about me

Charged multi-sheet wormhole solutions

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1. Introduction

Axisymmetric & stationary solutions can be analyzed in *the Weyl formalism.*

Application to wormhole spacetimes

-> *multi-sheet wormholes* [1]

connecting individual multiple spatial infinities.

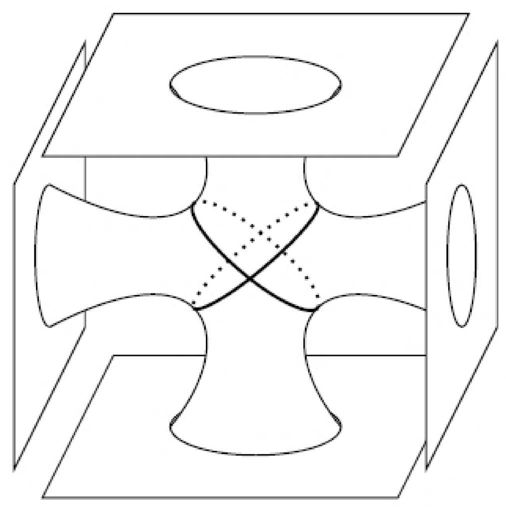
Wormholes require negative energy.

-> Assume *phantom* scalar field as an "exotic matter."

We aim for constructing

- *globally regular (non-singular)*
- *less negative energy*

solutions by a *charging* method.



Why charging?

Charged black hole spacetimes have wormhole-like structures, but its energy is still *positive*.

elemag charges ⟷ (corres-pondence (?)) negative energy

2. Multi-sheet wormholes

4-dim. Einstein- massless *phantom* scalar system :

$$S = \frac{1}{2\kappa}\int \left[R + 2g^{\mu\nu}(\partial_\mu\Psi)(\partial_\nu\Psi)\right]\sqrt{-g}\,\mathrm{d}^4x$$

Static & vacuum solution in Weyl formalism :

$$\mathrm{d}s^2 = -f(\rho,z)\mathrm{d}t^2 + \frac{\rho^2}{f(\rho,z)}\mathrm{d}\phi^2 + F(\rho,z)\left(\mathrm{d}\rho^2 + \mathrm{d}z^2\right)$$

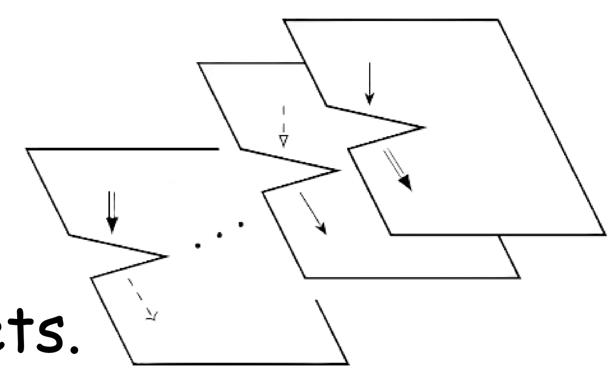
Solution Generating Procedure [1]

1. Find $f(\rho,z)$ & $\Psi(\rho,z)$ satisfying Laplace eq.

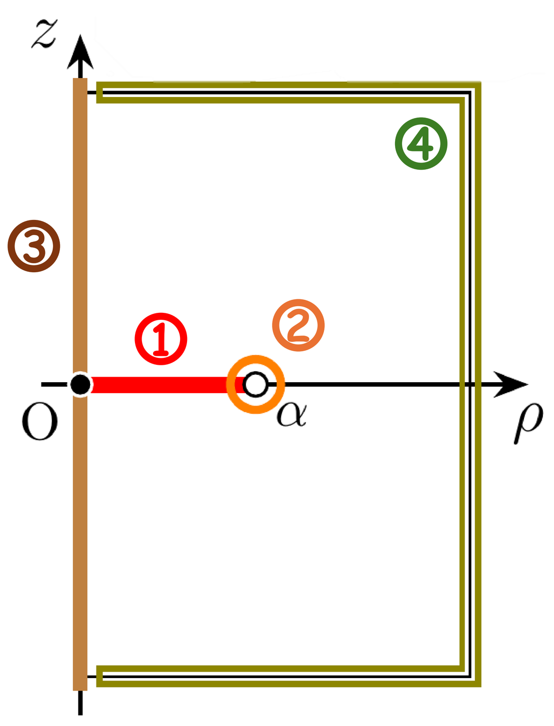
Einstein eq.
Klein-Gordon eq. $\xrightarrow{\text{reduce to}}$ Laplace eq. in flat space

2. Find $F(\rho,z)$ by integral.

$$(\partial \ln F) \sim (\partial \ln f)^2 - (\partial\Psi)^2$$



3. Analyze regularities in ρz-sheets.
   1. cut (continuity & smoothness)
   2. edge of the cut
      - i. geometry
      - ii. field eqs. (= Laplace eq.)
   3. the axis
   4. asymptotic flatness



If one wants charged solutions, apply *Harrison transformation* before step 2.

Harrison transformation

$$f(\rho,z) = \frac{f_0}{\left(1-|q|^2 f_0\right)^2} \qquad (q\in\mathbb{C})$$

$$A_t(\rho,z) = (\mathrm{Re}\,q)\,\frac{f_0}{1-|q|^2 f_0} \quad \triangleleft \text{ electric field}$$

$$\partial_j A_\phi(\rho,z) = (\mathrm{Im}\,q)\,\rho\epsilon_j{}^k\partial_k \ln f_0 \quad \triangleleft \text{ magnetic field}$$

3. The solution

Charged multi-sheet wormhole solutions :

$$\mathrm{d}s^2 = -f_\pm(\rho,z)\mathrm{d}t^2 + \frac{\rho^2}{f_\pm(\rho,z)}\mathrm{d}\phi^2 + F_\pm(\rho,z)\left(\mathrm{d}\rho^2 + \mathrm{d}z^2\right)$$

metric
$$\begin{cases} f_\pm(\rho,z) = \dfrac{M^2-Q^2}{Q^2}\dfrac{1}{\sinh^2\Theta_\pm(\rho,z)} \\[2ex] F_\pm(\rho,z) = \dfrac{1}{f_\pm(\rho,z)}\left[1 - \dfrac{\rho^2}{4\alpha^2}\left(1-w(\rho,z)\right)^2\right]^{-B} \end{cases}$$

Maxwell
$$\begin{cases} \partial_j A_t^\pm(\rho,z) = -\dfrac{Q_\mathrm{e}}{2\alpha} f_\pm(\rho,z)\,\partial_j \ln\left(\dfrac{\mu_{\mp i\alpha}}{\mu_{\pm i\alpha}}\right)^i \\[2ex] \partial_j A_\phi^\pm(\rho,z) = +\dfrac{Q_\mathrm{m}}{2\alpha} i\left(\partial_j\mu_{\mp i\alpha} - \partial_j\mu_{\pm i\alpha}\right) \end{cases}$$

scalar
$$\Psi_\pm(\rho,z) = \frac{P}{2\alpha}\ln\left(\frac{\mu_{\mp i\alpha}}{\mu_{\pm i\alpha}}\right)^i$$

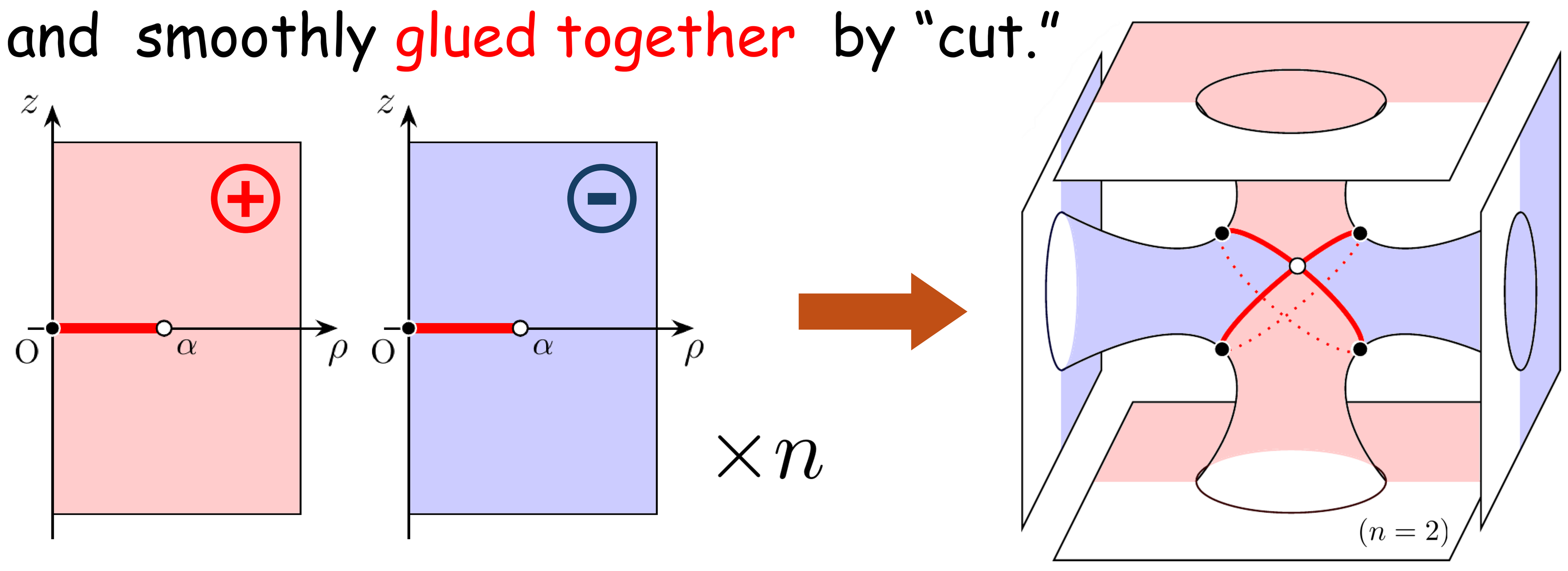
where

$$\Theta(\rho,z) = \frac{M^2-Q^2}{2\alpha}\ln\left(\frac{\mu_{\mp i\alpha}}{\mu_{\pm i\alpha}}\right)^i + \ln\left(\frac{M+\sqrt{M^2-Q^2}}{Q}\right)$$

$$\mu_{\pm i\alpha}(\rho,z) = -(z\mp i\alpha) + \sqrt{\rho^2 + (z\mp i\alpha)^2} \qquad Q^2 := Q_\mathrm{e}^2 + Q_\mathrm{m}^2$$

$$w(\rho,z) = -\frac{z^2+\alpha^2}{\rho^2} + \frac{\sqrt{(\rho^2+z^2-\alpha^2)^2 + 4\alpha^2 z^2}}{\rho^2}$$

(ρ,z)-coordinates are introduced in each sheet and smoothly glued together by "cut."



Physical quantities :

mass $M = \dfrac{1+|q|^2}{1-|q|^2}m$

electric charge $Q_\mathrm{e} = 2m\dfrac{\mathrm{Re}\,q}{1-|q|^2}$

magnetic charge $Q_\mathrm{m} = 2m\dfrac{\mathrm{Im}\,q}{1-|q|^2}$

throat radius α

scalar charge P

Constraints from regularity :

$$B := \frac{P^2+Q^2-M^2}{\alpha^2} = \frac{2n-1}{n} \qquad (Q^2 = Q_\mathrm{e}^2 + Q_\mathrm{m}^2)$$

4. Results

- Maxwell field & massless scalar are decoupled.
  -> charged versions of multi-sheet wormholes can be generated via Harrison transformation.
- Regularity analyzing procedure does *not* change.
  -> results in original one [1] are still valid.
- EM charges Q_e & Q_m behave as negative energy in a viewpoint of regularity condition.

◆ but...

- Wormholes *still* require negative energy even if EM charges are *extremely* large.
  -> one meets a singularity at the end of journey.
  (Reissner-Nordström-like naked singularity)

if lacking negative energy

[1] Y. Makita *et al.* (2025), Class. Quantum Grav. **42** 175024.