Real evolution from imaginary spin

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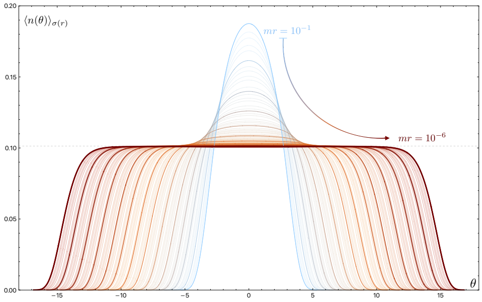
Based on WIP with Alexandre Homrich and David Simmons-Duffin
California Institute of Technology



Bootstrap 2026

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- Example: $e^+e^- \to$ hadrons in QCD

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4. Show how matching and evolution can be interpreted as evolution of a 1d thermodynamic gas for 2d integrable models
5. Test/demonstrate these ideas in the 2d Ising CFT

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- Detectors provide the analytic continuation of operators to complex spin [Caron-Huot '17], [Kravchuk, Simmons-Duffin '18]
- Has applications in conformal Regge theory [Costa et al. '12], conformal collider bounds [Hofman, Maldacena '08], QCD, and more (see [Moult, Zhu 2506.09119])

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- This talk: we will take $\mathcal{D}^{\mathrm{IR}}$ to be a massive detector, and $\mathcal{D}_i^{\mathrm{UV}}$ to be a massless detector

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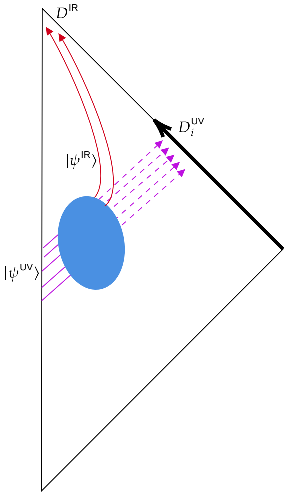

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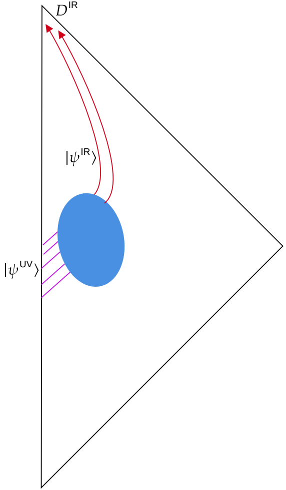

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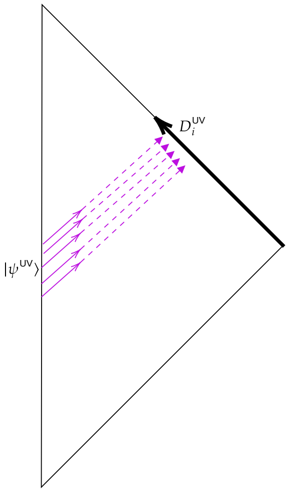

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- Decompose $n(\theta)$ into Lorentz irreps. In 2d, this is a Fourier transform:

$$\hat{n}(J_L) = \int_{-\infty}^{\infty} e^{J_L\theta} n(\theta)$$

  where $J_L \in i\mathbb{R}$ is a boost weight that later will be generalized to complex values

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- Detector matching respects Lorentz symmetry:

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- In the small r limit, the light-ray operators with minimal $\Delta_{L,i}$ will dominate.

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- This then implies the differential equation (defining $t \equiv -\log r$)

$$\left(\frac{\partial^2}{\partial t^2} - J_L^2\right) \langle \hat{n}(J_L) \rangle_{\mathcal{O}(r)} = 0 + \text{subleading}$$

  which after transforming back to θ space, becomes

$$\left(\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial \theta^2}\right) \langle n(\theta) \rangle_{\mathcal{O}(r)} = 0 + \text{subleading}$$

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- Derivation only depends on $\Delta_{L,i}$, which is a property of the UV theory
- Does not depend on the RG flow nor $\mathcal{O}$.
- Subleading effects come from subleading trajectories and RG-dependent terms coming from conformal perturbation theory
- Alternatively, including subleading trajectories modifies the differential operator on the LHS, but those modifications reduce back to the massless wave equation in the small r limit

Thermodynamic gas interpretation for integrable models

- Two point functions have the following form factor expansion:

$$\langle \mathcal{O}(r)\mathcal{O}(0)\rangle = \sum_N \frac{1}{N!}\int \frac{d^N\theta}{(2\pi)^N}\,\exp\left(-mr\sum_i \cosh\theta_i\right)|\langle \mathcal{O}(0)|\theta_1,\ldots,\theta_N\rangle|^2$$

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- Generically, form factors factorize in integrable models at large rapidity separation:

$$\lim_{\alpha\to\infty} F_n(\theta_1+\alpha,\ldots,\theta_m+\alpha,\theta_{m+1},\ldots,\theta_n) \propto F_m(\theta_1,\ldots,\theta_m)F_{n-m}(\theta_{m+1},\ldots,\theta_n).$$

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- Thus, the expansion can be thought of as a thermodynamic grand-partition function for a 1d gas with a short range potential given by

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- The term $z(\theta) \equiv \frac{1}{2\pi}\exp\left(-mr\cosh\theta\right)$ is a position-dependent fugacity that confines the gas to a box of size $2L = 2\log\frac{2}{mr}$

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- Then to leading order in r, the density is

$$\langle n(\theta) \rangle_{\mathcal{O}(r)} \approx n_{\text{kink}}(L-\theta) + n_{\text{kink}}(L+\theta) - n_c$$

which obeys the wave equation. In J_L space, this implies

$$\langle \widehat{n}(J_L) \rangle = (mr)^{J_L} \widehat{n}_{\text{kink}}(J_L) + (mr)^{-J_L} \widehat{n}_{\text{kink}}(-J_L) + \text{subleading}.$$

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- Thus, form factor factorization implies a thermodynamic gas in a box

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- Using integrability methods, the σ densities have been computed [Liu '26]:

$$\langle \widehat{n}(J_L) \rangle_{\sigma(r)} = A(J_L) r^{J_L} + B(J_L, y) r^{J_L+1} + (J_L \to -J_L) + O(r^2)$$

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- No $r^{\pm J_L + 1/8}$: $\hat{n}(J_L)$ does not match onto σ trajectory due to $\mathbb{Z}_2$ symmetry

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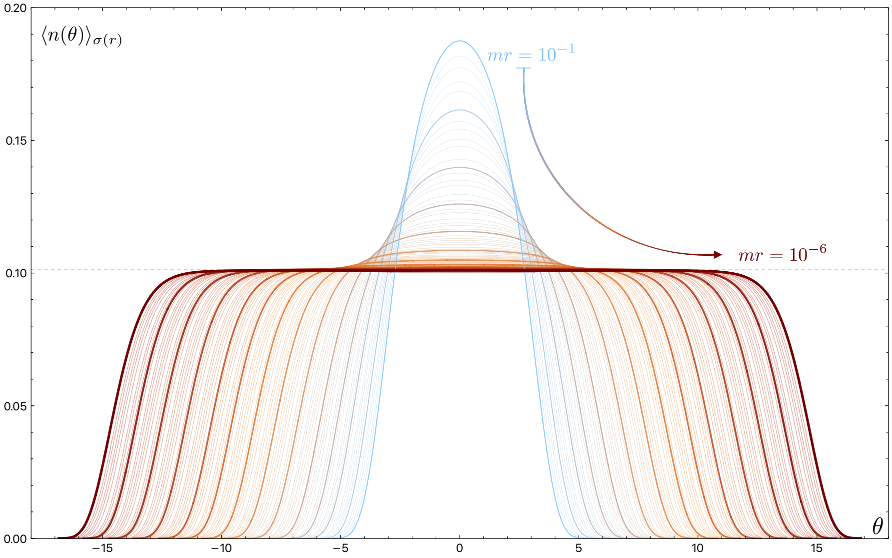

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- There is no $r^{\pm J_L+1}$ term: $\langle\epsilon\epsilon\epsilon\rangle = 0$
- The kink function is

$$\langle n_{\text{kink}}(x)\rangle = e^{x-e^x}$$

which goes to zero as $x \to \pm\infty$. Thus, $\langle\hat{n}_{\text{kink}}(J_L)\rangle$ has no pole at $J_L = 0$

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- Define a particle density for each particle type:

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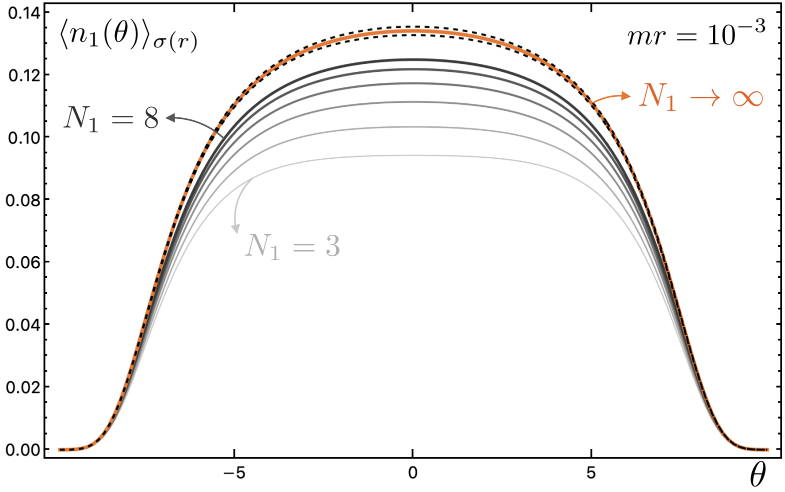
- At fixed t, we perform Monte-Carlo sampling to obtain densities, then extrapolate $N_1 \rightarrow \infty$

σ Densities in the Magnetic Ising



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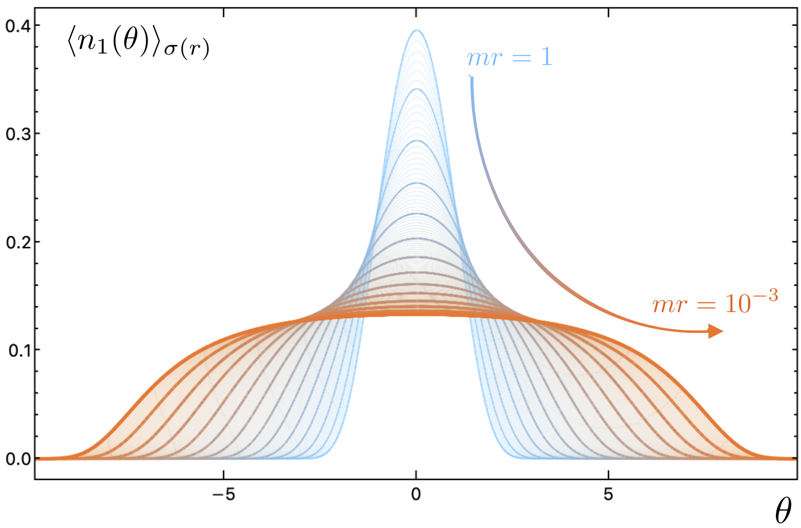

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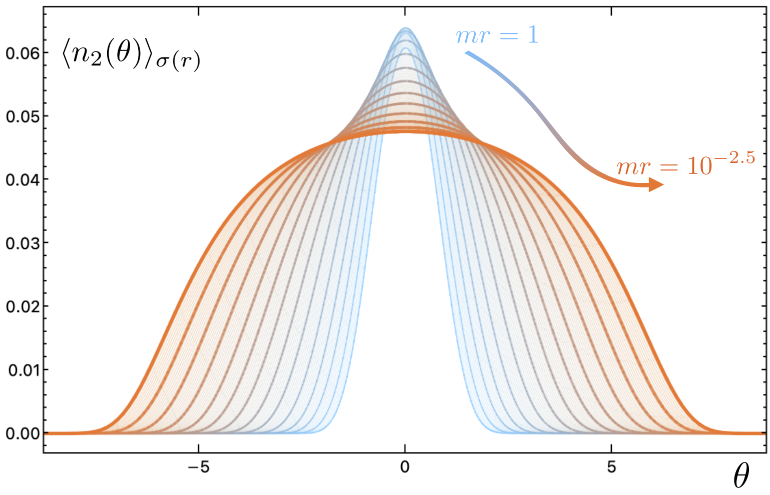

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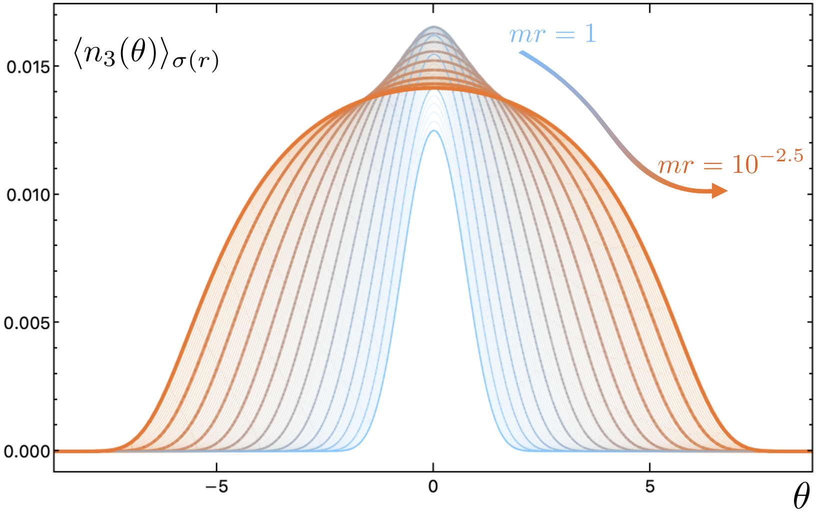

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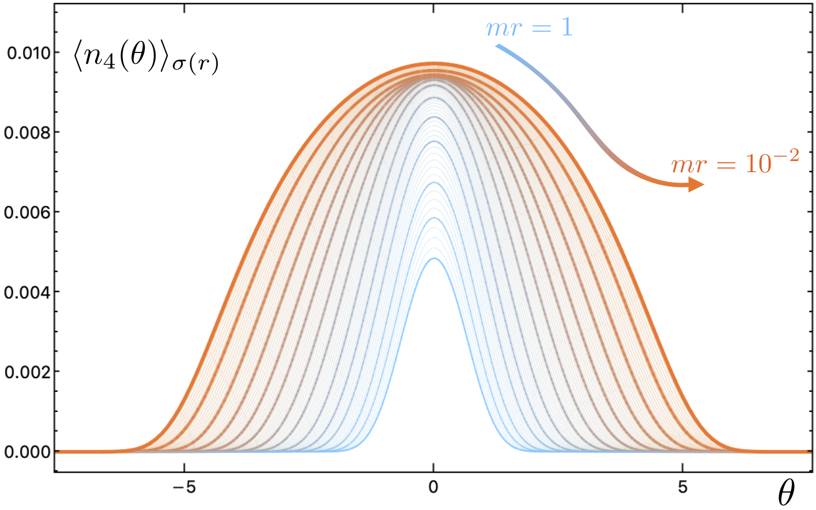

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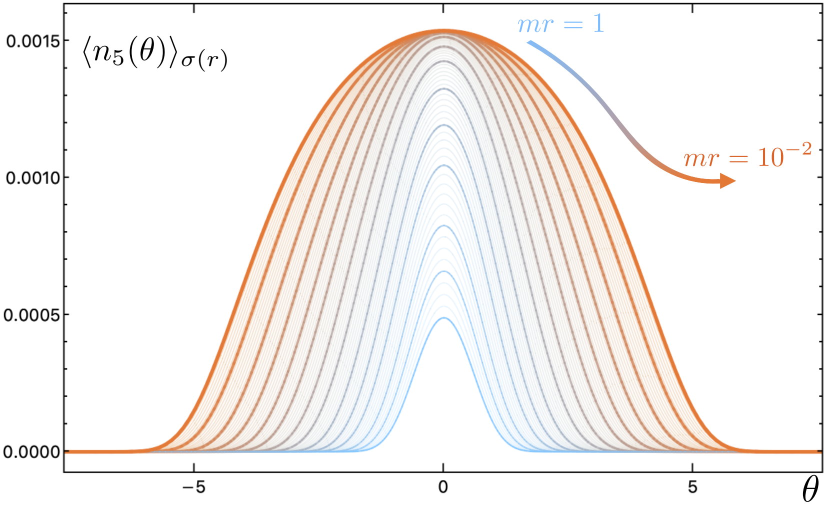
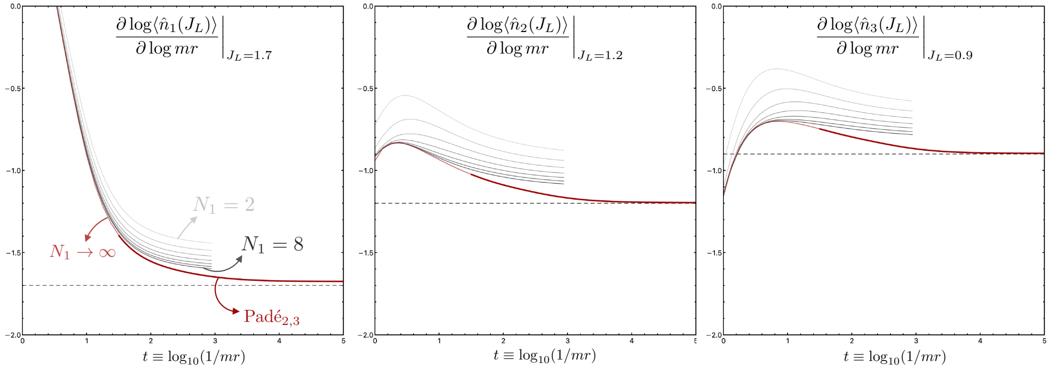


Magnetic Ising harmonic leading harmonic moments

We compute

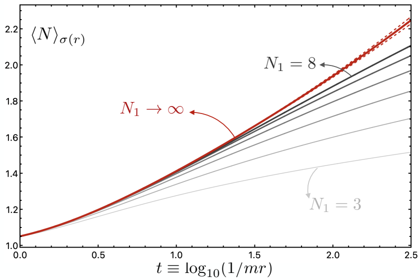
$$\frac{\partial}{\partial t}\log\langle \hat{n}_i(J_L)\rangle_{\mathcal{O}(r)}$$

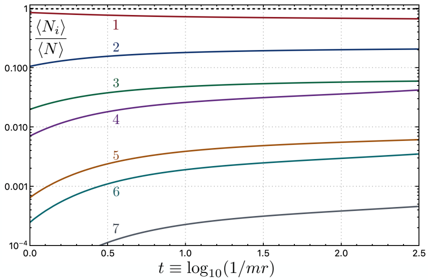
and extrapolate $t \to \infty$ by fitting a (2,3) Pade approximation to extract the leading power-law behavior, which we expect to be $r^{\pm J_L}$.



Magnetic Ising particle number

We also compute total particle number $\langle N \rangle$ and particle fraction $\frac{\langle N_i \rangle}{\langle N \rangle}$. Detector matching of each species individually tells us that in the $t \to \infty$ limit, the particle fractions should asymptote to a constant.





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Thank you!