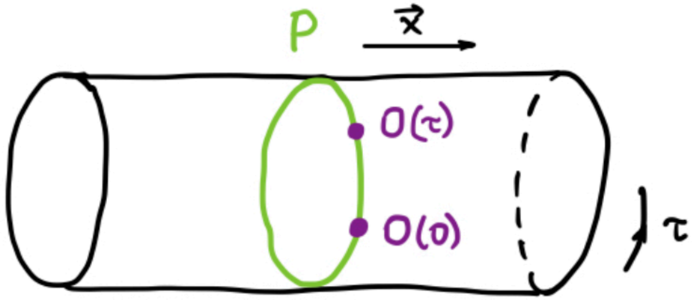
Bootstrap 2026

Bootstrapping the Maldacena-Polyakov loop in N=4 SYM at finite temperature



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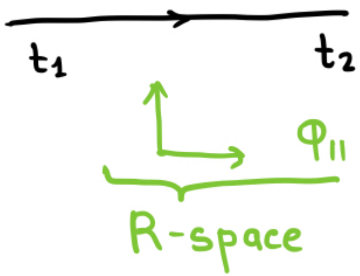
Based on work in progress with J. Barrat, E. Marchetto and E. Pomoni

Part I: Introducing the Maldacena-Wilson Line at finite temperature

MWL at zero temperature



$$= P\exp \int_{t_1}^{t_2} dt \left[i\, \underbrace{A_M(t)}\, \dot{x}^M + \underbrace{\varphi_\parallel(t)}\, |\dot{x}| \right]$$

FIELDS OF $\mathcal{N}=4$ SYM

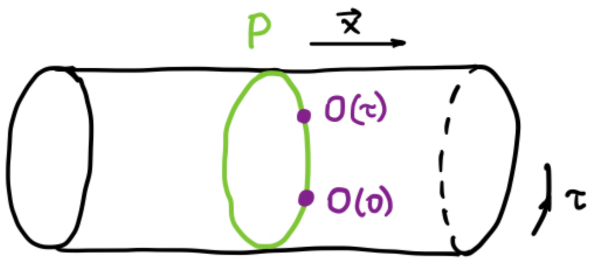
Symmetries: $\underbrace{SO(1,2)}_{\text{1d conformal group}} \times \underbrace{SO(3)}_{\text{space-time rotation}} \times SO(5)$

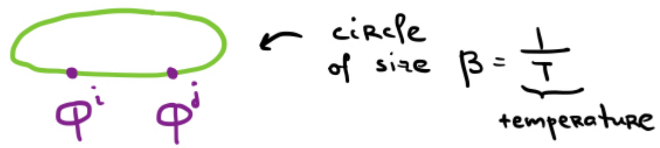
Scalar fields: $\varphi^m \cong \varphi^m_\perp \quad m = 1, \dots, 5 \qquad \varphi_\parallel$

operator insertions on the line

$$\longrightarrow \varphi_\parallel \longrightarrow \varphi_\parallel \longrightarrow \; = \; \frac{1}{|t_{12}|^{2\Delta_{\varphi_\parallel}}}$$

Introducing Temperature





- $g(\tau) = \langle \varphi^i(0)\, \varphi^j(\tau) \rangle_\beta$: non-trivial object after the compactification of the time direction

- OPE: $\varphi^i(0)\, \varphi^j(\tau) = \dfrac{1}{\tau^{2\Delta_\varphi}} \sum_{O} \underbrace{f_{\varphi\varphi O}}_{\text{zero-temperature OPE coefficient}}\, \tau^{\Delta}\, \underbrace{\Pi^{i_1 \dots i_s}\, O_\Delta^{i_1 \dots i_s}(\tau)}_{\text{one-point functions}}$

- $\langle \varphi^i(0)\, \varphi^j(\tau) \rangle_\beta = \dfrac{1}{\tau^{2\Delta_\varphi}} \sum_{\Delta} f_{\varphi\varphi O} \langle O_\Delta \rangle_\beta \left(\dfrac{\tau}{\beta}\right)^{\Delta} =$

$$= \frac{1}{\tau^{2\Delta_\varphi}} \sum_{\Delta} \underbrace{a_\Delta}_{\text{thermal OPE coefficient}} \left(\frac{\tau}{\beta}\right)^{\Delta}$$

$$a_\Delta = f_{\varphi\varphi O}\, b_O \qquad \langle O_\Delta(\tau) \rangle_\beta = \frac{b_O}{\beta^{\Delta}}$$

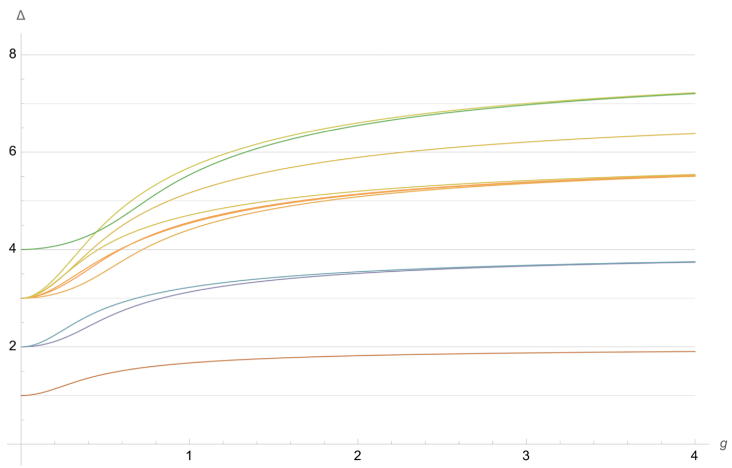
The zero-temperature data is rich

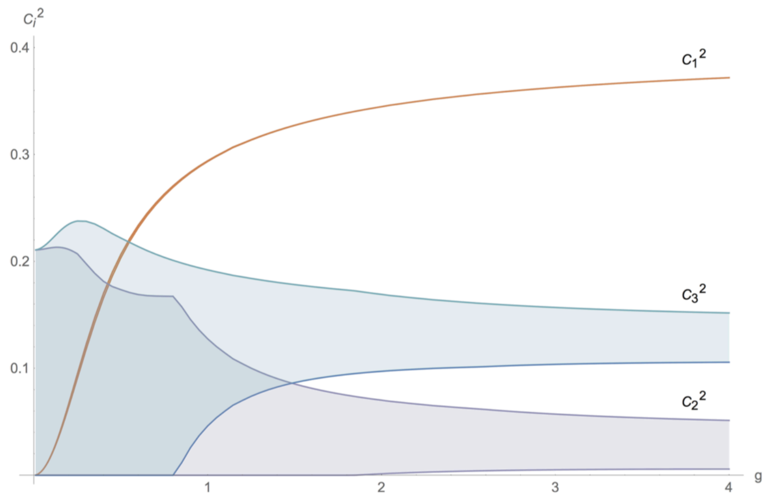
$$\langle \varphi^i(0)\,\varphi^j(\tau)\rangle_\beta = \frac{1}{\tau^{2\Delta_\varphi}} \sum_\Delta f_{\varphi\varphi O}\, b_O \left(\frac{\tau}{\beta}\right)^\Delta = \frac{1}{\tau^{2\Delta_\varphi}} \sum_\Delta a_\Delta \left(\frac{\tau}{\beta}\right)^\Delta$$

Δ : zero-temperature spectrum

$f_{\varphi\varphi O}$: zero-temperature OPE-coefficient

b_O - one-point function – "new" observable at finite temperature





Bootstrability program [Cavaglià, Gromov, Julius, Preti' 21-23]

Exchanged operators

Let us see in more details the exchanged spectrum in the planar limit (large-N) at finite temperature.

Multi-trace operators consisting of operators inserted into the line and bulk $\mathcal{N}=4$ operators projected.

} operator inserted to the line.

} m-trace bulk operators inserted next to the line

$$f_{\varphi\varphi\mathcal{O}_{\text{exchanged}}} \sim \frac{1}{N^m} \qquad b_{\mathcal{O}_{\text{exchanged}}} \sim N^m,$$

so that $a_{\mathcal{O}_{\text{exchanged}}} \sim 1$, and all operators can contribute in principle based on this counting.

Exchanged low-lying operators

$\Delta = 1$:

φ_{11} NON-protected operator
single-trace

$\Delta = 2$:

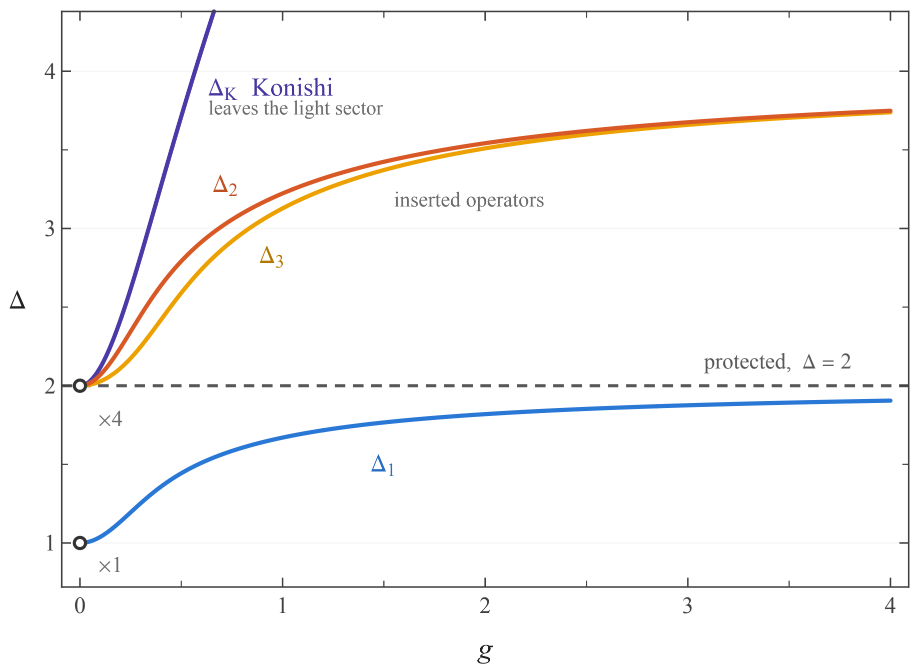
$O^+ = \varphi^i\varphi^i + \sqrt{5}\,\varphi_{11}^2$ non-protected operator
single-trace

$O^- = \varphi^i\varphi^i - \sqrt{5}\,\varphi_{11}^2$ single-trace

$O_K = \mathrm{tr}\left[\varphi^i\varphi^i + \varphi_{11}^2\right]$ double-trace

$O_{20'} = \mathrm{tr}\left[\varphi^i\varphi^i - 5\,\varphi_{11}^2\right]$ protected operator of Δ
double-trace

$\langle O_{20'} \rangle = 0$

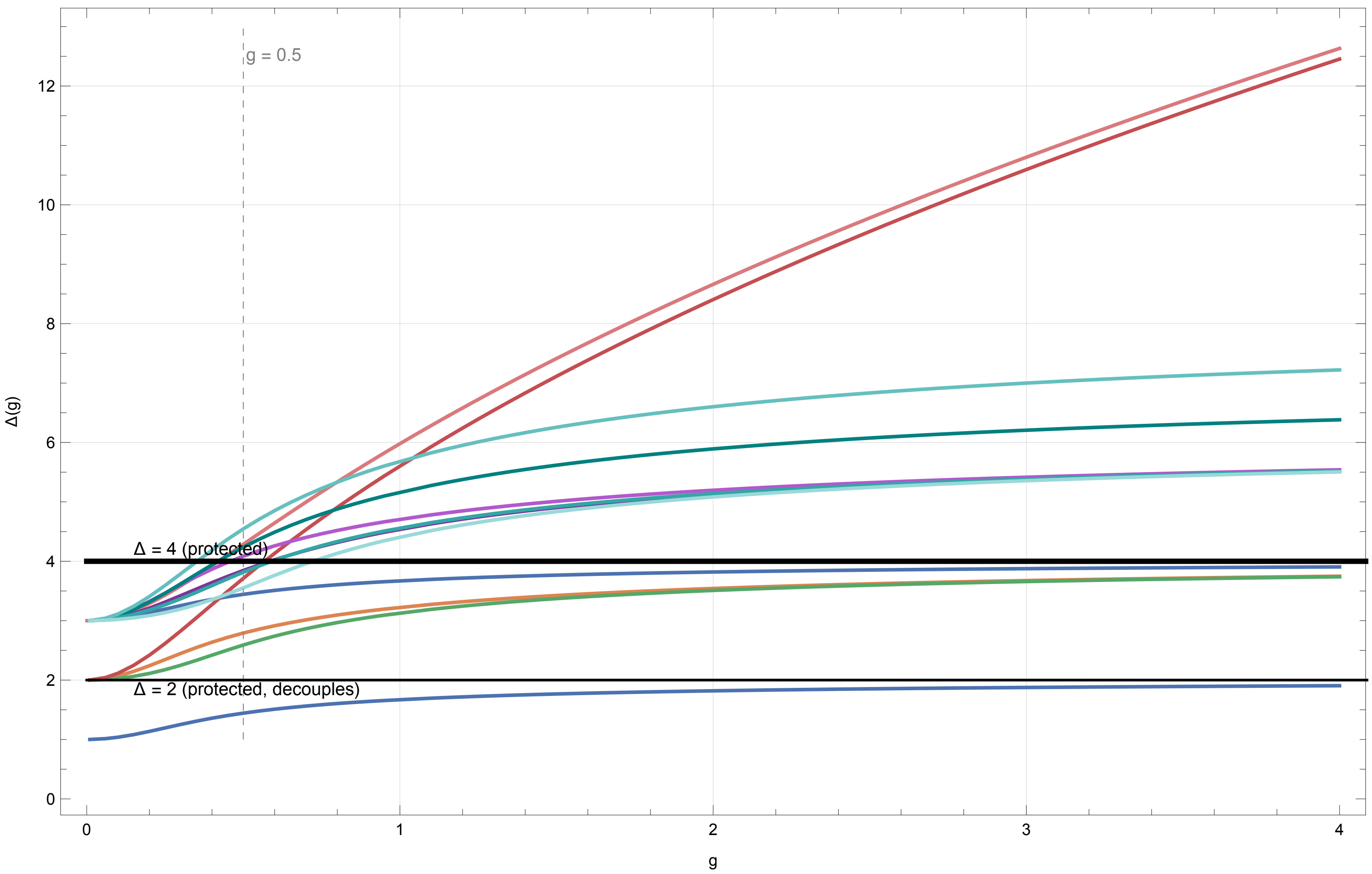


Coupling conventions: $g = 16\pi^2\lambda^2$

In the planar limit, Δ_{O^+}, Δ_{O^-}, Δ_K, are available from integrability, in particular from the Quantum Spectral Curve.

[Gromov, Kazakov, Vieira'13]
[Gromov, Kazakov, Leurent, Volin' 14]
[Cavaglià, Gromov, Julius, Preti' 21]

Spectral overview



Part II: Perturbative results

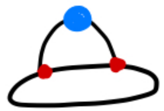
Introducing the perturbative computation

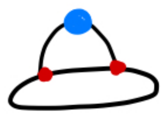
Tree-level Revision: THERMAL PROPAGATOR  $= \frac{1}{4\pi^2} \sum_m \frac{1}{(\tau + m\beta)^2}$

$g^{(0)}(\tau) = \frac{\lambda}{n_1^{(0)}}$  $= \left(\frac{\pi}{\beta}\right)^2 \csc^2\left(\frac{\pi\tau}{\beta}\right)$

$$a_\Delta^{(0)} = \begin{cases} 2(\Delta-1)\xi_\Delta & \text{if } \Delta \text{ even} \\ 0 & \text{if } \Delta \text{ odd} \end{cases} \quad ; \quad a^{(0)}_{\varphi_{11}} = 0$$

One-loop ORDER:

$g^{(1)}(\tau) = \frac{1}{n_1}\Big[\lambda$  $+ \lambda^2\Big($  $+$  $\Big)\Big] =$

$= \Big[\frac{\pi^2}{3}$  $+ 8\pi^2\Big($  $+$  $\Big)\Big]\lambda.$

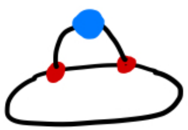
tree-level Self-energy Gluon exchange

The self-energy diagram

Self-energy diagram.

$$-\frac{5}{2} I_1(\tau) \qquad -2 I_1(\tau) \qquad \frac{1}{2} I_2(\tau) \qquad - I_3(\tau)$$

The integrals $I_j(\tau)$ can all be reduced to sums of images of zero-temperature integrals.



$$= -\frac{1}{2} \sum_{m=-\infty}^{\infty} B(\tau+m) + \sum_{m_1, m_2} Y(0, m_1, \tau+m_2)$$

$$- 2 \sum_{m_1, m_2} (-1)^{m_1+m_2} Y(0, \tau+m_1, \tau+m_2),$$

where $B(\tau)$ and $Y(\tau_1, \tau_2, \tau_3)$ are known zero-temperature integrals:

$$B(\tau) \sim \int \frac{d\tau'}{\tau'^2 (\tau-\tau')^2} \qquad Y(\tau_1,\tau_2,\tau_3) = \int \frac{d\tau'}{(\tau_1-\tau')^2 (\tau_2-\tau')^2 (\tau_3-\tau')^2}$$

The self-energy diagram

We can perform the remaining sums:

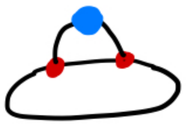
$$S_k^{\pm}(\tau) = \sum_{m=-\infty}^{\infty} (\pm 1)^m \frac{\log\left((\tau+m)^2\right)}{(\tau+m)^k}$$

$$= \frac{1}{64\pi^4}\Big[\pi^2 \csc^2(\pi\tau)\log(\epsilon^2)$$

$$-2\pi^2 \log\left(\pi^2 \csc^2(\pi\tau)\right) - 2\pi^2 \csc^2(\pi\tau) + S_2^{+}(\tau)$$

$$+ 2\left(\pi \cot(\pi\tau) S_1^{+}(\tau) - 2\pi \csc(\pi\tau) S_1^{-}(\tau)\Big] + C$$

Expand around $\tau \sim 0$:



$$\Bigg|_{\text{finite}} = \frac{1}{64\pi^4\tau^2}\sum_{m=0}^{\infty}\left(c_{m,0} + c_{m,1}\log(\tau)\right)\tau^{2m}$$

with $c_{0,0} = -2$ $\quad c_{1,0} = 12 S_2' + 2\pi^2 + \frac{4\pi^2}{3}\log(2) + C$

$c_{0,1} = -2$ $\quad c_{1,1} = \frac{4\pi^2}{3}$

$\vdots$

The gluon exchange diagram

Gluon exchange diagram.

 $= \frac{1}{4} I_4(\tau)$

$$I_4(\tau) = \tilde{I}_4(\tau) + 4 \sum_{m_1, m_2} Y(0, m_1, \tau + m_2)$$

$$\tilde{I}_4(\tau) = \sum_{m_1, m_2} \left(\partial_{\tau_1} - \partial_{\tau_2} \right) \int_0^1 d\tau_4 \ \epsilon(\tau_1, \tau_2, \tau_4) \ Y(\tau_1 + m_1, \tau_2 + m_2, \tau_4) \Big|_{(\tau_1, \tau_2) \to (0, \tau)}$$

The key identity:

$$Y(\tau_1 + m_1, \tau_2 + m_2, \tau_4) = \frac{1}{32\pi^4} \partial_{\tau_4} \mathrm{Re} \left[\frac{1}{\tau_{21} + m_{21}} \left(\mathrm{Li}_2\left(\frac{\tau_{41} - m_1}{\tau_{21} + m_{21}} \right) - \mathrm{Li}_2\left(\frac{\tau_{42} - m_2}{\tau_{12} + m_{12}} \right) \right) \right]$$

allows integration by parts.

Rethinking the integral

$$\tilde{I}_4(\tau) = \frac{1}{8\pi^4} \sum_{(m_1,m_2)\in\mathbb{Z}^2} \partial_u \left[\mathrm{Re}\, \frac{1}{\omega} \left[\mathrm{Li}_2 \frac{-m_1}{\omega} + \mathrm{Li}_2 \frac{m_2}{\omega} - \mathrm{Li}_2 \frac{u-m_1}{\omega} - \mathrm{Li}_2 \frac{u+m_2}{\omega} \right] \right]$$

with $\omega = u + m_2 - m_1$

Focusing on the log terms, they come from

$$(m_1, m_2) = \begin{matrix} (0, m) \\ (m, 0) \\ (m, m). \end{matrix}$$

$$\tilde{I}_4(\tau)\Big|_{\tau^{2n}\log\tau} = \frac{2n+1}{8\pi^4}\left(4 H_{2n+1} + \frac{2}{n+1}\right)\zeta(2n+2)$$

The two-point function at the one-loop order

Final one-loop assembly.

$g^{(1)}(\tau) = \frac{1}{n_1}\Big[\lambda$  $+ \lambda^2\Big($  $+$  $\Big)\Big] =$

$= \frac{\pi^2}{3}$  $+ 8\pi^2\Big($  $+$  $\Big).$

tree-level   Self-energy   Gluon exchange

$$g^{(1)}(\tau) = C + \frac{1}{4}\log(\tau) + \sum_{n\geq 1}\Big[\mathcal{I}_n \log\tau + \underbrace{C_n}_{\text{numerical}}\Big]\tau^{2n}$$

$$\mathcal{I}_n = \frac{\zeta(2n+2)}{4\pi^2}\Big[(2n+1)\Big(4H_{2n+1} + \frac{2}{n+1}\Big) - 16 + 4^{1-n}\Big] \qquad n \geq 1$$

$$C_1 = -1.1405323662 \qquad \mathcal{I}_1 = \frac{\pi^2}{36}, \quad \mathcal{I}_2 = \frac{19\pi^4}{2160}, \quad \mathcal{I}_3 = \frac{4813\pi^6}{3024000}$$

$$C_2 = -1.1211375969$$

Thermal OPE coefficients

OPE expansion: $$g(\tau) = \frac{1}{\tau^2} \sum_{\Delta} a_\Delta \left(\frac{\pi\tau}{\beta}\right)^{\Delta}$$

$$\mathcal{I}_n = \pi^{2n+2} \langle a^{(0)} \gamma^{(1)} \rangle_{\Delta=2n+2} \qquad C_n = \pi^{2n+2} \left[\langle a^{(1)} \rangle + \langle a^{(0)} \gamma^{(1)} \rangle \ln \pi \right]_{\Delta=2n+2}$$

$\Delta_0 = 1$: $\quad a^{(0)}_{\varphi_{11}} = 0 \qquad \langle a^{(0)} \rangle_{\Delta=3,5,7\ldots} = 0$

$\qquad a^{(1)}_{\varphi_{11}} = 0 \qquad \langle a^{(1)} \rangle_{\Delta=3,5,7\ldots} = 0$

$\Delta_0 = 2$: $$\boxed{\frac{1}{4}} = \pi^2 \langle a^{(0)} \gamma^{(1)} \rangle_{\Delta=2} = \pi^2 \Big(\underbrace{a^{(0)}_{O_+}}_{0} \gamma^{(1)}_{O_+} + \underbrace{a^{(0)}_{O_-}}_{0} \gamma^{(1)}_{O_-} + \underbrace{a^{(0)}_{K} \gamma^{(1)}_{O_K}}_{\frac{1}{3} \cdot \frac{3\lambda}{4\pi^2}} \Big) =$$

$$= \boxed{\frac{1}{4}}$$

One-loop prediction matches the anomalous dimension of bulk Konishi.

$\Delta_0 = 4$: $$\langle a^{(1)}_4 \rangle = -0.014930513\,\lambda$$
$$\langle a^{(1)}_6 \rangle = -0.0021864060\,\lambda$$

Thermal OPE coefficients at one-loop order

What about $\langle \Phi_{||} \rangle$?

Let us consider $\langle \varphi'' \rangle_\beta =$  .

scalar propagator: $\langle \varphi^i(\tau)\, \varphi^j(0) \rangle_\beta \sim \frac{\lambda}{2} \int d^3p \int d\omega \, \frac{1}{p^2+\omega^2+m_{th}^2} =$

$$= \frac{m_{th}}{4\pi^2} \sum_m \frac{K_1(m_{th}|\tau + m\beta|)}{|\tau + m\beta|}$$

$\langle \varphi'' \rangle_\beta = \frac{1}{\sqrt{n_{\varphi''}}} Z_{\varphi''}^{-1/2} \Big($  $+$  $+$  $+ \; \dots \Big) =$

$$= \frac{1}{\sqrt{n_{\varphi''}}} Z_{\varphi''}^{-1/2} \left(\frac{\lambda}{2} \frac{m_{th}}{4\pi^2} \sum_m \int_0^\beta d\tau \, \frac{K_1(m_{th}|\tau + m\beta|)}{|\tau + m\beta|} + \dots \right) =$$

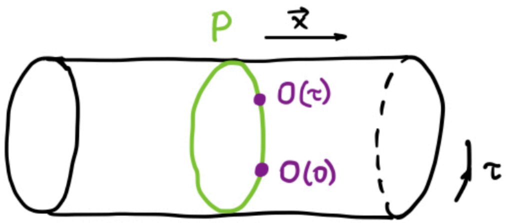
$$= -\frac{\sqrt{\lambda}\, m_{th}}{2\sqrt{2}} + \dots = -\frac{\lambda}{2\sqrt{2}\,\beta} + \dots$$

m_{th}^2 of $\mathcal{N}=4$ $= \frac{\lambda}{\beta^2} + \dots$

$$a_{\varphi''} = f_{\varphi\varphi\varphi''}\, b_{\varphi''} = \frac{\lambda^{3/2}}{8\pi} + \dots$$

Part III: Sum rules

Periodicity (KMS) condition



Periodicity. • If we move the operator $O(\tau)$ a full circle $O(\tau) \to O(\tau+\beta)$, we obtain the same configuration again (KMS condition).

• Combine this property with time reversal $\tau \to -\tau$: $\boxed{g(\tau) = g(\beta-\tau)}$

• The KMS condition around $\frac{\beta}{2}$: $g(\beta/2+\tau) = g(\beta/2-\tau)$ together with the OPE gives the set of sum rules:

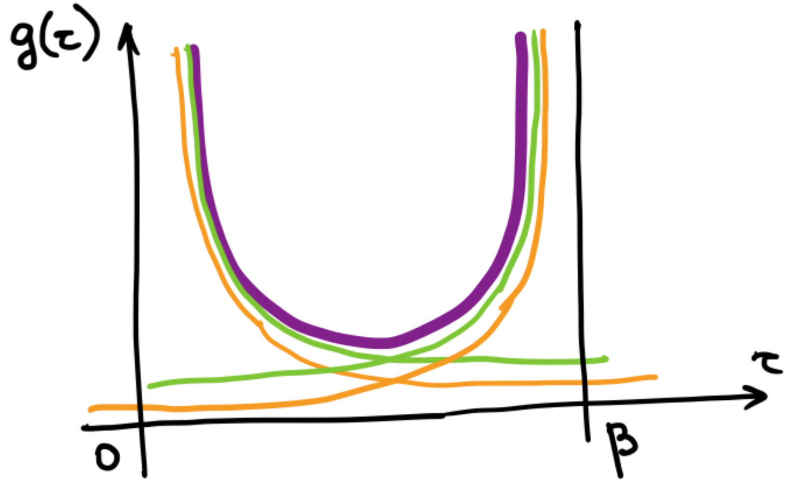
$$\sum_{\Delta} \binom{\Delta - 2\Delta_\varphi}{k} \frac{a_\Delta}{2^\Delta} = 0 \qquad \forall k \in 2\mathbb{N}+1$$

[Marchetto, Miscioscia, Pomoni' 23]
[Barrat, Fiol, Marchetto, Miscioscia, Pomoni' 24]
[Barrat, Marchetto, Miscioscia, Pomoni' 25]

Sum rules

- The idea is to truncate Δ up to Δ_{max}, and approximate the contributions of heavy operators with $\Delta > \Delta_{max}$.

The operators with $\Delta \le \Delta_{max}$ constitute the unknown light sector.



Adding more operators to the light sector improves the approximation.

$$\underbrace{\sum_{\Delta \le \Delta_{max}} \binom{\Delta - 2\Delta_\varphi}{k} \frac{a_\Delta}{2^\Delta}}_{\text{set of unknown } a_\Delta \text{ in the light sector}} + \sum_{\Delta_H > \Delta_{max}} \binom{\Delta_H - 2\Delta_\varphi}{k} \frac{a_{\Delta_H}}{2^{\Delta_H}} \sim 0 \qquad \forall k \in 2\mathbb{N}+1$$

$$a_{\Delta_H} = \frac{\Delta_H^{2\Delta_\varphi - 1}}{\Gamma(2\Delta_\varphi)} \delta\Delta_H$$

[Marchetto, Miscioscia, Pomoni' 23]
[Barrat, Fiol, Marchetto, Miscioscia, Pomoni' 24]
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Outlook

- To see if the sum rules can be used to extract the thermal OPE-coefficients a_Δ for the operators in the light sector. Now we can compare with the explicit one-loop result.

  Can we extract thermal OPE coefficients numerically?

- To consider the $\langle O^i(\tau)\, O^j(0)\rangle_\beta$ correlator at strong coupling.

  [Giombi, Li, Shan' 26]

- To consider $\mathcal{N}=4$ SYM theory.

  First, to do the perturbative computations at finite temperature and to do the spectrum.

Thank you!