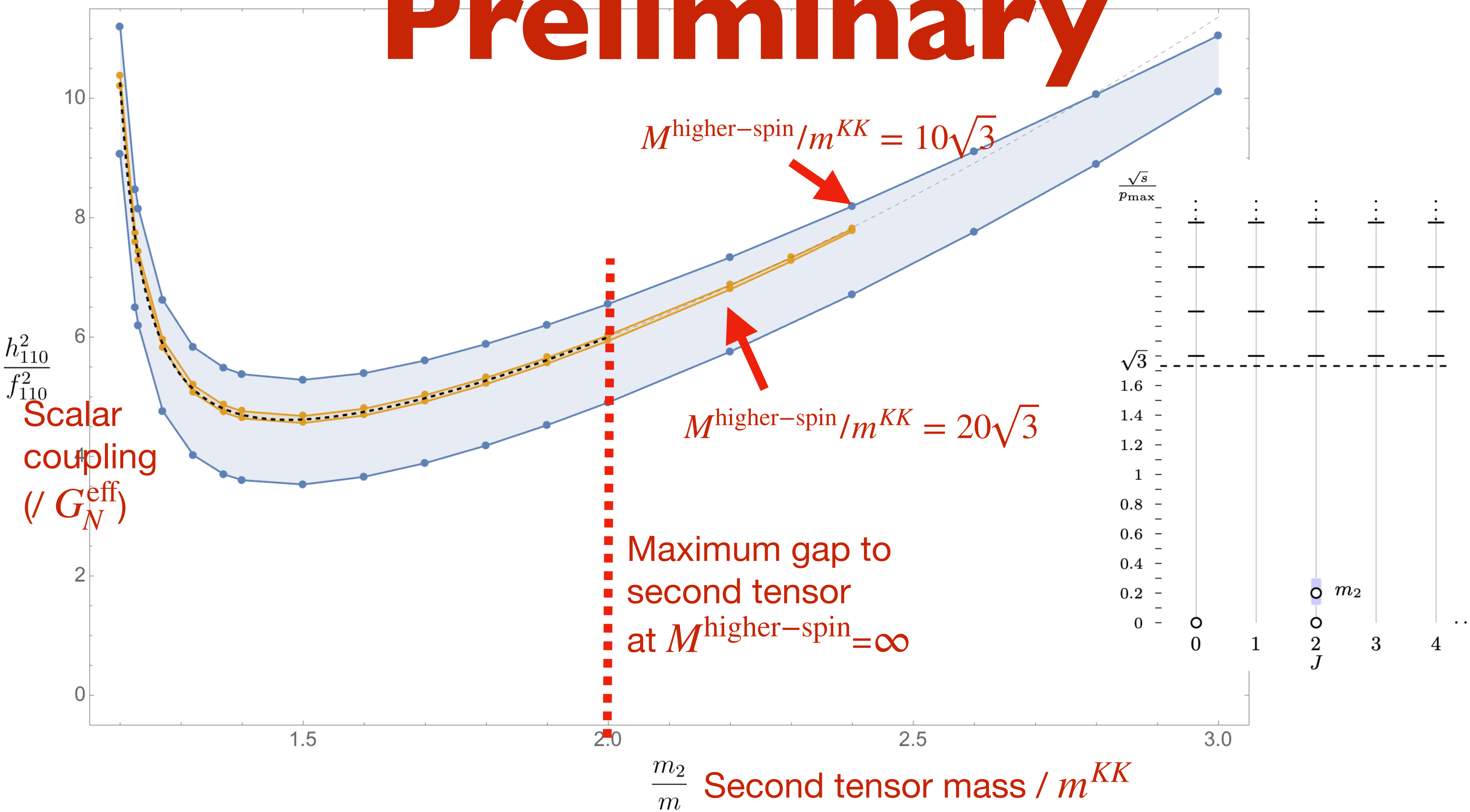
Seems to coincide with B-H sum rules+correction.

Ex: assuming S^1 KK spectrum, couplings get fixed:



Consistent with B-H sum rules with error $\sim (m/M^{\text{higher-spin}})^4$ as expected from dim. analysis

Preliminary



Other massive tensors cannot be decoupled (eg. cannot have isolated massive spin-2) if $M^{\text{higher-spin}}/m$ large enough

Summary & directions

- Studied scattering of massive spin-2 particles. Constraints previously derived by assuming $\sim E^2$ fixed-angle energy growth seem implied by causality+unitarity up to $(m/M^{\text{higher-spin}})^{\#}$ corrections

- Superconvergence relations + dual amplitudes: enable to study small (not O(100)) mixed system

  Other applications: massless gravitons in d>4, stress tensor correlators, higher-spins?

- What if $m_{\text{spin}-2} \sim H$? Do $1/m^4_{\text{spin}-2}$ effects cause local pathologies on lengths $1/M^{\text{higher-spin}} \ll H^{-1}$?