

Where is tree-level heterotic string theory?

Waltraut Knop

based on [M. Boisvert, WK, L. Rastelli: 2406.12959]



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Is string theory the unique solution?

$2 \rightarrow 2$ graviton scattering: basics

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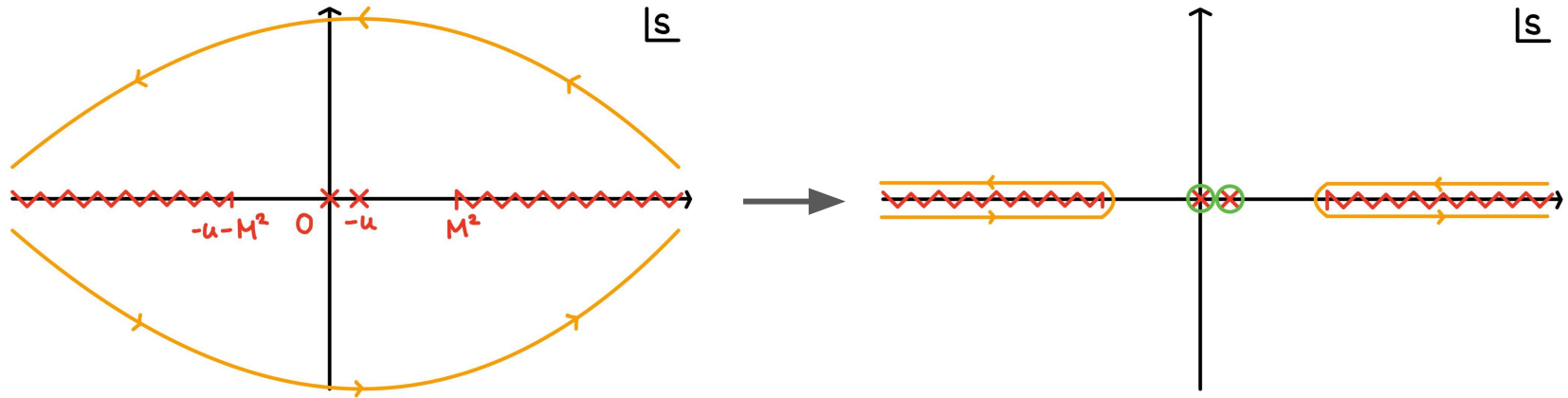
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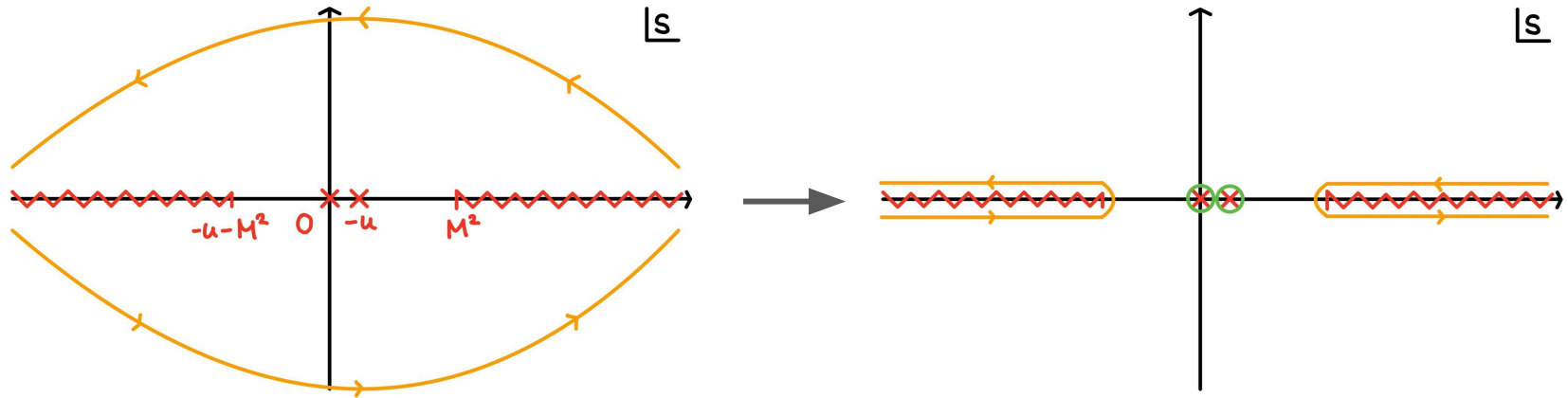
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❖ Tree-level: meromorphic

Lightning summary: sum rules

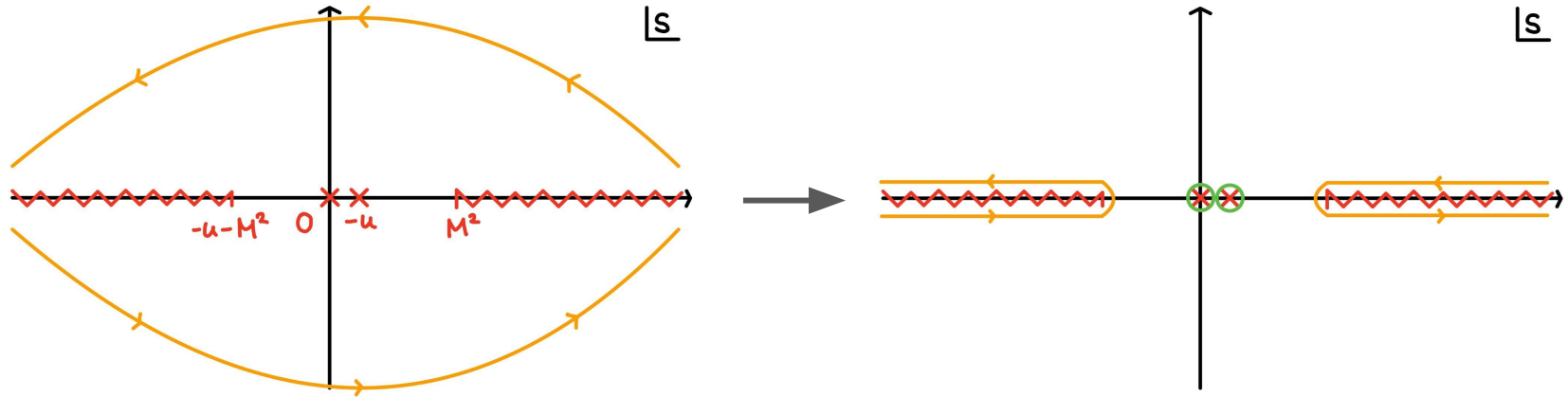


Lightning summary: sum rules



$$\text{Res}_{s=0, -u} \left[\frac{\mathcal{M}_{\text{IR}}(s, u)}{s[s(s+u)]^{k/2}} \right] = \left\langle \frac{2m^2 + u}{m^2 + u} \frac{\mathcal{P}_J \left(1 + \frac{2u}{m^2} \right)}{[m^2(m^2 + u)]^{k/2}} \right\rangle$$

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$$g_2 = \langle 2 \rangle$$

$$g_4 = \left\langle \frac{1}{m^4} \right\rangle$$

+ null constraints

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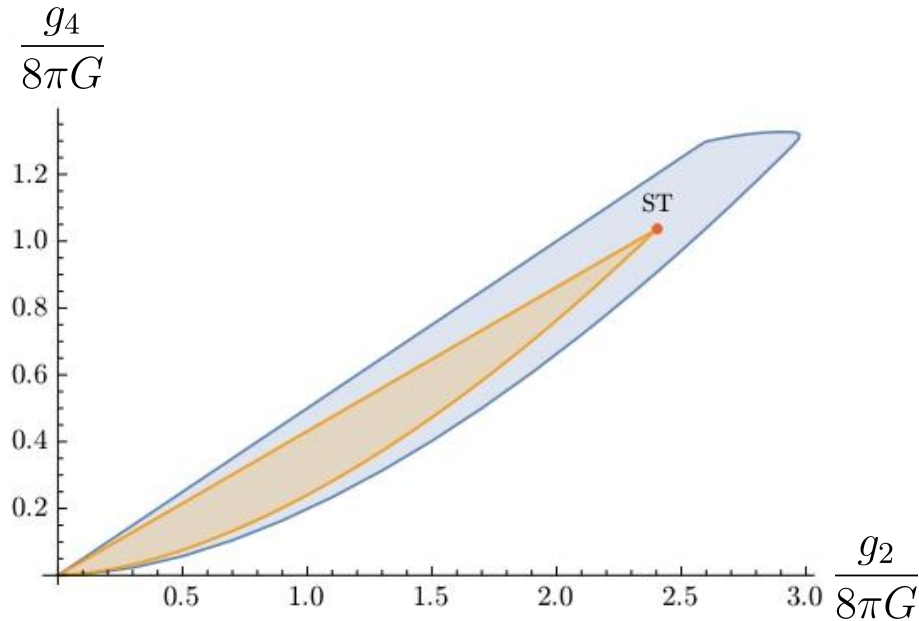
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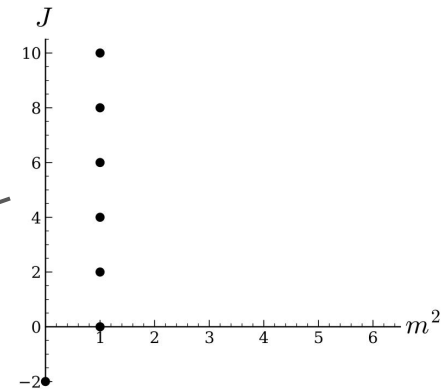
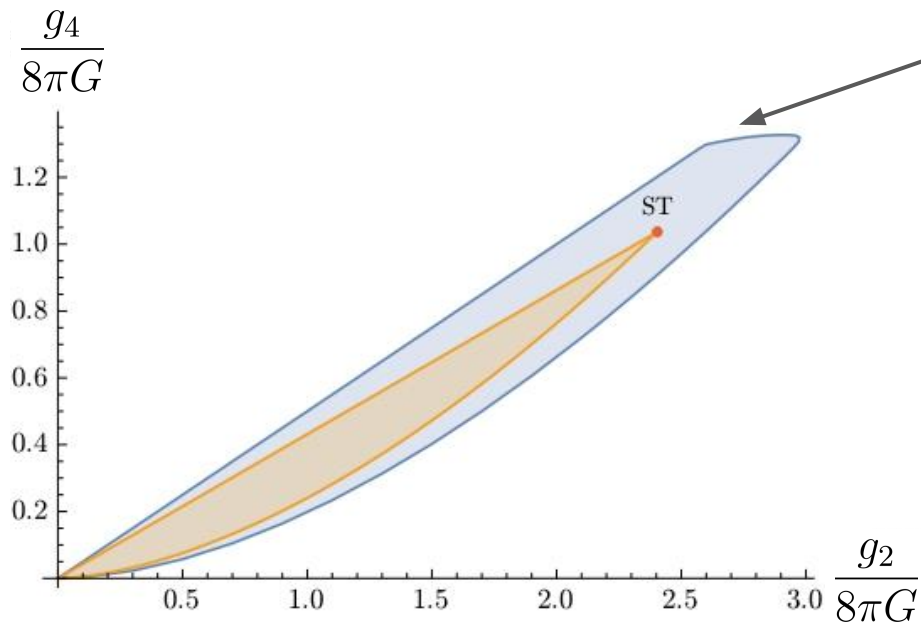


Goal: bound $\frac{g_2}{8\pi G}$ and $\frac{g_4}{8\pi G}$

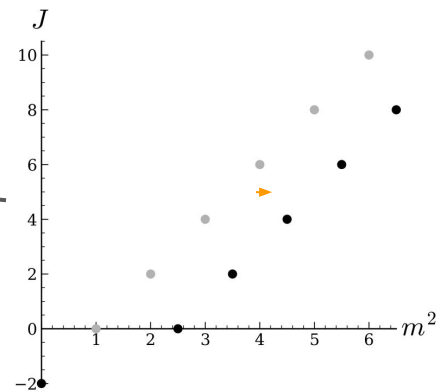
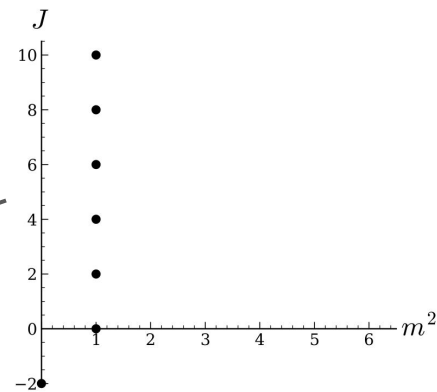
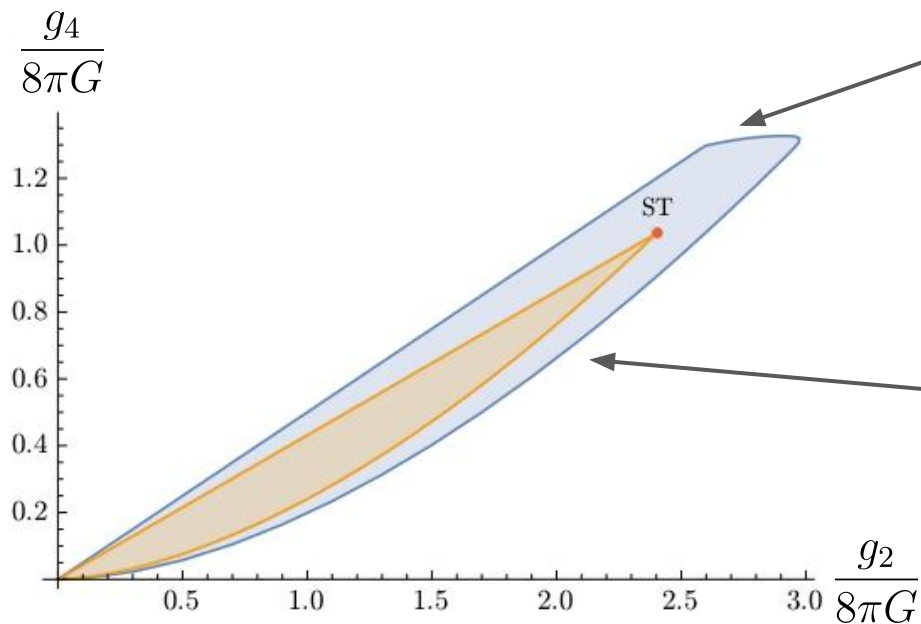
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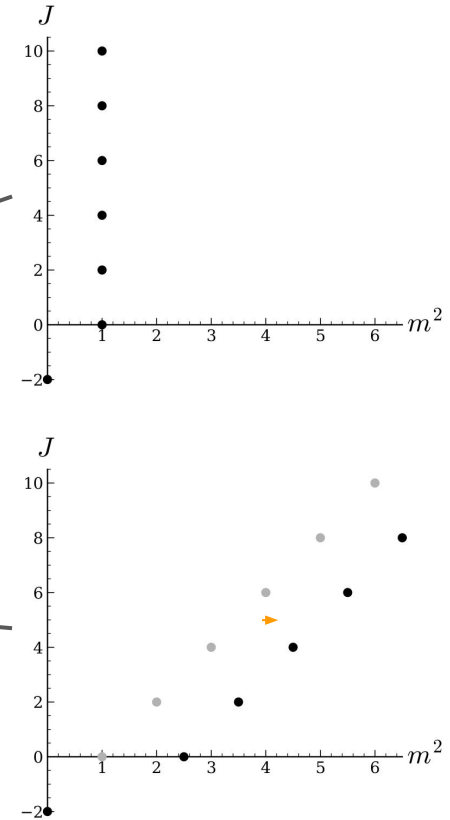
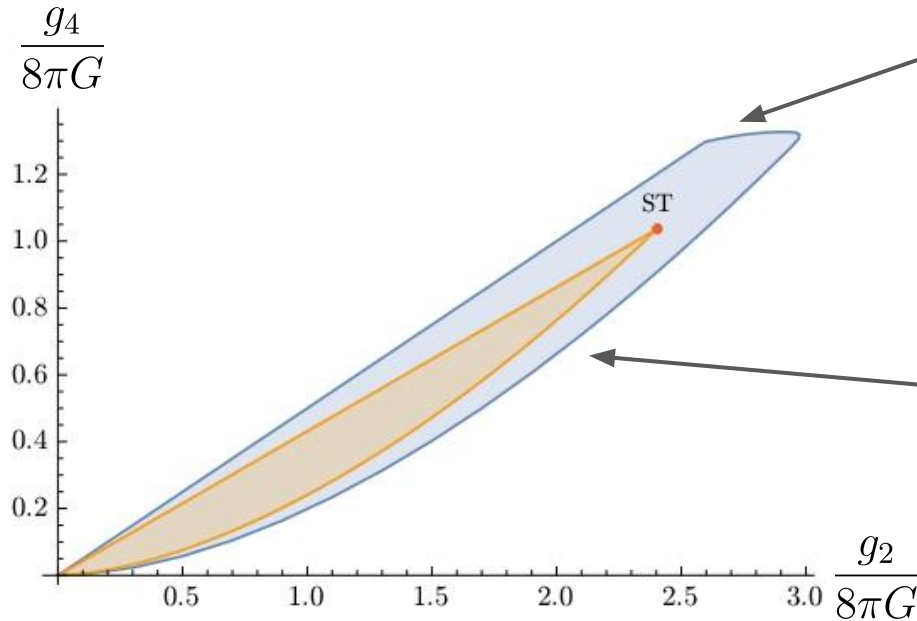
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Do these solutions extend to less susy?

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[Berman, Elvang, Geiser, Lin: 2412.13368]

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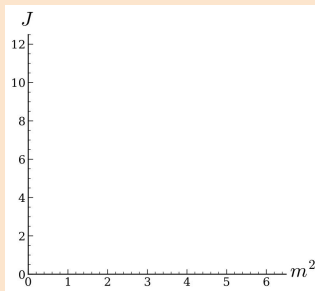
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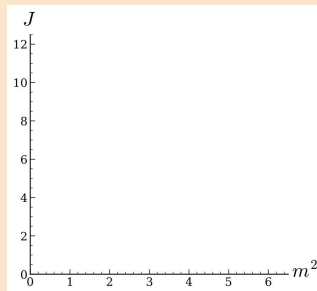
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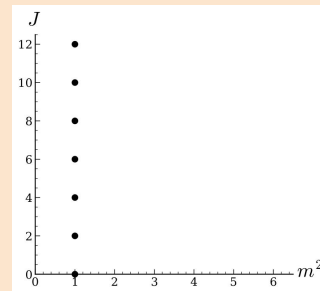
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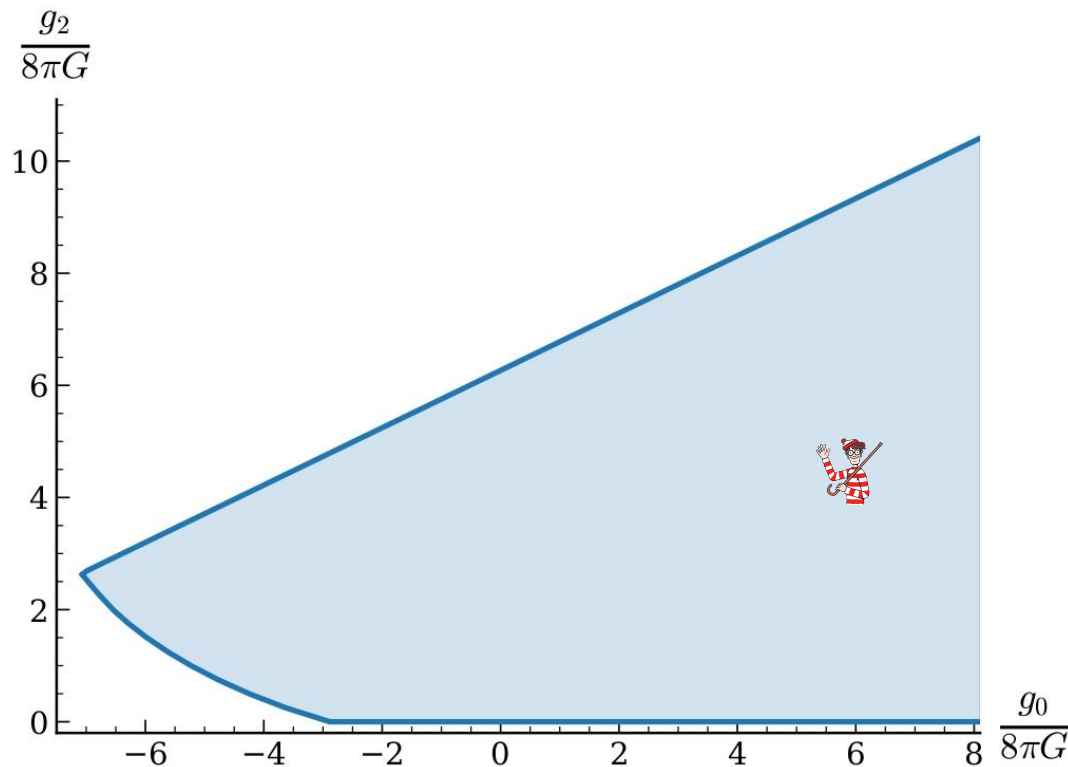


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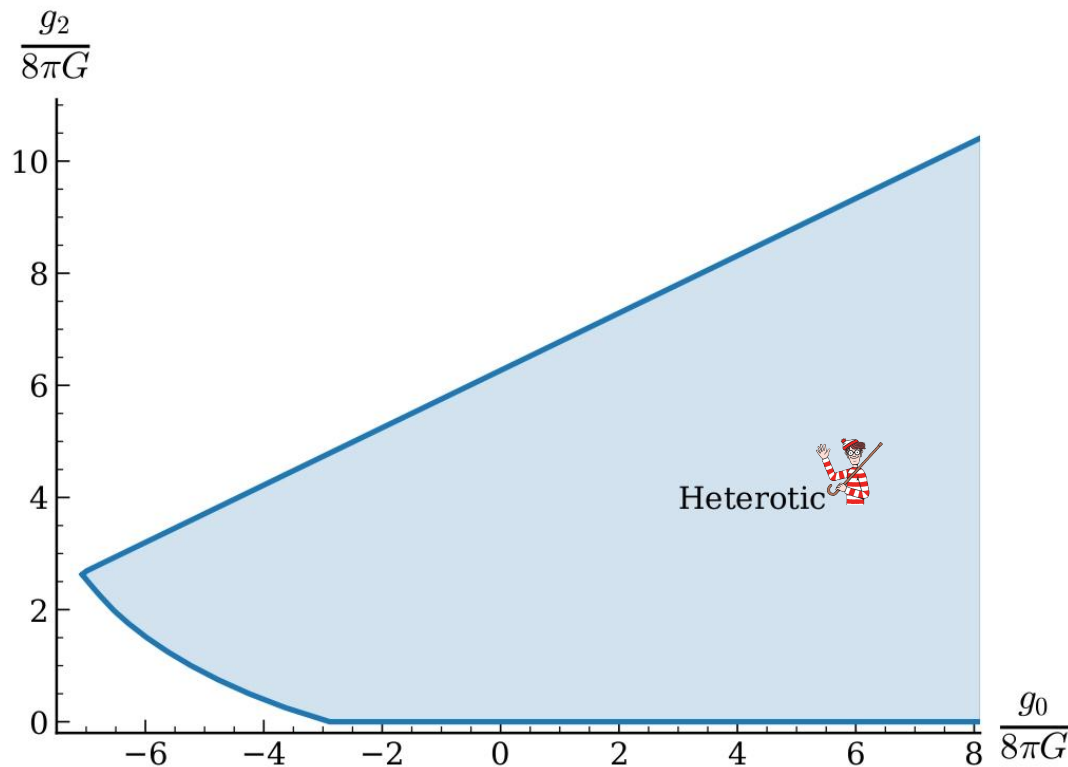


Where's tree-level heterotic string theory?

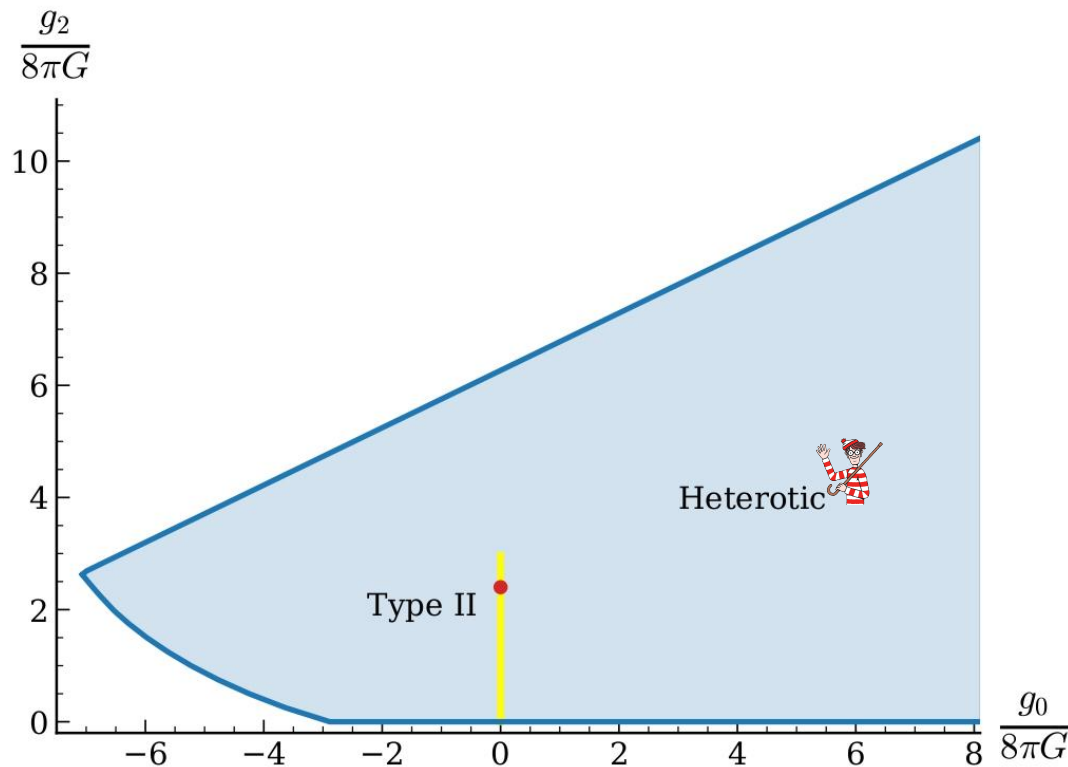
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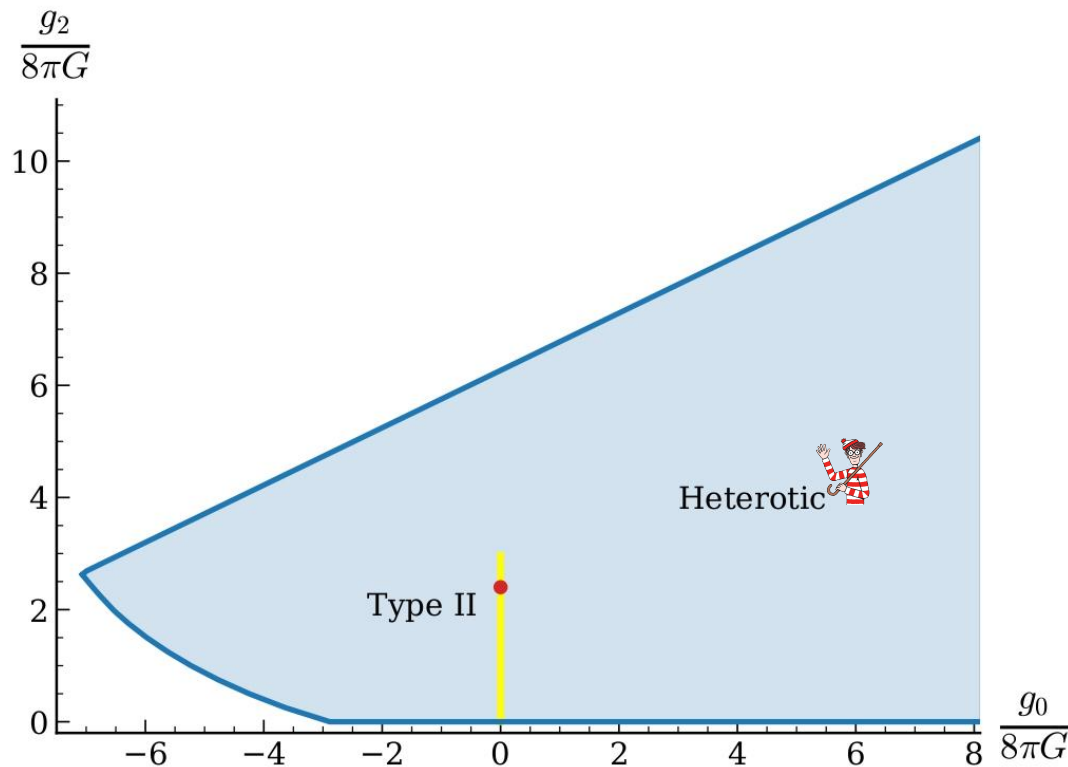
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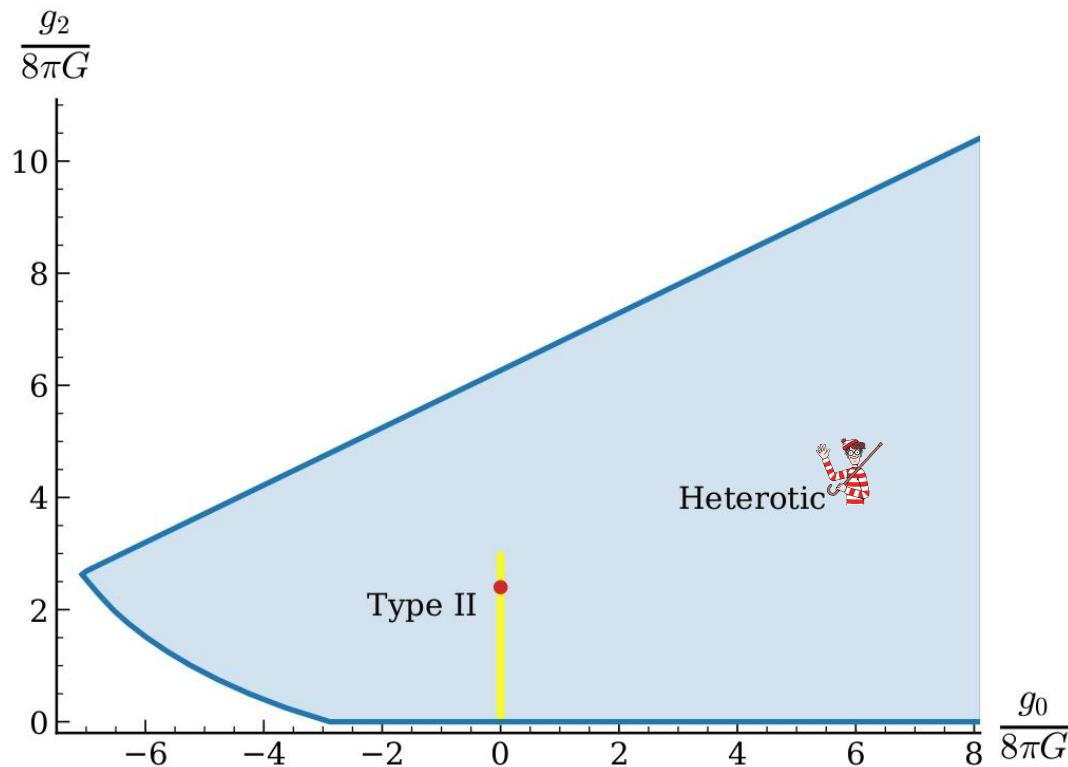


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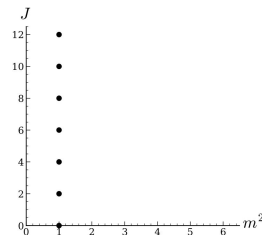


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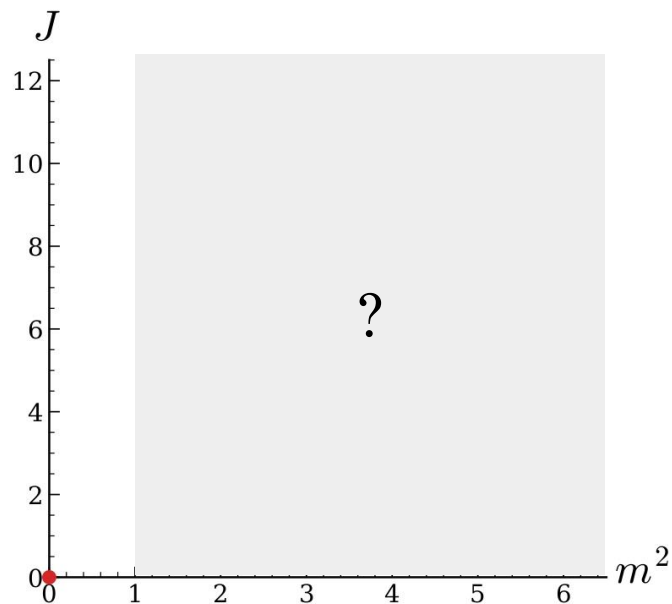


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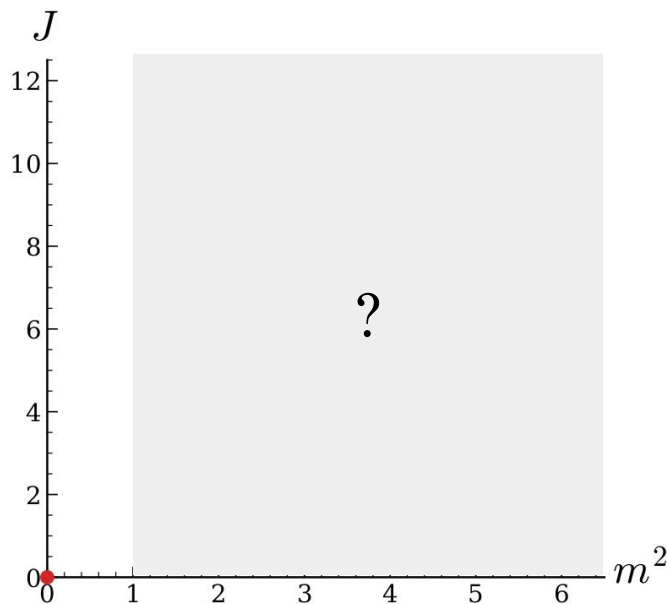
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Constraining the spectrum

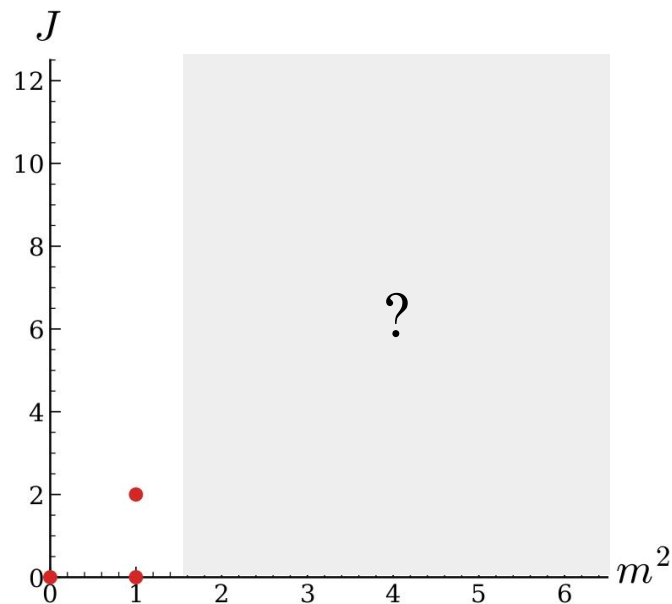


most general

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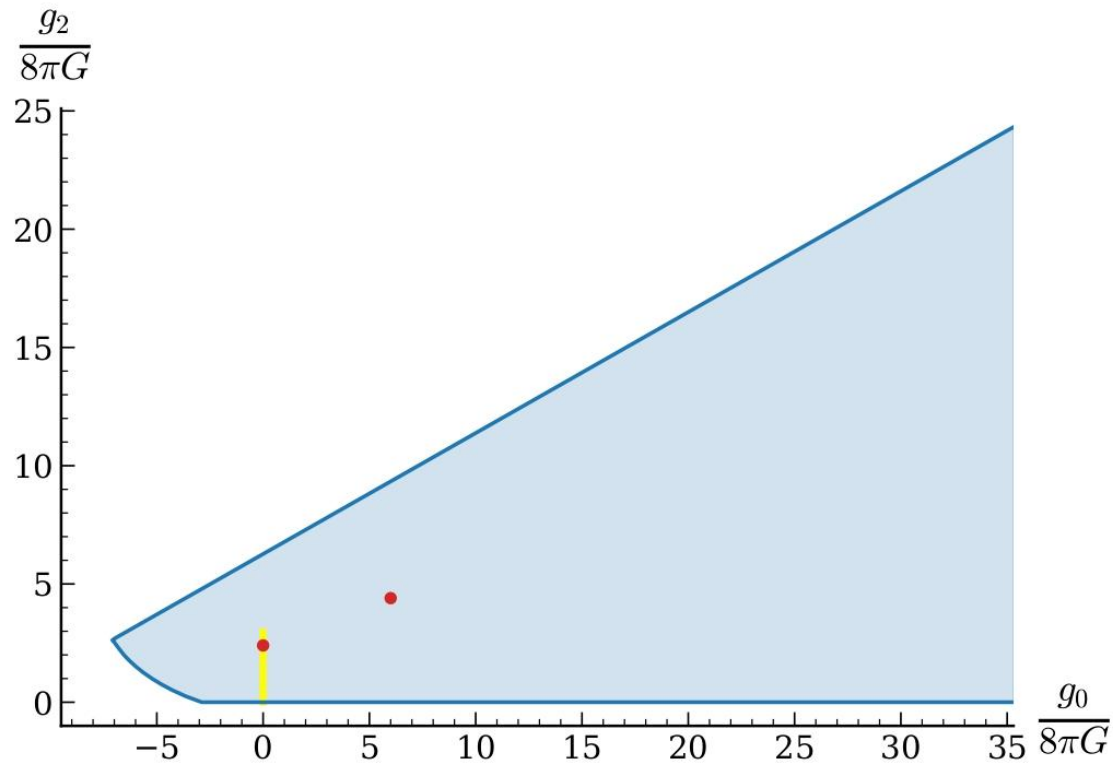
most general



excludes infinite spin tower

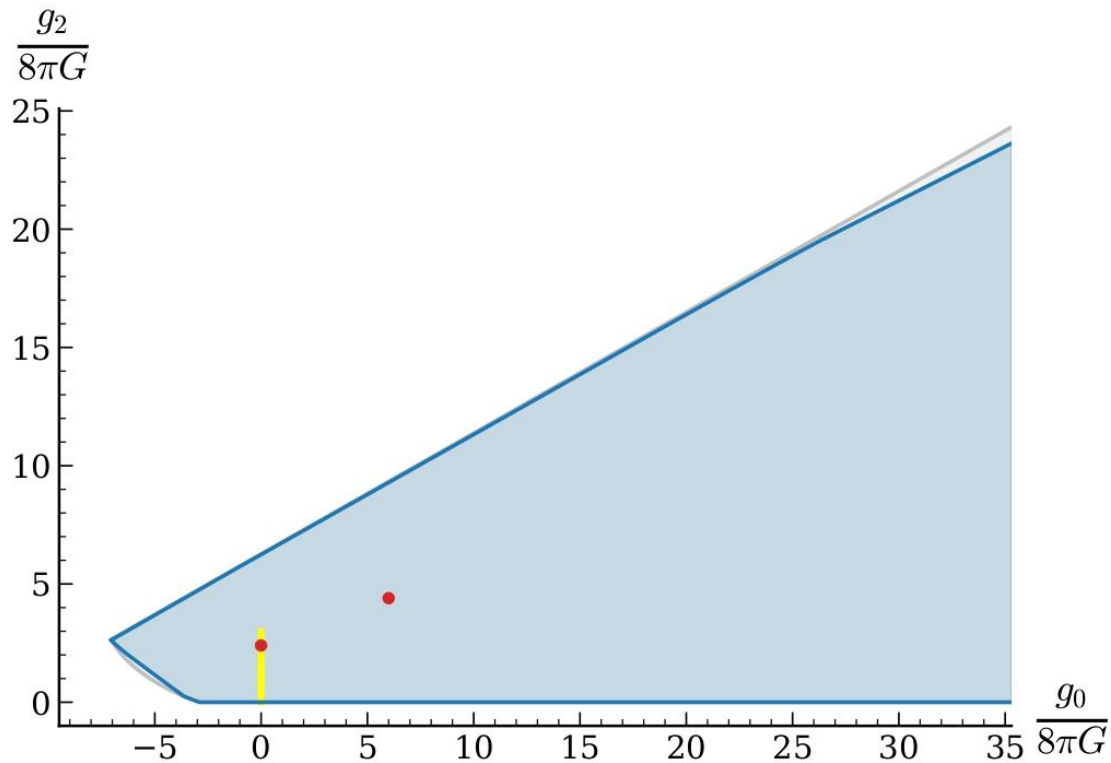
Constraining the spectrum

$$\mathcal{M}^2 = 1$$



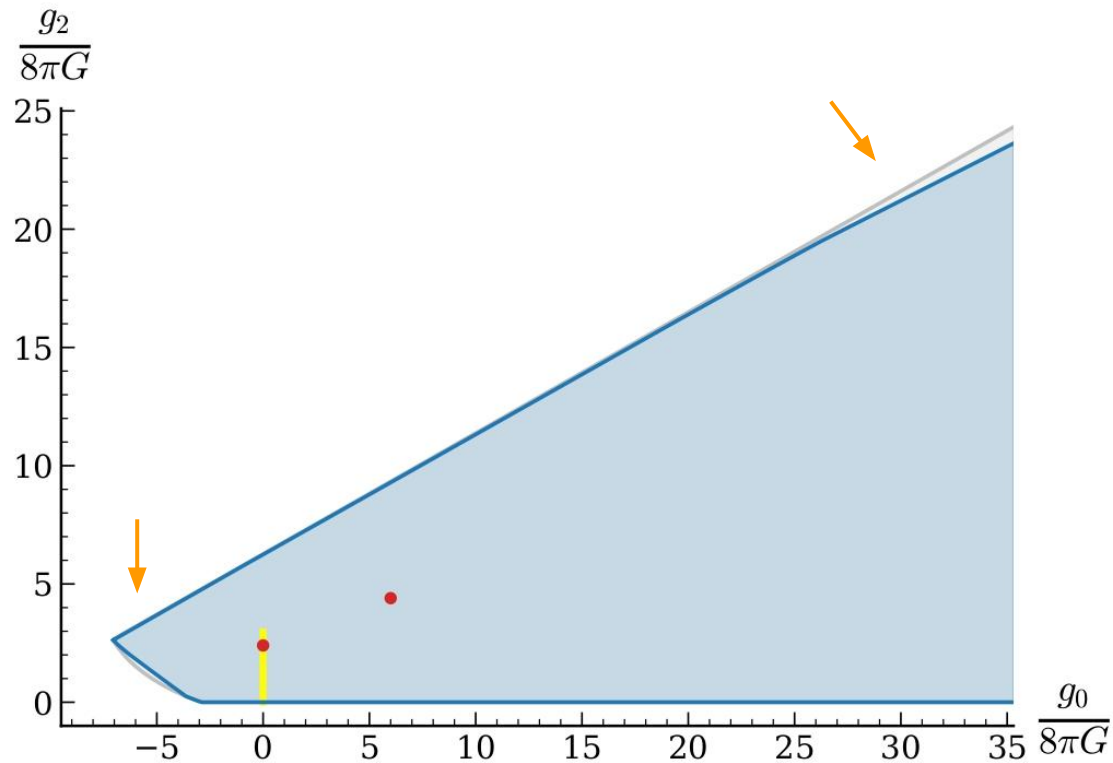
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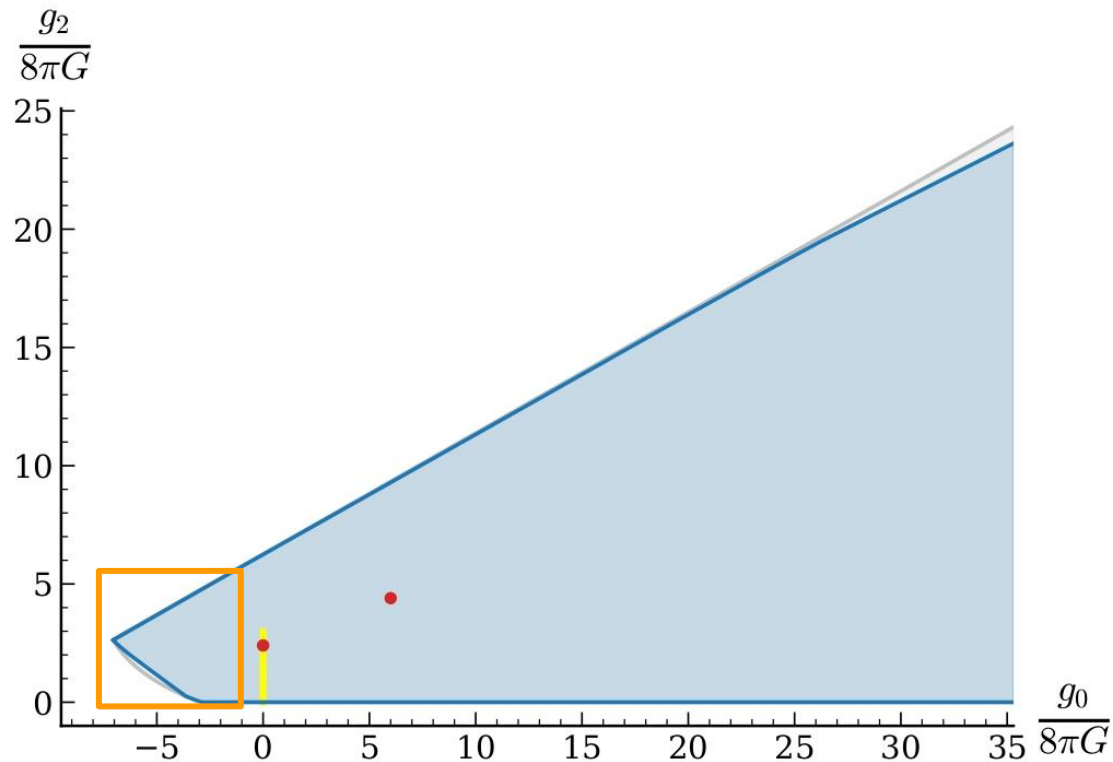
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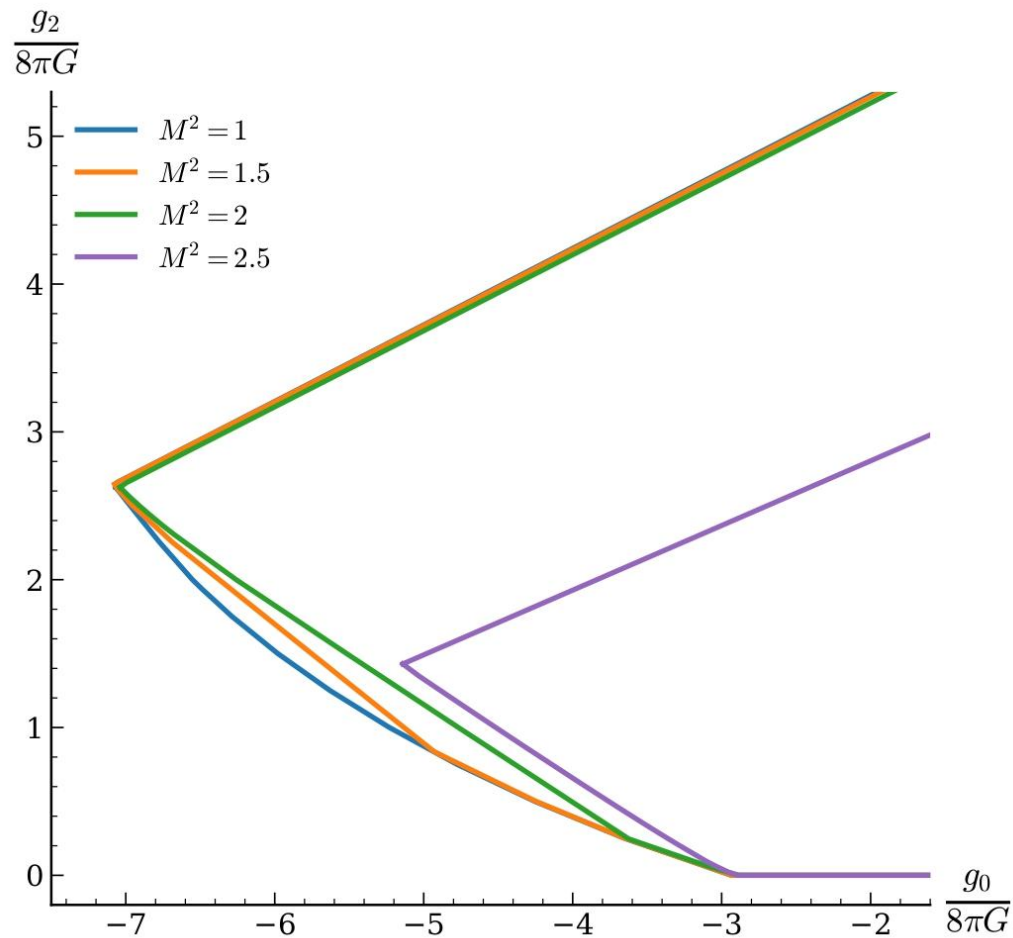


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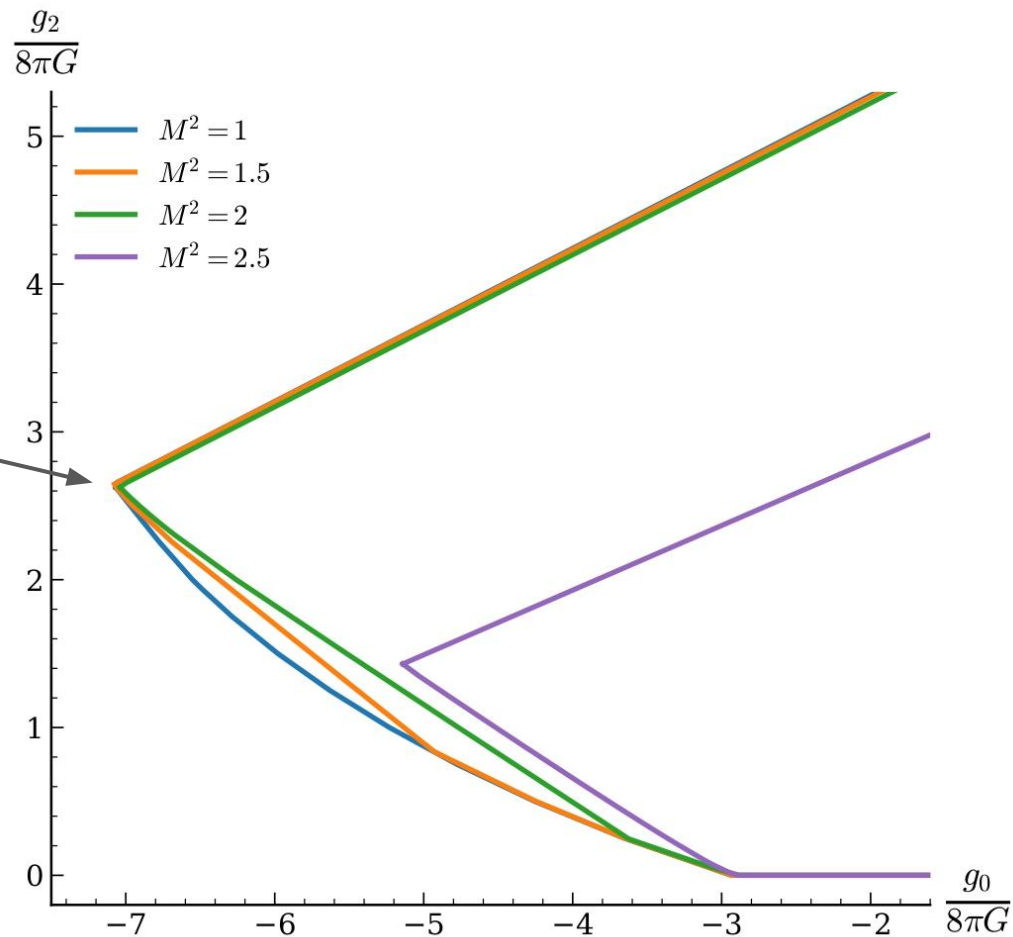
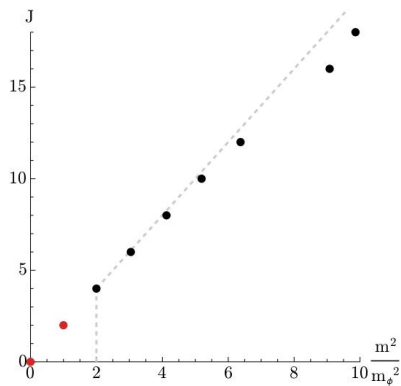
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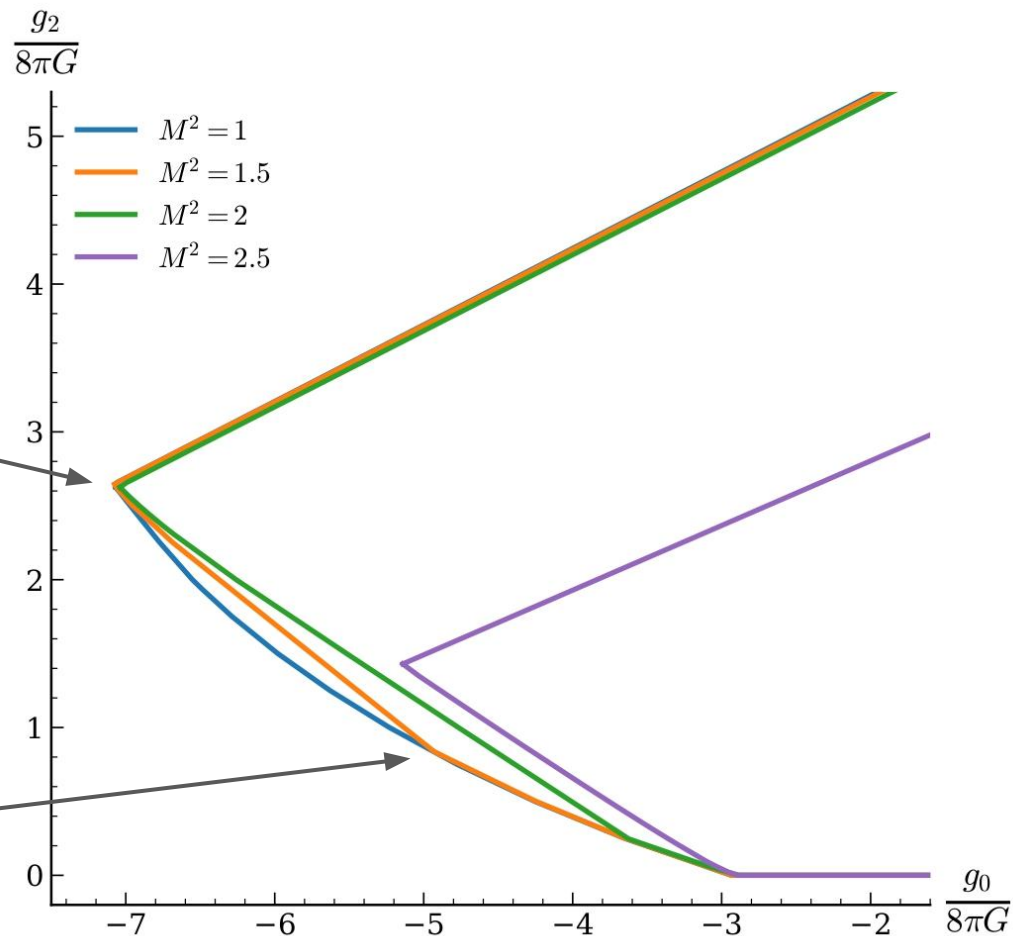
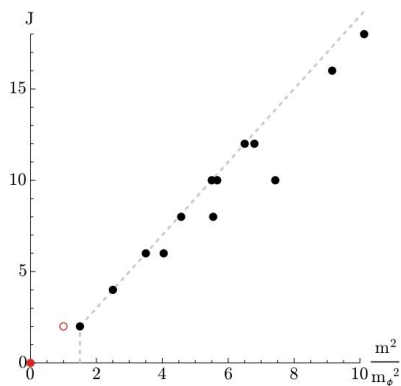
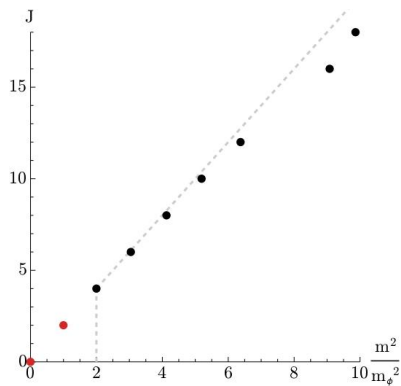
Zoom in: low g_0



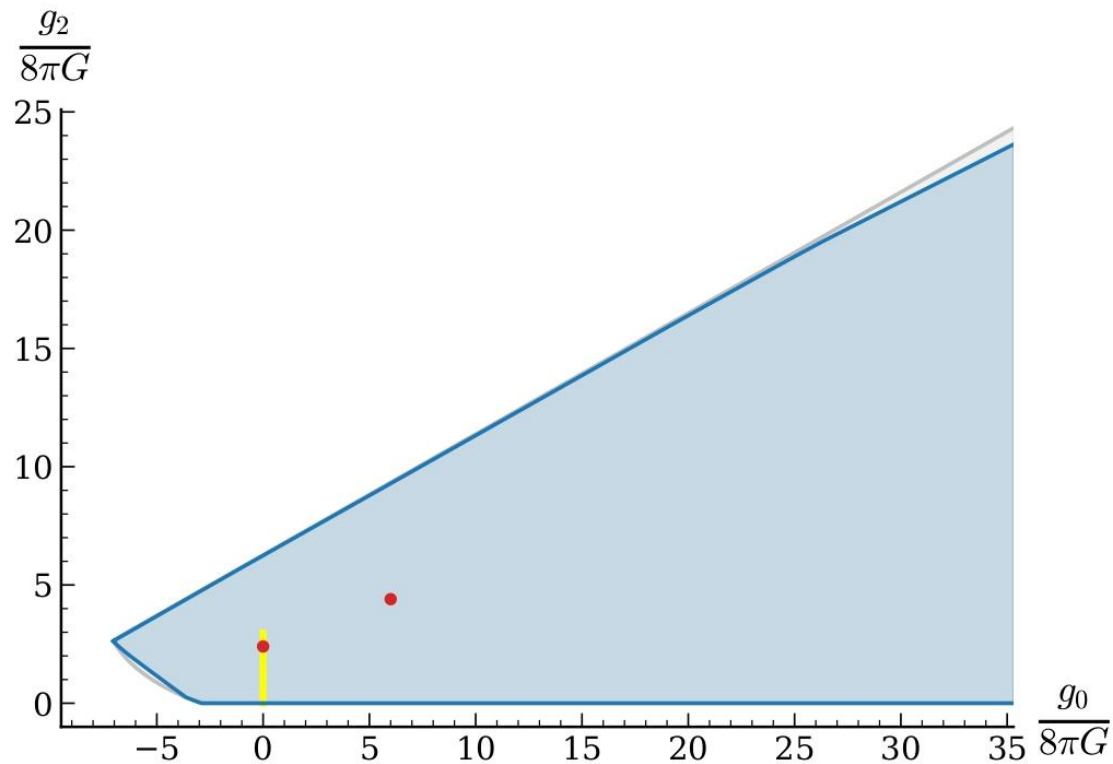
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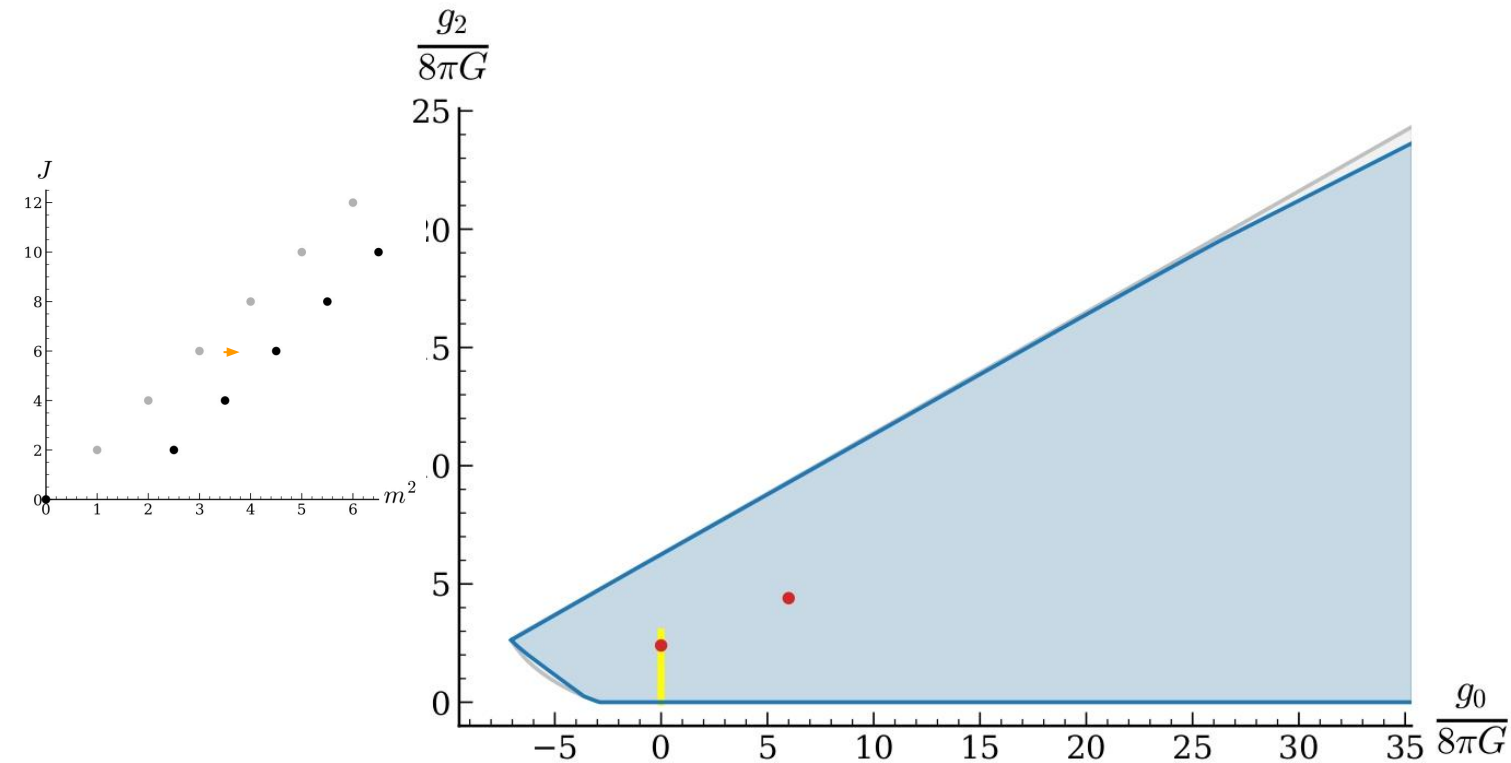
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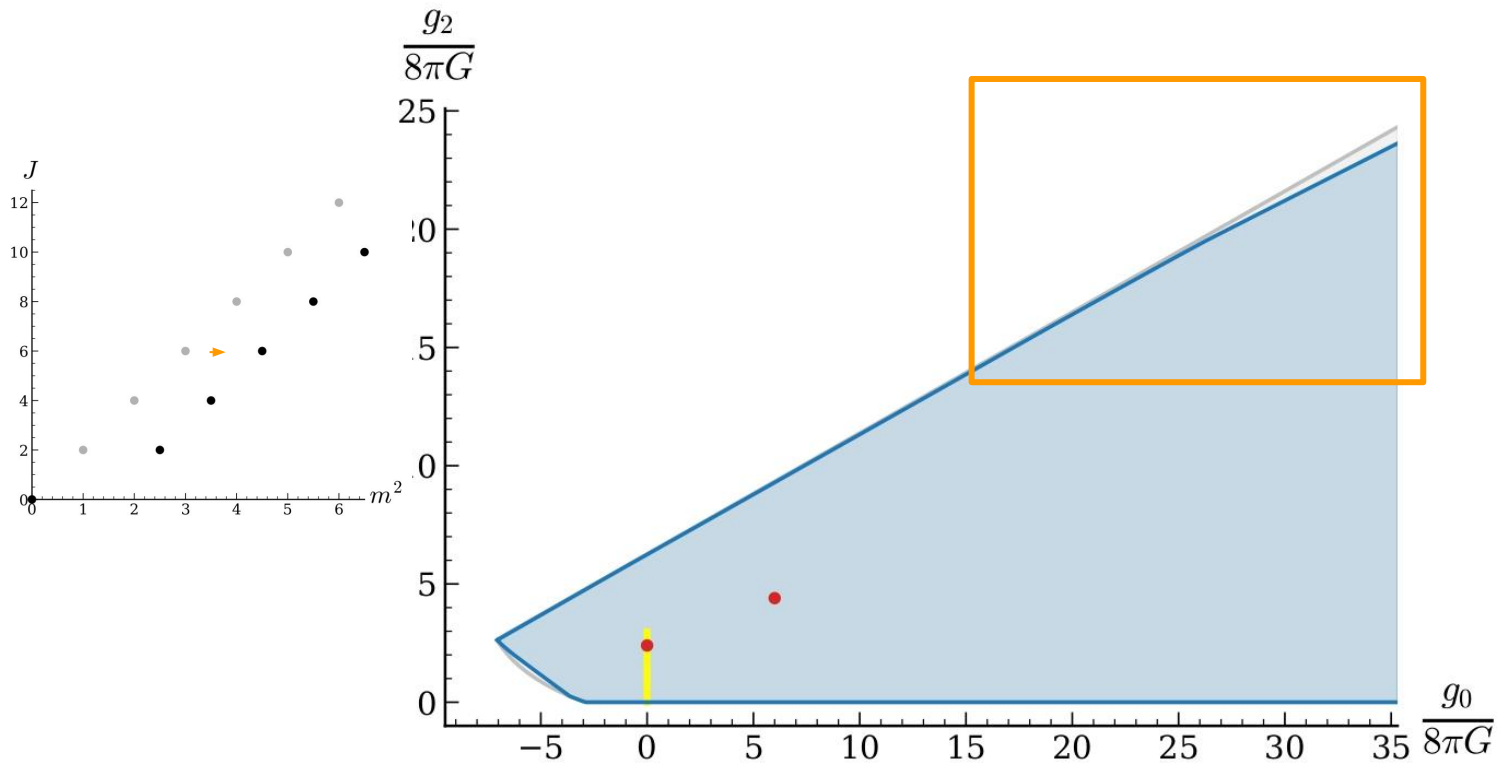
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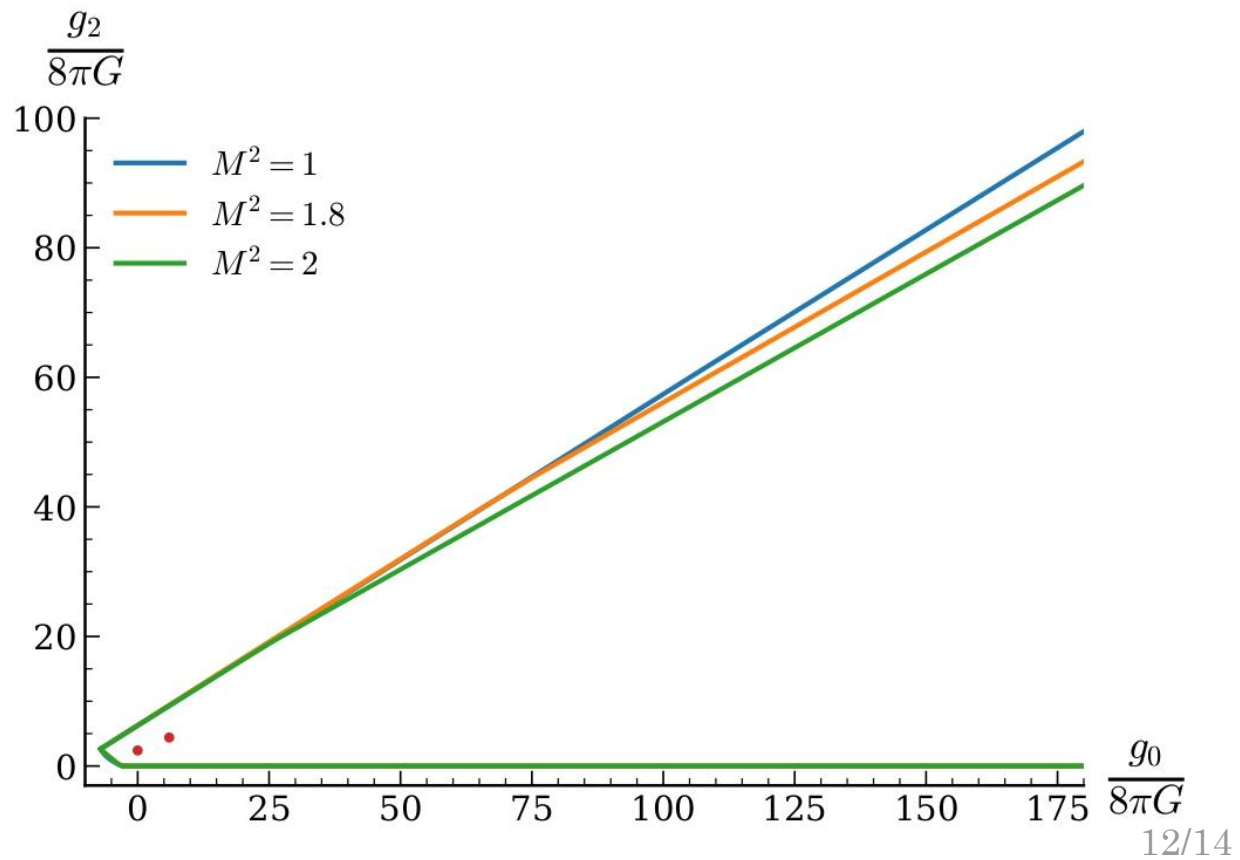
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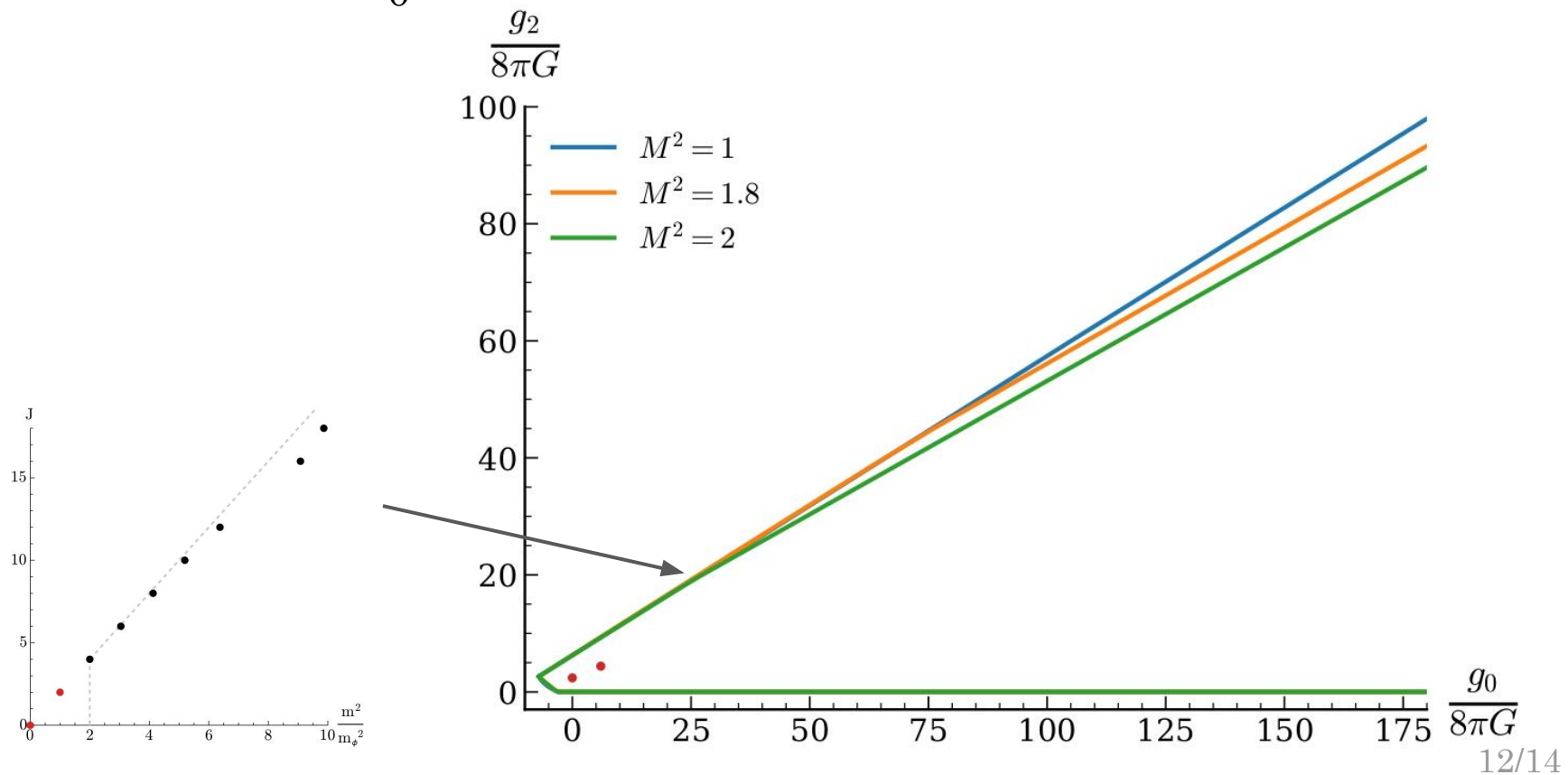
Constraining the spectrum



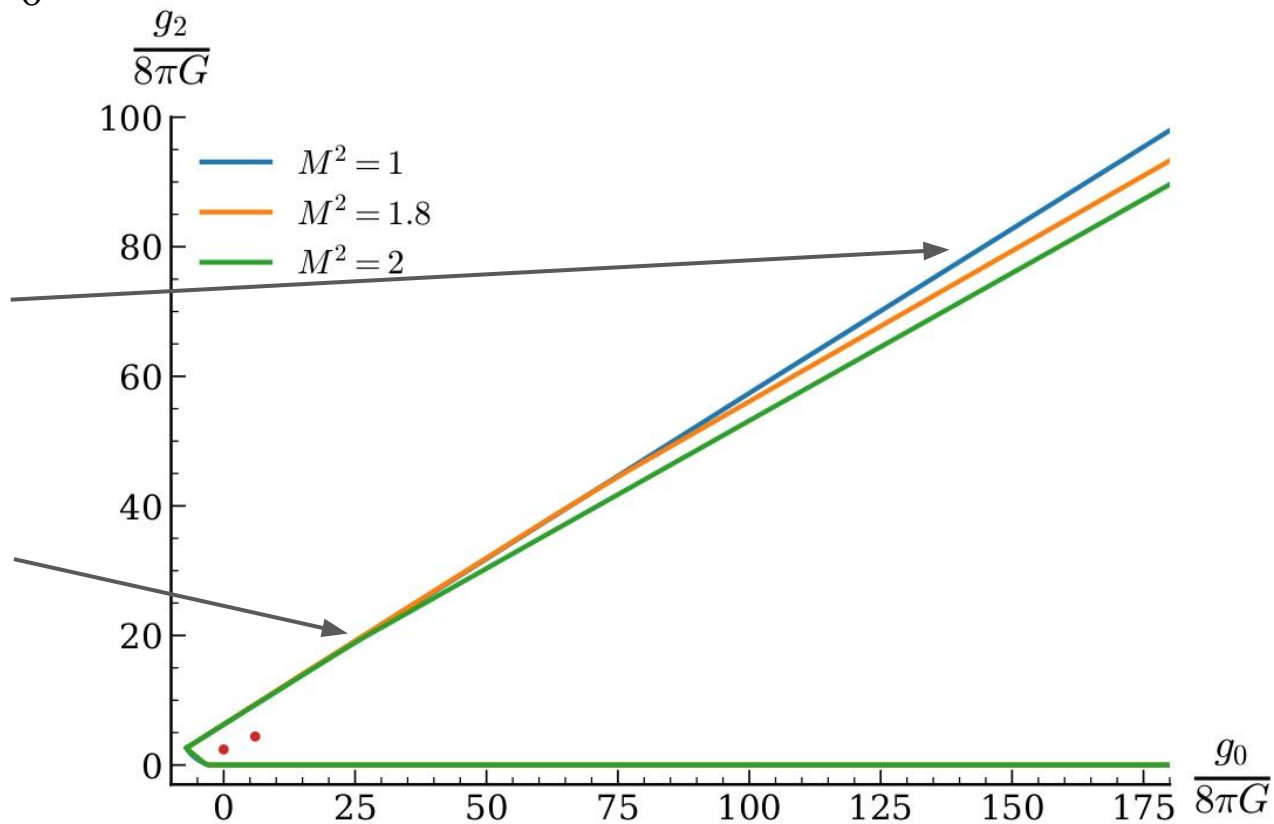
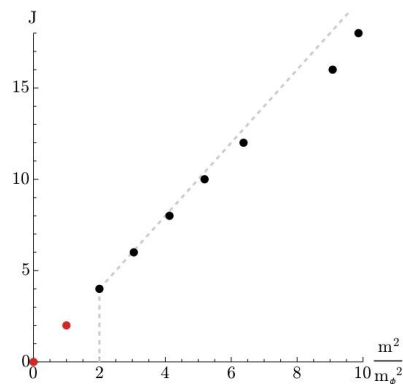
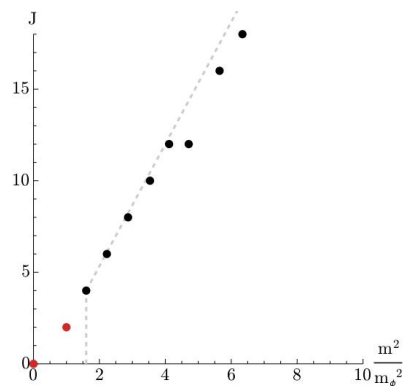
Zoom out: high g_0



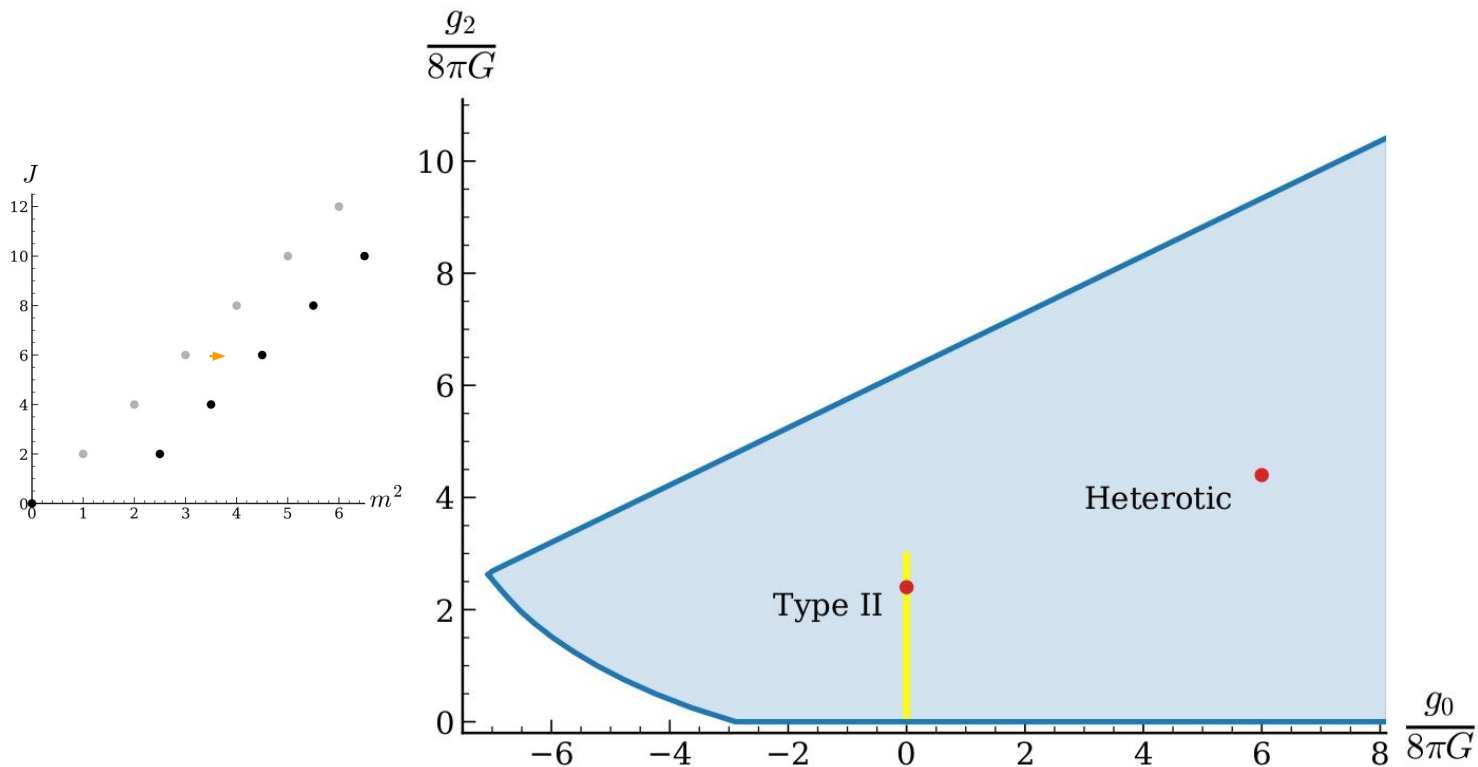
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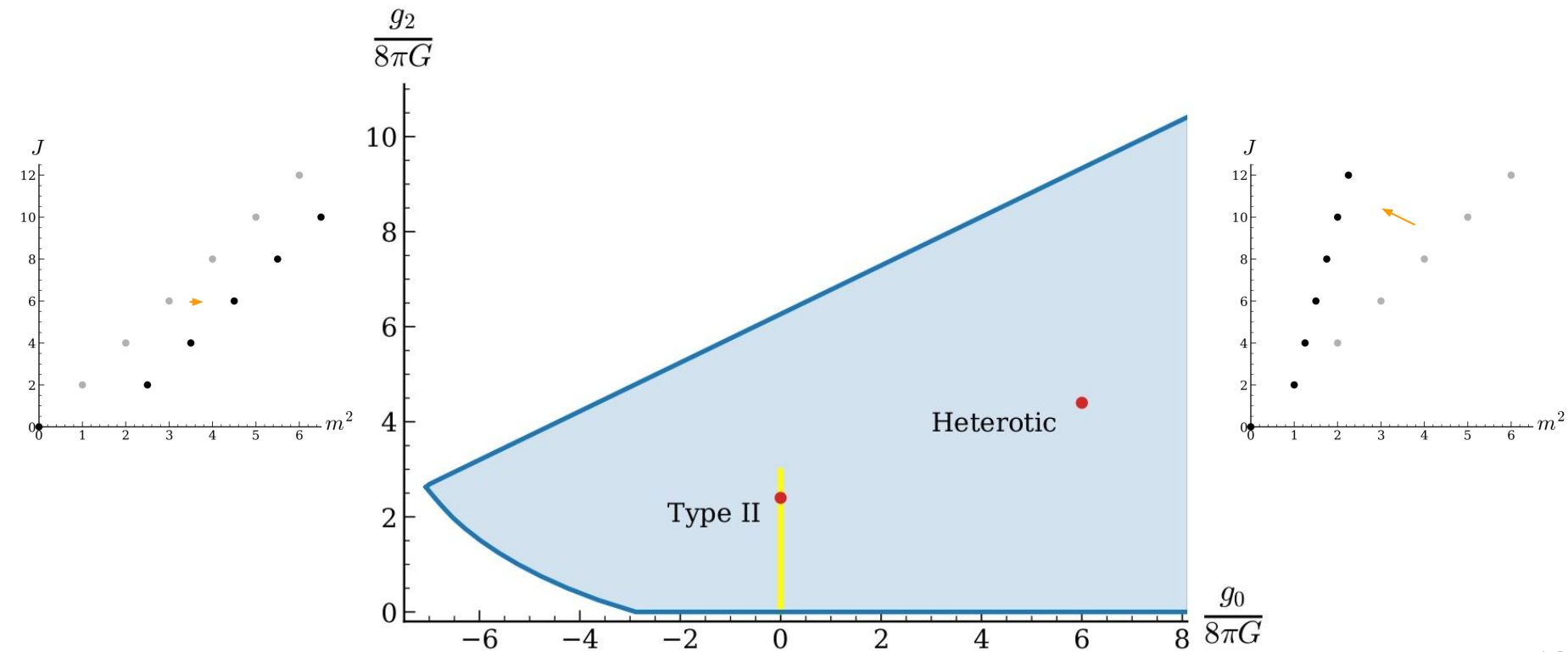
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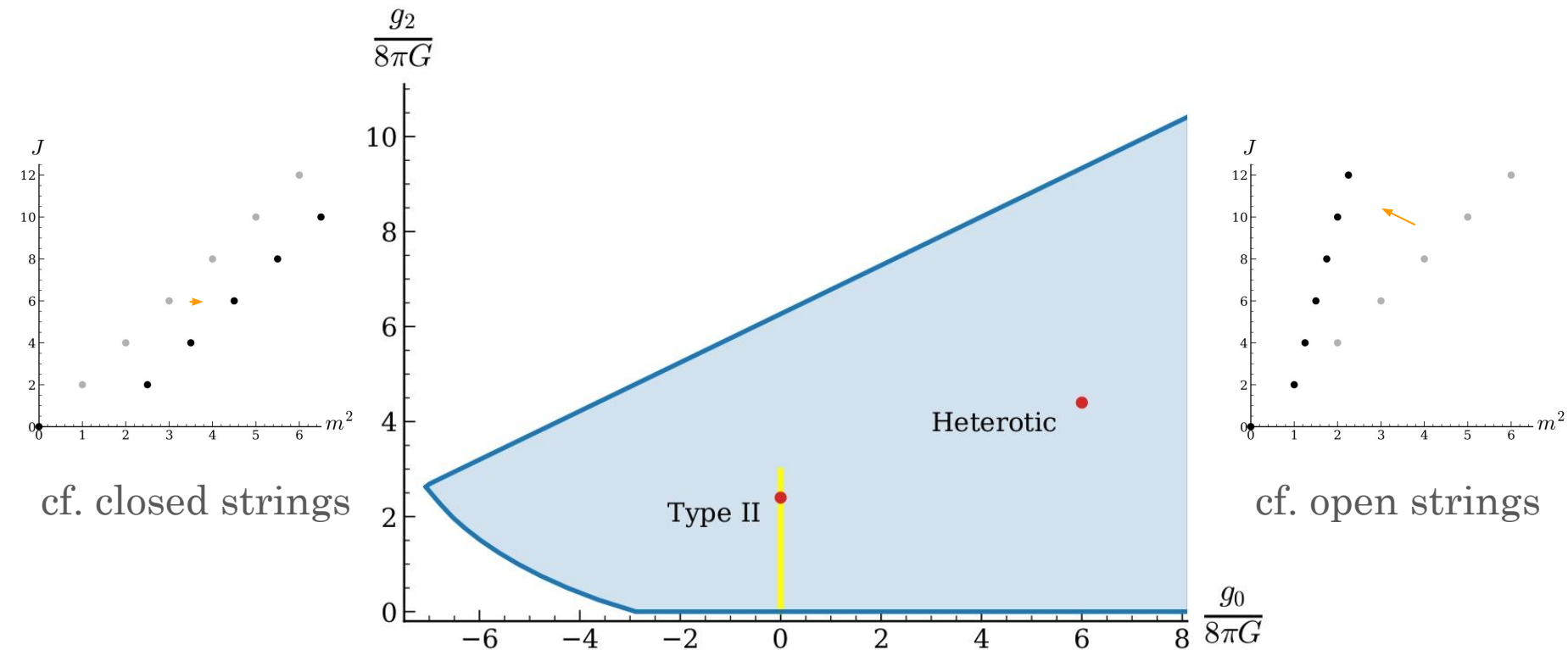
Summary



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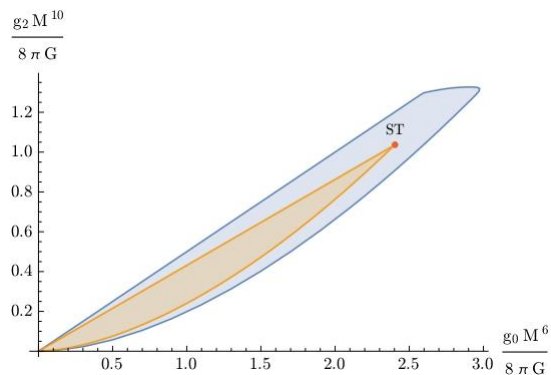
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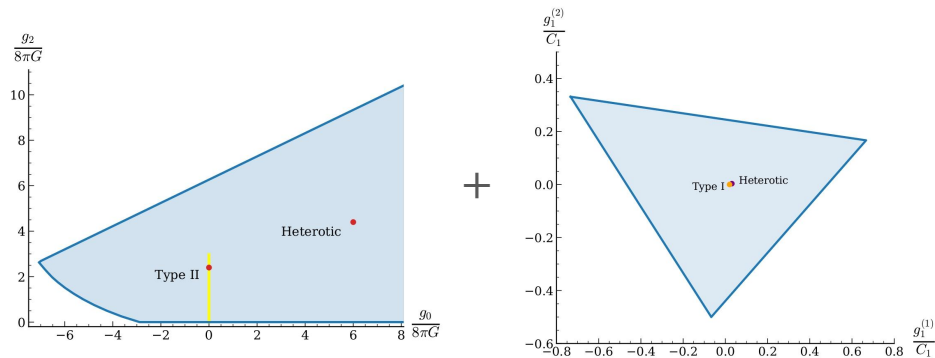
Next up: mixed correlators

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Maximum susy:
gravity + massive scalar

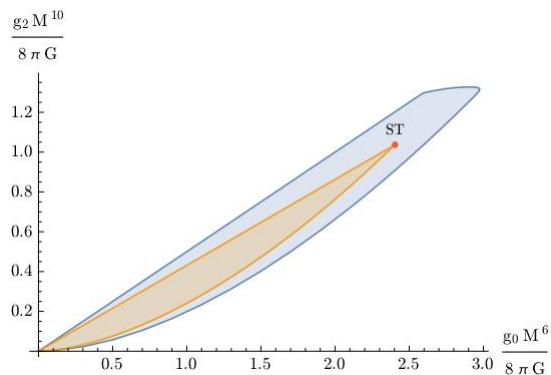


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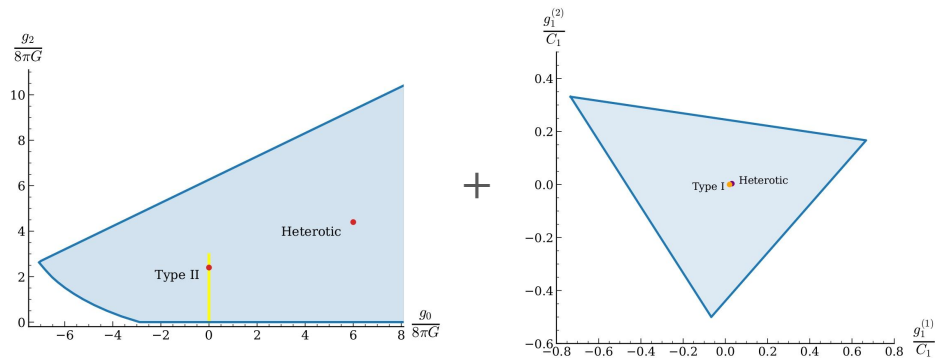


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Thank you

