NON RENORMALIZABILITY PROPERTIES OF EXTRENAL CORRELATORS OF 1/4 BPS OPERATORS

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MAIN GOALS:

- UNDERSTAND BETTER PROTECTED QUANTITIES IN $\mathcal{N}=4$ SYM $\rightarrow$ IN PARTICULAR 1/4 BPS OPERATORS
- PROTECTED SECTORS IN $\mathcal{N}=4$ SYM CONSTRAIN THE NON PROTECTED ONES.

OPERATORS IN $\mathcal{N}=4$ SYM

- $\frac{1}{2}$ BPS

$$\boxed{\begin{array}{l} \Delta = p \,, \quad \text{R-symmetry } [0,p,0] \\ \langle \mathcal{O}_{p_1}\, \mathcal{O}_{p_2}\, \mathcal{O}_{p_3} \rangle = f(N) \end{array}}$$

PROTECTED FROM QUANTUM CORRECTIONS

OPERATORS IN $\mathcal{N}=4$ SYM

- $\frac{1}{2}$ BPS $\quad \Delta = p$, R-symmetry $[0, p, 0]$

$$\langle \mathcal{O}_{p_1} \mathcal{O}_{p_2} \mathcal{O}_{p_3} \rangle = f(N)$$

e.g. $\quad p=2 \quad \mathcal{O}_2 \sim \mathrm{tr}\, \phi^2$

$\quad p=3 \quad \mathcal{O}_3 \sim \mathrm{tr}\, \phi^3$

$\quad p=4 \quad \mathcal{O}_4 \sim \mathrm{tr}\, \phi^4$

$\quad \mathcal{O}_4 \sim \mathrm{tr}\, \phi^2\, \mathrm{tr}\, \phi^2$

OPERATORS IN $\mathcal{N}=4$ SYM

- $\frac{1}{2}$ BPS $\quad \Delta = p$, R-symmetry $[0,p,0]$

$$\langle \mathcal{O}_{p_1} \mathcal{O}_{p_2} \mathcal{O}_{p_3} \rangle = f(N)$$

e.g.

$$\left.\begin{array}{ll} p=2 & \mathcal{O}_2 \sim \operatorname{tr} \phi^2 \\ p=3 & \mathcal{O}_3 \sim \operatorname{tr} \phi^3 \\ p=4 & \mathcal{O}_4 \sim \operatorname{tr} \phi^4 \end{array}\right\} \text{SINGLE TRACE}$$

$$\mathcal{O}_4 \sim \operatorname{tr} \phi^2 \operatorname{tr} \phi^2 \rightarrow \text{DOUBLE TRACE}$$

- ACTUALLY THE PROPER OPERATOR (EIGENSTATE OF THE DILATATION GENERATOR) IS A LINEAR COMBINATION, FOR INSTANCE

$p = 4 \,, \quad \Delta = 4$

$$\mathcal{O}_4^{SP} = \left(\frac{4(N^2+1)}{(N^2-1)(N^2-4)(N^2-9)} \right)^{1/2} \left(t_4 - \frac{2N^2-3}{N(N^2+1)} t_2^2 \right)$$

$$\mathcal{O}_4^{DT} = \left(\frac{2}{(N^4-1)} \right)^{1/2} t_2^2$$

WHERE $\quad t_p = \mathrm{Tr}(y \cdot \phi)^p$

[ALDAY, ZHOU]
[AB, FARDELLI, MANENTI]

- IN THE AdS DUAL THEORY

$\theta_2 \rightarrow$ GRAVITON SUPERMULTIPLET

$\theta_3 \rightarrow$ KALUZA KLEIN MODES

$\vdots$

$\theta_4^{DT} \rightarrow$ BOUND 2-PARTICLE STATE

• $\frac{1}{4}$ BPS $\qquad \Delta = 2q + p \qquad [q, p, q]_R$

An example: $\mathcal{O}_{pq}$

$$\mathcal{O}_{02} \sim \mathrm{Tr}\, \phi^i \phi^j \, \mathrm{Tr}\, \phi^k \phi^l \, P_{ijkl}$$

$$\Delta = 4 \qquad [2,0,2]_R$$

- LONG OPERATORS

THEIR DIMENSION AND OPE COEFFICIENTS AQUIRE CORRECTIONS AS FUNCTIONS OF λ AND N.

$\lambda = g_{YM}^2 N$

MOVING TO 4-POINT FUNCTIONS

- THEY GENERICALLY DEPEND ON λ, EVEN WHEN DEALING WITH $\frac{1}{2}$ BPS OPERATORS

- HOWEVER IT EXISTS A FAMILY OF 4-POINT CORRELATORS WHICH ARE PROTECTED.

EXTREMAL AND NEAR EXTREMAL CORRELATORS

$\langle \mathcal{O}_{p_1} \mathcal{O}_{p_2} \mathcal{O}_{p_3} \mathcal{O}_{p_4} \rangle$

$\mathcal{O}_p \to \frac{1}{2}$ BPS op

$\Delta = p$

if $\quad p_4 = p_1 + p_2 + p_3 - 2k$

$k = 0$ EXTREMAL

$k = 1$ NEAR EXTREMAL

- THESE CORRELATORS ARE PROTECTED (do not depend on λ).

[D'HOKER, FREEDMAN, MATHUR, MATUSIS, RASTELLI]
[D'HOKER, ERDMENGER, FREEDMAN, PEREZ-VICTORIA]
...

WHY IS IT THE CASE?

- THIS FACT HAS BEEN PROVEN IN SEVERAL DIFFERENT WAYS.

- I WOULD LIKE TO DISCUSS ONE APPROACH BASED ON R- SYMMETRY CONSIDERATIONS AND LATER I WILL INTRODUCE SUPERSPACE METHODS.

BRIEF RECAP:

- 4 POINT FUNCTIONS OF $\frac{1}{2}$ BPS OPERATORS

  (FOR SIMPLICITY OPERATORS WITH SAME DIMENSION)

$$\langle \mathcal{O}_p(x_1,y_1)\,\mathcal{O}_p(x_2,y_2)\,\mathcal{O}_p(x_3,y_3)\,\mathcal{O}_p(x_4,y_4)\rangle = \mathcal{G}(u,v,\sigma,\tau)$$

$$= \sum_{\Delta,\ell,R} a^R_{\Delta,\ell}\; g_{\Delta,\ell}(u,v)\; Y^R(\sigma,\tau)$$

BRIEF RECAP: [DOLAN, OSBORN]

- 4 POINT FUNCTIONS OF $\frac{1}{2}$ BPS OPERATORS (FOR SIMPLICITY OPERATORS WITH SAME DIMENSION)

$$\langle \mathcal{O}_p(x_1,y_1)\,\mathcal{O}_p(x_2,y_2)\,\mathcal{O}_p(x_3,y_3)\,\mathcal{O}_p(x_4,y_4)\rangle = \mathcal{G}(u,v,\sigma,\tau)$$

$$= \sum_{\substack{\Delta,\ell \\ R}} a^R_{\Delta,\ell}\, \underbrace{g_{\Delta,\ell}(u,v)}_{\text{CONFORMAL BLOCKS}}\, Y^R(\sigma,\tau) \rightarrow \text{R-SYMMETRY}$$

$[0,p,0]\times[0,p,0]$

CONFORMAL BLOCKS → PRIMARIES

- USE SUPERCONFORMAL WARD IDENTITIES:

$$\mathcal{G}(u,v,\sigma,\tau) = k + \underset{\substack{\downarrow \\ \text{SHORT MULTIPLETS}}}{\mathcal{G}^{f}(u,v,\sigma,\tau)} + R(u,v,\sigma,\tau)\, \underset{\substack{\downarrow \\ \text{LONG MULTIPLETS}}}{\mathcal{H}(u,v,\sigma,\tau)}$$

SUPERCONFORMAL PRIMARIES

$$\mathcal{H}(u,v,\sigma,\tau) = \sum_{\substack{\Delta,\ell \\ R}} \tilde{a}^{R}_{\Delta,\ell}\, Y^{R}(\sigma,\tau)\, \underbrace{G_{\Delta,\ell}(u,v)}_{\text{SUPERCONFORMAL BLOCKS}}$$

- FOR THE SPECIFIC CASE OF $\frac{1}{2}$ BPS, SUPERCONFORMAL PRIMARIES IN LONG REPRESENTATIONS APPEARING IN THE OPE OF $\frac{1}{2}$ BPS OPERATORS TRANSFORM UNDER

$$[0,p,0] \times [0,p,0] \underset{\text{LONG}}{\longrightarrow} \sum_{0 \leq m \leq n \leq p-2} [n-m,\, 2m,\, n-m]$$

- CAVEAT: RECOMBINATION OF MULTIPLETS AT THRESHOLD.

- LET US GO BACK TO OUR CASE OF EXTREMAL AND NEAR EXTREMAL CORRELATORS

$$\langle \mathcal{O}_p(x_1, y_1)\, \mathcal{O}_p(x_2, y_2)\, \mathcal{O}_p(x_3, y_3)\, \mathcal{O}_{3p}(x_4, y_4) \rangle$$

$$\langle \mathcal{O}_p(x_1, y_1)\, \mathcal{O}_p(x_2, y_2)\, \mathcal{O}_p(x_3, y_3)\, \mathcal{O}_{3p-2}(x_4, y_4) \rangle$$

- IDENTIFY THE R-SYMMETRY REPS OF THE INTERMEDIATE OPERATORS

$$[0,p,0] \times [0,p,0] \cap [0,p,0] \times [0,3p,0]$$

$$= [0,2p,0]$$

$$[0,p,0] \times [0,p,0] \cap [0,p,0] \times [0,3p-2,0]$$

$$= [0,2p,0] \oplus [0,2p-2,0] \oplus [1,2p-2,1]$$

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$$= [0,2p,0] \oplus [0,2p-2,0] \oplus [1,2p-2,1]$$

$$\boxed{\text{IN NONE OF THESE R-SYMMETRY REPS } \underline{\underline{\text{LONG}}} \text{ OPERATORS CAN TRANSFORM}}$$

- AS A DIRECT CONSEQUENCE, DUE TO THE FACT THAT 3 POINT FUNCTIONS OF $\langle \frac{1}{2}\text{BPS}\ \frac{1}{2}\text{BPS}\ \text{short} \rangle$ ARE PROTECTED, EXTREMAL AND NEAR EXTREMAL CORRELATORS OF $\frac{1}{2}$ BPS OPERATORS ARE PROTECTED (DO NOT DEPEND ON λ, ONLY ON N).

MOVE ON TO THE CASE OF $\langle \frac{1}{2}\text{BPS}\ \frac{1}{2}\text{BPS}\ \frac{1}{4}\text{BPS}\ \frac{1}{4}\text{BPS}\rangle$

- CONSIDER THE CASE IN WHICH

$$\frac{1}{4}\text{BPS} \to [q_i, 0, q_i]$$

- EXTREMALITY $\to$ $p_1 + p_2 + 2q_1 = 2q_2$ (ASSUMING $p_2 \geq q_1$)

- $$[0, p_1, 0] \times [0, p_2, 0] = \sum_{n=0}^{p_1-2} \sum_{m=0}^{n} [n-m, 2m+2\delta, n-m]$$

LONG OPERATORS

$2\delta = p_2 - p_1$

- BY INSPECTING THE $[q,0,q] \times [q+\frac{p_1+p_2}{2}, 0, q+\frac{p_1+p_2}{2}]$ IT IS POSSIBLE TO SEE THAT THERE IS NO OVERLAP WITH THE $[0,p_1,0] \times [0,p_2,0]$.

$$\langle \tfrac{1}{2}\,\text{BPS} \quad \tfrac{1}{2}\,\text{BPS} \quad \tfrac{1}{4}\,\text{BPS} \quad \tfrac{1}{4}\,\text{BPS} \rangle$$

IS ALSO NON RENORMALISED!

$$\left\langle \tfrac{1}{4}\,\mathrm{BPS} \quad \tfrac{1}{4}\,\mathrm{BPS} \quad \tfrac{1}{4}\,\mathrm{BPS} \quad \tfrac{1}{4}\,\mathrm{BPS} \right\rangle$$

- TO STUDY THIS PROBLEM WE NEED TO RESORT TO ANALYTIC SUPERSPACE.
- IN ANALYTIC SUPERSPACE SHORT AND SEMI SHORT MULTIPLET ARE AUTOMATICALLY IRREDUCIBLE
- SUPERCONFORMAL WARD IDENTITIES ARE SOLVED BY SIMPLE ALGEBRAIC CONSTRAINTS

ANALYTIC SUPERSPACE [HESLOP, APRILE HESLOP, HOWE HARTWELL ...]

- MINKOWSKI + R SYMMETRY $y^{aa'}$

$$X^{A\dot{A}} = \begin{pmatrix} x^{\alpha\dot{\alpha}} & \rho^{\alpha a'} \\ \bar{\rho}^{a\dot{\alpha}} & y^{aa'} \end{pmatrix}$$

ANALYTIC SUPERSPACE

- MINKOWSKI + R SYMMETRY $y^{aa'}$

$$X^{a\dot{a}} = \begin{pmatrix} \overbrace{x}^{\text{LORENTZ}}{}^{\alpha\dot{\alpha}} & \rho^{\alpha a'} \\ \bar{\rho}^{a\dot{\alpha}} & y^{\underbrace{aa'}_{SU(2)\subset SU(4)_R}} \end{pmatrix}$$

$\uparrow$

FERMIONIC COORDINATES GENERATING THE MULTIPLET

ANALYTIC SUPERSPACE

- MINKOWSKI + R SYMMETRY $y^{aa'}$

$$X^{a\dot{a}} = \begin{pmatrix} x^{\alpha\dot{\alpha}} & \rho^{\alpha a'} \\ \bar{\rho}^{a\dot{\alpha}} & y^{aa'} \end{pmatrix}$$

- SUPERPRIMARIES $\Theta^{\gamma}_{\lambda_L, \lambda_R}$

ANALYTIC SUPERSPACE

- MINKOWSKI + R SYMMETRY $y^{aa'}$

$$X^{A\dot{A}} = \begin{pmatrix} x^{\alpha\dot{\alpha}} & \rho^{\alpha a'} \\ \bar{\rho}^{a\dot{\alpha}} & y^{aa'} \end{pmatrix}$$

- SUPERPRIMARIES $\quad \mathcal{O}^{\gamma}_{\lambda_L, \lambda_R}$

$$\downarrow$$

$$\underset{\lambda_L}{SL(2|2)} \times \underset{\lambda_R}{SL(2|2)} \times \underset{\gamma}{\mathbb{C}^*}$$

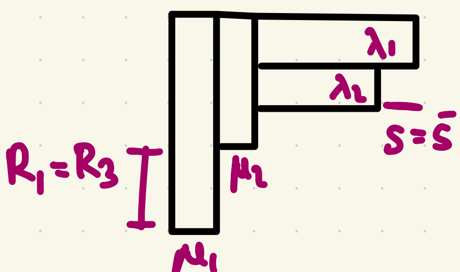
ANALYTIC SUPERSPACE

- MINKOWSKI + R SYMMETRY $y^{aa'}$

$$X^{A\dot{A}} = \begin{pmatrix} x^{\alpha\dot{\alpha}} & \rho^{\alpha a'} \\ \bar{\rho}^{a\dot{\alpha}} & y^{aa'} \end{pmatrix}$$

- SUPERPRIMARIES $\mathcal{O}^{\gamma}_{\lambda_L,\lambda_R} \to$ FOR SYMM REPS

$$\lambda_L = \lambda_R = \lambda = [1^{\mu_1}, 1^{\mu_2}, \lambda_1, \lambda_2]$$



STANDARD NOTATION $[R_1, R_2, R_3]$

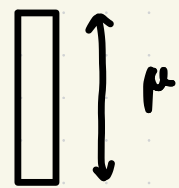
$(s, \bar{s})$

$$\Delta = \gamma + \sum_i \lambda_i$$

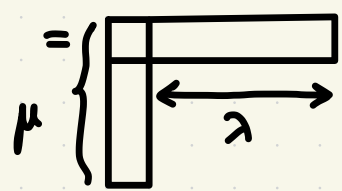
- FOR PROTECTED OPERATORS:

$\frac{1}{2}$ BPS $\quad \lambda = \bullet$ (OPERATORS ARE SCALARS IN ANALYTIC SUPERSPACE)

$\frac{1}{4}$ BPS $\quad \lambda = [1^{\mu}] =$



SEMI SHORT $\quad \lambda = [1^{\mu}, \lambda] =$



STRATEGY

- SET UP 3pt FUNCTIONS IN ANALYTIC SUPERSPACE
- USE THE OPE TO WRITE THE 4pt → 3pt
    - SOLVING SUPERCONFORMAL WARD IDENTITIES
    - HARMONIC ANALYTICITY
    - PROCEED AS IN THE FIRST PART OF TALK BY UNDERSTANDING WHICH MULTIPLETS APPEAR IN THE OPE.

3pt FUNCTIONS [HESLOP HOWE, HESLOP PUERTA-RAMISA]

$$\left\langle \theta^{\gamma_1}_{\lambda^1_L,\lambda^1_R} \; \theta^{\gamma_2}_{\lambda^2_L,\lambda^2_R} \; \theta^{\gamma_3}_{\lambda^3_L,\lambda^3_R} \right\rangle =$$

$$= C_{123} \; g_{12}^{\frac{1}{2}(\gamma_1+\gamma_2-\gamma_3)} \; g_{23}^{\frac{1}{2}(\gamma_2+\gamma_3-\gamma_1)} \; g_{13}^{\frac{1}{2}(\gamma_3+\gamma_1-\gamma_2)} \,\cdot$$

$$\left[\prod_{ij} I_{ij}^{a_{ij}} \prod_{k=1}^{3} Y_k^{a_k} \right]_{\lambda^k_L,\lambda^k_R}$$

3pt FUNCTIONS

$$\langle \Theta^{\gamma_1}_{\lambda^1_L,\lambda^1_R}\ \Theta^{\gamma_2}_{\lambda^2_L,\lambda^2_R}\ \Theta^{\gamma_3}_{\lambda^3_L,\lambda^3_R} \rangle =$$

$$= C_{123}\ g_{12}^{\frac{1}{2}(\gamma_1+\gamma_2-\gamma_3)}\ g_{23}^{\frac{1}{2}(\gamma_2+\gamma_3-\gamma_1)}\ g_{13}^{\frac{1}{2}(\gamma_3+\gamma_1-\gamma_2)}.$$

SUSY GENERALIZATION OF CONFORMAL INVERSION

$$\left[\prod_{ij} I_{ij}^{a_{ij}} \prod_{k=1}^{3} Y_k^{a_k}\right]_{\lambda^k_L,\lambda^k_R}$$

$a_{ij}+a_{ik}+a_i = |\lambda^i_R|$

$a_{ji}+a_{ki}+a_i = |\lambda^i_L|$

COMBINATION OF INVERSIONS

$$g_{ij} = \mathrm{sdet}\,(X_{ij}^{-1}) = \frac{\det y_{ij}}{\det x_{ij}} + \Theta(\rho,\bar\rho)$$

$[\cdots]$ YOUNG SYMMETRISATION

- FOR NON $\frac{1}{2}$ BPS OPERATORS THERE ARE SEVERAL STRUCTURES, SIMILARLY TO SPINNING OPS

HARMONIC ANALYTICITY

[HESLOP, HESLOP-PUERTA, RAMOS]

- SHORTENING CONDITIONS USING WARD IDENTITIES

$$\Updownarrow$$

ANALYTICITY IN y_{ij}

- POWERS OF SUPERDETERMINANT NEED TO BE POSITIVE

$$\gamma_i + \gamma_j - \gamma_k \geq 0$$

- I_{ij} AND Y_i ARE BUILT WITH y_{ij}^{-1}. THE DEPENDENCE NEEDS TO BE CANCELLED BY THE g_{ij}.

e.g. $$g_{ij}^{\gamma}\, I_{ij}^{q_1}\, I_{ji}^{q_2} \rightarrow \gamma \geq q_1 + q_2$$

- CAVEAT RELATED TO THE PROJECTION OF INDICES ACCORDING TO $\lambda_L^i \lambda_R^i \rightarrow$ NAIVE POLES CAN BE SOFTENED BY SPECIFIC YOUNG PROJECTION [HESLOP-PUERTA-RAMISA]

GO BACK TO THE EXTREMAL 4^{pt} OF $\frac{1}{4}$ BPS

$$\langle \mathcal{O}_{[q_1,0,q_1]} \, \mathcal{O}_{[q_2,0,q_2]} \, \mathcal{O}_{[q_3,0,q_3]} \, \mathcal{O}_{[q_4,0,q_4]} \rangle$$

$$q_1 = q_2 + q_3 + q_4$$

$$\propto \langle \mathcal{O}_{[q_1,0,q_1]} \, \mathcal{O}_{[q_2,0,q_2]} \, \mathcal{O}_\lambda^{2q_3+2q_4} \rangle \langle \mathcal{O}_\lambda^\gamma \, \mathcal{O}_{[q_3,0,q_3]} \, \mathcal{O}_{[q_4,0,q_4]} \rangle$$

1) POSITIVITY OF g_{ij} $\qquad \gamma = 2q_3 + 2q_4$

2) POLE-FREE STRUCTURE $\qquad |\lambda| = q_3 + q_4$

$\Updownarrow$

HARMONIC ANALYTICITY

3) YOUNG PROJECTION $\qquad$ ONLY POSSIBILITY IS TO HAVE ANTISYMMETRISED INDICES

$$\Downarrow$$

$$\theta^{2q_3+2q_4}_{\underline{\lambda}} = \theta_{[q_3+q_4,\, 0,\, q_3+q_4]}$$

ONLY PROTECTED OPERATORS!!

- EXTEND THIS PROCEDURE TO $\frac{1}{4}$ BPS OPERATORS TRANSFORMING UNDER $[q_i, p_i, q_i]$ $i = 1, \dots, 4$.

- ALSO FOR NEAR EXTREMAL (MORE INVOLVED HARMONIC ANALYTICITY BUT SAME IN SPIRIT)

MAIN POINT OF THE TALK

$$\left\langle \tfrac{1}{2}\text{BPS} \quad \tfrac{1}{2}\text{BPS} \quad \tfrac{1}{4}\text{BPS} \quad \tfrac{1}{4}\text{BPS} \right\rangle$$

EXTREMAL
&
NEAR EXTREMAL

$$\left\langle \tfrac{1}{4}\text{BPS} \quad \tfrac{1}{4}\text{BPS} \quad \tfrac{1}{4}\text{BPS} \quad \tfrac{1}{4}\text{BPS} \right\rangle$$

EXTREMAL
&
NEAR EXTREMAL

CONTAINS ONLY PROTECTED MULTIPLETS

↓

ALL 4pt PROTECTED IF

$\left\langle \tfrac{1}{4}\text{BPS} \ \tfrac{1}{4}\text{BPS} \ \text{short/semishort} \right\rangle$ IS PROTECTED

3pt FUNCTIONS:

- IN ORDER TO CONCLUDE THAT 4pt FUNCTIONS ARE NOT RENORMALISED WE HAVE TO MAKE SURE THAT

$$\left\langle \frac{1}{4}\text{BPS} \quad \frac{1}{4}\text{BPS} \quad \begin{matrix}\text{short}\\ \text{semi-short}\end{matrix} \right\rangle$$

IS PROTECTED.

- NOTE: THE SHORT | SEMI SHORT OPERATORS THAT WE CONSIDER ARE SUPERCONFORMAL PRIMARIES BY CONSTRUCTION

- IN [HESLOP, HOWE] IT HAS BEEN SHOWN USING SUPERSPACE APPROACH THAT SUCH 3pt FUNCTIONS ARE PROTECTED.

SEVERAL DIRECTIONS

- UNDERSTAND THE STRUCTURE OF SUPERCONFORMAL BLOCKS
- MAKE CONTACT w/ SUPERGRAVITY
- RELAX THE ASSUMPTION OF EXTREMALITY TO UNDERSTAND THE STRUCTURE OF THE $\frac{1}{4}$BPS OPE

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