Dynamics from Boundaries
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Review

Entanglement Entropy

Entanglement Entropy is a measure of the entanglement between a region A and its complement

Important Formulae

Density Matrix

$$\rho(\psi) = |\psi\rangle \langle\psi|$$

Reduced Density Matrix

$$\rho_A = \mathsf{Tr}_{A_c}\, \rho$$

Entanglement Entropy

$$S_A = \mathsf{Tr}(\rho_A \,\mathsf{ln}(\rho_A))$$

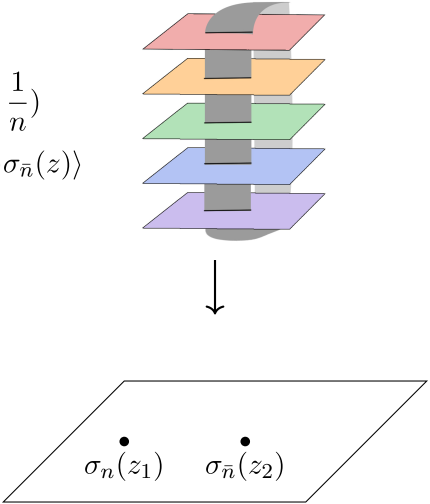
Replica Trick

$$S_A = \lim_{n\to 1} \partial_n \,\mathsf{Tr}(\rho_A^n)$$

Replica Surface and Twist Operators

$$\Delta = \frac{c}{12}(n - \frac{1}{n})$$

$$\mathsf{Tr}(\rho_A^n) = \langle \sigma_n(z)\sigma_{\bar{n}}(z) \rangle$$



Ryu-Takayanagi

The CFT answer looks like a geodesic length in hyperbolic space!

$$S_A = \frac{c}{3}\ln(\frac{l}{\epsilon})$$

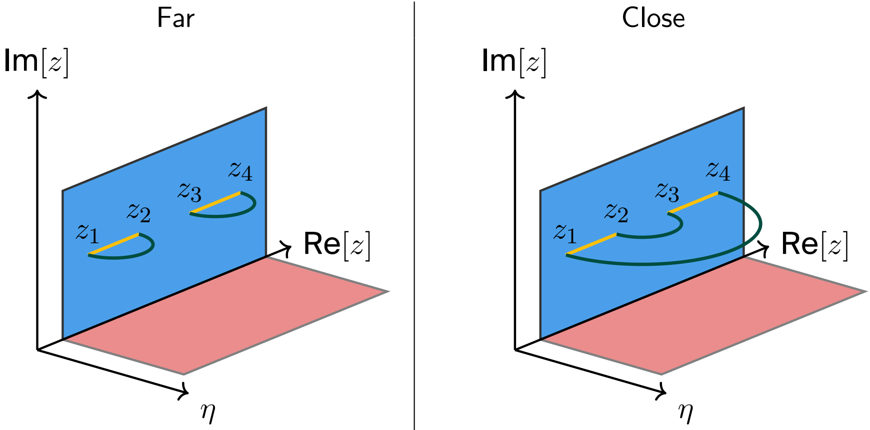
Ryu-Takayanagi Conjecture

S_A is the length of the smallest geodesic (homologous to A) in the holographic dual of the original CFT.

S. Ryu, T. Takayanagi. "Holographic Derivation of Entanglement Entropy from AdS/CFT" PRL (2006)

Different Phases

EPFL



We can consider disjoint subsystems $A \cup B$

Transitions from Conformal Blocks

$$\mathrm{Tr}(\rho^n_{A\cup B}) = \langle \sigma_n(0)\sigma_{\bar{n}}(x)\sigma_n(1)\sigma_{\bar{n}}(\infty)\rangle$$

T. Hartman, 2013 (BCFT extension in Sully, van Raamsdonk, Wakeham, 2020)

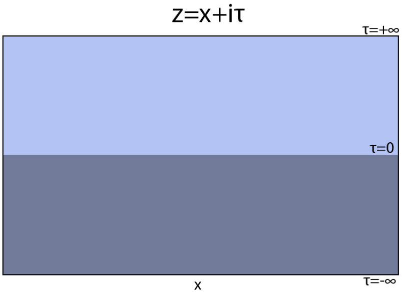
1. Take the OPE in the s-channel $x \to 0$ and the t-channel $x \to 1$
2. At large c, Virasoro blocks exponentiate (BPZ)
3. $S_{A\cup B} = \begin{cases} \frac{c}{3}\ln\frac{x}{\epsilon} & \text{s-channel} \\ \frac{c}{3}\ln\frac{1-x}{\epsilon} & \text{t-channel} \end{cases}$
4. At large c, and with some constraints on the spectrum, the vacuum block dominates and we get the holographic result

At smaller c we expect the transition to smooth

CFT on UHP

BCFT (*Cardy, 2008*)

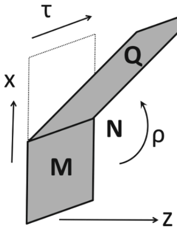
- No energy/momentum flow across boundary $\Rightarrow T(z) =_{\tau=0} \bar{T}(\bar{z})$
- One copy of the Virasoro Algebra
- For diagonal CFTs, one B.C. per primary



$AdS_3/BCFT_2$

M. Fujita, T. Takayanagi, E. Tonni JHEP 11 (2011)

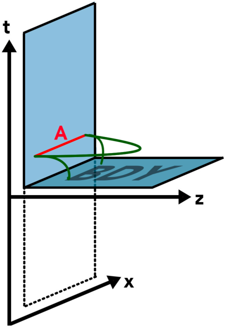
Put the BCFT in the path integral!

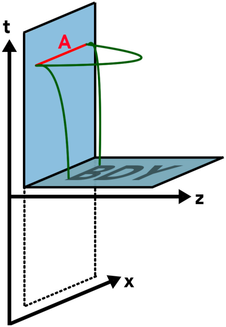


$$I_E = \frac{-1}{16\pi G_N}\int_N \sqrt{g}(R-2\Lambda) - \frac{1}{8\pi G_N}\int_Q \sqrt{h}(K-T)$$

Neumann Boundary conditions on Q viz. $K \propto T$

More Transitions





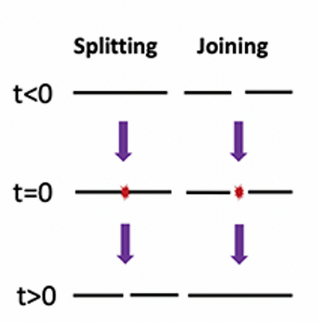
$S_A^{dis}(\tau) = \frac{c}{3} ln(\frac{2\tau}{\epsilon}) + S_{bdy}$

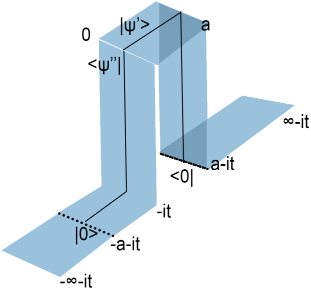
$S_A^{con}(\tau) = \frac{c}{3}\,\mathrm{ln}(\frac{l}{\epsilon}) = \mathrm{cnst}$

Quenches and Dynamical Phase Transitions

Splitting and Joining Quenches

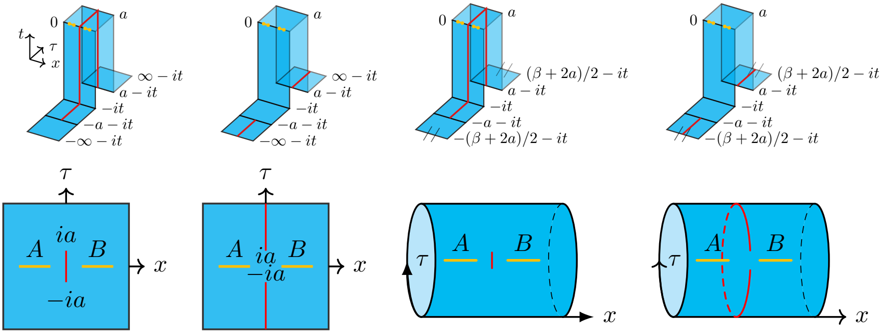
Quantum quenches: prepare a state with H, then time-evolve with H'





- AMO experiments: *Meinert et al., PRL 111 2013*
- OG: *P. Calabrese and J. Cardy, various papers (2004-2009)*
- AdS/BCFT: *T. Shimaji, T. Takayanagi, Z. Wei Holographic Quantum Circuits from Splitting/Joining Local Quenches JHEP 03 (2019)*

Uniformization to UHP

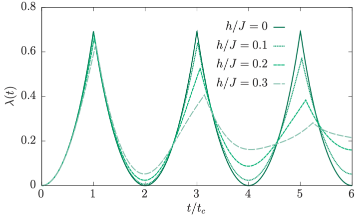


Dynamical Phase Transitions

EPFL

Main Idea

"Nonequilibrium phase transitions can also occur on transient time scales with physical quantities becoming nonanalytic as a function of time"



Quenches of coupling $h \int d^2x\, \sigma(x)$ in E_8 integrable theory (lattice equivalent)

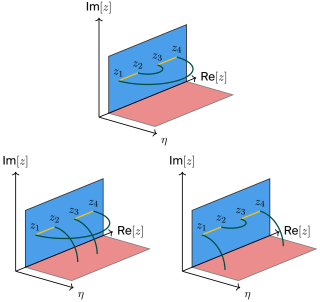
Review: *Heyl, Reports on Progress in Physics, 2018*
OG: *Heyl, et al. PRL 2013*

The Punchline

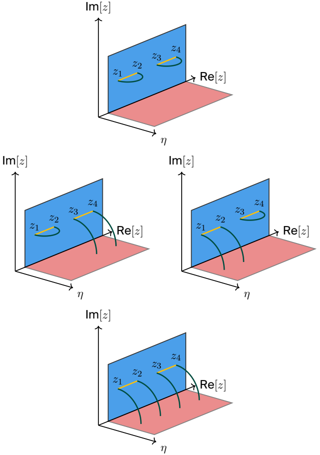
Phases of $S_{A\cup B}$

$$I(A:B) = S_A + S_B - S_{A\cup B}$$

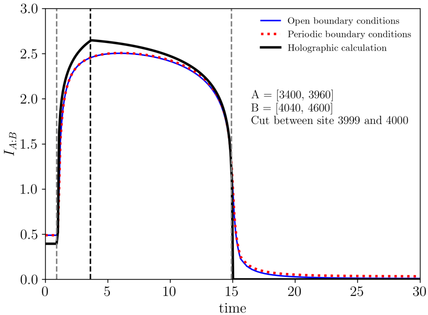
Has Mutual Info

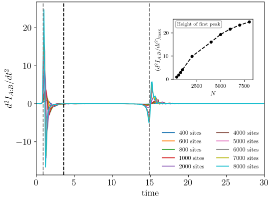


No Mutual Info



Non-Analyticities



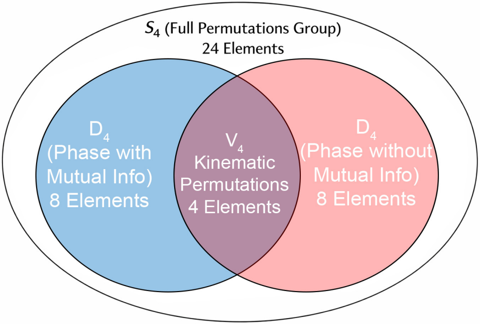


- Divergence of 2nd derivative $\Rightarrow$ non-analytic in the scaling limit
- Dynamical phase transition between MI & no-MI persists at $c = 1$

Question 1.

Does the non-analytic behavior come from the $c \to \infty$ limit, the replica limit ($n \to 1$), or the analytic continuation ($\tau \to i\tau$)?

Symmetry Structure



Question 2.

Can we pull this back to a more physical symmetry?

Main Takeaways

1. Mutual Information in certain quenches undergoes a dynamical phase transition
2. This phase transition is governed by which conformal blocks dominate in various OPE channels
3. BCFT allows us to probe dynamics with familiar CFT tools

Thank you!

Phases

$$S^{(1)}_{X\cup Y} = \frac{c}{6}\ln\frac{|z_{12}||z_{\bar{1}\bar{2}}||z_{34}||z_{\bar{3}\bar{4}}|}{\epsilon^4_{\text{UHP}}}$$
$$S^{(2)}_{X\cup Y} = \frac{c}{6}\ln\frac{|z_{1\bar{1}}||z_{2\bar{2}}||z_{3\bar{3}}||z_{4\bar{4}}|k^2}{\epsilon^4_{\text{UHP}}}$$
$$S^{(3a)}_{X\cup Y} = \frac{c}{6}\ln\frac{|z_{12}||z_{\bar{1}\bar{2}}||z_{3\bar{3}}||z_{4\bar{4}}|k}{\epsilon^4_{\text{UHP}}}$$
$$S^{(3b)}_{X\cup Y} = \frac{c}{6}\ln\frac{|z_{34}||z_{\bar{3}\bar{4}}||z_{1\bar{1}}||z_{2\bar{2}}|k}{\epsilon^4_{\text{UHP}}}$$

$$S^{(4)}_{X\cup Y} = \frac{c}{6}\ln\frac{|z_{14}||z_{\bar{1}\bar{4}}||z_{23}||z_{\bar{2}\bar{3}}|}{\epsilon^4_{\text{UHP}}}$$
$$S^{(5a)}_{X\cup Y} = \frac{c}{6}\ln\frac{|z_{14}||z_{\bar{1}\bar{4}}||z_{2\bar{2}}||z_{3\bar{3}}|k}{\epsilon^4_{\text{UHP}}}$$
$$S^{(5b)}_{X\cup Y} = \frac{c}{6}\ln\frac{|z_{23}||z_{\bar{2}\bar{3}}||z_{1\bar{1}}||z_{4\bar{4}}|k}{\epsilon^4_{\text{UHP}}}.$$