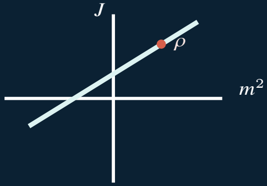
The bootstrap program for large N QCD

Part II: ρ meson scattering



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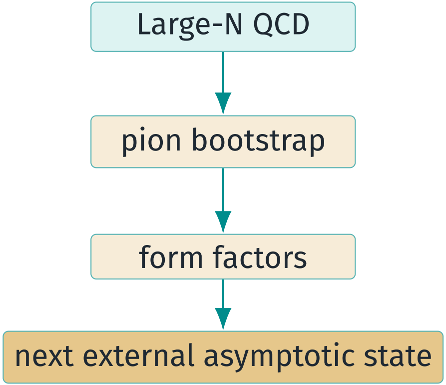
with Jan Albert, Johan Henriksson and Leonardo Rastelli (in progress)

London - Bootstrap 2026

Leonardo's talk: large-N QCD is a sharp S-matrix bootstrap problem.

- meromorphic amplitudes;
- data $\{m_k, J_k; \lambda_{ijk}\}$;
- crossing, unitarity, Regge boundedness;
- local probes can input asymptotic freedom.



Why scatter ρ mesons?

Large N gives a unique simplification:

$$\Gamma_{\text{meson}} \sim \frac{1}{N}, \qquad \mathcal{M}_{2\to 2} \sim \frac{1}{N},$$

Mesons are stable asymptotic states at $N = \infty$.

After the pion, the natural next state is the vector meson:

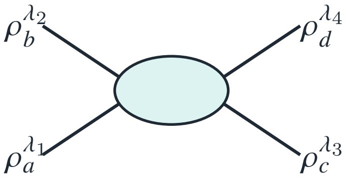
π: Goldstone, scalar $\longrightarrow$ ρ: massive spin one

The object

We focus on the flavor-ordered $\rho\rho \to \rho\rho$ amplitude:

$$\mathcal{T}_{abcd}^{\lambda_1\lambda_2\lambda_3\lambda_4} = 4\mathcal{M}(1^{\lambda_1}, 2^{\lambda_2}, 3^{\lambda_3}, 4^{\lambda_4}) \left[\mathrm{Tr}(T_aT_bT_dT_c) + \mathrm{Tr}(T_aT_cT_dT_b)\right] + \text{crossed}.$$

- massive vector: $\lambda = +, 0, -$;
- three flavor channels: S, P, D;
- 17 independent helicity structures in $d = 4$;
- 51 physical amplitudes.



$$\mathcal{M}_k(s,t), \quad k = 1, \ldots, 17$$

Numerology: why this is much larger than pion scattering

	pion scattering	ρ scattering
external spin	0	1
polarizations	1	3
independent $d = 4$ structures	1	17
amplitudes	3	51
positivity	scalar density	matrix density
Regge transfer	direct	kinematic matrix inversion

The payoff is that low-spin exchanges already stress the Regge assumption.

Exchange data: on-shell 3-point vertices

The exchanged spectrum is organized by J^{PC} and flavor representation. The covariant 3-point classification gives the allowed tensor structures:

	$C = +$				$C = -$			
	$J = 0$	even $J \geq 2$	$J = 1$	odd $J \geq 3$	$J = 0$	even $J \geq 2$	$J = 1$	odd $J \geq 3$
$P = +$	2	4	1	2	0	1	2	2
$P = -$	1	2	1	1	0	2	3	4

Pink sectors are absent in the pure $q\bar{q}$ large-N spectrum.

The ρ is 1^{--} (blue sector) and has only 2 structures.

Real-world mesons across the exchange sectors

The observed light-meson spectrum largely follows the same J^{PC} organization. To keep the comparison transparent, only representative states are shown below.

$P = +$

C	Representative states and families
+	$\boldsymbol{f_J/a_J}$, even J: $f_2(1270)$, $a_2(1320)$, $a_4(1970)$, $f_4(2050)$, $f_6(2510)$
+	$\boldsymbol{f_J/a_J}$, odd J: $a_1(1260)$, $f_1(1285)$, $a_3(1875)$, $f_3(2050)$
−	No standard $q\bar{q}$ mesons for even J
−	$\boldsymbol{h_J/b_J}$, odd J: $h_1(1170)$, $b_1(1235)$, $h_3(2025)$, $b_3(2030)$

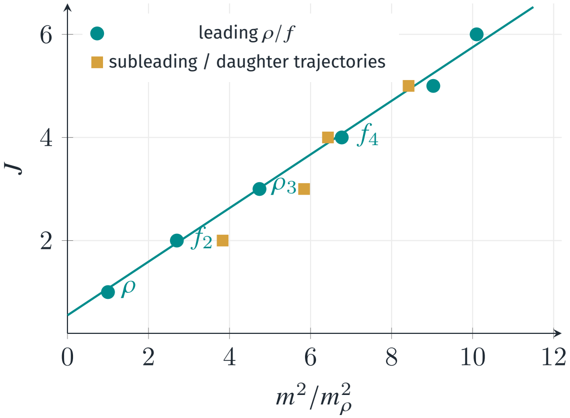
At large N famililies with same J^{PC} are expected to become degenerate

$P = -$

C	Representative states and families
+	$\boldsymbol{\pi_J/\eta_J}$, even J: π, $\pi_2(1670)$, $\eta_2(1870)$, $\pi_4(2250)$, $\eta_4(2330)$
+	No standard $q\bar{q}$ mesons for odd J
−	Even-spin vector families: $\rho_2(1940)$, $\omega_2(1975)$, $\rho_4(2230)$
−	$\boldsymbol{\rho_J/\omega_J/\phi_J}$, odd J: $\rho(770)$, $\omega(782)$, $\phi(1020)$, $\rho_3(1690)$, $\phi_3(1850)$, $\rho_5(2350)$

Radial or daughter candidates include $\pi(1300)$, $\rho(1450)$, $\omega(1420)$ and $\phi(1680)$.

Phenomenological input: Chew–Frautschi plots



The real-world spectrum organizes into approximately linear families.

$$J(t) \simeq \alpha_0 + \alpha' t, \qquad \alpha_0 < 1.$$

For fixed negative t this motivates

$$\mathcal{A}(s,t) \sim s^{J(t)} \quad \Rightarrow \quad \mathcal{A}(s,t) < s.$$

Regge assumption for physical amplitudes

The basic high-energy input is imposed on physical helicity amplitudes:

$$\boxed{\mathcal{A}_I(s,t) < s^1\,, \qquad t < 0 \text{ fixed.}}$$

This is stronger than generic spinning EFT behavior, but supported by Regge theory.

Technical challenge I: percolating Regge boundedness

Dispersion relations are writte in terms of stripped amplitudes $M_I(s,t)$:

$$M_I(s,t) = \sum_J (E^{-1})_{IJ}(s,t)\,\mathcal{A}_J(s,t), \qquad (E^{-1})_{IJ} = s\, e^{(-1)}_{IJ}(t) + e^{(0)}_{IJ}(t) + \cdots .$$

Naively,

$$\mathcal{A}_I < s \quad \Rightarrow \quad M_I < s^2.$$

But special linear combinations improve the growth:

Regge growth of combinations	$< s^1$	$< s^0$	$< s^{-1}$	$< s^{-2}$	$< s^{-3}$
number of combinations	48	30	24	6	3

These improved combinations generate anti-subtracted sum rules and null constraints.

Why the ρ system is expected to be nontrivial

In pion scattering a spin-two state has to be forced in to see genuine Reggeization: spin 0 and spin 1 exchanges are comparatively harmless.

For external ρ mesons, this changes:

ρ-exchange and π-exchange already violate the $J = 1$ Regge behavior.

pion scattering $\longrightarrow$ ρ scattering

need to insert f_2
to force a tower

π and ρ poles
already stress Regge

Low-energy EFT anchors the bootstrap data

The first few 3-point couplings can be written as

$$\begin{aligned}\mathcal{L} = &-\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu}_a - \frac{1}{2}m_\rho^2\rho^a_\mu\rho^\mu_a \\ &-\frac{1}{2}\lambda_{\rho\rho\pi}d_{abc}\epsilon^{\mu\nu\sigma\tau}F^a_{\mu\nu}F^b_{\sigma\tau}\pi^c \\ &-\frac{1}{2}\lambda^{(1)}_{\rho\rho\rho}f_{abc}\rho^a_\mu\rho^b_\nu F^{c,\mu\nu} - \frac{1}{3}\lambda^{(2)}_{\rho\rho\rho}f_{abc}F^{a,\nu}_\mu F^{b,\sigma}_\nu F^{c,\mu}_\sigma + \cdots\,.\end{aligned}$$

$\lambda_1 \equiv \lambda^{(1)}_{\rho\rho\rho}$: massive-gluon-like

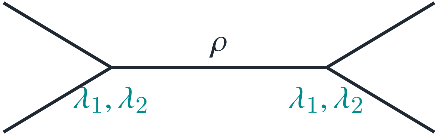
$\lambda_2 \equiv \lambda^{(2)}_{\rho\rho\rho}$: higher-derivative

$$\lambda_1 + m_2\lambda_2 \longleftrightarrow \text{charge}, \qquad \lambda_2 \longleftrightarrow (\mu_\rho - 2).$$

ρ self-exchange violates Regge boundedness

For pole exchange, the worst large-s terms behave schematically as

$$\mathcal{A}_{\rho\text{-exch}} \sim \lambda_1^2 s^1 + \lambda_1\lambda_2 s^3 + \lambda_2^2 s^4 + \cdots \, .$$



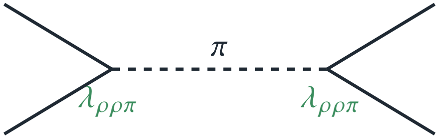
The quadratic term in λ_1 is local and can be canceled by a contact term (ex: HIggs mechanism)

Pion exchange also violates Regge boundedness

The parity-odd coupling

$$\lambda_{\rho\rho\pi}\ \mathrm{Tr}([\rho, \partial\rho]\partial\pi)$$

produces a massless pion pole in the crossed channel.



In the relevant reduced amplitude,

$$\bar{M}^{(5)}(s,t)\Big|_{\pi} = -\frac{4(s+t-2m_\rho^2)}{t}\,\lambda_{\rho\rho\pi}^2,$$

which creates a sum rules for $\lambda_{\rho\rho\pi}$.

Core expectation

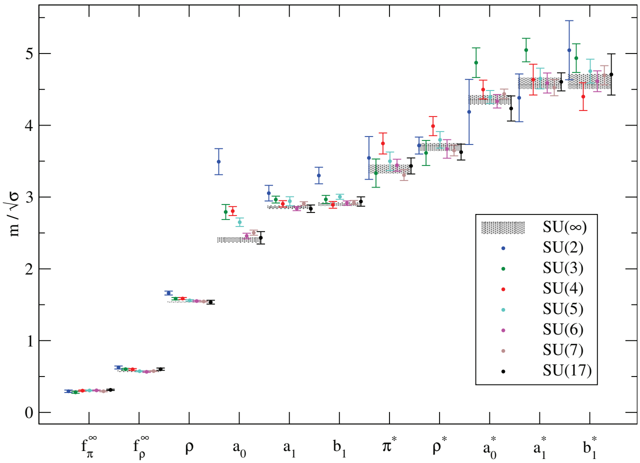
If $\lambda_{\rho\rho\pi}$ or $\lambda_{1,2}$ are nonzero, isolated low-energy exchange grows too fast. Therefore the amplitude needs towers of particles with increasing spin and mass:

$$\sum_J c_J \frac{s^J}{t - M_J^2} \quad \longrightarrow \quad s^{J(t)}, \qquad J(t) < 1 \text{ for fixed } t < 0.$$

Strategy

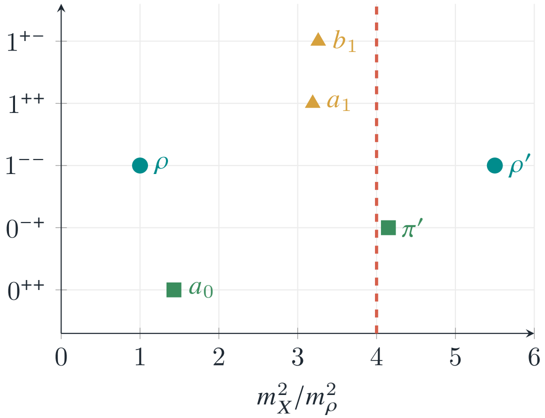
Raise the gap after the low states $\{\pi, \rho\}$: the offending couplings should be driven down.

Lattice input: low spectrum



S. Bali, Bursa, Castagnini, Collins, Del Debbio, Lucini, Panero ’13

Phenomenological + lattice input: low spectrum



A constraining working assumption:

below $2m_\rho$ the only parity-odd exchange is ρ .

S. Bali, Bursa, Castagnini, Collins, Del Debbio, Lucini, Panero '13

Bonanno, Pérez, González-Arroyo, Ishikawa, Okawa '25

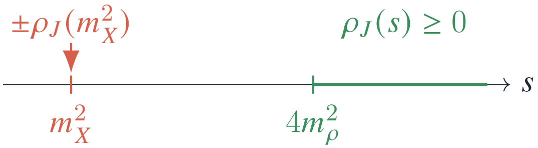
This matters because poles below the two-particle threshold are not constrained by unitarity in the same immediate way as the continuum.

Technical challenge II: unitarity below threshold

Above threshold the spectral density is positive semidefinite:

$$\rho_Q(s) \succeq 0, \qquad s \geq 4m_\rho^2.$$

Below threshold, factorization instead fixes the residue:

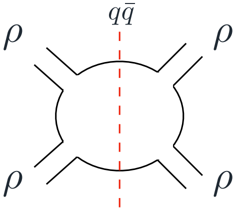


The $\pm$ is equal to the parity P of the state!

Luckily, under the lattice-motivated spectrum assumption, only the ρ itself has to be treated this way.

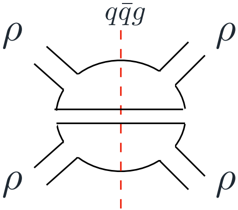
Quark model vs large-N topology

Pure $q\bar{q}$ quark-model truncation



- only conventional $q\bar{q}$ quantum numbers;
- exotic J^{PC} sectors projected out;
- pink cells are empty by assumption.

Leading large-N planar sector



- a cut still gives one color-singlet meson;
- internal gluons allow exotic J^{PC};
- these “exotic-hybrid” mesons populate the pink sectors.

Large-N counting: planar diagrams with one boundary quark loop and internal gluons are leading.

A numerical no-go theorem

Assumptions imposed simultaneously

1. Regge growth below spin one: $\mathcal{A}(s,t) < s$;
2. no parity-odd states below $4m_\rho^2$, except the ρ;
3. nonzero $\lambda_{\rho\rho\pi}$ or nonzero ρ self-coupling;
4. no exotic-hybrid, i.e. no pink-sector spectral weight.

Numerical result

No theory satisfies all four assumptions.

- the corresponding coupling vanishes, or
- one assumption is violated

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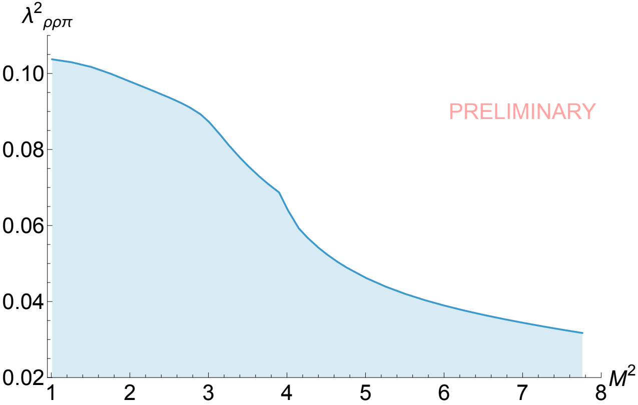
To proceed further, we relax assumption 4 and allow exotic-hybrid exchanges.

A possible real-world counterpart is the spin-exotic $\pi_1(1600)$, with $J^{PC} = 1^{-+}$, often discussed as a hybrid candidate and observed in the $\rho\pi$ channel. The large-N lattice has not targeted this hybrid/exotic sector yet.

PDG 2025 summary tables list $\pi_1(1600)$; COMPASS confirms the $\pi_1(1600) \to \rho(770)\pi$ amplitude.

Numerical result I: bound on $\lambda_{\rho\rho\pi}$ vs M^2/m_ρ^2



Physics message:

- pion exchange alone grows too fast;
- increasing the gap forces Regge-canceling states farther away;
- the coupling should be suppressed when no tower is available.

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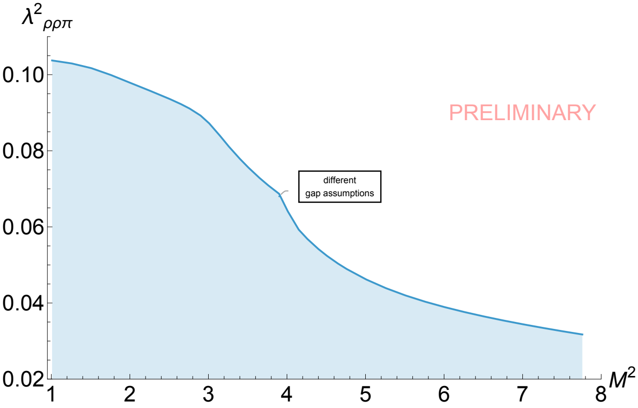

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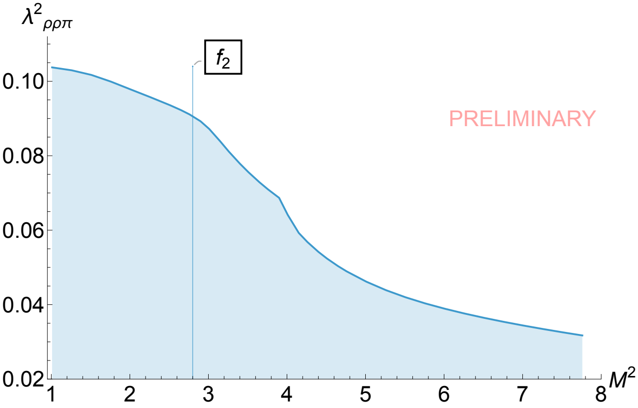

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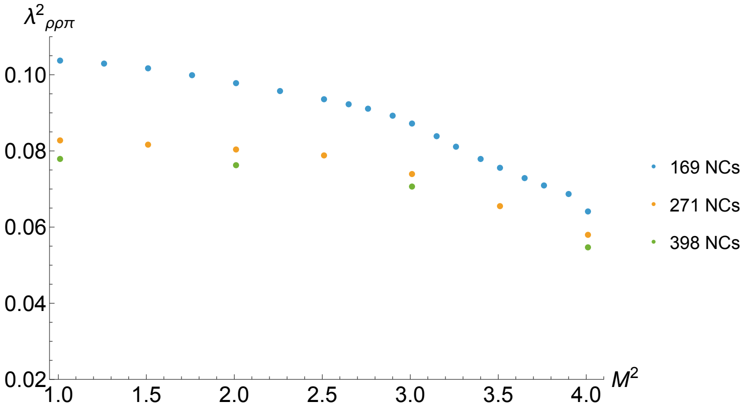
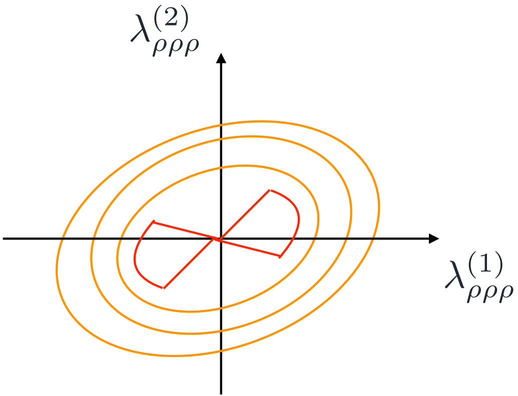


- Convergence is slow but it seems to happen

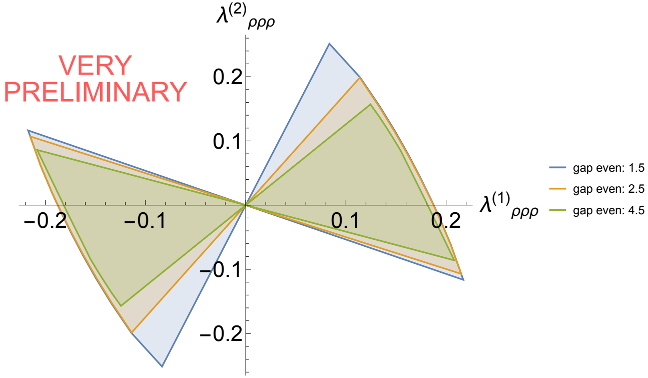
Numerical result II: ρ self-coupling

Schematically



- increasing gaps in P=- states reduces the allowed region
- when gap ≥ 4, not all ratios $\lambda^{(1)}_{\rho\rho\rho}/\lambda^{(2)}_{\rho\rho\rho}$ are allowed

Numerical result II: ρ self-coupling



- Recall $\mathcal{A}_{\rho\text{-exch}} \sim \lambda_1^2 s$
- but $\mathcal{A}_{\rho\text{-exch}} \sim \lambda_1 \lambda_2 s^3 + \lambda_2^2 s^4$

Expectation as gap $\to \infty$:

- bound on $\lambda_1 \longrightarrow$ finite
- bound on $\lambda_2 \longrightarrow 0$

Summary

1. Numerical evidence that both π and ρ exchange already stress Regge
2. Will get more bounds and spectra soon enough
3. No-Go theorem for Quark model: need exotic-hybrid mesons
4. Interplay with lattice is becoming fundamental: need to measure f_2 and other $J > 1$ states.

Thank you

Backup: explicit crossing convention

$$
\begin{aligned}
\mathcal{T}_{abcd}^{\lambda_1\lambda_2\lambda_3\lambda_4} &= 4\mathcal{M}(1^{\lambda_1},2^{\lambda_2},3^{\lambda_3},4^{\lambda_4})\left[\mathrm{Tr}(T_aT_bT_dT_c)+\mathrm{Tr}(T_aT_cT_dT_b)\right] \\
&+ 4\mathcal{M}(1^{\lambda_1},2^{\lambda_2},4^{\lambda_4},3^{\lambda_3})\left[\mathrm{Tr}(T_aT_bT_cT_d)+\mathrm{Tr}(T_aT_dT_cT_b)\right] \\
&+ 4\mathcal{M}(1^{\lambda_1},3^{\lambda_3},4^{\lambda_4},2^{\lambda_2})\left[\mathrm{Tr}(T_aT_cT_bT_d)+\mathrm{Tr}(T_aT_dT_bT_c)\right] .
\end{aligned}
$$

The numerical problem in one line

The low-energy vector is matched to positive spectral data:

$$\vec{L} = \vec{H}_0 + \sum_Q \langle \vec{V}^Q \rangle_{\mathtt{spec}} + \sum_X \boldsymbol{\lambda}_X^T \vec{V}^{Q_X} \boldsymbol{\lambda}_X .$$

We search for a functional $\vec{\alpha}$ such that

maximize $\quad \vec{\alpha} \cdot \vec{o}, \quad$ subject to $\quad \vec{\alpha} \cdot \vec{n} = 0, \quad \vec{\alpha} \cdot \vec{V}^Q \succeq 0.$

With exchange-state observables,

$$\boldsymbol{\lambda}_X = \sqrt{a_X}\, \boldsymbol{\mu}(\theta_i),$$

so one can bound either the norm a_X at fixed angle, or relax over angles.

Hidden local symmetry: the ρ as a gauge boson

$$U(x) = e^{2i\pi/F_\pi} = \xi_L^\dagger \xi_R$$

The factorization introduces a redundancy:

$$\xi_{L,R} \longrightarrow h(x)\, \xi_{L,R}\, g_{L,R}^\dagger, \qquad h(x) \in SU(N_f)_V.$$

Gauge this hidden local symmetry:

$$\rho_\mu \longrightarrow h \rho_\mu h^\dagger - i\, \partial_\mu h\, h^\dagger.$$

$$\mathcal{L}_{\mathrm{HLS}} = F_\pi^2 \,\mathrm{Tr}(\hat{\alpha}_{\perp\mu}^2) + F_\sigma^2 \,\mathrm{Tr}(\hat{\alpha}_{\|\mu}^2)$$

$$-\frac{1}{2}\,\mathrm{Tr}(\rho_{\mu\nu}^2)$$

$$a \equiv \frac{F_\sigma^2}{F_\pi^2}$$

$$m_\rho^2 = a g^2 F_\pi^2, \qquad g_{\rho\pi\pi} = \frac{a}{2} g.$$

The ρ is not a microscopic gauge boson of QCD, but the EFT can organize it as the gauge field of an emergent local $SU(N_f)_V$.

KSRF relations and the ρ self-coupling

KSRF I

$$g_\rho = 2F_\pi^2 g_{\rho\pi\pi}$$

Here

$$\langle 0|J_\mu^a|\rho^b\rangle = \delta^{ab} g_\rho\, \epsilon_\mu .$$

In HLS:

$$g_\rho = agF_\pi^2, \qquad g_{\rho\pi\pi} = \frac{a}{2}g .$$

So KSRF I is independent of a.

KSRF II

$$m_\rho^2 = 2g_{\rho\pi\pi}^2 F_\pi^2$$

This follows for

$$a = 2, \qquad g_{\rho\pi\pi} = g .$$

The same value gives tree-level vector-meson dominance.

The Yang–Mills kinetic term gives

$$\mathcal{L}_{\rho^3} \sim g\ \mathrm{Tr}([\rho,\rho]\partial\rho),$$