Solving the Strong CP problem with SUSY (and Gudrun Hiller)

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Solving the strong CP problem with SUSY

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collaboration with G. Hiller (SLAC)

+ hep-ph/0105254 (PLB)
+ hep-ph/0201251

Outline

- the strong CP problem in the SM
  - classic solutions
- new solution
  - spontaneous CP violation
  - SUSY, non-renormalization theorems
- predictions & summary

What is the strong CP puzzle ?

there are 2 CP violating parameters in the Standard Model

1. ϕ_{CKM} the CKM phase

2. $\bar{\theta}_{QCD}$ the strong CP phase

Experimentally we see

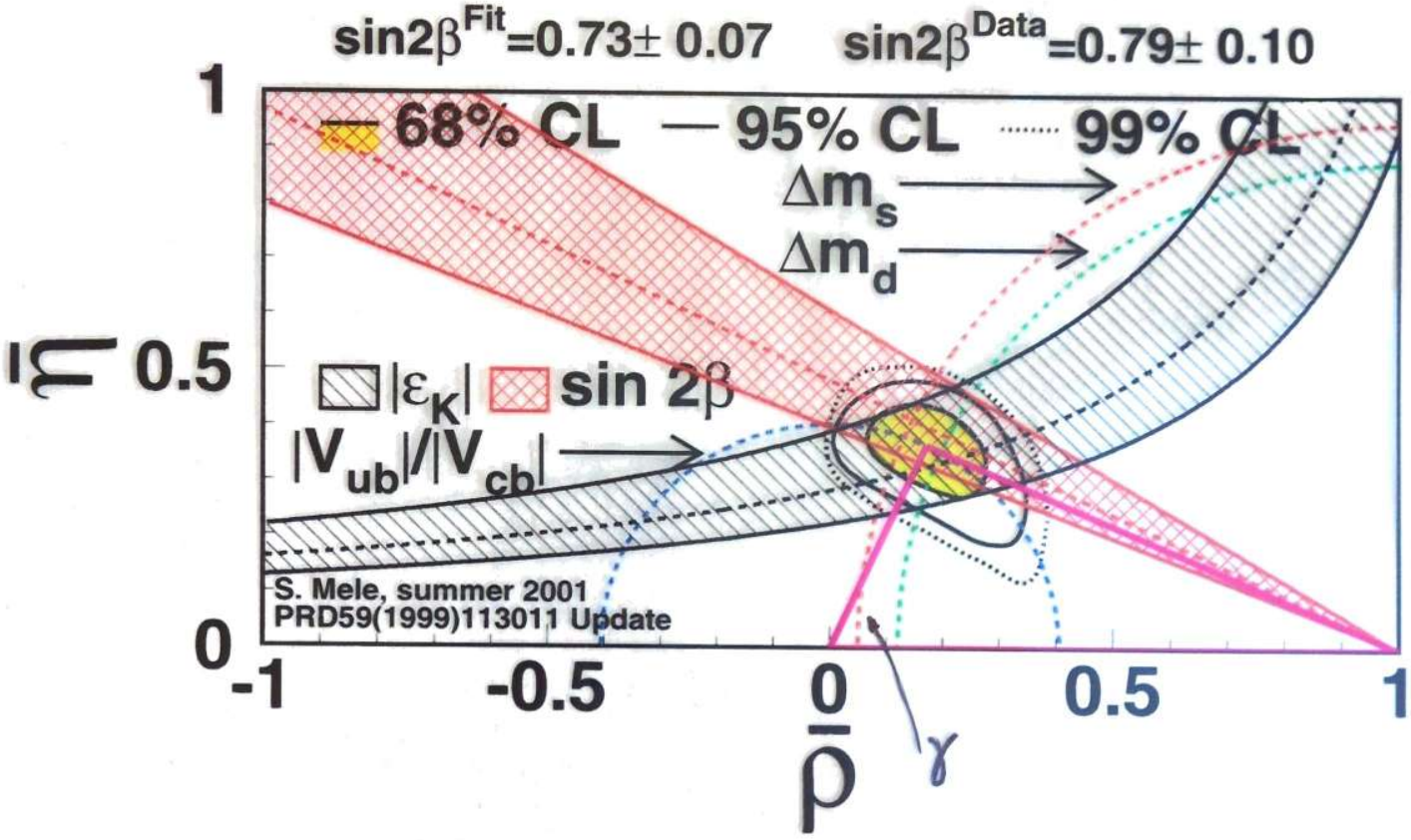
$$\phi_{CKM} \sim O(1)$$
$$\bar{\theta}_{QCD} \lesssim 10^{-10}$$

Naïve theory predicts

$$\bar{\theta}_{QCD} \sim \phi_{CKM}$$

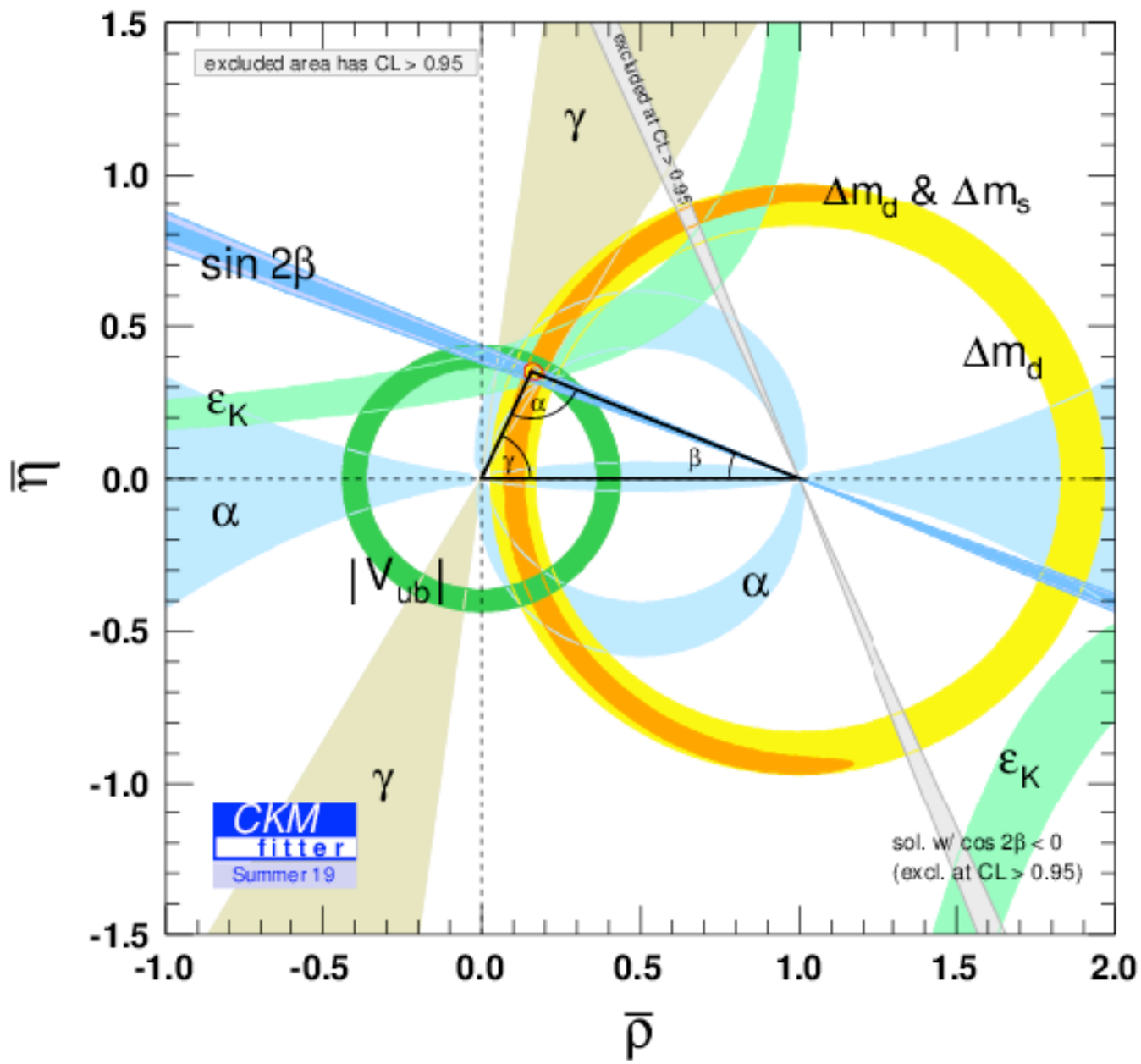
Experimental status of ϕ_{CKM}

$$\phi_{CKM} = \gamma = \arg(\bar{\rho} + i\bar{\eta})$$



if flavor-changing CP violation is explained by the CKM phase, then $\phi_{CKM} \sim O(1)$.

Experimental status of ϕ-CKM 2025



$$\phi_{CKM} = \gamma = O(1)$$

Experimental bounds on $\bar{\theta}_{QCD}$

Today:

$\bar{\theta}$ contributes to electric dipole moments.

e.g. $d_n \simeq 5.2 \cdot 10^{-16}\ \bar{\theta}\ e\,cm$

$d_n|_{\text{lattice QCD}} \approx 1.48(14)(31) \times 10^{-16}\bar{\theta}\,\mathrm{e}\cdot\mathrm{cm}$

the current upper bound is $d_n < 3 \cdot 10^{-25}\ e\,cm$

nEMD (Paul Scherrer 2020)

$|d_n| \lesssim 1.8 \times 10^{-26}\ \mathrm{e}\cdot\mathrm{cm}\,,$

best bound from ^{199}Hg (Romalis, Griffith, Fortson '00)

$\bar{\theta} < 1.5\ 10^{-10}$

$$|\bar{\theta}| \lesssim 10^{-10}$$

proposed Los Alamos experiment can reach sensitivity to $\bar{\theta} \sim 10^{-12-13}$ (Golub, Lamoreaux)

Strong CP Lagrangian

$$\theta \frac{g^2}{32\pi^2} F\tilde{F} \qquad + \qquad m\,\overline{Q}Q$$

rephrasing invariant: $\overline{\theta} = \theta - \arg m$

Standard Model: $\overline{\theta} = \theta - \arg\det Y_u - \arg\det Y_d$

unless fine tuning, strong CP bound: $\arg\det Y_{u,d} < 10^{-10}$

but weak CP violation requires: $\arg\det Y_{u,d} \sim 1$

classic solutions

- axion — $\bar{\theta}$ becomes a field, QCD dynamics gives it a potential with minimum at zero.

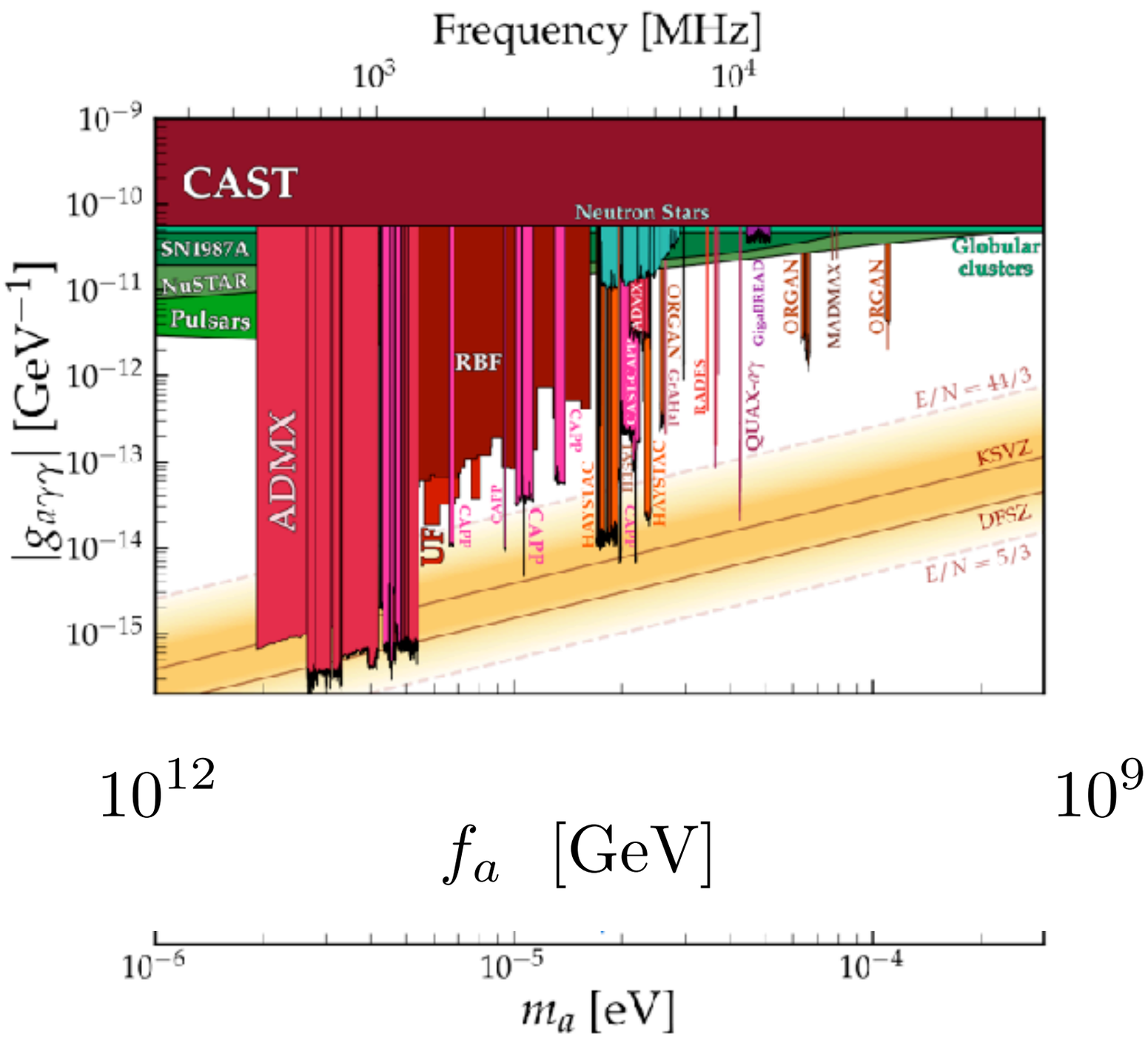
the "axion window" is slowly closing

- $m_u = 0$ — θ can be rotated away by re-defining the up-quark fields. It becomes unobservable.

  χPT fits strongly disfavor $m_u = 0$
  (Leutwyler, Cohen-Kaplan-Nelson)

- spontaneous CP violation — CP is a symmetry at high scales ($\Rightarrow \theta_0 = 0$, Y's are real) and is broken in such a way as to preserve $\bar{\theta} \approx 0$ while allowing $\phi_{CKM} \sim O(1)$
  (Nelson-Barr)

Axion searches



Wanted!

a framework in which we have

$$\Phi_{CKM} \sim O(1)$$

but

$$\arg \det Y \leq 10^{-10}$$

as well as $\Theta_0 \simeq 0$.

Sharpening the problem :

$$\bar{\theta} \lesssim 10^{-10}$$

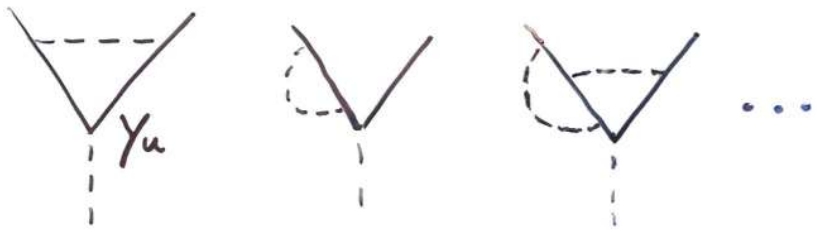
at what scale ? ($\theta(\mu)$ is not RGE invariant)

answer : it doesn't matter, $\bar{\theta}$ is almost RGE invariant. the biggest renormalization in the standard model gives $\bar{\theta} \sim 10^{-19}$

$\bar{\theta}$ renormalization in the SM

- compute radiative corrections to $\arg\det Y_{u/d}$
- $Y_{u/d}$ contain the only flavor- and CP-violating parameters in the SM $\Rightarrow$ compute loops involving them:

e.g.



$$Y_u \to Y_u \left(1 + \frac{Y_u^\dagger Y_u}{16\pi^2} \log\left(\frac{\Lambda}{\mu}\right) + \frac{Y_d^\dagger Y_d}{16\pi^2} \log\left(\frac{\Lambda}{\mu}\right) + \frac{Y_u^\dagger Y_u}{16\pi^2}\frac{Y_d^\dagger Y_d}{16\pi^2} \left(\log(\;)\right)^2 + \dots \right)$$

define $\quad \varepsilon_{u/d} \equiv \dfrac{Y_{u/d}^\dagger Y_{u/d}}{16\pi^2} \log\left(\frac{\Lambda}{\mu}\right) \longleftarrow$ hermitian, small

$$\rightarrow \quad Y_u^{ren} = Y_u^0 \left(1 + \varepsilon_u + \varepsilon_d + \varepsilon_u \varepsilon_d + \varepsilon_{u/d}^2 + \varepsilon^3 + \dots \right)$$

$$\arg\det Y_u^{ren} = \arg\det Y_u^0 + \mathrm{Im}\,\mathrm{Tr}\left[\varepsilon_u + \varepsilon_d + \varepsilon^2 + \varepsilon^3 + \dots \right]$$

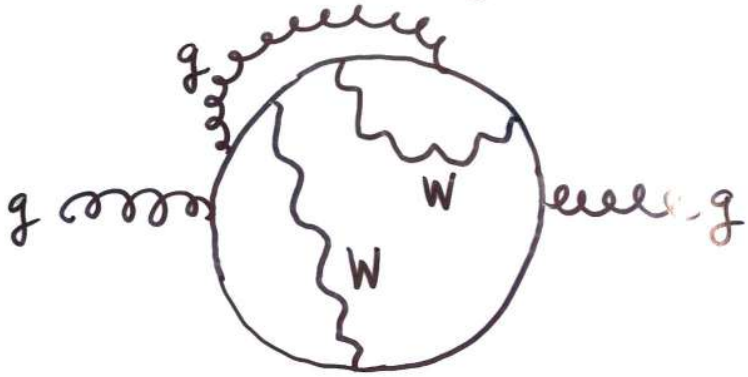
$$\delta\bar{\theta} = \mathrm{Im}\,\mathrm{Tr}\left[\varepsilon_u^2 \varepsilon_d^2 \varepsilon_u \varepsilon_d \right] + \text{smaller}$$

$$= \frac{m_t^4 m_b^4 m_c^2 m_s^2}{v^{12}} \underbrace{s_{12} s_{13} s_{23} \sin\delta_{CKM}}_{\text{Jarlskog Invariant}} \left[\frac{\log(\Lambda/\mu)}{16\pi^2} \right]^6 \approx 10^{-30}$$

$\bar{\theta}$ renormalization II

finite contributions (dominated by loop momenta $\approx$ GeV)

(Khriplovich '86, estimate by Ellis, Gaillard '79)



$$\delta\bar{\theta}_{SM} = -\frac{7 G_F^2 m_c^2 m_s^2 \alpha_s}{288\pi^5} s_{12} s_{23} s_{13} \sin\delta_{CKM} \{\text{Logs}\}$$

e.g. $\text{Log}\frac{m_t}{m_s}$

$$\approx 4 \cdot 10^{-19}$$

$\bar{\theta}=0$ radiatively stable in SM even with $\phi_{CKM} \sim 1$

→ a "solution" to the strong CP problem is

1. $\bar{\theta}(\Lambda) = 0$
2. ϕ_{CKM} large
3. use a CP invariant regulator/cutoff
4. assume SM valid up to Λ

<u>problems with this "solution":</u>

- $\bar{\theta}=0$ is not an "enhanced symmetry" point when CP is broken
- radiative stability depends crucially on assumptions 3+4

⇒ extensions of the SM which seek to explain the origin of flavor and CP violation lead to large corrections to $\bar{\theta}$.

key fact:

$\bar{\theta}$ is <u>not</u> renormalized in SUSY
(nonrenormalization thrm)

ϕ_{CSM} <u>is</u> renormalized in SUSY

'82 Ellis, Ferrara, Nanopoulos

(non) renormalization in SUSY

Superpotential $W \sim Q \hat{Y}_D D H_D + Q \hat{Y}_u U H_u + \dots$

(not renormalized)

Kähler potential $K \sim Z^D_{ij} D_i^\dagger D_j + Z^Q_{ij} Q_i^\dagger Q_j + \dots$

(is renormalized)

quark masses: $$M_D = \underbrace{\frac{1}{\sqrt{Z_Q}} \hat{Y}_D \frac{1}{\sqrt{Z_D}}}_{Y_D} \langle H_D \rangle$$

$\hat{Y}$ not renormalized

Y is renormalized $\Rightarrow$ ϕ_{CKM} is renormalized

$$\bar{\theta} \sim \arg\det Y = \underbrace{\arg\det \frac{1}{\sqrt{Z_Q}}}_{=0} + \arg\det \hat{Y} + \underbrace{\arg\det \frac{1}{\sqrt{Z_D}}}_{=0}$$

Z hermitian

not renormalized

Example: flavor from RH wave functions

before rescaling:

$$K = Q^\dagger Q + U^\dagger U + D^\dagger V_d^\dagger \frac{1}{\hat{m}_d^2} V_d D$$

$$W = Q \hat{m}_u U H_u + Q \mathbb{1} D H_d$$

after rescaling:

$$K = Q^\dagger Q + U^\dagger U + D^\dagger D$$

$$W = Q \hat{m}_u U H_u + Q V_d^\dagger \hat{m}_d D H_d$$

$$V_{CKM} = V_d \qquad \overline{\theta} = -\arg\det \hat{m}_u - \arg\det V_d^\dagger \hat{m}_d = 0$$

Strong CP SUSY Solution Schematic

CP+SUSY good symmetries => θ =0, Yukawa's real

spontaneous CP breaking, integrate out CP violating "sector"

CP violation feeds into $Y_{u,d}$ but not into $\overline{\theta} = \theta - \arg\det Y_{u,d}$

guaranteed to work if communicated by loops!

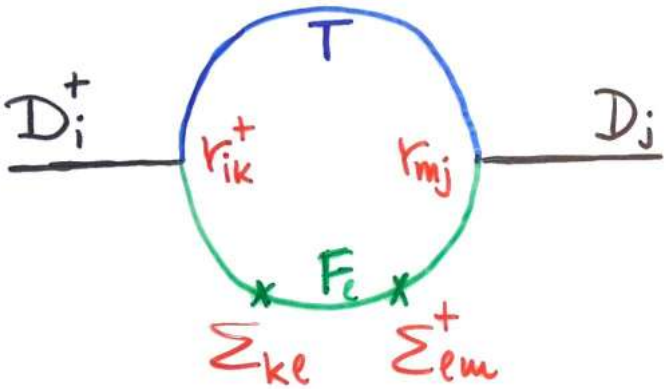
SUSY breaking communicated => soft masses real

MSSM with: $\overline{\theta} \sim 0$ $\phi_{CKM} = O(1)$

general framework

- CP is a good symmetry of the high energy theory, broken spontaneously by Σ vev
- Σ only couples to the MSSM and other colored superfields through loops or in the Kähler potential
- at the loop level, we generate complex kinetic terms (and real Yukawas)
- supersymmetry breaking is mediated at a lower scale $M_{SUSY} \ll \langle \Sigma \rangle$

An example model



T carries the SM gauge quantum #'s of D

F carries the flavor index

$\langle\Sigma\rangle$ breaks CP

$$W_{\not{CP}} = r_{ij} D_i \bar{F}_j \bar{T} + \langle\Sigma_{ij}\rangle F_i \bar{F}_j + M T \bar{T}$$

loop generates non-canonical kinetic term for D

$$\delta Z \simeq \frac{r^+ \Sigma \Sigma^+ r}{16\pi^2 \, \mathrm{tr}(\Sigma\Sigma^+)} \cdot \mathrm{Logs}$$

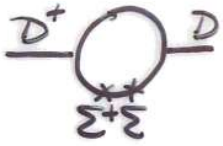
for $\langle\Sigma\rangle \gtrsim M$

need $r \sim 4\pi$ so that $\delta Z \approx O(1)$ which is required for $\phi_{CKM} \approx O(1)$.

$\Rightarrow$ strong coupling, cannot compute δZ.

Summary

M_{pl} — CP symmetry $\Rightarrow \begin{cases} \text{all couplings real} \\ \bar{\theta} = 0 \end{cases}$, SUSY

M_{CP} $\langle \Sigma \rangle$ — $\not{CP}$: 

hermitian Z factors $K \sim Z^D_{ij} D_i^+ D_j$

go to canonical basis : $Y = \frac{1}{\sqrt{Z_L}} \hat{Y} \frac{1}{\sqrt{Z_R}}$

$$\Rightarrow \begin{cases} \phi_{CKM} \neq 0 \\ \bar{\theta} = 0 \end{cases}$$

What about SUSY breaking?

Why low-scale SUSY breaking?

e.g. MSUGRA, assume flavor universal soft masses at M_{pl}

$$K \sim \quad D_i^\dagger \left(1 + \frac{S^\dagger S}{M_{pl}^2}\right) \delta_{ij} D_j \qquad \text{at } M_{pl}$$

but $$D_i^\dagger \left(Z^D_{ij} + X^D_{ij} \frac{S^\dagger S}{M_{pl}^2}\right) D_j \qquad \text{at } M_{GP}$$

$\Rightarrow$ soft masses $$m^2_{ij} = \left(\frac{F}{M_{pl}}\right)^2 \underbrace{\left(\frac{1}{\sqrt{Z^D}} X \frac{1}{\sqrt{Z^D}}\right)_{ij}}$$

$\lightning$ order 1 off-diagonals

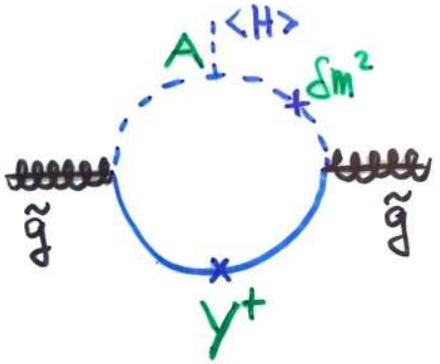
$\bar{\theta}$, FCNC, K-$\bar{K}$ mixing ...

(exception: anomaly mediation where $X = Z$)

$\delta\bar{\Theta}$ from soft SUSY breaking

$$\bar{\Theta} = \Theta - \arg\det Y_u Y_d - 3 \arg M_3$$

very strong constraints on phases in soft parameters



$$\sim \frac{\alpha_s}{4\pi} \mathrm{Tr}\left[Y^+ A \right] + \mathrm{Tr}\left[Y^+ \frac{\delta m^2}{m^2} A \frac{\delta m^2}{m^2} \right] + \dots$$

$$\rightarrow \delta\bar{\Theta} \sim \mathrm{Im}\,\mathrm{Tr}\left[Y^+ A \right]$$

most natural solution:

$$A_{ij} \sim Y_{ij} A_0$$
$$m^2_{ij} \sim m_0^2 \delta_{ij}$$
$$M_{1/2}, \mu, B, A_0 \text{ real}$$

with tiny corrections (δm^2, δA), more constraining than FCNC's

$\delta\bar{\theta}$ and gauge mediation

$$m^2 = m_0^2 \left(1 + \varepsilon_u + \varepsilon_d + \varepsilon^2 + \dots \right) \quad , \quad \varepsilon_u = \frac{Y_u^\dagger Y_u}{16\pi^2}$$

$$A = A_0 \frac{Y}{16\pi^2} \frac{\alpha}{4\pi} \left(1 + \varepsilon_u + \varepsilon_d + \varepsilon^2 + \dots \right)$$

similar to SM calculation of $\delta\bar{\theta}$ get

$$\delta\bar{\theta} \sim \mathrm{Im}\,\mathrm{Tr}\left[\varepsilon_u^2 \varepsilon_d^2 \varepsilon_u \varepsilon_d \right]$$ tiny $\left(10^{-30} (\tan\beta)^6 \right)$

Power corrections $\frac{M_{SUSY}}{M_{CP}}$

soft masses in gauge mediation vanish as powers of $\left(\frac{M_{Mess}}{\mu}\right)$ at high scales $\mu > M_{Mess}$

e.g.

$$m^2(\mu \gg M_{Mess}) \simeq \left(\frac{\alpha}{4\pi}\right)^2 \left(\frac{F}{\mu}\right)^2$$

M_{CP} — $$m^2_{ij} \sim \left(\frac{\alpha}{4\pi}\right)^2 \left(\frac{F}{M_{CP}}\right)^2 \left(\frac{1}{Z}\right)_{ij}$$

M_{Mess} (M_{SUSY}) — $$m^2_{ij} = \left(\frac{\alpha}{4\pi}\right)^2 \left(\frac{F}{M_{Mess}}\right)^2 \left[\delta_{ij} + \underbrace{\left(\frac{M_{Mess}}{M_{CP}}\right)^2 \left(\frac{1}{Z}\right)_{ij}}_{\frac{\delta m^2}{m^2} \lesssim 10^{-6} \quad (\bar{\theta} \lesssim 10^{-10})}\right]$$

$\rightarrow$ $M_{Mess} \lesssim 10^{-3} M_{CP}$ is safe

Predictions

$Y = \frac{1}{\sqrt{Z_L}} \hat{Y}_{real} \frac{1}{\sqrt{Z_R}}$ is general enough to allow arbitrary V_{CKM} and quark masses

$\longrightarrow$ no relations between masses and angles

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0. SUSY

1. minimal flavor violation, i.e. no flavor violation beyond the SM (B physics, $\mu \to e\gamma$, ...)

2. no CP violation BSM at collider energies, $\sin 2\beta$

3. 1st & 2nd generation scalars degenerate

4. $\bar{\Theta} \sim 10^{-10} - 10^{-19}$ (exp. will get to $10^{-12\,-13}$?)
no measurable lepton ^{E} dipole moments.

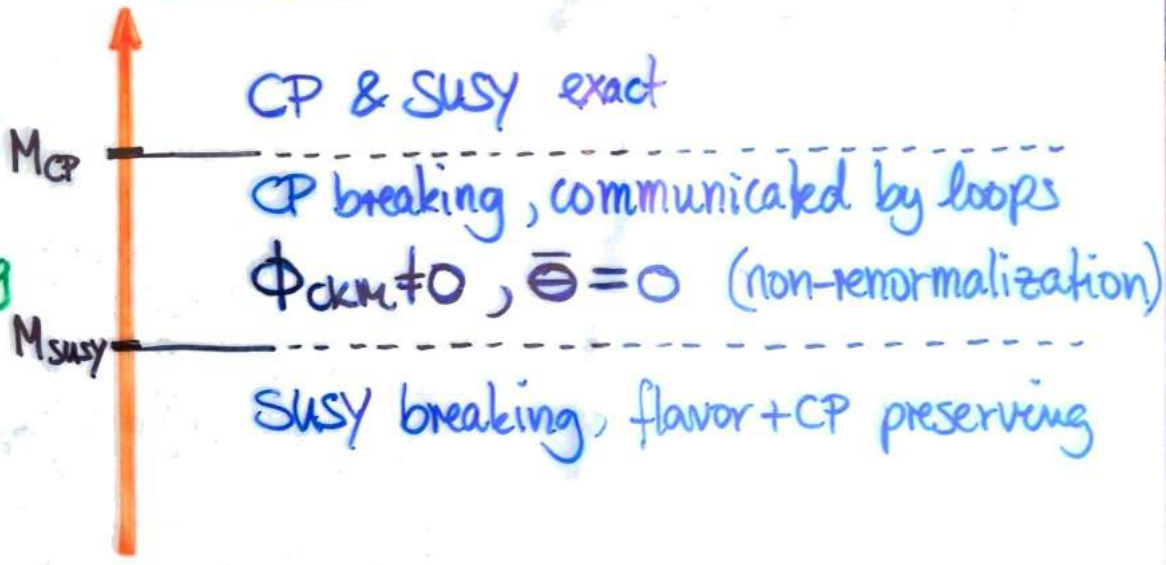
Conclusions

★ Strong CP problem is a huge fine-tuning problem

$$\bar{\theta} < 10^{-10}\, \phi_{CKM}\Big|_{\text{experiment}} \quad \text{vs.} \quad \bar{\theta} \sim \phi_{CKM}\Big|_{\text{naïve expectation}}$$

★ <u>solutions</u>:
- axion
- $m_u = 0$
- Nelson-Barr

Non-renormalization of $\bar{\theta}$ in SUSY as solution to strong CP problem



★ <u>predictions</u>:
- SUSY, low scale SUSY breaking (gravitino LSP)
- flavor universal
- SM CP, $\sin 2\beta$
- $\bar{\theta} \gtrsim 10^{-19}$ "large" if $\frac{M_{Mess}}{M_{CP}} \approx 10^{-3}$