Canonical Quantum Gravity

and the Problem of Time

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Literature

- C.K., *Quantum Gravity*, fourth edition (Oxford University Press, Oxford, 2025);
- C.K., Quantum gravity - an unfinished revolution, arXiv:2302.13047 [gr-qc];
- See also my recorded lectures on https://www.thp.uni-koeln.de/gravitation/courses/qg2024.html

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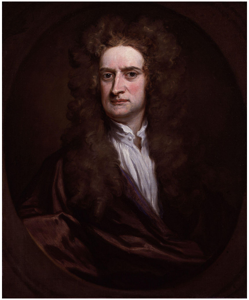
Newtonian gravity



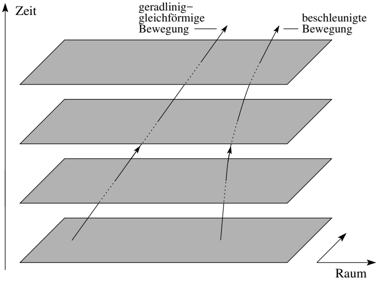
Painting by Sir Godfrey Kneller (1702),

National Portrait Gallery, London

$$\mathbf{F} = -\frac{GM_1M_2}{r^2}\hat{\mathbf{r}}$$

Isaac Newton (1687)

Absolute time and space



Isaac Newton 1687:

I. Absolute, true, and mathematical time, of itself, and from its own nature, flows equably without relation to anything external, and by another name is called duration. . . .

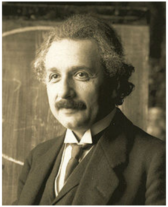
II. Absolute space, in its own nature, without relation to anything external, remains always similar and immovable. . . .

Newton to Bentley, February 25, 1692:

That gravity should be innate inherent and essential to matter so that one body may act upon another at a distance through a vacuum without the mediation of any thing else by and through which their action or force may be conveyed from one to another is to me so great an absurdity that I believe no man who has in philosophical matters any competent faculty of thinking can ever fall into it. Gravity must be caused by an agent acting constantly according to certain laws, but whether this agent be material or immaterial is a question I have left to the consideration of my readers.

Einsteinian gravity

- General theory of relativity



$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

(November 25, 1915)

- left: geometry (gravitational field)
- right: matter fields (Standard model of particle physics)

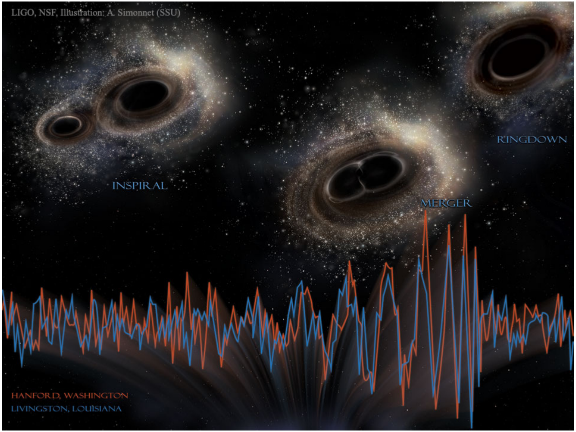


Figure credit: LIGO, NSF, Aurore Simonnet

Nobel Prize 2017 for Rainer Weiss, Barry Barish, Kip Thorne

Background independence

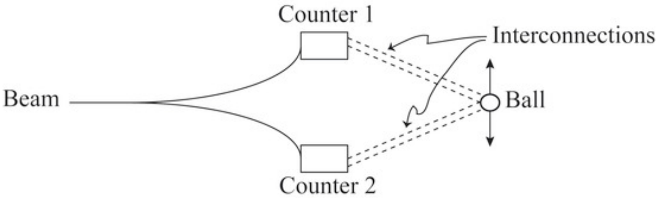
Albert Einstein:
Es widerstrebt dem wissenschaftlichen Verstande, ein Ding zu setzen, das zwar wirkt, aber auf das nicht gewirkt werden kann.

(It is contrary to the scientific mode of understanding to postulate a thing that acts, but which cannot be acted upon.)

There are no absolute fields in general relativity, that is, no background structure.

Richard Feynman 1957:

. . . if you believe in quantum mechanics up to any level then you have to believe in gravitational quantization in order to describe this experiment. . . . It may turn out, since we've never done an experiment at this level, that it's not possible . . . that there is something the matter with our quantum mechanics when we have too much *action* in the system, or too much mass—or something. But that is the only way I can see which would keep you from the necessity of quantizing the gravitational field. It's a way that I don't want to propose. . . .



Why quantum gravity?

- Superposition principle
- Unification of all interactions
- Singularity theorems
  - Black holes
  - ‘Big Bang’
- Problem of time
- Absence of viable alternatives

Max Planck, Über irreversible Strahlungsvorgänge, *Sitzungsberichte der königlich-preußischen Akademie der Wissenschaften zu Berlin, phys.-math. Klasse*, Seiten 440–80 (1899)

Planck units

$$
\begin{aligned}
l_{\mathrm{P}} &= \sqrt{\frac{\hbar G}{c^3}} \approx 1.62 \times 10^{-35}\ \mathrm{m} \\
t_{\mathrm{P}} &= \frac{l_{\mathrm{P}}}{c} = \sqrt{\frac{\hbar G}{c^5}} \approx 5.40 \times 10^{-44}\ \mathrm{s} \\
m_{\mathrm{P}} &= \frac{\hbar}{l_{\mathrm{P}} c} = \sqrt{\frac{\hbar c}{G}} \approx 2.17 \times 10^{-5}\ \mathrm{g} \approx 1.22 \times 10^{19}\ \mathrm{GeV}/c^2
\end{aligned}
$$

aussermenschliche Culturen nothwendig behalten und welche daher als »natürliche Maasseinheiten« bezeichnet werden können.

Die Mittel zur Festsetzung der vier Einheiten für Länge, Masse, Zeit und Temperatur werden gegeben durch die beiden erwähnten Constanten a und b, ferner durch die Grösse der Lichtfortpflanzungsgeschwindigkeit c im Vacuum und durch die der Gravitationsconstante f. Bezogen auf Centimeter, Gramm, Secunde und Celsiusgrad sind die Zahlenwerthe dieser vier Constanten die folgenden:

$$a = 0.4818\cdot 10^{-10}\,[\text{sec} \times \text{Celsiusgrad}]$$

$$b = 6.885\cdot 10^{-27}\left[\frac{\text{cm}^2\,\text{gr}}{\text{sec}}\right]$$

$$c = 3.00\cdot 10^{10}\left[\frac{\text{cm}}{\text{sec}}\right]$$

$$f = 6.685\cdot 10^{-8}\left[\frac{\text{cm}^3}{\text{gr. sec}^2}\right]^{1}.$$

Wählt man nun die »natürlichen Einheiten« so, dass in dem neuen Maasssystem jede der vorstehenden vier Constanten den Werth 1 annimmt, so erhält man als Einheit der Länge die Grösse:

$$\sqrt{\frac{bf}{c^3}} = 4.13\cdot 10^{-33}\,\text{cm},$$

als Einheit der Masse:

$$\sqrt{\frac{bc}{f}} = 5.56\cdot 10^{-5}\,\text{gr},$$

als Einheit der Zeit:

$$\sqrt{\frac{bf}{c^5}} = 1.38\cdot 10^{-43}\,\text{sec},$$

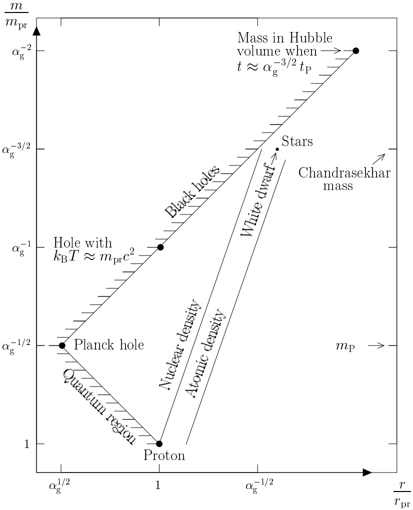
als Einheit der Temperatur:

$$a\sqrt{\frac{c^5}{bf}} = 3.50\cdot 10^{32}\,{}^\circ\,\text{Cels.}$$

Diese Grössen behalten ihre natürliche Bedeutung so lange bei, als die Gesetze der Gravitation, der Lichtfortpflanzung im Vacuum und die beiden Hauptsätze der Wärmetheorie in Gültigkeit bleiben, sie müssen also, von den verschiedensten Intelligenzen nach den verschiedensten Methoden gemessen, sich immer wieder als die nämlichen ergeben.

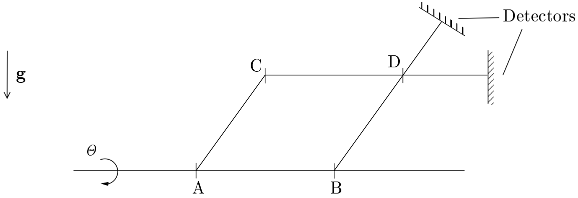
[1] F. Richarz und O. Krigar-Menzel, Anhang zu den Abhandlungen dieser Akademie vom Jahre 1898 S. 110, im Auszug: Wied. Ann. 66, S. 190, 1898.

Structures in the Universe



$$\alpha_{\rm g} = \frac{G m_{\rm pr}^2}{\hbar c} = \left(\frac{m_{\rm pr}}{m_{\rm P}}\right)^2 \approx 5.91 \times 10^{-39}$$

Neutron and atom interferometry



Experiments:

- Neutron interferometry in the field of the Earth
  (Colella, Overhauser, and Werner ("‘COW"’) 1975)
- Neutron interferometry in accelerated systems
  (Bonse and Wroblewski 1983)
- Discrete neutron states in the field of the Earth
  (Nesvizhevsky *et al.* 2002)
- Atom interferometry
  (z. B. Peters, Chung, Chu 2001: measurement of g with accuracy $\Delta g/g \sim 10^{-10}$)

Non-relativistic expansion of the Dirac equation yields

$$\mathrm{i}\hbar\frac{\partial\psi}{\partial t} \approx H_{\mathrm{FW}}\psi$$

mit

$$\begin{aligned} H_{\mathrm{FW}} \quad = \quad & \underbrace{\beta mc^2}_{\text{rest mass}} + \underbrace{\frac{\beta}{2m}\mathbf{p}^2}_{\text{kinetic energy}} - \underbrace{\frac{\beta}{8m^3c^2}\mathbf{p}^4}_{\text{SR correction}} + \underbrace{\beta m(\mathbf{a}\,\mathbf{x})}_{\text{COW}} \\ & - \underbrace{\boldsymbol{\omega}\mathbf{L}}_{\text{Sagnac effect}} - \underbrace{\boldsymbol{\omega}\mathbf{S}}_{\text{Mashhoon effect}} \\ & + \frac{\beta}{2m}\mathbf{p}\frac{\mathbf{a}\,\mathbf{x}}{c^2}\mathbf{p} + \frac{\beta\hbar}{4mc^2}\vec{\Sigma}(\mathbf{a}\times\mathbf{p}) + \mathcal{O}\left(\frac{1}{c^3}\right) \end{aligned}$$

Main approaches to quantum gravity

No question about quantum gravity is more difficult than the question, "What is the question?"
(John Wheeler 1984)

- Quantum general relativity
  - Covariant approaches (perturbation theory, path integrals including spin foams, asymptotic safety, . . .)
  - Canonical approaches (geometrodynamics, connection dynamics, loop dynamics, . . .)
- String theory
- Fundamental discrete approaches
  (quantum topology, causal sets, group field theory, . . .);
  have partially grown out of the other approaches

Linearized quantum gravity

Pioneered by Matvei Bronstein (1936)

Perturbation theory:

$$\boxed{g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}}$$

- $\bar{g}_{\mu\nu}$: classical background
- Perturbation theory with respect to $h_{\mu\nu}$ (Feynman rules)
- 'Particle' of quantum gravity: graviton (massless[1] spin-2 particle)
- perturbative non-renormalizability

[1] $m_{\mathrm{g}} \lesssim 7.7 \times 10^{-23}\ \mathrm{eV}/c^2$ from LIGO and Virgo, cf. $m_\gamma \leq 10^{-18} \mathrm{eV}/c^2$

Example: Transition rate from the $3d$ level to the $1s$ level in the hydrogen atom due to the emission of a graviton:

$$\Gamma_{\rm g} = \frac{G m_{\rm e}^3 c \alpha^6}{360 \hbar^2} \approx 5.7 \times 10^{-40}\ {\rm s}^{-1}$$

This corresponds to a life-time of

$$\tau_{\rm g} \approx 5.6 \times 10^{31}\ \text{years}\ .$$

Gravitons from the early Universe

Gravitons are created out of the vacuum during an inflationary phase of the early Universe ($\sim 10^{-34}$ s after the big bang); the quantized gravitational mode functions $h_{\mathbf{k}}$ in de Sitter space obey

$$\langle h_{\mathbf{k}} h_{\mathbf{k}'} \rangle = \frac{4}{k^3} \, (t_{\mathrm{P}} H)^2 \, \delta(\mathbf{k} + \mathbf{k}') \equiv P_{\mathrm{t}} \; \delta(\mathbf{k} + \mathbf{k}')$$

Power spectrum:

$$\Delta_{\mathrm{t}}^2(k) := \frac{k^3}{2\pi^2} P_{\mathrm{t}} = \frac{2}{\pi^2} \, (t_{\mathrm{P}} H)^2$$

(H is evaluated at Hubble-horizon exit, i.e. at $|k\eta| = 1$)

The BICEP2 experiment

"Background Imaging of Cosmic Extragalactic Polarization"

Figure credit: BICEP2 Collaboration

Most likely, the observed signal comes from a dust foreground (arXiv:1502.00612)

Quantum origin of perturbations

Power spectrum for the scalar modes (inflaton plus metric):

$$\Delta_{\mathrm{s}}^2(k) = \frac{1}{8\pi^2}\,(t_{\mathrm{P}}H)^2\,\epsilon^{-1} \approx 2\times 10^{-9}$$

ϵ: slow-roll parameter

Tensor-to-scalar ratio: $r := \frac{\Delta_{\mathrm{t}}^2}{\Delta_{\mathrm{s}}^2} = 16\epsilon$

The CMB spectrum from the PLANCK mission

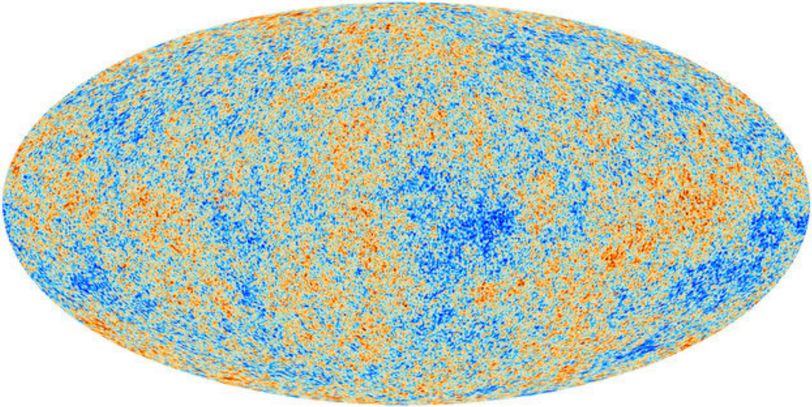


Figure credit: ESA/PLANCK Collaboration

First observational test of quantum gravity

- Within the inflationary scenario, the observed CMB fluctuations can only be understood from quantized metric plus scalar field modes.
- This is an indirect test of linearized quantum gravity.
- The observation of primordial B-modes would be an indirect confirmation of the existence of gravitons.
- The difference in the duration of inflation between the ‘cold spots’ and the ‘hot spots’ in the CMB spectrum is only of the order of the Planck time.

Effective field theory

One-loop corrections to the non-relativistic potentials obtained from the scattering amplitude by calculating the non-analytic terms in the momentum transfer

- Quantum gravitational correction to the Newtonian potential

$$V(r) = -\frac{Gm_1m_2}{r}\left(1 + \underbrace{3\frac{G(m_1+m_2)}{rc^2}}_{\mathrm{GR-correction}} + \underbrace{\frac{41}{10\pi}\frac{G\hbar}{r^2c^3}}_{\mathrm{QG-correction}}\right)$$

(Bjerrum-Bohr et al. 2003)

- Quantum gravitational effects to the Coulomb potential (scalar QED)

$$V(r) = \frac{Q_1Q_2}{r}\left(1 + 3\frac{G(m_1+m_2)}{rc^2} + \frac{6}{\pi}\frac{G\hbar}{r^2c^3}\right) + \dots$$

(Faller 2008)

Analogy: Chiral perturbation theory (small pion mass)

Divergences in perturbative quantum gravity

- Quantum general relativity: no divergence for pure gravity at one loop (*'t Hooft and Veltman 1974*); divergences are present if other fields are coupled to gravity (*Deser et al. 1975*)

$$\mathcal{L}^{(\mathrm{div})}_{1-\mathrm{loop}} = \frac{\hbar}{8\pi^2\epsilon}\sqrt{-\bar{g}}\left(\frac{\bar{R}^2}{120} + \frac{7}{20}\bar{R}_{\mu\nu}\bar{R}^{\mu\nu}\right)$$

- Quantum general relativity: divergences at two loops (*Goroff and Sagnotti 1986, van de Ven 1992*)

$$\mathcal{L}^{(\mathrm{div})}_{2-\mathrm{loop}} = \frac{209\hbar^2}{2880}\frac{32\pi G}{(16\pi^2)^2\epsilon}\sqrt{-\bar{g}}\bar{R}^{\alpha\beta}{}_{\gamma\delta}\bar{R}^{\gamma\delta}{}_{\mu\nu}\bar{R}^{\mu\nu}{}_{\alpha\beta}$$

- For higher loops one finds from dimensional analysis that the divergent part of the Lagrangian is of the form

$$\mathcal{L}^{(\mathrm{div})}_{L-\mathrm{loop}} \sim \sqrt{-\bar{g}}\hbar^L G^{L-1} \nabla^p \bar{R}^m \frac{1}{\epsilon}\ , \quad p+m=L+1$$

  (recall that the loop expansion is also a WKB expansion), where the symbolic notation $\nabla^p \bar{R}^m$ is a shorthand for all curvature terms and their derivatives that can occur at this order.
- $N=8$ supergravity (maximal supersymmetry!) is finite up to four loops (explicit calculation!) and there are arguments that it is finite also at five and six loops (and perhaps up to eight loops) (*Bern et al. 2009*) – new symmetry?
- There are theories that exist at the non-perturbative level, but are perturbatively non-renormalizable (e.g. non-linear σ model for $D>2$), see below

Beyond perturbation theory?

Example: self-energy of a thin charged shell

Energy of the shell using the bare mass m_0 is

$$m(\epsilon) = m_0 + \frac{Q^2}{2\epsilon} \ ,$$

which diverges for $\epsilon \to 0$. But the inclusion of gravity leads to

$$m(\epsilon) = m_0 + \frac{Q^2}{2\epsilon} - \frac{Gm^2(\epsilon)}{2\epsilon} \ ,$$

which leads for $\epsilon \to 0$ to a finite result,

$$m(\epsilon) \xrightarrow{\epsilon \to 0} \frac{|Q|}{\sqrt{G}} \ .$$

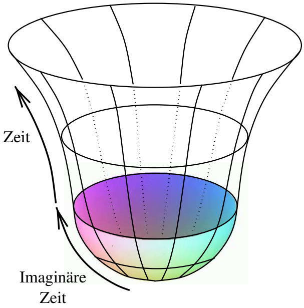
Path integrals

$$Z[g] = \int \mathcal{D}g_{\mu\nu}(x)\ \mathrm{e}^{\mathrm{i}S[g_{\mu\nu}(x)]/\hbar}$$

In addition: sum over all topologies?

- Euclidean path integrals
  (e.g. for Hartle–Hawking proposal or Regge calculus)
- Lorentzian path integrals
  (e.g. for dynamical triangulation)
- Asymptotic safety:
  search for non-trivial fixed points of renormalization group equations

Example: The proposal by Hartle and Hawking

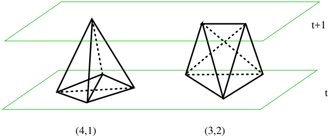


Stephen Hawking, Vatican Conference 1981:

There ought to be something very special about the boundary conditions of the universe and what can be more special than the condition that there is no boundary.

Dynamical triangulation

- makes use of *Lorentzian* path integrals
- edge lengths of simplices remain fixed; sum is performed over all possible combinations with equilateral simplices
- Monte-Carlo simulations



Preliminary results:

- Hausdorff dimension $H = 3.10 \pm 0.15$
- Spacetime two-dimensional on smallest scales (cf. asymptotic-safety approach)
- positive cosmological constant needed
- continuum limit?

(Ambjørn, Loll, Jurkiewicz from 1998 on)

Canonical quantum gravity

Central equations are constraints:

$$\boxed{\hat{H}\Psi = 0}$$

Different canonical approaches:

- Geometrodynamics –
  metric and extrinsic curvature
- Connection dynamics –
  connection (A_a^i) and coloured electric field (E_i^a)
- Loop dynamics –
  flux of E_i^a and holonomy

Problem of time

- External time t has vanished from the formalism.
- it holds for *any* theory that has no absolute time at the classical level.
- The Hilbert-space structure in quantum mechanics is connected with the probability interpretation, in particular with probability conservation *in time* t; what happens with this structure in a timeless situation?

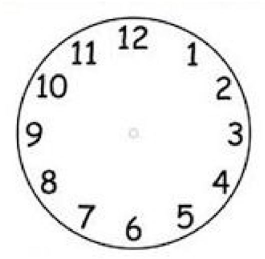


Figure: Absence of time in full quantum gravity

Figure from: C.K, B. Nikolić, *J.Phys.Conf.Ser.* **880**, 012002 (2017).