Quantum Cosmology

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Quantum Cosmology

Georges Lemaître 1931:

If the world has begun with a single quantum, the notions of space and time would altogether fail to have any meaning at the beginning . . . If this suggestion is correct, the beginning of the world happened a little before the beginning of space and time.

Closed Friedmann–Lemaître universe with scale factor a, containing a homogeneous massive scalar field ϕ (two-dimensional *minisuperspace*)

$$\mathrm{d}s^2 = -N^2(t)\mathrm{d}t^2 + a^2(t)\mathrm{d}\Omega_3^2$$

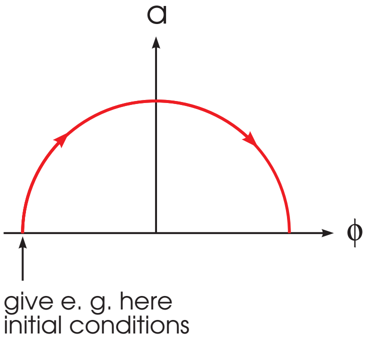
The Wheeler–DeWitt equation reads (with units $2G/3\pi = 1$)

$$\frac{1}{2}\left(\frac{\hbar^2}{a^2}\frac{\partial}{\partial a}\left(a\frac{\partial}{\partial a}\right) - \frac{\hbar^2}{a^3}\frac{\partial^2}{\partial \phi^2} - a + \frac{\Lambda a^3}{3} + m^2 a^3 \phi^2\right)\psi(a,\phi) = 0$$

Factor ordering chosen in order to achieve covariance in minisuperspace

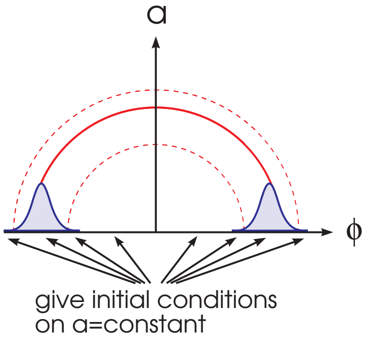
Determinism in classical and quantum theory

Classical theory



Recollapsing part is deterministic successor of expanding part

Quantum theory



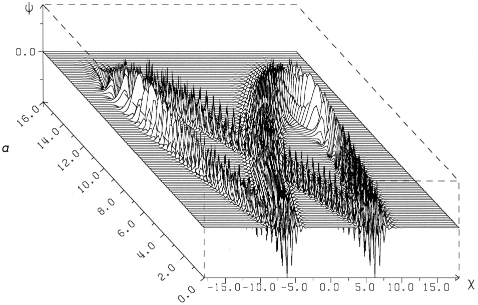
‘Recollapsing’ wave packet must be present ‘initially’

No intrinsic difference between ‘big bang’ and ‘big crunch’!

Example

Indefinite Oscillator

$$\hat{H}\psi(a,\chi) \equiv (-H_a + H_\chi)\psi \equiv \left(\frac{\partial^2}{\partial a^2} - \frac{\partial^2}{\partial \chi^2} - a^2 + \chi^2\right)\psi = 0$$



C.K. (1990)

Quantum-gravitational corrections

In the Born–Oppenheimer approximation to the Wheeler–DeWutt equation, we have (Lecture III)

$$\hat{H}^{\mathrm{m}} \to \hat{H}^{\mathrm{m}} + \frac{1}{m_{\mathrm{P}}^2}\,(\text{various terms})$$

(*C.K. and Singh* (1991); *Barvinsky and C.K.* (1998))

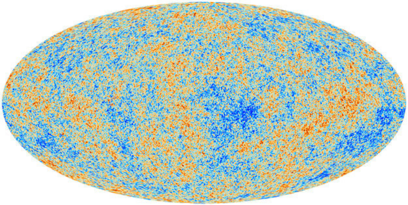


Figure credit: ESA/PLANCK Collaboration

C.K. and M. Krämer, *Phys. Rev. Lett.*, **108**, 021301 (2012); D. Bini, G. Esposito, C.K., M. Krämer, and F. Pessina, *Phys. Rev. D*, **87**, 104008 (2013); A. Y. Kamenshchik, A. Tronconi, and G. Venturi, *Phys. Lett. B* **734**, 72 (2014); D. Brizuela, C.K., M. Krämer, *Phys. Rev. D* **93**, 104035 (2016); *ibid.* **94**, 123527 (2016); D. Brizuela, C.K., M. Krämer, S. Robles-Pérez, *ibid.* **99**, 104007 (2019); L. Chataignier and M. Krämer, *Phys. Rev. D*, **103**, 066005 (2021); . . .

Uncorrected power spectra

The calculation gives the standard result for the power spectra:

- Scalar modes:

$$\mathcal{P}_{\mathrm{S};0}(k) = \left.\frac{(t_{\mathrm{P}} H_0)^2}{\pi\epsilon}\right|_{k=aH_0}$$

- Tensor modes:

$$\mathcal{P}_{\mathrm{T};0}(k) = \frac{16(t_{\mathrm{P}} H_0)^2}{\pi}$$

For the ratio of tensor and scalar spectrum one thus has $r := 16\epsilon$; combining the PLANCK data with the BICEP2 and KECK 2015 data, one finds $r < 0.044$.

Corrected power spectra

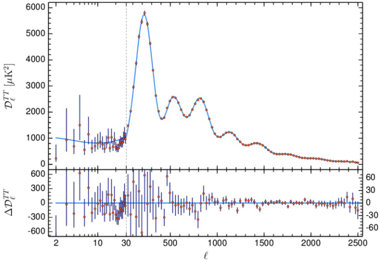
The quantum-gravitational corrections to the power spectra read

$$\mathcal{P}_{\mathrm{S,T}}(k) \simeq \mathcal{P}_{\mathrm{S,T;0}}(k) \left\{ 1 + \frac{4\pi}{3}(t_{\mathrm{P}} H_0)^2 \left(\frac{k_\star}{k}\right)^3 \left[2.85 - 2\log(-2k\chi)\right] \right\},$$

where $\chi \equiv \pm(H_0 a)^{-1}$ denotes conformal time, and k has been rescaled to have dimension of inverse length, with $k_\star$ being a reference scale, for example the pivot scale $k_\star = 0.002\ \mathrm{Mpc}^{-1}$ used in the data analysis of the CMB anisotropy spectrum, that is, the scale on which the CMB spectral index is measured.

Interpretation

- Concrete prediction from a conservative approach to quantum gravity (Wheeler–DeWitt equation); consistent with existing observational limits;
- quantum-gravitational corrections lead to an enhancement of power at large scales of the order 10^{-10}, which is too small to be observable by current missions;
- corrections break scale invariance: k^{-3}-dependence resp. ℓ^{-3}-dependence; this is confirmed by other groups, see e.g. Kamenshchik, Tronconi, and Venturi (2013–2016)



What is a singularity?

Hawking and Penrose 1996:

A spacetime is singular if it is timelike or null geodesically incomplete but cannot be embedded in a larger spacetime.

Ellis, Maartens, and MacCallum 2012:

We shall define a singularity as a boundary of spacetime where either the curvature diverges . . . or geodesic incompleteness occurs. The relation between these two kinds or aspects of singularities is still not fully clear; but often they will occur together.

DeWitt 1967

Provided it does not turn out to be ultimately inconsistent, this condition,[1] ... yields two important results. Firstly, it makes the probability amplitude for catastrophic 3-geometries vanish, and hence gets the physicist out of his classical predicament. Secondly, it may permit the Cauchy problem for the "wave equation" ... to be handled in a manner very similar to that of the ordinary Schrödinger equation.

[1] He refers to $\hat{H}\Psi = 0$.

Singularity avoidance in the quantum theory

No general agreement on the criteria!

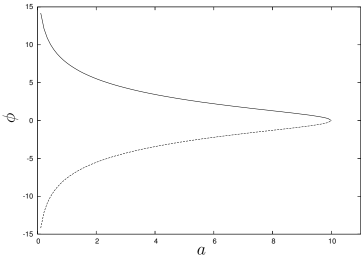
Sufficient criteria in quantum geometrodynamics:

- Vanishing of the wave function at the point of the classical singularity (dating back to DeWitt 1967)
- Spreading of wave packets when approaching the region of the classical singularity

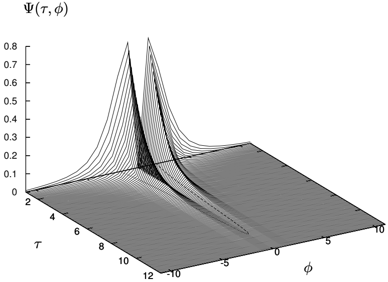
(These criteria were successfully applied in a number of models by Albarran, Alonso-Serrano, Bouhmadi-López, Dąbrowski, Kamenshchik, C.K., Kwidzinski, Krämer, Martín-Moruno, Marto, Moniz, Sandhöfer, and others)

Example I: Big-brake cosmology

Normalizable solutions of the Wheeler–DeWitt equation vanish at the classical singularity



(a) Classical trajectory



(b) Wave packet

(Kamenshchik, C.K., Sandhöfer 2007)

Example II: Quantum phantom cosmology

Classical model: Friedmann universe with scale factor $a(t)$ containing a scalar field with negative kinetic term ('phantom'): develops a big-rip singularity
(ρ and p diverge as a goes to infinity at a *finite time*)

Quantum model: Wave-packet solutions of the Wheeler–DeWitt equation disperse in the region of the classical big-rip singularity: time and the classical evolution come to an end; only a stationary quantum state is left

Exhibition of quantum effects at large scales!

(Dąbrowski, C.K., Sandhöfer 2006)

Review on phantom singularities and their avoidance:

M. Bouhmadi-López, C.K., P. Martín-Moruno, arXiv: 1904.01836

Direction of time

Paul Cézanne, *Nature morte au crâne* (Barnes Foundation, Pennsylvania)

Arrows of time

Almost all the fundamental laws of Nature are time-symmetric; but we observe classes of phenomena that distinguish a direction of time:

- Radiation (advanced versus retarded)
- Thermodynamics (increase of entropy)
- Quantum theory (measurement process)
- Gravity (expansion of the Universe;
  formation of structure; black holes)
- …

Master arrow of time?

Radiation

Fields that interact with local sources are usually desribed by retarded solutions:

$$\begin{aligned} A^\mu &= \text{source term plus boundary term} \\ &= A^\mu_{\text{ret}} + A^\mu_{\text{in}} \\ &= A^\mu_{\text{av}} + A^\mu_{\text{out}} \end{aligned}$$

Why is $A^\mu_{\text{in}} \approx 0$?

$\Longrightarrow$ Cosmology

Einstein und Ritz (1909):

" . . . Ritz considers the restriction to the form of the retarded potentials as one of the roots of the Second Law, while Einstein believes that the irreversibility is exclusively based on reasons of probability. "

Thermodynamics

Second Law of thermodynamics: Increase of entropy

$$\frac{\mathrm{d}S}{\mathrm{d}t} = \underbrace{\left(\frac{\mathrm{d}S}{\mathrm{d}t}\right)_{\mathrm{ext}}}_{\mathrm{d}S_{\mathrm{ext}}=\delta Q/T} + \underbrace{\left(\frac{\mathrm{d}S}{\mathrm{d}t}\right)_{\mathrm{int}}}_{\geq 0}$$

Statistical foundation?

- Loschmidt's *Umkehreinwand*
- Zermelo's *Wiederkehreinwand*

Need special boundary condition!

Quantum theory

The Schrödinger equation is time reversal invariant, but the measurement process distinguishes a direction of time, either by

- a dynamical collapse of the wave function, or
- a branching of the Everett paths.

Increasing entanglement leads to
Decoherence: irreversible emergence of classical properties through interaction with the environment

Arrow of time from cosmology

Where does the Sun come from?

$\Downarrow$

Gravitational instability of dust clouds

$\Downarrow$

Cosmology

Ludwig Boltzmann (1898):

That in Nature the transition from a probable to an improbable state does not happen equally often as the opposite transition, should be sufficiently explained by the assumption of a very improbable initial state of the whole Universe surrounding us . . . .

How special is the Universe?

Penrose (1981):

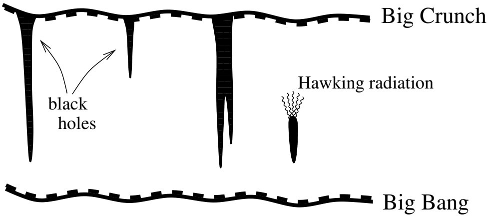
Entropy of the observed part of the Universe is maximal if all its mass is in one black hole; the probability for our Universe would then be (updated version from C.K. arXiv:0910.5836)

$$\frac{\exp\left(\frac{S}{k_{\mathrm{B}}}\right)}{\exp\left(\frac{S_{\mathrm{max}}}{k_{\mathrm{B}}}\right)} \sim \frac{\exp\left(1.2\times 10^{103}\right)}{\exp\left(1.8\times 10^{121}\right)} \approx \exp\left(-1.8\times 10^{121}\right)$$

(Gibbons–Hawking entropy connected with Λ is neglected here.)

Weyl curvature hypothesis

HYPOTHESIS (CLASSICAL)*:* The Weyl curvature vanishes at all past singularities, as the singularity is approached from future directions. *(This condition can be weakened by only demanding that the Weyl tensor be finite, rather than diverging)*



(Penrose 1979)

Quantum Weyl curvature hypothesis

HYPOTHESIS (QUANTUM)*:* The quantum states for the Weyl scalars describing scalar and tensor modes assume the form of adiabatic vacuum states in a (quasi-) de Sitter space, as the region of small-enough scale factors is approached from future directions.

C.K., *AVS Quantum Science* **4**, 015607 (2022).

Origin of irreversibility

Fundamental asymmetry with respect to “intrinsic time”:

$$\hat{H}\Psi = \left(\frac{\partial^2}{\partial\alpha^2} + \sum_i \left[-\frac{\partial^2}{\partial x_i^2} + \underbrace{V_i(\alpha, x_i)}_{\to 0 \text{ for } \alpha\to-\infty} \right] \right) \Psi = 0$$

Is compatible with simple boundary condition:

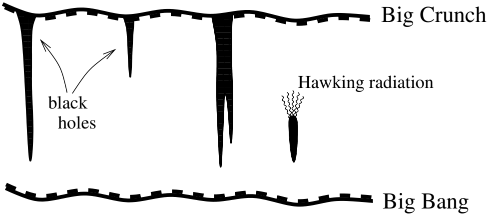
$$\Psi \stackrel{\alpha\to-\infty}{\longrightarrow} \psi_0(\alpha) \prod_i \psi_i(x_i)$$

Entropy $-k_{\mathrm{B}} \mathrm{Tr} \rho \ln \rho$ increases with increasing α, since entanglement with other degrees of freedom increases –
this defines the direction of time

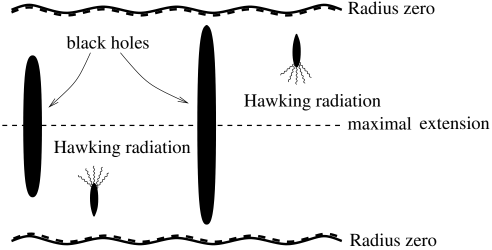
Is the expansion of the Universe a tautology?

See L. Chataigner, C. Kiefer, M. Tyugi, arXiv: 2503.10752 [gr-qc] and the references therein

Arrow of time in a recollapsing quantum universe



(Penrose 1979)



(C.K. and Zeh 1995)

John Wheeler (1968):

These considerations reveal that the concepts of spacetime and time itself are not primary but secondary ideas in the structure of physical theory. These concepts are valid in the classical approximation. However, they have neither meaning nor application under circumstances when quantum-geometrodynamical effects become important. . . . There is no spacetime, there is no time, there is no before, there is no after. The question what happens “next” is without meaning.

Bryce DeWitt (1999):

. . . one learns that time and probability are both *phenomenological* concepts.