New mechanism to solve fine-tuning in d>2?

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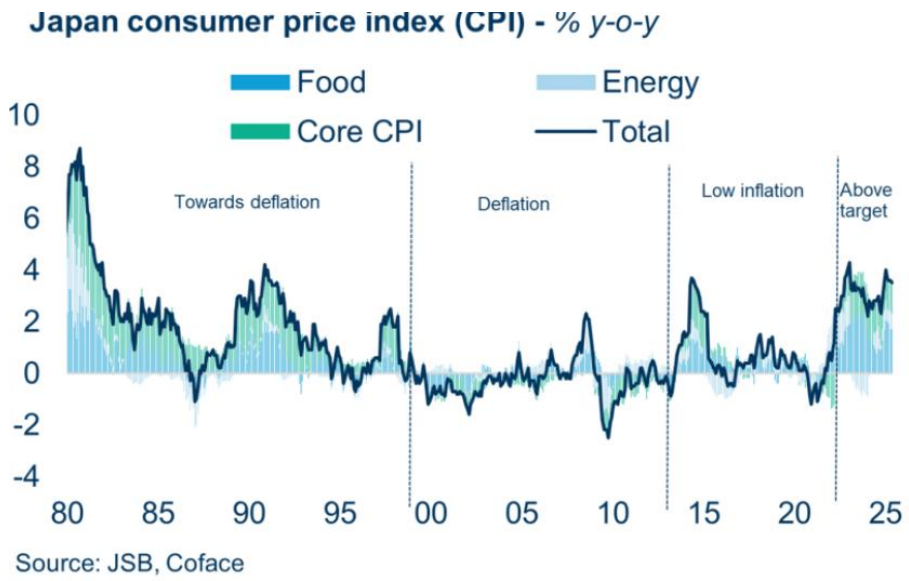
e-Print: 2511.00386

In Collaboration with Toshiki Onagi

Steven Hawking visiting Japan

When I was a student (how many years ago?), Steven Hawking gave a talk at the University of Tokyo



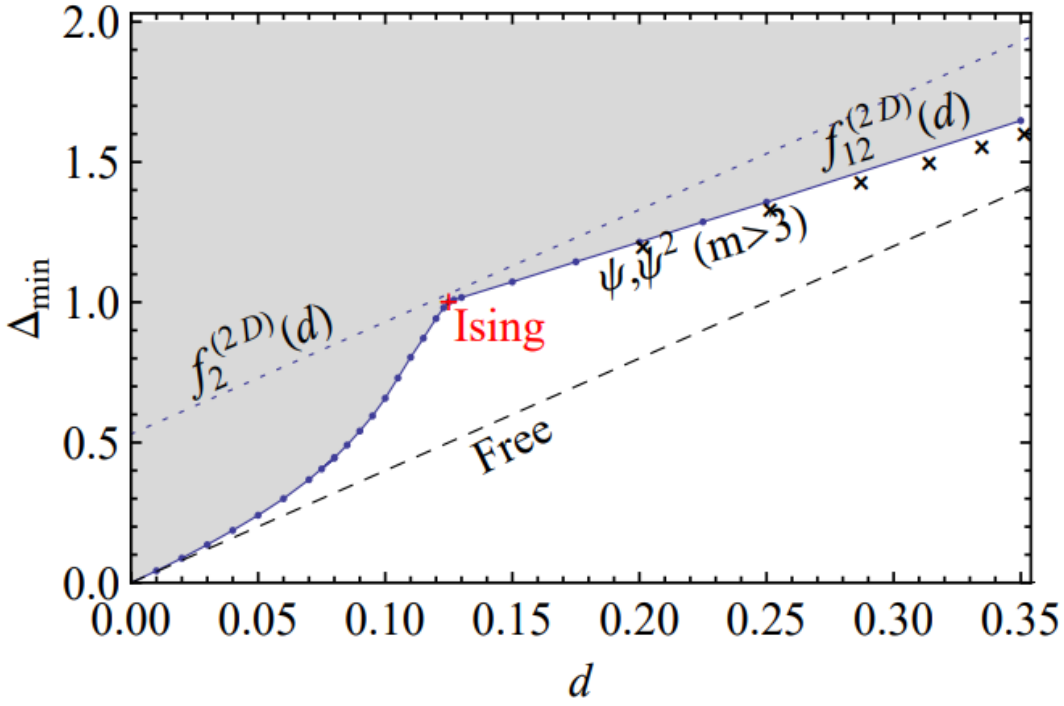
"Japanese government accepted Inflation"

I found this

Not there, but here.

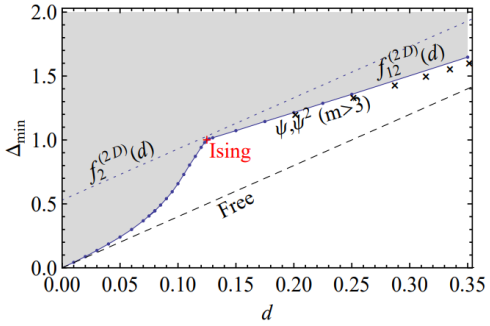
My first encounter with numerical bootstrap

$$\sigma \times \sigma = 1 + \epsilon$$



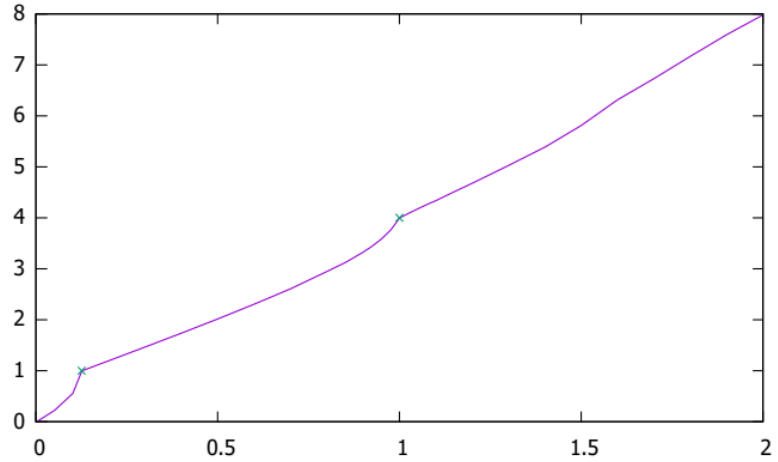
Rychkov-Vichi: 0905.2211

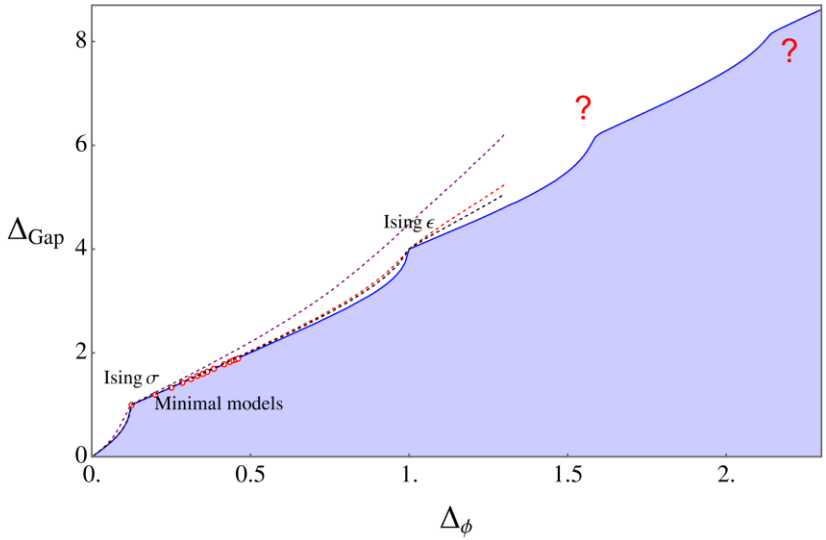
Trying harder (in d=2)



$$\sigma \times \sigma = 1 + \epsilon$$

YN: 1912.11748 Λ =55

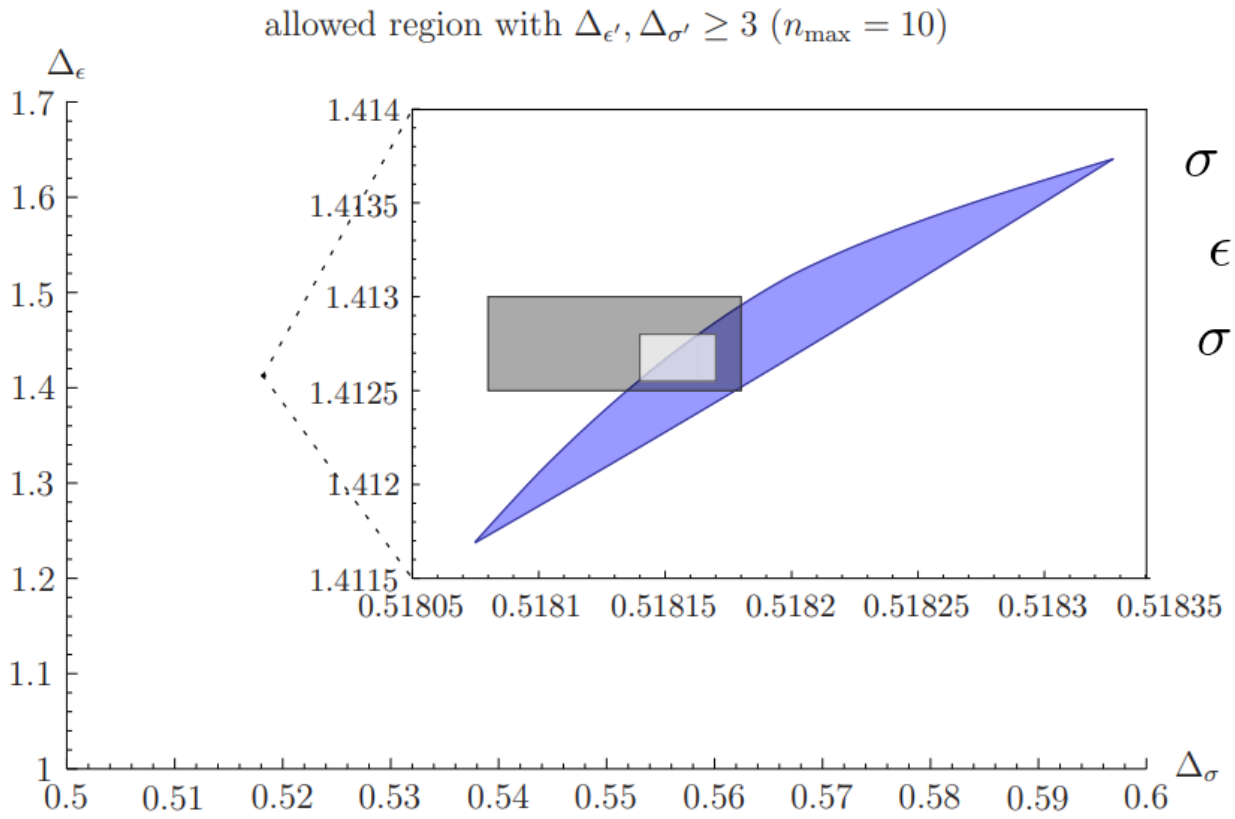




$$\sigma \times \sigma = 1 + \epsilon$$

Ghosh-Zheng 2307.11144 Λ =171

Mixed correlator bootstrap (in d=3)



$$\sigma \times \sigma = 1 + \epsilon + \cdots$$

$$\epsilon \times \epsilon = 1 + \epsilon + \cdots$$

$$\sigma \times \epsilon = \sigma + \cdots$$

$$\Delta_{\sigma'}, \Delta_{\epsilon'} > 3$$

Kos Poland Simmons-Duffin 1406.4858

Mixed correlator bootstrap (in d=2?)

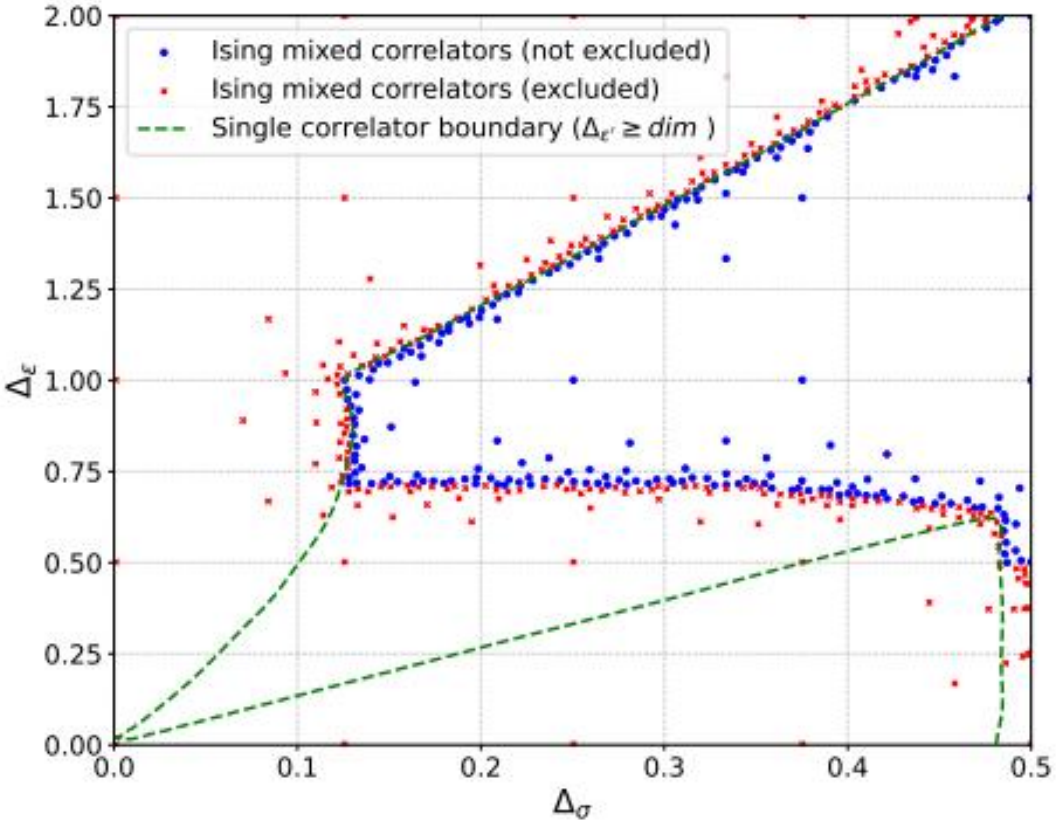
- So, one day, I asked my collaborator Tomoki Ohtsuki, "Do we see a similar island in the d=2 mixed correlator bootstrap?"
- With a puzzled look, he immediately replied "Of course not"

- I didn't contest it, but what did he mean by "Of course not"?

- Later I found the similar remark in PyCFTboot manual by Connor Behan:

"In the extreme case of d = 2, the same plot that follows purely from crossing symmetry and unitarity is returned with the extra assumptions here having no effect."

Anyway, I tried it eventually (YN-Onagi)



$\Delta_{\sigma'}, \Delta_{\epsilon'} > 2$ is assumed, which makes a difference from Behan's claim

$\lambda_{\sigma\sigma\epsilon} = \lambda_{\sigma\epsilon\sigma}$

Why "Of course, not"?

- The assumption cannot exclude (generalized) minimal models with identification $V(\phi) = \phi^{2m-2}$

$$\sigma = O_{1,2} \quad \phi^{m-2} \qquad \epsilon = O_{1,3} \quad \phi^{2(m-2)}$$

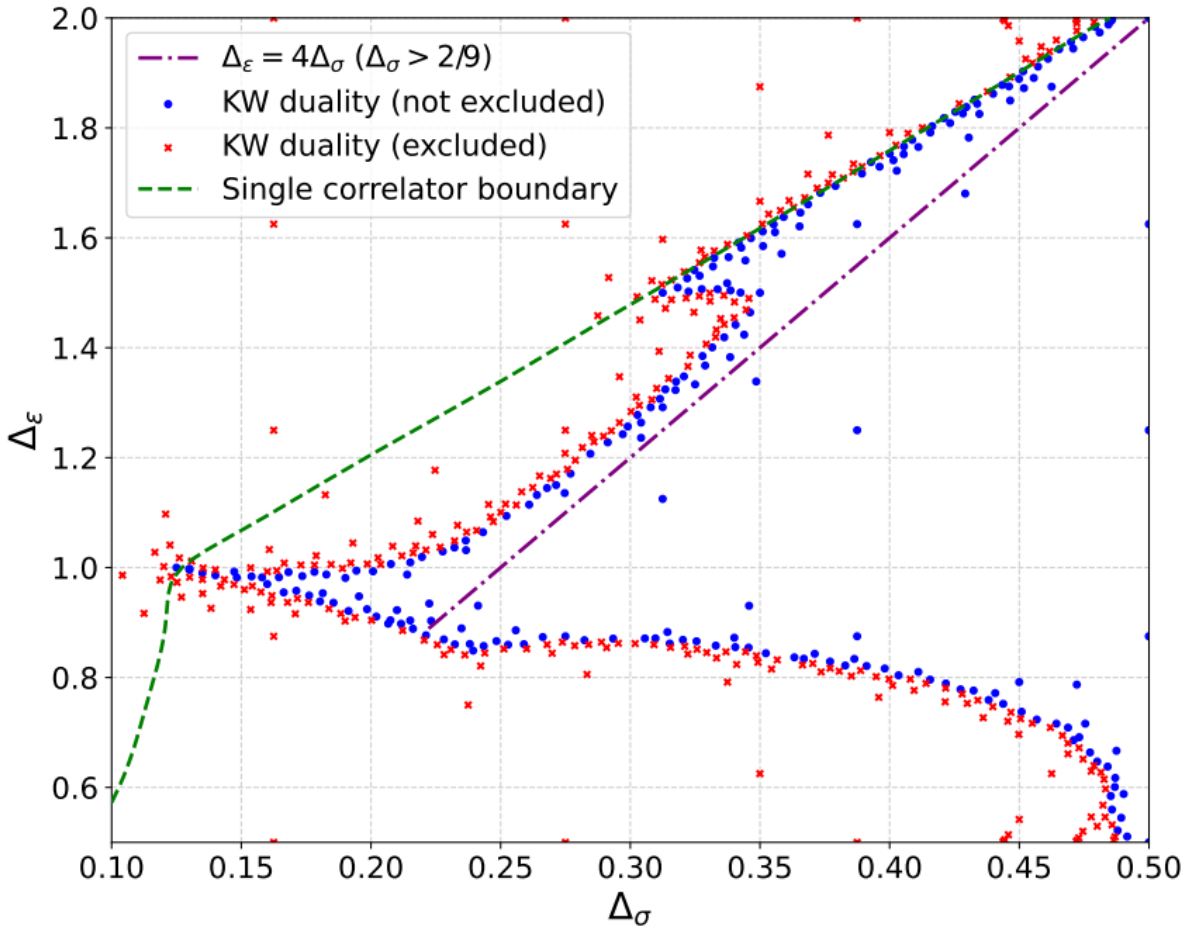
- OPE is consistent with all the assumptions made in mixed correlator bootstrap

$$O_{1,2} \times O_{1,2} = 1 + O_{1,3} + T\bar{T} + \cdots$$
$$O_{1,2} \times O_{1,3} = O_{1,2} + O_{1,4} + \cdots$$
$$O_{1,3} \times O_{1,3} = 1 + O_{1,3} + O_{1,5} + \cdots$$

- Can we isolate the Ising?
- Use the (non-Lagrangian) non-invertible symmetry

$$O_{1,3} \times O_{1,3} = 1 + 0 \times O_{1,3} + \cdots$$

Imposing non-invertible symmetry



$$\sigma \times \sigma = 1 + \epsilon + \cdots$$

$$\epsilon \times \epsilon = 1 + 0 \cdot \epsilon + \cdots$$

$$\sigma \times \epsilon = \sigma + \cdots$$

$$\Delta_{\sigma'}, \Delta_{\epsilon'} > 2$$

New ways to solve the fine-tuning problem?

Fine-tuning problem

- To find the criticality (= second order phase transition, or continuum limit), typically we have to fine-tune bare parameters (Temperature, bare Higgs mass etc)

- If we want to avoid this (self-organized criticality), typical strategy is
  - Find a symmetry so that the mass is charged (e.g. chiral symmetry for fermion mass)
  - Find the theory where there is no relevant parameters "dead-end CFT" (much more difficult…)
- New idea?
- In d=2, duality protects the OPE (we can avoid fine-tuning in d=2 Ising/TCI by KW duality)

Duality, non-invertible symmetry and beyond

- Such a new idea may be beyond the (invertible) symmetry used in conformal bootstrap
- KW-duality exchanges, spin and defect (non-local object)
- But some of the consequences can be implemented in OPE structures of local operators

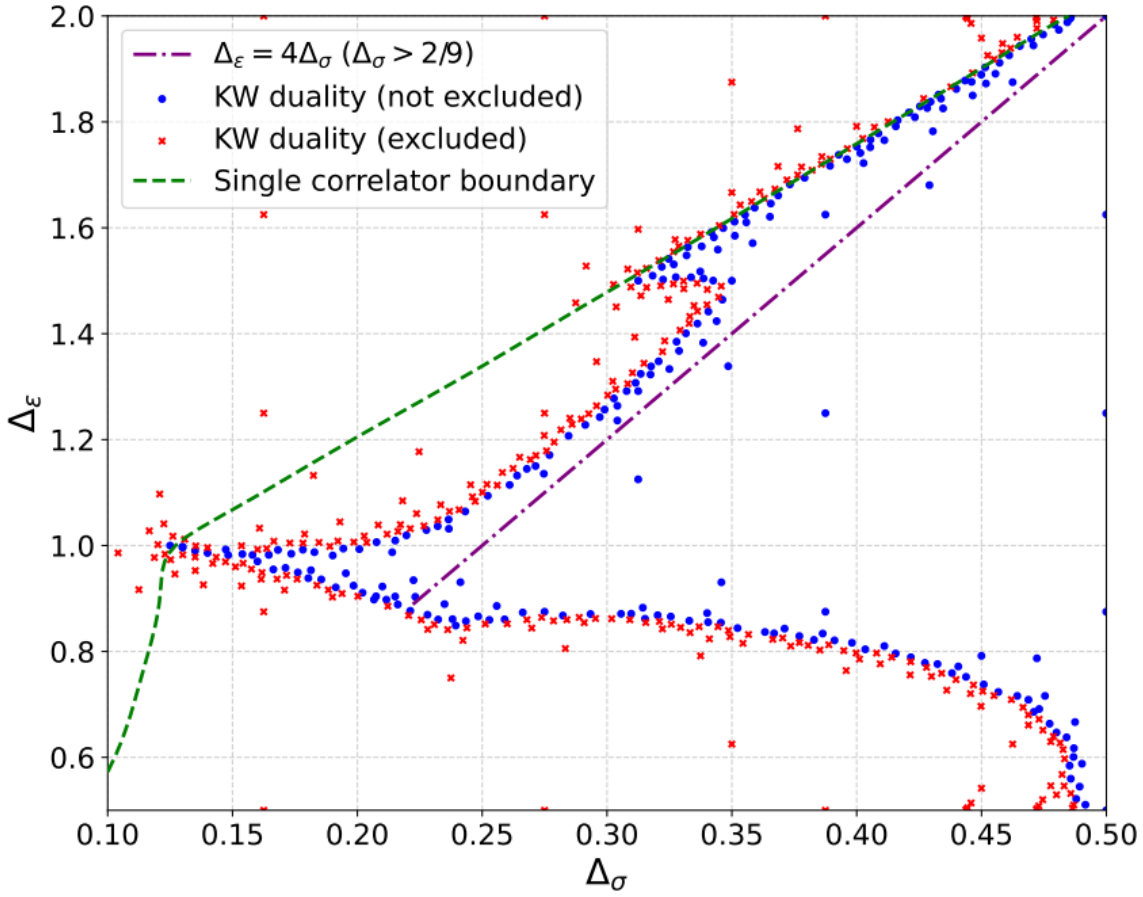
- After all, without knowing any underlying mechanism if

$$\epsilon \times \epsilon = 1 + 0 \times \epsilon + \text{irrelevant}$$

then fine-tuning problem is solved

- (We can think about the reason after that!)

Imposing non-invertible symmetry



$$\sigma \times \sigma = 1 + \epsilon + \cdots$$
$$\epsilon \times \epsilon = 1 + 0 \cdot \epsilon + \cdots$$
$$\sigma \times \epsilon = \sigma + \cdots$$

$$\Delta_{\sigma'}, \Delta_{\epsilon'} > 2$$
$$\lambda_{\sigma\sigma\epsilon} = \lambda_{\sigma\epsilon\sigma}$$

Imposing non-invertible symmetry

- 2d Ising is “isolated”
- Other allowed CFTs

(purple line)

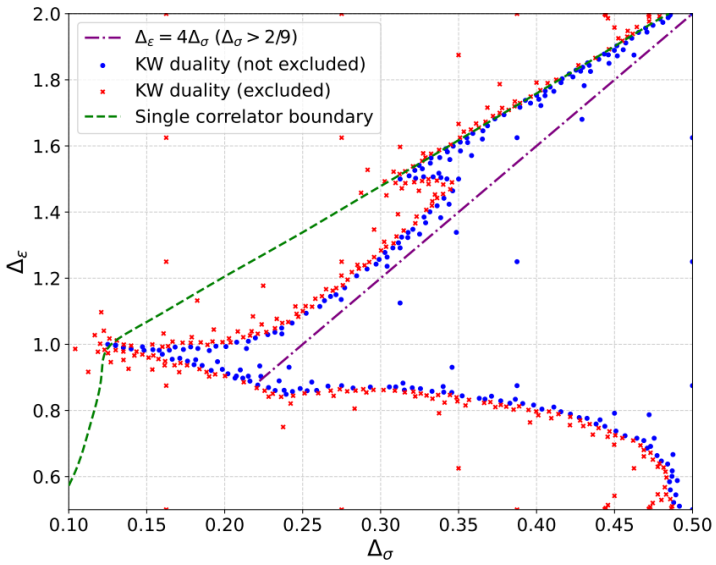
C=1 boson with

$$\sigma = \cos X \quad \epsilon = \cos(2X)$$

$$\sigma \times \sigma = 1 + \epsilon + \cdots$$

$$\epsilon \times \epsilon = 1 + 0 \cdot \epsilon + \cdots$$

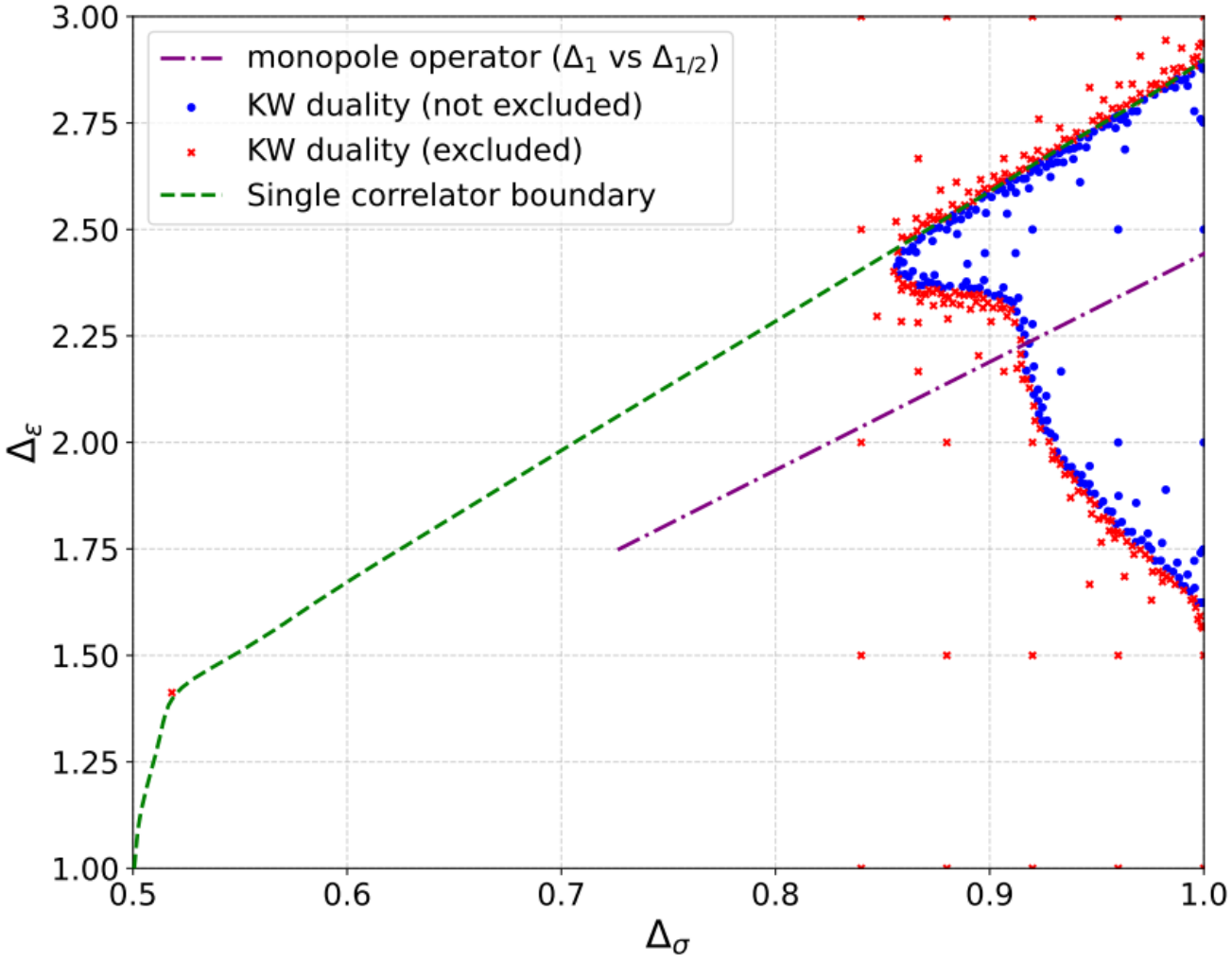
$$\sigma \times \epsilon = \sigma + \cdots$$



- Manifestation that this bound does not know the reason behind (symmetry or generalized symmetry or who knows what)
- (8,7) minimal model, hepta-critical Ising, look very suspicious (this theory DOES violate the assumption made)

d=3

$$\epsilon \times \epsilon = 1 + 0 \times \epsilon + \text{irrelevant}$$



Large N flavor QED

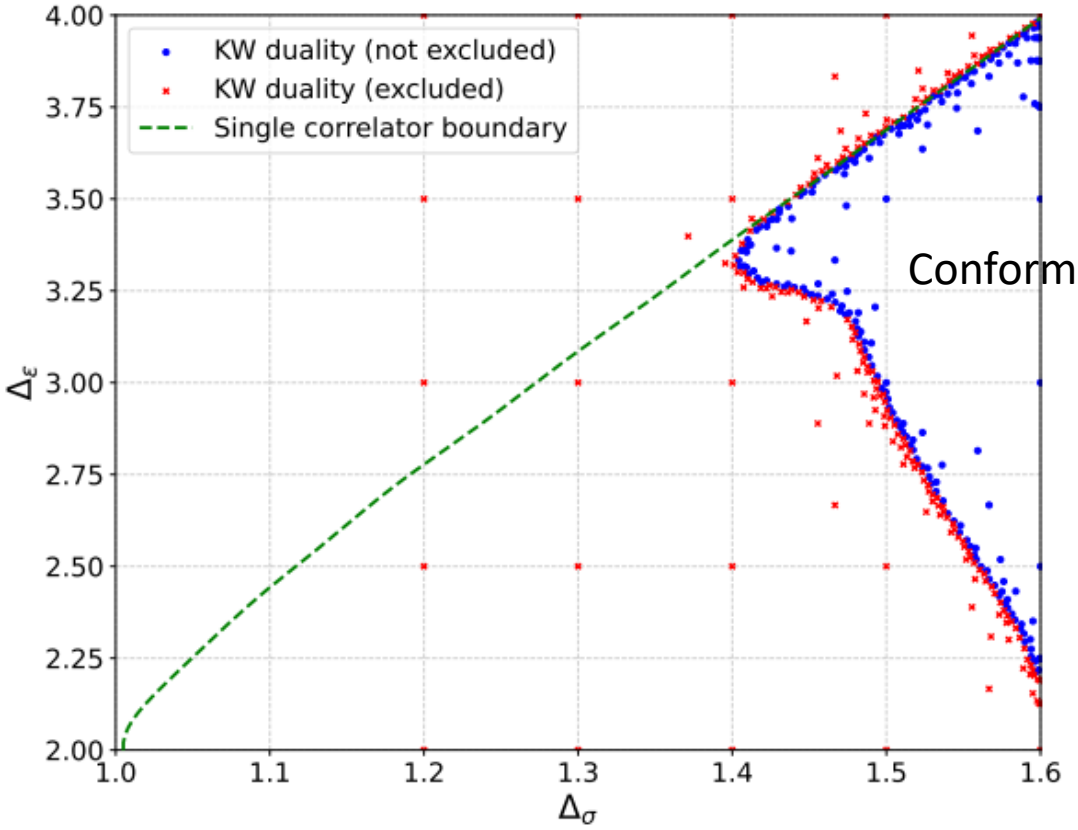
Dyer-Mezei-Pufu,

$$\sigma \times \sigma = 1 + \epsilon + \cdots$$

$$\epsilon \times \epsilon = 1 + 0 \cdot \epsilon + \cdots$$

$$\sigma \times \epsilon = \sigma + \cdots$$

d=4 $\epsilon \times \epsilon = 1 + 0 \times \epsilon + \text{irrelevant}$

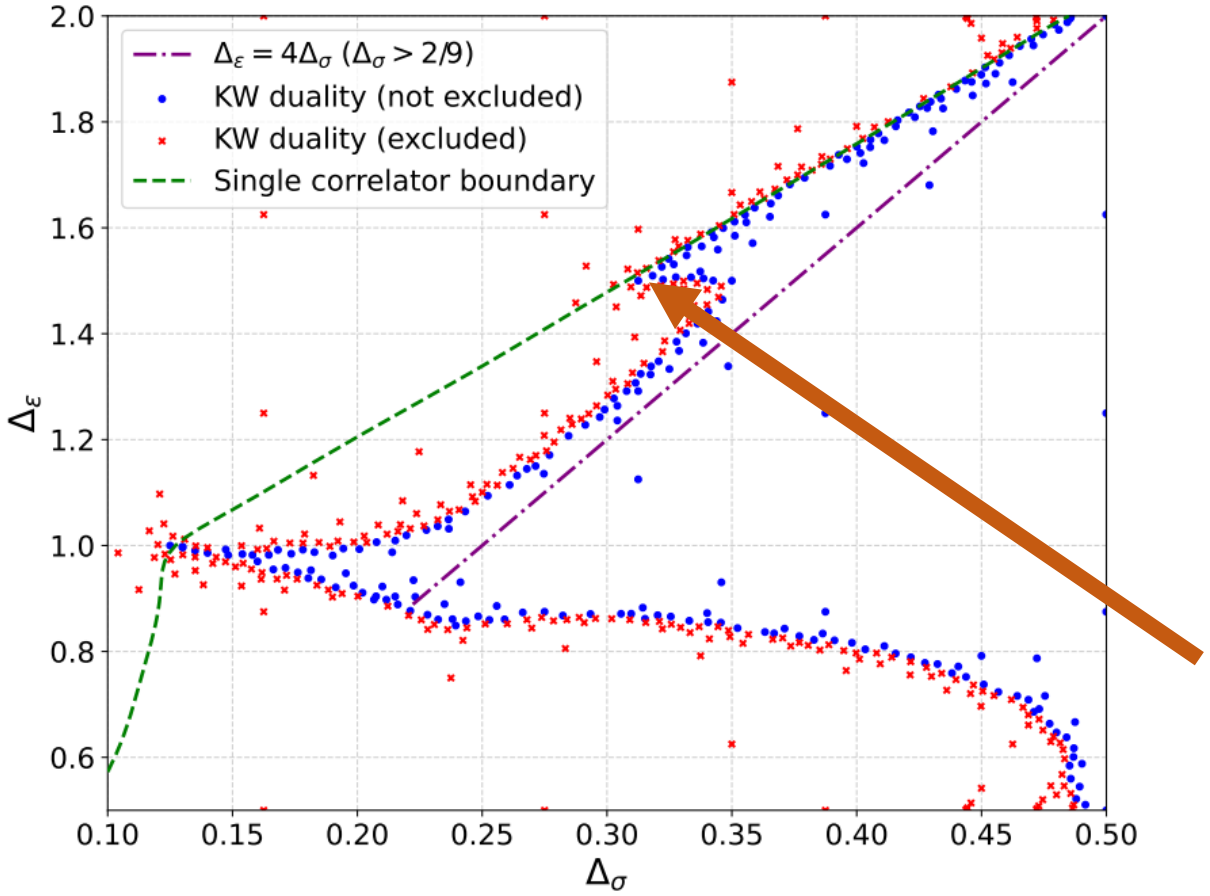


Conformal gauge theories?

Can we use this to solve the hierarchy problem in the SM?

Let’s now talk about the “trivial mix”

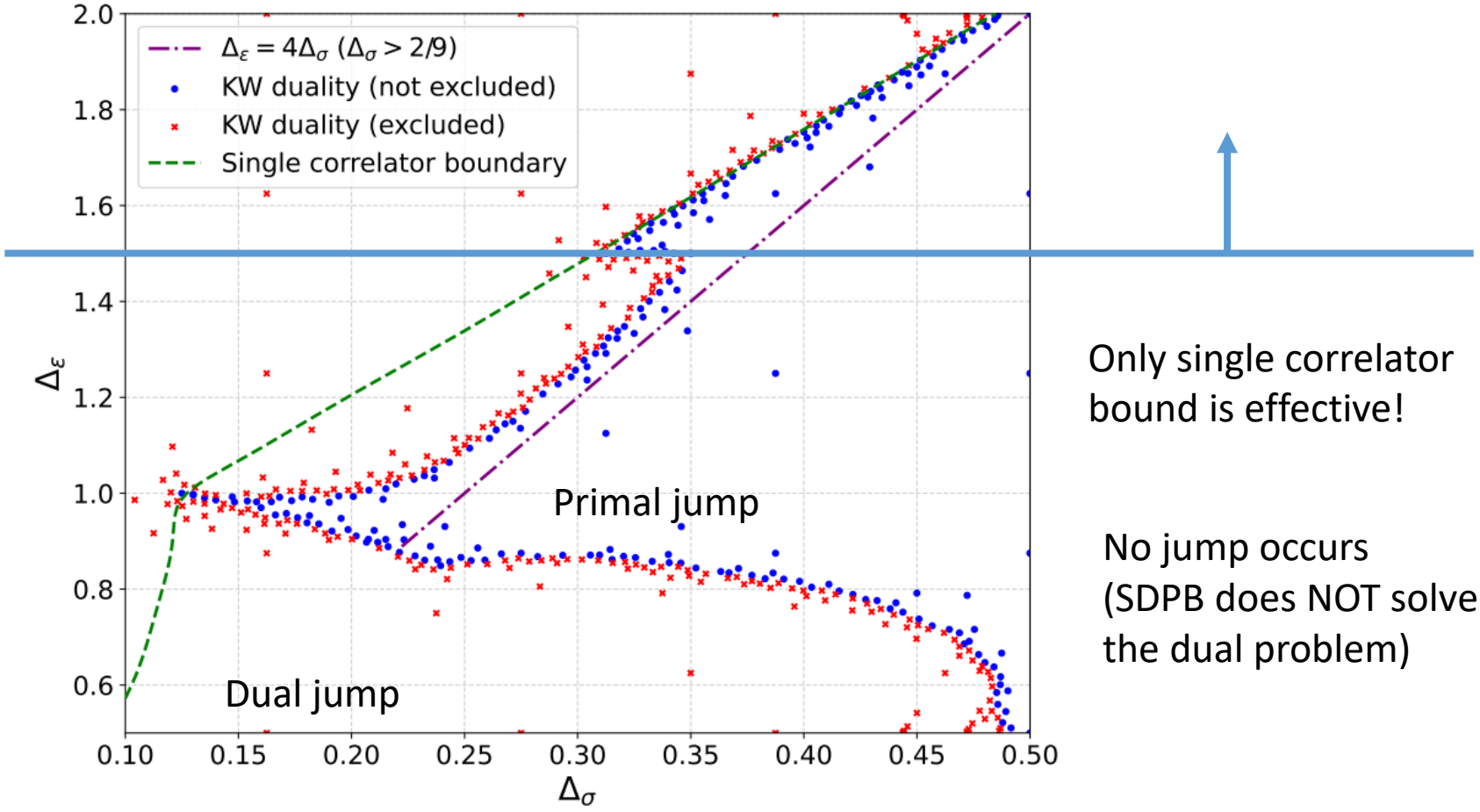
What happens at the nose?



Hepta-critical Ising should have been killed by the bootstrap!

Strictly speaking, it is not logically wrong… but….

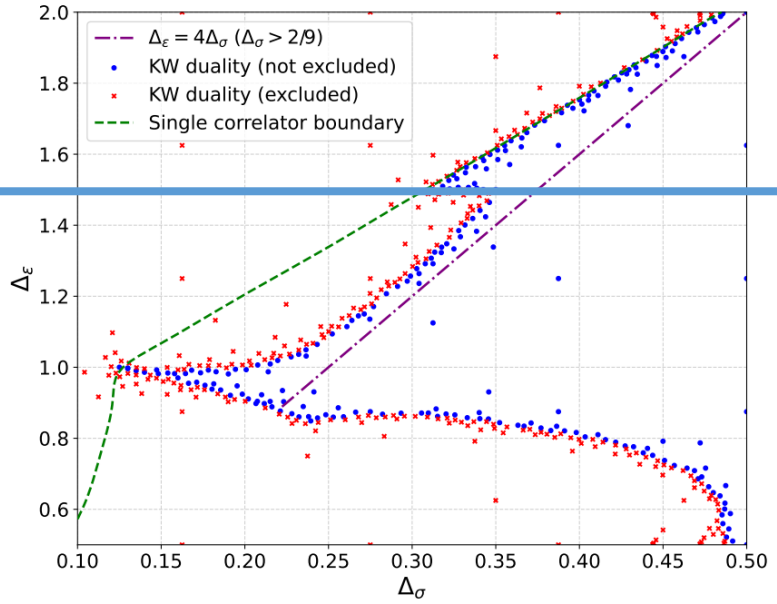
Trivial mix (Su's Perimeter lecture)



Only single correlator bound is effective!

No jump occurs (SDPB does NOT solve the dual problem)

Trivial mix (Su's Perimeter lecture)



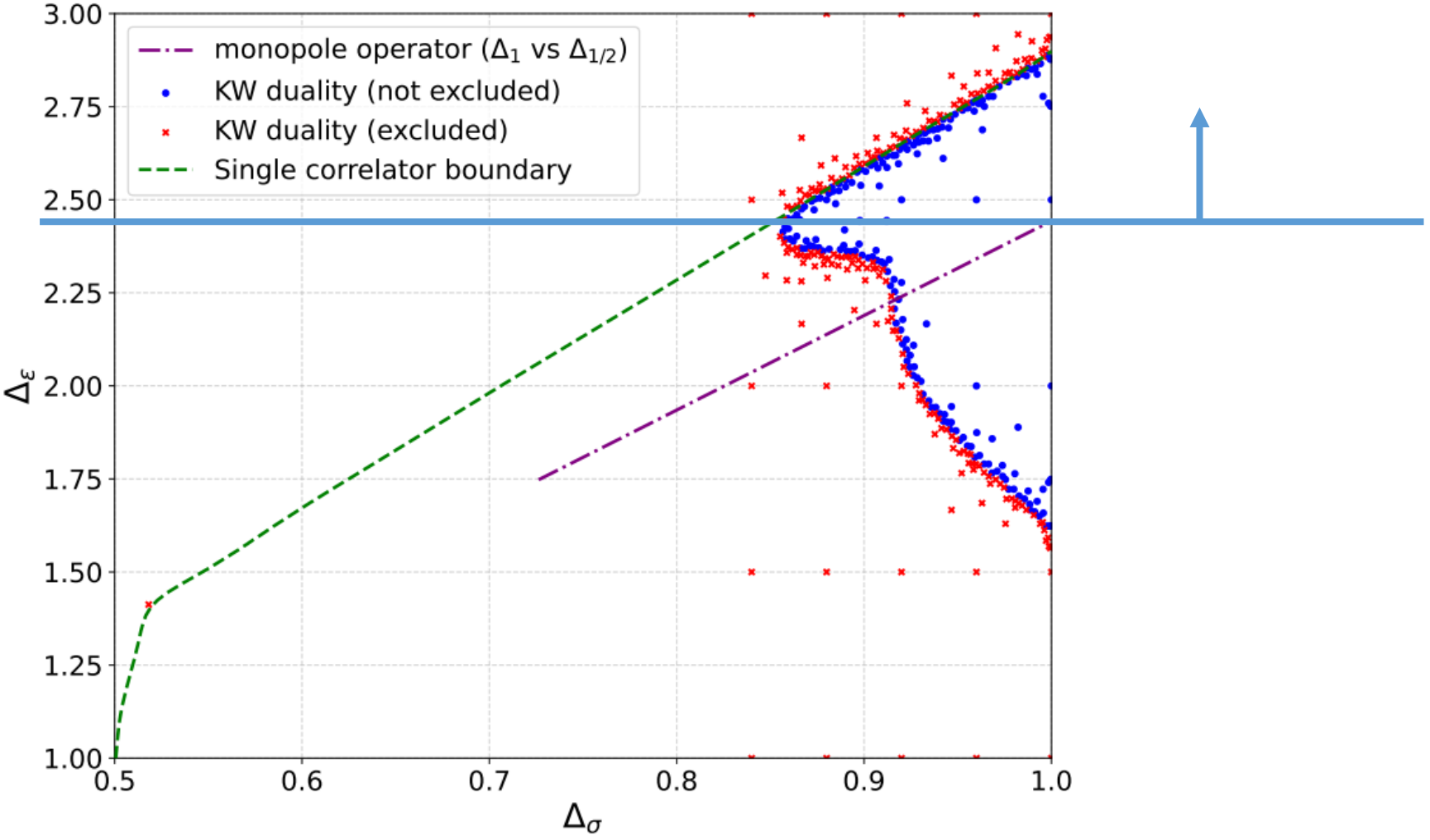
Only single correlator bound is effective!

No jump occurs (SDPB does NOT solve the dual problem)

- Apparently, this has happened in other studies (for larger dimensions)
- If we look at the alpha vector (constraint vector), it is only non-zero in single correlator block
- This does appear in PyCFT boot, Simpleboot, cboot... Maybe an issue with SDPB algorithm?
- According to Su, OPE scan, adding gaps may resolve the issue (in some cases)

d=3

$$\epsilon \times \epsilon = 1 + 0 \times \epsilon + \text{irrelevant}$$



Future directions

- OPE assumed bootstrap (with/without symmetry reasoning) is interesting
- New way to solve fine-tuning problem?

- Other rank 3 fusion category (do we have such CFTs?)
  - Rep S_3

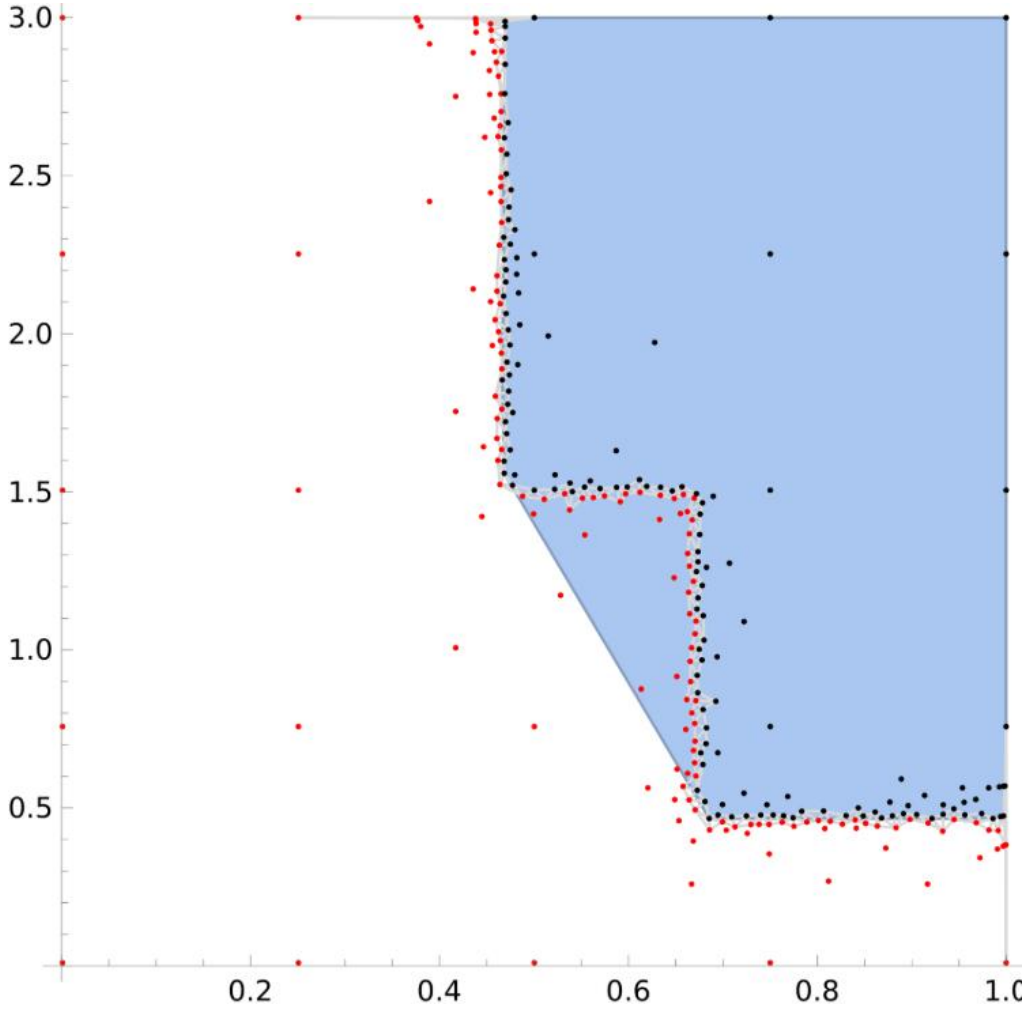
$$\sigma \times \sigma = 1 + \sigma + \epsilon$$
$$\epsilon \times \epsilon = 1$$
$$\sigma \times \epsilon = \sigma$$

  - SU(2)_5

$$\sigma \times \sigma = 1 + \epsilon$$
$$\epsilon \times \epsilon = 1 + \sigma + \epsilon$$
$$\sigma \times \epsilon = \sigma + \epsilon$$

The most ridiculous one (Fibonacci)

$$\sigma \times \sigma = 1 + \sigma + \text{irrelevant}$$
$$\epsilon \times \epsilon = 1 + \text{irrelevant}$$
$$\sigma \times \epsilon = 0$$



I can find the alpha functional that kills this model…