Dynamics of Heavy Operators and Holography

Based on recent work:

Aprile,Giusto,Russo, “Holographic correlators with BPS bound states in $\mathcal{N}=4$ SYM” Phys.Rev.Lett. 134 (2025)
Aprile,Giusto,Russo, “Four-point correlators with BPS bound states in AdS3 and AdS3” JHEP (2025)
Aprile, Giusto,Russo,VilasBoas “Multi-particle correlators with higher KK modes I: a bootstrap approach” (2026)
Aprile, Giusto,Russo,VilasBoas “Multi-particle correlators with higher KK modes I: a supergravity approach” (to appear)

Aprile, Dorigoni, Treilis,“Dynamics of Heavy operators in $\mathcal{N}=4$ SYM: integrated correlators and AdS bubbles” JHEP (2026)

Bootstrap2026

Francesco Aprile, August 4th

UCM, Madrid

First step towards solving a CFT is to study correlation of local operators (say scalar operators)

“Classic” problem in the bootstrap community

$$\langle O_{\Delta_1}(x_1)O_{\Delta_2}(x_2)O_{\Delta_3}(x_3)O_{\Delta_4}(x_4)\rangle$$

$$\langle \mathbb{H}(x_1)\mathbb{H}(x_2)O_{\Delta_3}(x_3)O_{\Delta_4}(x_4)\rangle$$

Make the operator heavier w.r.t the central charge

- **Semiclassics of the path integral**
- **Large charge expansion**

[Badel, Cuomo, Monin,Rattazzi]
[Hellerman,Orlando] [Rodriguez-Gomez,Russo]
[Benjamin,Lee,Ooguri,Simmons-Duffin]
[Poland,Rogelberg]

$$\langle O_{\Delta_1}(x_1)O_{\Delta_2}(x_1)O_{\Delta_3}(x_1)\ldots O_{\Delta_n}(x_n)\rangle$$

Higher-point correlation functions

Challenges:

- **kinematic space gets bigger;**
- **proliferation of tensor structures in the OPE**

4pt functions have non trivial space-time dependence and non trivial dynamics

$$\langle O_{\Delta_1}(x_1)O_{\Delta_2}(x_2)O_{\Delta_3}(x_3)O_{\Delta_4}(x_4)\rangle$$

$$\langle \mathbb{H}(x_1)\mathbb{H}(x_2)O_{\Delta_3}(x_3)O_{\Delta_4}(x_4)\rangle$$

Make the operator heavier w.r.t the central charge

$$\langle O_{\Delta_1}(x_1)O_{\Delta_2}(x_1)O_{\Delta_3}(x_1)\ldots O_{\Delta_n}(x_n)\rangle$$

Higher-point correlation functions

This talk

I will explain that with a judicious choice of heavy operator

There is a natural bridge connecting HHLL and higher-point correlation functions

4pt functions have non trivial space-time dependence and non trivial dynamics

$$\langle O_{\Delta_1}(x_1)O_{\Delta_2}(x_2)O_{\Delta_3}(x_3)O_{\Delta_4}(x_4)\rangle$$

$$\langle \mathbb{H}(x_1)\mathbb{H}(x_2)O_{\Delta_3}(x_3)O_{\Delta_4}(x_4)\rangle$$

Make the operator heavier w.r.t the central charge

$$\langle O_{\Delta_1}(x_1)O_{\Delta_2}(x_1)O_{\Delta_3}(x_1)\dots O_{\Delta_n}(x_n)\rangle$$

Higher-point correlation functions

Consider

$$\langle e^{\alpha O_\Delta}\, e^{\alpha O_\Delta}\, O_{\Delta_3}O_{\Delta_4}\rangle \qquad \sum_{n,m\geq 0}\frac{\alpha^{n+m}}{n!m!}\langle [O^n_\Delta][O^m_\Delta]O_{\Delta_3}O_{\Delta_4}\rangle$$

In the situation in which the semiclassical problem for the HHLL correlator can be solved

and under favourable analytic conditions

the idea is to extract from it 4pt-multiparticle correlators and get new data for the multipoint bootstrap

$$\langle e^{\alpha O_\Delta}\, e^{\alpha O_\Delta}\, O_{\Delta_3} O_{\Delta_4} \rangle \qquad \longleftrightarrow \qquad \sum_{n,m\geq 0} \frac{\alpha^{n+m}}{n!m!} \langle [O_\Delta^n][O_\Delta^m] O_{\Delta_3} O_{\Delta_4} \rangle$$

Heavy-Heavy-Light-Light — *four-pts with multi-particles*

In Holography this setup is particularly interesting!

because the insertion of $e^{\alpha O_\Delta}(0) e^{\alpha O_\Delta}(\infty)$ **backreacts on AdS and produces a new (asympt. AdS) geometry**

- **study dynamical properties of gravity from correlation functions**
- **understand the role of multi-particle operators**

Exponential and (more generally) coherent state operators appeared already in the literature

Eg. large N QCD [Yaffe] **Eg. Matrix Models** [Berenstein,Vazquez] [Vazquez] [Anempodistov, Holguin, Kazakov, Murali] **Etc…**

Today: I will use this class of operators $e^{\alpha O_\Delta}$ **in an original way. In particular,**

I will establish the connection

$$\langle e^{\alpha O_\Delta}\, e^{\alpha O_\Delta}\, O_{\Delta_3} O_{\Delta_4} \rangle \qquad \longleftrightarrow \qquad \sum_{n,m\geq 0} \frac{\alpha^{n+m}}{n!m!} \langle [O_\Delta^n][O_\Delta^m] O_{\Delta_3} O_{\Delta_4} \rangle$$

in $\mathcal{N}=4$ **SYM which is a paradigmatic example of both a strongly coupled CFT and Holography**

focusing on multi-graviton operators

Outline:

Part I

A CFT bootstrap approach to 4pt correlators with multi-particle operators

Part II

A gravity approach to the HHLL correlator built out of multi-gravitons

$$e^{\alpha \mathcal{O}_2} = \sum_{n=0}^{\infty} \frac{\alpha^n}{n!} \mathcal{O}_2^n$$

Part III

The integrated correlator: towards the emergence of spacetime

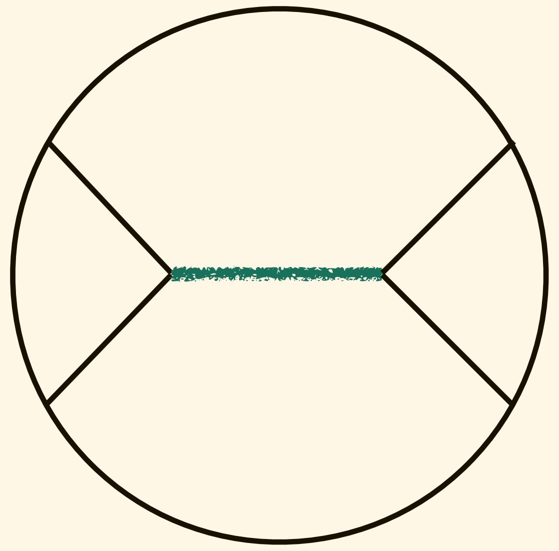
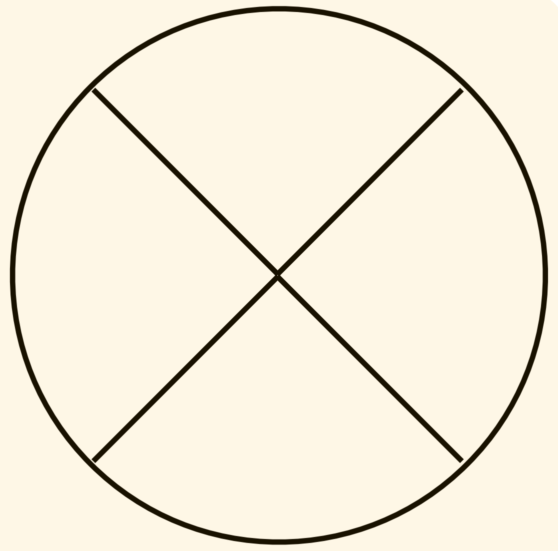
Let’s begin with part I

Part I

A CFT bootstrap approach to 4pt correlators with multi-particle operators

Preliminary comments on supergravity tree-level four-point correlators

from AdS

$\sum_{\text{Bulk } E(\text{xchanged})} C_{p_1 p_2 E} C_{E p_3 p_4}$  $+$ *crossing* $+ \lambda_{p_1 p_2 p_3 p_4}$  $= \quad \langle \mathcal{O}_{p_1} \mathcal{O}_{p_2} \mathcal{O}_{p_3} \mathcal{O}_{p_4} \rangle^{(1)}$

Result = a diagrammatic sum over Witten diagrams

Preliminary comments on supergravity tree-level four-point correlators

from CFT

$$\langle \mathcal{O}_{p_1}\mathcal{O}_{p_2}\mathcal{O}_{p_3}\mathcal{O}_{p_4}\rangle = \mathit{Free} + \begin{pmatrix} \mathit{Kinematic} \\ \mathit{Invariant} \end{pmatrix} \mathcal{H}_{\mathit{reduced}}$$

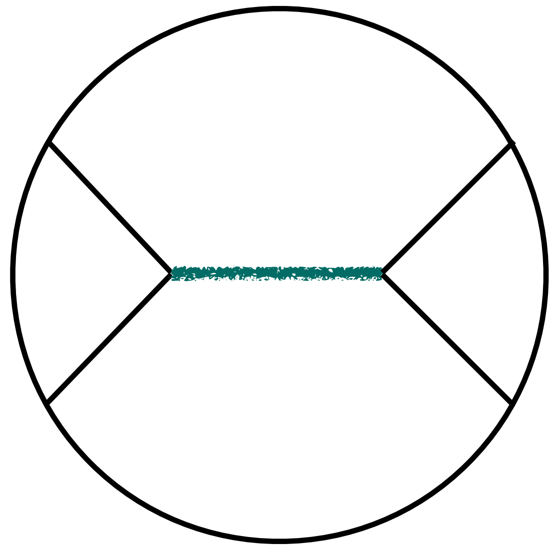
One can use properties of N=4 SYM in the CFT: First, there is a partial non-renormalisation theorem

[Eden,Petkou,Schubert,Sokatchev]
[Howe,Heslop]
[Nirschl,Osborn]

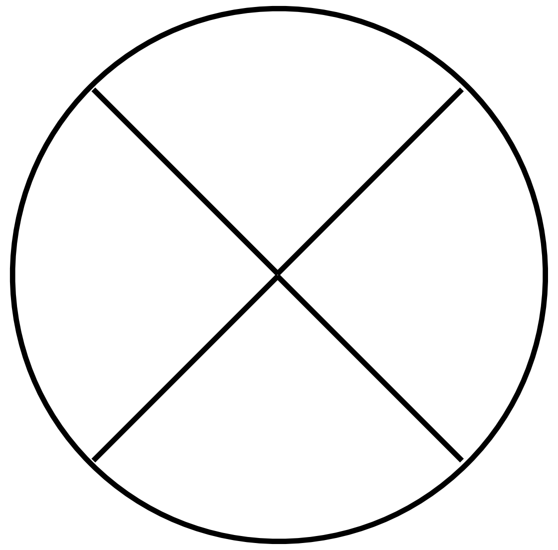
Preliminary comments on supergravity tree-level four-point correlators

from AdS = from CFT

$$\sum_{\text{Bulk } E(\text{xchanged})} C_{p_1 p_2 E} C_{E p_3 p_4}$$



$$+ \; \textit{crossing} \; + \; \lambda_{p_1 p_2 p_3 p_4}$$



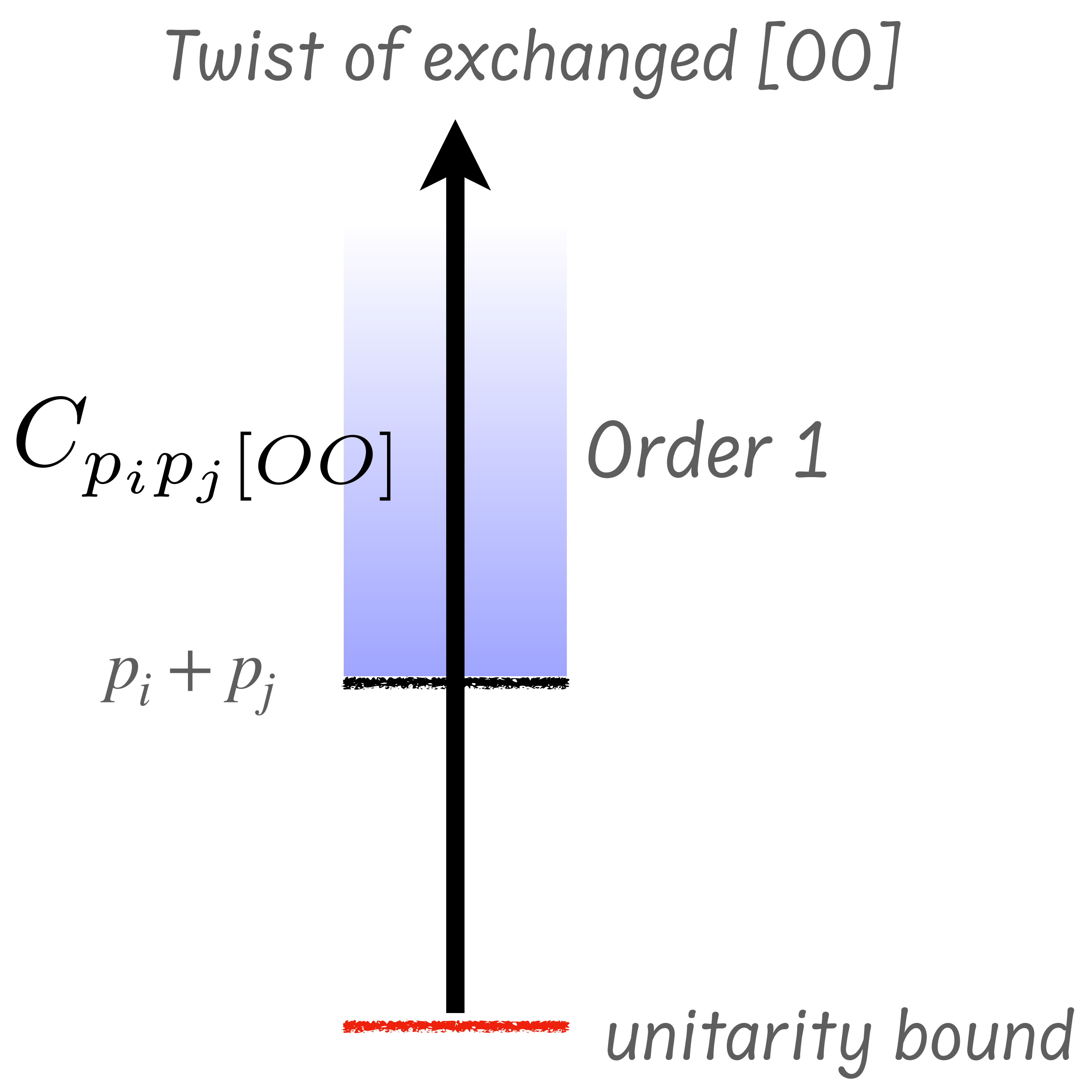
$$= \; \textit{Free} \; + \begin{pmatrix} \textit{Kinematic} \\ \textit{Invariant} \end{pmatrix} \times \mathcal{H}^{(1)}_{\textit{reduced}}$$

Compatibility of the partial non-renormalisation theorem and tree-level Witten diagrams

[Rastelli-Zhou]

is powerful enough to solve the single-particle correlators for arbitrary external single-particle operators

Preliminary comments on supergravity tree-level four-point correlators

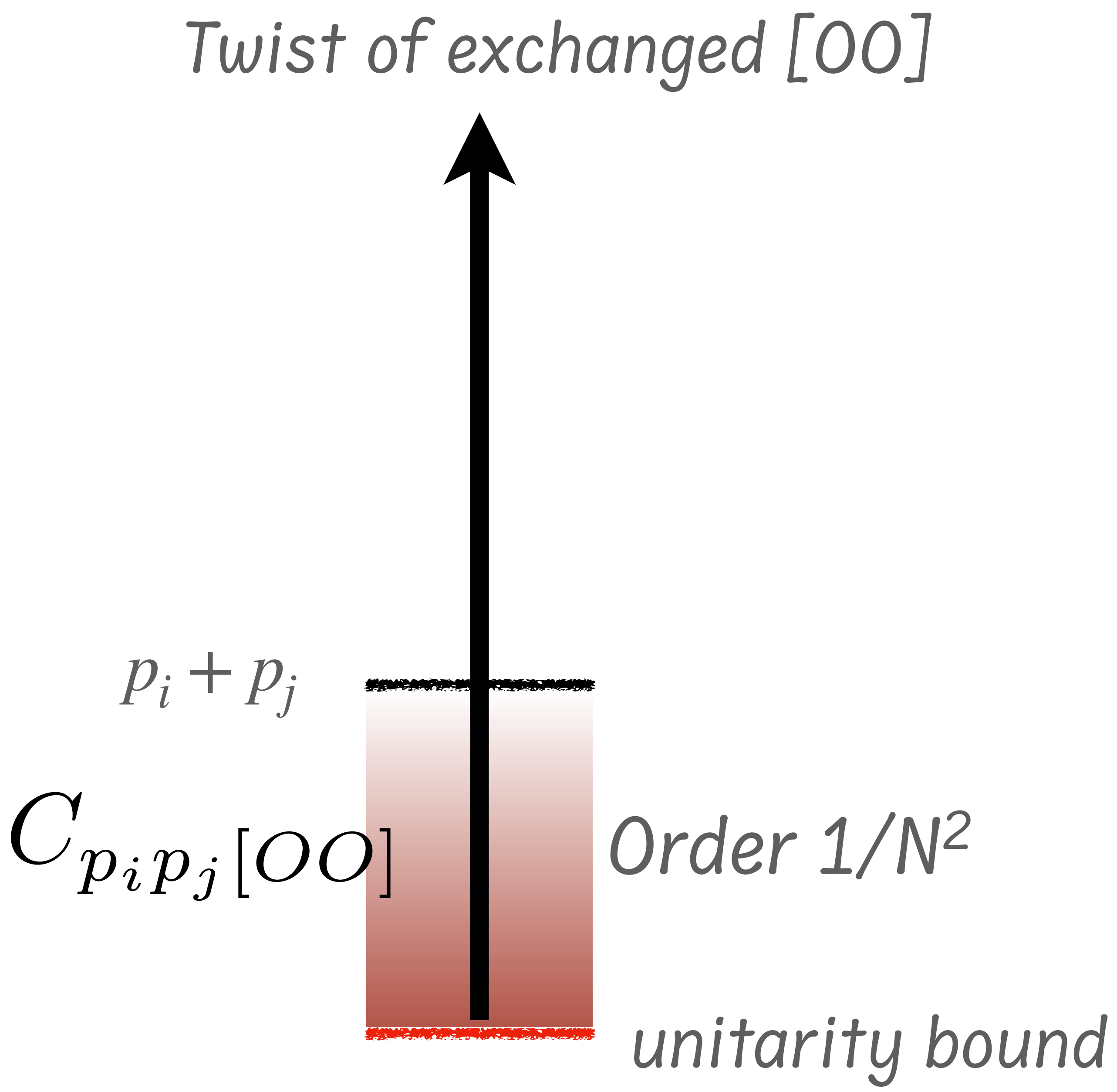


from CFT

$$\langle \mathcal{O}_{p_1} \mathcal{O}_{p_2} \mathcal{O}_{p_3} \mathcal{O}_{p_4} \rangle^{(1)} = \mathit{Free} + \begin{pmatrix} \mathit{Kinematic} \\ \mathit{Invariant} \end{pmatrix} \times \mathcal{H}^{(1)}_{\mathit{reduced}}$$

At tree-level one can use the consistency of the OPE of N=4 SYM

Preliminary comments on supergravity tree-level four-point correlators



from CFT

$$\langle \mathcal{O}_{p_1} \mathcal{O}_{p_2} \mathcal{O}_{p_3} \mathcal{O}_{p_4} \rangle^{(1)} = \text{Free} + \begin{pmatrix} \text{Kinematic} \\ \text{Invariant} \end{pmatrix} \times \mathcal{H}^{(1)}_{\text{reduced}}$$

At tree-level one can use the consistency of the OPE of N=4 SYM

$$\langle \mathcal{O}_{p_1} \mathcal{O}_{p_2} \mathcal{O}_{p_3} \mathcal{O}_{p_4} \rangle^{(1)} \Big|_{\tau = 2a+b+2} = 0$$

In particular, $C_{p_1 p_2 [OO]} C_{p_3 p_4 [OO]}$ **of protected double-particle (semishort) at the unitarity bound in any rep** $[a, b, a]$ **are further suppressed**

[Arutyunov, Frolov,Petkou]
[Dolan, Nirschl,Osborn]

Preliminary comments on supergravity tree-level four-point correlators

from CFT

$$\langle \mathcal{O}_{p_1}\mathcal{O}_{p_2}\mathcal{O}_{p_3}\mathcal{O}_{p_4}\rangle^{(1)} = \text{Free} + \begin{pmatrix}\text{Kinematic}\\ \text{Invariant}\end{pmatrix} \times \mathcal{H}^{(1)}_{\text{reduced}}$$

As a result one finds a

$$\mathcal{H}_{\text{reduced}} = \sum_{d_{ij}} \oint ds_{ij} \prod_{i<j} \frac{\vec{Y}_{ij}^{2d_{ij}}}{\vec{X}_{ij}^{2s_{ij}}} \frac{\Gamma[s_{ij}]}{\Gamma[d_{ij}+1]} \mathcal{M}_{\text{reduced}} \qquad \mathcal{M}^{(1)}_{\text{reduced}} = \frac{1}{(\mathbf{s}+1)(\mathbf{t}+1)(\mathbf{u}+1)}$$

$$\sum_{i\neq j} d_{ij} = p_i - 2 \qquad \sum_{i\neq j} s_{ij} = p_i + 2$$

$$-\mathbf{s} = s_{12} - d_{12} = s_{34} - d_{34}$$

$$-\mathbf{t} = s_{14} - d_{14} = s_{23} - d_{23}$$

$$-\mathbf{u} = s_{13} - d_{13} = s_{24} - d_{14}$$

The formula is remarkably simple in AdSxS Mellin space

[Aprile,Vieira]

A CFT bootstrap approach to tree level 4pt correlators with multi-particle operators

$$\langle [\mathcal{O}_a\mathcal{O}_b]\mathcal{O}_{p_2}\mathcal{O}_{p_3}\mathcal{O}_{p_4}\rangle^{(\frac{3}{2})} = \text{Free} + \begin{pmatrix}\text{Kinematic}\\ \text{Invariant}\end{pmatrix} \times \mathcal{H}^{(\frac{3}{2})}_{\text{reduced}}$$

One double-particle insertion

Then, at tree-level one can use again the consistency of the OPE of N=4 SYM

In particular, 3pt couplings of protected (semishort) double-particle operators at the unitarity bound

$$C_{p_ip_j[OO]_{\tau=2a+b+2}} = \frac{\#}{N^2} + \ldots$$

$$C_{[ab]p[OO]} = C_{bpO}\delta_{aO} + C_{apO}\delta_{bO} + \ldots$$

$$= \frac{\#}{N} + \ldots$$

$$\Longrightarrow \quad \langle [\mathcal{O}_a\mathcal{O}_b]\mathcal{O}_{p_2}\mathcal{O}_{p_3}\mathcal{O}_{p_4}\rangle^{(\frac{3}{2})}\Big|_{\tau=2a+b+2}$$

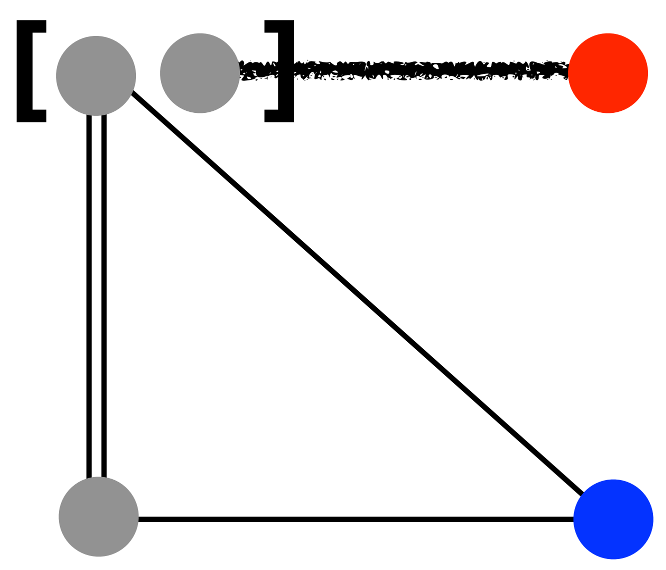
Non trivial prediction!

CFT data of double-particles is know from [Aprile, Drummond, Heslop, Paul]

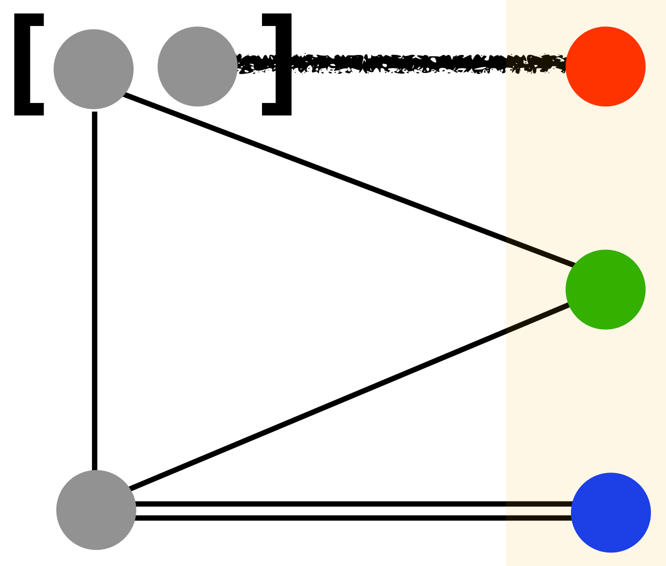
Non-trivial predictions take the following form:

$$\left\langle [\mathcal{O}_a \mathcal{O}_b] \mathcal{O}_{p_2} \mathcal{O}_{p_3} \mathcal{O}_{p_4} \right\rangle^{(\frac{3}{2})}$$

$$= \sum_{m,n} \frac{\left(\langle [\mathcal{O}_a \mathcal{O}_b] \mathcal{O}_{p_2} \mathcal{O}_m \mathcal{O}_n \rangle^{(\frac{1}{2})} \right) \times \left(\langle \mathcal{O}_m \mathcal{O}_n \mathcal{O}_{p_3} \mathcal{O}_{p_4} \rangle^{(1)} \right)}{\left(\langle \mathcal{O}_m \mathcal{O}_n \mathcal{O}_m \mathcal{O}_n \rangle^{(0)} \right)}$$



This contribution is non zero as long as the three-point coupling exists and it is non zero.



Triple-particles

More generally, disconnected contributions vanish as long as the connected lower-point function is near extremal. One can show that this is always the case.

$$\frac{-p_2 + \sum \text{others}}{2} \leq -3 + (n\text{-pts})$$

A non trivial property of single-particle correlators!

[Aprile,Drummond, Paul,Helsop, Sanfilippo Santagata, Stewart]

A CFT bootstrap approach to tree level 4pt correlators with multi-particle operators

$$\langle [\mathcal{O}_a \mathcal{O}_b] \mathcal{O}_{p_2} \mathcal{O}_{p_3} \mathcal{O}_{p_4} \rangle^{(\frac{3}{2})} = \text{Free} + \begin{pmatrix} \text{Kinematic} \\ \text{Invariant} \end{pmatrix} \times \mathcal{H}^{(\frac{3}{2})}_{\text{reduced}}$$

One double-particle insertion

$$\mathcal{M}^{\text{reduced}}_{[2^2]p_2p_3p_4} = \Big(\mathcal{M}_{su} + \text{crossing} \Big) + \mathcal{M}_{stu}$$

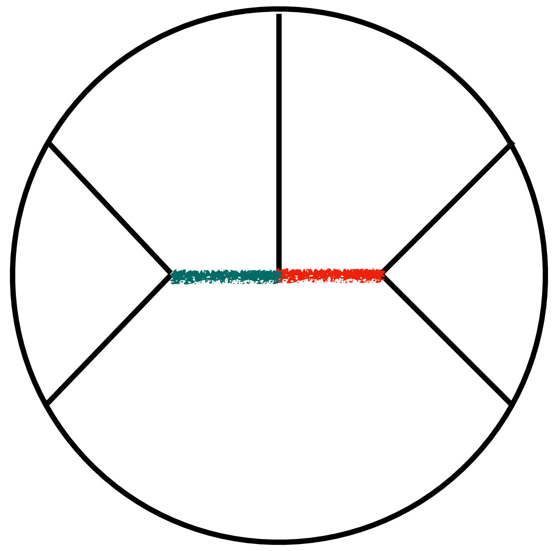
$$\mathcal{M}_{su} = \frac{p_{1,0,1}(d_{ij}, \vec{p})}{(\mathbf{s}+1)(\mathbf{t})(\mathbf{u}+1)} + \frac{p_{1,-1,1}(d_{ij}, \vec{p})}{(\mathbf{s}+1)(\mathbf{t}-1)(\mathbf{u}+1)} + \frac{p_{0,1,0}(d_{ij}, \vec{p})}{(\mathbf{s})(\mathbf{t}+1)(\mathbf{u})} \qquad \mathcal{M}_{stu} = \frac{p_{1,1,1}(d_{ij}, \vec{p})}{(\mathbf{s}+1)(\mathbf{t}+1)(\mathbf{u}+1)}$$

Polynomials of total degree at most 3, and degree at most 2 in the d_{ij}

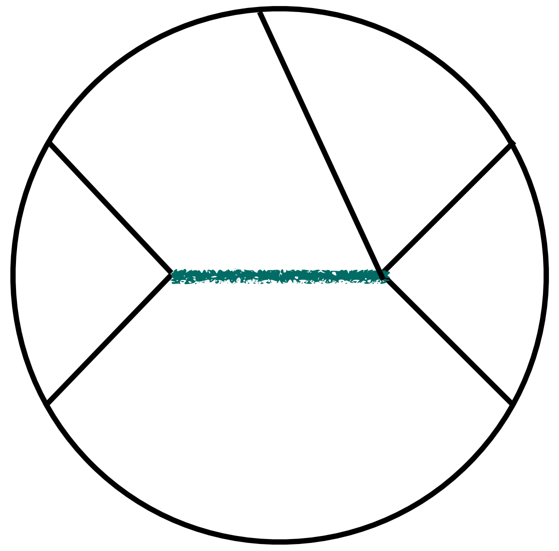
***see backup slides for the explicit expressions**

A CFT bootstrap approach to tree level 4pt correlators with multi-particle operators

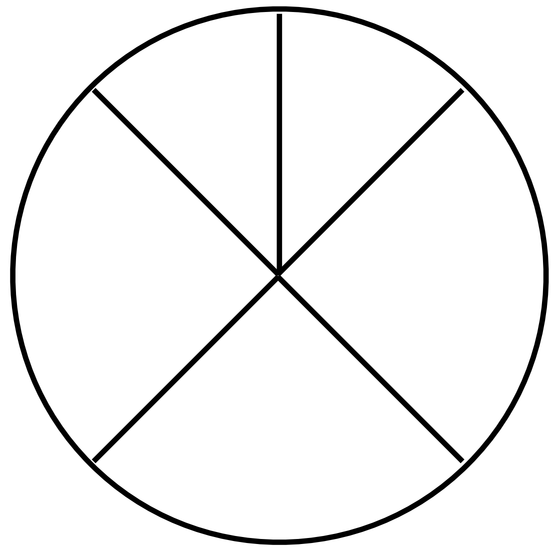
$$\sum_{\text{Bulk } E \text{ and } E} C_{ap_2E}C_{bEE'}C_{E'p_3p_4}$$



$$+ \quad C_{ap_2E}\lambda_{Ebp_3p_4}$$



$$+ \ \textit{crossing} \ + \lambda_{abp_2p_3p_4}$$



[Fernandes,Goncalves,Huang,Tang,VilasBoas,EllisYuan]

The (half-BPS) double-particle limit of 5pt functions

$$\lim_{x_a \to x_b} \left[\lim_{y_a \to y_b} \langle \mathcal{O}_a \mathcal{O}_b \mathcal{O}_{p_2} \mathcal{O}_{p_3} \mathcal{O}_{p_4} \rangle^{(\frac{3}{2})} \right] = \langle [\mathcal{O}_a \mathcal{O}_b] \mathcal{O}_{p_2} \mathcal{O}_{p_3} \mathcal{O}_{p_4} \rangle^{(\frac{3}{2})}$$

$$\lim_{x_a \to x_b} \left[\lim_{y_a \to y_b} \underbrace{\mathcal{M}^{(\frac{3}{2})}_{abp_2p_3p_4}}_{\text{non reduced}} \right] = \textit{Kinematic} \times \underbrace{\mathcal{M}^{(\frac{3}{2})}_{[ab]p_2p_3p_4}}_{\text{reduced}}$$

[Aprile,Giusto,Russo,VilasBoas] part I

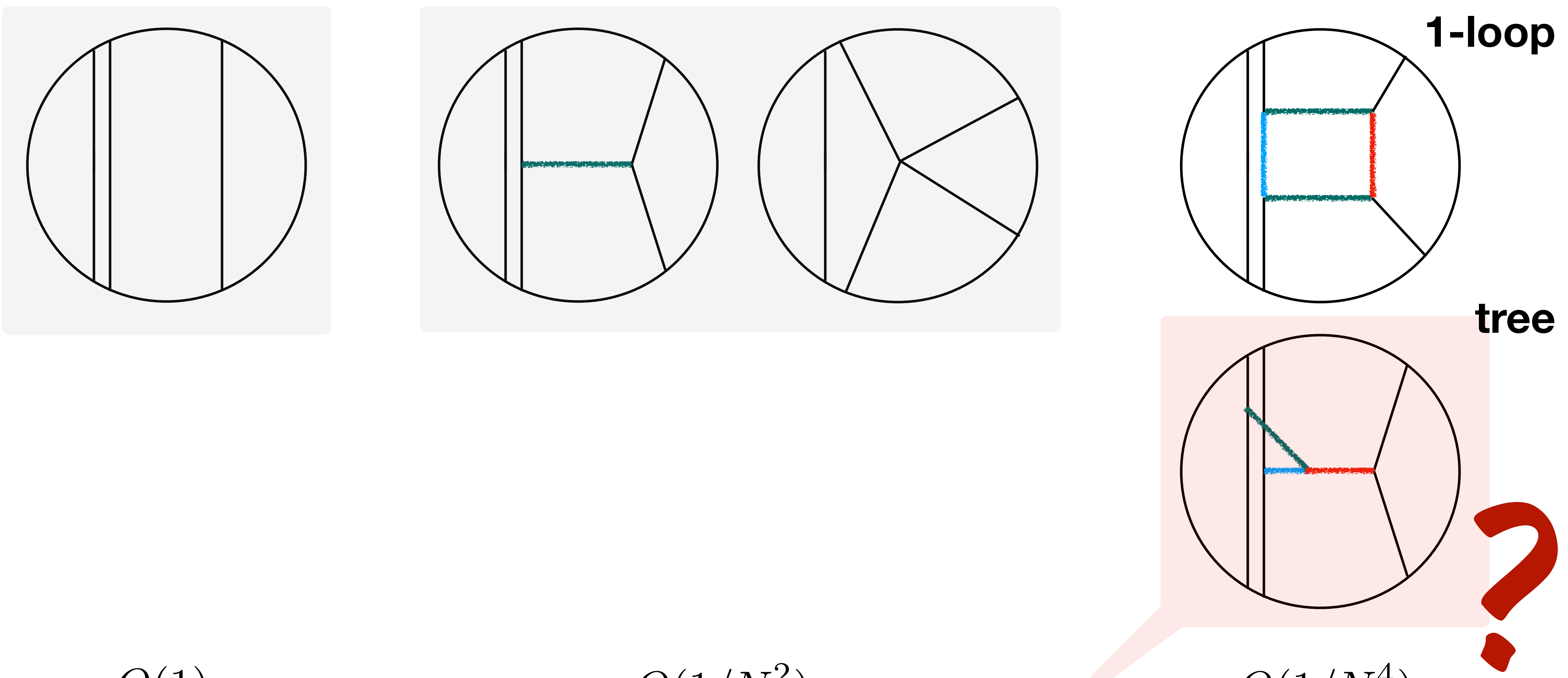
A CFT bootstrap approach to tree level 4pt correlators with multi-particle operators

$$\langle [\mathcal{O}_a\mathcal{O}_b][\mathcal{O}_c\mathcal{O}_d]\mathcal{O}_{p_3}\mathcal{O}_{p_4}\rangle$$

Two double-particle insertions

Disconnected

Connected



$O(1)$ $O(1/N^2)$ $O(1/N^4)$

The double-particle bootstrap is not enough!

Let's move to part II

Part I

A CFT bootstrap approach to 4pt correlators with multi-particle operators

Part II

A gravity approach to the HHLL correlator built out of multi-gravitons

$$e^{\alpha\mathcal{O}_2} = \sum_{n=0}^{\infty} \frac{\alpha^n}{n!}\mathcal{O}_2^n$$

In the next I will show that it is possible to solve the tree-level semiclassical HHLL correlators of the form

$$\langle e^{\alpha O_\Delta}\, e^{\alpha O_\Delta}\, O_{\Delta_3} O_{\Delta_4}\rangle$$

In particular, I will focus on the simplest case: $\langle e^{\alpha O_2}\, e^{\alpha O_2}\, O_p O_p\rangle$

To derive formulae for $\langle [\mathcal{O}_2]^2[\mathcal{O}_2]^2\mathcal{O}_p\mathcal{O}_p\rangle,\ \langle [\mathcal{O}_2]^3\mathcal{O}_2\mathcal{O}_p\mathcal{O}_p\rangle,\ \langle [\mathcal{O}_2]^3[\mathcal{O}_2]^3\mathcal{O}_p\mathcal{O}_p\rangle, \mathrm{etc}\ldots$

Moreover, for given multi-particle levels it will be possible to use CFT predictions from the double-particle spectrum to prove the validity of the analytic continuation when α is infinitesimal.

Let's see first why an heavy state made just by half-BPS CPOs in the graviton multiplet is the simplest

In Holography we construct an AdS background by specifying boundary vevs

$$\langle\langle T_{\mu\nu}\rangle\rangle, \; \langle\langle J_\mu\rangle\rangle, \; \langle\langle \mathcal{O}_2\rangle\rangle$$

... , $\langle\langle \mathcal{O}_p\rangle\rangle$ **with** $p > 2$ **but** $\langle \mathcal{O}_2^m \, \mathcal{O}_p \, \mathcal{O}_2^n \rangle = 0$

A background made just by CPOs in the graviton multiplet belongs to a consistent truncation of AdS5xS5

[Cvetic, Lu, Pope, Sadrzadeh and T. A. Tran]

$$ds_{10}^2 = \Delta^{\frac{1}{2}} ds_5^2 + \Delta^{-\frac{1}{2}} \left(T_{ij}\right)^{-1} D\mu^i D\mu^j \qquad F_{(5)} = G_{(5)} + \star G_{(5)}$$

With all fields in the stress-tensor multiplet turned on, depending only on the AdS coordinates

5d metric **scalars in the** $20'$ **gauge fields in the** 15

Let’s see first why an heavy state made just by half-BPS CPOs in the graviton multiplet is the simplest

In Holography we construct an AdS background by specifying boundary vevs

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... , $\langle\!\langle \mathcal{O}_p\rangle\!\rangle$ **with** $p > 2$ **but** $\langle \mathcal{O}_2^m\, \mathcal{O}_p\, \mathcal{O}_2^n \rangle = 0$

A unique half-BPS solution in this truncation: sometimes called “AdS bubble”

[Liu, Lu, Pope and Vazquez-Poritz]
[Giusto,Rosso]

$$ds_5^2 = H(r)^{-\frac{2}{3}}\big(1 + r^2 H(r)\big)d\tau^2 + H(r)^{\frac{1}{3}}\left(\frac{dr^2}{1+r^2H(r)} + r^2 d\Omega_3^2\right)$$

$$A = \frac{dt}{H(r)}$$

$$\varphi = \sqrt{\frac{2}{3}}\log H(r)$$

$$\eta_{\mathbb{R}} + i\eta_{\mathbb{I}} = \operatorname{arccosh}\left[\frac{1}{2r}\frac{d}{dr}\left[r^2 H(r)\right]\right]$$

$$H(r) = -\frac{1}{r^2} + \sqrt{\left(1+\frac{1}{r^2}\right)^2 + \frac{2Q}{r^2}}$$

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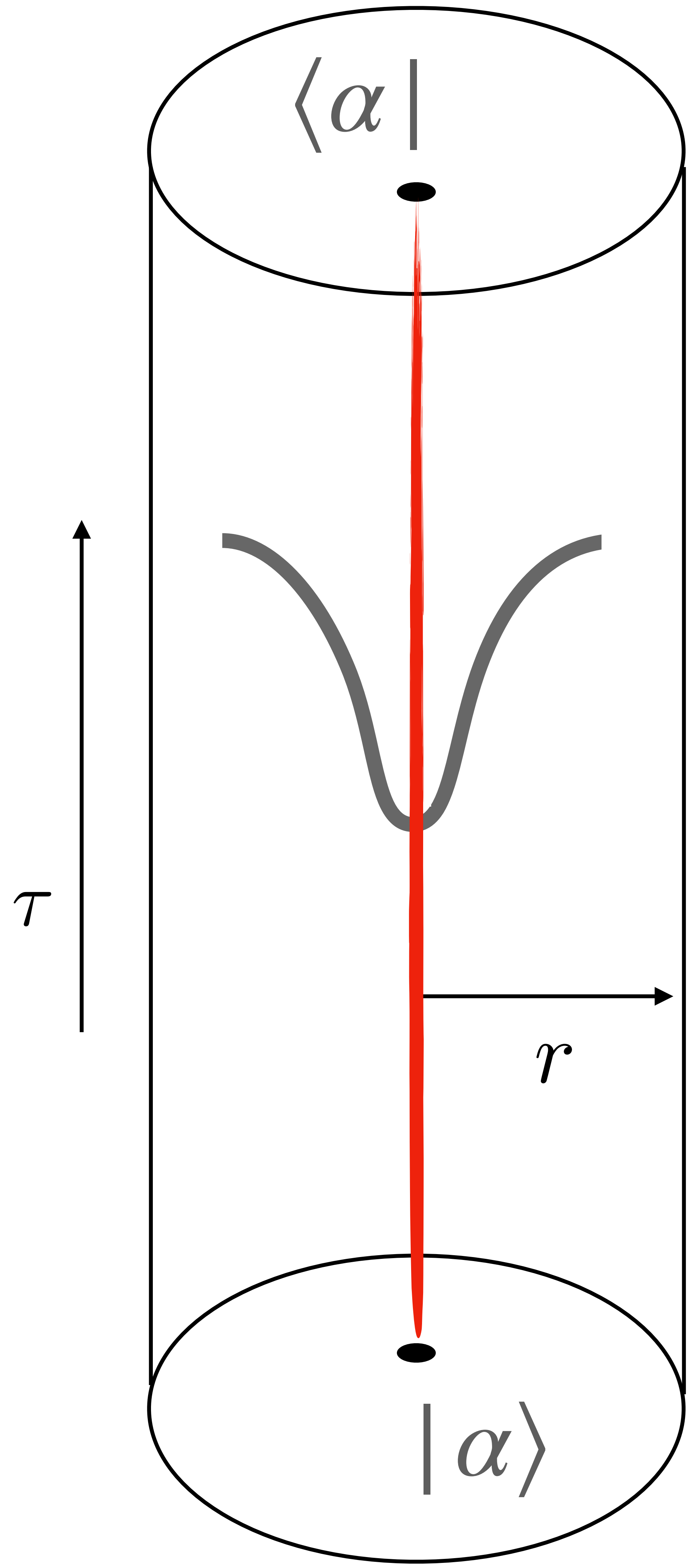
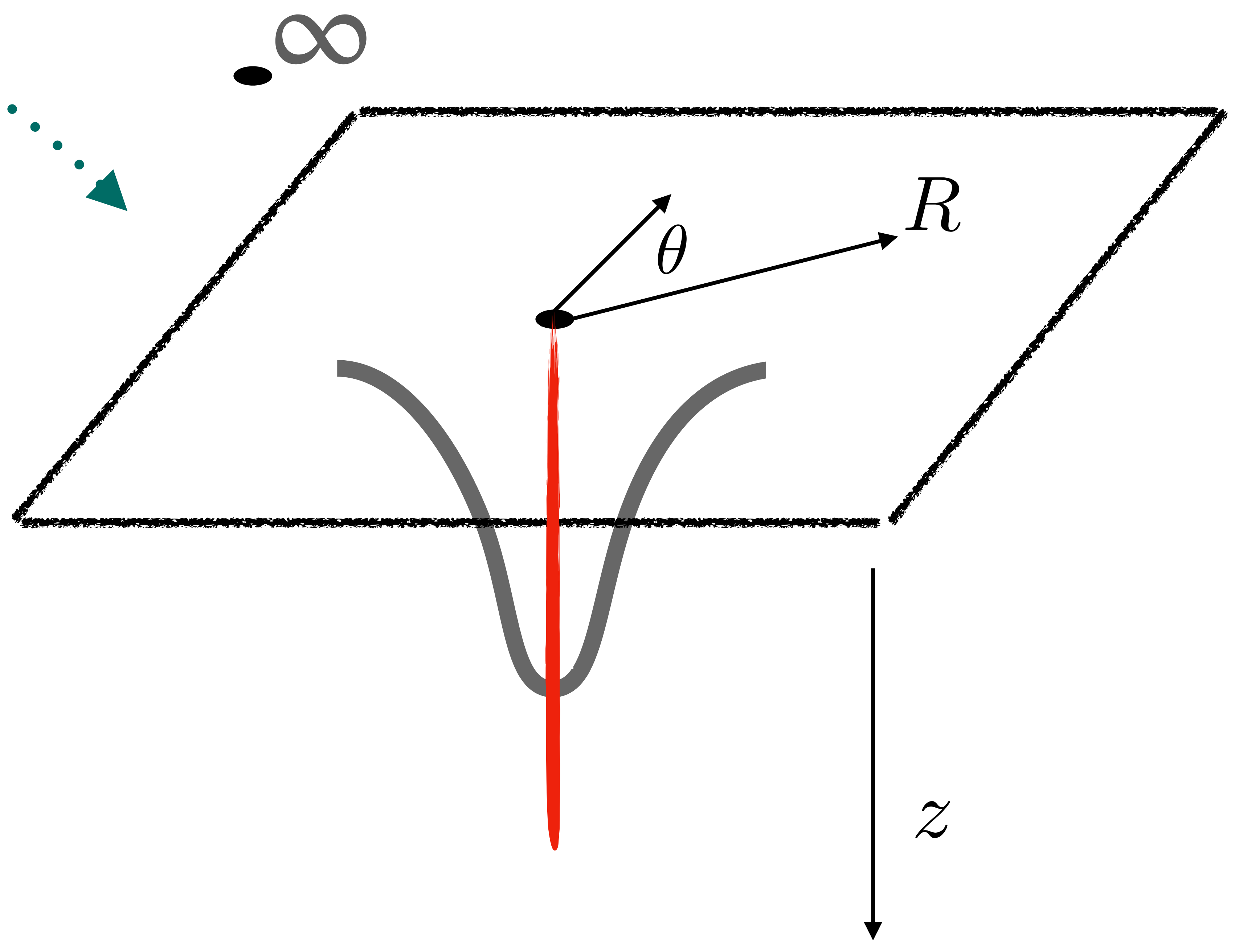


from global to Poincare

$$r = \frac{R}{z}$$

$$\tau = \frac{1}{2}\log(z^2 + R^2)$$



Reading off the holographic vevs
And matching with free theory (since 3pt couplings here are protected)

We discover that the heavy state is

$$e^{\frac{\alpha}{\sqrt{2}}\mathrm{TrX}^2} \qquad \textbf{with} \qquad \frac{2\alpha^2}{1-2\alpha^2} = \frac{Q}{2}$$

To compute the 4-pt HHLL correlator of interest we will do the following

HHLL of Light CPO, eg. $\mathrm{Tr} X^2$

To compute the 4-pt HHLL correlator of interest we will do the following

HHLL of Light CPO, eg. $\mathrm{Tr} X^2$

Take this way

HHLL of Lagrangians , eg. $\mathrm{Tr} F_+^2$

Use known Ward Identities

Solve the dilaton wave equation

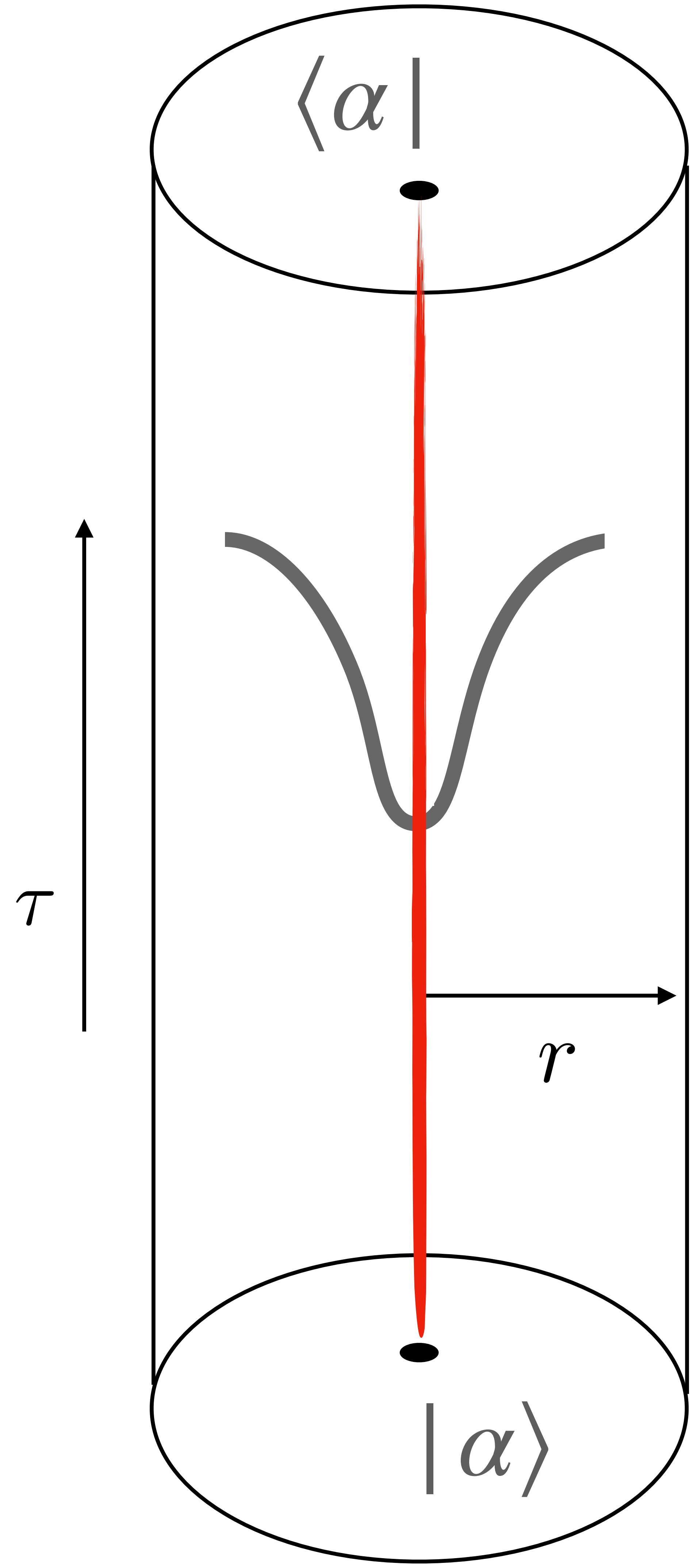
$$\Box_{10}\Phi = 0$$

[Giusto,Russo,Wen]

[Aprile,Giusto,Russo]

interesting `classic' problem in Holography

The Holographic computation



$$\Phi = \int \frac{d\omega}{2\pi} e^{i\omega\tau} \times \sum_{l=0}^{\infty} (l+1) Y_l(\theta) \times \phi_{\omega,l}(r, \Omega_5)$$

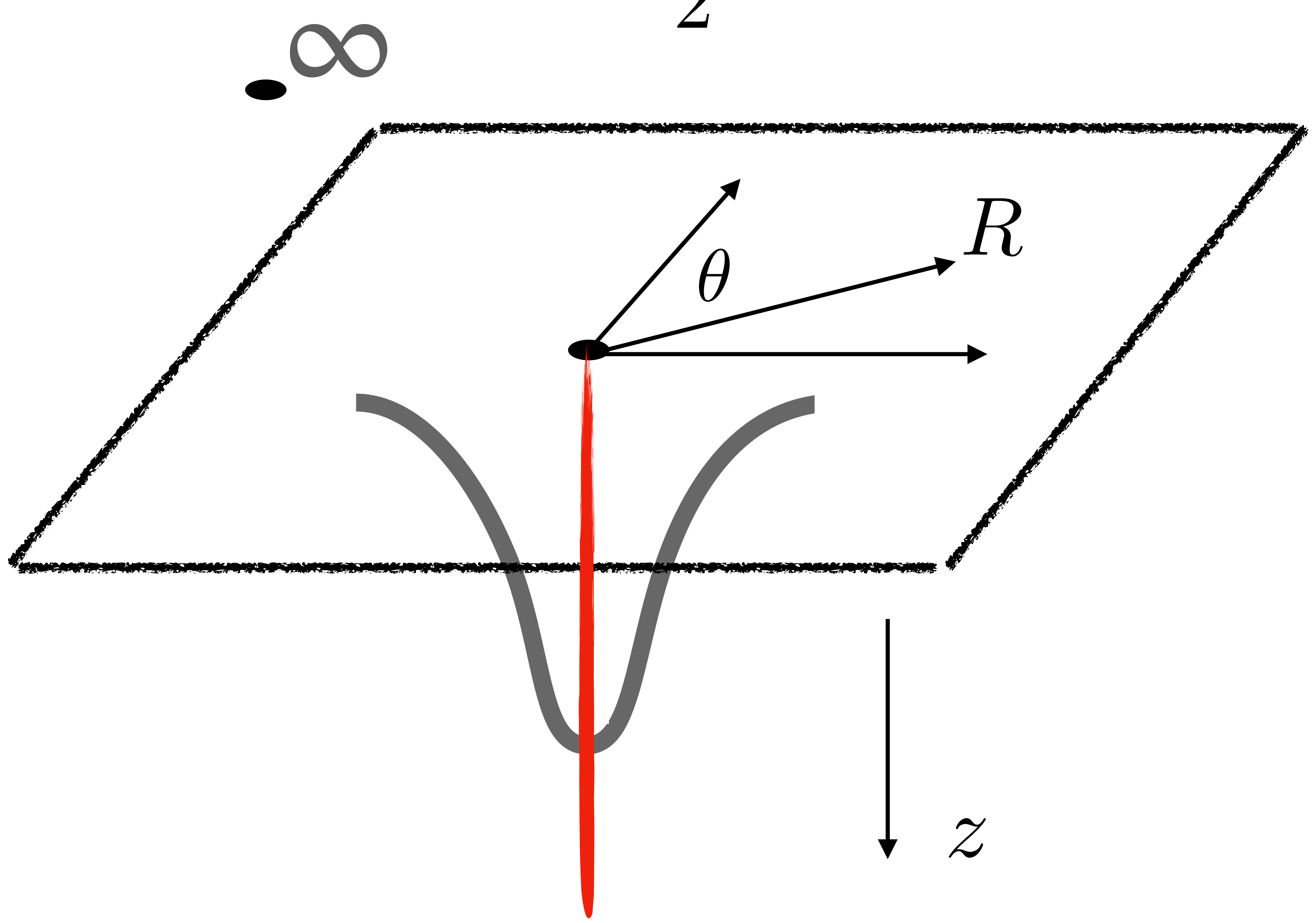
$$r = \frac{R}{z}$$

$$\tau = \frac{1}{2}\log(z^2 + R^2)$$

We shall use the boundary value of the non-normalisable mode to prepare a delta function source

$$\delta(\tau) = 0 \qquad \delta(\theta) = 0$$

In Poincare coordinates this gives $R = 1$ at the boundary in the direction $\theta = 0$



$$\frac{\langle\alpha|\bar{\mathcal{L}}_{p-2}(1)\mathcal{L}_{p-2}(R,\theta)|\alpha\rangle}{\langle\alpha|\alpha\rangle} = N^2 \frac{\partial^2}{\partial\bar{J}\partial J} S_{\text{on-shell}}\left[\Phi\Big|_{\boldsymbol{\Omega}_{p-2}}\right]$$

KK-level

$$\frac{\langle\alpha|\bar{\mathcal{L}}_{p-2}\mathcal{L}_{p-2}|\alpha\rangle}{\langle\alpha|\alpha\rangle} = N^2 \frac{\partial^2}{\partial\bar{J}\partial J} S_{\text{on-shell}}\left[\Phi\Big|_{\boldsymbol{\Omega}_{p-2}}\right]$$

As usual with Holography the result is proportional to the normalizable mode of Φ at the boundary of AdS

In the simplest case, $p=2$, there is single single harmonic, i.e. unity, and the bulk equation becomes the ODE

$$\left[\frac{1}{r^3}\frac{d}{dr}\left(r^3(1+r^2H_\alpha(r))\frac{d}{dr}\right) + \frac{\omega^2 H_\alpha(r)}{1+r^2H_\alpha(r)} - \frac{l(l+2)}{r^2}\right]\phi_{\omega,l}(r) = 0$$

To solve this equation one has to match the boundary expansion $r\to\infty$ (with the given source) with the regular bulk expansion from $r=0$, in other words: one has to solve a connection problem for our ODE

Interestingly: our ODE can be put into the form of a Heun equation. The nice thing about it is that

The connection problem has been solved in closed form by [Bonelli, Iossa, Lichtig, and Tanzini; 2201.04491]

"The Trieste formula"

From there one can derive the following result:

$$\frac{\langle\alpha|\mathrm{Tr}\bar{X}^2(1)\mathrm{Tr}X^2(x,\bar{x})|\alpha\rangle}{\langle\alpha|\alpha\rangle} = N^2 \oint \frac{ds}{2\pi i}(x\bar{x})^s \sum_{l\geq 0}(l+1)\frac{x^{l+1}-\bar{x}^{l+1}}{x-\bar{x}}\left[\Gamma[-s+\alpha^2\gamma(s,l,\alpha)]\times \mathrm{Res}(s,l,\alpha)\right]$$

Computed by the period of a SW curve. Physically it computes the dimension of the HL bound states

$$\underset{\substack{\text{reduced}\\ \text{unnormalised}}}{\mathcal{H}_{\alpha\alpha 22}} = \sum_{n=1}^{\infty}\frac{1}{(n!)^2}\left(\frac{\alpha}{\sqrt{2}}\right)^{2n}\underset{\substack{\text{reduced}\\ \text{unnormalised}}}{\mathcal{H}_{[2^n][2^n]22}}$$

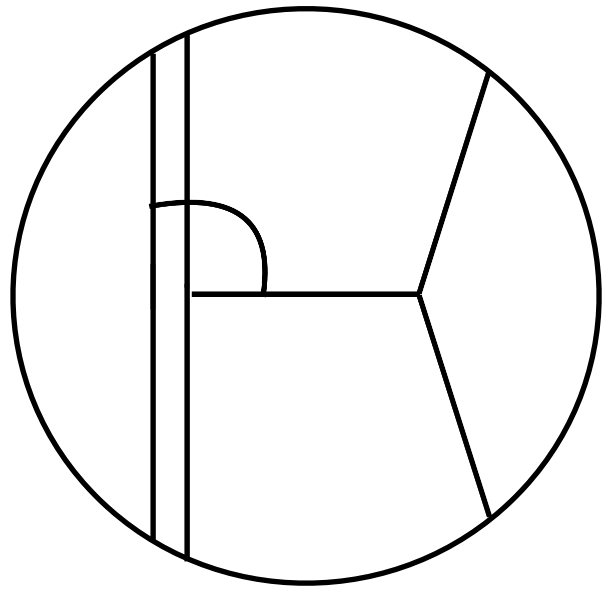
We can then test the limit $\alpha \to 0$

* *there are only diagonal contributions in the HHLL reduced correlator, i.e. $n=m$ in the sum*

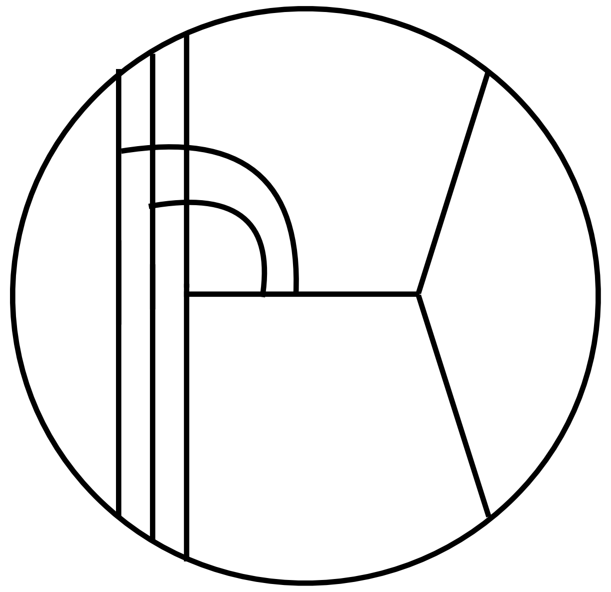
In the limit $\alpha \to 0$

$$\frac{\langle\alpha|\mathrm{Tr}\bar{X}^2(1)\mathrm{Tr}X^2(x,\bar{x})|\alpha\rangle}{\langle\alpha|\alpha\rangle} = N^2 \oint \frac{ds}{2\pi i}(x\bar{x})^s \sum_{l\geq 0}(l+1)\frac{x^{l+1}-\bar{x}^{l+1}}{x-\bar{x}}\left[\Gamma[-s+\alpha^2\gamma(s,l,\alpha)] \times \mathrm{Res}(s,l,\alpha)\right]$$

matches $\mathcal{H}^{(1)}_{2222} = -4\overline{D}_{2422}$ and produces new results for the (connected) tree-level correlators $[2^n][2^n]22$



n=2



n=3

...

It turns out the results have the form

$$\frac{poly_n(x,\bar{x})}{(x-\bar{x})^{4n+4-1}} Ladder_n(x,\bar{x}) +$$

$$+ \quad \textit{lower weight completion}$$

Using the double-particle OPE one can predict some 1-variable slices of this correlator and the results are consistent!

For $p > 2$ the KK modes from $\phi_{\omega,l}(r,\Omega_5)$ satisfy a mixed system of ODEs in the radial coordinate

REMARKABLY

For each KK level, the coupled ODEs can be unmixed and the problem reduces again to Heun equations

What we have computed so far:

- **4-point single particle correlators match the Heun formalism**
- **4-point single particle correlators with one double particle match the Heun formalism**
- **NEW explicit results for two double particle insertions**

In Mellin space we have a formula for $\mathcal{H}^{(2)}_{[2^2][2^2]pp \atop reduced}$ ***and all charges*** p

[Aprile,Giusto,Russo,VilasBoas], part II

Let's move to part III :

Part I

A CFT bootstrap approach to 4pt correlators with multi-particle operators

Part II

A gravity approach to the HHLL correlator built out of multi-gravitons

$$e^{\alpha \mathcal{O}_2} = \sum_{n=0}^{\infty} \frac{\alpha^n}{n!} \mathcal{O}_2^n$$

Part III

The integrated correlator: towards the emergence of spacetime

[Aprile,Dorigoni,Treilis]

Part III

Now that we have understood that the AdS bubble is predictive we would like to do something more:

The dream would be see the emergence of the holographic geometry from the CFT e.g. by plotting the HHLL correlator and staring at it. But we are not there yet!

Instead, we will make the dream more tractable by considering a simpler object

the so called integrated correlator

$$\int \frac{d^4x_1 \dots d^4x_4}{\mathrm{vol}(SO(2,4))} \mathcal{H}_{reduced}(x_{ij}; N, \tau)$$

Pioneered by

[Binder, Chester, Pufu, Wang]

Further studied by

[Dorigoni, Green, Wen]

$$\int \frac{d^4x_1 \dots d^4x_4}{\mathrm{vol}(SO(2,4))} \frac{\mathcal{H}_{\alpha\alpha 22}(x_{ij};N,\tau)}{\langle\alpha|\alpha\rangle} = \frac{1}{\langle\alpha|\alpha\rangle} \sum_{n=1}^{\infty} \frac{1}{(n!)^2} \left(\frac{\alpha}{\sqrt{2}}\right)^{2n} \int \frac{d^4x_1 \dots d^4x_4}{\mathrm{vol}(SO(2,4))} \mathcal{H}_{[2^n][2^n]}(x_{ij};N,\tau)$$

The study of these "maximal trace" integrated correlators was initiated by [Paul, Perlmutter, Raj]

They explain that the best way to present the integrated correlator is to use a $SL(2,\mathbb{Z})$ spectral decomposition

$$\int \frac{d^4x_1 \dots d^4x_4}{\mathrm{vol}(SO(2,4))} \underbrace{\mathcal{H}_{[2^n][2^n]}(x_{ij};N,\tau)}_{\text{normalised}} = \langle \mathcal{C}_{p,N} \rangle + \int_{\mathrm{Re}(s)=\frac{1}{2}} c_{n,N}(s)\, E^*(s;\tau) \frac{ds}{2\pi i}$$

spectral overlap

$$c_{n,N}(s) = \tilde{M}_N(s)\left[1 - {}_3F_2\left[-n, s, 1-s; 1, \tfrac{N^2-1}{2} | 1\right]\right]$$

non-holomorphic Eisenstein series

$$E^*(s;\tau) = E^*(1-s;\tau)$$

If you are interested in the maximal trace integrated correlators [2^n][2^n]22

$$c_{n,N}(s) = \tilde{M}_N(s)\left[1 - {}_3F_2\left[-n, s, 1-s; 1, \tfrac{N^2-1}{2}|1\right]\right]$$

1) the dependence in *s* and *N* is not factorized

2) treat carefully the double scaling limit : large *n* and large *N* !

n/N^γ = *fixed*

[Aprile,Dorigoni,Treilis]

If you are interested in the maximal trace integrated correlators $[2^n][2^n]22$

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[Aprile,Dorigoni,Treilis]

If you are interested in the HHLL integrated correlator, then from the very definition we get

$$c_{HHLL} = \frac{1}{\langle\alpha|\alpha\rangle}\sum_{n=1}^{\infty}\frac{\alpha^{2n}}{n!}\left[2^n\left(\frac{N^2-1}{2}\right)_n\right] \times c_{n,N}(s)$$

rewriting here the ${}_3F_2$

$$c_{HHLL} = (1-2\alpha^2)^{\frac{N^2-1}{2}}\sum_{n=1}^{\infty}\frac{\alpha^{2n}}{n!}\left[2^n\left(\frac{N^2-1}{2}\right)_n\right] \times \tilde{M}_N(s)\left[\sum_{k=1}^{n}\frac{n!(s)_k(1-s)_k(-1)^k}{(n-k)!(\frac{N^2-1}{2})_k}\frac{1}{(k!)^2}\right]$$

If you are interested in the maximal trace integrated correlators $[2^n][2^n]22$

$$c_{n,N}(s) = \tilde{M}_N(s)\left[1 - {}_3F_2\left[-n, s, 1-s; 1, \tfrac{N^2-1}{2} \middle| 1\right]\right]$$

1) the dependence in *s* and *N* is not factorized

2) treat carefully the double scaling limit : large *n* and large *N* !

[Aprile,Dorigoni,Treilis]

If you are interested in the HHLL integrated correlator, then from the very definition we get

$$c_{HHLL}(N,\alpha;s) = \tilde{M}_N(s)\left[1 - {}_2F_1\left[s, 1-s, 1 \middle| -\tfrac{2\alpha^2}{1-2\alpha^2}\right]\right]$$

FINAL RESULT IS REMARKABLY SIMPLE AND THE DEPENDENCE ON α and *N* FACTORIZED

In the large N limit @ genus 0. **For the maximal trace integrated correlators we consider**

$$c_{n,N}(s) = \sum_{g=0}^{\infty} N^{2-2g} \Big[N^{-s} \mathcal{A}^{(g)}(s,Q) + (s \leftrightarrow 1-s) \Big]$$

$$\frac{n}{N^2} = \frac{Q}{2} \quad \textit{fixed}$$

$$\mathcal{A}^{(0)} = \textit{prefactor} \times (s-\tfrac{1}{2})^2 \tan(\pi s) \Big[1 - {}_2F_1 \Big[s, 1-s, 1; -\tfrac{Q}{2} \Big] \Big]$$

[Aprile,Dorigoni,Treilis]

For the HHLL integrated correlator

$$c_{HHLL}(s) = \sum_{g=0}^{\infty} N^{2-2g} \Big[N^{-s} \mathcal{B}^{(g)}(s,Q) + (s \leftrightarrow 1-s) \Big]$$

$$\mathcal{B}^{(0)} = \textit{prefactor} \times (s-\tfrac{1}{2})^2 \tan(\pi s) \Big[1 - {}_2F_1 \Big[s, 1-s, 1; -\tfrac{2\alpha^2}{1-2\alpha^2} \Big] \Big]$$

we learn that $\mathcal{A}^{(0)} = \mathcal{B}^{(0)}$

provided we map

$$\frac{2\alpha^2}{1-2\alpha^2} = \frac{Q}{2}$$

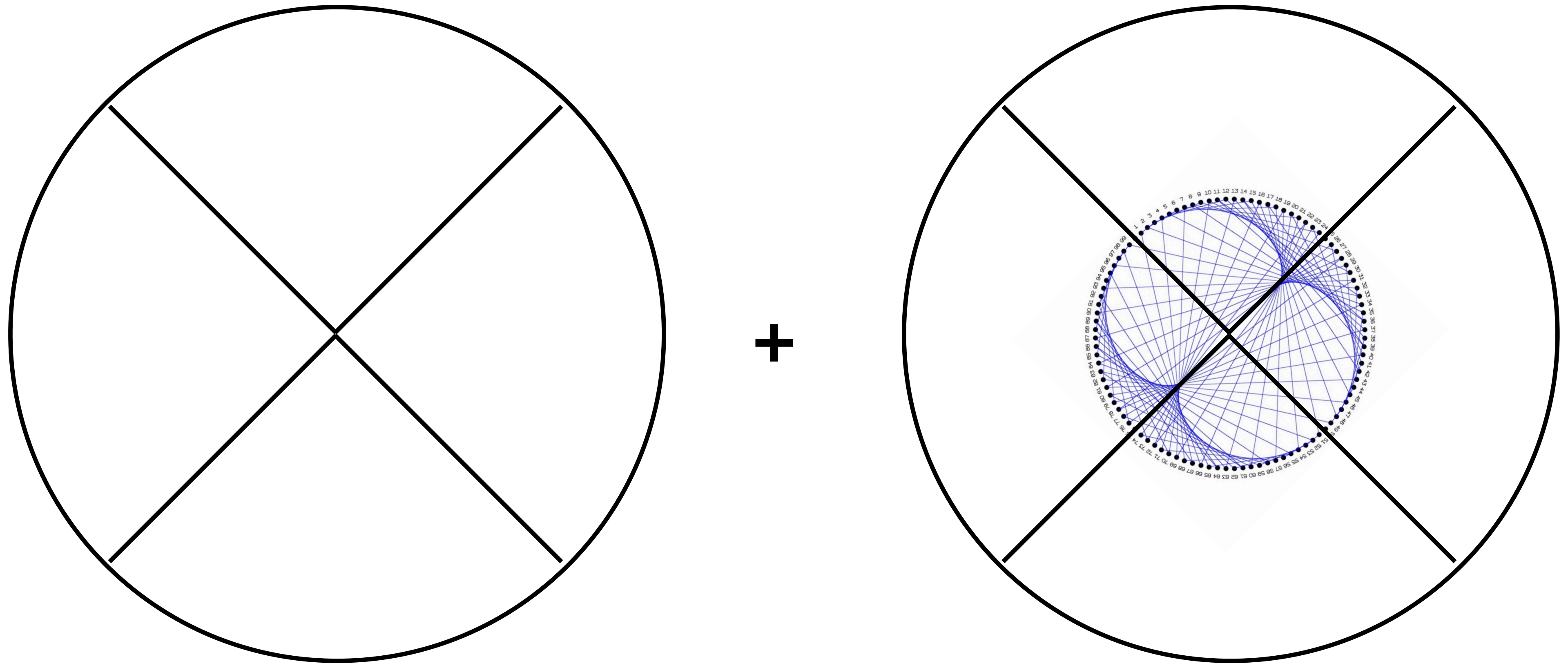
Of course $\mathcal{A}^{(g)} \neq \mathcal{B}^{(g)}$ **at higher genus because** $c_{n,N}(s)$ **is not factorized**

What’s the physical content these results ?

$$\mathcal{C}_{HHLL}(N,\alpha,\lambda) \sim -\frac{N^2}{4}\log(1-2\alpha^2) + N^2 \sum_{k=0}^{\infty} \tilde{\mathcal{B}}^{(0)}(k+\tfrac{3}{2};\alpha)\zeta_{2k+3}\,\lambda^{-k-\frac{3}{2}}$$
$$+\sum_{g=1}^{\infty} N^{2-2g}\left[\sum_{k=0}^{\infty}\tilde{\mathcal{B}}^{(0)}(k+\tfrac{3}{2};\alpha)\zeta_{2k+3}\,\lambda^{-k-\frac{3}{2}} + \sum_{k=0}^{g-1}\tilde{B}^{(g-k-1)}(k+\tfrac{3}{2};\alpha)\zeta_{2k+2}\lambda^{k+\frac{1}{2}}\right]$$

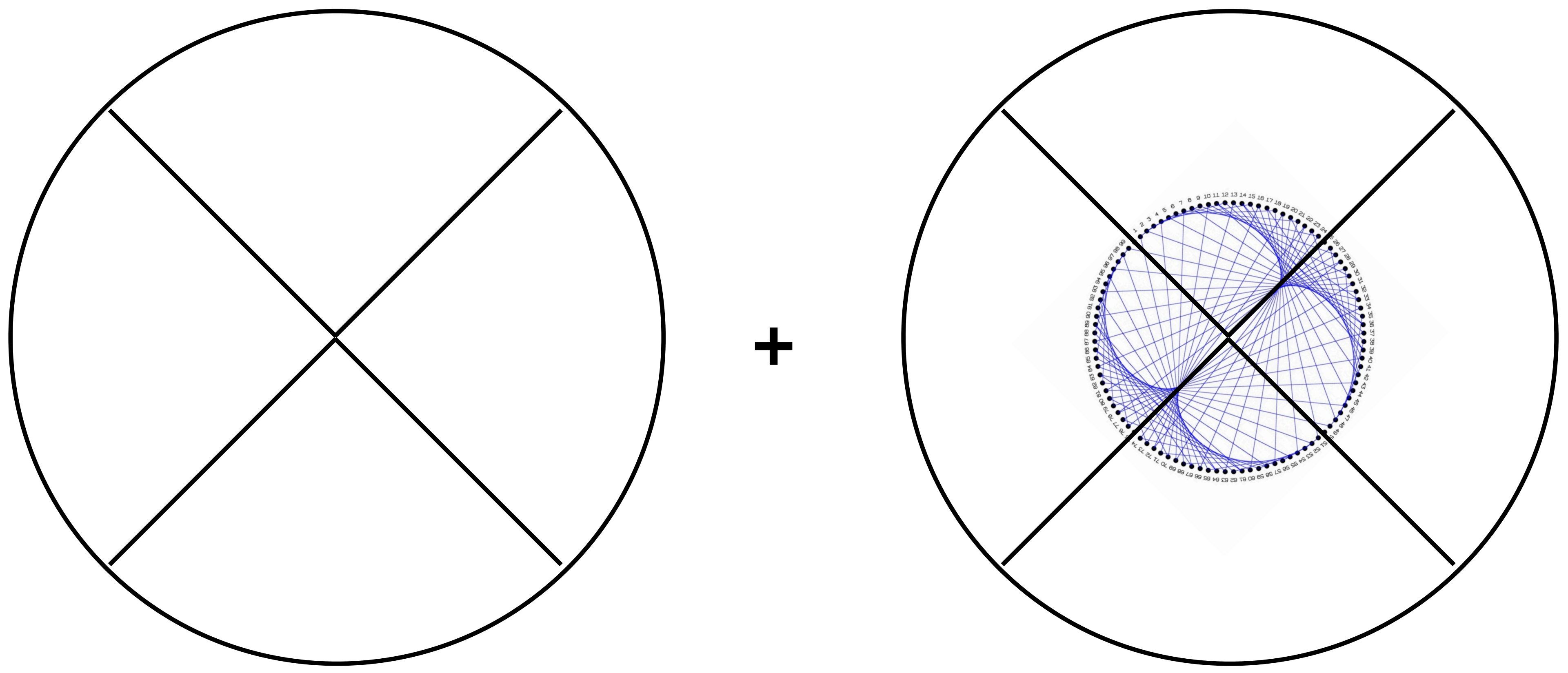
Higher genus & counterterms

SUGRA+ genus-zero Virasoro-Shapiro amplitude on the AdS Bubble.



short strings $\Delta \sim \lambda^{1/4}$ [Alday, Hansen,Silva]

The study of the integrated correlator reveals that the ’t Hooft expansion expansion is factorially divergent!



sugra $\Delta \sim 1$ short strings $\Delta \sim \lambda^{1/4}$ [Alday, Hansen, Silva]

The study of the integrated correlator reveals

the 't Hooft expansion expansion is factorially divergent!

On the other hand $\mathcal{C}_{HHLL}(N,\alpha,\tau)$ is a well defined modular function $\rightarrow$ resolve the divergences

This analysis goes under the framework of modular resurgence analysis and the outcome is that

$$\mathcal{C}_{HHLL}(N,\alpha,\lambda) = -\frac{N^2}{4}\log(1-2\alpha^2) + N^2 \sum_{k=0}^{\infty} \tilde{\mathcal{B}}^{(0)}(k+\tfrac{3}{2};\alpha)\zeta_{2k+3}\,\lambda^{-k-\frac{3}{2}} \quad + \quad \text{NON PERTURBATIVE COMPLETION}$$

Q1: What are the parameters of the NP completion? And Q2: how do they map to Holography?

Q1: What are the parameters of the NP completion?

[Aprile,Dorigoni,Treilis]

$$\mathcal{C}^{(0),I}_{HHLL,NP}(N,\tau) \sim \exp\left(-4\sqrt{\pi N}\,\frac{|p\tau+q|}{\sqrt{\tau_2}}\right)$$

$$\mathcal{C}^{(0),-}_{HHLL,NP}(N,\tau) \sim \exp\left(-4\sqrt{\pi N}\,\sqrt{\frac{1-\sqrt{2}\alpha}{1+\sqrt{2}\alpha}}\,\frac{|p\tau+q|}{\sqrt{\tau_2}}\right) \qquad \mathcal{C}^{(0),+}_{HHLL,NP}(N,\tau) \sim \exp\left(-4\sqrt{\pi N}\,\sqrt{\frac{1+\sqrt{2}\alpha}{1-\sqrt{2}\alpha}}\,\frac{|p\tau+q|}{\sqrt{\tau_2}}\right)$$

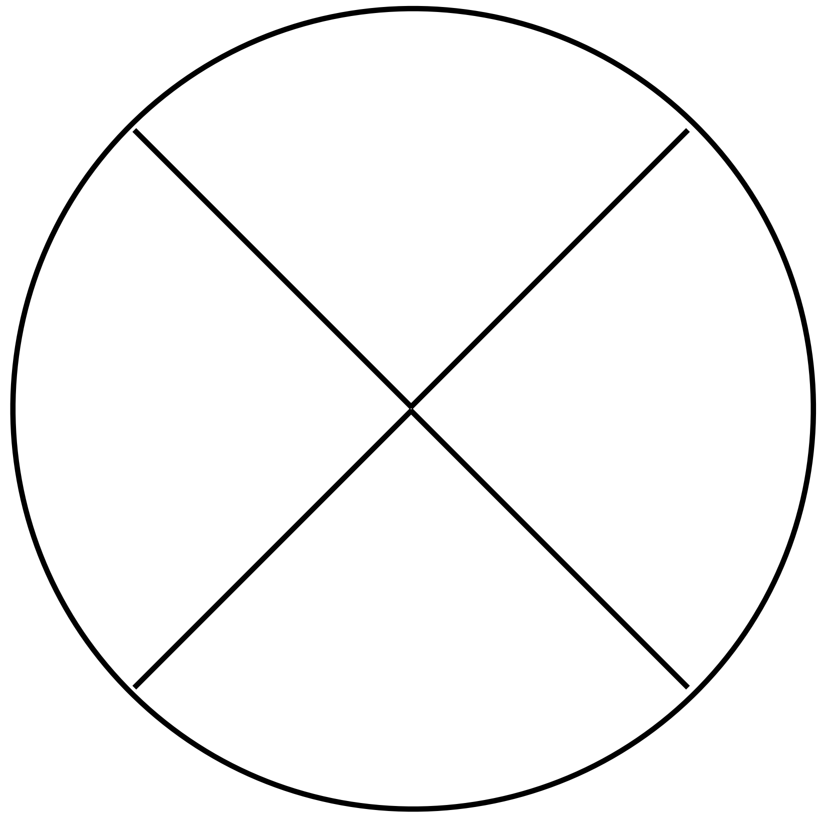
Rewriting the arguments of the exponentials in the regime of large 't Hooft coupling

$$\mathcal{C}_{HHLL}(N,\alpha,\lambda) = -\frac{N^2}{4}\log(1-2\alpha^2) + N^2\sum_{k=0}^{\infty}\tilde{\mathcal{B}}^{(0)}(k+\tfrac{3}{2};\alpha)\zeta_{2k+3}\,\lambda^{-k-\frac{3}{2}} \quad +$$

NON PERTURBATIVE COMPLETION

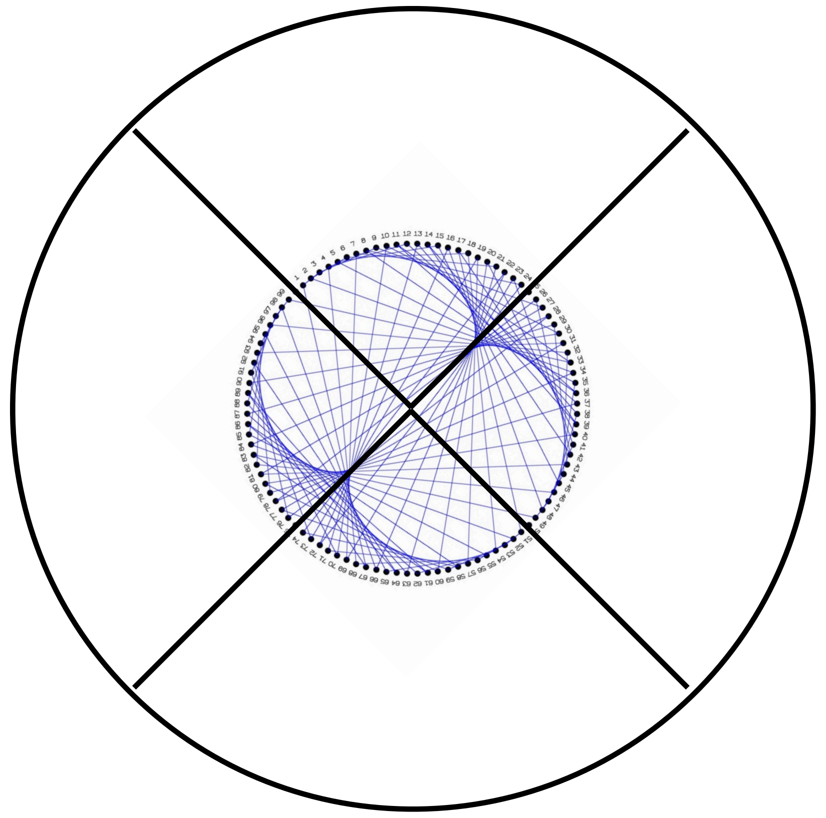
$$\exp\left(-2|q|\sqrt{\lambda}\,\sqrt{\frac{1+\sqrt{2}\alpha}{1-\sqrt{2}\alpha}}\right) \ll \exp\left(-2|q|\sqrt{\lambda}\right) \ll \exp\left(-2|q|\sqrt{\lambda}\,\sqrt{\frac{1-\sqrt{2}\alpha}{1+\sqrt{2}\alpha}}\right)$$

Q2: how do they map to Holography?



sugra $\Delta \sim 1$

+



short strings $\Delta \sim \lambda^{1/4}$

+

NON PERTURBATIVE COMPLETION

$$\exp\left(-2\sqrt{\lambda}\, L_{\pm}\right)$$

$$L_- = \sqrt{\frac{1-\sqrt{2}\alpha}{1+\sqrt{2}\alpha}} \qquad L_+ = \sqrt{\frac{1+\sqrt{2}\alpha}{1-\sqrt{2}\alpha}}$$

The idea is that these NP corrections correspond to contributions coming from big strings.

There are big strings in our geometry! These are the so called Maldacena-Hofman giant magnons.

These strings sits at the center of space in AdS, $r = 0$, they extend in global time, and move on the S⁵

$$(T_{ij})^{-1} D\mu^i D\mu^j \Big|_{\text{at the boundary}} \quad \to \quad \cos^2(\theta) d\tilde{\Omega}_3^2 + d\theta^2 + \sin^2(\theta)(d\phi + d\tau)^2$$

worldsheet $\sigma = \phi$ *embedding* $\theta = \theta(\sigma)$

The Maldacena-Hofman giant magnon

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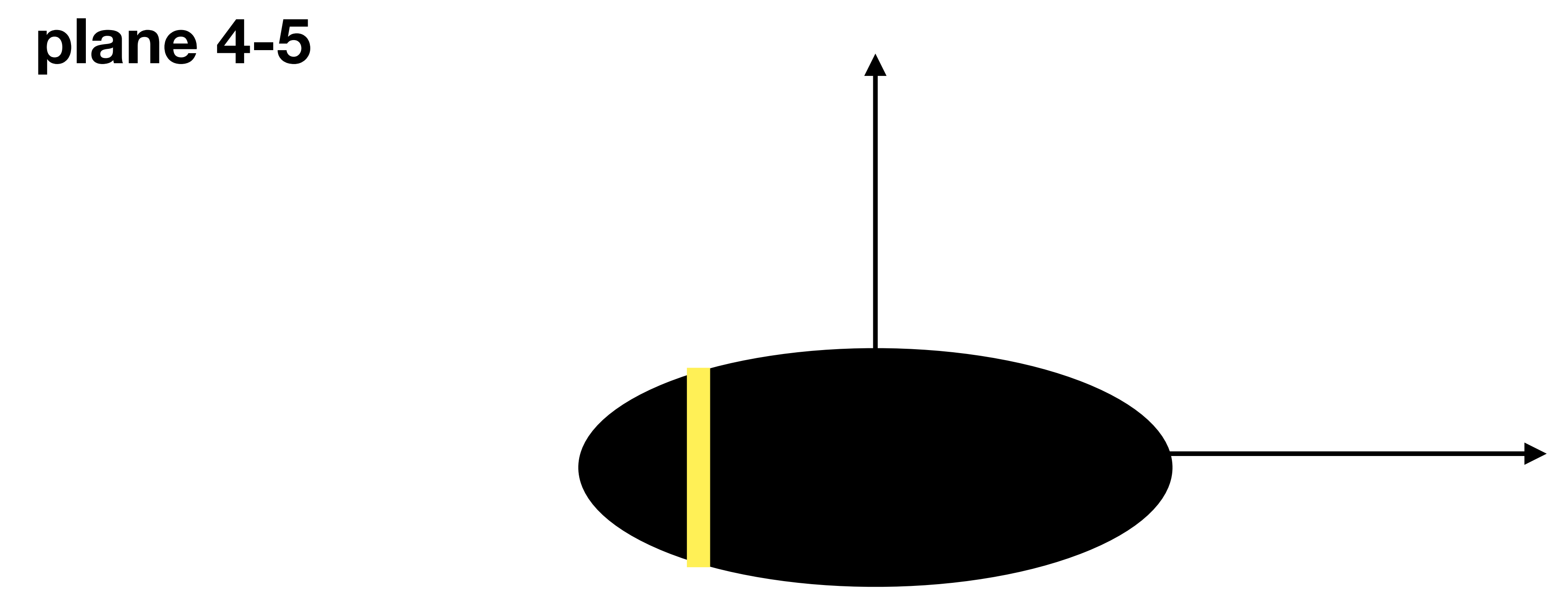
worldsheet time = global time *worldsheet* $\sigma = \phi$ *embedding* $\theta = \theta(\sigma)$

$$\sum_{i=0,1,2,3} (\mu^i)^2 = \cos^2(\theta)$$

$\mu^4 + i\mu^5 = \sin(\theta) e^{i\phi}$ **a way to draw the motion is to project on the `disk' in the the 4-5 plane**

$$S_{NG} = \frac{\sqrt{\lambda}}{2\pi} \int d\tau d\sigma \sqrt{(\dots)}$$

$$= \frac{\sqrt{\lambda}}{2\pi} \int d\tau d\sigma \sqrt{L_-^2 \left(\frac{d\mu^4}{d\sigma}\right)^2 + L_+^2 \left(\frac{d\mu^5}{d\sigma}\right)^2}$$



$L_\pm$ **are the lengths of the major/minor axis of the ellipse dual to the Heavy state in the LLM coordinates**

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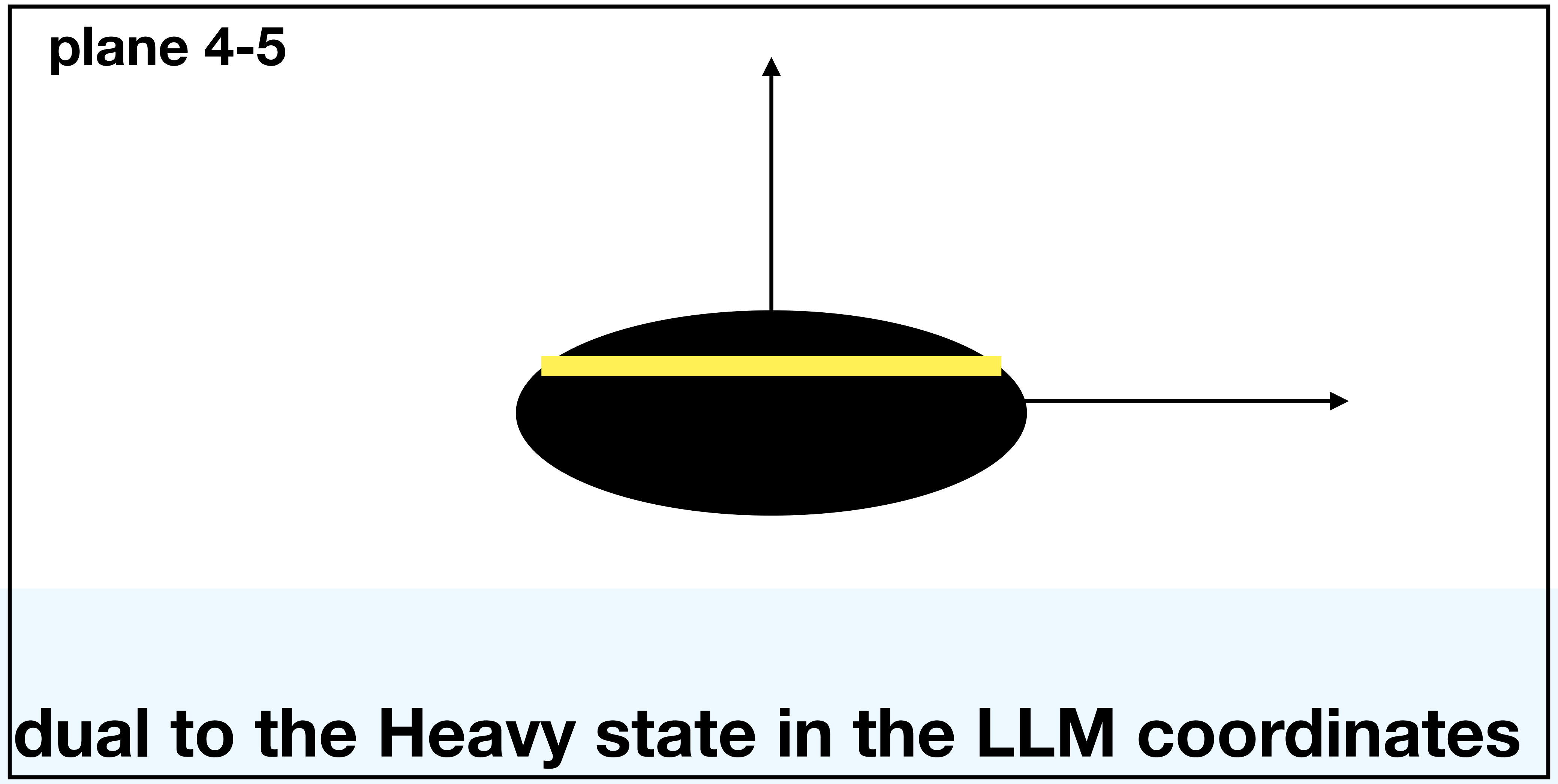

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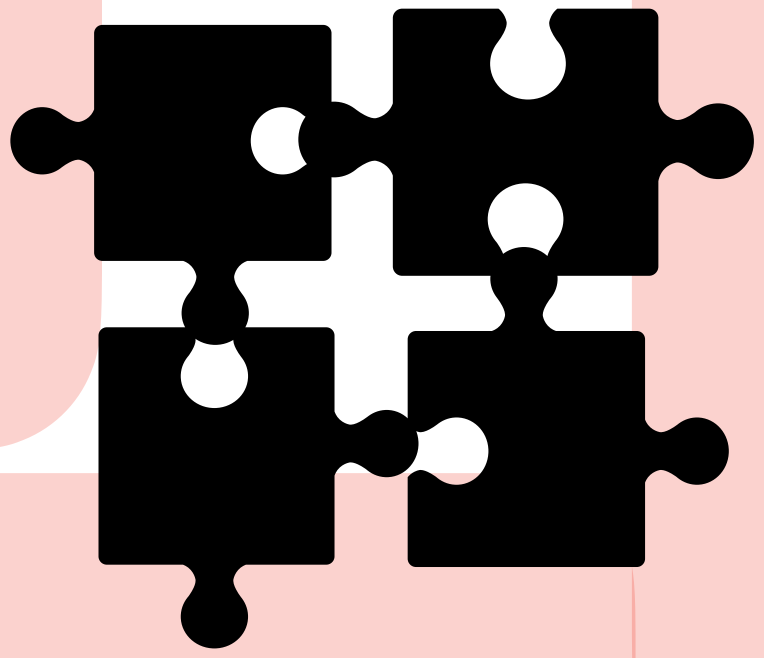
Summary

Part I

I showed how to use a CFT approach, i.e. the double-particle bootstrap to solve tree-level four-point correlators with one double-particle insertion and get data for two (or more) insertions



Part II

I showed how to use a gravity approach to compute HHLL in which the Heavy operator is

$$e^{\alpha \mathcal{O}_2} = \sum_{n=0}^{\infty} \frac{\alpha^n}{n!} \mathcal{O}_2^n$$

And I showed how to relate this to four-point correlators with two multi-particle insertions

Part III

I showed exact result for the integrated HHLL correlator and its implications for string theory : make predictions for the full VS amplitude in the AdS bubble, both P and NP.

Summary

Part I

I showed how to use a CFT approach, i.e. the double-particle bootstrap to solve tree-level four-point correlators with one double-particle insertion and get data for two (or more) insertions

Part II

I showed how to use a gravity approach to compute $\langle \exp(\alpha \mathcal{O}_2) \exp(\alpha \mathcal{O}_2) \mathcal{O}_2 \mathcal{O}_2 \rangle$ and its multiparticle expansion

Part III

I showed exact result for the integrated HHLL correlator and its implications for string theory : make predictions for the full VS amplitude in the AdS bubble, both P and NP.

Future directions

CFT bootstrap for one double-particle in diverse dimensions, e.g. 6d [[wip with H.Puerta-Ramirez and M.Woolley]]

More holographic correlators for exp(single particle): LLM geometry are known from [[Vazquez]]

Flat space limit

[[wip with J.Penedones]]

Higher dimensional symmetry?

Worldsheet approach to multi-particle correlators?

Energy-energy correlators?

Thank you for the attention.

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MINISTERIO DE CIENCIA E INNOVACIÓN

AGENCIA ESTATAL DE INVESTIGACIÓN

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Plan de Recuperación, Transformación y Resiliencia

Backup slides: on tree-level correlators with one double-particle

$$\mathcal{M}_{[2^2]p_2p_3p_4} = \left(\mathcal{M}_{[2^2]su} + crossing\right) + \mathcal{M}_{[2^2]stu}\,,$$

$$\mathcal{M}_{[2^2]su} = \frac{\frac{1}{4}\big((p_4-p_2-p_3)(f_t-3)+(f_t-4)\big)f_t}{(1+\mathbf{s})(\mathbf{t})(1+\mathbf{u})} + \frac{\frac{1}{8}(p_2+p_3-p_4)(f_t-2)f_t}{(1+\mathbf{s})(-1+\mathbf{t})(1+\mathbf{u})} + \frac{\frac{1}{8}(4-p_2-p_3-p_4)f_sf_u}{(\mathbf{s})(1+\mathbf{t})(\mathbf{u})}$$

$$\mathcal{M}_{[2^2]stu} = \frac{4-\frac{1}{4}(f_s^2+f_t^2+f_u^2)-\frac{1}{8}\big((4-p_2+p_3-p_4)f_sf_t+(4-p_2-p_3+p_4)f_sf_u+(4+p_2-p_3-p_4)f_tf_u\big)}{(1+\mathbf{s})(1+\mathbf{t})(1+\mathbf{u})}$$

$$f_{ij;mn} = \frac{p_i+p_j-p_m-p_n}{2} + d_{ij} + d_{mn},$$

$$f_s := f_{12;34}$$
$$f_t := f_{14;23}\,.$$
$$f_u := f_{13,24}$$