Classifying and Bootstrapping 3d GNY-like Models

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Based on 2512.11963 with David Poland,
and work in progress with David Poland and Robin Tsai

Bootstrap 2026

1. Why GNY?
2. Bootstrapping “ordinary” GNY
3. Perturbative classification of GNY-like models
4. Towards bootstrapping GNY-like models

What is the space of self-consistent 3d CFTs?

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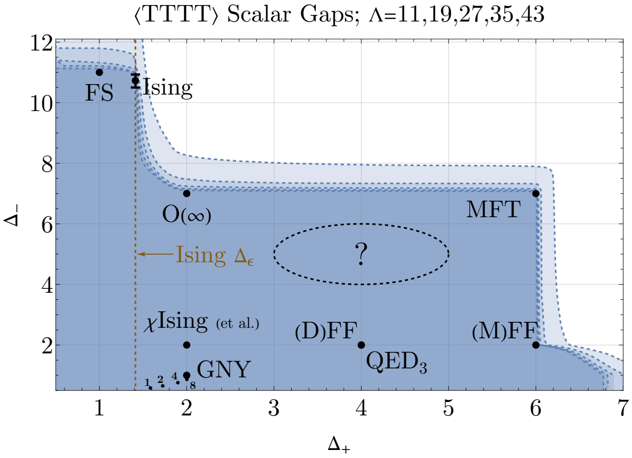

From 2602.13383 (MM and Rajeev Erramilli). Also based on 2411.15300 (Chang, Dommes, Erramilli, Homrich, Kravchuk, Liu, MM, Poland, and Simmons-Duffin).

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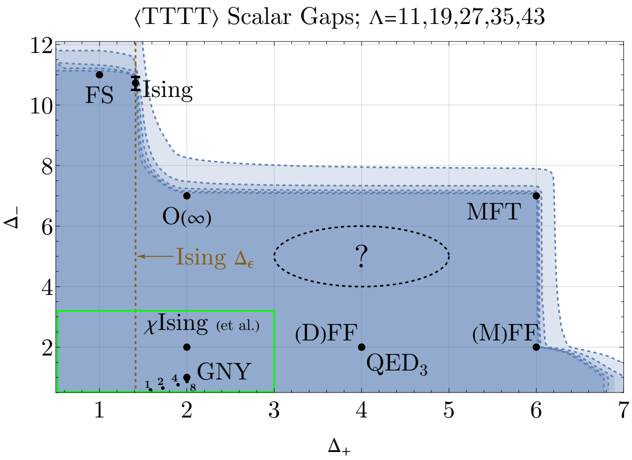

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Gross–Neveu–Yukawa

N Majorana fermions, Yukawa-coupled to scalars

$$\mathcal{L}_{\mathrm{GNY}} = -\frac{1}{2}(\partial\phi)^2 - i\frac{1}{2}\psi_a\partial\psi_a - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4}\phi^4 - i\frac{g}{2}\phi\psi_a\psi_a$$

$\mathrm{O}(N)$ action: $\quad \phi \to \phi, \quad \psi_a \to \Omega_{ij}\psi_j$

Parity: $\quad \phi \to -\phi, \quad \psi\psi \to -\psi\psi$

Typically: QCP in (2+1)d systems

- $N = 1$: Super-Ising
- $N > 1$: TRSB in graphene, d-wave superconductors, Chern insulators, and more

Spinor product shorthand

$$\chi\psi \equiv (\gamma^0)^{\alpha\beta}\chi_\alpha\psi_\beta = \psi\chi$$

Gross–Neveu–Yukawa

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Term	Upper crit. dimension	
ϕ^4	4	Relevant in 3d
$\phi\psi\psi$	4	
ϕ^6	3	
ϕ^{2n}	$(2, 8/3]$	
$\phi^{2n+1}\psi\psi$	$(2, 8/3]$	
$\psi\psi\psi\psi$	2	
$\phi^{2n}\psi\psi\psi\psi$	2	

GNY: Operator content

Operator	Parity	$O(N)$	Δ at large N	Δ in ϵ-exp.
ψ_i	$+$	V	$1+\frac{4}{3\pi^2 N}+\frac{896}{27\pi^4 N^2}+\frac{\#}{N^3}+\ldots$	$\frac{3}{2}-\frac{N+5}{2(N+6)}\epsilon+\ldots$
$\psi_i' \sim \phi^2\psi_i$	$+$	V	$3+\frac{100}{3\pi^2 N}+\ldots$	-
$\sigma \sim \phi$	$-$	S	$1-\frac{32}{3\pi^2 N}+\frac{32\left(304-27\pi^2\right)}{27\pi^4 N^2}+\ldots$	$1-\frac{3}{N+6}\epsilon+\ldots$
$\sigma' \sim \phi^3$	$-$	S	$3+\frac{64}{\pi^2 N}-\frac{128\left(770-9\pi^2\right)}{9\pi^4 N^2}+\ldots$	$3+\frac{\sqrt{N^2+132N+36}-N-30}{6(N+6)}\epsilon+\ldots$
$\epsilon \sim \phi^2$	$+$	S	$2+\frac{32}{3\pi^2 N}-\frac{64\left(632+27\pi^2\right)}{27\pi^4 N^2}+\ldots$	$2+\frac{\sqrt{N^2+132N+36}-N-30}{6(N+6)}\epsilon+\ldots$
$\epsilon' \sim \phi\partial^2\phi$	$+$	S	$4-\frac{64}{3\pi^2 N}+\frac{64\left(400-27\pi^2\right)}{27\pi^4 N^2}+\ldots$	-
ϕ^k	$(-1)^k$	S	$k+\frac{16(3k-5)k}{3\pi^2 N}-\frac{\#}{N^2}+\ldots$	-
$\sigma_T \sim \psi_{(i}\psi_{j)}$	$-$	T	$2+\frac{32}{3\pi^2 N}+\frac{4096}{27\pi^4 N^2}+\ldots$	-

GNY: Previous bootstrap work

Correlators
(up to ordering):

$$\langle\sigma\sigma\sigma\sigma\rangle$$
$$\langle\epsilon\epsilon\epsilon\epsilon\rangle$$
$$\langle\sigma\sigma\epsilon\epsilon\rangle$$
$$\langle\psi\psi\psi\psi\rangle$$
$$\langle\psi\psi\sigma\sigma\rangle$$
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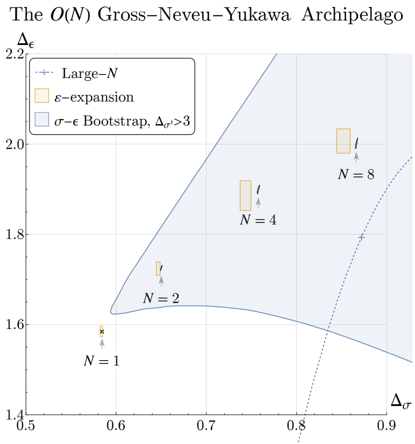

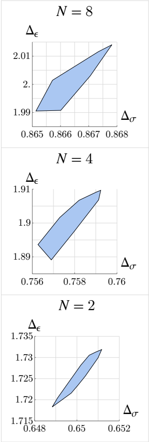

From 2210.02492 (Erramilli, Iliesiu, Kravchuk, Liu, Poland, and Simmons-Duffin)

GNY: Previous bootstrap work

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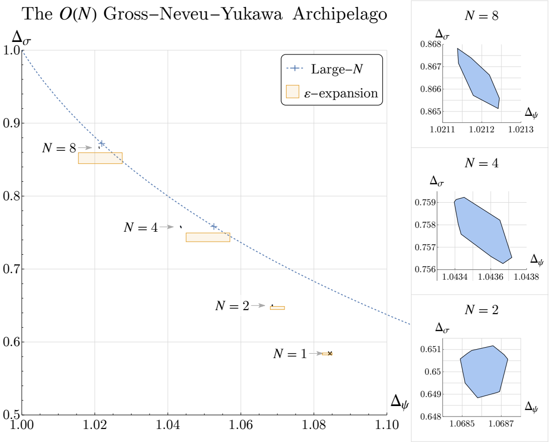

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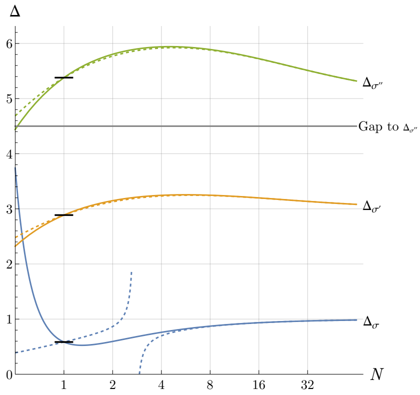
GNY: Tricriticality?

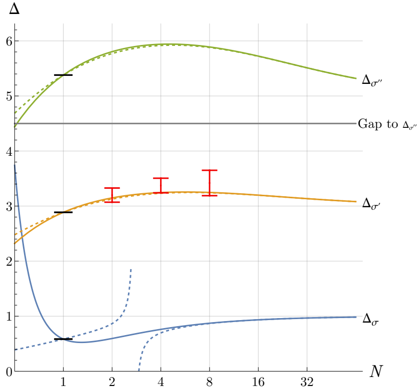
- What about higher operators, like $\sigma' \sim \phi^3$?
- Known to be relevant at $N = 1$ (super-Ising)
- For $N \geq 2$, can we flow to a new CFT? (potentially by explicitly breaking parity?)



2406.12974, MM and David poland

GNY: Tricriticality?

- What about higher operators, like $\sigma' \sim \phi^3$?
- Known to be relevant at $N = 1$ (super-Ising)
- For $N \geq 2$, can we flow to a new CFT? (potentially by explicitly breaking parity?)
- Answer: **No**



2406.12974, MM and David poland

What quantum critical points can we build with electrons in $(2+1)$ dimensions?

Classifying 3d GNY models

$$\mathcal{L}_{\text{GNY}} = \mathcal{L}_0 - \frac{\lambda}{4}\phi^4 - i\frac{g}{2}\phi\psi_a\psi_a$$

- What if $\langle\phi\rangle$ breaks a global $\mathbb{Z}_2$ instead of parity?
- E.g. the "chiral Ising" or "chiral GNY" model, which has $\mathrm{O}(N/2)^2 \rtimes \mathbb{Z}_2$ symmetry:

$$\mathcal{L}_{\chi I} = \mathcal{L}_0 - \frac{\lambda}{4}\phi^4 - i\frac{g}{2}\phi(\psi_a^L\psi_a^L - \psi_a^R\psi_a^R)$$

- What if ϕ is a flavor vector?
- Chiral XY ($\mathrm{O}(2)$ subgroup), Chiral Heisenberg ($\mathrm{O}(3)$ subgroup), and possibly others

$$\mathcal{L}_{\chi\text{Heisenberg}} = \mathcal{L}_0 - \frac{\lambda}{4}|\phi|^4 - \frac{i}{2}g\begin{pmatrix}\bar{\psi}_a^L & \bar{\psi}_a^R\end{pmatrix}(\vec{\phi}\cdot\vec{\sigma})\begin{pmatrix}\psi_a^L \\ \psi_a^R\end{pmatrix}$$

- Goal: Classify these models and bootstrap them

A group-theoretic aside

- Electrons are four-component **Dirac** spinors, Yukawa term is $\frac{ig}{2}\phi\Psi_a^\dagger\left[\ldots\right]\Psi_a$
- Basis in which first 3 gamma matrices are **real** and **block-diagonal**

$$\tilde{\gamma}^0 = \begin{pmatrix} \gamma^0 & 0 \\ 0 & -\gamma^0 \end{pmatrix}, \quad \tilde{\gamma}^1 = \begin{pmatrix} \gamma^1 & 0 \\ 0 & -\gamma^1 \end{pmatrix}, \quad \tilde{\gamma}^2 = \begin{pmatrix} \gamma^2 & 0 \\ 0 & -\gamma^2 \end{pmatrix}, \quad \tilde{\gamma}^3 = \begin{pmatrix} 0 & -i1 \\ i1 & 0 \end{pmatrix}$$

- In 3d: factorization into real, four-component Majorana spinors, **symmetry enhancement**
- Form of spinor Yukawa determines enhancement:
  - Chiral Ising: $\phi\bar{\Psi}_a\Psi_a \to \phi\left(\psi_a^{Lr}\psi_a^{Lr} + \psi_a^{Li}\psi_a^{Li} - \psi_a^{Rr}\psi_a^{Rr} - \psi_a^{Ri}\psi_a^{Ri}\right)$
    Symmetry: $U(N_D) \to \mathrm{O}(2N_D)^2 \rtimes \mathbb{Z}_2$
  - Standard GNY: $\phi\Psi_a^\dagger\Psi_a \to \phi\left(\psi_a^{Lr}\psi_a^{Lr} + \psi_a^{Li}\psi_a^{Li} + \psi_a^{Rr}\psi_a^{Rr} + \psi_a^{Ri}\psi_a^{Ri}\right)$
    Symmetry: $U(N_D) \to \mathrm{O}(4N_D)$

Classifying 3d GNY models

- Assume $\vec{\phi}$ has M components; $\mathrm{SO}(M)$ is not explicitly broken (for now)
- Start with N_D Dirac fermions, and find symmetry enhancement from $\mathrm{U}(N_D) \times \mathrm{O}(M)$ to $G \subseteq \mathrm{O}(4N_D) \times \mathrm{O}(M)$.
- General form:

$$\mathcal{L} = \mathcal{L}_0 - \frac{\lambda}{4}|\phi|^4 - \frac{i}{2}\phi_m \psi_a^A R_m^{AB} \psi_a^B$$

For real 4×4 matrices R_m and $\psi_a = \begin{pmatrix} \psi_a^{Lr} & \psi_a^{Li} & \psi_a^{Rr} & \psi_a^{Ri} \end{pmatrix}$.

- For GNY: $R_1 = g \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$. For chiral Ising: $R_1 = g \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$.
- Look for interacting fixed points **perturbatively** in $4 - \epsilon$ expansion

Classifying 3d GNY models

$$\mathcal{L} = \mathcal{L}_0 - \frac{\lambda}{4}\phi^4 - \frac{i}{2}\phi_m \psi_a^A R_m^{AB} \psi_a^B$$

Theorem: for a valid set of $\{R_m\}$, the symmetry group of a GNY-like model is

$$G = \left(\prod_j \mathrm{O}\left(N_D m_j^{\mathbb{R}}\right) \times \prod_j \mathrm{U}\left(N_D m_j^{\mathbb{C}}\right) \times \prod_j \mathrm{Sp}\left(N_D m_j^{\mathbb{H}}\right) \right) \rtimes \begin{cases} \mathrm{O}(M) & \text{if } M > 1 \text{, or } M = 1 \text{ and } R \text{ has an even spectrum} \\ 1 & \text{if } M = 1 \text{ and } R \text{ does not have an even spectrum} \end{cases}$$

where $m_j^{\mathbb{R},\mathbb{C},\mathbb{H}}$ are the multiplicities of each real, complex, and quaternionic module in the $C*$ algebra generated by the $\{R_m\}$.

All GNY-like models

12 models, 6 have fixed points!

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Name	Symmetry / Ψ rep	# of couplings / R-modules	Yukawa term / Coupling matrices R_m
GNY	$\mathrm{O}(4N_D)$	1	$-\frac{ig}{2}\phi\Psi_a^\dagger\Psi_a = \frac{-ig}{2}\phi(\psi_a^1\psi_a^1+\psi_a^2\psi_a^2+\psi_a^3\psi_a^3+\psi_a^4\psi_a^4)$
	$1^{\otimes 4}$	$1\oplus 1\oplus 1\oplus 1$	$\begin{pmatrix} g&0&0&0\\0&g&0&0\\0&0&g&0\\0&0&0&g\end{pmatrix}$
"Quarter GNY"	$\mathrm{O}(3N_D)\times\mathrm{O}(N_D)$	2	$-\frac{i}{2}g_1\phi(\psi_a^1\psi_a^1+\psi_a^2\psi_a^2+\psi_a^3\psi_a^3) -\frac{i}{2}g_2\phi\psi_a^4\psi_a^4$
	$1^{\otimes 4}$	$1_1\oplus 1_1\oplus 1_1\oplus 1_2$	$\begin{pmatrix} g_1&0&0&0\\0&g_1&0&0\\0&0&g_1&0\\0&0&0&g_2\end{pmatrix}$
	$\mathrm{O}(2N_D)^2$	2	$-\frac{i}{2}g_1\phi(\psi_a^1\psi_a^1+\psi_a^2\psi_a^2) -\frac{i}{2}g_2\phi(\psi_a^3\psi_a^3+\psi_a^4\psi_a^4)$
	$1^{\otimes 4}$	$1_1\oplus 1_1\oplus 1_2\oplus 1_2$	$\begin{pmatrix} g_1&0&0&0\\0&g_1&0&0\\0&0&g_2&0\\0&0&0&g_2\end{pmatrix}$
	$\mathrm{O}(2N_D)\times\mathrm{O}(N_D)^2$	3	$-\frac{i}{2}g_1\phi(\psi_a^1\psi_a^1+\psi_a^2\psi_a^2) -\frac{i}{2}g_2\phi\psi_a^3\psi_a^3-\frac{i}{2}g_3\phi\psi_a^4\psi_a^4$
	$1^{\otimes 4}$	$1_1\oplus 1_1\oplus 1_2\oplus 1_3$	$\begin{pmatrix} g_1&0&0&0\\0&g_1&0&0\\0&0&g_2&0\\0&0&0&g_3\end{pmatrix}$
	$\mathrm{O}(N_D)^4$	4	$-\frac{i}{2}g_1\phi\psi_a^1\psi_a^1-\frac{i}{2}g_2\psi_a^2\psi_a^2 -\frac{i}{2}g_3\phi\psi_a^3\psi_a^3-\frac{i}{2}g_4\phi\psi_a^4\psi_a^4$
	$1^{\otimes 4}$	$1_1\oplus 1_2\oplus 1_3\oplus 1_4$	$\begin{pmatrix} g_1&0&0&0\\0&g_2&0&0\\0&0&g_3&0\\0&0&0&g_4\end{pmatrix}$
Chiral Ising	$\mathrm{O}(2N_D)^2\rtimes\mathbb{Z}_2$	1	$-\frac{ig}{2}\phi\bar\Psi_a\Psi_a = \frac{-ig}{2}\phi(\psi_a^1\psi_a^1+\psi_a^2\psi_a^2-\psi_a^3\psi_a^3-\psi_a^4\psi_a^4)$
	$1^{\otimes 4}$	$1_1\oplus 1_1\oplus 1_2\oplus 1_2$	$\begin{pmatrix} g&0&0&0\\0&g&0&0\\0&0&-g&0\\0&0&0&-g\end{pmatrix}$
	$\mathrm{O}(N_D)^4\rtimes\mathbb{Z}_2$	2	$-\frac{i}{2}g_1\phi(\psi_a^1\psi_a^1-\psi_a^2\psi_a^2) -\frac{i}{2}g_2\phi(\psi_a^3\psi_a^3-\psi_a^4\psi_a^4)$
	$1^{\otimes 4}$	$1_1\oplus 1_2\oplus 1_3\oplus 1_4$	$\begin{pmatrix} g_1&0&0&0\\0&-g_1&0&0\\0&0&g_2&0\\0&0&0&-g_2\end{pmatrix}$

Table 2. An exhaustive list of $M=1$ GNY-like models and their symmetry groups. All named models have a perturbative fixed point in the $4-\epsilon$ expansion; the unnamed ones do not.

This model has perturbative fixed points when

$$g_1^2=g_2^2=g_3^2=g_4^2=\frac{16\pi^2}{4N_D+6}\epsilon+O(\epsilon^2) \tag{3.3}$$

Name	Symmetry / Ψ rep	# of couplings / R-modules	Yukawa term / Coupling matrices R_m
Chiral XY	$\mathrm{O}(2N_D)\rtimes\mathrm{O}(2)$	1	$-\frac{i}{2}g\bar\Psi_a\left(\phi_1+i\tilde\gamma_5\phi_2\right)\Psi_a$
	$2_q\oplus 2_{q+1}$	$2^{\mathbb{R}}\oplus 2^{\mathbb{R}}$	$\begin{pmatrix} g&0&0&0\\0&g&0&0\\0&0&-g&0\\0&0&0&-g\end{pmatrix},\begin{pmatrix} 0&0&0&g\\0&0&-g&0\\0&-g&0&0\\g&0&0&0\end{pmatrix}$
	$\mathrm{O}(N_D)^2\rtimes\mathrm{O}(2)$	2	$-\frac{i}{2}g_1\psi_a^{\dagger L}\left(\phi_1+i\phi_2\right)\psi_a^L+\text{h.c.} -\frac{i}{2}g_2\psi_a^{\dagger R}\left(\phi_1+i\phi_2\right)\psi_a^R+\text{h.c.}$
	$2_{1/2}\oplus 2_{1/2}$	$2^{\mathbb{R}}\oplus 2'^{\,\mathbb{R}}$	$\begin{pmatrix} g_1&0&0&0\\0&-g_1&0&0\\0&0&g_2&0\\0&0&0&-g_2\end{pmatrix},\begin{pmatrix} 0&g_1&0&0\\g_1&0&0&0\\0&0&0&g_2\\0&0&g_2&0\end{pmatrix}$
	$\mathrm{O}(N_D)\times\mathrm{O}(2)$	2	$-\frac{i}{2}\frac{g_1+g_2}{2}\bar\psi_a^L\left(\phi_1+i\phi_2\right)\psi_a^R+\text{h.c.} -\frac{i}{2}\frac{g_1-g_2}{2}\psi_a^{\dagger L}\left(\phi_1+i\phi_2\right)\psi_a^R+\text{h.c.}$
	$1\oplus 1\oplus 2_1$	$4^{\mathbb{R}}$	$\begin{pmatrix} 0&0&g_1&0\\0&0&0&g_2\\g_1&0&0&0\\0&g_2&0&0\end{pmatrix},\begin{pmatrix} 0&0&0&-g_1\\0&0&g_2&0\\0&g_2&0&0\\-g_1&0&0&0\end{pmatrix}$

Table 3. An exhaustive list of $M=2$ GNY-like models that preserve $\mathrm{O}(2)$ symmetry, along with their full symmetry groups.

Name	Symmetry / Ψ rep	# of couplings / R-modules	Yukawa term / Coupling matrices R_m
Chiral Heisenberg	$\mathrm{U}(N_D)\times\mathrm{O}(3)$	1	$-\frac{i}{2}g\left(\bar\psi_a^L\ \bar\psi_a^R\right)(\phi\cdot\sigma)\begin{pmatrix}\psi_a^L\\ \psi_a^R\end{pmatrix} = -\frac{i}{2}g\bar\Psi_a(i\tilde\gamma^3\phi_1+i\tilde\gamma^5\phi_2+\phi_3)\Psi_a$
	4	$4^{\mathbb{C}}$	$\begin{pmatrix} 0&0&g&0\\0&0&0&g\\g&0&0&0\\0&g&0&0\end{pmatrix},\begin{pmatrix} 0&0&0&g\\0&0&-g&0\\0&-g&0&0\\g&0&0&0\end{pmatrix},\begin{pmatrix} g&0&0&0\\0&g&0&0\\0&0&-g&0\\0&0&0&-g\end{pmatrix}$
"Orthogonal Heisenberg" (new fixed point)	$\mathrm{O}(N_D)\times\mathrm{O}(3)$	1	$-\frac{i}{2}g\chi_a\left(\phi_1\psi_a^1+\phi_2\psi_a^2+\phi_3\psi_a^3\right)$
	$1\oplus 3$	$4^{\mathbb{R}}$	$\begin{pmatrix} 0&g&0&0\\g&0&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix},\begin{pmatrix} 0&0&g&0\\0&0&0&0\\g&0&0&0\\0&0&0&0\end{pmatrix},\begin{pmatrix} 0&0&0&g\\0&0&0&0\\0&0&0&0\\g&0&0&0\end{pmatrix}$

Table 4. An exhaustive list of $M=3$ GNY-like models that preserve $\mathrm{O}(3)$ symmetry, along with their full symmetry groups. For these models, the $\mathrm{U}(N_D)$ and $\mathrm{O}(N_D)$ subgroups do not experience symmetry enhancement in (2+1)d, so the semidirect product reduces to a direct product.

and

$$\lambda^2=\frac{8\pi^2\left(\sqrt{16N_D^2+528N_D+36}-4N_D+6\right)}{3\left(4N_D+6\right)}\epsilon+O(\epsilon^2), \tag{3.4}$$

but we are free to choose the signs of each g_i. Up to isomorphism, this gives us three universality classes.

2512.11963, MM and David Poland

All GNY-like fixed points

12 models, 6 have fixed points!

For one scalar:

- GNY, $\mathrm{O}(4N_D)$ symmetry:

$$\mathcal{L} = \mathcal{L}_{\text{free}} - \frac{\lambda}{4}\phi^4 - i\frac{g}{2}\phi\left(\psi_a^1\psi_a^1 + \psi_a^2\psi_a^2 + \psi_a^3\psi_a^3 + \psi_a^4\psi_a^4\right)$$

- Chiral Ising, $\mathrm{O}(2N_D)^2 \rtimes \mathbb{Z}_2$ symmetry:

$$\mathcal{L} = \mathcal{L}_{\text{free}} - \frac{\lambda}{4}\phi^4 - i\frac{g}{2}\phi\left(\psi_a^1\psi_a^1 + \psi_a^2\psi_a^2 - \psi_a^3\psi_a^3 - \psi_a^4\psi_a^4\right)$$

- **"Quarter GNY"**, $\mathrm{O}(3N_D) \times \mathrm{O}(N_D)$ symmetry:

$$\mathcal{L} = \mathcal{L}_{\text{free}} - \frac{\lambda}{4}\phi^4 - i\frac{g}{2}\phi\left(\psi_a^1\psi_a^1 + \psi_a^2\psi_a^2 + \psi_a^3\psi_a^3 - \psi_a^4\psi_a^4\right)$$

(identical perturbative data at 2 loops)

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For two scalars:

- Chiral XY, $\mathrm{O}(2N_D) \rtimes \mathrm{O}(2)$ symmetry:

$$\mathcal{L} = \mathcal{L}_{\mathrm{free}} - \frac{\lambda}{4}|\phi|^4 - i\frac{g}{2}\bar{\Psi}_a \left(\phi_1 + i\phi_2\gamma_5\right) \Psi_a .$$

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For three scalars:

- Chiral Heisenberg, $\mathrm{U}(N_D) \times \mathrm{O}(3)$ symmetry:

$$\mathcal{L} = \mathcal{L}_{\text{free}} - \frac{\lambda}{4}\phi^4 - i\frac{g}{2}\bar{\Psi}_a\left(\phi_m\sigma_m\right)\Psi_a.$$

- **“Orthogonal Heisenberg”**, $\mathrm{O}(N_D) \times \mathrm{O}(3)$ symmetry:

$$\mathcal{L} = \mathcal{L}_{\text{free}} - \frac{\lambda}{4}\phi^4 - i\frac{g}{2}\chi_a\left(\phi_1\psi_a^1 + \phi_2\psi_a^2 + \phi_3\psi_a^3\right).$$

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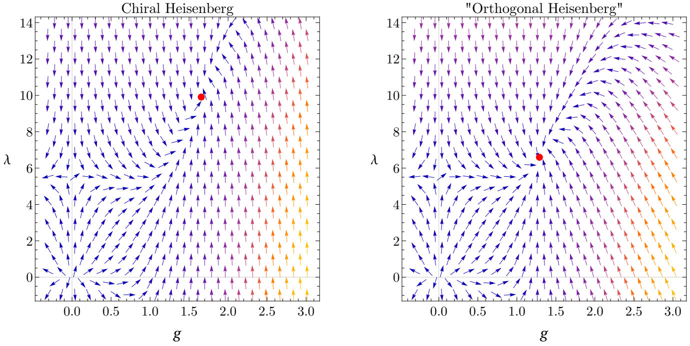

$$\mathcal{L}_{\text{Chiral Heisenberg}} = \mathcal{L}_{\text{free}} - \frac{\lambda}{4}|\phi|^4 - i\frac{g}{2}\bar{\Psi}_a\left(\phi_m\sigma_m\right)\Psi_a$$

$$\mathcal{L}_{\text{Orthogonal Heisenberg}} = \mathcal{L}_{\text{free}} - \frac{\lambda}{4}|\phi|^4 - i\frac{g}{2}\chi_a\left(\phi_1\psi_a^1 + \phi_2\psi_a^2 + \phi_3\psi_a^3\right)$$

All GNY-like fixed points

Critical point	Global symmetry	Breaking when $\langle\phi\rangle \neq 0$
GNY	$\mathrm{O}(4N_D)$	parity, time-reversal
Quarter GNY	$\mathrm{O}(3N_D) \times \mathrm{O}(N_D)$	parity, time-reversal
Chiral Ising	$\mathrm{O}(2N_D)^2 \rtimes \mathbb{Z}_2$	$\mathbb{Z}_2$
Chiral XY	$\mathrm{O}(2N_D) \rtimes \mathrm{O}(2)$	$\mathrm{O}(2) \to \mathbb{Z}_2$
Chiral Heisenberg	$\mathrm{U}(N_D) \times \mathrm{O}(3)$	$\mathrm{O}(3) \to \mathrm{O}(2)$
Orthogonal Heisenberg	$\mathrm{O}(N_D) \times \mathrm{O}(3)$	$\mathrm{O}(3) \to \mathrm{O}(2)$

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Next: Breaking the scalar symmetry

Explicit symmetry breaking and lattice deformation

- For 3d **Heisenberg** model: t_4 is relevant [2011.14647]
- Heisenberg magnets are unstable to anisotropic lattice deformations
- Is the same true in **chiral Heisenberg?**

Work in progress with David Poland and Robin Tsai

Explicit symmetry breaking and lattice deformation

- For 3d **Heisenberg** model: t_4 is relevant [2011.14647]
- Heisenberg magnets are unstable to anisotropic lattice deformations
- Is the same true in **chiral Heisenberg?**
- To leading order in ϵ: Yes. Nonperturbatively? Unclear

Chiral Heisenberg
$\mathrm{U}(N_D) \times \mathrm{O}(3)$

$\xrightarrow{+\lambda_2(\phi_1^4 + \phi_2^4 + \phi_3^4)}$ Cubic
$\mathrm{U}(N_D) \times \mathrm{Cubic}(3)$

$\xrightarrow{+\lambda_3(\phi_1^2 + \phi_2^2)^2 + \lambda_4 \phi_3^4}$ Biconical
$\mathrm{U}(N_D) \times (\mathrm{O}(2) \times \mathbb{Z}_2)$

Work in progress with David Poland and Robin Tsai

Towards boostrapping GNY-like models

We want to know:

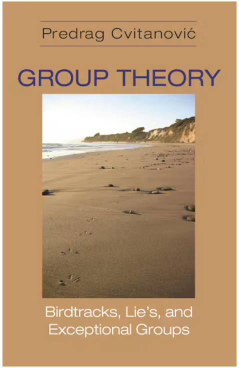
- Islands at various N_D for chiral Ising, quarter GNY, chiral XY, chiral Heisenberg, and orthogonal Heisenberg
- What operators are relevant at $d = 3$?

Challenges:

- Lots of tensor math!
- Conformal, flavor, spin statistics
- Rep theory is different for different models
- $\mathrm{U}(N_D)$ irreps are not self-conjugate; need to be careful

$$\begin{aligned}\langle \mathcal{O}_i \mathcal{O}_j \mathcal{O}_k \mathcal{O}_l \rangle &= \sum_{\mathcal{O}_m} \sum_{f f'} \lambda^{f}_{ijm} \lambda^{f'}_{klm} \left[Q_f \cdot Q_{f'} \right] \, G_{ijkl;\Delta_m,\ell_m}(z,\bar{z}) \\ &= \sum_{I} T_{I;ijkl} \; g^{I}_{ijkl}(z,\bar{z})\end{aligned}$$

Solution: Birdtracks



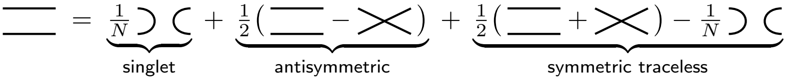
Every invariant tensor is a diagram: indices are lines, invariant tensors are vertices, and contracting an index pair means joining two lines.

$$\delta^i_{\ j} \;=\; i \;\text{——}\; j$$

$$\text{joined lines} \;=\; \text{contraction}$$

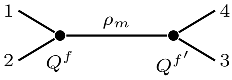
$$\bigcirc \;=\; \operatorname{tr}\mathbf{1} \;=\; N$$

Example: the three $O(N)$ invariants on $V \otimes V$ split it into irreps:



Solution: Birdtracks

1. Enumerate exchanged irreps ρ_m with multiplicity
2. Build orthogonal basis of three-point invariants Q^f
3. Build a four-point basis T_I;
4. Glue Qs; project onto $T_I \to$ crossing coefficients as rational function of N



$$= \sum_I c_I(N)\, T_I$$

Now implemented in `hyperion-bootstrap`
(Haskell-based conformal bootstrap library)

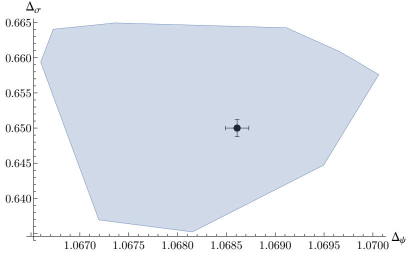
Reproducing GNY with minimal work

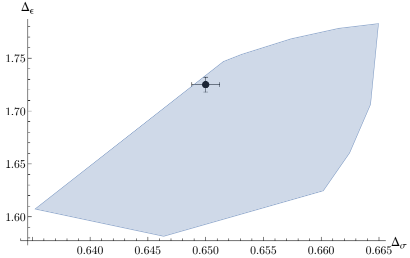
Specify operator representations;
`boostrap-math` handles everything else.

```
flavorRepOf :: ExternalOp s → IrrepOf ON
flavorRepOf Sig = singletRep
flavorRepOf Psi = vectorRep
flavorRepOf Eps = singletRep
```

```
crossingEqsList
  :: forall s a b . (Ord b, Fractional a, Eq a)
  ⇒ FourPointFunctionTerm (ExternalOp s) (B3d.Q4Struct, Flavor4PtStruct) b a
  → [(Taylors 'XT, FreeVect b a)]
crossingEqsList g = concat
  [ gen [ [Psi, Psi, Psi, Psi] ]
  , gen [ [Sig, Sig, Psi, Psi]
        , [Psi, Sig, Sig, Psi]
        , [Psi, Sig, Psi, Sig] ]
  , gen [ [Sig, Sig, Sig, Sig] ]
  , gen [ [Eps, Eps, Psi, Psi]
        , [Psi, Eps, Eps, Psi]
        , [Psi, Eps, Psi, Eps] ]
  , gen [ [Eps, Eps, Eps, Eps] ]
  , gen [ [Sig, Eps, Psi, Psi]
        , [Psi, Eps, Sig, Psi]
        , [Psi, Sig, Psi, Eps] ]
  , gen [ [Sig, Eps, Sig, Eps]
        , [Sig, Sig, Eps, Eps]
        , [Eps, Sig, Sig, Eps] ]
  ]
```

$N = 2$ GNY bootstrap, $\Lambda = 11$





What's next?

- More testing
- Handling semidirect products (chiral Ising)
- Finish automating "Spinning Conformal Correlators" and "Counting Conformal Correlators"
    - Ward identities
- Islands for new models
- Stability

CFT	Global symmetry
GNY	$\mathrm{O}(4N_D)$
Quarter GNY	$\mathrm{O}(3N_D) \times \mathrm{O}(N_D)$
Chiral Ising	$\mathrm{O}(2N_D)^2 \rtimes \mathbb{Z}_2$
Chiral XY	$\mathrm{O}(2N_D) \rtimes \mathrm{O}(2)$
Chiral Heisenberg $\longmapsto$ Cubic $\longmapsto$ Biconical	$\mathrm{U}(N_D) \times \mathrm{O}(3)$ $\mathrm{U}(N_D) \times \mathrm{Cubic}(3)$ $\mathrm{U}(N_D) \times (\mathrm{O}(2) \times \mathbb{Z}_2)$
Orthogonal Heisenberg	$\mathrm{O}(N_D) \times \mathrm{O}(3)$